RELIABILITY OF ROCK SLOPES WITH WEDGE MECHANISMS

by

BAK KONG LOW

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Signature of Author..
Department of Civil Engineering, August 24, 1979

Certified by..
Herbert H. Einstein, Thesis Supervisor

Certified by..
Daniele Veneziano, Thesis Supervisor

Accepted by..
Chairman, Department Committee

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ABSTRACT

A model is presented for reliability of wedge mechanisms in rock slopes. Only potential sliding along the line of intersection is considered and limit equilibrium analysis is used. The action of water and the effect of incomplete joint persistence are included. The factor of safety (ratio between mean resistance and mean driving force) is calculated as an explicit function of joint orientation angles, height, slope inclination, water and resistance parameters. If some or all of these parameters are random, then safety is better measured in terms of the second moment reliability index, β. A numerical procedure is developed and implemented for the calculation of this index. In actual calculations, only two sets of uncertain parameters are considered, one set includes joint orientation angles, the other includes resistance and water parameters.

Thesis Supervisor: Herbert H. Einstein
Title: Associate Professor

Thesis Supervisor: Daniele Veneziano
Title: Associate Professor

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CHAPTER 1

INTRODUCTION

The design of stable rock slopes is an important issue in many civil and mining engineering activities, such as cuts for transportation corridors, reservoirs, open-pit mine slopes and underground openings. The design is influenced not only by safety considerations but also by constraints on environmental impact and economic pressures to produce resources at low costs.

The present study deals with one aspect of the slope stability problem, namely the reliability of wedge mechanisms which might slide along the intersection of two joint planes. Situations where the wedge may fail by toppling, rotation or sliding on a single plane are not treated herein.

Chapter 2 described the mechanical model used in this study. A model for joints is presented first, followed by an idealization of water-induced forces. Underlying assumptions, limitations of the models, and alternative interpretation of some of its parameters are discussed thereafter.

Chapter 3 first shows how the Factor of Safety based on the model in Chapter 2 can be expressed explicitly as a function of joint orientation angles, height of wedge, and water and resistance parameters. Section 3.2 discusses the requirements for sliding along the line of intersection. Section 3.3 presents plots showing how the safe regions vary with changes in joint orientation angles and in water and resistance parameters. The physical meaning of the plots is also discussed.

An algorithm for calculating the second moment reliability index, β, is proposed in Chapter 4, first for the case of only geometric uncertainty, and then for the case of only joint resistance and water parameter uncertainty. Numberical results are given and samples of computer printout are attached.

A summary and conclusions follow in Chapter 5.

CHAPTER 2

DESCRIPTION OF MODEL

A model is presented herein for the analysis of rock slope stability with respect to wedge mechanisms. The underlying assumptions are outlined first, followed by description of rock and joint behavior and of the action of water.

The following general assumptions are made:

1. The rock mass which is subject to potential sliding failure is assumed to behave like a rigid body and the stability criterion is based on limit equilibrium analysis.
2. Water pressure and the weight of the wedge are the only two forces that may induce failure.
3. The presence of water in a joint has no effect on its strength.
4. Only tetrahedral wedges formed by 2 intersecting joints are considered. Hence, tension cracks are excluded from the study.
5. Potential sliding is considered only along the intersection of two joints. Situations where wedges may slide along one plane only are not analyzed here but they will be considered briefly in Chapter 3. Failure by rotation or toppling are excluded. The implicit assumption is that the lines of action of all the forces are concurrent at the centroid of the wedge, so that all moments are zero.
6. The crest of the slope is horizontal.
7. The frictional resistance of the joints and the intact strength of the rock are mobilized simultaneously when sliding failure occurs.

2.1 Joint Model

The model treats joint planes as consisting of a jointed portion and a set of intact rock bridges. The fraction of the joint plane area that is actually discontinuous is called the persistence of the joint plane; we shall denote this quantity by k. The fraction of the joint plane that is intact is denoted by I, hence $I = 1 - k$.

Usually, the relationship between shear strength of intact rock, τ_i, cohesion, c_r, and angle of internal friction, ϕ_i, can be approximated by the equation (see Figs. 2.1 and 2.2):

$$\tau_i = c_r + \sigma_n \tan\phi_i \tag{2.1}$$

where σ_n denotes normal stress at failure.

For the jointed portion, the shear strength, τ_j, is given by:

$$\tau_j = \sigma_n \tan\phi_j \tag{2.2}$$

where ϕ_j denotes the joint frictional angle.

In order for sliding failure to occur, all intact portions of the two joint planes have to be broken off. Assuming simultaneous mobilization of strength (Fig. 2.4), the combined resistance of jointed and intact portions can be expressed as (in terms of forces instead of stresses):

Resistance = (Joint Resistance) + (Intact Rock Resistance)

$$= (k_1\overline{N}_1\tan\phi_{1j}+k_2\overline{N}_2\tan\phi_{2j})+(C_{r1}I_1A_1+I_1\overline{N}_1\tan\phi_{1i}+C_{r2}I_2A_2+I_2\overline{N}_2\tan\phi_{2i})$$

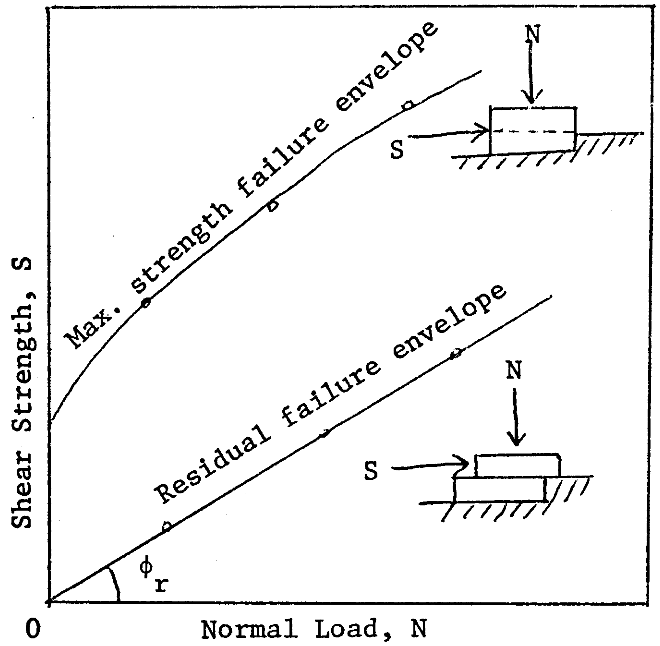


Fig. 2.1 Maximum strength and residual failure envelope for initially intact specimens. (From: Deere, Hendron, Patton, Cording)

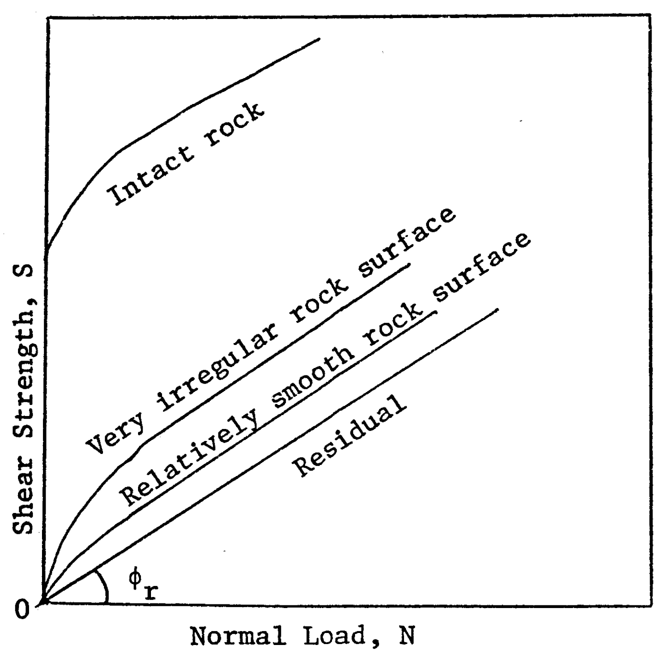


Fig. 2.2 Failure envelopes expected for rock masses. (From: Deere, Hendron, Patton, Aiyer).

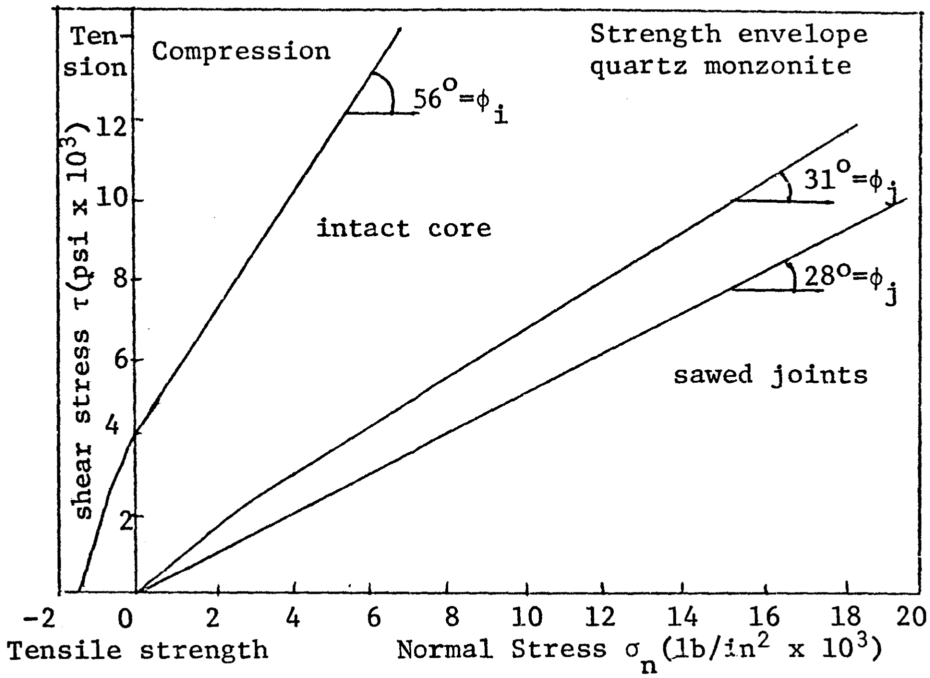


Fig. 2.3 Strength of intact and jointed speciments of quartz monzonite. (From: U.S. Corps Engrs.)

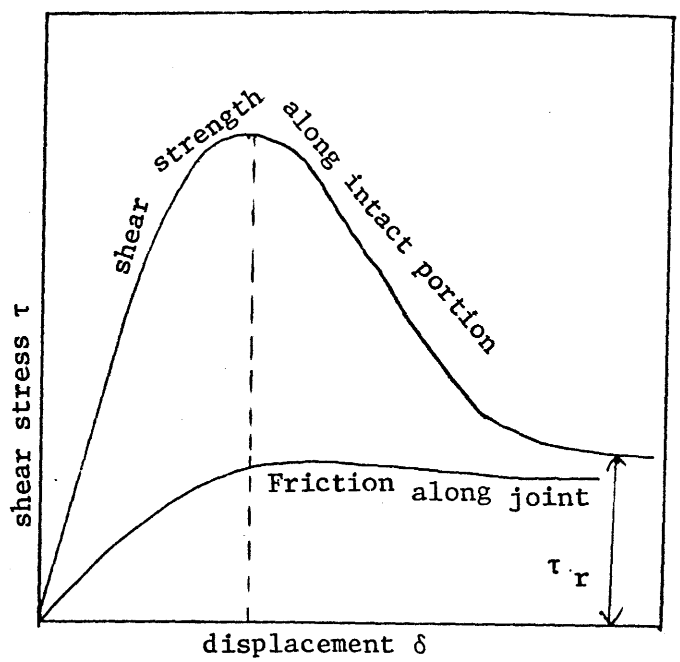


Fig. 2.4 Stress-strain relations that lead to simultaneous mobilization of intact rock and joint strength.

where $\overline{N}_1$, $\overline{N}_2$ = effective normal force on plane 1 and plane 2 respectively

ϕ_{1j}, ϕ_{2j} = joint frictional angle for plane 1 and plane 2 respectively

ϕ_{1i}, ϕ_{2i} = internal frictional angles for intact rock on planes 1 and 2.

Figure 2.3 shows that the internal frictional angle for intact rock, ϕ_i, may differ substantially from the joint frictional angle, ϕ_j. However, stability becomes questionable only when $I \simeq 0$ (when $k \simeq 1.0$), and under such circumstances the contribution from terms of the type $k\overline{N}\tan\phi_j$ dominates that from terms of the form $I_1\overline{N}\tan\phi_i$. Therefore, joint resistance will be calculated by setting $\phi_i = \phi_j$ in Equation 2.3. Since $k + I = 1$, it follows that

$$\text{Resistance} = \overline{N}_1\tan\phi_{1j} + \overline{N}_2\tan\phi_{2j} + C_{r1}I_1A_1 + C_{r2}I_2A_2 \tag{2.4}$$

From now on the subscript j will be dropped, it being understood that ϕ denotes the joint frictional angle.

Some typical shear strength parameters of intact rock are given below, from Stagg and Zienkiewicz (Rock Mechanics in Engineering Practice):

	Cohesion(1000psf)		ϕ_i (degrees)	
	Range	Average	Range	Average
Granite	200-840	500	51-58	55
Limestone	72-720	430	37-58	50
Sandstone	86-864	230-600	48-50	48

In the equation for the Factor of Safety (Eq. 3.3) it will become apparent that, due to high cohesion of the intact rock, a very small value of I is sufficient to ensure stability of the wedge.

2.2 Idealized Water Conditions

Water pressure is assumed to act only along the 2 joint planes, in direction normal to the planes. Its effect on the safety of tetrahedron wedges will be shown in this section to depend entirely on dimensionless parameters G_{w1} and G_{w2}, which, in terms of quantities defined in Fig. 2.5, are given by

$$G_{w1} = n_{w1}\left(\frac{hw}{h}\right)^3 \qquad 0 \leq n_{w1} \leq 1, \quad 0 \leq \frac{hw}{h} \leq 1$$

$$G_{w2} = n_{w2}\left(\frac{hw}{h}\right)^3 \qquad 0 \leq n_{w2} \leq 1, \quad 0 \leq \frac{hw}{h} \leq 1$$

These expressions refer to a horizontal water table (see Fig. 2.5) at height hw (the same for both joint planes) above the daylighting point O. Along the line of intersection BO, water pressure is assumed to increase hydrostatically from zero at the water surface to a maximum at a point U at depth $n_w h_w$ below the water table. Water pressure is assumed to decrease linearly from the maximum value $\rho_w n_w h_w$ at U to the value zero at the daylighting point O (Fig. 2.6), and to be zero along the segments EG, GO, OF, FE. The quantities, n_{w1} and n_{w2}, can take on different values to reflect different variations of permeability with depth on the triangular planes that bound the wedge.

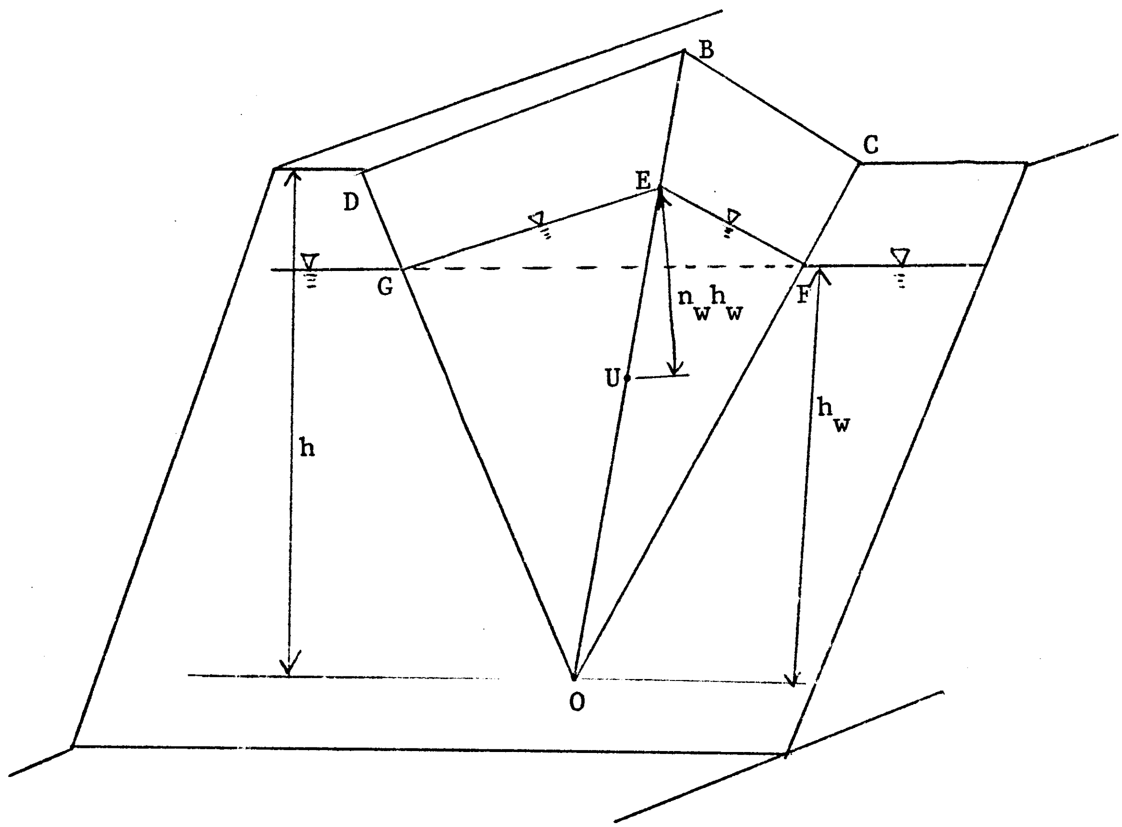


Fig. 2.5 Idealized Water Condition

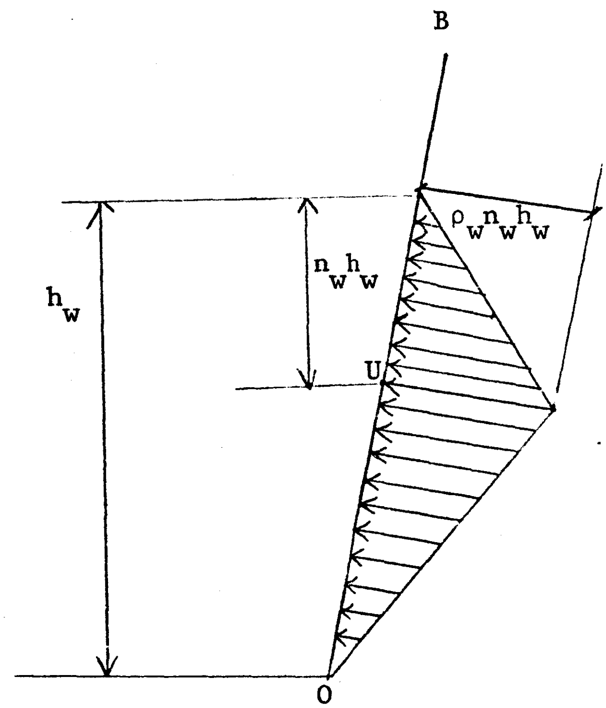


Fig. 2.6 Water pressure distribution along the line of intersection BO.

Within the triangles EGO and FOE, the water pressure distribution is assumed to be represented by pyramids with EGO and FOE as bases and with apices at distances $\rho_w n_{w1} h_w$ and $\rho_w n_{w2} h_w$ above points U_1 and U_2 respectively (Fig. 2.7).

In Figure 2.7, the height of pyramid, AU_1, is equal to $\rho_w n_{w1} h_w$. It represents the maximum value of water pressure on plane EGO. The total water force on that plane is given by the volume of the pyramid, which is equal to $\frac{1}{3}$ x (Area of Base) x (Height). Thus

$$\text{Total Water Force} = \frac{1}{3} \times (\text{Area of EGO}) \times \rho_w n_{w1} h_w$$

By properties of similar triangles, the ratio of area of triangles EGO to BDO in Fig. 2.5 is given by:

$$\frac{\text{Area EGO}}{\text{Area BDO}} = \frac{hw^2}{h^2}$$

Denote by A_1 the area BDO, then

$$\text{Area EGO} = \left(\frac{hw}{h}\right)^2 A_1$$

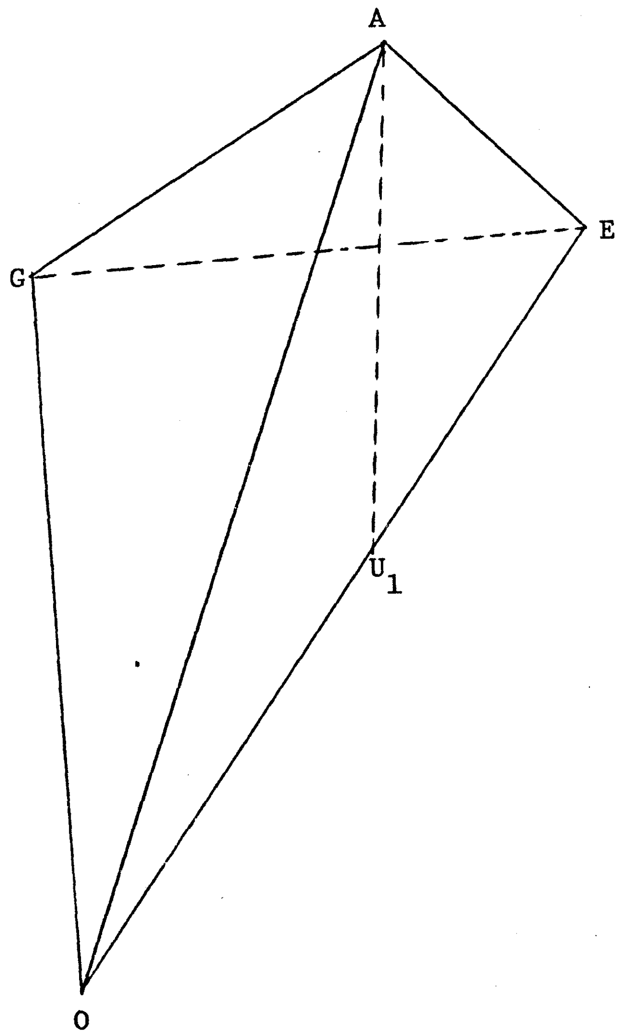
Hence,

$$F_{w1} = \frac{1}{3} \left(\frac{hw}{h}\right)^2 A_1 \times \rho_w n_{w1} h_w$$

$$= \frac{1}{3} \rho_w h A_1 n_{w1} \left(\frac{hw}{h}\right)^3$$

or

$$F_{w1} = \frac{1}{3} \rho_w h A_1 G_{w1} \tag{2.5}$$



$\overline{AU}$ = height of pyramid

= maximum value of water pressure

= $\rho_w n_w h_w$

Fig. 2.7 Water pressure distribution on triangular plane that bounds the wedge.

where F_{w1} denotes the total water force on triangle BDO and

$$G_{w1} = n_{w1}\left(\frac{hw}{h}\right)^3 \qquad (2.6)$$

Similarly,

$$F_{w2} = \frac{1}{3}\rho_w h A_2 G_{w2} \qquad (2.7)$$

where $$G_{w2} = n_{w2}\left(\frac{hw}{h}\right)^3 \qquad (2.8)$$

The water pressure distributions as presented above are idealizations of the complex groundwater flow process that occurs in reality. The assumption is that the wedge is impermeable and water acts only along the two joint planes that bound the wedge. Only steady state ground-water condition is modeled and transient flow is neglected. In reality, for porous or highly fractured material, transient variation in the groundwater regime can be critical, e.g. during rapid drawdown on reservoir slopes, rapid excavation of open pits and where there are changes in the groundwater regime brought about by earthquake activity or heavy precipitation. Perhaps more important, the actual variation of permeability on the joint planes has been highly idealized.

Possible presence of tension cracks and other fractures through the wedge have been ignored. These cracks and fractures, if present and filled with water, can greatly reduce the safety of the slope, e.g. by activating failure mechanisms other than those considered here.

Another water related effect that has not been considered is the expansive force from frost-wedging when joint water freezes during the cold season. The cumulative effect of repeated thawing and freezing can lead to deterioration of the rock and to significant reduction of wedge safety. More likely, the detrimental effect comes from breakage of the intact rock bridges on the joint planes and hence from an increase of joint persistence. Records of rock falls in a Canadian locality over several decades do show strong positive correlation between the number of rock fall incidents and the moist (snow precipitation) winter months.

So far, water pressure distribution around the wedge has been treated with the implicit assumption that water flows in a non-deformable medium. That is, that the joints (and fractures) have rigid, fixed openings and hence constant permeability in time.

Snow (1968) has discussed the effect of elasticity of fractured media in response to fluid pressure. Since fracture openings are very small (e.g. 100 μ) and fracture spacings very large (e.g. 10 ft.), the compression of blocks between fractures and the vertical extension of the medium that take place due to an increase in water pressure produces proportionately large increases of fracture openings. Therefore, for deformable rock masses, a dynamic model of mutual interaction between permeability and water pressure seems more appropriate: Permeability affects water pressure, and is in turn affected by it.

In view of the above discussion, it seems more meaningful to regard G_{w1} and G_{w2} as indices of average water pressure on the joint planes that bound the wedge and not as quantities with exactly the physical meaning implied by their derivation. One can give a more heuristic interpretation to G_{w1} and G_{w2} by considering the expressions for the average water pressure on a plane:

$$\text{Average water pressure on a plane} = \frac{\text{Total water force on that plane}}{\text{Area of plane}}$$

For plane 1,

$$\frac{F_{w1}}{A_1} = \frac{1}{3}\rho_w h G_{w1} \qquad (2.9)$$

For plane 2,

$$\frac{F_{w2}}{A_2} = \frac{1}{3}\rho_w h G_{w2} \qquad (2.10)$$

The range of G_{w1} and G_{w2} can be determined by the following considerations:

In Fig. 2.5, the worst that can happen is when water surface is up to the crest level DC (so that $\frac{hw}{h} = 1$), and that point U coincides with daylighting point O (so that $n_w = 1$). This water condition is possible when, for example, segments DO and CO are sealed by ice so that water pressure is entirely hydrostatic from crest to the daylighting point O. Under such circumstances, one obtains from the expressions for G_{w1} and G_{w2} (Eq. 2.6, 2.8):

$$G_{w1} = 1$$

$$G_{w2} = 1$$

On the other extreme, when joint planes are dry, $\frac{hw}{h} = 0$, so that

$$G_{w1} = 0$$

$$G_{w2} = 0$$

To sum up, the parameters G_{w1} and G_{w2} have values that range from 0 to 1, meaning that average water pressure for either of the two bounding planes (Eq. 2.9, 2.10) is always less or equal to $\frac{1}{3}\, \rho_w h$.

In Figure 2.8, G_w is plotted against $(\frac{hw}{h})$ for different values of n_w.

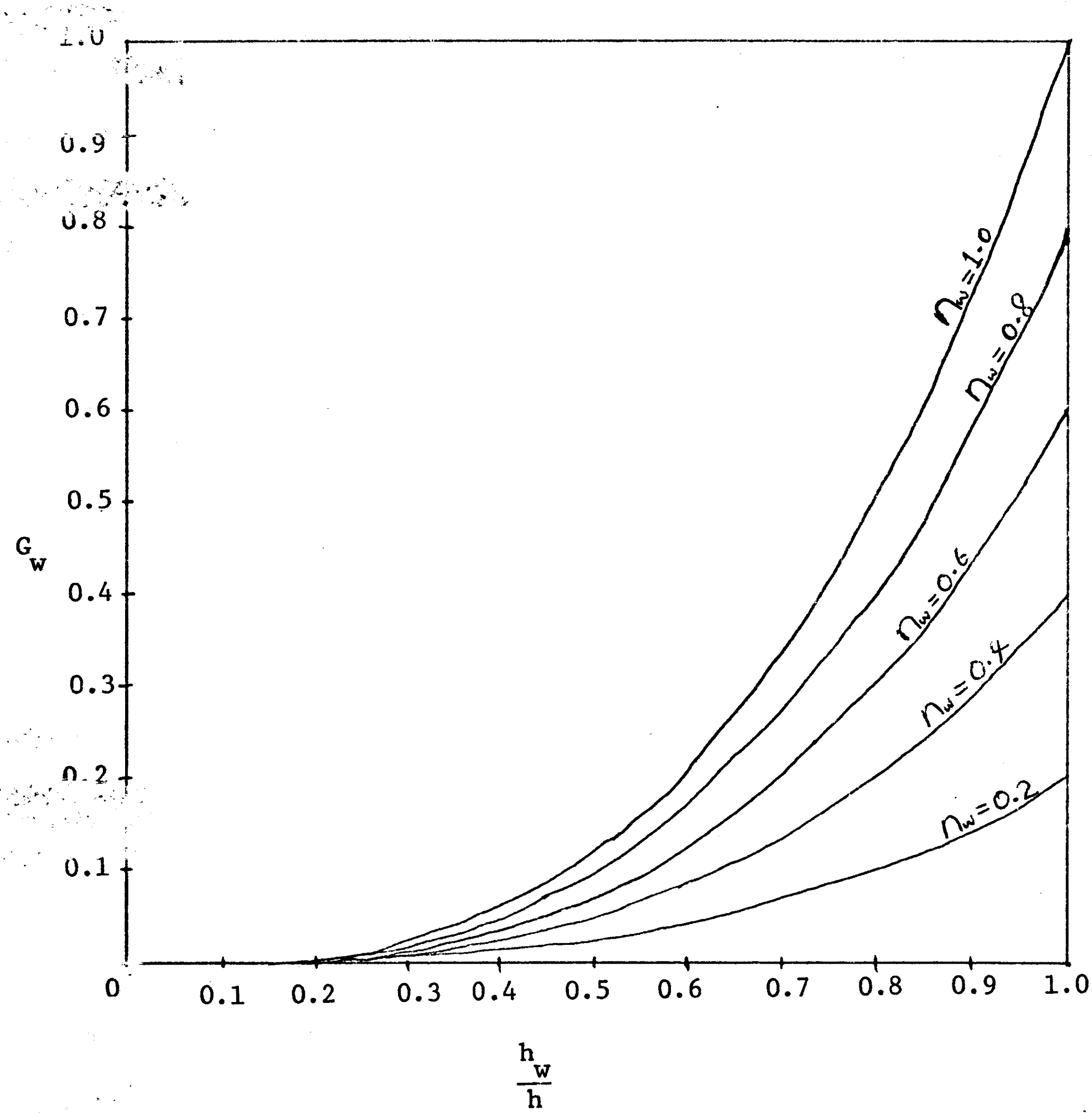


Figure 2.8 Water parameter G_w

CHAPTER 3

THE FACTOR OF SAFETY

3.1 Derivation of the Equation

In order for sliding along the line of intersection of two joint planes to be possible, such a line must daylight both on the slope and on the crest. For a horizontal crest, it is shown in Appendix A that this kinematic requirement leads to the following constraint on the orientation of the joints:

$$0 < \tan^{-1}\left\{\frac{\sin(\beta_2-\beta_1)}{\sin\beta_2\cot\gamma_1 - \sin\beta_1\cot\gamma_2}\right\} < \alpha \qquad (3.1)$$

where α is the inclination of the slope.

Wherever this condition is satisfied, the factor of safety for limit equilibrium analysis is:

$$\text{F.S.} = \text{Resistance/Driving Force} \qquad (3.2)$$

$$= \frac{(N_1-F_{w1})\tan\phi_1+(N_2-F_{w2})\tan\phi_2+C_{r1}I_1A_1+C_{r2}I_2A_2}{T_{12}}$$

with $(N_1-F_{w1}) \geq 0$, $(N_2-F_{w2}) \geq 0$

where N = Normal force on joint plane due to own weight

F_w = Water force (normal to joint plane)

ϕ = Joint frictional angle

I = $1-k$ = Fraction of joint plane that is intact

(k = persistence)

A = Area of the triangle that bounds the wedge

T_{12} = Driving force along the line of intersection.

(Subscripts 1 and 2 identify the joint plane)

The various terms N_1, N_2, F_{w1}, F_{w2}, A_1, A_2 and T_{12} are functions of combinations of the following:

Orientations of the two joints (β_1, γ_1, β_2, γ_2)

Inclination of the slope (α)

Height of wedge (h)

Water distribution parameters (G_{w1}, G_{w2})

Density of rock ρ_r

Density of water ρ_w

It is desirable to express the equation for the factor of safety as an explicit function of these parameters. Such an expression makes it possible to make sensitivity considerations about the Factor of Safety which would otherwise become apparent only after lengthy numerical work.

In Appendix B, A_1, A_2 and V are expressed as functions of the joint orientation angles β_1, γ_1, β_2, γ_2, slope inclination α and h. These expressions, together with the unit vector along the line of intersection (Eq. A.7 in Appendix A) are used herein to obtain the expressions for the following dimensionless terms in Eq. 3.2:

$$\frac{N_1}{T_{12}}, \quad \frac{F_{w1}}{T_{12}}, \quad \frac{N_2}{T_{12}}, \quad \frac{F_{w2}}{T_{12}}, \quad \frac{C_{r1} I_1 A_1}{T_{12}}, \quad \frac{C_{r2} I_2 A_2}{T_{12}}$$

It is then shown in this chapter that Eq. 3.2 can also be written as:

$$F.S. = (a_1-b_1G_{w1})\tan\phi_1+(a_2-b_2G_{w2})\tan\phi_2+3b_1n_\rho(\frac{C_{r1}I_1}{\rho_r h})+3b_2n_\rho(\frac{C_{r2}I_2}{\rho_r h}) \qquad (3.3)$$

where $G_{w1} = n_{w1}(\frac{hw}{h})^3$

$G_{w2} = n_{w2}(\frac{hw}{h})^3$

$n_\rho = \frac{\rho_r}{\rho_w}$ = specific density of rock

ρ_r = density of rock

C_r, I, ϕ as defined previously

and where a_1, b_1, a_2 and b_2 are dimensionless coefficients which depend only on the orientation of the joint planes and on the inclination of the slope. They are:

$$a_1 = \frac{N_1}{T_{12}} = (\sin\gamma_2\cot\gamma_1-\cos\gamma_2\cos(\beta_2-\beta_1))/[\sin\psi\sin(\beta_2-\beta_1)] \qquad (3.4)$$

$$a_2 = \frac{N_2}{T_{12}} = (\cos\gamma_1\cos(\beta_2-\beta_1)-\sin\gamma_1\cot\gamma_2)/[\sin\psi\sin(\beta_2-\beta_1)] \qquad (3.5)$$

$$b_1 = \frac{F_{w1}}{T_{12}G_{w1}} = a_0\sin\beta_2\sin\gamma_2 \qquad (3.6)$$

$$b_2 = \frac{F_{w2}}{T_{12}G_{w2}} = a_0\sin\beta_1\sin\gamma_1 \qquad (3.7)$$

in which

$$\sin\psi = \sqrt{1-[\sin\gamma_1\sin\gamma_2\cos(\beta_2-\beta_1)+\cos\gamma_1\cos\gamma_2]^2} \quad (3.8)$$

$$a_0 = \sin\psi/[n_\rho\sin^2(\beta_2-\beta_1)\sin^2\gamma_1\sin^2\gamma_2(\cot\varepsilon_x-\cot\alpha)] \quad (3.9)$$

$$\cot\varepsilon_x = (\sin\beta_2\cot\gamma_1-\sin\beta_1\cot\gamma_2)/\sin(\beta_2-\beta_1) \quad (3.10)$$

The various steps that lead from Eq. 3.2 to Eq. 3.3 are described in the rest of this chapter, together with discussions on the requirements for potential sliding along the line of intersection, and on how Factor of Safety varies with changes in joint orientation angles and resistance parameters.

The water forces, F_{w1} and F_{w2} in Eq. 3.2, act in a direction normal to planes 1 and 2, respectively. The line of intersection, along which the driving force T_{12} acts, is perpendicular to the normals to plane 1 and plane 2. Hence, the driving force along the intersection is not affected by the action of water in the two joint planes and, in the absence of other external forces, is given by the component of the weight of the wedge along the line of intersection. This component is

$$T_{12} = (V\rho_r)(-\hat{k}) \cdot \vec{W}_{12} \quad (3.11)$$

$$= -V\rho_r W_{12z}$$

where V = volume of wedge

ρ_r = density of rock

$\hat{k}$ = unit vector in the Z direction

$\vec{W}_{12}$ = unit vector along the line of intersection, pointing towards point 0.

The other component of the weight vector is perpendicular to the line of intersection. If one denotes this force by $\vec{N}_{12}$, then

$$\vec{N}_{12} = (V\rho_r)(-\hat{k}) - T_{12}\vec{W}_{12} \qquad (3.12)$$

The force $\vec{N}_{12}$ can be split further into components N_1 and N_2 acting normally to planes 1 and 2, respectively. First one writes,

$$\vec{N}_{12} = N_1\vec{W}_1 + N_2(-\vec{W}_2)$$

where $\vec{W}_1$ and $\vec{W}_2$ are the unit normal vectors to planes 1 and 2 respectively (see Fig. 3.1) and are given by Eq. A.1 and A.2 in Appendix A. Hence:

$$N_{12x} = N_1W_{1x} - N_2W_{2x}$$

$$N_{12y} = N_1W_{1y} - N_2W_{2y}$$

$$N_{12z} = N_1W_{1z} - N_2W_{2z}$$

Then one uses the first two equations to obtain

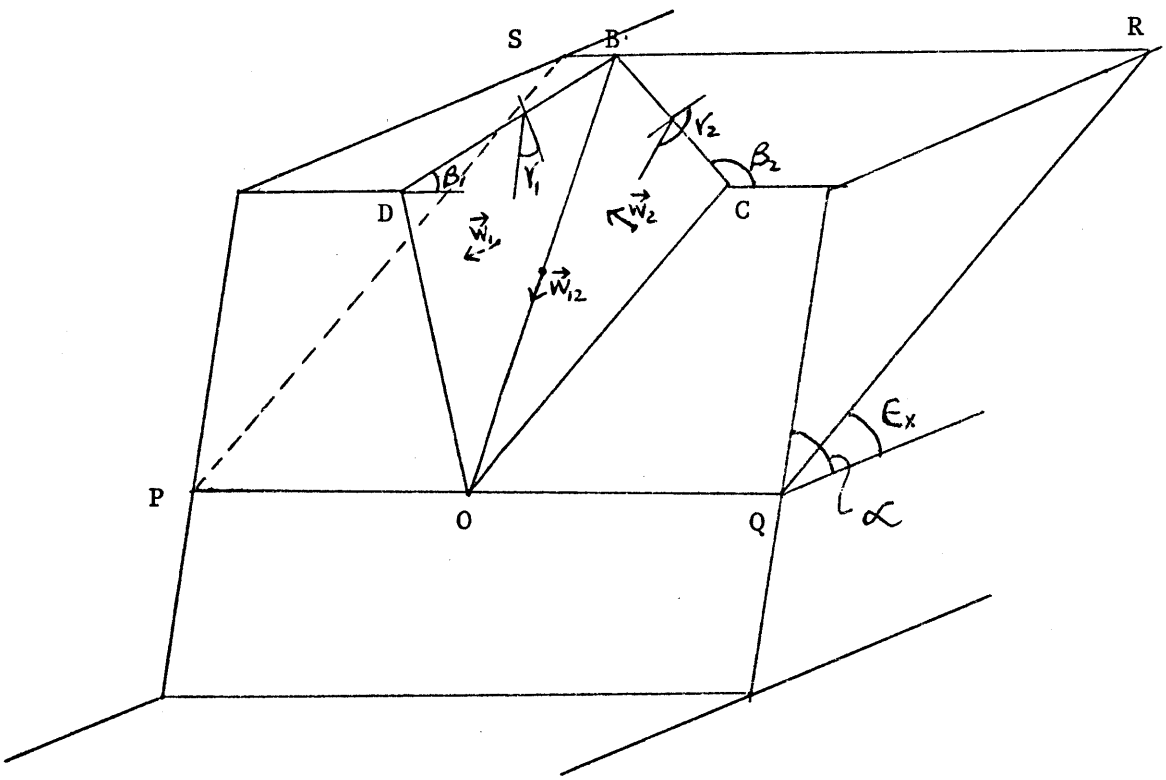


Figure 3.1 Notations

$$N_1 = \frac{(N_{12y}W_{2x} - N_{12x}W_{2y})}{(W_{1y}\ W_{2x} - W_{1x}\ W_{2y})}$$

$$N_2 = \frac{(N_{12y}W_{1x} - N_{12x}W_{1y})}{(W_{1y}\ W_{2x} - W_{1x}\ W_{2y})}$$

where $N_{12x} = -T_{12}W_{12x}$

$N_{12y} = -T_{12}W_{12y}$ (from Eq. 3.12)

The denominator, $(W_{1y}W_{2x} - W_{1x}W_{2y})$, equals X_{12z}, the component along Z of the vector product $(\vec{W}_2 \times \vec{W}_1 = \vec{X}_{12})$. Hence,

$$N_1 = \frac{[-T_{12}W_{2x}W_{12y} - (-T_{12}W_{2y}W_{12x})]}{X_{12z}}$$

and

$$\frac{N_1}{T_{12}} = [W_{2y}W_{12x} - W_{2x}W_{12y}]/X_{12z}$$

Using Eq. A.7 in Appendix A, one may rewrite this as

$$\frac{N_1}{T_{12}} = [W_{2y}X_{12x} - W_{2x}X_{12y}]/[X_{12z}\sin\psi]$$

Substituting from Eq. A.1 - A.5 in Appendix A, one obtains

$$\frac{N_1}{T_{12}} = \frac{\sin\gamma_2\cos\beta_2(\cos\beta_1\sin\gamma_1\cos\gamma_2-\cos\beta_2\cos\gamma_1\sin\gamma_2)-(-\sin\gamma_2\sin\beta_2)(\sin\beta_1\sin\gamma_1\cos\gamma_2-\sin\beta_2\cos\gamma_1\sin\gamma_2)}{\sin(\beta_1-\beta_2)\sin\gamma_1\sin\gamma_2\sin\psi}$$

$$= \frac{\cos\beta_2\cos\beta_1\cos\gamma_2-\cos^2\beta_2\sin\gamma_2\cot\gamma_1+\sin\beta_1\sin\beta_2\cos\gamma_2-\sin^2\beta_2\sin\gamma_2\cot\gamma_1}{\sin(\beta_1-\beta_2)\sin\psi}$$

$$= \frac{\cos\gamma_2\cos(\beta_2-\beta_1) - \sin\gamma_2\cot\gamma_1}{\sin(\beta_1-\beta_2)\sin\psi}$$

$$= \frac{\sin\gamma_2\cot\gamma_1 - \cos\gamma_2\cos(\beta_2-\beta_1)}{\sin(\beta_2-\beta_1)\sin\psi}$$

This is Equation 3.4, shown earlier in this chapter.

Similarly,

$$\frac{N_2}{T_{12}} = \frac{W_{1y}W_{12x} - W_{1x}W_{12y}}{X_{12z}}$$

$$= \frac{\cos\gamma_1 \cos(\beta_2-\beta_1) - \sin\gamma_1 \cot\gamma_2}{\sin(\beta_2-\beta_1)\sin\psi}$$

This is Eq. 3.5, shown earlier.

For the water condition assumed herein,

$$F_{w1} = \frac{1}{3}\rho_w h G_{w1} A_1$$

$$F_{w2} = \frac{1}{3}\rho_w h G_{w2} A_2$$

Therefore,

$$\frac{F_{w1}}{T_{12}} = \frac{\frac{1}{3}\rho_w h G_{w1} A_1}{-V\rho_r W_{12z}}$$

$$= \frac{1}{3}\left(\frac{hA_1}{V}\right) \frac{G_{w1}}{n_\rho(-W_{12z})}$$

$$= b_1 G_{w1}$$

$$\text{with } b_1 = \frac{1}{3n_\rho(-W_{12z})}\left(\frac{hA_1}{V}\right) \qquad (3.13)$$

From Equations B.2 and B.6 in Appendix B, one obtains

$$\frac{hA_1}{V} = \frac{3}{\sin\beta_1 \sin\gamma_1 (\cot\beta_1 - \cot\beta_2)(\cot\varepsilon_x - \cot\alpha)}$$

where $\cot\varepsilon_x$ is given by Eq. B.4 in Appendix B, so that

$$b_1 = \frac{\sin\Psi}{n_\rho \sin\beta_1 \sin\gamma_1 (\cot\beta_1 - \cot\beta_2)(\cot\varepsilon_x - \cot\alpha)\sin(\beta_2-\beta_1)\sin\gamma_1 \sin\gamma_2}$$

$$= \frac{\sin\beta_2 \sin\gamma_2 \sin\psi}{n_\rho \sin^2(\beta_2-\beta_1)\sin^2\gamma_1 \sin^2\gamma_2 (\cot\varepsilon_x - \cot\alpha)}$$

Similarly,

$$b_2 = \frac{\sin\beta_1 \sin\gamma_1 \sin\psi}{n_\rho \sin^2(\beta_2-\beta_1)\sin^2\gamma_1 \sin^2\gamma_2 (\cot\varepsilon_x - \cot\alpha)}$$

The expressions become

$$b_1 = a_0 \sin\beta_2 \sin\gamma_2, \qquad \text{hence Eq. 3.6}$$

and

$$b_2 = a_0 \sin\beta_1 \sin\gamma_1, \qquad \text{hence Eq. 3.7}$$

if one defines

$$a_0 = \frac{\sin\psi}{n_\rho \sin^2(\beta_2-\beta_1)\sin^2\gamma_1 \sin^2\gamma_2 (\cot\varepsilon_x - \cot\alpha)} \tag{3.9}$$

We now proceed to consider the remaining terms of Eq. 3.2. Dividing the third term of the numerator in Eq. 3.2 by the denominator, one obtains

$$\frac{C_{r1}I_1A_1}{T_{12}} = \frac{C_{r1}I_1A_1}{-V\rho_r W_{12z}}$$

$$= \left(\frac{A_1 h}{-VW_{12z}}\right) \left(\frac{C_{r1}I_1}{\rho_r h}\right)$$

From Eq. 3.13,

$$3n_\rho b_1 = \frac{A_1 h}{-VW_{12z}}$$

hence,

$$\frac{C_{r1}I_1A_1}{T_{12}} = 3n_\rho b_1 \left(\frac{C_{r1}I_1}{\rho_r h}\right)$$

and similarly,

$$\frac{C_{r2}I_2A_2}{T_{12}} = 3n_\rho b_2 \left(\frac{C_{r2}I_2}{\rho_r h}\right)$$

This completes the rewriting of Eq. 3.2 into Eq. 3.3.

3.2 Requirements for Sliding Along the Line of Intersection

The expressions of the Factor of Safety in Eqs. 3.2 and 3.3 have been derived under the assumption that failure can occur only by sliding of the wedge along the line of intersection of the bounding planes. For this to be true, the normal force component on each joint plane due to the weight of the wedge must exceed the water force on the same plane, i.e., it should be that

$$N_1 - F_{w1} \geq 0$$

and

$$N_2 - F_{w2} \geq 0$$

In the case where $F_{w1} = F_{w2} = 0$, the requirements can be expressed as conditions of positivity for the quantities a_1 and a_2 in Eqs. 3.4 and 3.5.

Since the terms $\sin\Psi$ (Eq. 3.8) and $\sin(\beta_2-\beta_1)$ are always positive, the requirements are equivalent to:

$$\sin\gamma_2\cot\gamma_1 - \cos\gamma_2\cos(\beta_2-\beta_1) \geq 0 \tag{3.14}$$

and

$$\cos\gamma_1\cos(\beta_2-\beta_1) - \sin\gamma_1\cot\gamma_2 \geq 0 \tag{3.15}$$

or, given that $0 < \gamma_1 \leq 90^\circ$, and $90^\circ \leq \gamma_2 < 180^\circ$,

$$|\sin\gamma_2\cot\gamma_1| + |\cos\gamma_2|\cos(\beta_2-\beta_1) \geq 0 \tag{3.16}$$

and

$$|\cos\gamma_1|\cos(\beta_2-\beta_1) + |\sin\gamma_1\cot\gamma_2| \geq 0 \tag{3.17}$$

One concludes that under the present constraints on γ_1 and γ_2, conditions 3.14 and 3.15 are always satisfied if $\cos(\beta_2-\beta_1) \geq 0$, i.e. if $\beta_2-\beta_1 \leq 90^\circ$.

In order to show what combinations of $(\beta_2-\beta_1) > 90^\circ$, γ_1 and γ_2 correspond to potential sliding along the line of intersection, we first rearrange Eqs. 3.14 and 3.15 and write them as:

$$\sin\gamma_2\cot\gamma_1 \geq \cos\gamma_2\cos(\beta_2-\beta_1) \quad (3.18)$$

and

$$\cos\gamma_1\cos(\beta_2-\beta_1) \geq \sin\gamma_1\cot\gamma_2 \quad (3.19)$$

Keeping in mind the constraints on γ_1, γ_2, expression 3.18 can be further rewritten as

$$\tan\gamma_2 \leq \tan\gamma_1\cos(\beta_2-\beta_1)$$

or

$$|\tan\gamma_2| \geq |\tan\gamma_1||\cos(\beta_2-\beta_1)| \quad (3.20)$$

Similarly, expression 3.19 can be rewritten as

$$|\tan\gamma_2| \leq \frac{|\tan\gamma_1|}{|\cos(\beta_2-\beta_1)|} \quad (3.21)$$

Combining Eqs. 3.20 and 3.21, one obtains

$$|\tan\gamma_1||\cos(\beta_2-\beta_1)| \leq |\tan\gamma_2| \leq \frac{|\tan\gamma_1|}{|\cos(\beta_2-\beta_1)|} \quad (3.22)$$

which is equivalent to the requirement of positivity for a_1 and a_2 when $(\beta_2-\beta_1) > 90^\circ$.

The plot of Fig. 3.2 shows which combinations of $(\beta_2-\beta_1)$, γ_1, γ_2

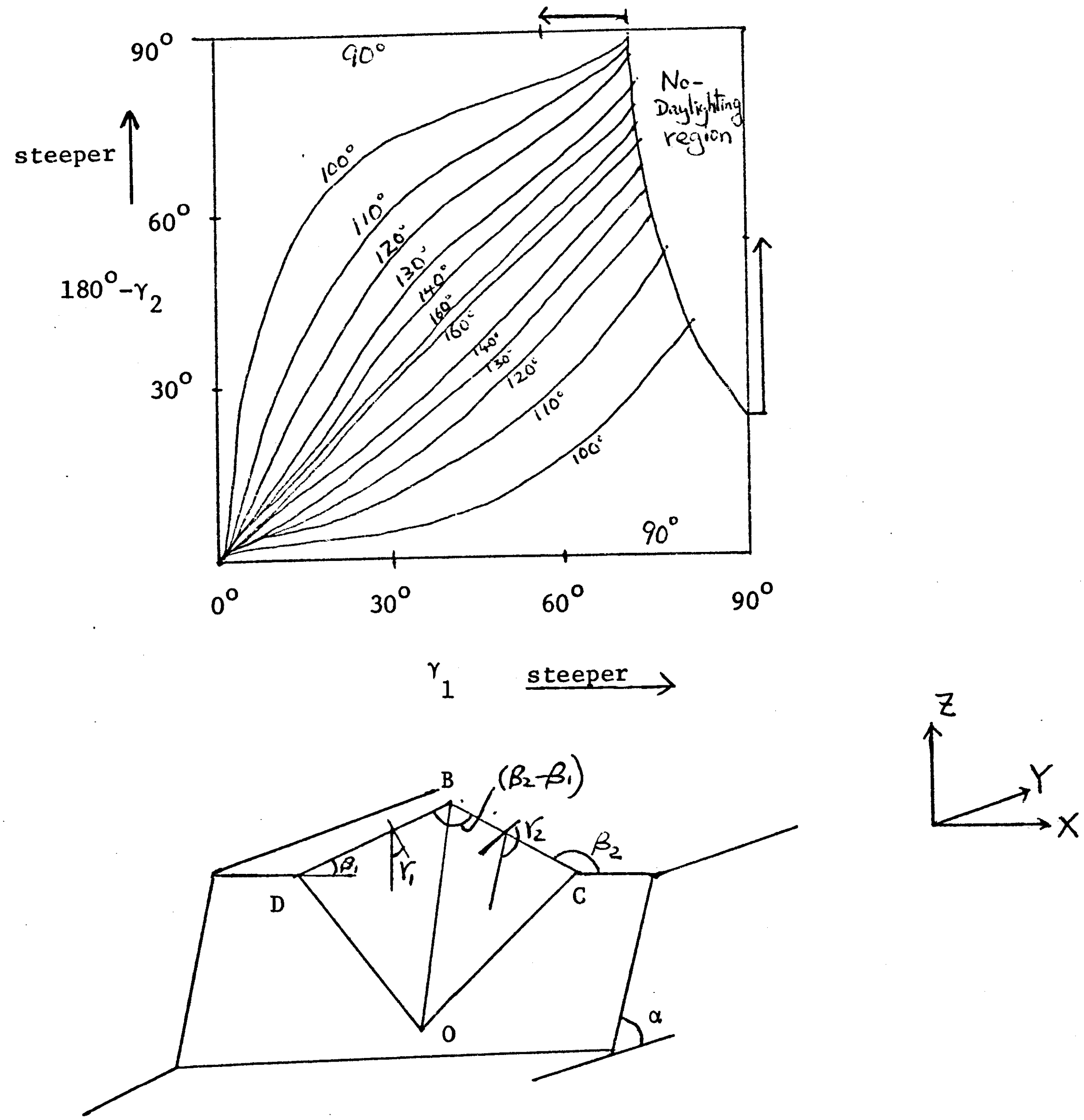


Figure 3.2 Joint Orientations for which wedge can slide along the line of intersection.

satisfy the inequality expression 3.22.

In the extreme case when $\beta_2-\beta_1$ approaches 180°, expression 3.22 can be satisfied only when $\gamma_2 \simeq \gamma_1$, as shown by the $\beta_2-\beta_1 = 160^\circ$ curves in Fig. 3.2.

One can show that the condition $a_1 \geq 0$ is equivalent to $C\hat{B}O \leq 90^\circ$ (Fig. 3.2) and that $a_2 \geq 0$ is equivalent to $D\hat{B}O \leq 90^\circ$, so that the requirements for sliding along the line of intersection actually means (in the dry state for which $G_{w1} = G_{w2} = 0$) that both DBO and CBO must be smaller than 90°. The expressions for $D\hat{B}O$ and $C\hat{B}O$ are obtained as follows:

A unit vector along $\overline{BD}$, $\vec{W}_{BO}$, has components

$$\vec{W}_{BD} = (-\cos\beta_1, -\sin\beta_1, 0)$$

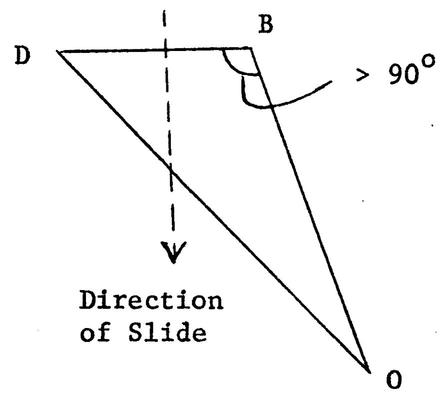
Therefore, $\cos D\hat{B}O = \vec{W}_{BD} \cdot \vec{W}_{12}$

$$= \frac{\cos\gamma_1 \sin\gamma_2 \cos(\beta_2-\beta_1) - \sin\gamma_1 \cos\gamma_2}{\sin\psi}$$

$$= \frac{\cos\gamma_1 \cos(\beta_2-\beta_1) - \sin\gamma_1 \cot\gamma_2}{\sin\psi/\sin\gamma_2}$$

and $\cos D\hat{B}O > 0$ if the numerator in the previous expression is itself greater than 0, i.e. if $\cos\gamma_1\cos(\beta_2-\beta_1) - \sin\gamma_1\cot\gamma_2 > 0$. This condition is identical to that in expression 3.15. Similarly, it can be shown that $C\hat{B}O \leq 90^\circ$ if and only if Eq. 3.14 is satisfied.

These conditions make physical sense: a weight placed on a slope always tends to slide in the dip direction (the direction of maximum gradient). Therefore, if $D\hat{B}O$ and $C\hat{B}O$ are both acute angles, potential sliding is along the line BO; if on the contrary $D\hat{B}O$ is obtuse, sliding is away from the line of intersection, on the plane BDO, as shown in the figure below.



Given the present constraints on γ_1, γ_2, the angles $D\hat{B}O$ and $C\hat{B}O$ are always smaller than 90^o if $D\hat{B}C$ (= β_2-β_1) is less than 90^o. Hence the curves in Fig. 3.2.

The shape of the no-daylighting-region changes with β_1 and β_2. That shown in Fig. 3.2 corresponds to $\beta_1 = 10^o$. The arrows bordering the Figure show shifting of the no-daylighting boundary as (β_2-β_1) increases from 90^o to 160^o.

3.3 Safe Regions in the $\gamma_1\gamma_2$ Plane

This section deals with the variation of the safe regions with joint orientation angles.

The plots in Fig. 3.3 show contour lines of the factor of safety function at the level FS = 1 (safe region boundary) on the $\gamma_1\gamma_2$ plane for different values of wedge angle (β_2-β_1) and other parameters fixed to the values given in the figure. The associated non-daylighting regions vary as (β_2-β_1) increases from 40^o to 90^o as indicated by the

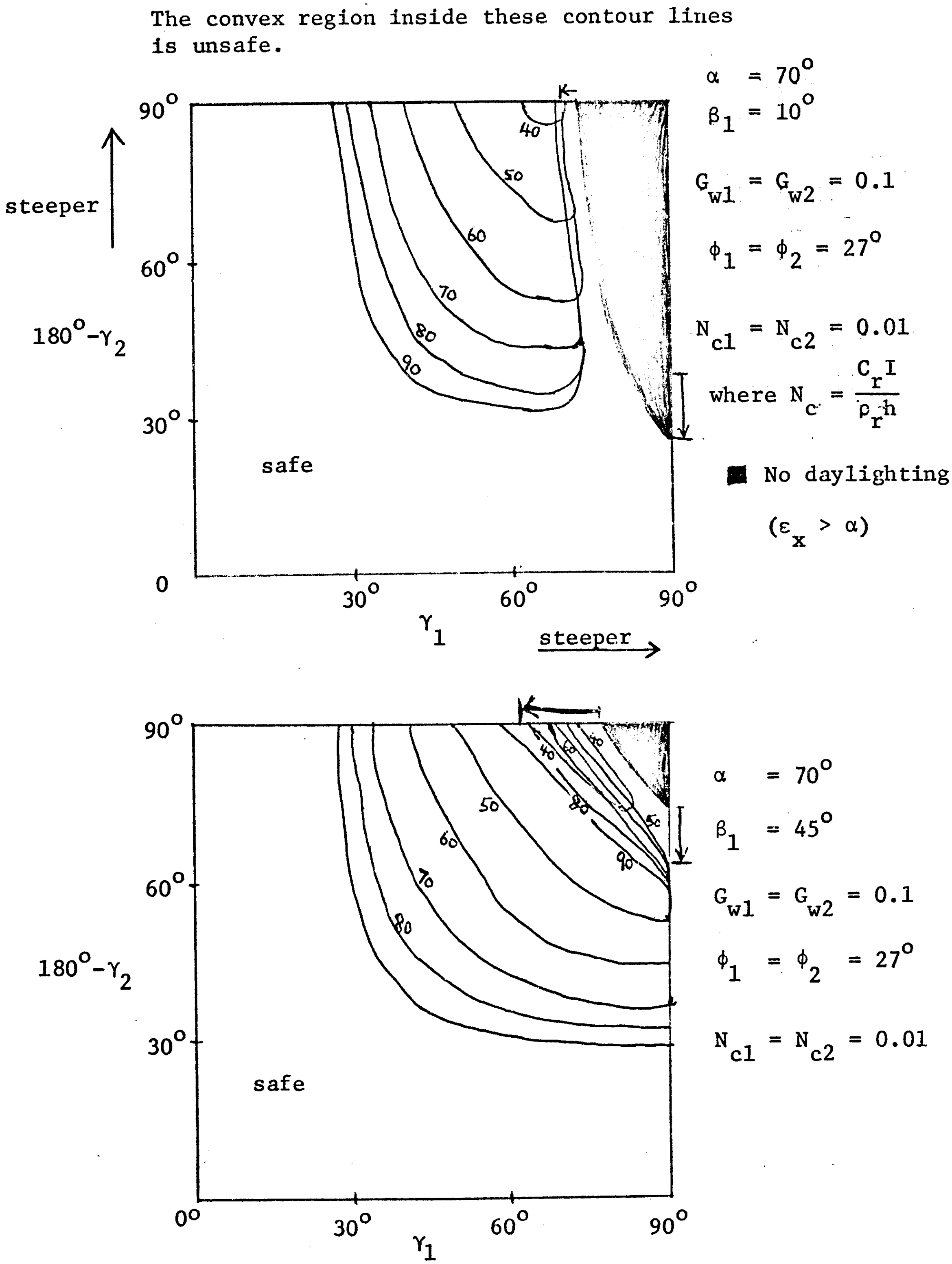
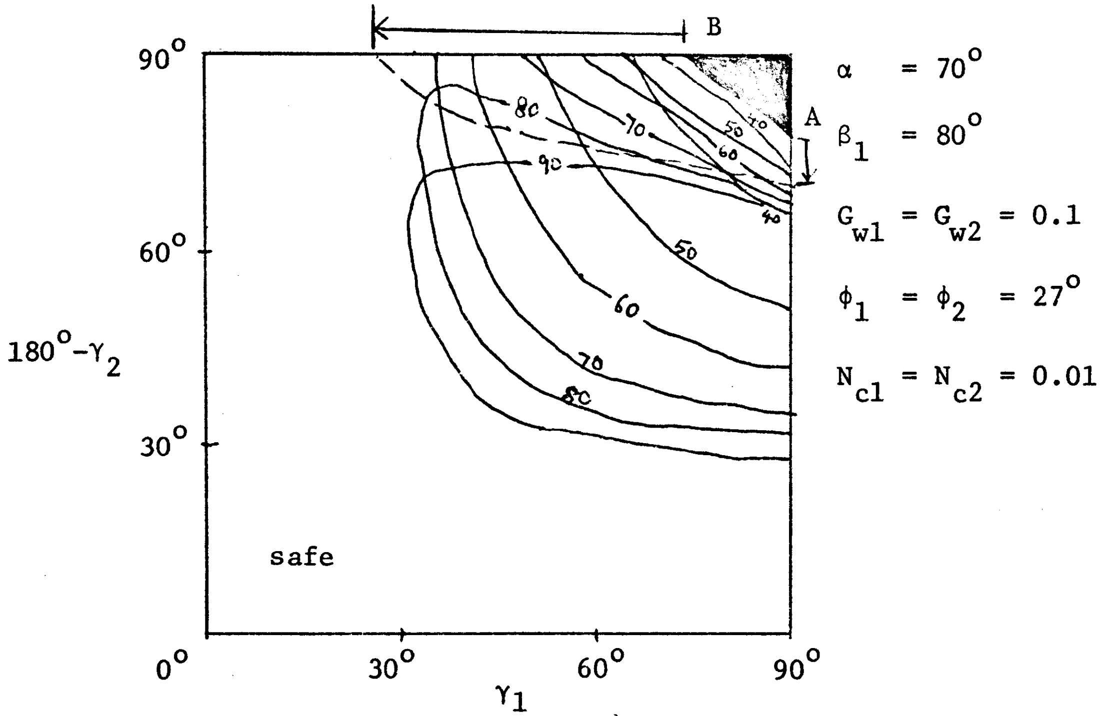


Figure 3.3 Variation of F.S. = 1 curves with joint orientation angles.



■ Non-daylighting region (for $\beta_2-\beta_1 = 40^\circ$)

---- Boundary of no daylighting region for $(\beta_2-\beta_1)=90^\circ$

Arrows show movements of A and B as $(\beta_2-\beta_1)$ increases.

Figure 3.3 (continued)

arrows bordering each figure. The parameter which varies from figure to figure is β_1, with values 10^o, 45^o, and 80^o.

In the calculations that led to the results of Fig. 3.3 as well as in those for the reliability index in Chapter 4, whenever the water parameter G_w is such that $bG_w > a$, the term $(a - bG_w)$ in Eq. 3.3 is set equal to zero and the Factor of Safety calculated accordingly. The reason for this operation is the likely occurrence of joint dilation, followed by a decrease in water pressure.

Fig. 3.3 shows that the unsafe region in the $\gamma_2\gamma_1$ plane expands rapidly as $(\beta_2-\beta_1)$ increases, whereas for the water and strength parameters given in the figure, wedges with $(\beta_2-\beta_1) \leq 30^o$ are safe for any combinations of γ_1 and γ_2 within the ranges shown.

The plots also show that the safe region in this problem is unlike those in most other problems because of its non-convexity.

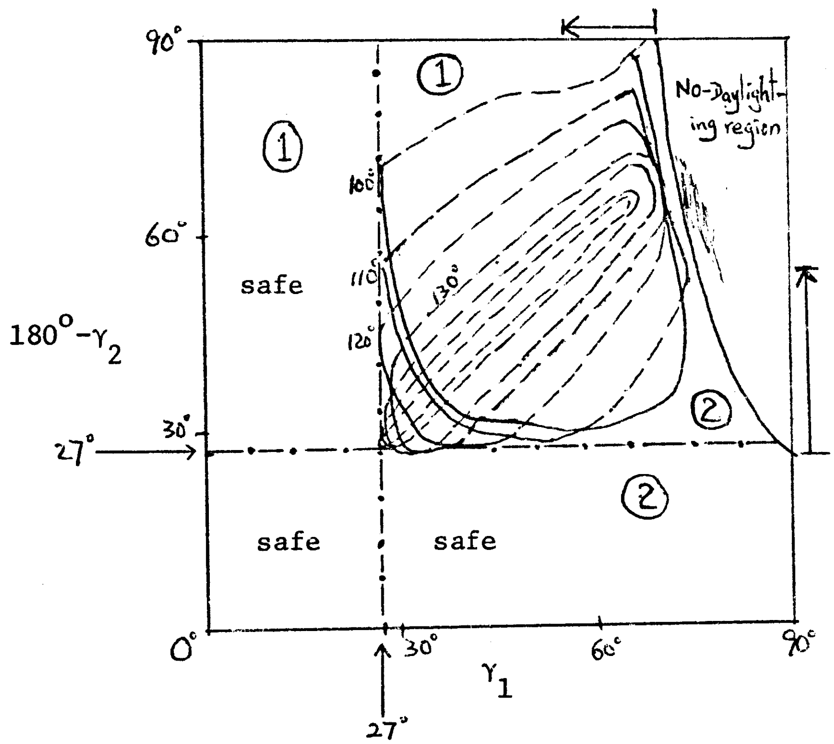
Fig. 3.4 shows FS = 1 contours for $(\beta_2-\beta_1) > 90^o$. The unsafe regions shown in the plot are for potential sliding along the line of intersection only. The dotted lines represent the boundaries between region where potential sliding is along the intersection and region where potential sliding is on one plane only (see Fig. 3.2). The lower plot in Fig. 3.4 shows how one such curve, $\beta_2-\beta_1 = 110^o$, is obtained.

Plots for $\beta_1 = 45^o$ and $\beta_1 = 80^o$ are nearly identical to those in Fig. 3.4.

For sliding along one plane only, no frictional resistance is contributed by the other joint plane, while water effect and intact rock on that plane may still have an influence. If one neglects both water

1: potential sliding is along plane 1 only.

2 : potential sliding is along plane 2 only.



$\alpha = 70^{\circ}$

$\beta_1 = 10^{\circ}$

$G_{w1} = G_{w2} = 0.1$

$\phi_1 = \phi_2 = 27^{\circ}$

$N_{c1} = N_{c2} = 0.01$

where $N_c = \dfrac{C_r I}{\rho_r h}$

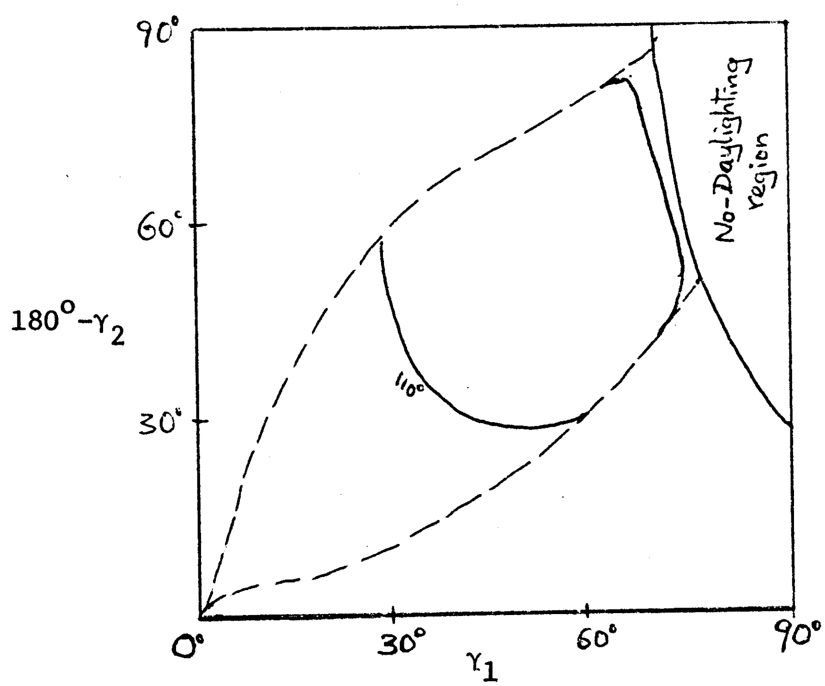


Figure 3.4 Variation of F.S. = 1 contours with joint orientation angles ($\beta_2-\beta_1 > 90^{\circ}$).

force and intact rock resistance on the two planes when considering sliding along one plane, then two lines, corresponding to $\gamma_1 = \phi_1$ and $\gamma_2 = \phi_2$, can be drawn to define the safe boundary. These lines are shown in Fig. 3.4. They are drawn on the basis that sliding along a single plane occurs if the plane dips at an angle greater than the frictional angle, provided there is no water or cohesion effect.

The 3 plots in Fig. 3.3 appear to be quite different primarily because of the different shape of the non-daylighting zones. For $\alpha = 90^o$, the non-daylighting region disappears and the 3 plots look very much the same, each one displaying the contour lines approximately as concentric loops with center at the top right corner.

From these results, the following conclusions can be drawn:

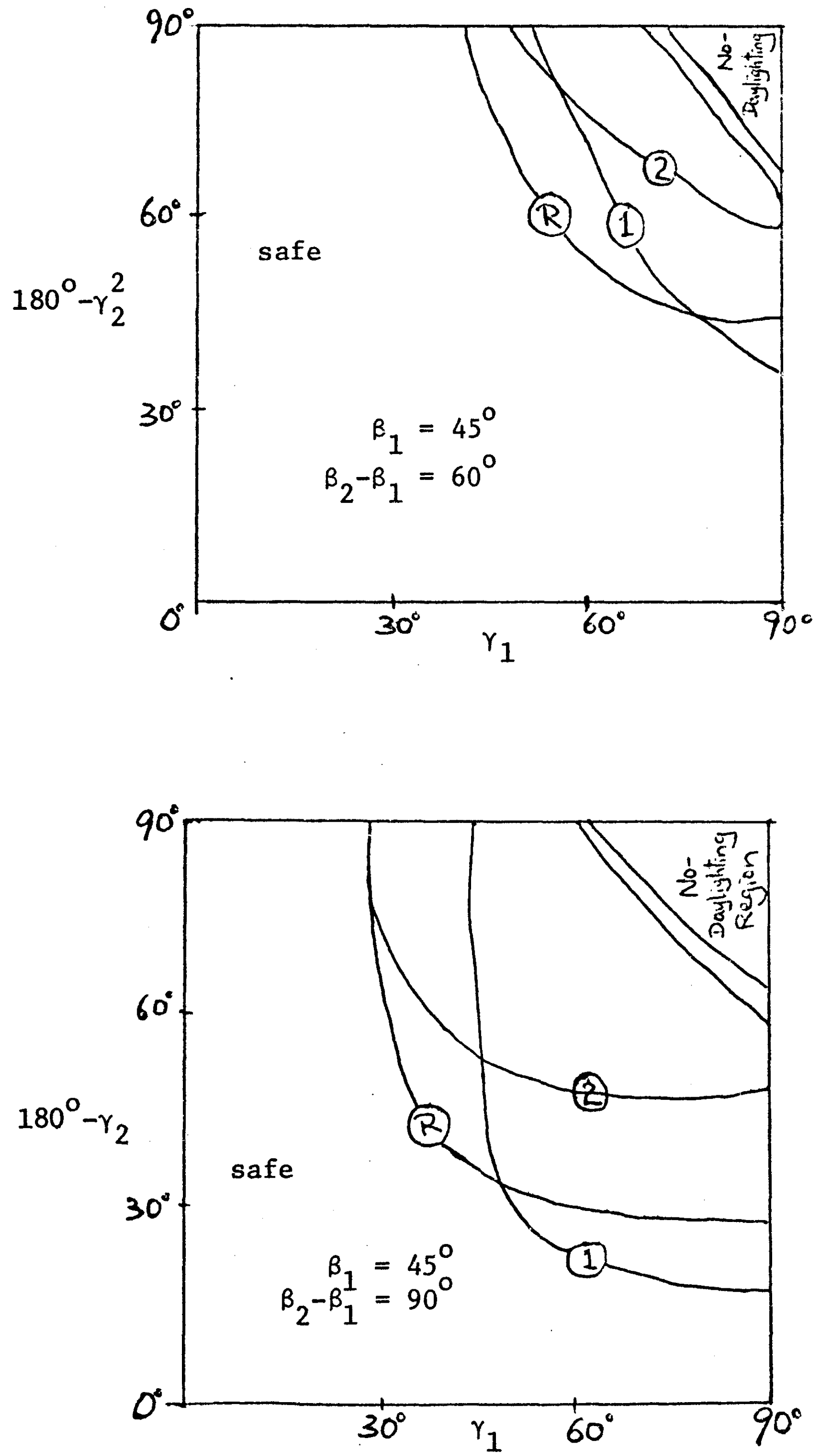
1. The Factor of Safety exceeds 1 (the wedge is safe) if either one or the following conditions applies:

   $\gamma_1 < \phi_1$ or $(180^o - \gamma_2) < \phi_2$

2. For given $(\beta_2 - \beta_1)$, the Factor of Safety increases as $\sqrt{(90^o - \gamma_1)^2 + (90^o - \gamma_2)^2}$ increases. However, the inequality expression 3.1 should first be checked to ensure daylighting.

3. The Factor of Safety decreases as $(\beta_2 - \beta_1)$ increases.

For wedges with different water and resistance parameters, the shape of the contours FS = 1 is the same except that the contours are compressed in the direction of the coordinate axis corresponding to the 'stronger' joint plane. The safe boundaries in Fig. 3.5 illustrate the above statement.



Curve R

Reference Case

$G_{w1} = G_{w2} = 0.1$

$\phi_1 = \phi_2 = 27^\circ$

$N_{c1} = N_{c2} = 0.01$

Curve 1

Stronger plane 1
Weaker plane 2

$G_{w1} = 0.1,\ G_{w2} = 0.3$

$\phi_1 = 45^\circ,\ \phi_2 = 20^\circ$

$N_{c1}=0.01, N_{c2}=0.005$

Curve 2

Stronger plane 2

$G_{w1} = G_{w2} = 0.1$

$\phi_1 = 27^\circ,\ \phi_2 = 45^\circ$

$N_{c1} = N_{c2} = 0.01$

Figure 3.5 Dependence of the safe region on 'joint strength'.

In Fig. 3.5, the difference between the boundary of the safe region for joints with equal strength (curve R) and the same boundary for joints with unequal strength (curves 1 and curve 2) can be anticipated by the following considerations:

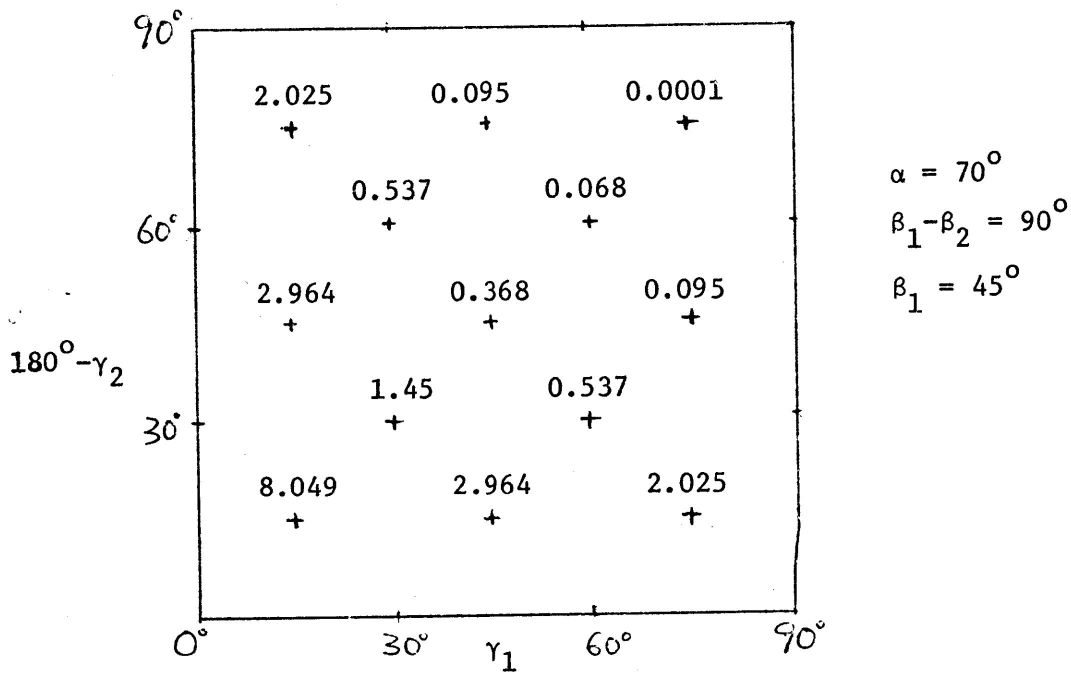
Wedges bounded by joint planes with higher strength become unsafe only for steeper dip. Hence, when compared with curve R, curve 1 (which corresponds to a stronger joint 1 and a weaker joint 2) is compressed to the right and extended downwards. On the contrary, curve 2 (which corresponds to a case with stronger joint 2 but joint 1 with equal strength as for curve R) is similar to curve R except that it is compressed upwards.

The thin strip of safe region between the non-daylighting zone and the unsafe zone can be explained by the rapid decrease in volume (and hence in driving force) as ε_x approaches the inclination of the slope, α. Cohesion of the intact rock is then sufficient to ensure stability. Figure 3.6 shows how the quantity (Volume/h^3) varies in the $\gamma_2\gamma_1$ plane. This term enters the formula for the Factor of Safety through the dimensionless quantity b_1 and b_2 (Eq. 3.13).

For given height, h, the wedge volumes for a symmetrical wedge with $\gamma_1 = 45^o$ and $\gamma_2 = 135^o$ and for a wedge bordering the non-daylighting zone can differ by several orders of magnitude. The expression for $\frac{V}{h^3}$, as given by Eq. B.6 in Appendix B, is

$$\frac{V}{h^3} = \frac{1}{6}(\cot\beta_1 - \cot\beta_2)(\cot\varepsilon_x - \cot\alpha)^2$$

with the square term accounting for dependence on ε_x.



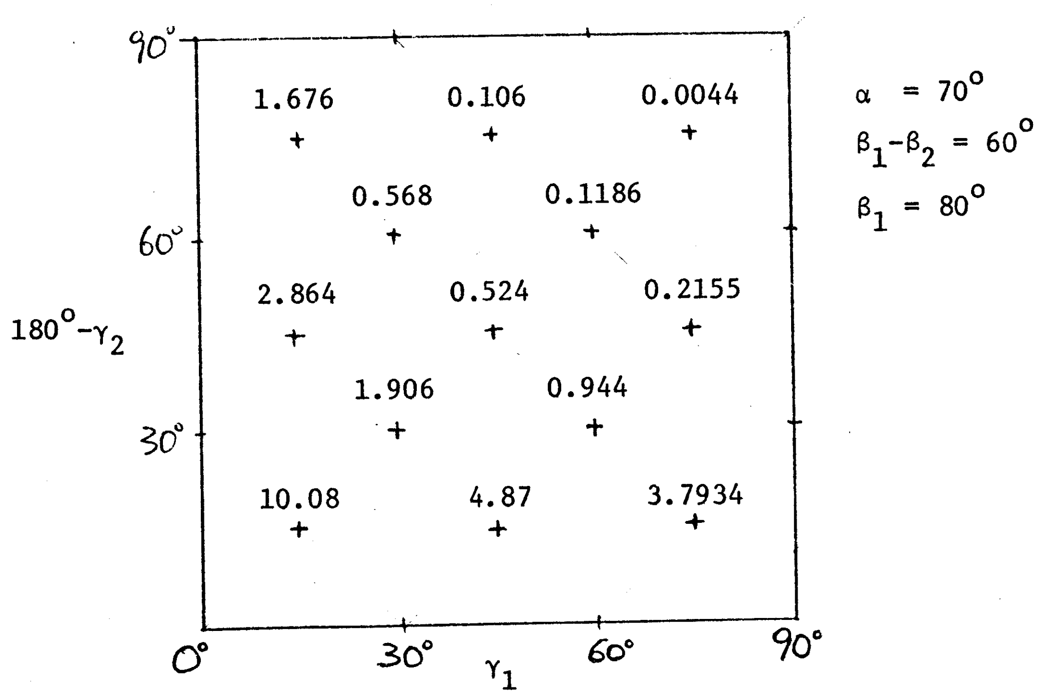


Figure 3.6. Variation of the quantity $\frac{\text{Volume}}{h^3}$ with γ_1 and γ_2

CHAPTER 4

CALCULATION OF THE SECOND MOMENT RELIABILITY INDEX

4.1 The Reliability Index, β

The probability distribution of joint orientation angles and that of resistance and water parameters are seldom known. However, the first two probabilistic moments of such variables can often be obtained with good accuracy, by processing joint survey data. It is now assumed that this information is available for the calculation of the so-called second-moment reliability index, β (Hasofer and Lind, 1974).

Usual design proceeds as follows. Given the mean value of all parameters, it is required that the factor of safety associated with it be larger than a given minimum value. This minimum value is larger than 1, to account for errors in the mathematical model and to secure against adverse values of the uncertain parameters.

A better approach would be to explicitly acknowledge the uncertainties and calculate reliability or at least a reliability index associated with the design.

Among various indices of reliability, one that is enjoying much popularity is the index β defined by Hasofer and Lind (1974): if safety depends on the realization of a random vector, $\underline{x}$, with mean $\underline{m}$ and covariance matrix $\underline{C}$ and if the system fails for $\underline{x}$ that belongs to a 'failure region', F, then β is defined as

$$\beta = \min_{\underline{x} \in F} \sqrt{(\underline{x}-\underline{m})^T \underline{C}^{-1} (\underline{x}-\underline{m})} \qquad (4.1)$$

The geometrical interpretation of β is illustrated in Fig. 4.1. Grossly speaking, β is the distance from $\underline{m}$ to the boundary of F, in units of (directional) standard deviations.

In the important case when the components of $\underline{x}$ are uncorrelated, the expression for β simplified to

$$\beta = \min_{\underline{x} \epsilon F} \left| \sum_j \frac{(x_j - m_j)^2}{\sigma_j^2} \right|^{\frac{1}{2}} \tag{4.2}$$

In Fig. 4.1, one defines the 1-σ dispersion ellipse by the following equation:

$$(\underline{x}-\underline{m})^T \underline{C}^{-1} (\underline{x}-\underline{m}) \leq 1 \tag{4.3}$$

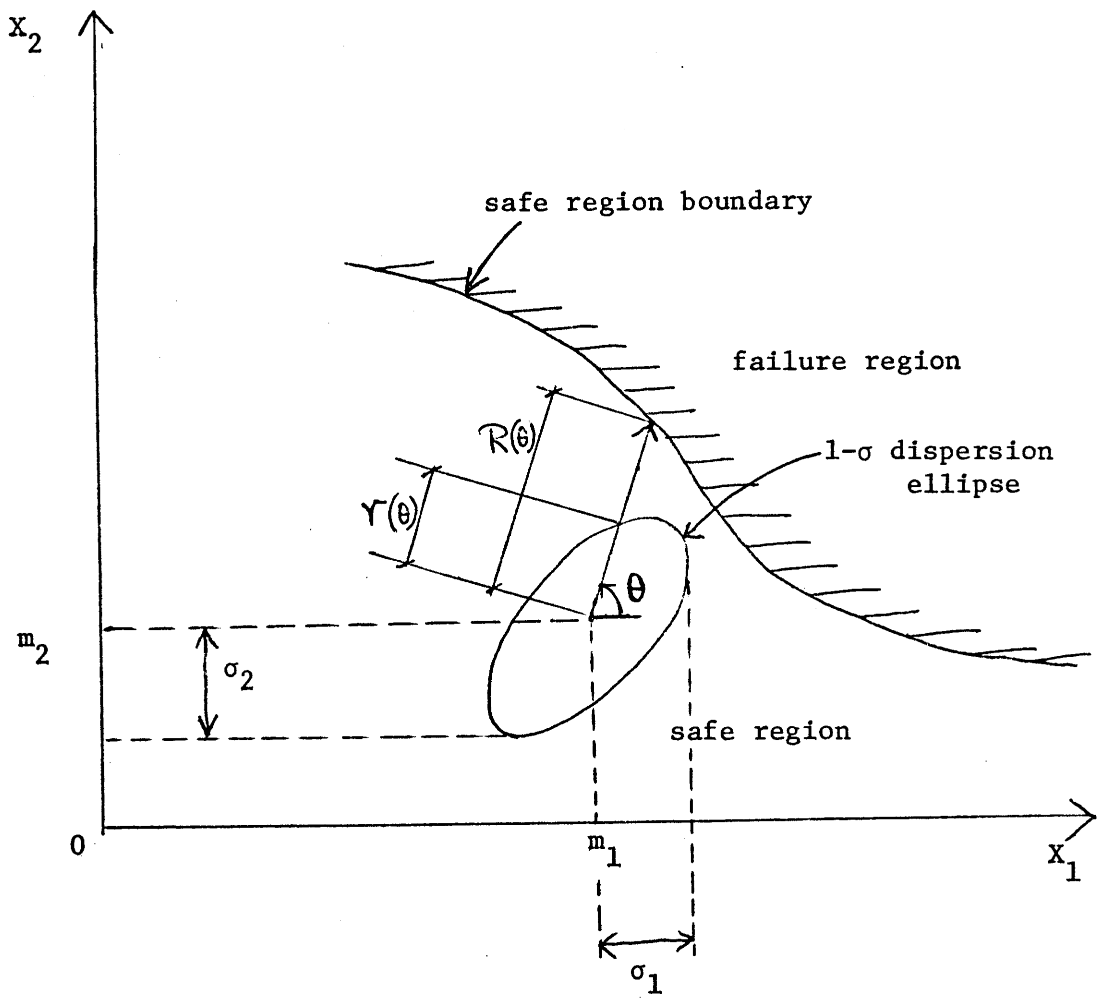
where $\underline{x}$ is the second-moment vector with two components,

$$\underline{x} = \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} \sim \left(\underline{m} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}, \; \underline{C} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \right)$$

Denote by $r(\theta)$ the distance from $\underline{m}$ to the boundary of the 1-σ dispersion ellipse (Eq. 4.3 above) in the direction θ, and let $R(\theta)$ be the distance between $\underline{m}$ and the critical region in the same direction.

Then $$\beta = \min_{\theta} \left(\frac{R(\theta)}{\gamma(\theta)}\right) \tag{4.4}$$

The critical direction, θ_{cr}, is defined as the value of θ that corresponds to the minimum in Eq. 4.4.



$$\beta = \min_{\theta} \left(\frac{R(\theta)}{r(\theta)}\right)$$

Figure 4.1 Illustration of β in the Plane

4.2 Approximate Calculation of

1. Only geometric uncertainty

We assume here that strength and water parameters and slope inclination are given, and study wedge reliability with respect to random variations in the joint orientation parameters, β_1, γ_1, β_2, and γ_2.

If these parameters are uncorrelated, as we assume for simplicity, the boundary of the 1-σ dispersion ellipse (an ellipsoid in R^4) satisfies

$$\frac{(\beta_1-m_{\beta 1})^2}{\sigma_{\beta 1}^2}+\frac{(\beta_2-m_{\beta 2})^2}{\sigma_{\beta 2}^2}+\frac{(\gamma_1-m_{\gamma 1})^2}{\sigma_{\gamma 1}^2}+\frac{(\gamma_2-m_{\gamma 2})^2}{\sigma_{\gamma 2}^2}=1$$

As a generalization of angle θ in Fig. 4.1, the generic direction in 4-dimensional space is characterized by three angles which we denote by θ, Ω and ψ. These angles are such that a unit vector in the direction identified by them, $\vec{S}(\theta,\Omega,\psi)$, has components:

$$S_x = \cos\theta\sin\Omega\sin\psi$$
$$S_y = \sin\theta\sin\Omega\sin\psi$$
$$S_z = \cos\Omega\sin\psi$$
$$S_v = \cos\psi$$

The approximate algorithm for the calculation of β discretizes the search points by giving equal increments to θ, Ω, ψ and to γ = distance of the point from the mean value point $\underline{m}$. The procedure articulates into nested searches:

The first search discretizes the entire four-dimensional space using large increments of the directional angles θ, Ω, ψ. The critical direction (the direction with minimum ratio $\frac{R}{\gamma}$) is identified and used as the central direction of the second search. This second search uses as many directional vectors as the first search, but the range of directions is half that of the first search. A total of 5 nested searches are made, always using the critical direction of the previous run as the central direction and each time halving the angular increments.

The search range and the increments of θ, Ω, ψ for each of the 5 searches are as follows:

Search No.	Range of Search	Increment in θ, Ω, ψ
1	360°	45°
2	180°	22.5°
3	90°	11.25°
4	45°	5.63°
5	22.5°	2.81°

In the case where all search vectors miss F (F may be within a rather small angular region), the critical direction is taken to be that along which the Factor of Safety is minimum. This is then the central direction for the next search.

Example runs showing the values of β, the critical direction, and the critical point of each nested search, are given in Tables 1 to 11.

The cases in Tables 1 and 2 have the same mean joint orientation angles but different standard deviations. Hence they have

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0   (STD DEV: 6.00)
   DIP1=    40.0   (STD DEV: 5.00)
   STRIKE2=100.0   (STD DEV: 5.00)
   DIP2=   130.0   (STD DEV: 4.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100      Gw2=  0.100
   PHI1=  30.0      PHI2=  30.0
   Nc1= 0.0100      Nc2= 0.0100
*** MEAN FS =   1.66 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

   MINIMUM     CRITICAL      CRITICAL
     RI        DIRECTION     ORIENTATIONS

     3.622     -0.500000     STRIKE1= 35.46
                0.707107        DIP1= 53.49
                0.500000     STRIKE2=109.54
                0.000000        DIP2=130.00

     3.251     -0.603553     STRIKE1= 34.67
                0.353553        DIP1= 46.05
                0.603553     STRIKE2=110.33
               -0.382683        DIP2=123.45

     3.173     -0.678058     STRIKE1= 33.27
                0.544895        DIP1= 49.42
                0.453064     STRIKE2=107.84
               -0.195090        DIP2=126.63

     3.142     -0.701715     STRIKE1= 32.99
                0.451099        DIP1= 47.72
                0.468871     STRIKE2=108.02
               -0.290285        DIP2=125.03

     3.140     -0.682466     STRIKE1= 33.39
                0.491966        DIP1= 48.37
                0.456009     STRIKE2=107.76
               -0.290285        DIP2=125.06
```

Table 1

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0    (STD DEV:10.00)
   DIP1=    40.0    (STD DEV:10.00)
   STRIKE2=100.0    (STD DEV:10.00)
   DIP2=   130.0    (STD DEV:10.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100        Gw2=  0.100
   PHI1=  30.0        PHI2=  30.0
   Nc1= 0.0100        Nc2= 0.0100
*** MEAN FS =   1.66 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
1.861	-0.353553	STRIKE1= 38.42
	0.500000	DIP1= 49.30
	0.353553	STRIKE2=106.58
	-0.707107	DIP2=116.84
1.711	-0.461940	STRIKE1= 37.10
	0.653282	DIP1= 51.18
	0.461940	STRIKE2=107.90
	-0.382683	DIP2=123.45
1.673	-0.543184	STRIKE1= 35.91
	0.513280	DIP1= 48.59
	0.543184	STRIKE2=109.09
	-0.382683	DIP2=123.60
1.673	-0.487327	STRIKE1= 36.84
	0.513280	DIP1= 48.59
	0.593809	STRIKE2=109.94
	-0.382683	DIP2=123.60
1.673	-0.487327	STRIKE1= 36.84
	0.513280	DIP1= 48.59
	0.593809	STRIKE2=109.94
	-0.382683	DIP2=123.60

Table 2

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0    (STD DEV: 6.00)
   DIP1=    40.0    (STD DEV: 5.00)
   STRIKE2=130.0    (STD DEV: 5.00)
   DIP2=   150.0    (STD DEV: 4.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100      Gw2=  0.100
   PHI1=  30.0      PHI2=  30.0
   Nc1= 0.0100      Nc2= 0.0100
*** MEAN FS =   1.42 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
2.736	-0.353553	STRIKE1= 40.48
	0.500000	DIP1= 46.40
	0.353553	STRIKE2=134.52
	-0.707107	DIP2=140.95
2.718	-0.603553	STRIKE1= 36.88
	0.270598	DIP1= 43.64
	0.250000	STRIKE2=133.36
	-0.707107	DIP2=140.49
2.609	-0.574830	STRIKE1= 37.37
	0.461940	DIP1= 46.13
	0.384089	STRIKE2=135.10
	-0.555570	DIP2=142.63
2.598	-0.646705	STRIKE1= 36.30
	0.391952	DIP1= 45.27
	0.345671	STRIKE2=134.65
	-0.555570	DIP2=142.53
2.597	-0.592984	STRIKE1= 37.13
	0.427461	DIP1= 45.67
	0.396219	STRIKE2=135.26
	-0.555570	DIP2=142.63

Table 3

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0    (STD DEV:10.00)
   DIP1=    40.0    (STD DEV:10.00)
   STRIKE2=130.0    (STD DEV:10.00)
   DIP2=   150.0    (STD DEV:10.00)
DIP OF SLOPE: 7Q.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100       Gw2=  0.100
   PHI1=  30.0       PHI2=  30.0
   Nc1= 0.0100       Nc2= 0.0100
*** MEAN FS =   1.42 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
1.280	-0.353553	STRIKE1= 40.48
	0.500000	DIP1= 46.40
	0.353553	STRIKE2=134.52
	-0.707107	DIP2=140.95
1.280	-0.353553	STRIKE1= 40.48
	0.500000	DIP1= 46.40
	0.353553	STRIKE2=134.52
	-0.707107	DIP2=140.95
1.280	-0.353553	STRIKE1= 40.48
	0.500000	DIP1= 46.40
	0.353553	STRIKE2=134.52
	-0.707107	DIP2=140.95
1.280	-0.353553	STRIKE1= 40.48
	0.500000	DIP1= 46.40
	0.353553	STRIKE2=134.52
	-0.707107	DIP2=140.95
1.280	-0.332379	STRIKE1= 40.75
	0.474864	DIP1= 46.08
	0.405005	STRIKE2=135.18
	-0.707107	DIP2=140.95

Table 4

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0    (STD DEV: 6.00)
   DIP1=    60.0    (STD DEV: 5.00)
   STRIKE2=100.0    (STD DEV: 5.00)
   DIP2=   130.0    (STD DEV: 4.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100       Gw2=  0.100
   PHI1=  30.0       PHI2=  30.0
   Nc1= 0.0100       Nc2= 0.0100
*** MEAN FS =   1.32 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
1.876	-0.707107	STRIKE1= 37.67
	0.000000	DIP1= 60.00
	0.707107	STRIKE2=107.33
	0.000000	DIP2=130.00
1.595	-0.603553	STRIKE1= 39.94
	0.353553	DIP1= 62.97
	0.603553	STRIKE2=105.06
	-0.382683	DIP2=126.79
1.570	-0.709704	STRIKE1= 38.98
	0.353553	DIP1= 63.00
	0.474209	STRIKE2=104.02
	-0.382683	DIP2=126.75
1.567	-0.761406	STRIKE1= 38.40
	0.277785	DIP1= 62.41
	0.508755	STRIKE2=104.41
	-0.290285	DIP2=127.48
1.561	-0.712048	STRIKE1= 38.96
	0.317197	DIP1= 62.69
	0.528091	STRIKE2=104.48
	-0.336890	DIP2=127.14

Table 5

THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
STRIKE1= 45.0 (STD DEV:10.00)
DIP1= 60.0 (STD DEV:10.00)
STRIKE2=130.0 (STD DEV:10.00)
DIP2= 150.0 (STD DEV:10.00)
DIP OF SLOPE: 70.0
SG OF ROCK: 2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
Gw1= 0.100 Gw2= 0.100
PHI1= 30.0 PHI2= 30.0
Nc1= 0.0100 Nc2= 0.0100
*** MEAN FS = 1.23 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
0.698	-0.353553	STRIKE1= 42.53
	0.500000	DIP1= 63.49
	0.353553	STRIKE2=132.47
	-0.707107	DIP2=145.06
0.670	-0.250000	STRIKE1= 43.32
	0.146447	DIP1= 60.98
	0.250000	STRIKE2=131.68
	-0.923880	DIP2=143.81
0.661	-0.326641	STRIKE1= 42.84
	0.308658	DIP1= 62.04
	0.326641	STRIKE2=132.16
	-0.831470	DIP2=144.50
0.661	-0.326641	STRIKE1= 42.84
	0.308658	DIP1= 62.04
	0.326641	STRIKE2=132.16
	-0.831470	DIP2=144.50
0.661	-0.326641	STRIKE1= 42.84
	0.308658	DIP1= 62.04
	0.326641	STRIKE2=132.16
	-0.831470	DIP2=144.50

Table 6

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 75.0    (STD DEV:10.00)
   DIP1=    40.0    (STD DEV:10.00)
   STRIKE2=100.0    (STD DEV:10.00)
   DIP2=   130.0    (STD DEV:10.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100       Gw2=  0.100
   PHI1=  30.0       PHI2=  30.0
   Nc1= 0.0100       Nc2= 0.0100
*** MEAN FS =   3.79 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
3.905	-0.353553	STRIKE1= 61.19
	0.500000	DIP1= 59.52
	0.353553	STRIKE2=113.81
	-0.707107	DIP2=102.39
3.398	-0.603553	STRIKE1= 54.49
	0.353553	DIP1= 52.02
	0.603553	STRIKE2=120.51
	-0.382683	DIP2=116.99
3.389	-0.543184	STRIKE1= 56.59
	0.513280	DIP1= 57.40
	0.543184	STRIKE2=118.41
	-0.382683	DIP2=117.03
3.370	-0.576143	STRIKE1= 55.58
	0.435514	DIP1= 54.68
	0.576143	STRIKE2=119.42
	-0.382683	DIP2=117.10
3.370	-0.576143	STRIKE1= 55.58
	0.435514	DIP1= 54.68
	0.576143	STRIKE2=119.42
	-0.382683	DIP2=117.10

Table 7

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 75.0   (STD DEV: 6.00)
   DIP1=    40.0   (STD DEV: 5.00)
   STRIKE2=130.0   (STD DEV: 5.00)
   DIP2=   150.0   (STD DEV: 4.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100      Gw2=  0.100
   PHI1=  30.0      PHI2=  30.0
   Nc1= 0.0100      Nc2= 0.0100
*** MEAN FS =   2.16 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

   MINIMUM    CRITICAL     CRITICAL
     RI       DIRECTION    ORIENTATIONS

     5.863    -0.353553    STRIKE1= 65.30
               0.500000       DIP1= 53.71
               0.353553    STRIKE2=139.70
              -0.707107       DIP2=130.61

     5.218    -0.788581    STRIKE1= 52.27
               0.353553       DIP1= 50.19
               0.326641    STRIKE2=139.42
              -0.382683       DIP2=138.97

     5.159    -0.709704    STRIKE1= 55.21
               0.353553       DIP1= 49.86
               0.474209    STRIKE2=143.23
              -0.382683       DIP2=139.33

     5.107    -0.677472    STRIKE1= 56.68
               0.337497       DIP1= 49.13
               0.452673    STRIKE2=142.24
              -0.471397       DIP2=137.25

     5.098    -0.683822    STRIKE1= 56.50
               0.377070       DIP1= 50.20
               0.409867    STRIKE2=141.09
              -0.471397       DIP2=137.25
```

Table 8

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 75.0   (STD DEV:10.00)
   DIP1=    60.0   (STD DEV:10.00)
   STRIKE2=130.0   (STD DEV:10.00)
   DIP2=   130.0   (STD DEV:10.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100       Gw2=  0.100
   PHI1=  30.0       PHI2=  30.0
   Nc1= 0.0100       Nc2= 0.0100
*** MEAN FS =   1.32 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

   MINIMUM    CRITICAL     CRITICAL
     RI       DIRECTION    ORIENTATIONS

     0.923    -0.353553    STRIKE1= 71.74
               0.500000       DIP1= 64.62
               0.353553    STRIKE2=133.26
              -0.707107       DIP2=123.47

     0.830    -0.603553    STRIKE1= 69.99
               0.353553       DIP1= 62.93
               0.603553    STRIKE2=135.01
              -0.382683       DIP2=126.82

     0.830    -0.603553    STRIKE1= 69.99
               0.353553       DIP1= 62.93
               0.603553    STRIKE2=135.01
              -0.382683       DIP2=126.82

     0.830    -0.603553    STRIKE1= 69.99
               0.353553       DIP1= 62.93
               0.603553    STRIKE2=135.01
              -0.382683       DIP2=126.82

     0.830    -0.603553    STRIKE1= 69.99
               0.353553       DIP1= 62.93
               0.603553    STRIKE2=135.01
              -0.382683       DIP2=126.82
```

Table 9

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 75.0    (STD DEV:10.00)
   DIP1=     40.0    (STD DEV:10.00)
   STRIKE2=130.0    (STD DEV:10.00)
   DIP2=    150.0    (STD DEV:10.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100      Gw2=  0.100
   PHI1=  30.0      PHI2=  30.0
   Nc1= 0.0100      Nc2= 0.0100
*** MEAN FS =   2.16 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
2.742	-0.353553	STRIKE1= 65.30
	0.500000	DIP1= 53.71
	0.353553	STRIKE2=139.70
	-0.707107	DIP2=130.61
2.723	-0.461940	STRIKE1= 62.42
	0.270598	DIP1= 47.37
	0.461940	STRIKE2=142.58
	-0.707107	DIP2=130.74
2.630	-0.488852	STRIKE1= 62.14
	0.461940	DIP1= 52.15
	0.488852	STRIKE2=142.86
	-0.555570	DIP2=135.39
2.630	-0.488852	STRIKE1= 62.14
	0.461940	DIP1= 52.15
	0.488852	STRIKE2=142.86
	-0.555570	DIP2=135.39
2.620	-0.478939	STRIKE1= 62.45
	0.427461	DIP1= 51.20
	0.528428	STRIKE2=143.85
	-0.555570	DIP2=135.44

Table 10

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 75.0    (STD DEV:10.00)
   DIP1=    60.0    (STD DEV:10.00)
   STRIKE2=100.0    (STD DEV:10.00)
   DIP2=   130.0    (STD DEV:10.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100       Gw2=  0.100
   PHI1=  30.0       PHI2=  30.0
   Nc1= 0.0100       Nc2= 0.0100
*** MEAN FS =   3.19 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
3.005	-0.353553	STRIKE1= 64.38
	0.500000	DIP1= 75.02
	0.353553	STRIKE2=110.62
	-0.707107	DIP2=108.75
2.620	-0.603553	STRIKE1= 59.19
	0.353553	DIP1= 69.26
	0.603553	STRIKE2=115.81
	-0.382683	DIP2=119.97
2.620	-0.603553	STRIKE1= 59.19
	0.353553	DIP1= 69.26
	0.603553	STRIKE2=115.81
	-0.382683	DIP2=119.97
2.620	-0.603553	STRIKE1= 59.19
	0.353553	DIP1= 69.26
	0.603553	STRIKE2=115.81
	-0.382683	DIP2=119.97
2.620	-0.603553	STRIKE1= 59.19
	0.353553	DIP1= 69.26
	0.603553	STRIKE2=115.81
	-0.382683	DIP2=119.97

Table 11

different reliability index: 3.14 for the case with smaller 1-σ dispersion volume (Table 1), and 1.67 for the case with larger 1-σ dispersion volume (Table 2). The Factor of Safety (calculated for the mean joint orientation angles and the given resistance parameters) is the same for both cases.

It is noticed that sometimes the β value appears to be the same from one search to the next while the critical direction and critical orientations change by a small amount. For example, between the third and fourth searches in Table 2 and between the fourth and fifth searches in Table 4. For such cases, the β value of the successive search is actually slightly smaller than that of the previous search, but the difference is too small (variation in the fourth or higher decimal places) to be revealed in the printout which exhibits 3 decimal places.

In most of the runs, the greatest reduction in the β value occurs between the first and the second search, and becomes quite stable after the third search.

The equal increments given to θ, Ω and ψ do not imply that the solid angles associated with the vectors are the same. This can be more easily visualized in a 3-D situation, where the direction of the search vectors are defined by 2 angles, θ and Ω, e.g., the spherical coordinates used in defining longitude and latitude on the surface of the earth. Clearly, the area covered by one degree of latitude and longitude is much larger near the equator than near the poles.

The error in the calculated β value due to discretization of the search directions has been evaluated by making 30 runs, each composed of 5 nested searches, holding $\underline{m}$, $\underline{C}$, water and resistance parameters constant. For each run, every vector in the first search was generated randomly with θ, Ω and ψ having independent and uniform probability distribution within a range of $\pm 22.5^{\circ}$ from the nominal values. The case used for this purpose is that of Table 4, and portions of the 30 runs are shown in Tables 12-17. Results of these 30 runs are summarized in Fig. 4.2, where the tail of each arrow indicates the value of β obtained in the first search and the head gives the final value. The final run correponds to non-randomized search directions (Table 4). The 30 β values show less than 1% variation, while the angles β_1, γ_1, β_2, γ_2 associated with the critical points on the boundary of the safe region each vary within a range of 1.5°.

Judging from the stability of these calculated β values using randomly modified angles, one may conclude that unevenness and discreteness of the search strategy introduces negligible inaccuracies for the problem at hand. The above statement is also a consequence of the fact that the boundary of the safe region is a smooth surface, as one can see from the plots in Chapter 3.

Fig. 4.3 compares two groups of cases, which differ in the values of the standard deviations. For each pair of points joined by a vertical line, the mean value, $\underline{m}$, and the factor of safety are the same. Clearly, β is not the same due to the differences in the standard deviations.

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0    (STD DEV:10.00)
   DIP1=    40.0    (STD DEV:10.00)
   STRIKE2=130.0    (STD DEV:10.00)
   DIP2=   150.0    (STD DEV:10.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100       Gw2=  0.100
   PHI1=  30.0       PHI2=  30.0
   Nc1= 0.0100       Nc2= 0.0100
*** MEAN FS =   1.42 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
1.392	-0.575536	STRIKE1= 36.99
	0.441569	DIP1= 46.15
	0.030631	STRIKE2=130.43
	-0.687631	DIP2=140.43
1.280	-0.385306	STRIKE1= 40.07
	0.441569	DIP1= 45.65
	0.428625	STRIKE2=135.49
	-0.687631	DIP2=141.20
1.280	-0.385306	STRIKE1= 40.07
	0.441569	DIP1= 45.65
	0.428625	STRIKE2=135.49
	-0.687631	DIP2=141.20
1.280	-0.385306	STRIKE1= 40.07
	0.441569	DIP1= 45.65
	0.428625	STRIKE2=135.49
	-0.687631	DIP2=141.20
1.280	-0.385306	STRIKE1= 40.07
	0.441569	DIP1= 45.65
	0.428625	STRIKE2=135.49
	-0.687631	DIP2=141.20

Table 12

THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0   (STD DEV:10.00)
   DIP1=   40.0   (STD DEV:10.00)
   STRIKE2=130.0   (STD DEV:10.00)
   DIP2=   150.0   (STD DEV:10.00)
DIP OF SLOPE: 70.0
SG OF ROCK: 2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1= 0.100   Gw2= 0.100
   PHI1= 30.0   PHI2= 30.0
   Nc1= 0.0100   Nc2= 0.0100
*** MEAN FS =   1.42 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
1.289	-0.359360	STRIKE1= 40.37
	0.540478	DIP1= 46.97
	0.381717	STRIKE2=134.92
	-0.658054	DIP2=141.52
1.289	-0.359360	STRIKE1= 40.37
	0.540478	DIP1= 46.97
	0.381717	STRIKE2=134.92
	-0.658054	DIP2=141.52
1.289	-0.359360	STRIKE1= 40.37
	0.540478	DIP1= 46.97
	0.381717	STRIKE2=134.92
	-0.658054	DIP2=141.52
1.280	-0.386303	STRIKE1= 40.06
	0.389108	DIP1= 44.98
	0.410337	STRIKE2=135.25
	-0.728689	DIP2=140.68
1.280	-0.386303	STRIKE1= 40.06
	0.389108	DIP1= 44.98
	0.410337	STRIKE2=135.25
	-0.728689	DIP2=140.68

Table 13

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0    (STD DEV:10.00)
   DIP1=    40.0    (STD DEV:10.00)
   STRIKE2=130.0    (STD DEV:10.00)
   DIP2=   150.0    (STD DEV:10.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100       Gw2=  0.100
   PHI1=  30.0       PHI2=  30.0
   Nc1= 0.0100       Nc2= 0.0100
*** MEAN FS =   1.42 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
1.345	-0.454792	STRIKE1= 38.88
	0.542141	DIP1= 47.29
	0.121259	STRIKE2=131.63
	-0.696092	DIP2=140.64
1.298	-0.373769	STRIKE1= 40.15
	0.542141	DIP1= 47.04
	0.286070	STRIKE2=133.71
	-0.696092	DIP2=140.96
1.280	-0.374641	STRIKE1= 40.21
	0.439898	DIP1= 45.63
	0.426133	STRIKE2=135.45
	-0.696092	DIP2=141.09
1.280	-0.374641	STRIKE1= 40.21
	0.439898	DIP1= 45.63
	0.426133	STRIKE2=135.45
	-0.696092	DIP2=141.09
1.280	-0.416804	STRIKE1= 39.67
	0.488874	DIP1= 46.26
	0.389414	STRIKE2=134.98
	-0.660025	DIP2=141.55

Table 14

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0    (STD DEV:10.00)
   DIP1=    40.0    (STD DEV:10.00)
   STRIKE2=130.0    (STD DEV:10.00)
   DIP2=   150.0    (STD DEV:10.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100       Gw2=  0.100
   PHI1=  30.0       PHI2=  30.0
   Nc1= 0.0100       Nc2= 0.0100
*** MEAN FS =   1.42 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
1.336	-0.193440	STRIKE1= 42.42
	0.450897	DIP1= 46.02
	0.283980	STRIKE2=133.79
	-0.823789	DIP2=138.99
1.327	-0.275857	STRIKE1= 41.34
	0.285083	DIP1= 43.78
	0.404972	STRIKE2=135.37
	-0.823789	DIP2=139.07
1.280	-0.383291	STRIKE1= 40.10
	0.474358	DIP1= 46.07
	0.376505	STRIKE2=134.82
	-0.697364	DIP2=141.08
1.280	-0.383291	STRIKE1= 40.10
	0.474358	DIP1= 46.07
	0.376505	STRIKE2=134.82
	-0.697364	DIP2=141.08
1.280	-0.383291	STRIKE1= 40.10
	0.474358	DIP1= 46.07
	0.376505	STRIKE2=134.82
	-0.697364	DIP2=141.08

Table 15

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0    (STD DEV:10.00)
   DIP1=    40.0    (STD DEV:10.00)
   STRIKE2=130.0    (STD DEV:10.00)
   DIP2=   150.0    (STD DEV:10.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100       Gw2=  0.100
   PHI1=  30.0       PHI2=  30.0
   Nc1= 0.0100       Nc2= 0.0100
*** MEAN FS =   1.42 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

   MINIMUM    CRITICAL      CRITICAL
     RI       DIRECTION     ORIENTATIONS

     1.336    -0.195275     STRIKE1= 42.39
               0.329626        DIP1= 44.40
               0.396062     STRIKE2=135.29
              -0.834476        DIP2=138.85

     1.317    -0.331977     STRIKE1= 40.63
               0.329626        DIP1= 44.34
               0.291185     STRIKE2=133.84
              -0.834476        DIP2=139.01

     1.280    -0.343036     STRIKE1= 40.61
               0.420675        DIP1= 45.38
               0.447131     STRIKE2=135.72
              -0.710938        DIP2=140.90

     1.280    -0.316286     STRIKE1= 40.95
               0.473888        DIP1= 46.06
               0.412263     STRIKE2=135.28
              -0.710938        DIP2=140.90

     1.280    -0.316286     STRIKE1= 40.95
               0.473888        DIP1= 46.06
               0.412263     STRIKE2=135.28
              -0.710938        DIP2=140.90
```

Table 16

```
THE MEAN JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0    (STD DEV:10.00)
   DIP1=    40.0    (STD DEV:10.00)
   STRIKE2=130.0    (STD DEV:10.00)
   DIP2=   150.0    (STD DEV:10.00)
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND JOINT STRENGTH PARAMETERS FOR THIS RUN ARE :
   Gw1=  0.100       Gw2=  0.100
   PHI1=  30.0       PHI2=  30.0
   Nc1= 0.0100       Nc2= 0.0100
*** MEAN FS =   1.42 ***

5 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL ORIENTATIONS
1.308	-0.412654	STRIKE1= 39.60
	0.447833	DIP1= 45.86
	0.525660	STRIKE2=136.87
	-0.594007	DIP2=142.23
1.308	-0.412654	STRIKE1= 39.60
	0.447833	DIP1= 45.86
	0.525660	STRIKE2=136.87
	-0.594007	DIP2=142.23
1.289	-0.345281	STRIKE1= 40.55
	0.374716	DIP1= 44.83
	0.439837	STRIKE2=135.67
	-0.739536	DIP2=140.47
1.280	-0.394530	STRIKE1= 39.95
	0.427720	DIP1= 45.47
	0.338361	STRIKE2=134.33
	-0.739536	DIP2=140.54
1.280	-0.394530	STRIKE1= 39.95
	0.427720	DIP1= 45.47
	0.338361	STRIKE2=134.33
	-0.739536	DIP2=140.54

Table 17

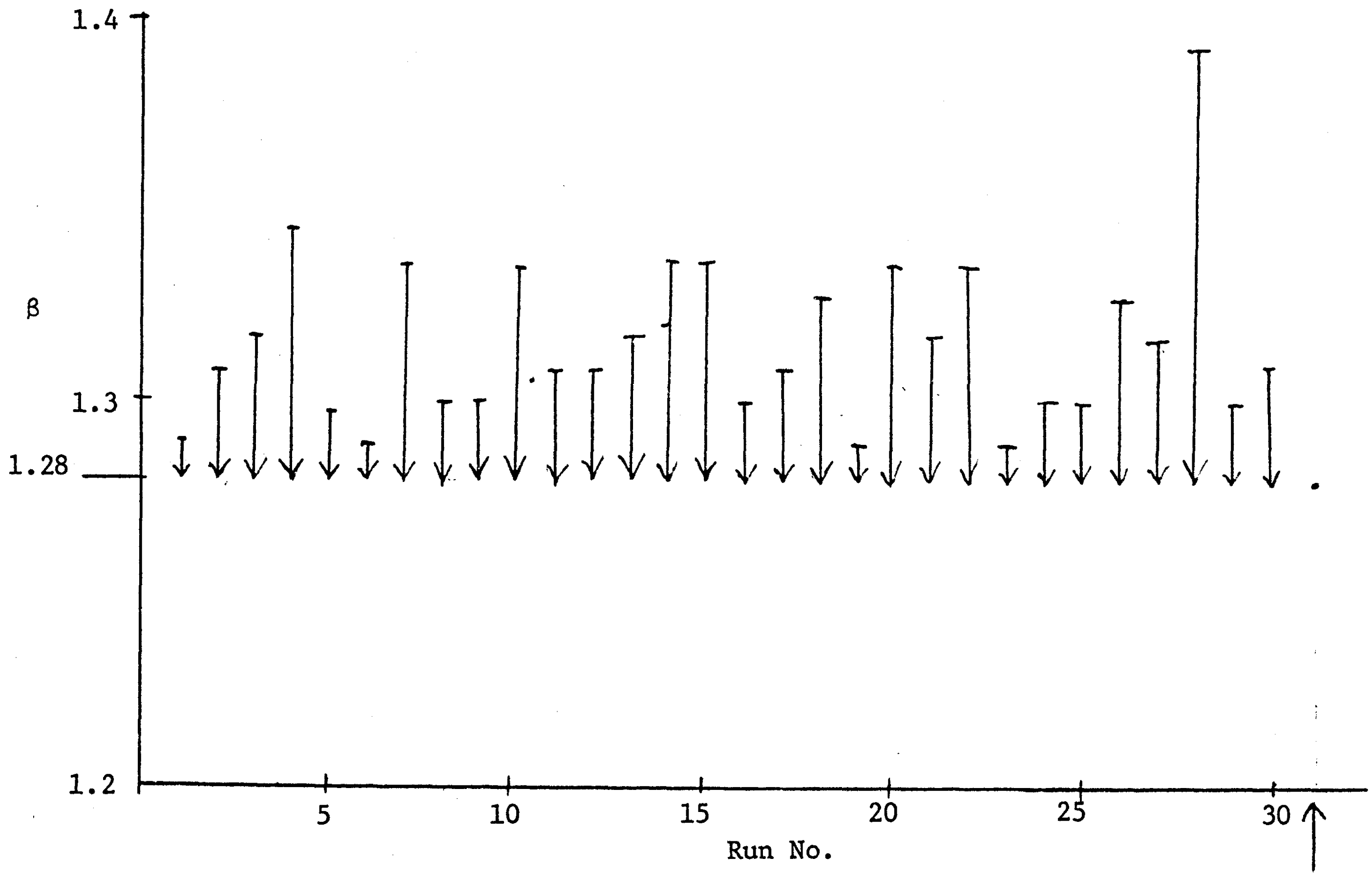
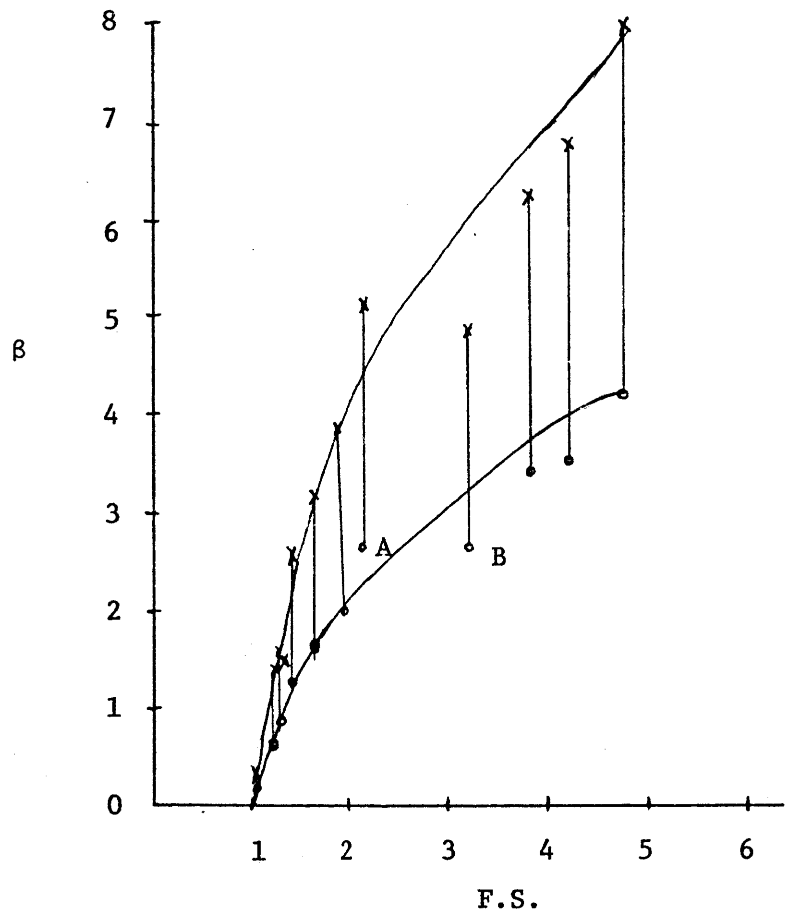


Figure 4.2 Geometric Uncertainty Only. 30 Runs with Randomized Directions During the First Search and Run with Deterministic Directions.

o: cases with $\underline{\sigma} = \sigma_{\beta 1} = 10^{\circ}$, $\sigma_{\beta 2} = 10^{\circ}$, $\sigma_{\gamma 1} = 10^{\circ}$, $\sigma_{\gamma 2} = 10^{\circ}$

x: cases with $\underline{\sigma} = \sigma_{\beta 1} = 6^{\circ}$, $\sigma_{\beta 2} = 5^{\circ}$, $\sigma_{\gamma 1} = 5^{\circ}$, $\sigma_{\gamma 2} = 4^{\circ}$



Same resistance and water parameter for both groups:

$G_{w1} = G_{w2} = 0.1$, $N_{c1} = N_{c2} = 0.01$,

$\phi_1 = \phi_2 = 30^{\circ}$

Figure 4.3 Geometric Uncertainty Only. Reliability Index β vs. F.S. for Two Sets of Cases

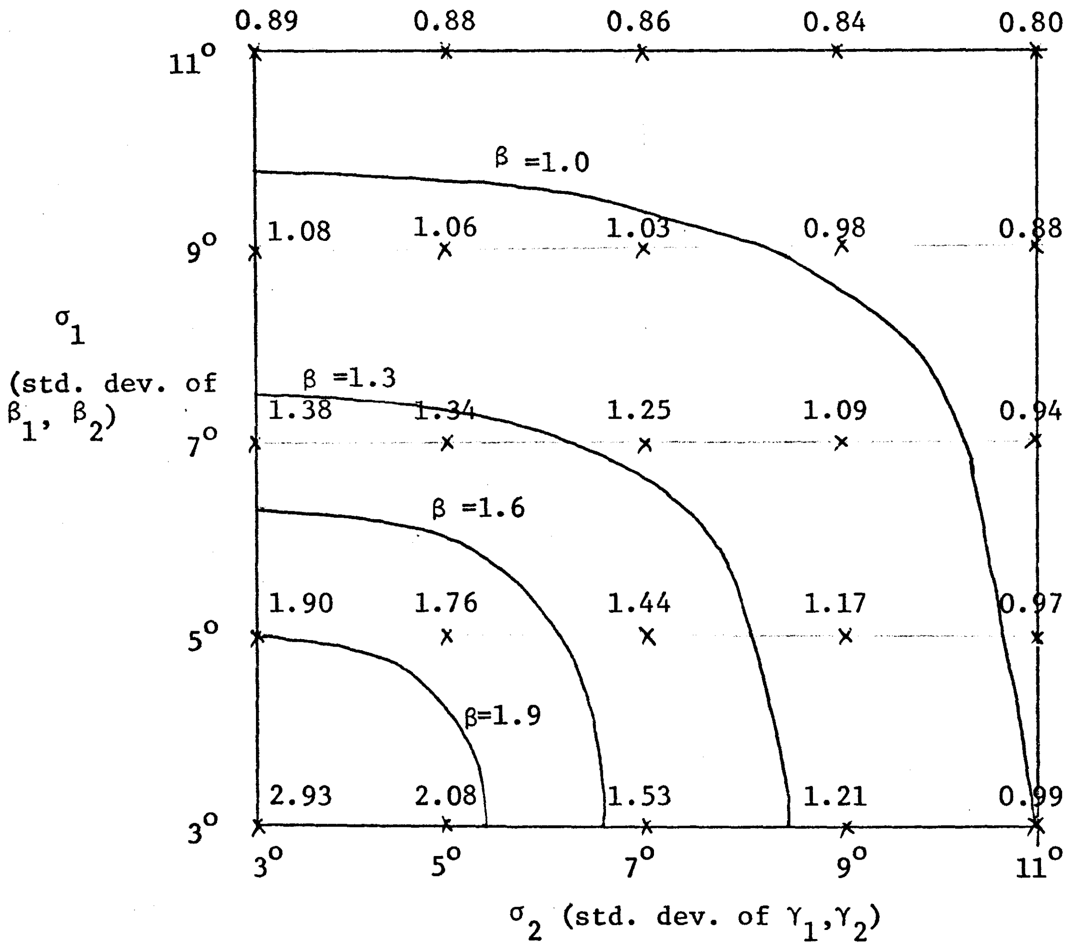
It is interesting that, for given values of standard deviation, the reliability index β does not necessarily increase with the Factor of Safety (e.g. compare cases A and B in Fig. 4.3, and their corresponding computer printout in Tables 10 and 11). This can only happen when the boundary of the safe region is nonlinear.

In all cases of Fig. 4.3, the critical direction is towards a wider angle $(\beta_2-\beta_1)$ and steeper dips γ_1, γ_2 with respect to the mean values. This is consistent with intuition and with plots in Chapter 3.

Figure 4.4 illustrates the variation of β with the standard deviation of the angles β_1, β_2, γ_1, and γ_2. Contour lines on the $\sigma_1\sigma_2$ plane (σ_1 = standard deviation of β_1 and β_2, σ_2 = standard deviation of γ_1 and γ_2) are nearly portions of circular arches.

2. Uncertainty on Resistance and Water Pressure Only

In this section we shall treat cases in which the parameters G_w, ϕ and $n_c = \frac{C_r I}{\rho_r h}$ are uncertain, whereas geometry of slope and wedge are given. In order to reduce the number of uncertain variables, we let $G_{w1} = G_{w2}$, $\phi_1 = \phi_2$, $N_{c1} = N_{c2}$. The search for β is therefore in a 3-D space with only 2 angles, θ and Ω, necessary to define search directions. The procedure is the same as in the previous section, except that it is much faster, not only because there are only three random variables but also because the quantities a_1, a_2, b_1, b_2 (all lengthy functions of joint orientation angles) need to be calculated only once. The number of nested searches for each run is six.



$\underset{\sim}{m}$: $m_{\beta 1} = 45^\circ$, $m_{\gamma 1} = 60^\circ$, $m_{\beta 2} = 100^\circ$, $m_{\gamma 2} = 135^\circ$

$G_{w1} = G_{w2} = 0.100$

$\phi_1 = \phi_2 = 30^\circ$

$N_{c1} = N_{c2} = 0.005$

$F.S.(\underline{m}) = 1.35$

Figure 4.4 Variation of β with standard deviations of strike and dip of the joint planes bounding the wedge.

Once more, the plot in Fig. 4.5 shows that higher Factor of Safety does not necessarily imply higher reliability. Each pair of points joined by a straight line segment in that figure corresponds to the same joint orientations, but to different mean values and standard deviations of G_w, ϕ and N_c.

Considering only the mean values of G_w, $\tan\phi$ and N_c, one might think that cases associated with solid (open) dots in Fig. 4.5 should be safer than the corresponding cases (same joint orientation) associated with crosses. However, Fig. 4.5 shows that this may not be true if one also considers covariances and if safety is measured in terms of the reliability index β. Whether one set of mean values and standard deviations corresponds to higher or lower reliability than another set depends highly on the value of the fixed orientation parameters.

If one decides that β should be at least equal to 1.5, then for the pairs a, b, c and d shown in Fig. 4.5, the cases with apparently higher resistance, weaker water effect and hence also higher F.S.($\underline{m}$) (the solid (open) dot cases) should be rejected as insufficiently safe, while their counterparts (crosses), which appear to be less safe on the basis of their F.S., are acceptable.

Example runs showing the values of β, the critical direction and the critical point of each nested search, are given in Tables 18 to 25.

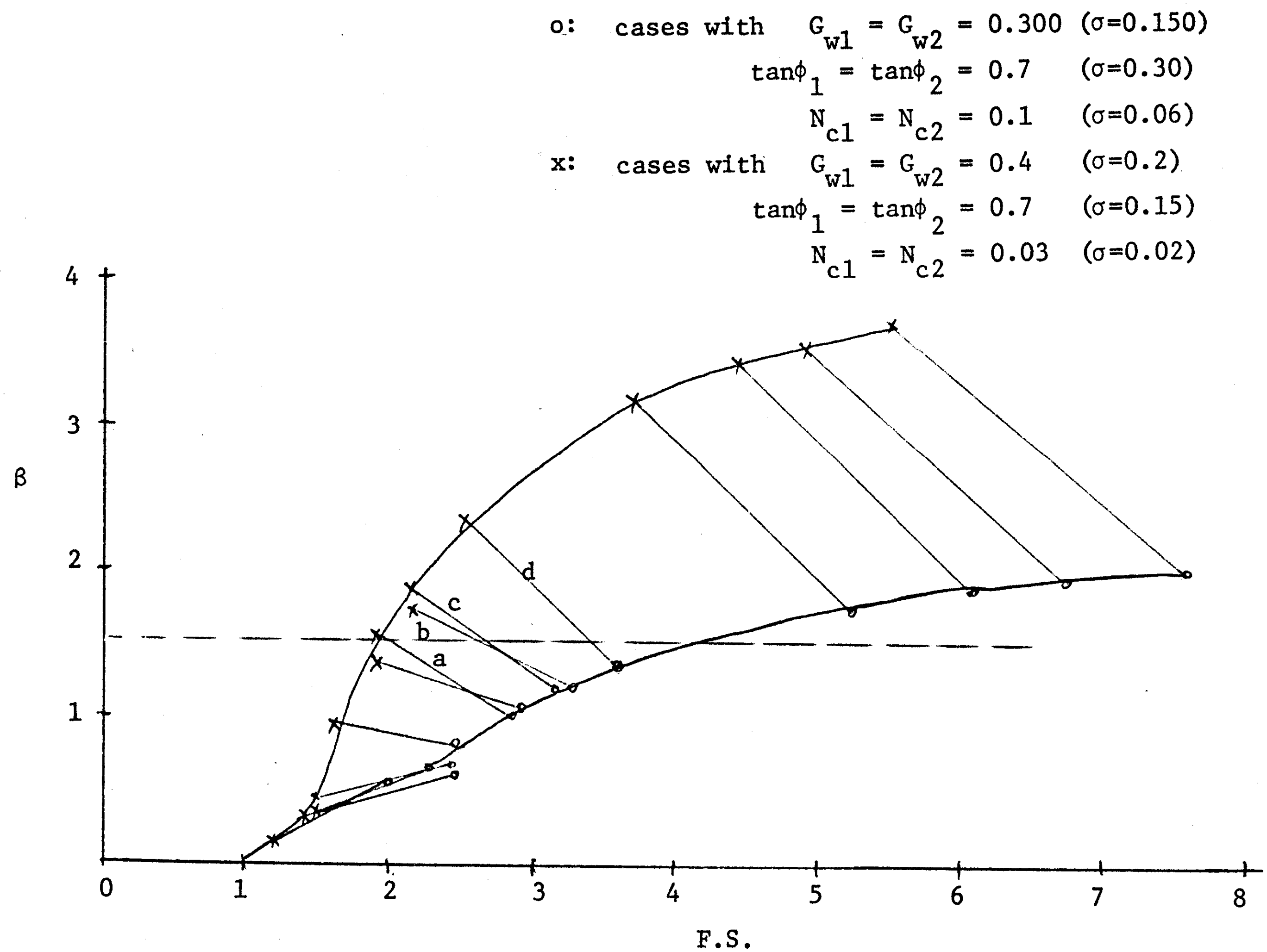


Figure 4.5 β vs. F.S. (Only joint resistance and water parameter uncertainty)

Cf = Coefficient of Friction = tan ϕ


```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0
   DIP1=     40.0
   STRIKE2=100.0
   DIP2=    130.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.300   (STD DEV: 0.150  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.300  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.1000  (STD DEV: 0.0600  RANGE: 1.0000  0.0000 )
*** MEAN FS =   2.86 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

   MINIMUM     CRITICAL      CRITICAL
     RI        DIRECTION     PARAMETERS

    1.215    -0.000000      Gw = 0.300
   (min.FS)  -1.000000      Cf = 0.000
             -0.000000      Nc = 0.1000

    1.051     0.353553      Gw = 0.392
   (min.FS)  -0.853553      Cf = 0.477
             -0.382684      Nc = 0.0001

    1.351    -0.000000      Gw = 0.300
             -0.980785      Cf = 0.310
             -0.195090      Nc = 0.0224

    1.114     0.093797      Gw = 0.330
             -0.952332      Cf = 0.396
             -0.290285      Nc = 0.0073

    1.036     0.092287      Gw = 0.327
             -0.937010      Cf = 0.426
             -0.336890      Nc = 0.0015

    1.036     0.092287      Gw = 0.327
             -0.937010      Cf = 0.426
             -0.336890      Nc = 0.0015
```

Table 18

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0
   DIP1=    40.0
   STRIKE2=100.0
   DIP2=   130.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.400   (STD DEV: 0.200  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.150  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.0300  (STD DEV: 0.0200  RANGE: 1.0000  0.0000 )
*** MEAN FS =   1.90 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

   MINIMUM     CRITICAL      CRITICAL
     RI        DIRECTION     PARAMETERS

    2.574      0.707107      Gw = 0.722
              -0.707107      Cf = 0.378
               0.000000      Nc = 0.0300

    2.552      0.382683      Gw = 0.555
              -0.923880      Cf = 0.327
               0.000000      Nc = 0.0300

    2.524      0.555570      Gw = 0.634
              -0.831470      Cf = 0.349
               0.000000      Nc = 0.0300

    1.546      0.703702      Gw = 0.591
              -0.703702      Cf = 0.509
              -0.098017      Nc = 0.0033

    1.545      0.737383      Gw = 0.603
              -0.668325      Cf = 0.516
              -0.098017      Nc = 0.0030

    1.545      0.737383      Gw = 0.603
              -0.668325      Cf = 0.516
              -0.098017      Nc = 0.0030
```

Table 19

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 75.0
   DIP1=    60.0
   STRIKE2=100.0
   DIP2=   150.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.300   (STD DEV: 0.150  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.300  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.1000  (STD DEV: 0.0600  RANGE: 1.0000  0.0000 )
*** MEAN FS =   6.77 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

   MINIMUM     CRITICAL      CRITICAL
     RI        DIRECTION     PARAMETERS

     2.408    -0.000000      Gw = 0.300
   (min.FS)   -1.000000      Cf = 0.000
              -0.000000      Nc = 0.1000

     2.408    -0.000000      Gw = 0.300
   (min.FS)   -1.000000      Cf = 0.000
              -0.000000      Nc = 0.1000

     1.240     0.191342      Gw = 0.398
   (min.FS)   -0.961940      Cf = 0.208
              -0.195090      Nc = 0.0003

     2.257    -0.000000      Gw = 0.300
              -0.995185      Cf = 0.029
              -0.098017      Nc = 0.0339

     2.004     0.048537      Gw = 0.329
              -0.987985      Cf = 0.113
              -0.146731      Nc = 0.0128

     1.901     0.024180      Gw = 0.314
              -0.984981      Cf = 0.146
              -0.170962      Nc = 0.0039
```

Table 20

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 75.0
   DIP1=    60.0
   STRIKE2=100.0
   DIP2=   150.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.400   (STD DEV: 0.200  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.150  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.0300  (STD DEV: 0.0200  RANGE: 1.0000  0.0000 )
*** MEAN FS =    4.86 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL PARAMETERS
4.354	-0.000000	Gw = 0.400
	-1.000000	Cf = 0.047
	0.000000	Nc = 0.0300
4.354	-0.000000	Gw = 0.400
	-1.000000	Cf = 0.047
	0.000000	Nc = 0.0300
4.351	0.195090	Gw = 0.529
	-0.980785	Cf = 0.050
	0.000000	Nc = 0.0300
4.347	0.098017	Gw = 0.464
	-0.995185	Cf = 0.049
	0.000000	Nc = 0.0300
3.572	0.289935	Gw = 0.560
	-0.955788	Cf = 0.172
	-0.049068	Nc = 0.0029
3.563	0.382222	Gw = 0.615
	-0.922767	Cf = 0.180
	-0.049068	Nc = 0.0024

Table 21

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0
   DIP1=    40.0
   STRIKE2=130.0
   DIP2=   150.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.400   (STD DEV: 0.200  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.150  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.0300  (STD DEV: 0.0200  RANGE: 1.0000  0.0000 )
*** MEAN FS =    1.62 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL PARAMETERS
1.990	0.707107	Gw = 0.649
	-0.707107	Cf = 0.451
	0.000000	Nc = 0.0300
1.990	0.707107	Gw = 0.649
	-0.707107	Cf = 0.451
	0.000000	Nc = 0.0300
1.974	0.555570	Gw = 0.583
	-0.831470	Cf = 0.426
	0.000000	Nc = 0.0300
1.214	0.769288	Gw = 0.569
	-0.631339	Cf = 0.561
	-0.098017	Nc = 0.0085
1.027	0.794514	Gw = 0.548
	-0.589252	Cf = 0.590
	-0.146730	Nc = 0.0026
0.956	0.776740	Gw = 0.534
	-0.606174	Cf = 0.596
	-0.170962	Nc = 0.0006

Table 22

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0
   DIP1=    60.0
   STRIKE2=100.0
   DIP2=   130.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.400   (STD DEV: 0.200   RANGE: 1.000   0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.150   RANGE: 2.000   0.000 )
   Nc1=Nc2= 0.0300  (STD DEV: 0.0200  RANGE: 1.0000  0.0000 )
*** MEAN FS =   1.48 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL PARAMETERS
1.586	0.707107	Gw = 0.598
	-0.707107	Cf = 0.502
	0.000000	Nc = 0.0300
0.406	0.853553	Gw = 0.462
	-0.353553	Cf = 0.674
	-0.382683	Nc = 0.0022
0.406	0.853553	Gw = 0.462
	-0.353553	Cf = 0.674
	-0.382683	Nc = 0.0022
0.379	0.681734	Gw = 0.441
	-0.559485	Cf = 0.666
	-0.471397	Nc = 0.0013
0.376	0.708366	Gw = 0.443
	-0.525360	Cf = 0.668
	-0.471397	Nc = 0.0012
0.356	0.678593	Gw = 0.438
	-0.503279	Cf = 0.672
	-0.534998	Nc = 0.0003

Table 23

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0
   DIP1=    40.0
   STRIKE2=100.0
   DIP2=   150.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.400   (STD DEV: 0.200  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.150  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.0300  (STD DEV: 0.0200  RANGE: 1.0000  0.0000 )
*** MEAN FS =    2.48 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL PARAMETERS
3.357	-0.000000	Gw = 0.400
	-1.000000	Cf = 0.196
	0.000000	Nc = 0.0300
3.256	0.382683	Gw = 0.597
	-0.923880	Cf = 0.224
	0.000000	Nc = 0.0300
3.256	0.382683	Gw = 0.597
	-0.923880	Cf = 0.224
	0.000000	Nc = 0.0300
3.256	0.382683	Gw = 0.597
	-0.923880	Cf = 0.224
	0.000000	Nc = 0.0300
2.551	0.554901	Gw = 0.636
	-0.830468	Cf = 0.347
	-0.049068	Nc = 0.0091
2.302	0.613565	Gw = 0.640
	-0.786210	Cf = 0.392
	-0.073565	Nc = 0.0012

Table 24

THE JOINT ORIENTATIONS FOR THIS RUN ARE:
STRIKE1= 75.0
DIP1= 40.0
STRIKE2=130.0
DIP2= 130.0
DIP OF SLOPE: 70.0
SG OF ROCK: 2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
Gw1=Gw2= 0.400 (STD DEV: 0.200 RANGE: 1.000 0.000)
Cf1=Cf2= 0.700 (STD DEV: 0.150 RANGE: 2.000 0.000)
Nc1=Nc2= 0.0300 (STD DEV: 0.0200 RANGE: 1.0000 0.0000)
*** MEAN FS = 1.89 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

MINIMUM RI	CRITICAL DIRECTION	CRITICAL PARAMETERS
2.567	0.707107	Gw = 0.721
	-0.707107	Cf = 0.379
	0.000000	Nc = 0.0300
2.559	0.382683	Gw = 0.555
	-0.923880	Cf = 0.326
	0.000000	Nc = 0.0300
2.526	0.555570	Gw = 0.634
	-0.831470	Cf = 0.349
	0.000000	Nc = 0.0300
1.521	0.769288	Gw = 0.611
	-0.631339	Cf = 0.527
	-0.098017	Nc = 0.0031
1.520	0.737383	Gw = 0.600
	-0.668325	Cf = 0.519
	-0.098017	Nc = 0.0034
1.396	0.684355	Gw = 0.567
	-0.718800	Cf = 0.525
	-0.122411	Nc = 0.0002

Table 25

In some of the runs, the first few of the 6 nested searches miss the F region. Under such circumstances, the lowest factor of safety encountered is printed below the column "MINIMUM RI", with the bracketed term "(min.FS)" printed to indicate that it is the factor of safety, not the β value. Examples are the first and second search in Table 18, and the first, second and third search in Table 20. It is seen that even though the first few searches may miss the unsafe region F, each successive search will bring the critical direction closer to the F region until finally the F region is hit. For instance in Table 20, the lowest factor of safety encountered decreased from 2.41 in the initial search to 1.24 (quite close to the F boundary which is F.S. = 1) in the third nested search. The next (fourth search) hit the F region, and the value 2.257 is the β value for that search.

One notices that in all cases in Tables 18-25, the final critical direction is towards an increase in the water parameter G_w, and a decrease in both $\tan\phi$ and N_c, as one would expect.

A comparison of the final critical directions between Tables 18 and 19 shows that in Table 18 the critical direction is mainly towards a reduction in $\tan\phi$, while in Table 19 an increase in water effect and a decrease in $\tan\phi$ are both about equally important. This has to do with the different mean value and standard deviations of G_w, ϕ and N_c between Tables 18 and 19.

The two cases corresponding to Tables 18 and 20 have been used to test the robustness of the search algorithm. This was done by randomizing uniformly the initial search directions within a range of $\pm 25^{o}$ from the nominal direction of search, as was done for the cases of only geometric uncertainty. Figs. 4.6 and 4.7 summarize results of these two cases, in each of which 20 runs were made. Also shown is the β value for the case when directions are not randomized (last run of each figure).

In both figures, no matter what critical direction is identified in the first randomized search, the β values and the critical points on the safe region boundary at the end of the sixth search are practically all the same (in the range 1.85 to 2.0 for Fig. 4.6 and 1.02 to 1.06 for Fig. 4.7).

Portions of the 20 randomized runs corresponding to Fig. 4.6 are shown in Tables 26 to 30, and those corresponding to Fig. 4.7, in Tables 31 to 35.

$\beta_1 = 75^\circ \qquad \beta_2 = 100^\circ$

$\gamma_1 = 60^\circ \qquad \gamma_2 = 150^\circ$

$\alpha = 70^\circ$

$G_{w1} = G_{w2} = 0.3 \ (\sigma = 0.15)$

$\tan\phi_1 = \tan\phi_2 = 0.7 \quad (\sigma = 0.03)$

$N_{c1} = N_{c2} = 0.1 \ (\sigma = 0.06)$

F.S.($\underline{m}$) = 6.8

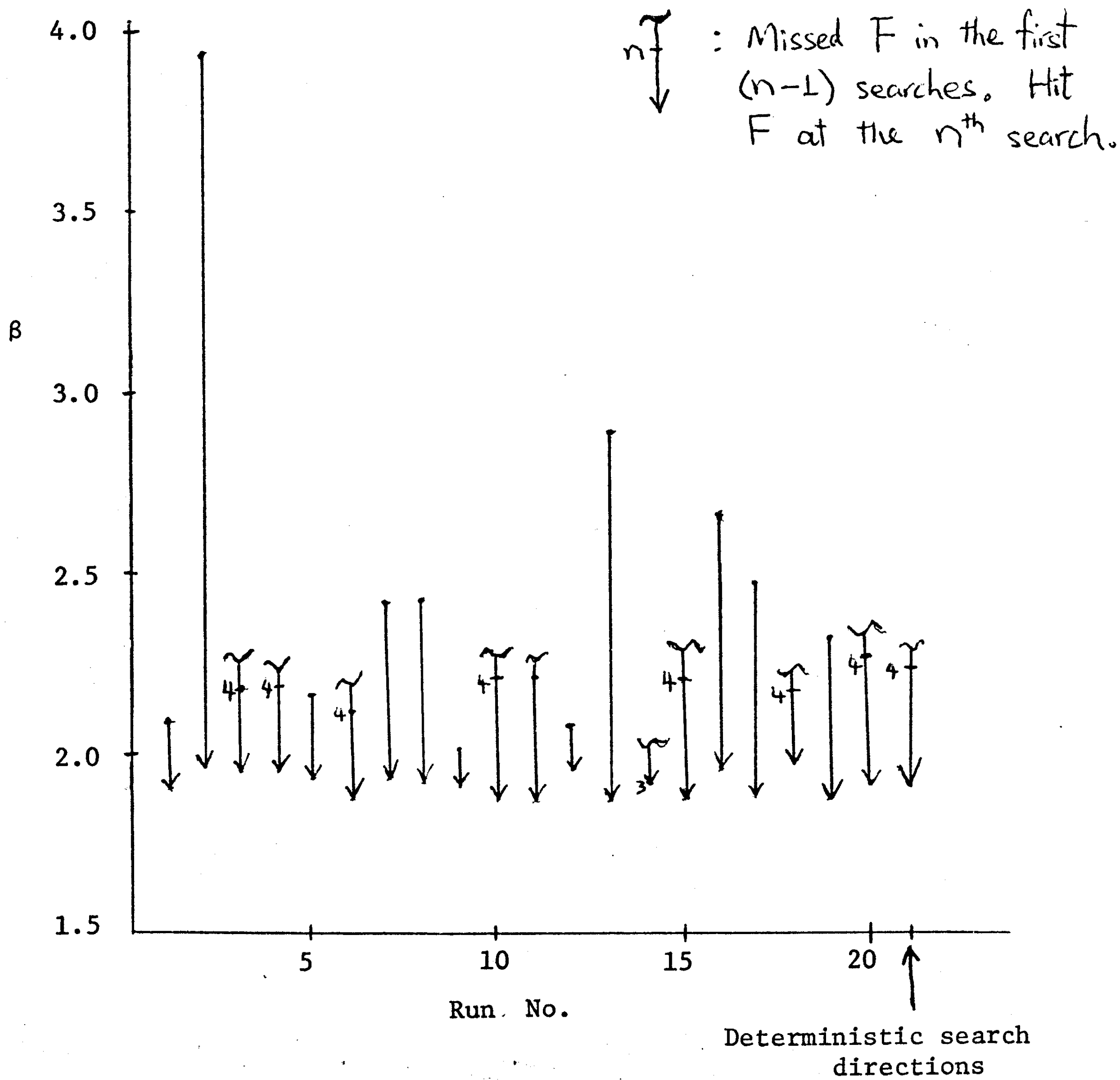


Figure 4.6 Only Joint Resistance and Water Pressure Uncertainty: 20 Runs with Randomized Directions During the First Search and Run with Deterministic Directions - Case 1

$\alpha = 70^{o}$

$\beta_1 = 45^{o}$ $\beta_2 = 100^{o}$

$\gamma_1 = 40^{o}$ $\gamma_2 = 130^{o}$

$G_{w1} = G_{w2} = 0.3$ (=0.15)

$\tan_{1} = \tan_{2} = 0.7$ (=0.3)

$N_{c1} = N_{c2} = 0.1$ (=0.06)

F.S.(m) = 2.86

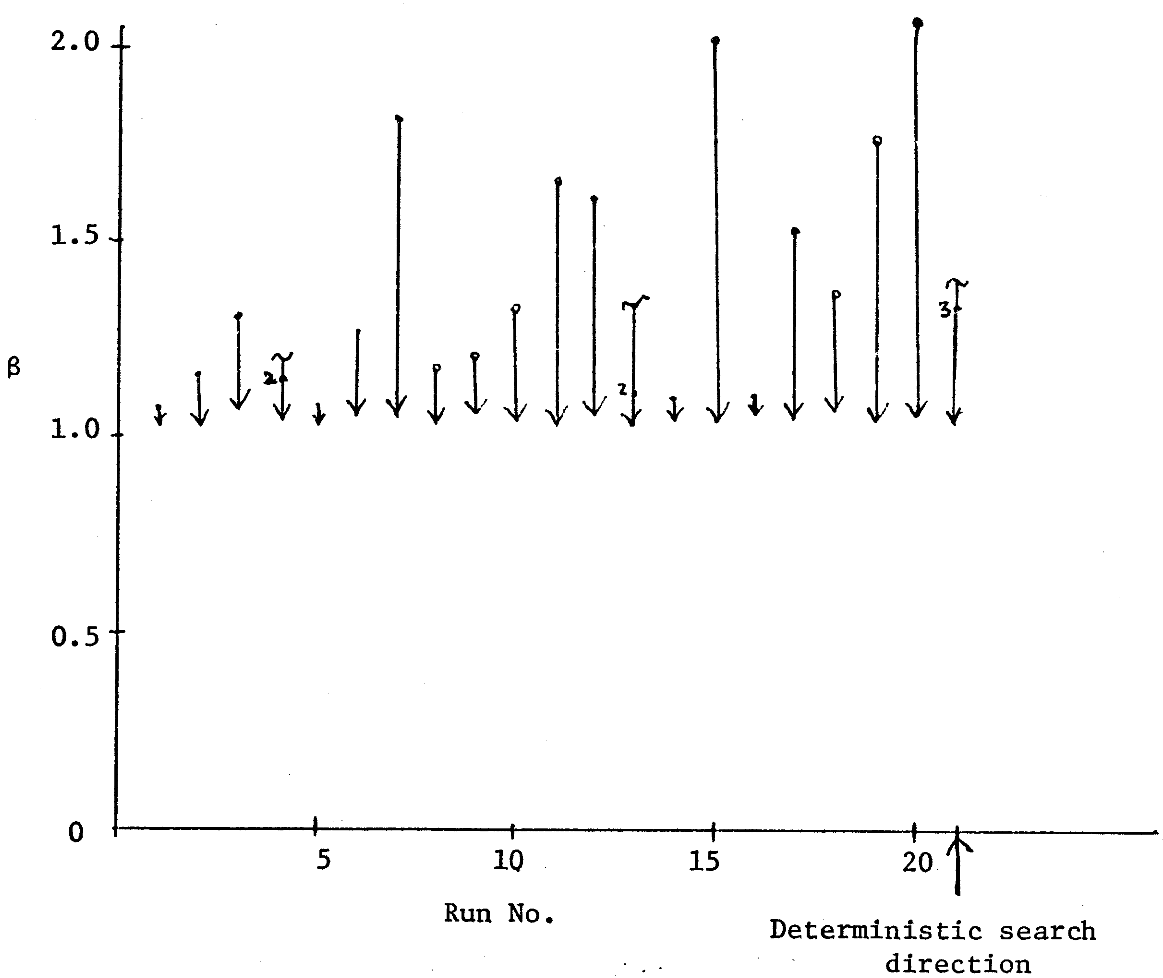


Figure 4.7 Only Joint Resistance and Water Pressure Uncertainty: 20 Runs with Randomized Directions During the First Search and Run with Deterministic Directions - Case 2

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 75.0
   DIP1=    60.0
   STRIKE2=100.0
   DIP2=   150.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.300   (STD DEV: 0.150   RANGE: 1.000   0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.300   RANGE: 2.000   0.000 )
   Nc1=Nc2= 0.1000  (STD DEV: 0.0600  RANGE: 1.0000  0.0000 )
*** MEAN FS =    6.77 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

   MINIMUM     CRITICAL      CRITICAL
     RI        DIRECTION     PARAMETERS

     3.937      0.709621     Gw = 0.959
               -0.700871     Cf = 0.050
               -0.072233     Nc = 0.0330

     3.937      0.709621     Gw = 0.959
               -0.700871     Cf = 0.050
               -0.072233     Nc = 0.0330

     3.195      0.559253     Gw = 0.767
               -0.825844     Cf = 0.010
               -0.072233     Nc = 0.0396

     1.942      0.198255     Gw = 0.412
               -0.965357     Cf = 0.154
               -0.169647     Nc = 0.0040

     1.906      0.054462     Gw = 0.331
               -0.983999     Cf = 0.146
               -0.169647     Nc = 0.0044

     1.906      0.030297     Gw = 0.317
               -0.985039     Cf = 0.145
               -0.169647     Nc = 0.0044
```

Table 26

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 75.0
   DIP1=    60.0
   STRIKE2=100.0
   DIP2=   150.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.300   (STD DEV: 0.150   RANGE: 1.000   0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.300   RANGE: 2.000   0.000 )
   Nc1=Nc2= 0.1000  (STD DEV: 0.0600  RANGE: 1.0000  0.0000 )
*** MEAN FS =    6.77 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL PARAMETERS
2.190	0.043805	Gw = 0.331
(min.FS)	-0.998953	Cf = 0.001
	-0.013167	Nc = 0.0908
2.164	0.422754	Gw = 0.626
(min.FS)	-0.906149	Cf = 0.000
	-0.013166	Nc = 0.0898
1.452	0.042851	Gw = 0.320
(min.FS)	-0.977193	Cf = 0.233
	-0.207986	Nc = 0.0006
2.183	0.043538	Gw = 0.328
	-0.992854	Cf = 0.054
	-0.111111	Nc = 0.0277
1.947	0.043247	Gw = 0.325
	-0.986211	Cf = 0.132
	-0.159741	Nc = 0.0079
1.946	0.019031	Gw = 0.311
	-0.986975	Cf = 0.131
	-0.159741	Nc = 0.0079

Table 27

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 75.0
   DIP1=    60.0
   STRIKE2=100.0
   DIP2=   150.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.300   (STD DEV: 0.150  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.300  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.1000  (STD DEV: 0.0600  RANGE: 1.0000  0.0000 )
*** MEAN FS =   6.77 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL PARAMETERS
1.444	0.116154	Gw = 0.355
(min.FS)	-0.971221	Cf = 0.236
	-0.207936	Nc = 0.0007
1.444	0.116154	Gw = 0.355
(min.FS)	-0.971221	Cf = 0.236
	-0.207936	Nc = 0.0007
1.444	0.116154	Gw = 0.355
(min.FS)	-0.971221	Cf = 0.236
	-0.207936	Nc = 0.0007
2.182	0.020725	Gw = 0.313
	-0.993598	Cf = 0.054
	-0.111060	Nc = 0.0278
1.947	0.020586	Gw = 0.312
	-0.986953	Cf = 0.131
	-0.159690	Nc = 0.0079
1.947	0.020586	Gw = 0.312
	-0.986953	Cf = 0.131
	-0.159690	Nc = 0.0079

Table 28

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 75.0
   DIP1=    60.0
   STRIKE2=100.0
   DIP2=   150.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.300   (STD DEV: 0.150  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.300  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.1000  (STD DEV: 0.0600  RANGE: 1.0000  0.0000 )
*** MEAN FS =   6.77 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL PARAMETERS
2.086	-0.212192	Gw = 0.171
	-0.966140	Cf = 0.112
	-0.146793	Nc = 0.0107
2.034	0.173686	Gw = 0.404
	-0.973799	Cf = 0.119
	-0.146794	Nc = 0.0124
2.006	-0.019631	Gw = 0.288
	-0.988972	Cf = 0.111
	-0.146794	Nc = 0.0126
2.006	-0.019631	Gw = 0.288
	-0.988972	Cf = 0.111
	-0.146794	Nc = 0.0126
2.003	0.028920	Gw = 0.317
	-0.988744	Cf = 0.112
	-0.146794	Nc = 0.0127
1.900	0.028806	Gw = 0.316
	-0.984846	Cf = 0.147
	-0.171025	Nc = 0.0039

Table 29

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 75.0
   DIP1=     60.0
   STRIKE2=100.0
   DIP2=    150.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.300   (STD DEV: 0.150  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.300  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.1000  (STD DEV: 0.0600  RANGE: 1.0000  0.0000 )
*** MEAN FS =    6.77 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL PARAMETERS
2.879	0.653946	Gw = 0.760
	-0.744729	Cf = 0.177
	-0.133166	Nc = 0.0064
2.077	-0.064193	Gw = 0.260
	-0.989013	Cf = 0.090
	-0.133167	Nc = 0.0178
2.077	-0.064193	Gw = 0.260
	-0.989013	Cf = 0.090
	-0.133167	Nc = 0.0178
2.067	0.033056	Gw = 0.320
	-0.990542	Cf = 0.091
	-0.133167	Nc = 0.0182
1.859	0.032798	Gw = 0.318
	-0.982818	Cf = 0.161
	-0.181637	Nc = 0.0003
1.859	0.032798	Gw = 0.318
	-0.982818	Cf = 0.161
	-0.181637	Nc = 0.0003

Table 30

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0
   DIP1=    40.0
   STRIKE2=100.0
   DIP2=   130.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.300   (STD DEV: 0.150  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.300  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.1000  (STD DEV: 0.0600  RANGE: 1.0000  0.0000 )
*** MEAN FS =    2.86 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

   MINIMUM     CRITICAL      CRITICAL
     RI       DIRECTION     PARAMETERS

    1.008      0.762670     Gw = 0.554
   (min.FS)  -0.575439     Cf = 0.508
              -0.295304     Nc = 0.0016

    1.106      0.132392     Gw = 0.342
              -0.946186     Cf = 0.402
              -0.295304     Nc = 0.0069

    1.106      0.132392     Gw = 0.342
              -0.946186     Cf = 0.402
              -0.295304     Nc = 0.0069

    1.106      0.132392     Gw = 0.342
              -0.946186     Cf = 0.402
              -0.295304     Nc = 0.0069

    1.034      0.175735     Gw = 0.351
              -0.923185     Cf = 0.434
              -0.341828     Nc = 0.0014

    1.032      0.153026     Gw = 0.344
              -0.927220     Cf = 0.432
              -0.341828     Nc = 0.0013
```

Table 31

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0
   DIP1=    40.0
   STRIKE2=100.0
   DIP2=   130.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.300   (STD DEV: 0.150  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.300  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.1000  (STD DEV: 0.0600  RANGE: 1.0000  0.0000 )
*** MEAN FS =   2.86 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL PARAMETERS
1.762	-0.520948	Gw = 0.060
	-0.831569	Cf = 0.317
	-0.192630	Nc = 0.0112
1.369	0.219643	Gw = 0.387
	-0.956374	Cf = 0.322
	-0.192630	Nc = 0.0238
1.355	0.028843	Gw = 0.312
	-0.980847	Cf = 0.309
	-0.192630	Nc = 0.0231
1.119	0.121841	Gw = 0.339
	-0.949883	Cf = 0.396
	-0.287884	Nc = 0.0078
1.040	0.073888	Gw = 0.322
	-0.939485	Cf = 0.424
	-0.334527	Nc = 0.0016
1.040	0.073888	Gw = 0.322
	-0.939485	Cf = 0.424
	-0.334527	Nc = 0.0016

Table 32

THE JOINT ORIENTATIONS FOR THIS RUN ARE:
STRIKE1= 45.0
DIP1= 40.0
STRIKE2=100.0
DIP2= 130.0
DIP OF SLOPE: 70.0
SG OF ROCK: 2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
Gw1=Gw2= 0.300 (STD DEV: 0.150 RANGE: 1.000 0.000)
Cf1=Cf2= 0.700 (STD DEV: 0.300 RANGE: 2.000 0.000)
Nc1=Nc2= 0.1000 (STD DEV: 0.0600 RANGE: 1.0000 0.0000)
*** MEAN FS = 2.86 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

MINIMUM RI	CRITICAL DIRECTION	CRITICAL PARAMETERS
1.156	0.216232	Gw = 0.371
	-0.936921	Cf = 0.393
	-0.274632	Nc = 0.0100
1.156	0.216232	Gw = 0.371
	-0.936921	Cf = 0.393
	-0.274632	Nc = 0.0100
1.147	0.029293	Gw = 0.310
	-0.961103	Cf = 0.382
	-0.274632	Nc = 0.0090
1.145	0.123357	Gw = 0.341
	-0.953604	Cf = 0.386
	-0.274632	Nc = 0.0097
1.060	0.075254	Gw = 0.323
	-0.943921	Cf = 0.416
	-0.321482	Nc = 0.0032
1.025	0.074604	Gw = 0.322
	-0.935772	Cf = 0.430
	-0.344624	Nc = 0.0005

Table 33

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0
   DIP1=    40.0
   STRIKE2=100.0
   DIP2=   130.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.300   (STD DEV: 0.150  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.300  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.1000  (STD DEV: 0.0600  RANGE: 1.0000  0.0000 )
*** MEAN FS =   2.86 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :

   MINIMUM    CRITICAL     CRITICAL
     RI       DIRECTION    PARAMETERS

    1.307      0.116154    Gw = 0.344
              -0.971221    Cf = 0.329
              -0.207936    Nc = 0.0206

    1.307      0.116154    Gw = 0.344
              -0.971221    Cf = 0.329
              -0.207936    Nc = 0.0206

    1.307      0.116154    Gw = 0.344
              -0.971221    Cf = 0.329
              -0.207936    Nc = 0.0206

    1.092      0.113174    Gw = 0.335
              -0.946308    Cf = 0.406
              -0.302809    Nc = 0.0058

    1.091      0.066605    Gw = 0.321
              -0.950721    Cf = 0.403
              -0.302809    Nc = 0.0055

    1.053      0.089189    Gw = 0.327
              -0.941116    Cf = 0.419
              -0.326107    Nc = 0.0027
```

Table 34

```
THE JOINT ORIENTATIONS FOR THIS RUN ARE:
   STRIKE1= 45.0
   DIP1=    40.0
   STRIKE2=100.0
   DIP2=   130.0
DIP OF SLOPE: 70.0
SG OF ROCK:  2.56
THE WATER AND RESISTANCE PARAMETERS ARE :
   Gw1=Gw2= 0.300   (STD DEV: 0.150  RANGE: 1.000  0.000 )
   Cf1=Cf2= 0.700   (STD DEV: 0.300  RANGE: 2.000  0.000 )
   Nc1=Nc2= 0.1000  (STD DEV: 0.0600  RANGE: 1.0000  0.0000 )
*** MEAN FS =   2.86 ***

6 SUCCESSIVE SEARCHES WITH DIMINISHING SEARCH RANGE
(RANGE= 360, 180, 90, 45, 22.5, 11.25 DEGREES RESPECTIVELY)
RESULTS OF EACH SEARCH :
```

MINIMUM RI	CRITICAL DIRECTION	CRITICAL PARAMETERS
1.650	0.758073	Gw = 0.564
	-0.591062	Cf = 0.494
	-0.275627	Nc = 0.0039
1.143	0.118095	Gw = 0.339
	-0.953983	Cf = 0.387
	-0.275627	Nc = 0.0095
1.143	0.118095	Gw = 0.339
	-0.953983	Cf = 0.387
	-0.275626	Nc = 0.0095
1.143	0.118095	Gw = 0.339
	-0.953983	Cf = 0.387
	-0.275626	Nc = 0.0095
1.059	0.070056	Gw = 0.321
	-0.943987	Cf = 0.416
	-0.322461	Nc = 0.0030
1.024	0.092394	Gw = 0.327
	-0.933824	Cf = 0.431
	-0.345594	Nc = 0.0005

Table 35

CHAPTER 5

SUMMARY AND CONCLUSIONS

A model of wedge stability based on limit equilibrium has been proposed. The associated Factor of Safety against sliding along the line of intersection is an explicit and relatively simple function of joint orientation angles, height of wedge, slope inclination and water and resistance parameters. A computer program has been developed which calculates the second moment reliability index β, for cases with only geometric uncertainties and with only water parameter and resistance uncertainties.

The following general conclusions can be drawn:

1. For given resistance and water parameters, the Factor of Safety of a wedge formed by 2 intersecting joint planes decreases as the angle $(\beta_2-\beta_1)$ increases and as the dips steepen, provided daylighting is still possible.
2. For any combinations of the dips within the range $0 < \gamma_1 \leq 90^o$, $90^o \leq \gamma_2 < 180^o$, sliding will be along the line of intersection of the two joint planes, provided $(\beta_2-\beta_1) \leq 90^o$. For $(\beta_2-\beta_1) > 90^o$, there are certain combinations of dips which will lead to sliding along one plane only. For $\beta_2-\beta_1 \simeq 180^o$, sliding along the intersection can only be realized if the two joint planes are equally steep.
3. From results in Chapter 4 on the second-moment reliability index β,

uncertainties associated with the water and resistance parameters are in general more critical than those associated with joint orientation angles.

The reliability index has been calculated by assuming no correlation or perfect correlation between the random variables and by separately testing joint orientation uncertainty and resistance and water parameter uncertainty. A possible and relatively simple extension of the study would be to take correlation into account and to increase the number of random variables that can be considered simultaneously.

Since consequences of wedge failure depend on the volume of the moving rock body, another possible area of further research is to make reliability comparisons while also accounting for wedge volume.

APPENDIX A

KINEMATIC REQUIREMENT FOR SLIDING ALONG THE LINE OF INTERSECTION OF TWO JOINT PLANES

The requirement in the title of this appendix can be stated as: the line of intersection must be able to surface both on the slope (point O in Fig. A.1) and on the crest (point B).

Given a horizontal crest and a slope inclination α, this is the same as requiring that the inclination, ε_x, of the plane PQRS be greater than zero and less than α and that the line PQ belong to the slope plane.

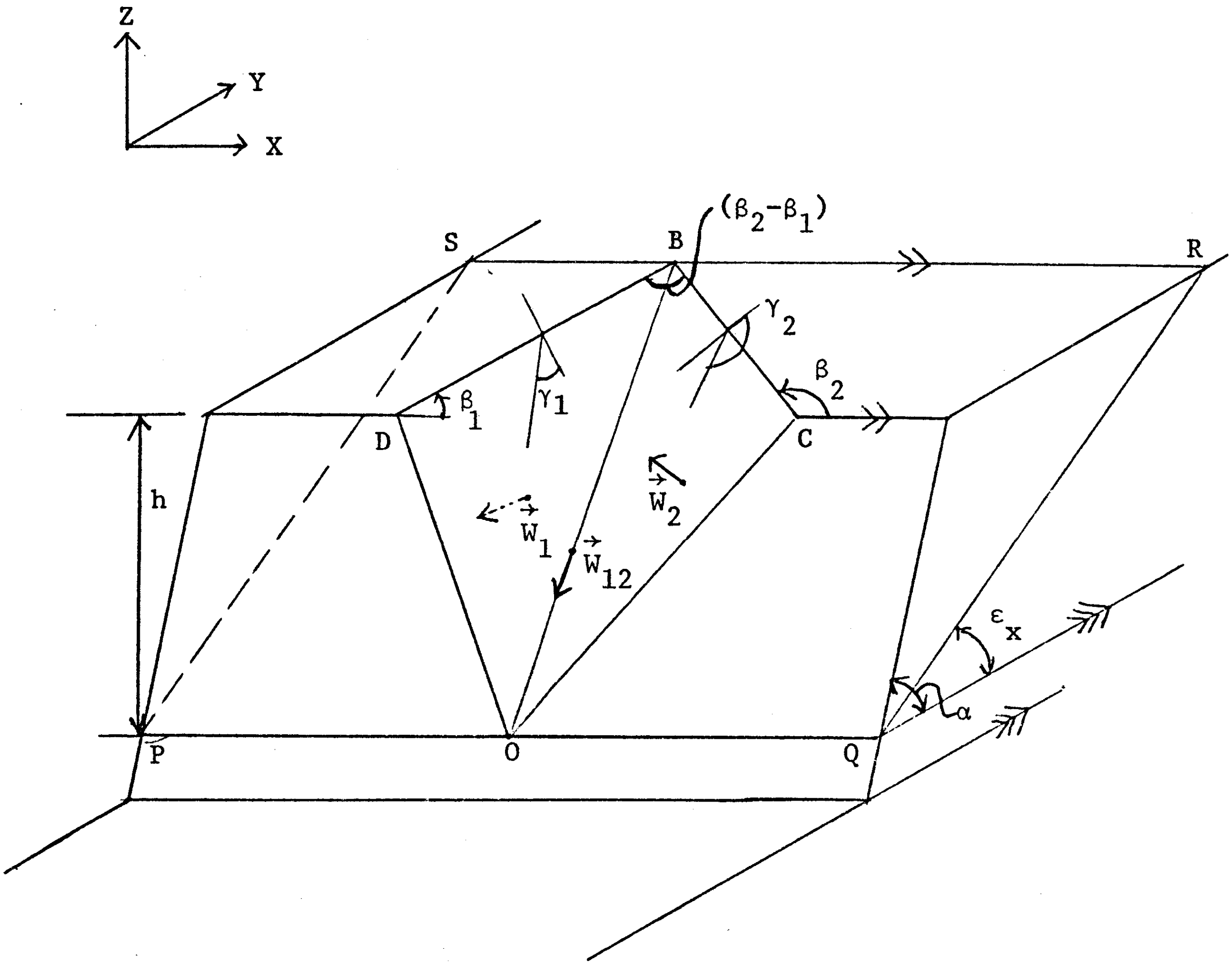
Since BO lies on PQRS, ε_x is the arctangent of $\frac{X_{12z}}{X_{12y}}$, where X_{12z} and X_{12y} are the Z and Y components respectively of a vector X_{12} which points in the direction BO,

$$\varepsilon_x = \tan^{-1} \frac{X_{12z}}{X_{12y}}$$

As shown in Hendron, Cording, Aiyer (1971), the vector $\vec{X}_{12}$ is given by the cross-product:

$$\vec{X}_{12} = \vec{W}_2 \times \vec{W}_1$$

where $\vec{W}_2$ is a unit normal vector to plane 2 (triangle BCO) and points toward the wedge, and $\vec{W}_1$ is a unit normal vector to plane 1 (triangle BDO) and points away from the wedge.



- Plane PQRS contains the line of intersection BO and strikes parallel to the slope.
- $\vec{W}_{12} = \dfrac{\vec{X}_{12}}{|X_{12}|}$ where X_{12} is a vector along BO.

$= \vec{X}_{12}/\sin\psi$ (see Fig. A.2 and Eq. A.7)

Figure A.1 Kinematic requirement for sliding.

$$\vec{W}_2 = \hat{i}(-\sin\gamma_2\sin\beta_2) + \hat{j}(\sin\gamma_2\cos\beta_2) + \hat{k}(-\cos\gamma_2) \qquad (A.1)$$

$$\vec{W}_1 = \hat{i}(-\sin\gamma_1\sin\beta_1) + \hat{j}(\sin\gamma_1\cos\beta_1) + \hat{k}(-\cos\gamma_1) \qquad (A.2)$$

Therefore,

$$X_{12x} = \cos\beta_1\sin\gamma_1\cos\gamma_2 - \cos\beta_2\cos\gamma_1\sin\gamma_2 \qquad (A.3)$$

$$X_{12y} = \sin\beta_1\sin\gamma_1\cos\gamma_2 - \sin\beta_2\cos\gamma_1\sin\gamma_2 \qquad (A.4)$$

$$X_{12z} = \sin(\beta_1-\beta_2)\sin\gamma_1\sin\gamma_2 \qquad (A.5)$$

Using these results, the kinematic requirement becomes

$$0 < \tan^{-1}\left\{\frac{\sin(\beta_2-\beta_1)}{\sin\beta_2\cot\gamma_1 - \sin\beta_1\cot\gamma_2}\right\} < \alpha \qquad (A.6)$$

For later purposes, we calculate also the components of a unit vector along the line of intersection. Call this vector $\vec{W}_{12}$. Then

$$\vec{W}_{12} = \frac{\vec{X}_{12}}{|\vec{X}_{12}|} = \frac{\vec{X}_{12}}{|\vec{W}_2||\vec{W}_1|\sin\psi}$$

where ψ = angle between $\vec{W}_2$ and $\vec{W}_1$

= dihedral angle of wedge (see Fig. A2).

Since $\vec{W}_1$ and $\vec{W}_2$ are unit vectors, $|\vec{W}_2||\vec{W}_1| = 1$ and

$$\vec{W}_{12} = \vec{X}_{12}/\sin\psi$$

where $\sin\psi = \sqrt{1 - \cos^2\psi}$

$$\cos\psi = \frac{\vec{W}_2 \cdot \vec{W}_1}{|\vec{W}_2|\ |\vec{W}_1|}$$

$$= \vec{W}_2 \cdot \vec{W}_1$$

$$= \sin\gamma_1 \sin\gamma_2 \cos(\beta_2 - \beta_1) + \cos\gamma_1 \cos\gamma_2$$

Therefore,

$$\vec{W}_{12} = \frac{\vec{X}_{12}}{\sin\psi} \qquad \text{(A.7)}$$

where $\sin\psi = \sqrt{1-[\sin\gamma_1 \sin\gamma_2 \cos(\beta_2-\beta_1)+\cos\gamma_1 \cos\gamma_2]^2}$

APPENDIX B

AREAS OF BOUNDING TRIANGLES AND VOLUME OF WEDGE

To express the Factor of Safety directly in terms of joint orientation angles, slope inclination and height of wedge, it is first necessary to have expressions for the areas of bounding triangles and volume of wedge. This appendix shows how these expressions (Eq. B.2, B.3, B.6) are obtained. They are needed in Chapter 3.

The expressions for A_1, A_2 and V were initially obtained using vector analysis. For instance

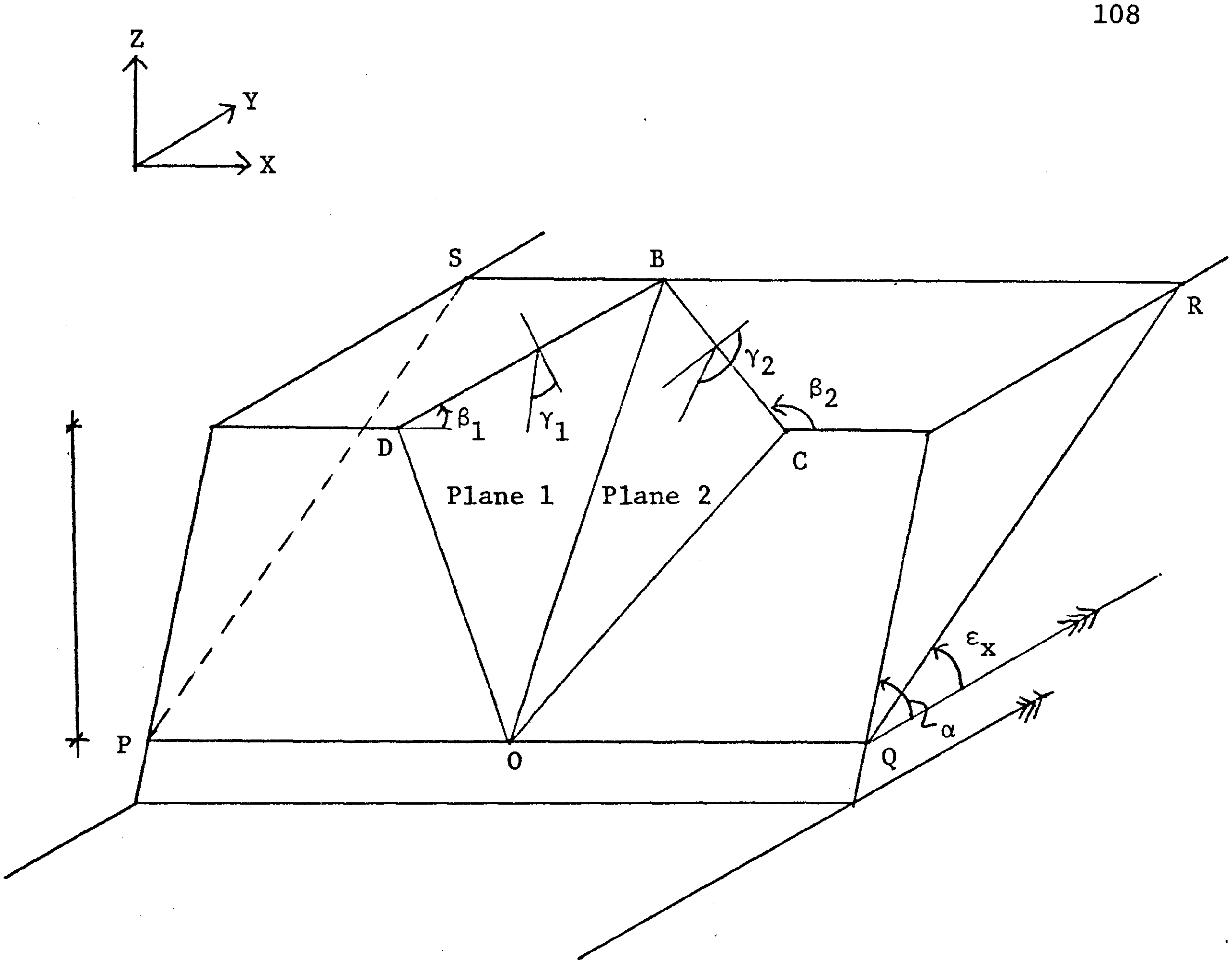
$$A_1 = \frac{1}{2} |\vec{OD} \times \vec{OB}|$$

Many tedious algebraic manipulations were involved in condensing the expressions from vectorial cross-products and dot-products. The condensed expressions have been used herein and checked by direct geometrical argument.

In Fig. B.1, PQRS is a plane that contains the line of intersection BO, and that strikes parallel to the slope.

Denote by $d_{p,KL}$ the perpendicular distance from a point p to a line KL, and by KL the length of the segment from K to L. Then the area of triangle BOD is:

$$A_1 = \frac{1}{2} \cdot BD \cdot d_{O,BD}$$



$$A_1 = \text{Area BDO} = \frac{1}{2}h^2 \frac{(\cot\varepsilon_x - \cot\alpha)}{\sin\beta_1 \sin\gamma_1}$$

$$A_2 = \text{Area BCO} = \frac{1}{2}h^2 \frac{(\cot\varepsilon_x - \cot\alpha)}{\sin\beta_2 \sin\gamma_2}$$

$$V = \text{Volume of tetrahedron BDOCB}$$

$$= \frac{1}{6}h^3 \ (\cot\beta_1 - \cot\beta_2)(\cot\varepsilon_x - \cot\alpha)^2$$

Figure B.1 Area of bounding planes and Volume of wedge.

The distance $d_{0,BD}$ can be calculated as:

$$d_{0,BD} = h/\sin\gamma_1$$

and BD can be found from the following developments. By geometry,

$$\sin\beta_1 = \frac{d_{B,DC}}{BD}$$

Let OZ be the vertical line through point 0, then

$$d_{B,DC} = d_{B,OZ} - d_{DC,OZ}$$

Dividing by h, one gets

$$\frac{d_{B,DC}}{h} = \frac{d_{B,OZ}}{h} - \frac{d_{DC,OZ}}{h}$$

$$= \cot\varepsilon_x - \cot\alpha \qquad \text{(B.1)}$$

hence

$$BD = \frac{h(\cot\varepsilon_x - \cot\alpha)}{\sin\beta_1}$$

with the result that

$$A_1 = \frac{1}{2} \cdot \frac{h(\cot\varepsilon_x - \cot\alpha)}{\sin\beta_1} \cdot \frac{h}{\sin\gamma_1}$$

or

$$\frac{A_1}{h^2} = \frac{1}{2} \frac{(\cot\varepsilon_x - \cot\alpha)}{\sin\beta_1 \sin\gamma_1} \qquad \text{(B.2)}$$

Similarly, one can show that

$$\frac{A_2}{h^2} = \frac{1}{2} \frac{(\cot\varepsilon_x - \cot\alpha)}{\sin\beta_2 \sin\gamma_2} \qquad \text{(B.3)}$$

In both Eq. (B.2) and Eq. (B.3) $\cot\varepsilon_x$ is given by $\frac{X_{12y}}{X_{12z}}$.

From Eq. (A.4) and Eq. (A.5), one obtains:

$$\cot\varepsilon_x = \frac{\sin\beta_2\cot\gamma_1 - \sin\beta_1\cot\gamma_2}{\sin(\beta_2-\beta_1)} \qquad \text{(B.4)}$$

Equations (B2) and (B3) are valid for $0 < \varepsilon_x < \alpha$ which is the requirement for the line of intersection to daylight both on the slope face and on the crest.

We now turn to the calculation of the wedge volume. For a tetrahedron, the volume is given by the product

$$\frac{1}{3} \cdot \text{(Area of base)} \cdot \text{Height}$$

so that for the wedge in Fig. B.1,

$$V = \frac{1}{3} \cdot (\frac{1}{2} \times DC \times d_{B,DC}) \cdot h \qquad \text{(B.5)}$$

where, from Eq. (B.1)

$$d_{(B,DC)} = h(\cot\varepsilon_x - \cot\alpha)$$

Looking in the direction perpendicular to triangle BCD (Fig. B.2),

$$\overline{DC} = \overline{DX} + \overline{XC}$$

Hence

$$\frac{\overline{DC}}{\overline{BX}} = \frac{\overline{DX}}{\overline{BX}} + \frac{\overline{XC}}{\overline{BX}} = \cot\beta_1 + \cot\theta$$

$$= \cot\beta_1 - \cot\beta_2$$

and

$$\overline{DC} = BX \cdot (\cot\beta_1 - \cot\beta_2)$$

$$= h(\cot\varepsilon_x - \cot\alpha)(\cot\beta_1 - \cot\beta_2)$$

Substituting for $d_{B,DC}$ and $\overline{DC}$ in Eq. (B.5),

$$V = \frac{1}{3} \times (\frac{1}{2} \times h(\cot\varepsilon_x - \cot\alpha)(\cot\beta_1 - \cot\beta_2) \times h(\cot\varepsilon_x - \cot\alpha)) \times h$$

or $$\frac{V}{h^3} = \frac{1}{6}(\cot\beta_1 - \cot\beta_2)(\cot\varepsilon_x - \cot\alpha)^2 \qquad (B.6)$$

This equation has been proved to be correct for Fig. B.2, where β_1 is acute and β_2 is obtuse. It also remains valid when both strike angles are acute (Fig. B.3) or when they are both obtuse (Fig. B.4).

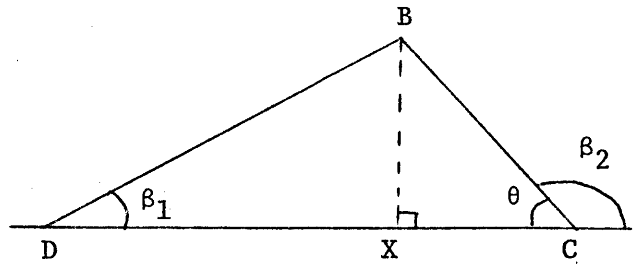


Figure B.2

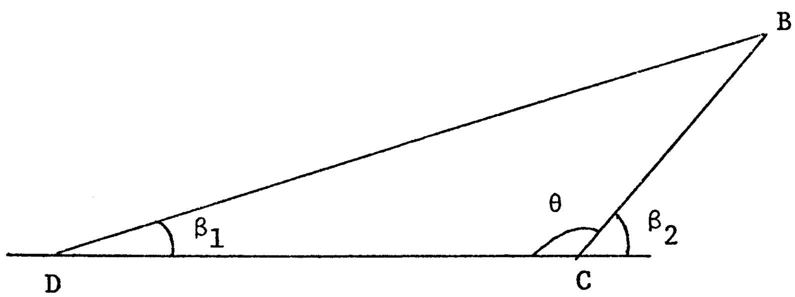


Figure B.3

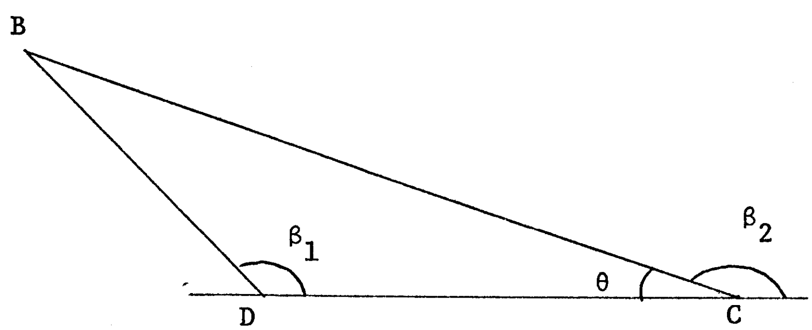


Figure B.4

REFERENCES

1. Campbell, D.S. (1974), "Analytical Method for Analysis of Stability of Rock Slopes," S.M. Thesis, M.I.T., 134 pp.

2. Deere, D.U., Hendron, A.J. Jr., Patton, F.D., and Cording, E.J., "Design of Surface and Near-Surface Construction in Rock," Symposium Rock Mech., 8th, Minnesota, 1966.

3. Glynn, E.F. (1978), "A Probabilistic Approach to the Stability of Rock Slopes," Ph.D. Thesis, M.I.T., 463 pp.

4. Hasofer, A.M., and Lind, N.C. (1974), "Exact and Invariant Second Moment Code Format," Journal of the Engineering Mechanics Division, ASCE, Vol. 100, No. EM1, Proc. Paper 10376, Feb., 1974, pp. 111-121.

5. Hendron, A.J., Cording, E.J., Aiyer, A.K. (1971), "Analytical and Graphical Methods for the Analysis of Slopes in Rock Masses," U.S. Army Engineering Nuclear Cratering Group Technical Report No. 36, 168 pp.

6. Hoek, E., and Bray, J. (1977), "Rock Slope Engineering," Inst. Min. Metall., London, 309 pp.

7. Snow, D.T. (1968), "Fracture Deformation and Changes of Permeability and Storage Upon Changes of Fluid Pressure," Q. of the Colorado School of Mines (63), pp. 201-244.

8. Snow, D.T. (1968), "Hydraulic Character of Fractured Metamorphic Rocks of the Front Range and Implications to the Rocky Mountain Arsenal Well," Q. of the Colorado School of Mines (63), pp. 167-198.

9. Stagg, K.G., and Zienkiewicz, O.C., "Rock Mechanics in Engineering Practice."

10. U.S. Corps of Engineers, "Strengthening of Rock Against Shock Effects, Piledriver Project," MRD Lab, No. 64/90, U.S. Army Corps Engrs., Mo. River Div. Lab., Omaha, Nebraska, 1964.

11. Veneziano, D. (1978), "Probabilistic Model of Joints in Rock," Unpublished Manuscript, Dept. of Civil Engineering, M.I.T.

12. Veneziano, D. (1979), "New Index of Reliability," Journal of the Engineering Mechanics Division, ASCE, Vol. 105, No. EM2, Proc. Pap. 14515, April, 1979, pp. 277-296.