Massachusetts Institute of Technology
6.042J/18.062J, Fall '02: Mathematics for Computer Science
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In-Class Problems — Week 8, Wed

Problem 1. (a) Prove that at a party where there are at least two people, there are two people who know the same number of people there. Assume that "knowing" is a two-way relationship and that knowing yourself does not count.

(b) Restate the problem from part (b) as a graph theorem.

Problem 2. Use the generalized pigeonhole principle to prove the following statement:

> Among any set of 150 natural numbers, there must be three numbers, a, b, and c, such that all of the pairwise differences, $(a - b)$, $(a - c)$, $(b - c)$, are multiples of 70.

(a) What are the pigeons?

(b) What are the holes?

(c) What is the function mapping the pigeons to the holes?

(d) Carefully prove the statement using the pigeonhole principle.

Problem 3. Counting Numbered Trees.

A *numbered tree* is a tree whose vertex set is $\{1, 2, \ldots, n\}$ for some $n \geq 2$. We define the *code* of the numbered tree to be a sequence of $n - 2$ integers from 1 to n obtained by the following recursive process:

> If $n = 2$, stop—the code is the empty sequence. Otherwise, write down the *father* of the largest leaf, delete this *leaf*, and continue the process on the resulting smaller tree.

For example, the codes of a couple of numbered trees are shown in Figure (1).

(a) Describe a procedure for reconstructing a numbered tree from its code.

(b) How many numbered trees with n vertices are there?

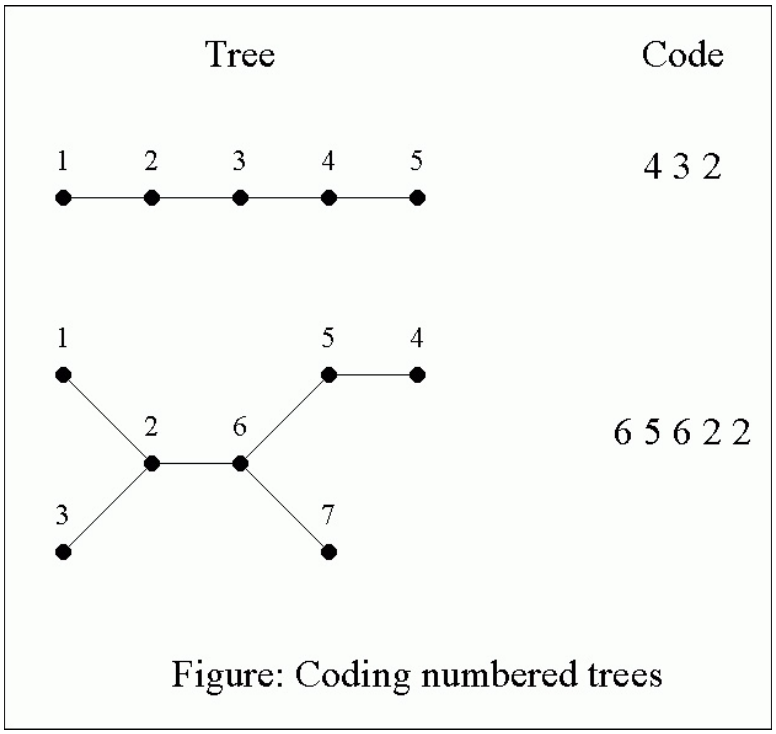

Figure 1: Coding Numbered Trees