Massachusetts Institute of Technology
6.042J/18.062J, Fall '02: Mathematics for Computer Science
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In-Class Problems — Week 8, Fri

Problem 1.

(a) In how many ways can 10 customers line up at a supermarket checkout?

(b) In how many ways can 10 customers line up at two supermarket checkouts?

(c) In how many ways can 10 customers line up at three supermarket checkouts?

(d) (Optional.) What is the general case for n customers and m supermarket checkouts?

Problem 2. An n-input, m-output *boolean* function is a mapping $f : \{0, 1\}^n \to \{0, 1\}^m$.

(a) How many n-input, 1-output boolean functions are there? *Hint:* Two boolean functions are different if there exists an n-bit input on which they output different values.

(b) How many n-input, m-output boolean functions are there?

Problem 3. On a set S of n elements, how many of the following types of relations are there? (An appendix is included if you need a reminder of the definitions.)

(a) binary relations

(b) symmetric binary relations

(c) reflexive binary relations

(d) symmetric and reflexive binary relations

(e) symmetric or reflexive binary relations

Problem 4. Consider the set of undirected graphs on the set $V = \{1, 2, \ldots, n\}$ of vertices. (Recall that undirected graphs have no self-loops.) Count the number of such graphs by exhibiting a bijection with one of the types of relations in Problem 3 . Prove that your mapping is a bijection.

1 Appendix

1.1 Relations

A binary relation R on a set A is a subset $R \subseteq A \times A$. A binary relation R is

- *reflexive* if $(a, a) \in R$ for every $a \in A$;
- *symmetric* if aRb implies bRa for every $a, b \in A$.

1.2 Functions

A function $f : A \to B$ is

- *injective (one-to-one)* if $f(x) = f(y)$ imples that $x = y$ for all x and y in the domain of f;
- *surjective (onto)* if for every element $b \in B$, there exists an element $a \in A$ such that $f(a) = b$;
- *bijective (one-to-one correspondence)* if f is both injective and surjective.