Emergency Braking Using Two Independent Steering Actuators
While Maintaining Directional Control

by

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Abstract

"By-wire" systems are currently an area of significant interest in the automotive industry. In particular, there is substantial work being done to investigate the use of steer-by-wire and brake-by-wire systems in automobiles. Since these systems replace traditional mechanical connections with digital controllers, ensuring the safety of these systems in the event of a failure is critical.

In this work, we investigated the feasibility of using two independent steering actuators to provide emergency braking capability in the event of a primary brake system failure. The work conducted consisted of three major areas. First, a vehicle model was developed for predicting vehicle response to various steering and braking inputs. The derivation, implementation and validation of the model are presented. Once validated, this model was used to run simulations of vehicle response while attempting to stop the vehicle with the steering system. Open-loop simulations were run first to determine how the vehicle responds to given inputs. The results of these simulations indicated that it is possible to achieve a reasonable level of deceleration with the steering system while still maintaining some level of directional control.

With the understanding obtained from the open-loop simulations, a closed-loop control strategy was developed for achieving desired performance. In this approach the steering wheel and brake pedal inputs from the driver are used to determine the appropriate steer angles at each of the front wheels. The control strategy attempts to provide the driver with the same response as if the braking and steering systems were functioning normally, requiring no change in the inputs from the driver. The results indicate that it is possible, for low to moderate levels of lateral acceleration and longitudinal deceleration, to provide performance similar to that under normal operation.

Although further work needs to be done, the results confirm that it is possible to provide a reasonable level of emergency braking capability from the steering system in the event of a brake system failure. The results suggest that this approach could be used to either reduce the level of redundancy needed for such by-wire systems, or to add an additional level of safety to an existing level of redundancy.

Thesis Supervisor: Kamal Youcef-Toumi
Title: Professor of Mechanical Engineering
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I must also thank Nicholas for several things. Nicholas was born July 2, 1999, about half way through my time at MIT. Nicholas, thank you for being a joy in my life. It has been wonderful to watch you grow and to see that big smile when I come home from working at the lab. Though you may never remember the time that we spent out here, it certainly will always have a special place in my heart for these were the days that I first got to know you. Nicholas is the source of the name, NAVDyn, for the vehicle model I created as part of this thesis. Although it may seem that the acronym came from the model itself, in reality I looked at Nicholas’ initials first, and then tried to come up with a name for the model that would match those initials. Whenever I use this model, I will always think of you, Nicholas.

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"From everyone who has been given much, much will be demanded; and from the one who has been entrusted with much, much more will be asked." Luke 12:48

I pray that I may be a good steward of the many gifts I have received.
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1. INTRODUCTION

1.1. State of the Industry

With the continued advancement of digital controllers and other electrical hardware, there continue to be new applications of this technology in the automotive industry. Digital control has started to be used in more and more safety-related areas of the automobile. In most systems to date, this has involved providing some enhancement to existing mechanical systems. Examples of this include the addition of anti-lock braking systems (ABS) and recent development in electric power steering (EPS). Typically with these systems, in the event that something goes wrong with the digitally controlled portion of the system, the vehicle reverts back to the base mechanical system. For example, if the ABS system has a failure of some type, you simply revert back to standard braking without the anti-lock feature. In an EPS system, if the digitally controlled portion of the system fails, which provides assist to the driver, then you revert back to manual steering of the vehicle. In both of these cases, you still have manual braking and steering control of the vehicle.

Recent advances in dependable embedded system technology, as well as continuing demand for improved handling and passive and active safety improvements, have led vehicle manufacturers and suppliers to actively pursue development programs in computer-controlled, by-wire subsystems. In particular, the automotive industry is currently looking into the application of digital controllers in more safety critical areas. These subsystems include steer-by-wire and brake-by-wire, and are composed of mechanically decoupled sets of actuators and controllers connected through multiplexed, in-vehicle computer networks; there is no mechanical link to the driver. They are referred to as "by-wire" systems because there is no longer a mechanical connection. These systems would have some type of driver input device that sends the driver command to the controller. The controller then determines the desired response based on the driver input and other vehicle information. The controller then sends the command signals to the actuators that actually steer or brake the vehicle. Usually there is also feedback of information from the actuators through the controller and back to the driver so that the driver knows what is happening. Figure 1-1 shows a conceptual design for a steer-by-wire system.
Activity to Date in the Automotive Industry

There has been significant work going on in the automotive industry to date related to by-wire systems. Nearly every automotive manufacturer has investigated these systems to varying degrees with the greatest interest being in the European market. Specifically, a European consortium was created several years ago to study many issues related to by-wire systems. The consortium consisted of the following companies and institutions: the car manufacturers DaimlerChrysler, Fiat, Ford Visteon, and Volvo, the suppliers Bosch, Magneti-Marelli, and Mecel, as well as the Chalmers University of Gothenburg and the Vienna University of Technology. According to the final report of this consortium:

"The objective of this project was to achieve a framework for the introduction of safety related fault tolerant electronic systems without mechanical or hydraulic backup in vehicles. These systems are called x-by-wire systems, while the "x" in "x-by-wire" represents the basis of any safety related application, e.g., steering, braking, powertrain or suspension control. These systems will increase overall active and passive safety by liberating the driver from routine tasks and assisting the driver to find solutions in critical situations" [1].
Within this project an architecture for fault tolerant electronic systems in vehicles was worked out and implemented in a prototype. Specifically, a prototype steer-by-wire system was created and implemented for use in evaluating the fault tolerant architecture. The belief was that an architecture that meets the requirements for a steer-by-wire system, believed to be the most difficult, would meet the requirements for most other by-wire systems as well. The work of this consortium has received wide attention throughout the automotive industry and is an indication of the importance and reality of the introduction of by-wire systems for future automobiles.

Although the European consortium has received the most attention, there are numerous other projects going on throughout the industry, [2][3], both publicly and privately. The U.S. National Highway Traffic Safety Administration (NHTSA) has started its own project and is building a prototype steer-by-wire vehicle for use in evaluating this trend toward by-wire systems. In addition, most automotive chassis suppliers have their own projects ongoing to investigate and/or develop by-wire systems. Changes are even being looked at for the automotive electrical system to accommodate such by-wire systems by changing from today’s 14 volt systems to 42 volts to accommodate the increased electrical demands [4].

Benefits of “By-Wire” Systems

With all the attention being given to by-wire systems, the obvious first question is what the benefits are of pursuing such systems. There are numerous secondary benefits to by-wire systems, but the major reason they are being pursued is for use in active safety systems. Figure 1-2 is from the final report of the European X-By-Wire Consortium discussed above and shows their assessment of the direction active safety is headed. Many of the more advanced active safety systems require the ability to de-couple the driver input from the resulting action. This is exactly where by-wire systems come into play. Traditional mechanical systems do not allow inputs that are independent from the driver.
In addition to their use for active safety systems, by-wire systems provide a number of packaging and assembly advantages over their conventional counterparts. For example, an electro-mechanical brake-by-wire system requires no hydraulic brake fluid being stored in the vehicle or loaded at the assembly plant. Most by-wire systems permit more modular assembly and thus reduce the number of parts that must be handled during production. Steer-by-wire systems have no steering column and may also eliminate the need for a cross-car steering assembly. This provides passive safety benefits as well. A steer-by-wire system not having a traditional steering column allows the possibility of designing better structures for absorbing energy during a collision.

Finally, there are a number of driver interface and performance benefits offered by by-wire systems. These systems provide great flexibility in tuning of the vehicle through software as well as the possibility of new features such as stability enhancement, cross-car wind correction, etc. The switch to by-wire systems allows numerous possibilities for integrating the performance of different systems to improve the performance of the overall vehicle. For example, the steering and braking systems can work together to optimize the performance of a vehicle.

Relevance of This Research

In order for improvements from the safety features described above to be realized, by-wire systems themselves must be safe. As can be seen with such systems, in the event that there is
some failure in the digital controller or the electrical hardware of the system, there is no longer a mechanical backup for the system and potentially there could be a complete loss of steering or braking. Obviously with steering and braking being such safety-critical systems, this can not be allowed to happen. One approach to addressing this issue is to add redundancy to the system in the form of backup controllers, wiring, actuators, etc. This approach is often used for such systems in the aircraft and military arenas. The problem is that this approach can be very expensive. For a multi-million dollar aircraft, this may be an acceptable route, but the automotive industry is very cost sensitive. If adding hundreds of dollars in redundancy is required for such systems, that would probably be enough to prevent such a system from being implemented. There is also research indicating that for increasingly complex and safety-critical systems, redundancy is not adequate to ensure safety [5].

Thus, a major issue with the implementation of these by-wire systems is addressing what to do in the event of a failure. The ability to maintain control of the vehicle during a failure is required. In addition, being able to do so at a cost that is not prohibitive is also a significant consideration. The work of this thesis is aimed directly at helping to address these issues.

1.2. Thesis Overview

The proposal for this thesis is to investigate the feasibility of using the steering and braking systems as back-ups for each other in the event of a failure. The idea is to use the functionality of another existing system in the vehicle to provide vehicle control in the event that the primary system fails. If such an approach is found to provide adequate control to safely operate the vehicle, this could reduce the need for redundancy and so reduce the overall cost of the system or alternatively provide an additional level of safety. (It would be unlikely that some level of redundancy would not still be required, but hopefully not to the extent needed if the primary system must always provide some level of functionality.) If the backup system can control the vehicle until it can be brought to a stop, or serve as a “limp-home” mode to get the vehicle to a place of service, then the chance of an accident can be reduced and/or the driver will not be stranded.
There are many different aspects of this topic area that could be pursued. The research covered in this thesis specifically looks into the feasibility of using the steering system to provide emergency braking control in the event of a failure in the primary braking system. The process to investigate this involved three major steps:

1. Develop a vehicle model that has enough complexity to allow for simulations to be run showing the effects of various control inputs,
2. Run simulations using open-loop control inputs to determine the response of a vehicle to independent steering inputs,
3. Use the open-loop results to develop a closed-loop control strategy for combined steering and braking of the vehicle using only the independent steering actuators.

This thesis will cover the details of each of these steps. As shown, the first step was the development of a vehicle model for conducting simulations of vehicle response. An attempt was made to find an existing model that was adequate for this study, but none were found. As a result a new model, referred to as Non-Linear Analysis of Vehicle Dynamics (NAVDyn), was created, implemented and validated. The assumptions behind the model and the derivation of the equations of motion and other supplemental equations are covered in Chapter 2. The implementation and validation of this model for use in simulations are covered in Chapter 3.

Once the model was developed, simulations were conducted to show the response of a vehicle to independent front steering wheel angles. Some of the specific areas looked at were:

1. Investigate the overall level of braking that is possible with the steering,
2. Investigate various scenarios: straight line braking, combined steering and braking, etc., and
3. Determine the feasibility of steering and stopping at the same time with only two independent steering actuators.

From these simulations a sound understanding of how the vehicle responds to independent steer angles was obtained. In particular, there were some initially unexpected results found and explained when the wheels are turned toward/away from each other. The open-loop simulations and results will be covered in detail in Chapter 4.

With the open-loop results in hand, the final step was generating a closed-loop control strategy to provide directional control and stopping capability at the same time in response to driver
commands. Chapter 5 covers the development of this closed-loop control strategy. It also gives results of simulations showing vehicle response using this strategy. As will be seen in this chapter, it is possible to steer and stop a vehicle at the same time using only independent steering actuators.

Finally, conclusions will be presented from this work in Chapter 6 as well as recommendations for areas of further study. The appendices cover additional details, particularly related to NAVDyn, that are not covered in the main text.
2. DEVELOPMENT OF VEHICLE MODEL

2.1. Introduction

When originally deciding on the topic of the feasibility of stopping a vehicle with the steering, the intent was to find an existing vehicle model that could be used to conduct the study. Two sources referenced during the process of deciding on a model to use were [6] and [7]. For this study, the model needed to have the following features:

- Competent for vehicle handling,
- Easily customizable,
- 4-wheel independent steering/braking, and
- Capable of adding in control systems.

Since the study would involve a wide range of steering inputs, it was necessary for the model to be capable of covering a wide range of vehicle handling maneuvers, including operation at high slip angles. It was also decided that the model needed to be capable of covering non-linearities both in the vehicle dynamics and in the suspension and steering system kinematics and compliances.

Originally it was thought that the study might get into two-wheel versus four-wheel steering. In addition the closed-loop portion of this research would involve making modifications to the steering system. As a result, the model needed to be easily customizable for making such changes. For example, it should be easy to change from two-wheel to four-wheel steering. It should also be easy to change the existing steering system in the model to represent a steer-by-wire system. Along these lines, the intent was to look at independent steering actuators, so the model needed to be capable of independent steering at each of the steerable wheels.

Also inline with customization, it needed to be easy to add the control systems necessary to implement the closed-loop strategy that is part of the study. It should be possible to do this without having to write software. This means the model should have block diagrams available for doing control systems.
When looking at existing models for use in this study, none were found that met all the requirements described above. Section 2.2 describes the review of existing models. The decision was made to create a new model for the purpose of conducting this study. The development of this model is covered in Section 2.3. The new model meets all of the above requirements and provides excellent results for a wide variety of vehicle handling scenarios.

As an added note, above it was mentioned that the original intent in conducting this research was to use an existing model. Since this did not happen and a new model was created from scratch, a significant portion of this thesis is dedicated to the development, implementation and validation of this model. This portion of the research obtained as much importance as the original idea of stopping a vehicle with the steering. This is mentioned here so the level of attention paid to the vehicle model does not confuse the reader. This model is already being used by other people and it became clear that good documentation of the model was necessary. Information about the development of this model is also covered in [8] and [9].

2.2. Review of Existing Vehicle Models

It was found that there are many existing models for simulating the motion of an automobile. The majority of these come at the two ends of the spectrum: very simplified models for a specific use (the most common being the "bicycle" model) or high-order models covering the full degrees-of-freedom for a vehicle. The simplified models do not have enough complexity to be used for many scenarios. Some of them are targeted at specific scenarios and are good at simulating those circumstances, but have poor results for general use. These models make many simplifying assumptions. If the circumstances to be studied are not within the scope of these assumptions, the results may be inaccurate. Many of them are linear so they give poor results for maneuvers where the effects of non-linearities in the vehicle become significant. For these reasons, these models were not adequate for conducting this research.

The other end of the spectrum involves the high-order vehicle models. These models are certainly more complete, have good general applicability, and can provide excellent results. The down side is that they are much more complicated and require a significant amount of information about the vehicle being modeled. In particular, to model several of the degrees-of-
freedom requires detailed information about suspension design, which may not be readily available. In addition, for studying vehicle handling on a smooth road (the focus for this research), many of the degrees-of-freedom are not necessary.

Another issue with many of the high-order models is that most are commercially available programs that have well-defined user interfaces. These interfaces make assumptions about the structure of the vehicle itself and/or the inputs and outputs the user desires. As an example, a standard hydraulic steering system is assumed in many of these models. Since it was desired to study a steer-by-wire system with independent actuators, making the necessary changes would have been quite difficult. Customization of these programs requires understanding the underlying programming of the model, which, being commercially available products, is often not easily accessible. Even if access is available, I did not want to undertake such a significant task. In addition, such models were cost prohibitive for the study being conducted.

When studying vehicle handling (in this thesis handling will be used to mean any combination of braking and steering inputs), the ideal model lies somewhere in between. If you assume the vehicle is traveling on a smooth road, which is often true for handling, several degrees-of-freedom can be eliminated. At the same time, handling can involve fairly complex, non-linear behavior. Thus the ideal model for studying many vehicle handling scenarios is one with only those degrees-of-freedom that are relevant, but for those degrees-of-freedom there should be very few, if any, simplifying assumptions. All significant non-linearities should be included whenever possible. In reviewing the models in existence, three were investigated more thoroughly for their potential use, although others, such as [10] and [11] were looked at as well.

Clemson Model

One of two models that were found that had a level of complexity in between the two extremes described above is one developed at Clemson University, [12]. This model has eight degrees-of-freedom, which is the same as the eventual model that was developed for this study. When reviewing the Clemson model, there were two issues that came up. First, although the model was implemented in Matlab/Simulink, it was done in such a way that was quite difficult to modify. Second, when reviewing the derivation of equations used in the model, some of the assumptions
that were made did not match with the desire for this study. For these reasons, the Clemson model was not used. However, this did prove to be a good resource for comparison against when NAVDyn was being developed.

**Ashok's Model**

The second model that was reviewed was developed by Ashok Chandy of Delphi Automotive Systems as part of his own personal work towards a Ph.D. Ashok’s model was also an eight degree-of-freedom model, again the same as NAVDyn. The down side to this model is that when it was created, it was only intended to be directionally correct for various influences on vehicle handling. It accomplished this goal, but there were several simplifying assumptions made to the model that did not match with the desire for this study. The positive side of this model is that it was implemented in a way that was exactly what was needed for this research. The implementation of NAVDyn follows very closely the same structure that was used in Ashok’s model. Further details of this implementation are given in 3.2 and [9].

**CarSim**

One high-order model was reviewed for potential use. This model was CarSim, a fourteen degree-of-freedom model developed at the University of Michigan [13]. CarSim was found to be quite competent for vehicle handling simulations, but it had a significant down side: it is a commercially available program that has a very well defined user interface. This prevented making changes to the steering system for investigating independent actuators. It also precluded the easy addition of a control system – one of the requirements for a model. For these reasons, CarSim was not viewed to be acceptable for use in this research. However, since CarSim had already been validated and was a good program for vehicle handling analysis, the decision was made to use CarSim for the purpose of validating NAVDyn. In 3.3 NAVDyn is compared against CarSim to show how well it compares against a higher-order model.

**2.3. Development of New Vehicle Model - NAVDyn**

So as mentioned, since no existing models were found that met the requirements for conducting this study of using the steering to stop the vehicle, the decision was made to create a new model.
As mentioned previously, this model was given the name Non-Linear Analysis of Vehicle Dynamics (NAVDyn). Creating a vehicle model consists of several steps. The dynamics of the vehicle must be defined describing how the vehicle responds to given force inputs. A model of the tires for generating the forces acting on the vehicle must be selected. The tire slip angles, slip ratios, inclination angles and normal forces must be calculated as inputs to the tire model. Finally, the influence of the steering and suspension systems on the tire angles must be modeled. In addition, models for the specific steering and braking systems desired must also be developed.

2.3.1. Generating the Lumped Mass Dynamic Equations

The first step in creating NAVDyn was generating the equations of motion. As described above, the desired model should only cover the degrees-of-freedom that are relevant for vehicle handling on a smooth road, but for those degrees-of-freedom should have as few simplifying assumptions as possible. The first step is to define the assumptions that the model will be based upon.

2.3.1.1. Assumptions/Degrees-of-Freedom

Before generating the equations of motion for the vehicle, the assumptions about the model need to be clearly defined. For this model, the single most significant assumption is that the vehicle is traveling on a smooth road such that there are no vertical motions of the wheels. With this assumption, the degrees-of-freedom associated directly with vertical motion can be neglected. The vehicle is assumed to be made of three masses connected together through the vehicle roll axis. The three masses are the sprung mass, the front unsprung mass and the rear unsprung mass. These masses represent the vehicle body, the front suspension and the rear suspension respectively.

The decision was made to exclude the pitch degree-of-freedom for two reasons. It is important to note, however, that the influence of longitudinal weight transfer is very important and has been included. The first reason for excluding pitch is that it has a less significant impact on vehicle response than the other degrees-of-freedom shown below, especially for low to moderate deceleration levels. The second reason is that the pitch degree-of-freedom is significantly more difficult to model since detailed information is needed about suspension design. Due to anti-lift and anti-dive features in the suspension, it is difficult to find a "pitch-axis" equivalent to the
"roll-axis" commonly used for the roll degree-of-freedom. The pitch degree-of-freedom also adds significant complexity to the model due to secondary influences such as changing the angle of the roll axis. Since a "simplified" model that still provides adequate results for vehicle handling was desired, pitch was excluded.

Given the above assumptions, the vehicle model is assumed to have the following eight degrees-of-freedom, which are shown by white arrows in Figure 2-1:

1. Translation in the longitudinal direction,
2. Translation in the lateral direction,
3. Body roll relative to the chassis about the roll axis,
4. Yaw about the vertical axis,
5. Rotation of the left front wheel,
6. Rotation of the right front wheel,
7. Rotation of the left rear wheel, and
8. Rotation of the right rear wheel.

When discussing degrees-of-freedom, sometimes the slip angle of each tire is considered a degree-of-freedom. With this approach, there would be four additional degrees-of-freedom, for a total of twelve.

The variables associated with each of the eight degrees-of-freedom above are as follows:

\[
\begin{align*}
V_{ox} &= \text{longitudinal velocity of chassis origin} \\
V_{oy} &= \text{lateral velocity of chassis origin} \\
\phi &= \text{body roll angle from vertical} \\
\psi &= \text{vehicle heading angle} \\
\omega_{lf} &= \text{angular velocity of left front wheel} \\
\omega_{rf} &= \text{angular velocity of right front wheel} \\
\omega_{lr} &= \text{angular velocity of right front wheel} \\
\omega_{rr} &= \text{angular velocity of right rear wheel}
\end{align*}
\]

For these eight degrees-of-freedom, only two minor simplifying assumptions have been made. It is assumed that the angle of the roll axis relative to horizontal is negligible, so the roll axis and the longitudinal axis are essentially the same. For the majority of vehicles, this is a reasonable assumption. As an example, for the Ford Taurus used to validate the model, the roll axis has an angle of 0.43 degrees relative to the x-axis.
The only other assumption made is that the center-of-gravity is located laterally at the center of the vehicle, which is generally a reasonable assumption. For the Ford Taurus, the CG of the vehicle as tested, with driver plus instrumentation only, was 17.5mm left of center. In generating the equations of motion, no other simplifications were made.
2.3.1.2. Coordinate Systems

To generate the equations of motion, three major coordinate systems were used. The first is the inertial reference frame or global coordinates, the second is fixed to the vehicle chassis and the third is fixed to the vehicle body. All equations are generated from the vehicle chassis reference frame. In addition, wheel-fixed coordinate frames are used for calculation of tire forces and moments. Conventions per SAE J670e are used for the coordinate systems. This means positive x-axis forward, positive y-axis to the right and positive z-axis down. Positive rotations are determined by the right-hand rule for these axes. For further details, see SAE J670e [14].

The locations of the coordinate systems are as follows. First, the chassis coordinate system is fixed to the chassis such that the z-axis is always parallel to the inertial z-axis. The origin is located longitudinally at the center of gravity of the vehicle. Laterally it is located at the center of the vehicle (which is assumed to be the location of the CG.) Vertically it is located below the total center of gravity of the vehicle at the roll axis. Figure 2-1 shows the location of the chassis coordinate system origin and positive directions for each of the axes.

The body coordinate system has the same origin as the chassis coordinate system, but is allowed to roll about the x-axis relative to the chassis coordinate system. Finally, it is assumed that at time \( t=0 \), the x-axis and z-axis of the chassis coordinate system are aligned with the inertial reference frame and the roll angle is zero. Figure 2-2 shows the location of the body coordinates relative to the chassis coordinates. They are only different by the roll angle. Figure 2-3 shows the location of the chassis reference frame relative to the inertial reference frame.

![Front View](image)

**Figure 2-2:** Location of Body Coordinates Relative to Chassis Coordinates
The parameters used to describe the vehicle are defined below. Many important parameters are shown in Figure 2-4 below.

\begin{align*}
    h_{cgf} &= \text{height of front unsprung mass CG above ground} \\
    h_{cgr} &= \text{height of rear unsprung mass CG above ground} \\
    h_{gs} &= \text{height of sprung mass CG above ground} \\
    h_o &= \text{height of chassis origin above ground} \\
    h_s &= \text{height of sprung mass CG above origin} (h_{gs} - h_o) \\
    h_{cf} &= \text{height of front unsprung mass CG above origin} (h_{cgf} - h_o) \\
    h_{cr} &= \text{height of rear unsprung mass CG above origin} (h_{cgr} - h_o) \\
    I_{sx} &= \text{sprung mass moment of inertia about x-axis} \\
    I_{sy} &= \text{sprung mass moment of inertia about y-axis} \\
    I_{sz} &= \text{sprung mass moment of inertia about z-axis} \\
    I_{sxz} &= \text{sprung mass product of inertia about x and z-axes} \\
    I_{sxo} &= \text{sprung mass moment of inertia about roll axis} \\
    I_{szx} &= \text{sprung mass product of inertia about roll axis} \\
    I_{z} &= \text{total vehicle moment of inertia about z-axis} \\
    I_{w} &= \text{moment of inertia of one wheel} \\
    l_f &= \text{longitudinal distance from total CG to front axle}
\end{align*}
$l_r$  = longitudinal distance from total CG to rear axle
$l_{cgs}$  = longitudinal distance from total CG to sprung mass CG
$L$  = wheelbase (distance from front axle to rear axle)
$M_s$  = sprung mass
$M_{uf}$  = front unsprung mass
$M_{ur}$  = rear unsprung mass
$M$  = total mass of the vehicle
$R_w$  = rolling radius of tire
$t_f$  = front track width
$t_r$  = rear track width

Figure 2-4: Definition of Vehicle Parameters
2.3.1.3. **Nomenclature**

Before getting into the actual derivation of the equations, the nomenclature used needs to be defined. The three coordinate systems used are defined by $OXYZ$ for the inertial frame, $oxyz$ for the chassis coordinate system, and $o'x'y'z'$ for the body coordinate system. For each of these coordinate systems, the associated unit vectors are defined as:

- Inertial coordinates: $u_x, u_y, u_z$
- Chassis coordinates: $i, j, k$
- Body coordinates: $i', j', k'$

For the variables used, several different subscripts are used to further define them. For variables associated with each of the wheels, the subscripts $lf, rf, lr, \text{ and } rr$ will be used to denote the left front, right front, left rear, and right rear respectively. For variables associated with the three different masses used, the subscripts $s, uf, \text{ and } ur$ will be used to denote the sprung mass, front unsprung mass and rear unsprung mass respectively. Finally, for variables associated with the coordinate systems, $o$ and $b$ will be used to denote the chassis origin and the body origin respectively. Other subscripts will be defined as they are used.

One other item to note is that in the early development of NAVDyn, the variable $r$ was used to denote the yaw rate of the vehicle. Later this was changed to $\psi$ for consistency and to remove any confusion as to the relation between heading angle and yaw rate. An attempt was made to make this change throughout all the documentation for NAVDyn. Although this was done for all of the equation derivations, there may be places in the Matlab implementation and in the plots from Matlab where yaw rate is still denoted by $r$.

### 2.3.1.4. Derivation of Equations of Motion Using Newton-Euler

In this section the complete derivation of the equations of motion for the above eight degrees-of-freedom will be presented. The first step is to define the movement of the chassis origin (same as the body origin) relative to the inertial reference frame. (The Newton-Euler approach, which can be found in almost any dynamics textbook, such as [15], is used for the derivations. Appendix A shows the same derivation using Euler-Lagrange.) To start with, the position vector from the inertial reference frame origin to the chassis origin is given by
\[ \mathbf{R}_o = X \mathbf{u}_x + Y \mathbf{u}_y. \]  

(2.1)

The conversion from inertial coordinates to chassis coordinates is given by

\[ \mathbf{u}_x = \cos \psi \mathbf{i} - \sin \psi \mathbf{j} \]
\[ \mathbf{u}_y = \sin \psi \mathbf{i} + \cos \psi \mathbf{j}. \]  

(2.2)

Converting (2.1) into chassis coordinates gives

\[ \mathbf{R}_o = (X \cos \psi + Y \sin \psi) \mathbf{i} + (-X \sin \psi + Y \cos \psi) \mathbf{j}. \]  

(2.3)

Taking the time derivative of this position vector gives the velocity of the chassis origin

\[ \mathbf{V}_o = \frac{d\mathbf{R}_o}{dt} = \dot{X} \mathbf{u}_x + \dot{Y} \mathbf{u}_y = (\dot{X} \cos \psi + \dot{Y} \sin \psi) \mathbf{i} + (-\dot{X} \sin \psi + \dot{Y} \cos \psi) \mathbf{j}. \]  

(2.4)

From this we define the longitudinal and lateral velocity of the vehicle relative to chassis coordinates as

\[ V_{ax} = \dot{X} \cos \psi + \dot{Y} \sin \psi \]
\[ V_{oy} = -\dot{X} \sin \psi + \dot{Y} \cos \psi. \]  

(2.5)

Thus we end up with the velocity of the chassis origin in chassis coordinates as

\[ \mathbf{V}_o = V_{ax} \mathbf{i} + V_{oy} \mathbf{j}. \]  

(2.6)

Since the body and chassis coordinates have the same origin, both coordinate systems have the same linear velocity. However, since the body rolls relative to the chassis, the angular velocities of the chassis and body will be different and are given by

\[ \Omega_c = \dot{\psi} \mathbf{k} \]  

(2.7)

\[ \Omega_b = \dot{\phi} \mathbf{i} + \dot{\psi} \mathbf{k} \]  

(2.8)

where \( \dot{\phi} \) is the roll rate and \( \dot{\psi} \) is the yaw rate.
The acceleration of the chassis (and body) origin can be calculated from

\[
\mathbf{a}_o = \frac{d^2 \mathbf{R}_o}{dt^2} = \frac{\partial \mathbf{V}_o}{\partial t} + \Omega \times \mathbf{V}_o. \tag{2.9}
\]

Using equations (2.6), (2.7) and (2.9) we end up with the acceleration of the chassis origin as

\[
\mathbf{a}_o = (\dot{V}_x - \psi V_y) \mathbf{i} + (\dot{V}_y + \psi V_x) \mathbf{j}. \tag{2.10}
\]

2.3.1.4.1. Linear Motion of Masses

From the above equation we have the motion of the chassis coordinate system defined and can use this information to derive the equations for the motion of the three masses. The first step is to calculate the linear velocities and accelerations for each of the three masses.

**Front Unsprung Mass Linear Motion**

The first step in defining the motion of the front unsprung mass is to define its location. This can be done by taking its global position directly, or more appropriately, by defining its position relative to the chassis origin and adding this to the position of the chassis origin as

\[
\mathbf{R}_{uf} = \mathbf{R}_o + \mathbf{r}_{uf}. \tag{2.11}
\]

From Figure 2-4 and the definition of parameters above, the position of the center of gravity for the unsprung mass is given by

\[
\mathbf{r}_{uf} = l_i \mathbf{i} - h_{uf} \mathbf{k}. \tag{2.12}
\]

With the position known, the velocity of the front unsprung mass is determined from

\[
\mathbf{V}_{uf} = \frac{d \mathbf{R}_{uf}}{dt} = \mathbf{V}_o + \frac{\partial \mathbf{r}_{uf}}{\partial t} + \Omega \times \mathbf{r}_{uf}. \tag{2.13}
\]

Using (2.6), (2.7) and (2.12), this results in the velocity of the front unsprung mass as

\[
\mathbf{V}_{uf} = V_{ax} \mathbf{i} + (V_{oy} + l_i \psi) \mathbf{j}. \tag{2.14}
\]
With this velocity known, the acceleration can be calculated from

$$a_{uf} = \frac{dV_{uf}}{dt} = a_o + \dot{\Omega} \times r_{uf} + \Omega \times (\Omega \times r_{uf}). \quad (2.15)$$

Using (2.7), (2.10) and (2.12), the resulting acceleration of the front unsprung mass is

$$a_{uf} = (\dot{V}_x - \psi V_y - l_x \dot{\psi})i + (\dot{V}_y + \psi V_x + l_x \dot{\psi})j. \quad (2.16)$$

**Rear Unsprung Mass Linear Motion**

The equations describing the motion of the rear unsprung mass follow exactly the same process as for the front unsprung mass. The only difference between the two is that the position of the rear unsprung mass relative to the chassis origin is different. The resulting equations for the rear unsprung mass are

$$R_{ur} = R_o + r_{ur} \quad (2.17)$$

$$r_{ur} = -l_r i - h_{ur} k \quad (2.18)$$

$$V_{ur} = \frac{dR_{ur}}{dt} = V_o + \frac{\partial r_{ur}}{\partial t} + \dot{\Omega} \times r_{ur} \quad (2.19)$$

$$V_{ur} = V_x i + (V_y - l_x \dot{\psi})j \quad (2.20)$$

$$a_{ur} = \frac{dV_{ur}}{dt} = a_o + \dot{\Omega} \times r_{ur} + \Omega \times (\Omega \times r_{ur}) \quad (2.21)$$

$$a_{ur} = (\dot{V}_x - \psi V_y + l_x \dot{\psi})i + (\dot{V}_y + \psi V_x - l_x \dot{\psi})j. \quad (2.22)$$

**Sprung Mass Linear Motion**

Next the linear motion of the sprung mass is calculated. This follows a similar process to that for the unsprung masses, except that the sprung mass moves relative to the chassis coordinate system. The position of the sprung mass is defined relative to the body-fixed coordinate system...
and then must be converted to chassis coordinates. The position of the sprung mass center of
gravity is given by

$$R_s = R_o + r_s . \quad (2.23)$$

The conversion from body-fixed coordinates to chassis coordinates is given by

$$i' = i$$

$$j' = \cos \phi j + \sin \phi k \quad (2.24)$$

$$k' = -\sin \phi j + \cos \phi k$$

From this the position of the sprung mass center of gravity can be converted from body
coordinates to chassis coordinates

$$r_s = l_{gs}i' - h_s k' = l_{gs}i + h_s \sin \phi j - h_s \cos \phi k . \quad (2.25)$$

As with the unsprung masses, the velocity of the sprung mass is determined from

$$V_s = \frac{dR_s}{dt} = V_o + \frac{\partial r_s}{\partial t} + \Omega \times r_s . \quad (2.26)$$

Using (2.6), (2.7) and (2.25), the resulting velocity is

$$V_s = (V_o - h_s \omega \sin \phi) i + (V_o + h_s \omega \cos \phi + l_{gs} \omega) j + h_s \omega \cos \phi k . \quad (2.27)$$

Since the unsprung mass moves relative to the chassis coordinate system, the calculation for the
acceleration has some additional terms compared with those for the sprung masses and is given
by

$$a_s = \frac{dV_s}{dt} = a_o + \frac{\partial^2 r_s}{\partial t^2} + 2 \Omega \times \frac{\partial r_s}{\partial t} + \dot{\Omega} \times r_s + \Omega \times (\Omega \times r_s) . \quad (2.28)$$

Using (2.7), (2.10), and (2.25), the final result for the acceleration of the sprung mass is
\[ a_s = \left( \dot{V}_{ax} - \dot{\psi} V_{oy} - 2h_s \dot{\phi} \psi \cos \phi - h_s \dot{\psi} \sin \phi - l_{cg} \dot{\psi}^2 \right) \mathbf{i} \]
\[ + \left( \dot{V}_{oy} + \dot{\psi} V_{ax} + h_s \ddot{\phi} \cos \phi - h_s \dot{\phi}^2 \sin \phi + l_{cg} \dot{\psi} - h_s \dot{\psi}^2 \sin \phi \right) \mathbf{j} \]
\[ + \left( h_s \ddot{\phi} \sin \phi + h_s \dot{\phi}^2 \cos \phi \right) \mathbf{k} \]  

(2.29)

2.3.1.4.2. Angular Motion of Masses

With the equations for linear motion complete, the next step is calculating the equations for the angular motion of each of the three masses.

Front Unsprung Mass Angular Motion

The first step is to calculate the angular momentum of the front unsprung mass. Since the front unsprung mass does not rotate about its own center of gravity, then the angular momentum is given by

\[ H_{uf} = R_{uf} \times M_{uf} \mathbf{V}_{uf} + H_{uf} \]  

(2.30)

where \( H_{uf} \) is the angular momentum of the front unsprung mass about its own center of gravity, which is given by

\[ H_{uf} = I_{uf} \Omega_c \]  

(2.31)

Since the unsprung masses are assumed to be symmetric about the y-axis and also about the x-z plane, all products of inertia are zero and the inertia tensor of the front unsprung mass is

\[ I_{uf} = \begin{bmatrix} I_{xxuf} & 0 & 0 \\ 0 & I_{yyuf} & 0 \\ 0 & 0 & I_{zzuf} \end{bmatrix} \]  

(2.32)

Substituting (2.11), (2.14) and (2.31) into (2.30) gives the angular momentum of the front unsprung mass as

\[ H_{uf} = \left[ M_{uf} h_{uf} V_{oy} + M_{uf} l_f h_{uf} \psi \right] \mathbf{i} - M_{uf} h_{uf} V_{ax} \mathbf{j} + \left[ \left( I_{zzuf} + M_{uf} l_f^2 \right) \psi + M_{uf} l_f V_{oy} \right] \mathbf{k} \]  

(2.33)
The final step is to calculate the time derivative of the angular momentum. Since all the equations are given relative to chassis coordinates, the time derivative of the angular momentum can be calculated from

\[
\frac{dH_{af0}}{dt} = \frac{\partial H_{af0}}{\partial t} + \Omega \times H_{af0}.
\]  

(2.34)

Using (2.7) and (2.33) and conducting the above calculations, the resulting time derivative of the front unsprung mass angular momentum is

\[
\frac{dH_{af0}}{dt} = \left[ M_{af} h_{af} \dot{V}_{oy} + M_{af} l_f h_{af} \ddot{\psi} + M_{af} h_{af} V_{oz} \dot{\psi} \right] i
\]

\[
+ \left[ -M_{af} h_{af} \dot{V}_{oz} + M_{af} h_{af} V_{oz} \dot{\psi} + M_{af} l_f h_{af} \dot{\psi}^2 \right] j.
\]  

(2.35)

\[
+ \left[ M_{af} l_f \dot{V}_{oy} + \left( I_{zzr} + M_{af} l_f^2 \right) \dot{\psi} \right] k
\]

Rear Unsprung Mass Angular Motion

As with the linear motion, the angular motion of the rear unsprung mass follows the exact same process as that for the front unsprung mass. Using the values associated with the rear unsprung mass, the resulting equations are

\[
H_{ur0} = \left[ M_{ur} h_{ur} V_{oy} - M_{ur} l_r h_{ur} \dot{\psi} \right] i - M_{ur} h_{ur} V_{ox} j + \left[ \left( I_{zzr} + M_{ur} l_r^2 \right) \dot{\psi} - M_{ur} l_r V_{oy} \right] k
\]  

(2.36)

\[
\frac{dH_{ur0}}{dt} = \left[ M_{ur} h_{ur} \dot{V}_{oy} - M_{ur} l_r h_{ur} \dot{\psi} + M_{ur} h_{ur} V_{ox} \dot{\psi} \right] i
\]

\[
+ \left[ -M_{ur} h_{ur} \dot{V}_{ox} + M_{ur} h_{ur} V_{ox} \ddot{\psi} - M_{ur} l_r h_{ur} \dot{\psi}^2 \right] j.
\]  

(2.37)

\[
+ \left[ -M_{ur} l_r \dot{V}_{oy} + \left( I_{zzr} + M_{ur} l_r^2 \right) \dot{\psi} \right] k
\]

Sprung Mass Angular Motion

For the sprung mass, the procedure is similar, but again is more involved since the sprung mass moves relative to the chassis coordinate system. As with the unsprung masses, the angular momentum of the sprung mass is calculated from
\[ H_{so} = R_s \times M_s V_s + H_s \]  

(2.38)

where \( H_s \) is the angular momentum of the sprung mass about its own center of gravity, which is given by

\[ H_s = I_s \Omega_b . \]  

(2.39)

It is important to note here that (2.39) uses the angular velocity of the body, not the chassis as in all previous calculations. Since the inertia tensor is calculated relative to body-fixed coordinates, then the angular velocity must also be in body-fixed coordinates. The conversion from chassis coordinates into body coordinates is given by

\[
\begin{align*}
    i &= i' \\
    j &= \cos \phi j' - \sin \phi k' \\
    k &= \sin \phi j' + \cos \phi k'
\end{align*}
\]  

(2.40)

Using (2.40) to convert (2.8) results in the angular velocity of the body in body coordinates as

\[ \Omega_b = \dot{i}' + \dot{j}' \sin \phi + \dot{k}' \cos \phi . \]  

(2.41)

The sprung mass is symmetric about the x'-z' plane, so the inertia tensor of the vehicle body is

\[ I_s = \begin{bmatrix} I_{xxs} & 0 & I_{xzs} \\ 0 & I_{ys} & 0 \\ I_{xzs} & 0 & I_{zzs} \end{bmatrix} . \]  

(2.42)

Substituting (2.41) and (2.42) into (2.39) and then calculating (2.38) results in an angular momentum of the sprung mass about the origin of

\[ H_{so} = H_{sox} i + H_{soy} j + H_{soz} k \]  

(2.43)

where

\[ H_{sox} = \left( I_{xxs} + M_s h_s^2 \right) \dot{\phi} + \left( I_{xzs} + M_s I_{cgs} h_s \right) \dot{\psi} \cos \phi + M_s h_s V_{oy} \cos \phi \]  

(2.44)
\[
H_{soy} = \left( I_{xx} + M_z h_s^2 \right) \dot{\phi} + \left( I_{yy} + M_z I_{rgs} h_s \right) \dot{\psi} \cos \phi + M_z h_s V_{oy} \cos \phi \\
H_{sov} = \left( I_{xx} + M_z I_{rgs} h_s \right) \dot{\phi} \cos \phi + \left( I_{yy} \sin^2 \phi + I_{zxx} \cos^2 \phi + M_z I_{rgs}^2 + M_z h_s^2 \sin^2 \phi \right) \dot{\psi} - M_z h_s V_{ox} \sin \phi + M_z I_{rgs} V_{oy} \cos \phi
\] (2.45)

Without reproducing all of the calculations and results here, the time derivative of the angular momentum is given by

\[
\frac{dH_{so}}{dt} = \frac{\partial H_{so}}{\partial t} + \Omega \times H_{so} .
\] (2.47)

2.3.1.4.3. The Final Equations - Rigid Body Dynamics

The final equations describing the motion of the vehicle are obtained by applying the dynamic principles for a rigid body. For linear motion, this is simply applying the force-momentum relation that comes from Newton’s Second Law to the results of the above calculations. For angular motion, it is more complex, but involves applying the torque-angular-momentum relation. This relation is an extension of Newton’s Second Law for a particle to rigid bodies.

Linear Motion Equations

The first step in obtaining the final equations of motion is the application of the force-momentum relation to the linear motion equations derived above. The relation says that the total force acting on the vehicle will be equal to the mass times the acceleration for each of the three masses, or

\[
\sum F = M_s a_s + M_{sf} a_{sf} + M_{ur} a_{ur} .
\] (2.48)

This vector equation can be broken down into three scalar equations for motion in each of the three directions. The first of the eight equations of motion is obtained from the x-direction component of (2.48)

\[
\sum F_x = M_s a_{sx} + M_{sf} a_{sfx} + M_{ur} a_{urx} .
\] (2.49)
Substituting in the x-direction components of (2.16), (2.22) and (2.29) into (2.49) and using the fact that \( M_s l_{cg} + M_{sf} l_f - M_{ur} l_r = 0 \) results in the longitudinal translation equation of

\[
\dot{V}_{ax} = \frac{\sum F_x + M_s \left( 2h_1 \dot{\phi} \psi \cos \phi + h_1 \dot{\psi} \sin \phi \right)}{M} + V_{oy} \dot{\psi}.
\] (2.50)

where \( M = M_s + M_{sf} + M_{ur} \) is the total mass of the vehicle.

The second final equation of motion comes from the y-direction component of (2.48), which is given by

\[
\sum F_y = M_s a_{xy} + M_{sf} a_{sfy} + M_{ur} a_{ury}.
\] (2.51)

Substituting in the y-direction components of (2.16), (2.22) and (2.29) into (2.51) and again using the fact that \( M_s l_{cg} + M_{sf} l_f - M_{ur} l_r = 0 \) results in the lateral translation equation of

\[
\dot{V}_{oy} = \frac{\sum F_y - M_s \left( h_1 \ddot{\phi} \cos \phi - h_1 \dot{\phi}^2 \sin \phi - h_1 \dot{\psi}^2 \sin \phi \right)}{M} - V_{ox} \dot{\psi}.
\] (2.52)

Angular Motion Equations

The angular motion equations come from applying the torque-angular-momentum relation, which for the three masses that are part of the vehicle is given by

\[
\sum T_o = \frac{dH_{so}}{dt} + \frac{dH_{sfo}}{dt} + \frac{dH_{uro}}{dt} + V_o \times (M_s V_s + M_{sf} V_{sf} + M_{ur} V_{ur}).
\] (2.53)

Similar to the force-momentum relations, the vector equation in (2.53) can be separated into three scalar equations for rotations in each of the three directions.

Since roll only occurs in the sprung mass, the x-direction portion of (2.53) can be simplified to look at only the torques acting on the sprung mass about the x-axis. This results in

\[
\sum T_{ox} = \frac{dH_{s0x}}{dt} + (V_o \times M_s V_s)_x.
\] (2.54)
Making the appropriate substitutions from (2.6), (2.27) and (2.44) into (2.54) and solving for \( \dot{\phi} \) results in the roll equation of

\[
\ddot{\phi} = \frac{\sum T_x - I_{xx0} \ddot{\psi} \cos \phi - M_s h_s a_{sy} \cos \phi + \left( I_{yy} - I_{zz} + M_s h_s^2 \right) \ddot{\psi} \sin \phi \cos \phi}{I_{xx0}}
\] (2.55)

where two new parameters are defined as

\[
I_{xx0} \equiv I_{xx} + M_s h_s^2
\]
\[
I_{zz0} \equiv I_{zz} + M_s l_{cgs} h_s
\] (2.56)

The fourth equation of motion comes from the z-direction portion of (2.53), which is given by

\[
\sum T_z = \frac{dH_{x\phi}}{dt} + \frac{dH_{y\phi}}{dt} + \frac{dH_{w\phi}}{dt} + \left[ V_x \times \left( M_s V_z + M_{uf} V_{uf} + M_{ur} V_{ur} \right) \right]_z.
\] (2.57)

To get the fourth equation of motion, substitutions are made from (2.6), (2.14), (2.20), (2.27), (2.35), (2.37), and (2.46) into (2.57). Simplifications are made again using the fact that \( M_s l_{cgs} + M_{uf} l_f - M_{ur} l_r = 0 \) and one additional new parameter is defined as

\[
I_{zz0} \equiv I_{zz} \cos^2 \phi + M_s l_{cgs}^2 + \left( I_{yy} + M_s h_s^2 \right) \sin^2 \phi + I_{zz0} + M_{uf} l_f^2 + I_{zz} + M_{ur} l_r^2.
\] (2.58)

The end result is the yaw equation given by

\[
\ddot{\psi} = \frac{\sum T_z - I_{zz0} \ddot{\phi} \cos \phi + I_{zz0} \ddot{\phi}^2 \sin \phi - 2 \left( I_{yy} - I_{zz} + M_s h_s^2 \right) \ddot{\psi} \sin \phi \cos \phi + M_s h_s a_{sy} \sin \phi}{I_{zz0}}.
\] (2.59)

### 2.3.1.4.4. Motion Equations for the Four Wheels

The equations describing the angular motion of the four wheels are quite straightforward. Since the wheels rotate about their center of gravity, the torque-angular-momentum relationship is simplified to the normal form that is seen equating the net moment being equal to the moment of inertia times the angular acceleration. For each wheel, the moments acting on the wheel are the
longitudinal force on the tire times the radius of the tire and the moment applied by the brake system. The resulting equation for a tire is then given by

$$\dot{\phi} = \frac{F_{\text{ax}} R_w - M_m}{I_w}.$$  \hspace{1cm} (2.60)

The specific equations for each wheel are just this equation with the appropriate subscripts from 2.3.1.3 added to represent the wheel in question.

### 2.3.1.4.5. Summary of Equations of Motion

To summarize the above derivation of the equations of motion, there are eight degrees-of-freedom for the vehicle. For each of these degrees-of-freedom, there is one differential equation describing the motion of the vehicle for the associated degree-of-freedom. As mentioned previously, these eight degrees-of-freedom are

1. Translation in the longitudinal direction ($V_{ax}$),
2. Translation in the lateral direction ($V_{ay}$),
3. Body roll relative to the chassis about the roll axis ($\phi$),
4. Yaw about the vertical axis ($\psi$),
5. Rotation of the left front wheel ($\omega_{lf}$),
6. Rotation of the right front wheel ($\omega_{rf}$),
7. Rotation of the left rear wheel ($\omega_{lr}$), and
8. Rotation of the right rear wheel ($\omega_{rr}$).

In the same order, the differential equations associated with each of these degrees-of-freedom are

$$\dot{V}_{ax} = \frac{\sum F_x + M_x \left( 2 h_x \dot{\phi} \psi \cos \phi + h_y \dot{\psi} \sin \phi \right)}{M} + V_{ay} \dot{\psi}$$ \hspace{1cm} (2.61)

$$\dot{V}_{ay} = \frac{\sum F_y - M_y \left( h_y \dot{\phi} \cos \phi - h_x \dot{\phi}^2 \sin \phi - h_y \dot{\psi}^2 \sin \phi \right)}{M} - V_{ax} \dot{\psi}$$ \hspace{1cm} (2.62)

$$\dot{\phi} = \frac{\sum T_x - \sum \left( I_{xx \phi} \dot{\psi} \cos \phi - M_f h_{a_{xy}} \cos \phi + \left( I_{yy \phi} - I_{zz \phi} + M_l h_{x \phi}^2 \right) \dot{\psi}^2 \sin \phi \cos \phi \right)}{I_{xxx \phi}}$$ \hspace{1cm} (2.63)
\[
\dot{\psi} = \frac{\sum T_i - I_{xx0} \dot{\phi} \cos \phi + I_{xx0} \dot{\phi}^2 \sin \phi - 2 \left(I_{yy} - I_{zz} + M h_2^2\right) \dot{\psi} \sin \phi \cos \phi + M h_2 \alpha \sin \phi}{I_{xx0}}
\]

(2.64)

\[
\dot{\omega}_f = \frac{F_{xlf} R_w - M_{bfl}}{I_w}
\]

(2.65)

\[
\dot{\omega}_r = \frac{F_{xrf} R_w - M_{bfr}}{I_w}
\]

(2.66)

\[
\dot{\omega}_l = \frac{F_{xlr} R_w - M_{blr}}{I_w}
\]

(2.67)

\[
\dot{\omega}_r = \frac{F_{xrr} R_w - M_{br}}{I_w}
\]

(2.68)

These equations form the core of NAVDyn. With these equations developed, the next step is to define the forces and moments acting on the vehicle.

2.3.1.4.6. External Forces and Moments Acting on Vehicle

In the above equations of motion, the external forces and moments acting on the vehicle are assumed to be known. In this section the actual forces and moments used in the model are described.

The forces acting in (2.61) are given by

\[
\Sigma F_x = F_{df} + F_{xrf} + F_{xlr} + F_{xrr} + F_{sw}
\]

(2.69)

which is the sum of the longitudinal forces in chassis coordinates from all four tires plus the longitudinal aerodynamic forces. Since the tire forces are generated in a local wheel reference frame, the lateral and longitudinal tire forces must be multiplied by the appropriate sine and cosine of the steer angle for each wheel to convert to chassis coordinates. A description of how tire forces are generated is described in more detail in 2.3.2. Any other x-direction forces of interest could be added (rolling resistance of tires for example.)
The forces acting in (2.62) are similar and given by

$$\Sigma F_y = F_{yf} + F_{yrf} + F_{yfr} + F_{yrr} + F_{yw} \quad \text{ (2.70)}$$

In (2.63) the sum of the moments acting on the sprung mass about the roll axis (assumed the same as the x-axis) is given by

$$\Sigma T_{xs} = T_{\phi f} + T_{\phi r} + M_s g h_s \sin \phi + T_{\phi w} \quad \text{ (2.71)}$$

where the last term is the aerodynamic roll moment. The first two terms are given by

$$T_{\phi f} + T_{\phi r} = -(K_{\phi f} + K_{\phi r}) \phi - (B_{\phi f} + B_{\phi r}) \dot{\phi} \quad \text{ (2.72)}$$

where $K_{\phi}$ and $B_{\phi}$ are the roll stiffness and roll damping respectively.

For (2.64) the sum of the moments acting on the vehicle about the z-axis is given by

$$\Sigma M_z = (F_{yf} + F_{yrf}) l_f - (F_{yfr} + F_{yrr}) l_r + \frac{t}{2} \left( F_{yf} - F_{yrf} \right)$$

$$+ \frac{t}{2} \left( F_{yfr} - F_{yrr} \right) + M_{df} + M_{rf} + M_{df} + M_{rr} + M_{zr} \quad \text{ (2.73)}$$

where the first four terms are the influence of tire forces, again in chassis coordinates, the second four terms are the tire aligning moments and the final term is the aerodynamic yaw moment.

For the actual implementation of NAVDyn, aerodynamic forces and moments were not taken into consideration, but could be added at a later date. As seen in the above equations, adding the influence from the aerodynamics is quite straightforward. In addition, if there are any other external forces acting on the vehicle that a user may be interested in they simply need to be inserted into the appropriate equations above.

2.3.2. Tire Model

The selection of a tire model was very important for proper performance of the overall model. The vehicle model was created to investigate the influence of independent steering angles and
more specifically, the ability to stop the vehicle with the steering system by steering the wheels towards or away from each other. For this reason, it was very important to have a tire model capable of generating forces and moments for high slip angles. Under normal operation with both wheels steering together, a vehicle rarely ever experiences slip angles higher than ten degrees and most of the time operates below five degrees. Because of this, many tire models only look at the linear portion of the tire operating range. (See 3.3.1 for sample tire output curves.) For high slip angles, such models are not valid. Since a typical vehicle has a maximum steer angle of 35-40 degrees, the tire model used needed to be capable of generating reasonable output for slip angles at least to this level. The preference was to have a tire model that could cover the entire range from 0 to 90 degrees of slip angle. In addition, it was desired to have NAVDyn be useful for braking simulations also, so the tire model used needed to have full capability for tire slip ratios from 0 to 1.0 also. Several tire models were reviewed before selecting STIREMOD, [16], as the final choice.

2.3.2.1. Review of Tire Models

As with vehicle models, there is a broad range of tire models available. The most basic models are simple linear models that relate slip angle to lateral force through what is referred to as the cornering stiffness. Obviously such models would not work for this study. Beyond the linear models, several models with greater functionality, [17][18][19][20], were reviewed before STIREMOD was selected as the final model. The main ones reviewed are discussed below. In addition the work of Sakai in [21] was reviewed to obtain an overall understanding of the properties of tires. In [21] Sakai reviews a wide range of properties for rubber itself and for tires. Most of these characteristics are covered in the tire models reviewed below. However, some of these characteristics, such as change in tire output with velocity and with temperature, are not typically covered in tire models.

Pacejka’s “Magic Formula”

One of the commonly used tire models is referred to as the "Magic Formula" and was developed by Pacejka, [17]. This model has many of the same features as STIREMOD. By adjusting the parameters in the equation used to generate forces, the model can be made to fit a broad range of
tire data. The major downside to this model was that there was no information available as to the parameter values for the tires used on the Ford Taurus. As such, to use this model would have involved a significant amount of work to find the best parameters to fit the tire data for the Ford Taurus. This is the major reason the "Magic Formula" was not used. In contrast, STIREMOD is the model that was used in much of the work done for the National Advanced Driving Simulator program. Because of this, the parameters for STIREMOD were readily available. STIREMOD also is designed around the Calspan coefficients that are readily available for many different tires. Add to this that STIREMOD has all the functionality of any other model, and in some cases more, and STIREMOD was an easy choice for use in NAVDyn.

"Combinator" Model

Another model that was reviewed is referred to as the "COMBINATOR". This model was developed to predict tire forces using only information from straight-line acceleration/deceleration and free-rolling cornering data, [18][22]. The benefit of this model is that it requires significantly less test data in order to develop the parameters for the model. For the same reason that parameter data was not readily available for the Ford Taurus tires, this model was not chosen. However, it was from a discussion with Marion Pottinger, one of the developers of this model, that the approach for how to address high slip angles was found. In [22], the same approach as described in 2.3.2.4 below was used to cover high slip angles.

2.3.2.2. The Final Selection - STIREMOD

The final tire model chosen for use in NAVDyn is one referred to as STIREMOD. STIREMOD was developed by Systems Technology Inc. under a contract from NHTSA. There are several very nice features about STIREMOD that resulted in it being selected as the final model. First is that STIREMOD has been used in vehicle rollover and spinout simulations and has capability of generating reasonable output for the full 0 to 90 degree range of slip angle. Another significant reason STIREMOD was selected is that, as mentioned above, all of the necessary parameters for the tires used on the Ford Taurus were readily available. In addition, much of the simulation work that has been done as part of the NADS project used this model so information was readily available to compare against when validating the tire model once it was implemented.
The complete equations for STIREMOD are given in Appendix B. The basic features of the model are presented here. STIREMOD has two sets of equations that are used. STIREMOD basically takes slip angle, slip ratio, inclination angle and vertical load as inputs and generates lateral force, longitudinal force, and aligning moment. For full details of STIREMOD see [16] and [23]. The first set of equations is for the load varying parameters used in the main equations. Since a tire's output depends upon the vertical load on the tire, the first step is to calculate the values of the tire parameters for the current vertical load. Once this is done, these parameters are used in the main equations to calculate the outputs. The three main output equations relate slip angle to lateral force and aligning moment and relate slip ratio to longitudinal force. Tire inclination angle also has an influence on lateral force. Validation of the model and representative plots are shown in 3.3.1.

2.3.2.3. Tire Dynamics - Slip Angle and Slip Ratio

The tire models in use, including STIREMOD, do not typically include any dynamics in the generation of forces and moments. Most are simply mathematical relationships between the inputs, like slip angle and slip ratio, and the force and moment outputs. In reality there are lags involved in the generation of tire forces and moments. There is a delay from the time a steer angle or brake torque is applied to a wheel until the resulting forces and moments are developed.

The way this is handled is that these dynamics are modeled separately, typically as first order lags with appropriate time constants. Most models in the literature have included these lags on the output from the tire model and are only concerned with high speeds, [24]. There are two problems with this approach. First, due to the way slip angles and ratios are calculated using this approach, there is a division by the longitudinal speed of the wheel. This approach results in a divide by zero problem when running a simulation where the vehicle is brought to a stop. The typical equations for slip angle and slip ratio are shown below.

\[ \alpha = \tan^{-1}\left(\frac{V_{wy}}{V_{wx}}\right) \]  

\[ s = \frac{V_{wx} - R_0 \omega}{V_{wz}} \]
and are the longitudinal and lateral velocity of the wheel in local coordinates. As can be seen from both of these equations, when \( V_{wx} \) goes to zero, you have a divide by zero issue.

The second problem with this approach is that lags are typically only placed on lateral force, or maybe lateral and longitudinal force, but rarely on the aligning moment. This approach of adding the lags on the output from the tire is opposite from what actually happens in a vehicle. In reality, the lags occur directly on the slip angle and slip ratio. For example, when the steer angle of a wheel changes, there is a period of time required for the new slip angle to develop. As the slip angle changes, this changes the lateral force from the tire, not the other way around.

Bernard and Clover, in [25], describe a procedure for putting the lag on the slip angle and slip ratio directly, which is more in line with what actually happens with the tire. The two main advantages to this approach are that it addresses the mathematical issues in calculating the slip angle and slip ratio down to zero vehicle speed and it also applies lags to all tire outputs. The equations used to do this are:

\[
\frac{d}{dt} \left( \tan \alpha \right) + \frac{V_{wx}}{K_{sly}} \tan \alpha = \frac{V_{wy}}{K_{sly}} \tag{2.76}
\]

\[
\dot{s} + \frac{V_{wx}}{K_{sls}} s = \frac{V_{wx} - R_s \omega \text{sgn}(V_{wx})}{K_{sls}} \tag{2.77}
\]

where \( \alpha \) is the slip angle of the tire and \( s \) is the slip ratio. \( V_{wx} \) and \( V_{wy} \) are the longitudinal and lateral velocities of the wheel in wheel-fixed coordinates. \( K_{sly} \) is the tire side force lag characteristic distance and \( K_{sls} \) is the longitudinal force lag characteristic distance. The recommended values from [25] are used for both of these constants.

As can be seen from these equations, there are no divide by zero problems when vehicle speed goes to zero. In addition the dynamics involved are applied directly to the slip angle and slip ratio of the tire, which means that the lags involved will influence all tire outputs, as they should.
2.3.2.4. Limitations of Tire Test Data and Method of Addressing Limitations

As mentioned above, vehicles rarely ever operate in the region of slip angles above 10 degrees. Because of this, there is a significant lack of information about tire performance in this region. Several individuals in the tire industry were contacted to find information about how to address this region. In discussions with them, several things came up. First, within the automotive industry there are two different types of tire test machines in use. Most of the machines are only capable of testing a tire up to around 12 degrees of slip angle. This is more than adequate for most people since this fully covers the normal range of operation. There are a smaller number of tire test machines that are capable of slip angles up to around 30 degrees. However, because of potential damage to the machine if operated right to the limit, testing usually stops around 25 degrees. Due to these testing limitations, tire test data is not available in the range from 25 to 90 degrees of slip angle. Since the study for this thesis could involve slip angles as high as 40 degrees, this presented an issue.

Another issue that came up was the question of whether there are significant longitudinal tire forces developed when a tire is operated at high slip angles. Many people were consulted on this issue, but most did not know the answer since this is an unusual situation and as mentioned, tires are not generally tested much past 12 degrees of slip angle. After many discussions, it was suggested to look at a paper written by Amnon Sitchin from Ford Motor Company, [26]. In this paper, longitudinal force data is presented as a function of slip angle for slip angles up to 30 degrees. This data shows that there are not significant longitudinal forces developed over this range. It also appears that longitudinal force tends to stay constant or even go down as slip angle increases. Although this does not cover the full range to 90 degrees, it strongly suggests that any longitudinal forces developed are not significant in comparison to the lateral forces developed. The longitudinal forces that are developed appear to be mostly due to rolling resistance.

Methodology to Address Limits in Available Tire Data

An interesting approach was used to address the limitation in tire test data described above. As mentioned above, this approach comes from [22]. The assumption that forms the basis to this approach is this: a tire operating at 90 degrees of slip angle is essentially the same as a tire
operating at a slip ratio of 1.0. In other words, sliding the tire sideways on the road is essentially the same as a rolling tire that locks up due to braking. This seems like a reasonable assumption in that if you had a block of rubber and slid it on the ground, the force to slide it longitudinally or laterally should be the same. Without test data available, this seemed like a good approach to take.

The benefit of this approach is that test data is available for slip ratios of 1.0. So with the above assumption, data is available for slip angles up to 25 degrees and then a single point at 90 degrees. The final step is to assume a smooth curve connecting the data between 25 and 90 degrees. Again, this seems like a reasonable assumption. Tires typically reach their peak lateral force at around 7 degrees of slip angle. Beyond the peak, the force vs. slip angle curve has a fairly constant negative slope. There is no reason to think that something unusual would happen between 25 and 90 degrees. This approach is supported by the work of Sakai in [21], which shows smooth curves of lateral force versus slip angle for the entire operating range of 0 to 90 degrees of slip angle.

So the end result is that using STIREMOD, the full range of tire operation from 0 to 90 degrees of slip angle can be simulated. Using the actual test data for a tire, the model parameters can be set to provide good results in the range the data is available. Then using the data point at 90 degrees the model can be adjusted to get a smooth curve from the end of the actual tire data out to the point at 90 degrees. Although it has not been discussed here, since tires can be tested for the full range of slip ratios, STIREMOD, as well as many other tire models, provides very good predictions of longitudinal force.

There are a couple other things to note related to the lack of tire test data in the 25 to 90 degrees of slip angle range. First, this is an obvious area that could use further study. Since there is no significant need for such data with current steering systems there is no push to develop testing strategies to cover the entire operating range. However, with increased interest in steer-by-wire systems within the automotive industry, there may become increased interest in this information in the near future.
The second item of note is that the influences of temperature and tire wear when operating a tire at these high slip angles is unknown. This is potentially a significant factor. The influence of temperature on tire output is discussed in [21] showing that tire output actually goes up at high slip angles as temperature increases from 10 to 100 degrees Celsius, but the relationship between tire slip angle, time and temperature is not is not covered. This relationship is necessary before any attempt at including the influence of temperature in a tire model could be added. It is also possible that temperatures beyond 100 degrees Celsius may occur. Most tire models, including STIREMOD, do not cover the influence of temperature. Fortunately the intent in this research is to only operate at high slip angles for emergency conditions. Still, how the tire responds when operated at high slip angles is not understood. It is possible that such operation even for a short period of time could result in high temperatures and/or significant tire wear. How this impacts the generation of forces and moments from the tire is an area that needs further study.

2.3.2.5. Tire Normal Forces

Another input for the tire force calculations is the normal force on each tire. For these calculations, an approach was taken that looks at the weight transfers that occur during dynamic maneuvers. These include longitudinal acceleration and lateral acceleration weight transfers plus the weight transfer that is due to the roll stiffness and roll damping. The roll dynamics are not taken into consideration directly, but only through the roll angle and roll rate. With this approach, you start from the static weight at each tire and then add or subtract the effects of each of the weight transfers.

Normal loads are calculated quasi-statically by summing the moments about the tire contact patches. For the longitudinal weight transfer, start by summing the moments about the rear axle contact patch:

\[
(F_{zf} + F_{cf})L - Mg l + M_{s} a_{sx} h_{cg} + M_{sf} a_{sy} h_{cg} + M_{ur} a_{ux} h_{cgur} = 0 .
\] (2.78)

Letting \( F_{zf} = F_{zf} + F_{cf} \) and solving for \( F_{zf} \) gives
\[ F_{d} = \frac{Mgl_{r}}{L} - \frac{(M_{r}a_{x_{r}}+h_{r_{x_{r}}} + M_{u_{f_{r}}}a_{u_{f_{r}}h_{f_{g_{y_{f_{r}}}}}} + M_{u_{r_{r}}}a_{u_{r_{r}}h_{r_{g_{y_{r}}}}})}{L} \]  \hspace{1cm} (2.79) \\

Assuming the effect of the roll angle on the longitudinal weight transfer is negligible, then

\[ F_{df} = F_{cf} = \frac{1}{2} F_{d} = \frac{Mgl_{r}}{2L} - \frac{(M_{r}a_{x_{r}}+h_{r_{x_{r}}} + M_{u_{f_{r}}}a_{u_{f_{r}}h_{f_{g_{y_{f_{r}}}}}} + M_{u_{r_{r}}}a_{u_{r_{r}}h_{r_{g_{y_{r}}}}})}{2L} \]  \hspace{1cm} (2.80) \\

where the first term is simply the static weight on each front wheel and the second term is the longitudinal weight transfer. For the rear wheels, the result is simply the static weight on each rear wheel plus the longitudinal weight transfer:

\[ F_{dr} = F_{cr} = \frac{1}{2} F_{r} = \frac{Mgl_{f}}{2L} + \frac{(M_{r}a_{x_{r}}+h_{r_{x_{r}}} + M_{u_{f_{r}}}a_{u_{f_{r}}h_{f_{g_{y_{f_{r}}}}}} + M_{u_{r_{r}}}a_{u_{r_{r}}h_{r_{g_{y_{r}}}}})}{2L} \]  \hspace{1cm} (2.81) \\

As can be seen, positive acceleration transfers weight from the front wheels to the rear wheels.

The next step is to calculate the lateral acceleration weight transfer. Summing the moments about the left front tire gives

\[ -F_{cf_{f}}t_{f} + \frac{Mgl_{r}a_{y_{f}}}{2L} = -M_{r}a_{y_{r}}l_{f}h_{f} - M_{u_{f}}a_{u_{f}}h_{f_{g_{y_{f_{r}}}}} = 0 \]  \hspace{1cm} (2.82) \\

Solving for \( F_{cf} \) gives

\[ F_{cf} = \frac{Mgl_{r}}{2L} - \frac{1}{t_{f}} \left( \frac{a_{y_{f}}l_{f}h_{f}}{L} + M_{u_{f}}a_{u_{f}}h_{f_{g_{y_{f_{r}}}}} \right) \]  \hspace{1cm} (2.83) \\

Again the first term is the static weight on the right front wheel and the second term is the weight transfer. Following a similar process for the other three wheels, the weight on each of them is

\[ F_{df} = \frac{Mgl_{r}}{2L} + \frac{1}{t_{f}} \left( \frac{a_{y_{f}}l_{f}h_{f}}{L} + M_{u_{f}}a_{u_{f}}h_{f_{g_{y_{f_{r}}}}} \right) \]  \hspace{1cm} (2.84)
\[ F_{zr}^{\text{dr}} = \frac{Mgl_f}{2L} \pm \frac{1}{t_r} \left( \frac{M_{ag}L_jh_r}{L} + M_{ur}a_{ury}h_{gur} \right). \] (2.85)

The final weight transfer is due to the roll stiffness and roll damping and is given by

\[ F_{zr}^{\text{sf}} = \frac{Mgl_r}{2L} \pm \left( \frac{K_{\phi} \dot{\phi} + B_{\phi} \dot{\phi}}{t_f} \right). \] (2.86)

\[ F_{zr}^{\text{dr}} = \frac{Mgl_f}{2L} \pm \left( \frac{K_{\phi} \dot{\phi} + B_{\phi} \dot{\phi}}{t_r} \right). \]

As can be seen again, the first term is the static weight and the second is the weight transfer.

Summarizing the results from above we get the following equations for the weight on each tire:

\[ F_{zf} = \frac{Mgl_f}{2L} - F_{zax} + F_{zxy} + F_{z\phi f} \] (2.87)

\[ F_{zf} = \frac{Mgl_r}{2L} - F_{zax} - F_{zxy} - F_{z\phi f} \] (2.88)

\[ F_{zf} = \frac{Mgl_f}{2L} + F_{zax} + F_{zxy} + F_{z\phi r} \] (2.89)

\[ F_{zf} = \frac{Mgl_f}{2L} + F_{zax} - F_{zxy} - F_{z\phi r}. \] (2.90)

The individual weight transfers are given by

\[ F_{zax} = \frac{M_i h_i a_{sx} + M_{if} h_{if} a_{gyx} + M_{ur} h_{ur} a_{xy}}{2L} \] (2.91)

\[ F_{zxy} = \frac{1}{t_f} \left( \frac{M_i h_i a_{sy} + M_{if} h_{if} a_{dy}}{L} \right) \] (2.92)

\[ F_{zxy} = \frac{1}{t_r} \left( \frac{M_i h_i a_{sy} + M_{ur} h_{ur} a_{xy}}{L} \right). \] (2.93)
\[ F_{z\phi} = -\frac{1}{f_f} (K_{\phi f} \phi + B_{\phi f} \dot{\phi}) \] (2.94)

\[ F_{z\theta} = -\frac{1}{f_r} (K_{\phi r} \phi + B_{\phi r} \dot{\phi}) \] (2.95)

where the vehicle accelerating in a right turn results in positive values for all of the weight transfers.

2.3.2.6. Tire inclination angles

The final inputs for the tire force calculations are the tire inclination angles. Tire inclination angle is the same as the camber angle with the exception that camber angle is measured as positive when the top of the wheels lean away from the center of the car. Inclination angle is positive when the wheels have a positive rotation about the x-axis. The angles are calculated by taking the nominal camber angle and adding to it the influences of roll camber and compliance camber. The result of this is multiplied by the cosine of the steer angle since the tire inclination angle changes as the wheel turns about the kingpin.

For simplicity, two assumptions have been made. First, it is assumed that the nominal kingpin axis angle from the front view is the same as the nominal camber angle. Second, the influence of caster angle has been neglected, meaning it is assumed that the kingpin is vertical from the side view. Since these are secondary effects on the camber angle and camber has only a minor influence on tire forces, these should be reasonable assumptions.

With the above assumptions, the inclination of each wheel is given by

\[ \gamma_f = \left( \gamma_{0f} + K_{r1f} \phi - K_{r2f} \phi^2 - K_{camberf} F_{yf} \right) \cos \delta_f \] (2.96)

\[ \gamma_r = \left( \gamma_{0r} + K_{r1r} \phi + K_{r2r} \phi^2 - K_{camberr} F_{yf} \right) \cos \delta_r \] (2.97)

\[ \gamma_l = \left( \gamma_{0l} + K_{r1l} \phi - K_{r2l} \phi^2 - K_{cambler} F_{yl} \right) \cos \delta_r \] (2.98)
\[
\gamma_{rr} = \left( \gamma_{0rr} + K_{y1}\phi + K_{y2}\phi^2 - K_{ccamb}F_{yrr} \right) \cos \delta_{rr}
\] (2.99)

where \( \gamma_0 \) is the nominal angle, the \( K_y \) terms are from a quadratic curve fit of the vehicle roll camber data with appropriate terms for front and rear, \( K_{ccamb} \) is the linear compliance camber, and \( \delta \) is the steer angle of the wheel in question.

### 2.3.2.7. Suspension and Steering Effects

There are numerous secondary effects to vehicle handling that are caused by the kinematics and compliances in the suspension and steering systems. The effects included in this vehicle model include:

- roll steer,
- roll camber,
- suspension compliance steer,
- suspension compliance camber,
- steering kinematics, and
- steering compliance steer.

Roll camber and suspension compliance camber both influence the tire inclination angle and were already discussed above. The other influences will be described here and all have an impact on the actual steer angle of each wheel. To get the final steer angle of each wheel, start with the nominal toe angle plus the steer angle from the steering system and then add each of the above steer effects.

The first influence to be discussed is the steering kinematics. With most steering systems there is assumed to be a constant steer ratio that relates the input angle from the steering wheel to the output angle of the road wheel. Due to the kinematic linkages of the steering system, this ratio actually changes as the road wheel rotates. A second order fit of the actual vehicle data was done to capture the influence of the steering system kinematics. Since this study only looks at front wheel steering, this influence only affects the front wheels. The implementation is shown in Figure 4-1.
The second influence is roll steer. When the body of the vehicle rolls relative to the chassis, the steer angle of the wheels changes. This is due to the fact that the linkage between the steering gear and the steer arm is typically not horizontal and its angle changes as the body rolls. This can be seen in Figure 2-5. This change in angle causes movement of the steer arm, which in turn changes the angle of the road wheel. A second order curve fit of the actual vehicle data was done to capture this influence. Roll steer affects both front and rear wheels, although the influence is different due to differences in suspension geometry. Even though the rear does not have a steering gear, there is a similar influence due to the suspension kinematics.

There is a similar influence due to compliance in the suspension and steering systems. Due to the tire forces and moments acting on the wheel, the suspension and steering systems both experience some deflection. This deflection causes a change in the steer angle of the wheel. These influences are captured through three linear coefficients, one for longitudinal force, one for lateral force, and the third for aligning moment. The coefficients are determined from vehicle data.

Combining all of the above influences, the equations for the final steer angle of each road wheel are shown below. The terms in the equations are in the same order as described above: nominal toe angle, steer angle from steering system, steering kinematics, roll steer, longitudinal force toe-in, lateral force compliance steer, and aligning moment compliance steer.

\[
\delta_y = \delta_{0y} + \delta_{1y} + K_4 \delta_{2y}^2 - \epsilon_1 \phi + \epsilon_2 \phi^2 + \epsilon_3 F_{x\delta} + \epsilon_4 F_{y\delta} + \epsilon_5 M_{\delta} \quad (2.100)
\]
\[ \delta_d = \delta_{0d} + \delta_{uf} - K_{uf} \delta_{uf}^2 - \epsilon_{f1} \phi - \epsilon_{f2} \phi^2 - \epsilon_{fg} F_{xuf} + \epsilon_{fg} F_{yuf} + \epsilon_{mg} M_{zuf} \quad (2.101) \]

\[ \delta_r = \delta_{0r} + \delta_{sur} - \epsilon_{s1} \phi + \epsilon_{s2} \phi^2 + \epsilon_{sgr} F_{xsr} + \epsilon_{sgr} F_{ysr} + \epsilon_{msr} M_{zsr} \quad (2.102) \]

\[ \delta_r = \delta_{0rr} + \delta_{srr} - \epsilon_{s1} \phi - \epsilon_{s2} \phi^2 - \epsilon_{sgr} F_{xrr} + \epsilon_{sgr} F_{yrr} + \epsilon_{msr} M_{zrr} \quad (2.103) \]

2.3.3. Steering System

The default steering system in NAVDyn was created to represent the characteristics of the hydraulic steering system of the 1994 Ford Taurus GL. This system will be described here. In Chapter 5, the implementation of the closed-loop steering control algorithm is covered. For the 1994 Ford Taurus GL, both front wheels steer together through a single steering gear. This makes the steering system very simple to model. In NAVDyn, the dynamics of the steering system are not modeled, but it is simply treated as a kinematic relationship between steering wheel angle and road wheel angle. The assumption is that the steering system dynamics are fast enough relative to the vehicle dynamics that they can be neglected. In addition, a filter is placed on the steering wheel input to prevent dramatic changes in steering wheel angle and angular velocity. With this approach, modeling the steering system is done by simply dividing the steering wheel angle by the steering ratio to get the nominal road wheel angle. This nominal angle is then modified by the steering influences in 2.3.2.7 to get the final road wheel angle.

Since this study involves only front wheel steering, the nominal steer angle of the rear wheels is set to zero. However, if four wheel steering were desired, the relationships defining the rear steer angles would simply need to be defined and they could easily be implemented. See 3.2.1.2 for the actual implementation of the steering system.

2.3.4. Braking System

The brake system was developed to represent the brake system in the 1994 Ford Taurus GL. The major difference is that the Taurus has anti-lock brakes and NAVDyn does not. For this reason NAVDyn is only valid for braking up to the limit of adhesion. Once wheel lock-up is achieved NAVDyn will no longer follow the performance of the actual vehicle.
The brake system is modeled with components that follow those in the actual vehicle. Brake pedal force is the input to the brake system. Based upon actual vehicle data there is a small dead zone in pedal force before any braking occurs. After the dead zone, there is a constant used to convert brake pedal force into brake system pressure. Brake system dynamics are modeled as a first order lag in the generation of brake pressure.

As is typical of most automotive brake systems, the full brake system pressure is applied to the front brakes, but a proportioning valve is used to reduce the pressure on the rear brakes. The pressure to the front and rear brakes is then multiplied by the appropriate brake gain for the front and rear brakes, which converts brake pressure into brake torque. All of the parameters used to define the brake system are generated from vehicle data for the Ford Taurus.

The equations used to model the brake system as described above are given here:

\[
\dot{P}_{br} = \frac{K_{brip} \left( F_{br} - B_{db} \right) - P_{br}}{\tau_b}, \quad F_{br} \geq B_{db}
\]

(2.104)

\[
P_{br} = 0, \quad F_{br} < B_{db}
\]

\[
T_{wfr} = T_{wrr} = -K_{brkf} P_{br}
\]

(2.105)

\[
T_{wfr} = T_{wrr} = -K_{brkr} P_{br}, \quad P_{br} < P_{prop}
\]

\[
T_{wfr} = T_{wrr} = -K_{brkr} \left[ P_{prop} + K_{prip} \left( P_{br} - P_{prop} \right) \right], \quad P_{br} \geq P_{prop}
\]

(2.106)

The variables and parameters used in the above equations are:

- \( B_{db} \) = Brake system deadband in pedal force before pressure begins,
- \( F_{br} \) = Brake pedal force,
- \( K_{brip} \) = Gain relating brake pedal force to brake system pressure,
- \( K_{brkf} \) = Gain relating pressure to torque for front brakes,
- \( K_{brkr} \) = Gain relating pressure to torque for rear brakes,
- \( K_{prop} \) = Proportion of brake pressure to rear brakes after \( P_{prop} \) is reached,
- \( P_{br} \) = Brake system pressure,
- \( P_{prop} \) = Pressure after which the rear brake pressure is proportioned, and
- \( T_{w} \) = Brake torque for a wheel.
For the equations above, (2.104) relates pedal force to brake pressure and includes the dead zone, the pedal force to brake pressure conversion and the brake system dynamics. (2.105) is the simple relationship between brake pressure and front wheel torque. (2.106) gives the relation between brake pressure and rear brake torque and includes the influence of the proportioning valve. The implementation of the brake system is shown in 3.2.1.2.

2.4. Summary

In summary, this chapter has covered the generation of all the important equations that define the vehicle model called NAVDyn. The equations of motion for the lumped masses were derived for given forces acting on the vehicle. The tire model, STIREMOD, that generates the tire forces that act on the vehicle was reviewed, including how the inputs to the tire model are calculated. Finally the steering and braking systems were covered discussing how the driver inputs of steering wheel angle and brake pedal force are converted to steer angles of the road wheel and brake torques on each of the wheels.
3. MODEL IMPLEMENTATION AND VALIDATION

3.1. Introduction

In this chapter the implementation of the model for use in simulation and the validation of the model for correct output are covered. In 3.2 the actual implementation of NAVDyn is covered in detail showing how the equations generated in Chapter 2 were put into use for simulation using Matlab/Simulink. Then in 3.3 the detailed validation of NAVDyn against two data sources is presented showing how NAVDyn compares against actual vehicle data as well as a high-order model.

3.2. Model Implementation

With all of the equations of the model complete, the next step was implementation of the model for use in simulation. The model was implemented using Matlab/Simulink [27]. In particular, Matlab v5.2.1 and Simulink v2.2 were used. As mentioned in 2.2 NAVDyn was implemented following closely the format developed by Ashok Chandy. The implementation was done in a way that is easy to customize. The model is broken up into several connected subsystems that mirror those found in an actual vehicle. Each subsystem in turn is itself made up of subsystems. The flow of information between subsystems is easy to follow and each individual subsystem can be customized without impacting the rest of the model. The end result is a model that provides excellent results for most handling scenarios and is easy to customize for a specific study.

Before getting into the structure of the implementation, it is worth noting that the implementation was done in a way that is generic to any particular vehicle or tire. Every parameter that is used within the model to define the vehicle and the tires is simply given a variable name. See Appendix C for the tire parameters used and Appendix D for the parameters used for the default vehicle, the 1994 Ford Taurus GL. All of the vehicle parameters are defined within a single M-file within Matlab. Likewise, all of the tire parameters are defined within another M-file. With this approach, the user can define the parameters within these files for any vehicle and/or tires
desired. Simply use the M-file that represents the particular vehicle desired and the model then
represents that vehicle. The same is true for the tires.

3.2.1. Overall Model Structure

The model was implemented using connected subsystems, which clearly show the flow of
information between subsystems. Each subsystem is generally composed itself of several
connected subsystems such that each layer deeper you go in the model, the more detail is
available. Every connection between subsystems is labeled defining the output from one
subsystem and the input to the next. In this way, at all levels it is easy to follow the flow of
information and to know what the inputs and outputs are. This makes later customization easier
since when changes are desired it is easy to find the needed information.

3.2.1.1. Top Level User Interface

The highest level of the model simply has the driver and the overall vehicle, as shown in Figure
3-1. The driver generates steering wheel angle and brake pedal force, which are inputs to the
vehicle. The vehicle then generates outputs describing the response of the vehicle to the driver
inputs. The vehicle model block contains the implementation of all of the equations developed in
Chapter 2. This block is itself composed of several subsystems described in the next section.

The user defines the steering and braking inputs from the driver. There are several default
choices for inputs, but the user could define any input they desire. The default input choices for
the steering system are shown in Figure 3-2 and are a sine wave, a ramp, a step, a triangular
pulse, a square pulse and an input from the workspace. The step and the two pulses are given as
combinations of ramps due to the fact that a real driver can never put in a pure step change in
steering wheel angle. The slope of the ramps used should be selected to represent a reasonable
handwheel speed. The choice of getting the input from the workspace allows the use of inputs
generated from other sources. For example, when conducting validation on the model, the actual
steering wheel input used on the test vehicle was loaded into the workspace and then used as the
input for the model.
For all of the inputs except getting the input from the workspace, the result is run through a simple first order filter to take out all sharp corners since an actual driver would have a smooth input. Since the data loaded from the workspace is usually measured from an actual vehicle, there is no need to filter this data. The brake pedal inputs are shown in Figure 3-3 and are virtually the same except there is no sine wave and a limit block is used to restrict pedal force to be between 0 and 200 N. Since the steering actuators have maximum angle limits on them there is no limit block placed on the steering wheel angle directly.

![NAVDyn v1.4 Diagram]

Figure 3-1: NAVDyn Top-Level User Interface
Figure 3-2: NAVDyn Steering Wheel Inputs

Figure 3-3: NAVDyn Brake Pedal Inputs
3.2.1.2. Vehicle Model Showing Steering and Braking Subsystems

Going one level deeper, the vehicle is composed of several connected subsystems: steering system, braking system, cruise control, and vehicle dynamics. This is shown in Figure 3-4. The driver steering and braking inputs go into the steering and braking systems respectively to generate steer angle and brake torque at each of the wheels. These then become inputs to the vehicle dynamics. The vehicle dynamics generate the response of the vehicle to the given inputs.

**Figure 3-4: Vehicle Model Block**

At this level, the steering and braking systems are black boxes. The user can put any systems they desire within these boxes without requiring any additional changes to the rest of the model. For example, two-wheel or four-wheel steering, standard hydraulic, manual or electric steering, or any other desired steering system that can be thought of could be used, so long as the outputs of the steering system are the four desired road wheel angles. The same is true for the braking system, as long as the outputs are the four wheel-torques from the braking system. Figure 3-5 shows the steering system used for this project. The Normal Steering block is shown in Figure 3-6. The other blocks will be described in Chapter 5. Figure 3-7 shows the brake system used in NAVDyn.
From these figures it can be seen that any additional inputs needed for a particular system can be easily added. For example, if it is desired that the brake system perform some type of yaw control by applying differential braking, it may be necessary for the brake system to have information about steering wheel angle and vehicle yaw rate. This simply requires creating two more inputs within the brake system and then connecting them to the desired signals, which are readily available. As seen in the steering and braking systems above, additional inputs have been added that were needed for this study.
The cruise control block is shown in Figure 3-8. Cruise control was added for the ability to maintain a specific vehicle speed during lateral response tests. This was particularly necessary when comparing NAVDyn against the actual vehicle data since the actual vehicle had the cruise control engaged during lateral response testing. The values of the parameters used in the cruise control are the same as those used in CarSim. The cruise control is basically a proportional-integral control of vehicle speed. The set speed for the cruise can be either a constant value or a value from the workspace. For validation of the model, the actual vehicle speed was used as the set speed so that the speed of the vehicle in NAVDyn would track the actual vehicle’s speed. The only other features of the cruise control are the switches. The cruise control is only activated if the cruise status in Figure 3-1 is ON and there is no brake input.

This implementation was adequate for the purposes of this study, but could be improved. In particular, for a real cruise control system, once there is a brake pedal input, the cruise is deactivated, even after the brake input is removed. For this implementation, the cruise control will come back on if the brake input is removed. This implementation also allows for acceleration and deceleration, where an actual cruise control system only provides acceleration. This was done because NAVDyn does not include aerodynamic forces and other losses so the vehicle will not decelerate as it would in reality. To be able to track the speed of the test vehicle, this was necessary. In addition, since NAVDyn does not include any powertrain dynamics, the cruise should only be used to operate in close proximity to the given set speed. The dynamics

**Figure 3-7: NAVDyn Brake System**
from the cruise control are not valid if the cruise is used to make large accelerations or decelerations.

Gains based upon those used in CarSim
converted for proper units
\[ K_p = 0.141 \text{ rev/s} = 0.277 \text{ m/s} \]
\[ K_i = 0.157 \text{ rev/s}^2 = 0.308 \text{ m/s}^2 \]
Driveline gain = 500 Nm/driver input (one wheel)
Time constant = 0

Previous values
\[ K_p = 200 \]
\[ K_i = 20 \]
Filter gain = 1
Filter time const = 0.5

Figure 3-8: Cruise Control

3.2.1.3. Vehicle Dynamics Implementation

The next subsystem is the vehicle dynamics block, which contains most of the equations developed in Chapter 2. The vehicle dynamics block is split up into several subsystems, shown in Figure 3-9, that follow what occurs in the actual vehicle. First, the various steer effects from the suspension and steering, as described in 2.3.2.7, influence the ideal steer angles as shown in Figure 3-10. The resulting actual steer angles, as well as the brake torques, are inputs into the wheels, along with additional information generated by the model.

The four main blocks of the entire model are those for the wheels, tires, body, and normal force. These form the core of NAVDyn. These blocks contain the information you may expect. The wheel block does all of the conversion between wheel coordinates and chassis coordinates and also contains the calculations for slip angles, slip ratio, and tire inclination angles. The tire block contains the implementation of STIREMOD. The body block contains the equations of motion for the vehicle. Finally, the normal force block calculates the vertical load on each tire.
Figure 3-9: Vehicle Dynamics Subsystem

Figure 3-10: Suspension Steer Effects
3.2.1.3.1. Wheels

The wheel subsystem is shown in Figure 3-11 and contains many different calculations. The first is calculation of the angular velocity of the four wheels. Next the longitudinal and lateral speeds of each wheel are calculated. This information, along with the angular velocities, is then used to calculate the slip angle and slip ratio for each wheel. Due to the fact that STIREMOD has some numerical problems with slip angles of \( \pm 90 \) degrees and slip ratios of \( \pm 1.0 \), limit blocks are used to limit slip angle to \( \pm 89 \) degrees and slip ratios to \( \pm 0.99 \). This should have virtually no impact since the difference between 89 and 90 degrees slip angle and 0.99 and 1.0 slip angle is negligible.

**Figure 3-11: Wheels**
The next calculations are the conversion of tire forces from wheel coordinates to chassis coordinates. This is necessary since the equations of motion in the body block are based upon forces in chassis coordinates. The final calculations are for the tire inclination angles.

### 3.2.1.3.2. Tires

The next subsystem is that for the tires and is shown in Figure 3-12. As mentioned, this is simply an implementation of the equations for STIREMOD. These equations can be found in Appendix B. The first thing to notice is that there are several conversions for the inputs and outputs in this block. STIREMOD was developed using English units and so all of the equations and the parameter values for the tires are based upon English units. Since an equivalent metric version of STIREMOD was not available, rather than try to conduct the conversion myself and risk making an error, the inputs are simply converted to English, the calculations made, and then the outputs are converted back to metric.

Figure 3-12: Tire Subsystem

One other item to note is that the normal force going into the tires goes through a memory block. There is an algebraic loop formed when trying to calculate tire forces and normal forces in the
same time step. To resolve this issue, the normal force from the previous time step is used in the
tire calculations. Since the normal force does not change rapidly and since it has a secondary
influence on tire outputs by changing the parameters used in the tire model, this has a negligible
influence.

Going one step deeper, the implementation of STIREMOD for one tire is shown in Figure 3-13.
As seen, the inputs are slip angle, slip ratio, normal force, inclination angle, and longitudinal
speed. It can be seen that the longitudinal force from the tire is used itself as an input to the tire
parameter calculations. Obviously this can not be done in the same time step and so a memory
block is used so that the tire input parameters are calculated based on the longitudinal force from
the previous time step. One additional item to note is that a switch is used on the normal force.
This is because the tire equations have a division by the normal force. This causes a problem if
normal force goes to zero and so the switch is used so that if normal force drops below one, then
a value of one is used. This condition should rarely occur since it means that a wheel is lifting off
of the ground. Even if it does occur, this approach prevents any numerical problems.

![Figure 3-13: Implementation of STIREMOD for One Tire](image)

The calculations for the input parameters are shown in Figure 3-14. As discussed in Appendix B,
the input parameters vary as a function of normal load. In addition, two of the parameters vary
with longitudinal force as well. One difference from the equations shown in Appendix B is that in the calculations for the peak coefficient of friction, instead of using the skid numbers, the nominal coefficient of friction of the road surface times a constant is used. These are equivalent, but it is more convenient to use the actual coefficient of friction than to use the skid number. The skid number is simply the coefficient of friction times 100. In the equations in Appendix B, the skid number of the simulation road surface is divided by the skid number for the tire test machine used to measure the tire. The value for most tire test machines is 85 or a coefficient of friction of 0.85. If the simulation road surface also has a coefficient of friction of 0.85, then the result is unity. For the implementation below, the simulation coefficient of friction is multiplied by 1.1765, which is simply 1 divided by 0.85. With this implementation, if the tires were tested on a surface with other than 0.85 coefficient of friction, then the constant used would need to be changed accordingly.

![Diagram of Tire Input Parameters](image)

**Figure 3-14: Tire Input Parameters**

The calculations of the tire outputs are shown in Figure 3-15. Again this is just an implementation of the equations in Appendix B, with one exception. In the equations for STIREMOD the coefficients of decay in friction with slip angle and slip ratio, \( K_{\mu x} \) and \( K_{\mu y} \), are constant. However, it has been shown by Sakai, [21], that these coefficients actually vary with vehicle speed. For the implementation of STIREMOD here, these coefficients have been
implemented as functions of vehicle speed using the method found in [12]. The effect of this is that friction level decays more with slip as vehicle speed increases. The result of this can be seen in some of the plots in 3.3.1 where tire forces increase as vehicle speed decreases for the same level of slip. One final note about the implementation of these speed-varying parameters is that the value of vehicle speed is limited to being greater than or equal to one. Once vehicle speed drops below one, the parameters start to get larger as vehicle speed decreases, which is not correct. This is the reason for the limit. Other than the difference in the coefficients of decay in friction, the equations shown are the equations for STIREMOD from Appendix B.

Figure 3-15: Tire Output Calculations

3.2.1.3.3. Body

The next block to cover is the body, which is shown in Figure 3-16. Within the body block are all of the equations of motion for the vehicle that are found in 2.3.1.4.5, excluding the wheel speeds, which were already covered in 3.2.1.3.1. As can be seen, there is a separate block for each of the four body degrees-of-freedom and all of the appropriate connections are shown. The inputs are
the tire forces and moments acting on the vehicle. The outputs are the associated accelerations, velocities and displacements. The longitudinal dynamics are shown in Figure 3-17. This is an implementation of (2.61) with one addition. There is a feature added that stops the simulation once vehicle speed reaches zero. Figure 3-18, Figure 3-19 and Figure 3-20 give the implementations of (2.62), (2.63), and (2.64) respectively.

Figure 3-16: Body Subsystem
LONGITUDINAL DYNAMICS

Figure 3-17: Longitudinal Dynamics

LATERAL DYNAMICS

Figure 3-18: Lateral Dynamics
3.2.1.3.4. Normal Force

The final portion of the implementation is for the normal force, shown in Figure 3-21. This is simply the implementation of the equations from 2.3.2.5. First each of the appropriate accelerations is calculated. The results are then used to calculate the individual weight transfers, which are added together with appropriate signs to the static weight for each tire.
3.3. Model Validation

Two separate steps were taken to validate NAVDyn. First the implementation of STIREMOD was validated for correct output. Once the tire model was known to be correct, the overall vehicle model was then validated. This approach made debugging easier since it was already known that the tires were giving correct output. As an added note, the above implementation of NAVDyn had the benefit that debugging of the model was simplified. Since each subsystem can itself be checked for correct input/output relations, this made finding errors easier. If the overall output of the model was not correct, you simply needed to go down one level deeper to find the subsystem causing the problem. This process is continued until the actual error is found and results in not needing to check the portions of the model that are providing correct output.

3.3.1. Tire Model Validation

Before starting validation of the complete model, the decision was made to validate the implementation of STIREMOD for correct output. This helped in debugging the overall model since it was already known that the tires were providing proper input/output relations. There were
two portions to this validation. First the model output was objectively compared against the plots in [16]. In addition plots relating tire inputs and outputs were sent to Jeffrey Chrstos who conducted some of the testing for the NADS project while he worked for the National Highway Traffic Safety Administration (NHTSA). He compared them against data he has for the tires used on the Ford Taurus. The second part of the validation was more subjective. For those portions of the tire model for which plots were not available for comparison against, simulations were conducted and the results checked to confirm directional correctness and that the magnitude of the response was reasonable.

As described below, the 1994 Ford Taurus GL was selected as the target vehicle for validation. For this reason the Ford Taurus tires were used in the model. The Taurus tires are General Ameritech P205/65R15 steel belted radials. Jeffrey Chrstos provided a file containing all of the parameters needed by STIREM for these tires. The parameters used can be found in Appendix C.

3.3.1.1. Objective Tire Simulation Comparisons

In [16] there were four different conditions evaluated comparing STIREM against actual tire data. These conditions were lateral force versus slip angle, aligning moment versus slip angle, longitudinal force versus slip ratio, and lateral force versus longitudinal force. The same plots were created using the implementation of STIREM found in NAVDyn. The results are shown in Figure 3-22 through Figure 3-25. The results match those in [16] very well. The first three plots show the force generation for slip angle only or slip ratio only. Figure 3-25 shows the interaction between lateral and longitudinal force generation.

As mentioned above, these plots were sent to Jeffrey Chrstos and he compared them against actual test data for the Taurus tires as well as comparing them against his own version of STIREM. He confirmed that the results were as expected.
Figure 3-22: STIREMOD Lateral Force vs. Slip Angle

Figure 3-23: STIREMOD Aligning Moment vs. Slip Angle
Figure 3-24: STIREMOD Longitudinal Force vs. Slip Ratio

Figure 3-25: STIREMOD Lateral Force vs. Longitudinal Force
3.3.1.2. **Subjective Tire Simulation Comparisons**

The plots above and in [16] do not give a complete picture for the tire model. To begin with, they only cover slip angles to ±15 degrees and slip ratios to ±0.5. In addition, they do not show the influence of camber angle, variation in output with normal load, or variation in output with vehicle speed. They also do not show directly how longitudinal force varies with changing slip ratio and how lateral force varies with changing slip angle. Figure 3-26 through Figure 3-33 show all of these influences. As can be seen in these figures, there are smooth curves across the complete tire operating range. In addition, the tire outputs change with the inputs as expected. For example, in Figure 3-26 the lateral force generated decreases as the longitudinal slip increases. In addition, the tire forces in Figure 3-31 and Figure 3-32 decrease as vehicle speed increases. This is the expected response. The only response that seems unusual is that in Figure 3-30 where the lateral force increases first then decreases with normal force. This is the only response that does not change in only one direction. However, when looking at the tire parameters and the plots in [16], this is the expected response. In addition to being directionally correct, the magnitudes of the responses shown are all within reason.

![Graph](image)

**Figure 3-26:** Lateral Tire Force for Varying Slip Ratios
**Figure 3-27:** Aligning Moment for Varying Slip Ratios

**Figure 3-28:** Longitudinal Tire Force for Varying Slip Angles
Figure 3-29: Variation of Tire Lateral Force with Normal Load

Figure 3-30: Variation of Tire Longitudinal Force with Normal Load
Figure 3-31: Variation of Tire Lateral Force with Vehicle Speed

Figure 3-32: Variation of Tire Longitudinal Force with Vehicle Speed
3.3.2. Vehicle Model Validation

With NAVDyn implemented and the tire model validated, the next step was to validate the overall vehicle model. Validation was conducted against actual vehicle data and against a high-order model. In order to conduct the validation, and for that matter, to even be able to use the model, a target vehicle had to be selected. Three different vehicles were considered before the 1994 Ford Taurus GL was selected. The other two vehicles were a Buick LeSabre and a Honda Accord. The LeSabre and Accord were considered because they were the vehicles used in Ashok Chandy's model and the Clemson from 2.2 model respectively. However, there were many reasons to use the 1994 Ford Taurus GL.

There has been a very significant national project going on in the United States to develop what is called the National Advanced Driving Simulator (NADS). This project has been funded by the National Highway Traffic Safety Administration and has involved many different aspects of vehicle modeling and simulation. There were three particular parts of the NADS project that lead to the selection of the Ford Taurus. The Ford Taurus was selected as the target vehicle to be used.
in validation and testing of NADS. As such there were several projects funded to capture the performance of the Taurus and use this information in simulation. The three that were significant for use in validation of NAVDyn were the vehicle testing to capture the performance of the Taurus, measurement of the vehicle parameters that characterize the Taurus, and a study comparing two other simulation tools against the actual Taurus test data, [28][29][30].

The first and possibly the most important information available from the NADS project is the field testing of an actual vehicle. [28] gives complete details of the testing program that was conducted. It includes a full description of the instrumentation used and how the data was collected. It also gives details of the test maneuvers that were conducted to capture the characteristics of the Taurus. Finally it describes the data reduction process to get the data into usable form. Prior to this work, very little vehicle test data was publicly available. Most testing is conducted by vehicle manufacturers and is not available to the general public. Since this is exactly the type of information that is needed to validate a vehicle model (this is why the testing was conducted), for this reason alone the Ford Taurus was a natural choice for validating NAVDyn. The report describing the testing and a CD-ROM with all of the processed data from this test program were obtained from Paul Grygier of the Vehicle Research and Test Center, which is part of NHTSA.

The second project that was key in the validation of NAVDyn was the measurement of all the necessary parameters characterizing the Taurus for use in simulation. [29] contains a complete description and results of the parameter measurement. This report provided the data needed to define values for all the parameters used in NAVDyn. The actual values used for NAVDyn are given in Appendix D. CarSim also used this same data in creating its default vehicle, which is also the Ford Taurus. Since CarSim uses the same information, some of the parameter values were taken from CarSim and others were obtained from the report. The parameters used in CarSim are shown in Appendix F.

The final NADS project that was helpful in validating NAVDyn was an evaluation of two different simulation tools against the Taurus test data. This evaluation is described in [30]. The two simulation tools used were VDANL and VDM RoaD. The details of these tools can be found in the report, but two aspects of the evaluations were useful. First, this evaluation was used as a
template for the validation of NAVDyn. Since [30] uses the same Taurus test data that was used to validate NAVDyn, the same tests and procedures were followed. The other information from this evaluation that was helpful is that VDM RoaD is a simulation tool developed by the University of Michigan. This tool is very similar to, and in many cases identical to, CarSim.

CarSim is the high-order model that was selected for comparison with NAVDyn, [13]. Specifically, version 4.0.2 was used for conducting this validation. CarSim is a commercially available simulation tool developed at the University of Michigan. CarSim is very similar to VDM RoaD, which as mentioned was used in the NADS evaluations, [30]. In addition to the eight degrees-of-freedom described above for NAVDyn, CarSim includes six additional degrees of freedom: vertical motion of the body, pitch of the body, and vertical motion of each of the four wheels. This results in fourteen degrees-of-freedom. If you add the slip angle of each of the four tires, this brings the total to eighteen for CarSim and twelve for NAVDyn. With both NAVDyn and CarSim using the same Taurus data, comparing them helped show if the assumptions that went into the simplification of NAVDyn to eight degrees-of-freedom were valid. In addition, there were some conditions of interest that were evaluated, which will be described in 3.3.2.2, that were not included in the test program for the Ford Taurus.

For purposes of validation, aerodynamic effects were neglected in both NAVDyn and CarSim. Based upon the results, this does not appear to have had a significant influence. This decision may account for some of the differences seen between the simulations and the vehicle test data.

3.3.2.1. Comparison With Taurus Test Data

Since NAVDyn is only intended to represent handling performance on a smooth road, test maneuvers relevant to vehicle handling and braking were chosen for validation. Since test data exists for the actual Taurus, the list of tests conducted for the NADS program was reviewed and only those tests viewed as relevant were selected, [28]. In particular, the same tests that were used to compare VDANL and VDM RoaD in [30] were used for this validation. The tests used include the following:

1. Pulse-Steer Maneuvers,
2. Slowly Increasing Steer Maneuvers,
3. Constant Speed J-Turn Maneuvers,
4. Double Lane-Change Maneuvers, and
5. Straight-Line Braking Maneuvers.

Each of these maneuvers is conducted under several different conditions and is described in
detail in [28]. The measured inputs on the actual test vehicle were used as the inputs to NAVDyn
and CarSim to see how well these models predicted the actual performance. The pulse-steer
maneuvers were used for frequency-domain comparisons while the other four maneuvers were
used for the time domain. The results of these comparisons are shown below.

3.3.2.1.1. Frequency Domain Comparisons

The pulse steer maneuvers are used to characterize the lateral response in the frequency domain.
The vehicle is operated at a constant speed and a pulse is input at the steering wheel to achieve a
maximum lateral acceleration of approximately 3 m/s². This level is chosen to have adequate
excitation, but for the vehicle to still be in the linear range of operation. A description of the
pulse steer test is found in [28] and the methodology for computing the frequency response
characteristics from this test is described in [28] and [31]. The M-files from Matlab that were
used to compute the frequency responses are given in Appendix E along with M-files for
generating standard plots.

Frequency responses to steering inputs are computed for lateral acceleration at the sprung mass
center of gravity, yaw rate, and roll rate. These comparisons provide a good overall view of a
simulation's ability to predict the dynamic characteristics of a vehicle. In the low frequency range
the responses show how well the simulations predict the vehicle's steady-state gains in the linear
operating range. The magnitude comparisons show if the simulation's damped natural frequency
and system damping are close to the actual vehicle. The phase angle comparisons indicate if the
simulations are of a reasonable order to predict vehicle response for a particular frequency. The
coherence is used only for the experimental data and is an indicator of measurement noise or
system non-linearity. In each of the plots for magnitude and phase, 95% confidence intervals are
shown on the vehicle data. Testing was conducted at three vehicle speeds of 11, 22 and 33 m/s,
with the cruise control engaged, and for both negative and positive pulses. Figure 3-34 through
Figure 3-51 show the results from the frequency response comparisons.

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Figure 3-34: Lateral Acceleration Frequency Response for 11.4 m/s and positive pulse

Figure 3-35: Yaw Rate Frequency Response for 11.4 m/s and positive pulse
Figure 3-36: Roll Rate Frequency Response for 11.4 m/s and positive pulse

Figure 3-37: Lateral Acceleration Frequency Response for 11.5 m/s and negative pulse
Figure 3-38: Yaw Rate Frequency Response for 11.5 m/s and negative pulse

Figure 3-39: Roll Rate Frequency Response for 11.5 m/s and negative pulse
Figure 3-40: Lateral Acceleration Frequency Response for 22.8 m/s and positive pulse

Figure 3-41: Yaw Rate Frequency Response for 22.8 m/s and positive pulse
Figure 3-42: Roll Rate Frequency Response for 22.8 m/s and positive pulse

Figure 3-43: Lateral Acceleration Frequency Response for 22.9 m/s and negative pulse
Figure 3-44: Yaw Rate Frequency Response for 22.9 m/s and negative pulse

Figure 3-45: Roll Rate Frequency Response for 22.9 m/s and negative pulse
Figure 3-46: Lateral Acceleration Frequency Response for 33.7 m/s and positive pulse

Figure 3-47: Yaw Rate Frequency Response for 33.7 m/s and positive pulse
**Figure 3-48:** Roll Rate Frequency Response for 33.7 m/s and positive pulse

**Figure 3-49:** Lateral Acceleration Frequency Response for 33.5 m/s and negative pulse
Figure 3-50: Yaw Rate Frequency Response for 33.5 m/s and negative pulse

Figure 3-51: Roll Rate Frequency Response for 33.5 m/s and negative pulse
In looking at the frequency response comparisons, subjectively both models have an overall shape that looks very similar to the vehicle data. For the lateral acceleration responses above, both models' predictions are quite good with two areas to note. Typically a vehicle will show an area of low magnitude in lateral acceleration response somewhere past 10 rad/sec. In a swept sine test this is the frequency where the vehicle is basically rotating about the accelerometer. Both models show close agreement with vehicle data for frequencies below 10 rad/sec. Above 10 rad/sec and for low vehicle speed, both models appear to predict this area of low magnitude at a lower frequency than the actual vehicle. In conjunction with this, the phase angle magnitude is also less than the actual vehicle in this range. At higher speeds there is better agreement between the simulations and vehicle data.

Both simulations predict the yaw rate magnitude very well across the frequency range, but with a slightly lower phase angle magnitude than actual. The opposite is true for roll rate, where both simulations have good phase angle predictions across the frequencies, but slightly overestimate the roll rate magnitude. Overall, both simulations show very similar results to the experimental data.

An important result is that NAVDyn and CarSim have nearly identical results. This indicates that the simplifications made to generate NAVDyn are valid for lateral response. In many cases, NAVDyn actually has slightly better results than CarSim. From the results it appears that the argument that the extra six degrees-of-freedom in CarSim that are not in NAVDyn are not necessary is true for lateral response.

3.3.2.1.2. Time Domain Comparisons

For the time domain, comparisons were done for constant-speed slowly increasing steer input, constant speed J-turn input, double lane-change and straight-line braking.

**Slowly Increasing Steer:**

The constant speed, slowly increasing steer test is used to evaluate the simulation’s ability to predict the steady-state gain of the vehicle from low lateral accelerations all the way up to the limit of adhesion. As the name implies, vehicle speed is held constant while the steering wheel
angle is slowly increased. Figure 3-52, Figure 3-53, and Figure 3-54 show the lateral acceleration gain comparison for vehicle speeds of 11 m/s (40 kph), 22 m/s (80 kph) and 33 m/s (120 kph) respectively. As can be seen, the model predictions are very good up to the limit of adhesion. Both model predictions are slightly below the actual data at the limit, with NAVDyn being closer than CarSim. The higher level of the actual vehicle may be due to a difference between the road surface friction level for the test vehicle and the 0.85 value used for simulation. If the actual friction level during testing were higher than 0.85, this could account for the difference. It appears there is still some other difference between CarSim and NAVDyn. For moderate acceleration levels, the results confirm the frequency response, which shows good correlation between simulation and vehicle for lateral acceleration at low frequency.

![Graph showing lateral acceleration gain comparison](image)

**Figure 3-52:** Lateral Acceleration Gain for Slowly Increasing Steer at 11 m/s
Figure 3-53: Lateral Acceleration Gain for Slowly Increasing Steer at 22 m/s

Figure 3-54: Lateral Acceleration Gain for Slowly Increasing Steer at 33 m/s
**Constant Speed J-Turns:**

The constant speed J-turns are used to evaluate both vehicle transient behavior and steady-state gain up to the limit of adhesion. As the name implies, vehicle speed is held constant and then an approximate step input is given at the steering wheel in order to achieve a desired level of lateral acceleration. Twelve maneuvers were conducted, run at vehicle speeds of 11, 22 and 33 m/s, each with four target severity levels ranging from 0.2 to 0.8 g. Plots are shown below in Figure 3-55 through Figure 3-72 for lateral response of each run and for roll response for every other run. The lateral response plots show both lateral acceleration and yaw rate while the roll response plots show both roll angle and roll rate.

**Figure 3-55:** Lateral Response for 11.2 m/s J-Turn at 0.2 g
Figure 3-56: Roll Response for 11.2 m/s J-Turn at 0.2 g

Figure 3-57: Lateral Response for 11.3 m/s J-Turn at -0.4 g
Figure 3-58: Lateral Response for 11.5 m/s J-Turn at 0.6 g

Figure 3-59: Roll Response for 11.5 m/s J-Turn at 0.6 g
Figure 3-60: Lateral Response for 11.3 m/s J-Turn at -0.8 g

Figure 3-61: Lateral Response for 22.7 m/s J-Turn at -0.2 g
Figure 3-62: Roll Response for 22.7 m/s J-Turn at -0.2 g

Figure 3-63: Lateral Response for 22.4 m/s J-Turn at 0.4 g
Figure 3-64: Lateral Response for 22.6 m/s J-Turn at -0.6 g

Figure 3-65: Roll Response for 22.6 m/s J-Turn at -0.6 g
Figure 3-66: Lateral Response for 22.0 m/s J-Turn at 0.65 g

Figure 3-67: Lateral Response for 33.0 m/s J-Turn at 0.2 g
Figure 3-68: Roll Response for 33.0 m/s J-Turn at 0.2 g

Figure 3-69: Lateral Response for 33.5 m/s J-Turn at -0.4 g
Figure 3-70: Lateral Response for 33.2 m/s J-Turn at 0.6 g

Figure 3-71: Roll Response for 33.2 m/s J-Turn at 0.6 g
As can be seen from the plots, both simulations do a reasonable job of predicting the vehicle response. The simulations have very good lateral response, especially for the lower vehicle speeds. At 33 m/s there appears to be more of a difference between the simulations and the actual vehicle. Although the actual reason for this is not known, it is possible that the differences are due to the steering system and/or the steer effects. Since at higher speeds less angle is required to get the same lateral acceleration, a small difference in road wheel angle will show a larger difference in lateral acceleration than it would at lower speeds. The differences do not seem to be consistent, making it difficult to determine the cause. Even with the differences, subjectively the simulations follow the vehicle response well with a few exceptions.

First, in Figure 3-66 and Figure 3-72 it can be seen that CarSim appears to have a loss of adhesion since the yaw rate in these figures diverges from the actual vehicle. This would go along with the results from the constantly increasing steer tests where it was seen that CarSim did not reach as high a level of lateral acceleration as did NAVDyn or the actual vehicle. It appears that at the limit of adhesion NAVDyn does a better job of predicting vehicle response. Another
item to note is that in Figure 3-61, although there is a significant difference between simulation and actual for yaw rate on a percentage basis, if you compare the scale on this plot with those for other plots, the difference is not as significant on an absolute scale. Finally, looking at yaw rate in Figure 3-67 there is a difference between simulation and actual. However, the difference exists even before the steering input. In this case there was a measured steering wheel angle in the actual vehicle while driving straight. Removing this difference, the simulations match very well with the actual vehicle. Looking back through the other plots, although less significant, there are other places where the same influence can be see. Finally, from the plots it can be seen that NAVDyn appears to be slightly underdamped and have a slightly higher frequency in its lateral response compared with the actual vehicle.

Moving on to the roll response we again see that the simulations do a reasonable job of predicting the actual vehicle. However, from the roll plots it can be seen that in almost every case, NAVDyn is closer to the actual vehicle response than CarSim. Even so, it still appears that NAVDyn is slightly underdamped in its roll response. This may explain some of the reason why the lateral response also appears to be underdamped. Since the roll angle changes the road wheel angle through the influence of roll steer, oscillations in the roll angle will cause oscillations in the road wheel angles, which will in turn cause oscillations in the lateral response of the vehicle.

**Double Lane-Change:**

The double lane-change maneuver is an attempt to recreate in a test environment a real world crash-avoidance maneuver. This maneuver is also used for comparing lateral response of the simulations with the actual vehicle. The double lane-change maneuver for the actual vehicle involved driving through a fixed course, trying to keep the vehicle within the course boundaries defined by cones. Severity of the test is adjusted by running maneuvers at increasing vehicle speeds. The simulations use the actual measured steering wheel angle and vehicle speed from the test vehicle. Figure 3-73 through Figure 3-78 show the comparisons for the double lane-change maneuvers.
Figure 3-73: Double Lane Change at 11 m/s

Figure 3-74: Double Lane Change at 13 m/s
Figure 3-75: Double Lane Change at 15.5 m/s

Figure 3-76: Double Lane Change at 16 m/s
Figure 3-77: Double Lane Change at 18 m/s

Figure 3-78: Double Lane Change at 20 m/s
As can be seen from these figures, both models do a very good job of predicting vehicle response for the low to moderate speeds. The simulations follow the actual vehicle data very closely up to 15.5 m/s. Starting at 16 m/s in Figure 3-76 CarSim begins to diverge from the actual vehicle data at the maximum yaw rate. As vehicle speed increases in Figure 3-77 and Figure 3-78, this becomes even more pronounced to the point where CarSim appears to have a significant loss of adhesion at 20 m/s.

In contrast, NAVDyn appears to follow the actual vehicle very well through the entire speed range. Even though the responses do not follow exactly for the high speeds, they are very close and NAVDyn has the same response shape as the actual vehicle. Compared against CarSim, NAVDyn does not appear to experience a loss of adhesion, or at least if it does it matches that seen by the actual vehicle. Both simulations use a 0.85 coefficient of friction, so the difference must come from the models themselves.

**Straight Line Braking:**

The final maneuvers used for validation are straight line braking. These maneuvers are intended to characterize the basic vehicle brake model performance. For all maneuvers, the vehicle is driven straight ahead at a speed of 80 kph and then an approximate step input in brake pedal force is applied to achieve a desired level of deceleration. Figure 3-79 through Figure 3-83 show the longitudinal response of the vehicle for deceleration levels from 0.2 g up to the maximum deceleration possible.

As can be seen, the simulation results are similar to the actual vehicle data in most cases. For all the runs except maximum deceleration, both simulations have approximately the same steady state deceleration level as the actual vehicle. However, there are several differences that are significant. First, CarSim has a time response considerably faster than the actual vehicle. By comparison, NAVDyn matches the time response well. This can likely be attributed to the fact that NAVDyn includes lags in the brake system and CarSim does not. One related note is that the actual vehicle appears to have a response that looks more like a second-order or higher system. In NAVDyn a simple first-order lag is used. These differences appear to be minor in comparison to the overall response. Another interesting thing to note is that CarSim appears to have some type
of computation issue with the deceleration level at low speeds. However, it appears the model is still valid since vehicle speed responds as expected. The issue must be only with the output and/or plotting.

There are two other interesting things to note. First, when looking at the figures it can be seen that as pedal force increases, the actual vehicle has a deceleration level that increases with time. Both simulations predict relatively constant steady state deceleration level. Evidently there are some brake system dynamics at high deceleration levels that are not modeled in NAVDyn or CarSim. One possible explanation for this is a change in brake lining friction level with temperature. For low to moderate deceleration levels, the lining temperature may not change significantly during a run. For high deceleration levels the change in brake temperature may be more significant.

Figure 3-83 shows the results for maximum deceleration and there are two things to note. Since the actual vehicle had anti-lock brakes, there is no drop-off in deceleration as is seen in the simulations. This occurs in the simulations because the wheels lock and so the longitudinal force from the tires drops to the level for sliding. Comparing the two simulations we see that both the peak and steady-state values for CarSim are significantly lower than those for NAVDyn. This goes along with what was seen in the lateral testing where CarSim experienced a loss of adhesion before the actual vehicle and NAVDyn. Although the actual cause is unknown, these differences may be due to the way the tires are modeled in CarSim, which is done using look-up tables and interpolation/extrapolation instead having a continuous model for the entire operating range of the tire. It is possible that this approach may cause errors under limit conditions. The final item of note is that the peak deceleration value for NAVDyn is very close to the actual vehicle, indicating that if both had anti-lock brakes the results would probably match.
Figure 3-79: Straight-Line Braking with 40 N Pedal Force

Figure 3-80: Straight-Line Braking with 57 N Pedal Force
Figure 3-81: Straight-Line Braking with 110 N Pedal Force

Figure 3-82: Straight-Line Braking with 150 N Pedal Force
3.3.2.2. Comparisons Between NAVDyn and CarSim

As can be seen from the above comparisons, test data was only available for conditions involving steering only or braking only, but not for combinations of steering and braking. In [28] there were brake-in-a-turn tests conducted on the Ford Taurus, but there appears to be a problem with the test data. When looking at the data from these runs, the level of deceleration does not match with the amount of brake pedal force applied when comparing against the straight-line braking runs. The differences can not be explained by the added deceleration caused by the steering input. The vehicle data was examined to attempt to find the source of the problem. There was no clear problem found that accounted for the difference. It appears from the data that there is an offset in the relationship of pedal force versus front brake pressure when comparing straight-line braking with brake-in-a-turn data. Since the brake system did not change, this relationship should be the same between the two data sets. Initially it was thought this might be the cause. However upon further investigation there was also an offset between the brake pressure and deceleration level relationship. It could not be determined if there was a problem in the pedal force measurement,
front brake pressure measurement, both of these, or neither of these. Since the brake-in-a-turn data was not used in [30] for the comparisons with VDANL and VDM RoaD, it is assumed that the same issue was found by that author as well.

Since the exact cause for this difference was not known, no attempt was made to try to correct the vehicle data. As a result, there is no vehicle data available for comparison. This is unfortunate as the condition of combined steering and braking, especially at the limit of adhesion, is exactly the place where differences between NAVDyn and CarSim may show up due to the influence of pitch. NAVDyn's lateral response was closer to the actual vehicle than CarSim near the limit of adhesion, but CarSim includes pitch. For this reason, it will be hard to determine which simulation is more accurate under combined steering and braking, especially near the limit, without actual data to compare against. However, some runs were conducted to try to gain insight into the significance of the pitch degree-of-freedom.

One thing to note is that because this study only involves low to moderate levels of longitudinal and lateral accelerations, the validation conducted above shows that NAVDyn will provide good results under these conditions. The time was not taken to thoroughly study the conditions of combined braking and steering near the limit of adhesion. It may be possible that with further investigation the problems with the Taurus test data could be determined and the data used to evaluate these conditions. As mentioned, this was not necessary for this study and so it was not done. For future use of NAVDyn, if a user wants to investigate these conditions, further work should be done to validate NAVDyn. If the problem with the Taurus data cannot be determined then additional vehicle testing should be conducted.

That being said, comparisons were made between NAVDyn and CarSim for normal force on the tires. This is the area where the pitch degree-of-freedom will have the largest impact. As mentioned during the derivation, NAVDyn does include the effects of weight transfers, but since pitch is not modeled, the weight transfers due to pitch are not included. Since normal force is an input to the tire model, this will have a secondary effect due to changes in the tire forces caused by changes in normal force.
Figure 3-84 shows a comparison of vertical load on the tires between CarSim and NAVDyn for the 150 N pedal force braking run. This data was not available for the actual vehicle. As can be seen, there are dynamics in the CarSim results that are not found in NAVDyn. These differences are due to the pitch degree-of-freedom, which is not included in NAVDyn. The steady state levels are nearly the same. This difference will have a secondary transient influence on the vehicle response. Since tire loading influences the forces and moments generated by the tires, this will in turn influence the dynamics of the vehicle. For all of the maneuvers used for validation, this difference does not appear to be significant. Further investigation is necessary into maneuvers involving combined braking and steering. Combined braking and steering maneuvers conducted at the limit of adhesion are likely to be the most influenced. Plots of normal force are not shown for any other maneuvers since normal force has only a secondary effect and is not one of the main variables under consideration. However, CarSim and NAVDyn have very similar results for all other conditions not involving heavy braking.

![Figure 3-84: Tire Normal Force During Braking with 150 N Pedal Force](image-url)
3.4. Summary

This chapter covered both the implementation and validation of NAVDyn. The details of how the equations for NAVDyn were implemented using Matlab/Simulink were presented. The implementation presented addresses the issues related to customization that were discussed previously. In addition, it has some added benefits that prove to be quite useful. By breaking the model up into connected subsystems, the model is easily customized by making changes only to the applicable subsystem. Since this does not require any changes to the rest of the model, then only the subsystem that has been changed needs to be validated for proper output. Since the flow of information is easy to follow, adding or modifying the inputs and outputs for a particular subsystem is very straightforward.

This approach has the added benefit of making debugging when initially implementing a model easier since each individual subsystem can be checked for correct input-output relationships. In addition, if changes are made to the model after it has been implemented initially, validation only requires checking the subsystem that has been changed since these changes do not, in general, influence calculations in other parts of the model.

With the ease of customization described and the added benefits of this approach, it could be useful to use this method for other models, both more and less complicated than NAVDyn. Since the benefits of this implementation are not specific to the model being used, this approach has general application to almost any model of interest.

As can be seen from the validation, NAVDyn provides excellent results for the conditions it was designed to represent. In most cases it matches almost exactly with CarSim, in some instances providing even better results most likely due to some features/dynamics included in NAVDyn that are not found in CarSim. These results support the simplifications that were made to reduce NAVDyn to an eight degree-of-freedom model. In addition, NAVDyn has excellent agreement with the actual test data for the Ford Taurus. This is even more significant than having good correlation with another model. An automobile is a complex system and it would be difficult to capture every single influence on vehicle handling in a model. As such there will almost always be some differences between a model and the actual vehicle under certain circumstances. Even
so, the results presented show that NAVDyn does a very good job of following the response of the actual vehicle within a reasonable level of accuracy.

The most significant difference between the two models was found in the tire vertical loads under heavy braking. This is due to the pitch degree-of-freedom, which is not included in NAVDyn. Further investigation needs to be conducted to determine the significance of this difference. For the maneuvers above, this did not appear to have a significant influence, but may for combined steering and braking maneuvers near the limit of adhesion. Still, for braking only, steering only, and combined braking and steering below the limit of adhesion, NAVDyn provides very good results. Considering the amount of complexity necessary to add pitch due to the influence of suspension geometry and features like anti-lift and anti-dive, this seems like a reasonable trade-off.

So as can been seen from the above results, for a wide variety of steering and braking inputs on a smooth road, NAVDyn does very well at predicting actual vehicle response. This in combination with the fact that it has a straightforward implementation that is easy follow and allows easy customization for future studies makes NAVDyn a very worthwhile model for use in a variety of vehicle handling simulations. In particular, NAVDyn has been shown to have the features necessary for conducting the study of using the steering system to stop the vehicle.
4. OPEN-LOOP INVESTIGATION

4.1. Introduction

With NAVDyn validated, it could then be used to begin the study of using two independent steering actuators to stop a vehicle while maintaining directional control. The first step in this process was to conduct numerous open-loop simulations in order to gain a fundamental understanding of how a vehicle responds to independent steer angles on the front wheels. All simulations were conducted at an initial vehicle speed of 100 kph unless otherwise noted. Before starting the simulations, modifications needed to be made to the steering system to allow for the various open-loop inputs. These modifications are discussed in 4.2. The first issue to address with these simulations was the maximum deceleration capability using the steering. This determined if the idea was even worth pursuing and is covered in 4.3. Next the response to a single wheel input holding the other wheel fixed was simulated and then the response to independent inputs at each wheel was investigated. These conditions are covered in 4.4 and 4.5 respectively. From all of the simulations conducted, some unexpected and very interesting results were found, which are explained in 4.6. Finally 4.7 covers conclusions and implications from these simulations on obtaining desired vehicle response.

4.2. Modifications to the Steering System

The steering system shown in Figure 3-5 is the final version obtained by the end of this research. This steering system shows two independent steering actuators, which for normal steering have the same input going to both. Under normal operation, this provides the same steering system performance as if both wheels are connected through a single steering gear and turn by the same amount, which matches the standard Ford Taurus. Obviously for this study, independent actuators were necessary. Although, as discussed earlier, the dynamics of the steering system were not modeled directly, it was desired to place some reasonable bounds on the output the steering system could provide. As such, rate limits and angle limits were placed on the steering actuators.
Figure 4-1 shows the details of one of the actuators. As can be seen, the commanded steer angle is run through a rate limiter, then the influence of steering kinematics is added and the result is run through a maximum angle limit. The main purpose of the rate limiter was for the closed-loop control. Since realistic driver inputs are used, under normal operation the steering system should always be able to provide the rate the driver commands. However, under closed-loop control the steering controller may command rapid changes in steer angle. The rate limit was used to prevent unrealistic changes in the steer angle due to bandwidth limitations of the steering actuator. This will be discussed in Chapter 5. The kinematics block is simply the implementation of the steering kinematics described in 2.3.2.7.

![Diagram of LF ROADWHEEL ACTUATOR](image)

**Figure 4-1: Detail of Left Front Road Wheel Actuator**

The maximum angle block was added to limit the maximum amount the wheels are allowed to turn. As discussed in the next section, maximum deceleration using the steering occurs at the maximum steer angle of the road wheel. As a result, when using the steering to stop, the wheels will often be operated near this limit. Obviously you can not turn a full 90 degrees. A realistic limit was needed for the maximum angle. After talking with some colleagues at Delphi, it was found that most production vehicles have a maximum steer angle in the range of 35-40 degrees. The decision was made to set the maximum steer angle to ±40 degrees. This was done in NAVDyn by using the limit block on the output from the steering actuators as shown. Since the suspension effects are added to the output from the steering system, the actual road wheel angles vary slightly from this amount.

In order to steer the wheels toward or away from each other, the switch in Figure 3-6 is used so that the steer angle of the wheels have opposite signs. By doing this, toe-in and toe-out are
obtained by simply changing the sign of the steering wheel angle input. This method was used for the simulations conducted in the next section. For the single wheel inputs in 4.4, the steering system was simply modified by deleting the input to one of the road wheel actuators. For the independent steer inputs used in 4.5 a few different approaches were used that will be discussed in that section.

4.3. Equal Toe-In/Toe-Out Steering Inputs at Both Wheels

The original thought when deciding to pursue this idea was that you could stop a vehicle with the steering by turning the wheels toward or away from each other. This is the same concept as "snow-plowing" to stop when you are skiing. Although an alternative approach that will be discussed later was also developed, this method provides the greatest level of deceleration. (It is not really feasible to slide a car sideways in a controlled manner to stop like an accomplished skier does.) So the first step in the process was to run simulations where the wheels are simply turned toward or away from each other by the same angle. By doing so, the y-direction component of the tire forces will cancel each other so the vehicle will remain going straight and the x-direction component will act to decelerate the vehicle. Figure 4-2 and Figure 4-4 show the results of these simulations for toe-in and toe-out respectively. As expected, turning the wheels to their maximum angle provides the greatest level of deceleration.

One thing to note is that for these simulations the input was steering wheel angle and the output was deceleration. Since the purpose of these simulations was to determine the relation between road wheel angles and deceleration in order to develop a relationship between brake pedal force and deceleration, brake pedal force could not be used at this point. The simulations were run using steering wheel angle instead of road wheel angle because it was easier to do it this way than to modify the model to use road wheel angle as the input. In addition, during the early part of this study, results from NAVDyn and CarSim were compared to confirm that NAVDyn gave the expected results. (A trick was found in CarSim that could be used to do toe-in/toe-out steering.) Steering wheel angle was the only available input for CarSim. Even though steering wheel angle is the input, the resulting road wheel angle is shown in Figure 4-3 and Figure 4-5 for toe-in and toe-out respectively.
Figure 4-2: Longitudinal Response for Maximum Toe-In of Road Wheels

Figure 4-3: Steer Input for Maximum Toe-In of Road Wheels
Figure 4-4: Longitudinal Response for Maximum Toe-Out of Road Wheels

Figure 4-5: Steer Input for Maximum Toe-Out of Road Wheels
There are a few things of note from these curves. First, it can be seen that the maximum deceleration level achievable is on the order of 0.3-0.4 g. Although this is certainly not as much as the normal braking system can provide, it is certainly enough to be worth pursuing. Many drivers under normal conditions rarely brake at levels higher than 0.3 g. The second thing to note is that toe-in and toe-out provide virtually the same results, as would be expected due to symmetry. The differences can be explained by differences between actual road wheel angles due to the steering kinematics and suspension compliances. Since these influences in NAVDyn were based upon a hydraulic steering system with both wheels turning together, they may not exist for true independent steering actuators. This says that from a stopping capability standpoint, there is no difference between the two scenarios and so if some other aspect of performance suggests that one is better than another then the better one should be used.

Within the automotive braking community it is known that when braking on a split coefficient of friction surface it is desirable to have some toe-in of the front wheels. The reason for this is that the side with the higher coefficient of friction will have higher braking forces than the low coefficient side, which will induce a yaw moment to turn the vehicle toward the high coefficient side. If you toe-in the front wheels, the steer angle on the wheel with the higher coefficient of friction will create a lateral force that will counteract the yaw moment from the braking forces. Using this same line of reasoning, toe-in is preferable to toe-out when using the steering to stop the vehicle.

The final item to note from the figures is that deceleration level increases as vehicle speed decreases. This is due to the same relationship between tire/road friction level and vehicle speed that was seen in 3.3.1.2.

Although the above figures show only the results for the maximum road wheel angles, simulations were run across the range of angles in order to determine the relationship between toe angle and deceleration. The results are summarized in the figures below. First, Figure 4-6 shows the relationship between steering wheel angle and road wheel angle during toe-in. Then Figure 4-7 shows the relationship between toe angle and deceleration level. From these results it can be seen that other than the initial portion at low angles, there is a fairly linear relationship between road wheel angle and deceleration. This relationship is not obvious since the portion of
the lateral tire forces acting along the longitudinal direction of the vehicle is a function of the sine of the road wheel angle. In addition, the lateral force versus slip angle relationship for the tires is very non-linear. Although not obvious, the result is useful since it indicates that a simple linear relationship between brake pedal force and toe angle of the front wheels can be used.

Figure 4-8 helps explain the results by showing the lateral force from the left front tire and the portion of this force that acts in the longitudinal direction. For angles from 0-40 degrees, the sine function is nearly linear. This gets multiplied by the tire lateral force in Figure 4-8 to produce the vehicle longitudinal force. The initial non-linear portion of the curve is due to the fact that tire forces start at zero then increase towards the peak. Near and beyond the peak since tire lateral force does not change significantly, the sine function has a greater influence on the amount of longitudinal force. As a result the relationship is close to linear after the initial force buildup. It can be seen from the results that the magnitude of the slope is slightly decreasing. This is expected since the slope of the sine function is slightly decreasing and the magnitude of the tire lateral force is slowly decreasing.

![Figure 4-6: Road Wheel Angle vs. Steering Wheel Angle for Toe-In Steering](image-url)
Figure 4-7: Longitudinal Acceleration vs. Toe-In Angle

Figure 4-8: Vehicle Longitudinal Force and Tire Lateral Force from LF Tire During Toe-In
4.4. Single Wheel Inputs

The next simulations conducted were for a single wheel input while holding the other wheel fixed. This was done first for the other wheel fixed straight ahead. This was the first condition looked at to see how the vehicle responds to differences between the left and right wheels. Then simulations were run for the second wheel fixed at its maximum angle. Since the lateral tire forces left and right no longer offset each other, both lateral and longitudinal responses are covered.

First, simulations were run for steer input to the left wheel at two different rates for both positive and negative angles while holding the right wheel straight ahead. The results are shown in Figure 4-9 through Figure 4-20. Due to symmetry the results are the same for steering the right wheel only. Reviewing the figures shows some expected results. First it can be seen that for both handwheel speeds the magnitude of response in both the lateral and longitudinal directions is greater for positive angles. This is expected since a positive angle means turning the left wheel to the right, which causes the vehicle to turn to the right. Due to weight transfer from lateral acceleration and roll, this puts more weight on the left wheel allowing it to generate higher forces. In contrast, for negative angles weight is transferred off of the left wheel, which reduces the amount of force that can be generated. The simulations with positive angles show higher deceleration levels, lateral acceleration levels and yaw rates. This is most noticeable in comparing Figure 4-16 with Figure 4-19 and Figure 4-17 with Figure 4-20. This is true even though the negative inputs result in a higher actual angle at the road wheel.

As expected, the peak lateral response occurs around 7-10 degrees of road wheel angle, which corresponds to the peak in lateral force for the tires. In addition, the peak deceleration occurs at the maximum road wheel angle, the same as was seen in 4.3. The magnitude of the longitudinal and lateral accelerations is also of note. In Figure 4-16 the maximum deceleration is approximately 0.17 g, which is half of the maximum deceleration when both wheels were steered in the previous section, as expected. For the 10 deg/sec runs the wheel never reaches its maximum angle so the same comparison can not be made. The other interesting thing to note is that steering with one wheel with the other fixed straight ahead, you can still achieve almost 0.6 g
peak lateral acceleration, although the level drops below 0.3 g at the point where maximum deceleration is achieved.

The final thing to note is the shape of the lateral response curve. Looking at Figure 4-17 you see that the lateral response looks very much the same shape as the lateral force versus slip angle curve for a tire up until around the peak. Then the curve drops off much quicker than a tire curve would due to the fact that the steer angle of the wheel causes an increasing amount of the tire lateral force to act in the longitudinal vehicle direction instead of the lateral direction. Finally the lateral acceleration slowly decreases as the vehicle speed drops.

![Figure 4-9: Steer Input for LF Wheel Only at 10 deg/sec](image-url)
Figure 4-10: Longitudinal Response for LF Wheel Only at 10 deg/sec

Figure 4-11: Lateral Response for LF Wheel Only at 10 deg/sec
Figure 4-12: Steer Input for LF Wheel Only at -10 deg/sec

Figure 4-13: Longitudinal Response for LF Wheel Only at -10 deg/sec
Figure 4-14: Lateral Response for LF Wheel Only at -10 deg/sec

Figure 4-15: Steer Input for LF Wheel Only at 100 deg/sec to Max Angle
Figure 4-16: Longitudinal Response for LF Wheel Only at 100 deg/sec

Figure 4-17: Lateral Response for LF Wheel Only at 100 deg/sec
Figure 4-18: Steer Input for LF Wheel Only at -100 deg/sec to Max Angle

Figure 4-19: Longitudinal Response for LF Wheel Only at -100 deg/sec
The results from these single wheel inputs already give an indication of one of the important results from this chapter. It is possible to achieve a still reasonable 0.17 g of deceleration with one wheel and still have the possibility of using the other wheel to provide directional response. The results show that if doing so, the outside wheel during a turn should be the one to provide the braking. This will be seen in greater detail in 4.5, but to investigate the performance limits, simulations were run with the left front wheel turned in to its maximum angle while varying the right front wheel from approximately –8 degrees to +8 degrees, which corresponds to the peaks for lateral force. The results are shown in Figure 4-21 through Figure 4-23 below. The major result from this simulation is that you can achieve on the order of 0.2 g deceleration while being capable of 0.3 g lateral acceleration in one direction and nearly 0.7 g in the other. This result shows the second method of combined steering and stopping – using one wheel to stop while steering with the other.

**Figure 4-20:** Lateral Response for LF Wheel Only at -100 deg/sec
Figure 4-21: Steer Angles for LF Wheel at Max Angle and Vary RF Wheel

Figure 4-22: Longitudinal Response for LF Wheel at Max Angle and Vary RF Wheel
4.5. Independent Inputs at Each Wheel

With the basic understanding from the previous sections, the next step was to begin to input independent steer angles at each of the front wheels. Three different scenarios were investigated. The first was turning the right front wheel in the opposite direction and a given percentage of the angle for the left front wheel. The second scenario looked at combinations of toe-in of the front wheels with normal steer angle at both wheels. This was done for toe-in first and then steer as well as steer first and then toe-in. The final scenario looked at plotting a complete map relating front steer angles with both lateral and longitudinal response.

4.5.1. Toe-In One Wheel a Percentage of the Other

The first step was looking at steer angles that are different, but still related. More specifically, the first simulations were run with the right front wheel turning the opposite direction of the left front, but at only a fixed percentage of the left wheel angle. This was done by simply placing a gain block on the input to the right front steering actuator and changing the value of the gain.
Simulations were run for gain values of 0.1, 0.3, 0.5, 0.7, and 0.9. Results are shown below for only the 0.1, 0.5, and 0.9 runs. The same steering wheel input of 750 degrees was used for all of the runs, which makes sure the left front wheel reaches its maximum angle, with the right front wheel turning at the given percentage of the left front. Steer input, longitudinal response and lateral response are all shown only for the run with gain of 0.1. For the others only the steer input and lateral response are shown since the longitudinal response simply increases as expected the more the right front wheel turns.

Some very interesting and unexpected results came out of these simulations. As can be seen in each of the lateral responses, the vehicle begins going to the right and then switches directions and starts going to the left. The relative magnitude of the response right and left changes as the gain value for the right front wheel input changes. The less the right front wheel steers compared to the left front, the larger the response in each direction. This response was not understood at first and further simulations were conducted to gain a better understanding. Although unusual, there is a very clear reason why this response occurs. This will be described in detail in 4.6.

Figure 4-24: Steer Input for RF Toe-In at 10% of LF
Figure 4-25: Longitudinal Response for RF Toe-In at 10% of LF

Figure 4-26: Lateral Response for RF Toe-In at 10% of LF
Figure 4-27: Steer Input for RF Toe-In at 50% of LF

Figure 4-28: Lateral Response for RF Toe-In at 50% of LF
Figure 4-29: Steer Input for RF Toe-In at 90% of LF

Figure 4-30: Lateral Response for RF Toe-In at 90% of LF
4.5.2. Combined Toe-In and Steering at Fixed Angles

The next step in the investigation was to look at the two different scenarios of stopping and steering at the same time using the steering actuators. This was done assuming that toe-in of the front wheels is used to stop the vehicle. The first scenario is to start with the vehicle going straight, toe-in the wheels to stop and then turn both wheels by the same amount to steer. The second is to look at the vehicle going into a turn by steering both wheels in the same direction, then toe-in both wheels by the same amount to stop. In order to do these simulations, the steer angle is used to steer both wheels as with normal steering and then a step input of opposite signs is used to toe-in the wheels for stopping. Since the step is added before the rate limiter, the actual change in angles occurs at the rate limit. The steering wheel angle used for all of the simulations is 60 degrees. This change to the steering system is shown in Figure 4-31. The results of the simulations are shown in Figure 4-32 through Figure 4-43.

![NORMAL STEERING SYSTEM](image)

**Figure 4-31**: Steering System Modifications for Combined Steering and Stopping
Figure 4-32: Steer Angles for 10 deg Toe-In Then Steer

Figure 4-33: Longitudinal Response for 10 deg Toe-In then Steer
Figure 4-34: Lateral Response for 10 deg Toe-In then Steer

Figure 4-35: Steer Angles for 20 deg Toe-In Then Steer
Figure 4-36: Longitudinal Response for 20 deg Toe-In then Steer

Figure 4-37: Lateral Response for 20 deg Toe-In then Steer
Figure 4-38: Steer Angles for Steer First Then 10 deg Toe-In

Figure 4-39: Longitudinal Response for Steer First Then 10 deg Toe-In
Figure 4-40: Lateral Response for Steer First Then 10 deg Toe-In

Figure 4-41: Steer Angles for Steer First Then 20 deg Toe-In
Figure 4-42: Longitudinal Response for Steer First Then 20 deg Toe-In

Figure 4-43: Lateral Response for Steer First Then 20 deg Toe-In
Several very interesting results come out of these simulations. First, for toe-in first and then steer, the longitudinal response is very much as expected. The unusual results are seen in Figure 4-34 and Figure 4-37 for the lateral response. On first inspection of Figure 4-34 it appears that there is nothing unusual. The vehicle continues going straight when toe-in of the wheels occurs and then there is a lateral response once the steering wheel input occurs. However upon looking at the lateral response magnitude we see it is very small in comparison with the expected response for a 60 degree steering wheel input. This is explained by looking at Figure 4-32 and remembering the tire lateral force curve from Figure 3-22. The initial toe-in causes both wheels to operate around ±8 degrees, which is near the point of peak lateral force. Once the steer input is added, the left wheel angle goes to 12 degrees and the right wheel changes to -4 degrees. The tire forces still act in opposite directions and have similar magnitude, but since lateral force drops off faster before the peak than after the peak, the left wheel prevails and there is a small response to the right.

The even more surprising result is seen in Figure 4-37. Again on first inspection it seems that nothing is too unusual. The vehicle continues straight during toe-in and then has a lateral response once the steering wheel input comes. The magnitude of the response is larger than in Figure 4-34 as expected since the tires are operating away from the peak tire force point. However, upon looking more closely, we see that the car responds in the opposite direction expected. The steering wheel is turned right, but the vehicle makes a left turn. Although unexpected, this response has an explanation, which will be discussed in detail in 4.6. Looking at Figure 4-35 the quick explanation is that since the wheels turn in opposite directions, the one operating closest to its peak lateral force point determines the direction of response.

Moving on to the simulations with steering first and then toe-in, we see some similar results. In Figure 4-40 we see the initial lateral response as expected from the steering wheel input, then once the toe-in is added, the lateral response goes almost to zero. This is due to the exact same reason as the previous case for 10 degrees of toe-in. As seen in Figure 4-38, the steer angle of the road wheels go from initially both being 2-3 degrees to 12 degrees for the left wheel and -4 degrees for the right, the same as before. This results in the same level of lateral acceleration as was seen in Figure 4-34.
Finally, in Figure 4-43 we see the most alarming response. The vehicle starts by responding as expected with a right turn from the steering wheel input. Then when the toe-in occurs the vehicle actually changes direction and starts going to the left. Obviously this is something you would not want to have happen when you are already in a turn. As before, this will be explained more fully in 4.6, but in short it is due to the wheel operating closest to its peak lateral force determining the direction of response when the wheels are turned in opposite directions.

4.5.3. Vehicle Response to Fully Independent Variable Road Wheel Angles

With the above results showing some unusual characteristics for some specific inputs, the next step was to try and characterize the response of the vehicle for any combination of road wheel angles. Plots were generated showing the longitudinal acceleration, lateral acceleration and yaw rate for all combinations of commanded road wheel angles. To do this the steering system in NAVDyn was modified to allow direct inputs into the road wheel actuators. The inputs are therefore the commanded road wheel angles going into the actuators. The actual road wheel angles will vary slightly due to steering and suspension influences. Numerous simulations were conducted to determine the steady state values for longitudinal acceleration, lateral acceleration and yaw rate for varying commanded road wheel angles. The results are shown in Figure 4-44, Figure 4-45, and Figure 4-46. Since the steady state values are shown, these plots do not show the results seen previously where the vehicle may start turning in one direction and then switch and go in the other direction.

There are a few things to note about these figures. First it can be seen that all the figures are symmetric as anticipated. For the deceleration level in Figure 4-44, as would be expected, the minimum level occurs when both wheels are commanded straight ahead. There is a very small nominal level of deceleration due to the influence of things like nominal toe angle of the wheels, but the value is so small to be insignificant. Also as expected, the higher the angles of the road wheels, the larger the magnitude of the longitudinal acceleration. As was seen in 4.3, the maximum levels occur when both wheels are at their maximum angle, in this case a commanded value of 40 degrees. It is interesting to note that the levels are similar for the cases where the wheels are steered towards/away from each other and when they are both steered in the same direction. The magnitude is slightly higher when the wheels are turned in opposite directions, but
turning them in the same directions still provides a significant level of deceleration. The level is lower when the wheels are turned in the same direction because of the weight transfers reducing the total force generated from both wheels when the vehicle turns. Finally, the figure confirms the results from previous sections. The maximum deceleration level is $0.3-0.4 \, g$. Steering one wheel and holding the other straight ahead gives a maximum deceleration of around $0.17 \, g$. Similarly a deceleration of $0.2 \, g$ can be achieved if one wheel is held at its maximum and the other can vary between $\pm 8$ degrees.

![Long. Accel. vs. Commanded Road Wheel Angle](image)

**Figure 4-44: Steady State Longitudinal Acceleration for Independent Road Wheel Angles**

The lateral acceleration in Figure 4-45 provides the most significant information about how the vehicle responds. Although maybe not obvious from the figure, there is a line of zero lateral acceleration going through all points where the wheels are turned in opposite directions by the same angle. As expected, the highest levels of lateral acceleration come when both wheels are turned in the same direction and the peak occurs for commanded values of around 10 degrees, which results in the front tires operating near the peak of lateral force. The figure confirms the results seen in the previous sections. With the outside wheel at its maximum angle and steering
with the inside wheel, a lateral acceleration level of 0.7 g can be obtained, which occurs when the steered wheel is operating at the peak lateral force in the same direction as the other wheel. Conversely, when the steered wheel is operating at the peak of lateral force in the opposite direction of the other wheel, the lateral acceleration level is around 0.3 g.

Figure 4-45: Steady State Lateral Acceleration for Independent Road Wheel Angles

Figure 4-46 shows the results for yaw rate. The first thing to note is that for the portions of the surface where both wheels are steered in the same direction, the surface is fairly rough, especially near the maximum values for yaw rate. The rest of the surface is fairly smooth. The reason is that when operating both wheels in the same direction, especially at angles near or beyond the point of maximum lateral force, the yaw rate response of the vehicle is very oscillatory. Added to this is the fact that yaw rate changes with vehicle speed, as will be seen in the next chapter in Figure 5-4. Since the figures are for steady state values, this gave some problems for yaw rate. It took some time for the yaw response to settle out to its steady state value. Over this same period of time, vehicle speed decreased due to the deceleration from Figure 4-44. The longer it took for yaw rate to settle out, the more the vehicle speed would change, which in turn causes the yaw rate
to change. This interaction is the reason for the roughness of the curve in the areas of peak yaw rate. Other than this roughness, yaw rate and lateral acceleration have very similar shapes in their response.

![Graph showing Yaw Rate vs. Commanded Road Wheel Angle](image)

Figure 4-46: Steady State Yaw Rate for Independent Road Wheel Angles

### 4.6. Explanation of Results

As discussed above, some very unusual results were found. In particular, for large enough toe-in angles if you toe-in first to stop and then turn both wheels in the same direction to steer, the vehicle responds in the opposite direction the wheels are turned. This means if you turn both wheels right, the vehicle goes left and vice versa. Additionally, if you are already in a turn and you then toe-in both wheels by the same angle, if it is large enough the vehicle will change directions. This means that if you are already in a right turn and you toe-in the wheels, the vehicle will change direction and start going left. Obviously these are very significant results and need to be fully understood before a closed-loop approach to providing the desired response can be created.
The explanation of these results is best explained by looking at the lateral force versus slip angle curve for a tire. For reference, Figure 3-29 is repeated here as Figure 4-47. Recall that the lateral force has an initial positive slope for low slip angles, reaches a peak force around 7-10 degrees and then has a negative slope beyond the peak. Once the slip angle of a tire goes beyond the peak, the amount of force goes down as the slip angle continues to increase. The influence of this on lateral response is compounded by the fact that as the steer angle of the wheel continues to increase, the portion of the lateral tire force acting in the lateral vehicle direction goes down. The end result is that if the wheels are being turned in opposite directions, the one operating closest to the peak lateral force will prevail and the lateral response will be in the direction determined by the prevailing wheel.

![Figure 4-47: Variation of Tire Lateral Force with Normal Load](image)

This is most significant as the wheel angles are changing and one tire goes through the peak first and then the other reaches the peak. The vehicle will begin responding in the direction determined by the first tire when it is closest to its peak, then will switch and respond in the direction of the second tire when it is closest to its peak. This is not an issue when both wheels...
are turning in the same direction, but when they are turned in opposite directions then the overall response of the vehicle will change directions. Understanding this fact is key to providing the desired response with the closed-loop system in the next chapter.

As an example refer back to Figure 4-41 and Figure 4-43. Here the vehicle initially makes a right turn and both wheels operate around 2-3 degrees. Then toe-in of 20 degrees occurs on both wheels resulting in the left front wheel operating at around 22 degrees and the right front at –14 degrees. The vehicle changes directions because after the toe-in the right front wheel is operating closer to the peak of lateral tire force and since it is turned to the left, the vehicle changes direction and starts going left.

Similar results are seen in Figure 4-35 and Figure 4-37. Here the initial input was toe-in of 20 degrees, resulting in actual road wheel angles of around ±18 degrees. This places both tires well beyond the peak of the lateral force curve. Then as both wheels are turned to the right, the right wheel moves further down the tire curve to an angle of around 22 degrees. The left wheel straightens out some to around –14 degrees. Since the left wheel is operating closer to the peak of lateral tire force it prevails and since it is turned to the left, the vehicle responds by turning left. (One thing to keep in mind is that when the wheels are being steered in opposite directions, the slip angle the tires see is roughly the same as the angle of the wheel.)

This same line of reasoning holds when trying to explain all of the other simulations as well. Figure 4-44 through Figure 4-46 show the full spectrum where it can be seen that for every combination of road wheel angles, when the wheels turn in opposite directions the one operating closest to the peak at 7-10 degrees determines the direction of response. The difference between how close one wheel is to the peak relative to the other determines the magnitude of the response.

This understanding confirms a previous result that was seen in Figure 4-23. The maximum lateral acceleration possible when the wheels are turned in opposite directions will occur when one wheel is at its maximum angle and the other is operating at the peak of lateral force. From Figure 4-23 we see that the peak lateral acceleration for this condition is 0.3 g.
4.7. Conclusions and Implications on Obtaining Desired Vehicle Response

From the above results and discussions many things were learned. The major points will be repeated here for summary.

1. The maximum level of deceleration possible with the steering system is on the order of 0.3-0.4 g and is obtained when the wheels are turned in opposite directions to their maximum angle.

2. Assuming that the maximum toe-in angle is the same as the maximum toe-out angle, there is no difference in deceleration level between toe-in and toe-out. As a result toe-in is preferred because of its preference when operating on split coefficient of friction surfaces.

3. By using one wheel to stop and the other to steer a deceleration level of 0.2 g can be obtained while providing 0.7 g of lateral acceleration.

4. Similarly, by using toe-in of both wheels to stop and then turning both wheels the same direction to steer the maximum lateral acceleration possible is 0.3 g and occurs with one wheel at its maximum angle and the other at the peak lateral force point.

5. The results suggest two possible methods of providing combined steering and stopping using only two independent steering actuators: toe-in both wheels to stop then adjust the angle of both wheels to steer; toe-in one wheel to stop and use the other to steer.

6. To provide a desired level of performance, the plots showing the relation between wheel angles and longitudinal acceleration, lateral acceleration and yaw angle can be used to find the optimal wheel angles.

7. Whenever the wheels are turned in opposite directions, the one operating closest to the peak lateral force point will determine the direction of response.

With these results and the understanding of how the vehicle responds to independent road wheel angles we can move on to creating a closed-loop system to provide the desired vehicle response.

4.8. Summary

In this chapter the open-loop investigations were reviewed showing how the vehicle responds to independent road wheel angles. Equal toe-in at both wheels, steering only one wheel and combined toe-in and steering of each wheel were all covered with the final result being a complete map of road wheel angles to lateral and longitudinal response. From these simulations some unexpected results were found and explained. Finally some conclusions and the implications of these results on obtaining the desired response of the vehicle were discussed.
5. CLOSED-LOOP CONTROL STRATEGY

5.1. Introduction

With the open-loop simulations complete, the information is now available to develop a closed-loop control strategy for providing the commanded vehicle response based upon the driver inputs. The desire is to have the system be as transparent as possible to the driver. Therefore the closed-loop control system should attempt to provide lateral and longitudinal response with failed brakes that is as close as possible to that seen when the steering and brakes are both operating normally. Before getting into the strategy to accomplish this, some modifications that were made to NAVDyn are covered in 5.2. In 5.3 the general strategy and two methods for providing combined stopping and steering based on the results from the previous chapter are discussed. Then in 5.4 the approach to provide stopping based upon brake pedal force is covered. 5.5 describes the development of the control strategy for providing directional control (or lateral response) based upon steering wheel angle. In 5.6 the methodology for making transitions among the various modes of operation is covered. Finally, once the overall control strategy is covered, simulations showing the performance provided by this strategy are shown in 5.7.

5.2. Modifications to NAVDyn

In order to implement the closed-loop emergency control of the steering system, some modifications had to be made to the initial versions of NAVDyn that only had normal steering operation. Some of these modifications were already shown in the figures from 3.2, but were not described at that point. The first modification was the addition of the brake status input in the top-level interface of NAVDyn, as shown in Figure 3-1. The user selects between normal brake operation and failed brakes. When "normal brakes" is selected, the steering system operates normally. When "failed brakes" is selected, the steering system switches to "Failed Brake Steering Control" and the output from the brake system is set to zero. This can be seen in Figure 3-4 and Figure 5-1. The transition from Normal Steering Control to Failed Brake Steering Control is covered in 5.6.
Figure 5-1: Overall NAVDyn Steering System

In addition to the brake status signal, several other inputs to the steering system were added, which can also be seen in Figure 3-4 and Figure 5-1. These include, besides steering wheel angle, vehicle speed, yaw rate and brake pedal force. The use of these signals will be described in later sections. For the yaw rate input to the steering system, there is a switch for choosing between actual yaw rate and yaw rate estimated by the speeds of the rear wheels. The estimate can be used in cases where the vehicle does not have a yaw sensor. 5.7.5 describes a comparison between actual and estimated yaw rate.

Figure 5-2 shows the details of the Failed Brake Steering Controller. Within the Failed Brake Steering Controller are the individual control strategies for steering then braking and braking then steering. These will be covered in 5.5. The rest of the blocks in Figure 5-2 are used in selecting which control strategy to use and making the transition from normal steering to steering with failed brakes. As mentioned previously, this transition is covered in 5.6.
5.3. General Strategy

As seen in the previous chapter, the response of the vehicle to independent road wheel angles led to two approaches for providing combined stopping and directional control of the vehicle. In addition, it is desirable to maintain normal steering control when there is no brake input. As a result, the general approach summarized here is used.

1. Maintain normal steering system operation until there is a brake pedal input.
2. For stopping first and then turning, toe-in the wheels to stop and then steer the vehicle by turning both wheels in the opposite direction you want to turn the vehicle.
3. For stopping after you are already in a turn, toe the outside wheel in to stop and then adjust the angle of the inside wheel to steer the vehicle.
4. After the brake pedal input is removed, transition back to normal steering operation.

The first approach to combined steering and stopping described above provides the greatest level of deceleration capability, but provides limited lateral acceleration. From here it will be referred to as Method I. In contrast, the second approach provides high levels of lateral acceleration at the expense of reduced deceleration. This approach will be referred to as Method II. The issue comes...
in using the correct method for the conditions. The above approach was selected for two major reasons. To begin with, Method I provides the greatest level of deceleration and provides good results for lateral accelerations up to 0.3 g. As will be seen later, this approach is easier to control and provides the same performance for a left or right turn. For straight line braking this is definitely the better approach.

By contrast Method II provides greater lateral acceleration in one direction than the other. In a turn of greater than 0.3 g lateral acceleration, using Method I would not work since it would not be able to maintain the present course. As a result, Method II is preferred if the vehicle is already in a turn. The issue with this method is that although it can provide 0.7 g in one direction, it can only provide 0.3 g in the other direction, in addition to having reduced deceleration capability. In addition, this 0.3 g is only possible when the wheel used for stopping is at its maximum angle.

For deceleration levels below 0.2 g, this wheel will operate closer to the lateral force peak and will reduce the lateral acceleration capability towards zero. In the worst case scenario, if the wheel used for stopping is operating at the peak lateral force point, then this method will only allow 0.7 g down to 0 g, or straight ahead, but cannot turn in the other direction. This may cause a problem if the driver assumes he has the same turning capability in both directions. This brings up an issue that must be addressed. If Method II is used and the driver wants to change directions, either a transition must be made to Method I, the angle of the wheel used for stopping must be increased or else the wheel used for stopping and the wheel used for steering in Method II must switch. This issue will be discussed in 5.6. For both methods, the desire is to maintain normal steering operation when there is no brake pedal input.

5.4. Control Strategy for Stopping

From the open-loop simulations we saw that other than the initial part of the curve at low angles, there is a nearly linear relationship between toe-in angle of the front wheels and the level of deceleration provided. As a result, the decision was made to use a simple open-loop control strategy for stopping that relates brake pedal force to toe angle, which in turn is related to deceleration. Besides being simpler, this approach also eliminates a conflict that was found under some conditions when trying to provide a closed-loop level of deceleration at the same time as
providing closed-loop lateral response with only two actuators. One downside is that since the road wheel angles get adjusted as necessary to provide directional control, the nominal level of deceleration provided by this approach gets changed as the road wheel angles change. This was viewed to be acceptable since the thought is that if the actual deceleration level changes from what the driver wants, he will simply adjust the amount of pedal force being applied to either increase or decrease the deceleration level as desired.

The result of this approach is that there is a simple gain that is used to convert brake pedal force to road wheel angle. This gain is different depending upon which of the above methods is used. Since the level of deceleration provided is double when using toe-in of both wheels compared with only one wheel, the gain for a single wheel is twice that for both wheels. The gain was determined by comparing Figure 4-7 with the relation between brake pedal force and deceleration under normal braking, as shown in Figure 5-3. From Figure 5-3 the slope relating pedal force to deceleration is -0.0045 g/N. From Figure 4-7 the relation between toe angle of the wheels and deceleration is -0.0080 g/deg. From these two numbers we get the relation between pedal force and toe-in angle as 0.56 deg/N for toe-in of both wheels. For a single wheel this number is doubled to 1.12 deg/N. These gains are used in the Failed Brake Steering Controller to relate pedal force to toe-in angle for the front wheel(s).

A few other features are used for the stopping control. First, since the normal brake system has a deadband in brake pedal force before brake output begins, this same deadband is implemented here. Since it is desired to keep the wheel(s) being used to stop operating past the peak lateral force since operating before the peak results in opposite influence on lateral response, an angle limit is used to keep the commanded toe angle between 10 and 40 degrees. No more deceleration is possible once the wheels reach their maximum toe-in angle. For Method I, stopping with both wheels, the pedal force input is limited to the value that achieves this maximum angle. This is done to prevent an interaction with the directional control strategy if such a limit is not imposed. This is not an issue with Method II. For Method II, using one wheel to stop and the other to steer, the wheel used to stop changes depending upon if the vehicle is in a left turn or right turn. As a result switches are used to select the correct wheel for stopping based upon the steering wheel angle. Finally, for both methods a rate limit is used on the speed of the stopping control. This was
done to improve the overall performance due to limits in the bandwidth of the steering actuators. The implementation of the open-loop stopping control for each of the methods is shown in Figure 5-6 and Figure 5-7.

![Graph](image)

**Figure 5-3:** Deceleration vs. Brake Pedal Force for Normal Braking

### 5.5. Control Strategy for Directional Control

While the stopping control was quite straightforward, the strategy for directional control was not as simple. Each of the two methods (stopping and steering with both wheels and stopping with one wheel and steering with the other) was implemented separately. As mentioned previously, Method I is used when stopping first and then steering. Method II is used when steering first and then stopping. The methodology for how to select which method is used and the transitions between modes of operation are covered in the next section. The first item covered in this section is the use of closed-loop yaw control to provide desired lateral performance based upon driver input. Next the individual implementations of the two methods are covered. These are shown later in Figure 5-6 and Figure 5-7.
5.5.1. Closed-Loop Yaw Rate Control for Providing Desired Directional Response

Both lateral acceleration and yaw rate were looked at to determine which would be used to provide the desired vehicle response. After first implementing closed-loop control of lateral acceleration, it was realized that the driver is more concerned with the direction the vehicle is heading than with the level of lateral acceleration. This was confirmed by talking with some colleagues at Delphi who communicated that yaw rate control is used for stability control systems currently in production. Thus closed-loop control of vehicle yaw rate was used as the method for providing the driver with the desired performance.

The way this was done is that numerous simulations were run showing the relationship between steering wheel angle, vehicle speed and yaw rate with the steering system operating normally. An initial speed of 250 kph was used, then the desired steering wheel angle was input and the vehicle speed allowed to drop until the simulation was ended at 20 kph. The initial speed of 250 kph was used to allow the vehicle response time to settle out before the speeds of interest were obtained. The results are shown in Figure 5-4 for vehicle speeds from 20 to 160 kph and steering wheel angles from 15 to 180 degrees. For each of the methods of Failed Brake Steering Control, this "normal" performance was used to generate the control signal for the desired yaw rate of the vehicle. As is seen in Figure 5-6 and Figure 5-7, steering wheel angle and vehicle speed are fed into a block that determines the control input. The details of this block are shown in Figure 5-5. This block simply uses a 2-D lookup table based upon the results shown in Figure 5-4 to determine the desired yaw rate. The lookup table is based upon positive values only since vehicle response is symmetric. For negative steering wheel angle, the input is made positive, fed into the lookup table and the result turned back to negative. The resulting control input from Figure 5-4 is then compared against actual vehicle yaw rate and the error is used to make adjustments to the road wheel angles. The values used in the lookup are shown in Appendix D.

The results in Figure 5-4 are based upon the steady state values of yaw rate. Since the actual vehicle response resembles a second-order system, the steady state yaw rate is first run through a second order filter in order to more closely resemble the performance with normal steering. The values for the second order filter were determined from the response of the vehicle with normal steering control.
Figure 5-4: Vehicle Yaw Rate vs. Vehicle Speed for Varying Steering Wheel Angle

Figure 5-5: Closed-Loop Control Reference Signal
5.5.2. Directional Control for Stopping First Then Steering – Method I

As mentioned previously, Method I involves toe-in of both front wheels to stop and then adjustment of the angle of both wheels to steer. This method is used when the vehicle is driving straight and the brake pedal is applied before a steering input. As was seen previously, with this approach, when the wheels are turned in one direction, the vehicle responds in the opposite direction. As a result, the control system is set up to steer the wheels in the opposite direction of the steering wheel input. The first thing to occur is that once the brake pedal is applied and the transition to Failed Brake Steering Control is made, the wheels are immediately turned inward to a minimum angle. This angle is chosen to guarantee that the tires will operate at or beyond the peak lateral tire force. As mentioned previously, the amount of toe-in angle is proportional to the brake pedal force. Added to the toe angle for each wheel is a steer angle that is determined from the closed-loop yaw control.

As was seen before, in order to steer the vehicle when the wheels are turned towards each other, one wheel needs to be turned further away from the peak lateral force point and the other towards the peak lateral force point. The lateral response is determined by the wheel closer to its peak and the magnitude is determined by the difference between how close each tire is operating to its peak. The maximum lateral acceleration with this method is achieved with one wheel at its maximum angle and one at the peak lateral force point. This methodology can be seen in Figure 5-6. The reference command is determined as described above based upon steering wheel angle and vehicle speed. This is compared against the actual yaw rate and the error is used to turn both wheels in the opposite direction that the vehicle needs to go. A combination of proportional and integral control is used to obtain the final commanded angle for the road wheels. The gains for the PI control are adjusted to obtain desired performance. The resulting angle is subtracted from the angle each wheel is operating at due to the brake pedal force. For both methods the rear wheels continue to have zero input angle, the same as with normal steering.
One item to note is the “anti-windup” feature that has been added. Since the angle of the road wheels is confined to minimum and maximum values, once these values are reached, the input to the integrator for the integral portion of the control signal is set to zero. Without this feature the integrator continues to command larger and larger angles if the error does not go to zero. The problem occurs when a change in direction is desired. If the integrator has reached a large value, when changing directions it takes some period of time for this integral error to reduce to zero and start changing in the other direction. This causes a lag in the response of the vehicle while this is occurring. The way this was addressed is that once the magnitude of the commanded road wheel angle reaches the maximum or minimum value, the input to the integrator is set to zero. This feature was not present for all the simulations, but was added when transitions between operating modes, described in 5.6 and 5.7.3, were being addressed. It was during these transitions that the effect of wind-up was noticed and the anti-windup feature was added. For driver inputs within the capability of the Failed Brake Steering Controller, this feature does not do much. It becomes important for cases where the driver is commanding performance greater than can be provided. It is in these instances where the actuators get saturated and integral error builds up without the anti-windup feature.
5.5.3. Directional Control for Steering First Then Stopping – Method II

Method II is used when the vehicle is already in a turn when the brakes are applied. For this method the outside wheel in the turn is used to stop the vehicle, as described in 5.4, while the inside wheel is used for steering. Figure 5-7 shows the implementation of this method. As mentioned before, since the selection of which wheel is used to stop and which is used to steer is determined by which direction the vehicle is turning, the controller must take this into consideration. This is done through the use of switches that change the input for each wheel based upon the sign of the steering wheel angle. As mentioned in 5.4 the brake input is simply proportional to the brake pedal force. For steering, the closed-loop yaw control is used to adjust the angle of the steered wheel.

![Figure 5-7: Failed Brake Steering Controller for Method II: Steer Then Brake](image)

As discussed in 5.5.1, the steering wheel angle and vehicle speed are fed into a lookup table to get the desired yaw rate for the vehicle. This is then compared against the actual yaw rate of the vehicle and the error is used to adjust the angle of the steered wheel. For Method II, a combination of proportional-integral control and feedforward of the steering wheel angle divided by the steering ratio are used. The PI control is very much the same as that for Method I with the
proportional and integral gains adjusted to achieve desired performance. Since the vehicle is already in a turn when the brake pedal is applied with Method II, initially the yaw rate error should be close to zero since the vehicle has been operating with normal steering. Without the feedforward term, this would cause the controller to try and straighten out the steered wheel, which would then cause a yaw error taking the angle back near its original position.

The feedforward term is used to have the nominal angle of the steered wheel, when yaw error is zero, be the same as it would be under normal steering operation. This improves performance since the steered wheel should already be close to the necessary angle when closed-loop control begins. Without the feedforward, the wheel starts at the correct angle, is driven to zero angle, and then is driven back near the original angle due to the yaw error. The angle of the steered wheel is limited to ±10 degrees so that it does not go into the negative slope portion of the lateral force curve. In addition, the same "anti-windup" feature as described above for Method I was added here for Method II.

5.6. Transitions Between Various Operating Modes

In implementing the closed-loop control strategy, the realization came quickly that figuring out how to make smooth transitions between the various modes of operation was a significant issue. (Keep in mind that the entire closed-loop control strategy assumes that you know the normal brake system has failed. Being able to detect this is an area of research all on its own.) It is not enough simply to have each mode operate well on its own, but lose control of the vehicle when changing between modes. As such this area received significant attention with many different approaches tried before the final methods shown here were decided upon. Even so, this is an area that could possibly be improved with even further work. There are several transitions that need to be managed. The first is the transition from normal steering operation into Failed Brake Steering Control once there is a brake pedal input. As part of this, the correct method must be selected based upon the conditions of the vehicle – namely if it is driving straight or in a turn. Similarly, once the brake pedal force is removed, a smooth transition needs to occur back to normal steering control. In addition there may be conditions where you want to transition from one method of closed-loop control to the other and this needs to be managed. As mentioned previously, there is
an issue when Method II is used and the driver tries to change directions. This is another transition that must be addressed. All of these transitions are discussed below.

5.6.1. Transition Between Normal Steering and Failed Brake Steering Control

The first transition that must occur is that from Normal Steering Control to Failed Brake Steering Control. As mentioned, the assumption is made in this research that the failure in the brake system can be detected and that information is communicated to the steering controller. Normal steering operation is maintained even with the brakes failed until there is a brake pedal input. Once the brake pedal input is received, at that point the transition to Failed Brake Steering Control must occur. The implementation of this approach can be seen in Figure 5-1. Normal Steering Control is used exclusively if the brakes are operating normally. If the brakes have failed, the choice between Normal Steering Control and Failed Brake Steering Control is determined by the output of the Failed/Normal Transition block. The details of this block are shown in Figure 5-8. This block also covers the opposite transition from Failed Brake Steering Control back to Normal Steering Control.

The basic operation of this block is as follows:

1. The output of the transition block is based upon the state of both the brake pedal and the angle of the road wheels.
2. The angle of each front wheel is added together and the absolute value taken. If the result is above an upper threshold value, the relay switches ON. Once the relay is ON it only switches OFF if the result is below the lower threshold value.
3. The same approach is taken for brake pedal force. The relay switches ON if the brake pedal force is above an upper threshold and only switches OFF if the value drops below the lower threshold.

4. The output from the steering relay and brake pedal relay are added together and fed into a third relay. This relay also switches ON once the upper threshold is reached and switches OFF once the value drops below the lower threshold. The key is that the upper threshold can only be obtained when the brake pedal relay is ON. Once the relay is ON, the input to it can only drop below the lower threshold if both the steering and brake pedal relays are switched OFF.

The intent is that the transition to Failed Brake Steering Control only occurs once there is a brake pedal input. The transition back to Normal Steering Control only occurs once both the steering wheel input and the brake pedal input are removed. The role of the road wheel angles is that the transition from Failed Brake Steering Control back to Normal Steering Control should only occur when the vehicle is driving nearly straight ahead. If the vehicle is operating at anything above a small level of lateral acceleration, making this transition results in erratic vehicle response. This is the reason the transition is not made until the combined angle of the road wheels is nearly straight ahead.

Once the brake pedal force reaches the threshold described and the transition is made to Failed Brake Steering Control, the correct method must be selected based upon the state of the vehicle. This selection is shown in Figure 5-2. As described previously, if the vehicle is driving straight when the brake pedal is applied, Method I should be used. If the vehicle is in a turn when the brake pedal is applied, Method II should be used. This is implemented through two basic ideas. First, the method used is determined based upon the state of the vehicle at the point when the Failed/Normal Transition output relay is turned ON. At this point in time, if the steering wheel input angle is above a threshold and the brake pedal is applied, Method II is used. If the steering wheel angle is below the threshold, then Method I is used. The second part of the implementation is that once a particular method is selected, you should maintain that method until the Failed/Normal Transition output relay is turned OFF. This prevents changing the method used in the middle of a maneuver, which would cause erratic vehicle response. This is accomplished through the use of the memory block. Once the Failed/Normal Transition output relay is turned ON, the method used becomes the same as that used in the previous time step. This does not get reset until the relay turns OFF.
5.6.2. Transitions Between Method I and Method II

As mentioned previously, the method used depends upon the state of the vehicle at the time the brake pedal is applied. One potential issue is that the state of the vehicle may change at some future point. This is an area that needs further consideration. The choice of Method I or Method II basically comes down to making a trade-off between longitudinal response and lateral response. Since this is somewhat subjective and all of the different possible scenarios seen by the vehicle need to be considered, this is an area that needs further discussions in deciding on a final approach, particularly with the vehicle manufacturer that may be the customer. This discussion will not be made here. Instead, an approach for making the transition between methods will be discussed, which could be used whenever such a transition is deemed necessary.

There are only two operating points that are shared in common between Method I and Method II. These operating points are the natural place to make a smooth transition between methods. The first point in question is the one where one wheel is operated at its maximum toe-in angle and the other wheel has a toe-in angle that corresponds to the peak lateral force for that tire. The second operating point is where both wheels have a toe-in angle corresponding to the peak lateral tire force. Once the vehicle is operating at either of these points, either method could be used and the transition could be made smoothly without any unusual changes in road wheel angles. Making the transition at any other operating point would result in a discontinuity in the commanded angle of the road wheels between the methods.

Obviously there are tradeoffs between the two operating points described above. The first operating point allows you to maintain a deceleration level of around 0.2 g while making the transition, but you have to be in a turn in order to do so. In contrast, the second operating point described above has the vehicle driving straight, but deceleration must be reduced to a minimal level in order to make the transition. So once again, there is a condition where a decision must be made between lateral response and longitudinal response.

Using the two operating points described as the transition points, making the transition between methods involves basically three steps. First, the decision must be made that a change between methods is needed. Then a decision must be made about which operating point will be used to
make the transition. Once these decisions are made, the vehicle needs to be taken to the appropriate operating point in order to make the transition. Once at the operating point the transition is nothing more than simply making the switch. To show that this approach works, it was implemented and simulations run to show the results, which can be found in 5.7.3.2. The actual conditions for making the transition were not defined. It was simply assumed that a transition was needed in order to show that it could be done. The implementation for each of the operating points will be shown below. The implementations are shown only for transitions from Method I to Method II. The transitions in the other direction are very similar and are not repeated here.

**Transition Between Method I and Method II While in a Turn**

As mentioned, the complete process of deciding under what conditions of the vehicle to make the transitions will not be covered here, but is an area of further work. The work here only shows that the transitions can be made once they are desired. There are two transitions for going from Method I to Method II. One is for the operating point when you are in a turn and the other is for the operating point when the vehicle is going straight ahead. The operating point in a turn will be covered first.

There is one issue when making this transition. If you start in Method I and are making a right turn to get to the first operating point described above, the left wheel will be at the peak lateral force point and the right wheel will be at the maximum toe-in angle. As it stands, Method II uses the outside wheel during a turn to provide braking. Since in this case it is a right turn, the left wheel would be used for braking and the right wheel for steering. This is the opposite of what we need to make the transition. For the transition the selection of wheels for braking and steering using Method II must be reversed. The result is that if you start in Method I by braking, then make a right turn to get to the first operating point, you will be at around 0.3 g lateral acceleration. Then you transition to Method II, but the role of the wheels is reversed from normal. This allows you to maintain the 0.3 g lateral acceleration and then you can change directions and achieve up to 0.7 g lateral acceleration in the other direction.
The way this was implemented is shown in Figure 5-9. This is a very simplified implementation just to show that the transition can be made and it depends upon the correct steering and braking inputs. Assuming you start with a brake input so that you enter Method I, then a steering input is needed to guarantee that you achieve the operating point with one wheel at its maximum angle and the other at the peak lateral force point. Once the wheels reach each of these points, the output of the relay after the sum turns ON. This causes a switch to Method II for the remainder of the simulation. See 5.7.3.2 for the results of the simulation for this transition.

**Figure 5-9: Implementation of Transition from Method I to Method II in a Turn**

**Transition Between Method I and Method II While Driving Straight**

The second way to get from Method I to Method II is if the vehicle is driving straight. For this transition to occur, both wheels need to have the same toe-in angle and this angle needs to be the one corresponding to peak lateral force. The implementation of this transition is exactly the same as that in Figure 5-9, but with some of the values changed for the relays. Again, this is a very simplified approach just to show that the transition can be made. Starting with a brake input to enter Method I, the vehicle needs to drive straight and then the road wheel angles have to be reduced to the minimum toe-in angle for Method I. Once these angles have been reached, the
transition to Method II can occur and then the road wheel angles can be changed as needed by Method II to obtain the desired performance. Simulation results showing this transition can be found in 5.7.3.2.

5.6.3. Method II – Transition During Change in Direction of Steering Wheel Input

As mentioned previously, when using Method II if the direction the vehicle is turning changes, the magnitude of the lateral response capability depends upon the amount of brake pedal force being applied. In the worst case the vehicle would only be capable of going straight ahead and could not turn in the opposite direction at all. Obviously this is not a desirable condition. To address this, one of two approaches needs to be taken. The simpler approach is to turn the wheel being used for stopping to its maximum angle once the driver changes directions for the steering wheel input. Doing so will allow the steered wheel to provide up to $0.3\,g$ lateral acceleration in the new direction. This is the same level that can be provided by Method I and occurs at the same operating point, as described above. As a side note, once at that operating point a transition to Method I could be made if desired as described in the previous section.

If more than $0.3\,g$ lateral acceleration is needed, the only option is to make a switch between the wheel used for steering and the one used for stopping. This transition is very difficult to make smoothly. To make this change at any operating point other than driving straight ahead would require large angle changes of the road wheels and unwanted vehicle response. The only way to make this transition without having the vehicle make unwanted changes in direction is to have the vehicle going straight ahead. Under this condition each wheel can make the change in operation to the other side of the lateral force peak in such a way that the lateral force from each of the tires always balance each other. For anything other than straight ahead driving the switch can not be made while maintaining the directional response of the vehicle.

Even if done at the point where the vehicle is driving straight ahead, bandwidth limitations of the actuators prevents this from happening instantaneously and the transition may not be able to occur as fast as the driver command to change direction of the vehicle. The other significant downside to this approach is that at some point both wheels must be turned to the point of peak lateral force. At this point deceleration is minimal. Making the transition as quickly as possible
reduces the time spent at this reduced level of deceleration. The influence of actuator bandwidth on making these transitions is covered in 5.7.4.

The implementation of the first of the two approaches just described is shown in Figure 5-10. This first approach of increasing the wheel used for stopping is done by adding additional steering angle to the wheel used for stopping once the steering wheel angle changes signs. This approach is seen in Figure 5-10. This is a very simplified implementation just to show feasibility. The basic approach is that the vehicle starts out turning in one direction with normal steering. Once the brake pedal is applied, the transition to Method II occurs. When the sign of the steering wheel angle changes, the angle of the wheel used for stopping is commanded to its maximum angle. This is accomplished through the relay shown which changes state when the sign of the steering wheel angle changes. At that point an angle is added large enough for the road wheel to reach the maximum angle. One other modification made is that the role of each wheel is maintained even when the steering wheel angle changes sign. This was done by simply changing the value of the threshold used in the switches at each wheel.

Figure 5-10: Implementation for Change in Direction Using Method II by Adding Angle
The second approach of making a switch between the wheel used for steering and the one used for stopping is accomplished by switching the role of the front wheels at the point the steering wheel angle changes signs. Method II was originally implemented by making this switch instantaneously when the sign of the steering wheel angle changes. Simulation results showing the switch in roles of the front wheels using this approach are shown in 5.7.3.3. Simply making the change in roles of the front wheels when the steering wheel angle changes sign does not provide a very smooth transition.

Although not actually implemented, it is easy to imagine an improved way of managing this transition. In order to make the transition predictable, the commanded yaw rate is set to zero during the transition. As the steered wheel and the wheel used for stopping are swapped, the angles are adjusted so that the lateral forces offset each other and the vehicle continues going straight until the transition is over. Although this approach has not yet been implemented it was simulated in 5.7.3.3 simply by the correct use of steering and braking inputs. Obviously it is possible to change the angles of the road wheels in such a way that the lateral force from each tire is the same and the vehicle continues going straight. The down side to this approach is that due to the rate limit of the actuators, making this transition will take some time. During this period of time the vehicle will continue going straight even if the driver is commanding a turn. The goal is to keep the time for this transition as short as possible. The rate limit of the actuators has a direct influence.

5.7. Simulations of Closed-Loop Performance

Once the above closed-loop approach to combined steering and emergency braking using only two independent steering actuators was developed, simulations were conducted to evaluate performance. First simulations were run for conditions using only Method I and then only Method II. Then simulations were conducted to evaluate the transitions between the various modes of operation. Finally there were some additional simulations conducted to investigate two different topics that influence the overall performance. The first investigation was looking at the influence of steering actuator bandwidth on performance. The second compared using actual vehicle yaw rate with using a yaw rate estimate based upon rear wheel speeds.
5.7.1. Closed-Loop Performance for Stopping First Then Steering – Method I

As mentioned, Method I is used when the vehicle is driving straight at the time the brake pedal is applied. Two different scenarios were investigated. First simulations were conducted for straight-line braking and the results were compared against performance when the brake system is operating normally. Then simulations were conducted showing conditions of combined braking and steering.

Straight-Line Braking

If the primary brake system fails, the main objective of Failed Brake Steering Control is to bring the vehicle to a stop in a controlled manner. In many instances this means simply stopping the vehicle while going in a straight line. Although this is the simplest scenario, the performance under these conditions is significant. Figure 5-11 shows the results for straight-line braking using the steering system. Figure 5-12 shows the relationship between brake pedal force and vehicle deceleration. As can be seen in both figures, the Failed Brake Steering Control provides performance that matches very closely with that under normal braking. The initial difference when the brakes are first applied is due to the fact that the first step for Failed Brake Steering Control is to toe-in the wheels to a minimum angle that keeps the tires operating at or beyond the peak lateral force point. This causes the initial deceleration level that is greater than that seen with normal brakes. Once the pedal force reaches a level that causes the toe-in of the wheels to exceed this minimum level, the performance matches nicely with normal brake operation.

For these simulations, the maximum brake pedal force applied was 78 N, which corresponds to the force needed to reach the maximum toe-in angle of the wheels. For pedal force greater than 78 N the Failed Brake Steering Control would not be able to provide any more deceleration. Obviously this would not be the case for the normal brakes, which would continue to decelerate the vehicle more as brake pedal force is increased beyond 78 N. Similarly, if the brake pedal force is applied faster than the capability of the steering actuators, there would be a lag in the response with Failed Brake Steering Control compared with normal braking. However, for deceleration levels and rates within the capability of Failed Brake Steering Control, the steering system provides performance that matches closely with that for normal braking.
Figure 5-11: Longitudinal Response Comparison for Straight-Line Braking

Figure 5-12: Deceleration vs. Pedal Force Comparison for Straight-Line Braking
Combined Braking and Steering

With straight-line braking covered, the next step was to look at conditions where there is a steering wheel input some time after the brake pedal input. Again, the simulations are conducted for inputs that correspond to acceleration levels that are within the capability possible with Method I, namely below 0.3 g lateral acceleration and longitudinal deceleration. Obviously the steering system could not match normal vehicle operation for performance outside of its capability. Two different runs were conducted. First, Figure 5-13 through Figure 5-15 show the response for a brake pedal force of 40 N applied at 2 seconds followed by a 30 degree steering wheel input at 5 seconds. The figures show results for both normal operation and Failed Brake Steering Control. Second, a pedal force of 60 N is applied at 2 seconds and then a 30 degree approximate square wave is input at 5 seconds. This square wave simulates a change in direction of the steering input similar to what may be seen in a lane-change maneuver. The results are shown in Figure 5-16 through Figure 5-18.

Figure 5-13: Longitudinal Response Comparison for Brake First Then 30 deg Step Steer
Figure 5-14: Lateral Response Comparison for Brake First Then 30 deg Step Steer

Figure 5-15: Steering Angles for Brake Then 30 deg Step Steer
Figure 5-16: Longitudinal Response Comparison for Brake First Then 30 deg Square Wave

Figure 5-17: Lateral Response Comparison for Brake First Then 30 deg Square Wave
As can be seen, the Failed Brake Steering Control provides a response that is very close to the response with normal steering and brakes. The longitudinal response occurs slightly faster than that with normal brakes, but this should not be a problem as faster stopping should be desirable. If it is decided that a response closer to the normal response is desired, a first order filter could be added to give the same response. The lateral response matches well except during the initial transient periods where normal operation has a slightly faster response and more overshoot. This may partially be due to the performance limits of the actuators. Even so, the responses match quite well considering the fact that the steering actuators are providing both stopping and steering capability.

5.7.2. Closed-Loop Performance for Steering First Then Stopping – Method II

The next simulations were run to evaluate Method II – steering first then stopping. The simulations in this section will cover only steering inputs in a single direction. Simulations showing a change of direction will be covered in the next section about transitions since, as discussed previously, making a change of directions with Method II requires some type of
transition. Again the inputs are limited to be within the performance capability of Method II. The first run was for a steering input of 60 degrees at 2 seconds, then a brake pedal input of 30 N at 5 seconds. The results are shown in Figure 5-19 through Figure 5-21.

As can be seen, there are several interesting results. The longitudinal response for failed brakes matches very well with that for normal brakes. The overall deceleration level is nearly identical other than the initial faster response as was seen before. The lateral response shows some more unusual results. As can be seen, when the transition from Normal Steering Control to Failed Brake Steering Control occurs, there are some initial oscillations in the lateral response. The more surprising response is what occurs as the vehicle speed approaches zero. Here there are some very significant oscillations. For the small oscillations at the initial transition it is hard to know whether these would be subjectively acceptable without evaluating them in an actual vehicle. The large oscillation at the end need to be further understood.

There are two possible explanations for this response. The first is that in an actual vehicle there is a difference between the way a vehicle responds to steering inputs for moderate to high speeds versus what occurs at low speeds. For very low speeds, as is the case when the large oscillations occur, kinematics start to have a larger influence since the vehicle no longer has much lateral acceleration and the dynamics of the tires are no longer as significant. Since this change in response at low speeds is not modeled there may be an issue when operating at low speeds. The more likely explanation, but also related, is that the speed of response of the tires is a function of the speed of rotation – the slower the tires turn the slower the response. What is most likely happening is that for low speeds the response of the tires becomes too slow for the needed response. This results in the command and the response getting out of phase and the system becomes unstable. This most likely does not actually happen for a real vehicle. Since, as mentioned, the response of the vehicle tends to change at low speeds and this is not covered in the model, this unusual response at low speeds can probably be neglected. However, to be sure further work would need to be done to either model the low speed response of the vehicle or do testing on an actual vehicle, or both.
Long. Accel. vs. time: Steer Input = 60 deg @ 300 deg/s Brake Input = 30 N @ 100 N/s

Figure 5-19: Longitudinal Response for Steer First Then 40 N Brake Pedal Input

Lat. Accel vs. time: Steer Input = 60 deg @ 300 deg/s Brake Input = 30 N @ 100 N/s

Figure 5-20: Lateral Response for Steer First Then 40 N Brake Pedal Input
5.7.3. Simulations Showing Transitions Between Modes of Operation

As mentioned, managing the transitions between the various modes of operation of Failed Brake Steering Control is a very important area. Several different transitions were discussed in 5.6 for which simulations were conducted to evaluate the performance of the Failed Brake Steering Control. The first are the transitions between Normal Steering Control and Failed Brake Steering Control. Next are the transitions from Method I to Method II and vice versa. Finally, the transitions that occur when changing directions of the steering wheel input under Method II are simulated.

5.7.3.1. Transitions Between Normal and Failed Brake Steering Control

Simulations were conducted to show how smoothly the transitions occur going from normal steering to Failed Brake Steering Control and then back again to normal steering. This was done for both Method I and Method II. The transitions from normal steering to Failed Brake Steering Control were already seen in the evaluations of Method I and Method II above since this has to
happen when the brake pedal is applied. For the simulations below the same results will be seen, but the simulations will also show the transition back to normal steering. For both methods simulations were conducted for removing the brake pedal input first then the steer input and for removing the steer input first then the brake pedal input.

**Method I Transitions**

The first simulation conducted was for simple straight-line braking. For this simulation, the brake pedal force was ramped up to 50 N at a rate of 50 N/sec, held constant for 3 seconds, then ramped back down to zero. The rate of 50 N/sec was selected so that the pedal force would change at a rate slower than the change in road wheel angles. This shows the transitions better since for rates faster than the capability of the steering actuators the response is dominated by the rate limit of the actuators. The results for longitudinal response are shown in Figure 5-22. Since there is no steering wheel input, the vehicle simply drives straight ahead and so the lateral response is not shown.

![Graph showing longitudinal acceleration and velocity](image)

**Figure 5-22:** 50 N Step in Brake Pedal Force Then Back to Zero
As can be seen, once the pedal force is applied, the actuators immediately go to their minimum angles, giving the initial response shown. Deceleration stays at that level until pedal force reaches a level requiring road wheel angles greater than the minimum levels. The response then ramps up to the deceleration corresponding to the 50 N pedal force. Then as pedal force drops the deceleration level decreases until the minimum angles are reached again. Deceleration stays at this level until the pedal force drops below the lower threshold for the Failed/Normal Transition. At this point the transition is made back to Normal Steering Control and the road wheel angles return to their "normal" positions, which in this case is straight ahead. This response is exactly as expected based upon the implementation of the transitions described previously.

The next simulation looks at combined braking and steering where a pedal force is applied, the steering wheel is turned, the pedal force is removed and then the steer input is removed. For this simulation, the same brake pedal force profile is used. The steering wheel input is 30 degrees starting at 4 seconds and then back to 0 degrees at 8 seconds. The results are shown in Figure 5-23 through Figure 5-25. There are several things to note from these results. As can be seen, the differences between normal steering and Failed Brake Steering Control are more significant.

First in Figure 5-23 you can see the interaction between steering control and stopping control. Since there is a simple open-loop gain relating brake pedal force to road wheel angle for stopping, as the wheel angles are adjusted as necessary for steering, in some cases, as is seen here, the deceleration level changes. The maximum deceleration level corresponding to the 50 N pedal force decreases once the steering wheel input occurs. The reason for this can be seen in Figure 5-25. Initially both wheels have the same toe-in angle. Then to turn right, both wheels are turned left. This results in the reduction in deceleration that is seen between 4 and 6 seconds. At 6 seconds the brake pedal force is removed causing another reduction in deceleration. As can be seen, the drop in deceleration when the pedal force is removed is not very significant. This is also due to the interaction with steering, but this time in the opposite direction. Instead of dropping to zero as would happen if the vehicle were going straight, the deceleration stays at the level caused by the angles needed to steer the vehicle. Then at 8 seconds the steering wheel angle returns to zero and the deceleration level drops to that corresponding to the minimum toe-in angles for
Method I. This level remains until around 12.5 seconds, which is the point at which the transition is made back to Normal Steering Control.

Looking at the lateral response in Figure 5-24 we see some additional interactions. The initial response to the steering wheel input is similar to what was seen before. However, this time when the brake pedal force is removed the road wheel angles change and the steering control has to compensate. This causes a drop in the yaw rate until the steering control can compensate. This is a case where faster steering actuators would probably improve performance. When the steering wheel angle goes to zero at around 8 seconds we see the lateral response do the same. The final item to note is that once the vehicle has been driving straight for some period the conditions are met for the transition back to Normal Steering Control, which occurs around 12.5 seconds. As the road wheel angles transition to their normal angles, there is a small disturbance in the lateral response that quickly settles out.

This set of figures has shown one of the limitations of Method I. In order to get any reasonable level of lateral response from the vehicle using Method I, large changes in road wheel angles are required. This causes two problems. First, since braking is done with open-loop control, the changes in angle caused by the steering function changes the level of deceleration. Second, since the bandwidth of the road wheel actuators is limited, making large changes in angle takes some time and may result in a lag from driver input to resulting vehicle response. Method II by contrast has the braking and steering functions separated, one for each wheel. In addition, the steered wheel only operates within around ±10 degrees so actuator speed is not as much of an issue. However, as has been and will be seen, Method II has its own issues.
Figure 5-23: Longitudinal Response for Brake Input Removed Before Steering Input - Method I

Figure 5-24: Lateral Response for Brake Input Removed Before Steering Input - Method I
The final Method I simulation is for removing the steering input before the brake input. For this simulation, the same brake input is used except that it is removed at 8 seconds instead of 6 seconds. The steering input is again 30 degrees, occurring at 4 seconds, but removed at 6 seconds. The results are shown in Figure 5-26 through Figure 5-28. There are really no unusual results here. As can be seen from these figures, the response matches that for normal brakes better when the steer input varies for a fixed brake pedal force than it does for the previous case of a fixed steer input and varying brake pedal force. This is a good result since under emergency conditions of trying to bring the vehicle to a stop, it seems more likely that the driver will be varying the steering angle while maintaining a relatively constant brake pedal force than the other way around.
Figure 5-26: Longitudinal Response for Steer Input Removed Before Brake Input - Method I

Figure 5-27: Lateral Response for Steer Input Removed Before Brake Input - Method I
Method II Transitions

For Method II similar simulations were run with the difference being the steer input always starts before the brake input, as required to enter Method II. The transition from normal steering to Failed Brake Steering Control is the same as in previous Method II simulations since this must occur to get to Method II. The simulations below show this transition as well as the transition back to normal steering again. The first simulation has a 60 degree steer input starting at 2 seconds and getting removed at 8 seconds. The brake pedal input is 30 N at 4 seconds and getting removed at 6 seconds. The results are shown in Figure 5-29 through Figure 5-31 and are mostly as expected.

In Figure 5-30 we see that just as was seen previously, there is some oscillation in lateral response during the initial transition into Method II, but after that the two responses are very similar. The only significant difference is seen in the longitudinal response, but is not unexpected. When the brake pedal force is removed at 6 seconds, the deceleration level drops only to that due to the minimum angle of the road wheel used for stopping. This remains until the
steering input is removed at 8 seconds. As the steering input is removed, the steered wheel starts to turn inward to offset the lateral force from the other wheel, as seen in Figure 5-31. This causes the deceleration level to increase until the conditions are met for transitioning back to Normal Steering Control. When this occurs, both wheels return to the angles for normal steering, which in this case are straight ahead. Other than these transitions at the end, the longitudinal response follows that for normal operation fairly well.

**Figure 5-29:** Longitudinal Response for Brake Input Removed Before Steer Input - Method II
Figure 5-30: Lateral Response for Brake Input Removed Before Steer Input - Method II

Figure 5-31: Steer Angles for Brake Input Removed Before Steer Input - Method II
The second simulation has the steer input being removed before the brake input. For this case the steer input goes to 0 degrees at 6 seconds and the brake input goes to 0 N at 8 seconds. The results are shown in Figure 5-32 through Figure 5-34. For this case of removing the steering input before the brake input, the results for failed brakes and normal brakes match better than those just seen for removing the brake pedal force first. This is the same as was seen for Method I. The longitudinal responses in Figure 5-32 match quite well other than the fact that the Failed Brake Steering Control has a faster response. As discussed previously, if desired this could be adjusted by adding a first-order filter to match the time response for normal braking.

The lateral response in Figure 5-33 again shows the oscillations during the initial transition into Method II and then follows the normal response quite well. One small difference here is seen during the transition from Method II back to Normal Steering Control. As can be seen there are some minor disturbances in the lateral response as the wheels return to their “normal” angles. The parameters used in the Failed/Normal Transition were adjusted to minimize these effects while still providing a reasonably fast transition back to normal operation.

![Graph showing longitudinal acceleration and velocity responses](image)

**Figure 5-32:** Longitudinal Response for Steer Input Removed Before Brake Input - Method II
Figure 5-33: Lateral Response for Steer Input Removed Before Brake Input - Method II

Figure 5-34: Steer Angles for Steer Input Removed Before Brake Input - Method II
5.7.3.2. Transitions Between Method I and Method II

The next simulations conducted were for making transitions between Method I and Method II. As discussed in 5.6.2, there are only two operating points shared in common between the two methods. The first is where one wheel is operating at its maximum toe-in angle and the other is at a toe-in angle corresponding to the peak lateral tire force. The second point is when both wheels have a toe-in angle corresponding to the peak lateral force. These are exactly the conditions in the two simulations where the transitions occur. The transitions from Method I to Method II will be simulated. The transitions from Method II to Method I are very similar and are not shown here.

Simulation of Transition from Method I to Method II While in a Turn

The first simulation is for the operating point where one wheel is at its maximum angle and the other is at the peak lateral force point. The brake input occurs first and is a 50 N pulse starting at 2 seconds and ending at 8 seconds. The steering input is 60 degrees starting at 4 seconds then a change to -60 degrees at 6 seconds. The results are shown in Figure 5-35 through Figure 5-37. The best part of the results is that the transition from Method I to Method II is so smooth that it can not be detected. Looking at Figure 5-37, the transition occurs when the left front wheel reaches its minimum angle and the right front wheel reaches its maximum angle. This occurs at around 4.5 seconds. Although within Matlab this could be detected, the responses shown do not give any indications of the exact point it occurs.

What can be seen from the results is that the transition did occur. Prior to 4 seconds it can be seen that Method I is used since both wheels have the same toe-in angle. Then after 6 seconds we see that Method II is in use since the right wheel is being used to stop and the left wheel to steer. One other item to note is seen in Figure 5-36. For the right turn, the commanded yaw rate, shown as a dashed line, can not be achieved because the steering input is beyond the Failed Brake Steering Control’s capability. This is because when using Method I or Method II when the wheels are turning towards each other, the level of lateral response is much lower than when Method II is used and both wheels turn together. The wheels turn in the same direction beyond 6.5 seconds and we see that the actual yaw rate follows the command very well in this region.
Figure 5-35: Longitudinal Response for Transition from Method I to II While in a Turn

Figure 5-36: Lateral Response for Transition from Method I to II While in a Turn
Steer Input vs. time: Steer Input = 60 deg then -60 deg @ 100 deg/s Brake Input = 50 N pulse @ 50 N/s

Road Wheel Angle vs. time: Steer Input = 60 deg then -60 deg @ 100 deg/s Brake Input = 50 N pulse @ 50 N/s

01-May-2000 NAVDyn v1.5 Vehicle=Taurus GL Tire=205/65R15 Brakes=Failed Initial Speed=100 kph

Figure 5-37: Steer Angles for Transition from Method I to II While in a Turn

Simulation of Transition from Method I to Method II While Driving Straight

The second simulation conducted was for the transition from Method I to Method II at the operating point discussed previously where the vehicle is driving straight. For this simulation the brake input is again 50 N starting at 2 seconds, then reducing to 5 N at 4 seconds and finally going up to 55 N at 6 seconds. The steering input is 0 degrees up until 8 seconds when it goes to 60 degrees. The results are shown in Figure 5-38 through Figure 5-40. Again there are no real surprises.

The longitudinal response in Figure 5-38 is very much as expected. The initial deceleration level is that corresponding to the 50 N input, then when the pedal force is reduced it drops down to the level corresponding to the minimum toe-in angles for Method I. The transition to Method II occurs next and then the pedal force goes up to 55 N. The deceleration goes to the maximum level for one wheel and varies slightly from that level going forward due to the changing angle of the steered wheel. Unlike the previous simulation, the lateral response in Figure 5-39 shows the point where the transition from Method I to Method II occurs at around 5 seconds. There is a
very small disturbance in the lateral response at this point. The slightly larger disturbance in the lateral response comes at around 6 seconds when the brake pedal force increases. This is due to the steered wheel having to compensate for the change in angle of the wheel used for stopping. Since the correction in angle of the steered wheel can not happen instantaneously the resulting response occurs. Once the transition to Method II occurs and after the initial correction from the brake pedal input, the yaw rate tracks almost identically with the commanded value, providing excellent lateral response. All of the results just discussed can also be seen and described by looking at the road wheel angles in Figure 5-40.

![Graphs showing Long. Accel. vs. time and Long. Velocity vs. time](image)

**Figure 5-38:** Longitudinal Response for Transition from Method I to II While Driving Straight

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Figure 5-39: Lateral Response for Transition from Method I to II While Driving Straight

Figure 5-40: Steer Angles for Transition from Method I to II While Driving Straight
5.7.3.3. Change of Direction Using Method II

The final transition is that which occurs when using Method II and changing the sign of the steering wheel input. As mentioned previously, the issue is that although capable of 0.7 g when both wheels turn in the same direction, when the steered wheel turns in the opposite direction of the wheel used for stopping a maximum of 0.3 g can be obtained. However, this level depends upon the amount that the wheel used for stopping is turned. The 0.3 g corresponds to the wheel being operated at its maximum angle. For operation at its minimum angle, the best that can be achieved is driving straight ahead. Obviously if the driver starts in a turn in one direction and then changes directions, this is a problem. The simulations in this section show the two different approaches to addressing this issue, which were described in 5.6.3. The first is to increase the angle of the wheel used to stop to its maximum angle in to achieve 0.3 g lateral acceleration. The second approach is to swap the role of the two wheels when making a change in direction.

Change in Direction with Method II by Increasing Angle of Wheel Used to Stop

The first simulation shows the results when the angle of the wheel used for stopping is increased to its maximum angle when the sign of the steering wheel changes. This allows 0.3 g lateral acceleration in the new direction. The steering wheel input for this simulation is 60 degrees at 2 seconds followed by -60 degrees at 6 seconds. The brake pedal input is a constant 10 N applied at 4 seconds. The results are shown in Figure 5-41 through Figure 5-44 and show the comparison between Method II operation without any modifications and Method II with the modification described in 5.6.3 of increasing the angle of the wheel used to stop. The results are as expected.

The area of interest is when the steering wheel input changes direction at 6 seconds. Figure 5-41 shows that the deceleration level increases due to the increase in angle of the wheel used for stopping. Figure 5-42 shows that without modification, the vehicle can only to return to straight ahead while with modification −0.3 g lateral acceleration can be achieved. This is less than the 0.6 g achieved for the initial right turn, as is expected due to the limitations of Method II. Figure 5-43 and Figure 5-44 show the steer angles for Method II without modification and Method II with modification respectively. As can be seen in Figure 5-44, once the steering wheel angle crosses zero, the angle of the left front wheel increases to its maximum angle.
Figure 5-41: Longitudinal Response Comparison When Increasing Angle of Wheel Used to Stop

Figure 5-42: Lateral Response Comparison When Increasing Angle of Wheel Used to Stop
Figure 5-43: Steer Angles for Method H Without Modification

Figure 5-44: Steer Angles for Method H When Angle of Wheel Used to Stop is Increased
Change in Direction with Method II by Swapping Role of the Wheels

The next simulation is for swapping roles between the steered wheel and the wheel used for stopping when the steering wheel angle changes. The initial implementation of Method II has the roles of the two wheels changing whenever the steering wheel angle changes signs. This is certainly a simple approach, but it does not provide very good performance. The first simulation shown is for this scenario. A steering input of 60 degrees at 2 seconds and then −60 degrees at 6 seconds is used. The brake pedal input is 50 N applied at 4 seconds. The results are shown in Figure 5-45 through Figure 5-47.

As can be seen, the transition from a left turn to a right turn is not very smooth. Since the road wheel angles are simply swapped and nothing is done to manage them, the condition occurs, as can be seen in Figure 5-47, where one wheel goes through the peak lateral force point and then the other does. This causes the initial oscillation around zero lateral acceleration and zero yaw rate seen in Figure 5-46. The subsequent oscillations are caused by the limited steering actuator bandwidth. It can be seen in Figure 5-47 that the angle of the left front wheel changes at exactly the rate limit of the actuator. This lag in response causes the oscillations. This influence of actuator bandwidth will be investigated further in the next section. As can be seen, this level of performance is not acceptable. In order to make the transition from left turn to right more smooth, the angles of the road wheels need to be managed as described in 5.6.3.

For this modified approach, the road wheels switch roles in a controlled manner with the vehicle going straight. This approach was not implemented, but has been demonstrated through the correct use of steering wheel and brake pedal inputs. The vehicle begins in a right turn with a steering wheel angle of 60 degrees starting at 2 seconds. Then at 4 seconds a 30 N brake pedal force is applied causing the transition into Method II. At around 6 seconds the steering wheel angle goes to zero, then at 6.6 seconds the brake pedal force goes to zero. Finally at 7.6 seconds the steering wheel angle goes to −60 degrees and at 10 seconds the brake pedal force returns to 30 N. The results of this simulation are shown in Figure 5-48 through Figure 5-50.
Figure 5-45: Longitudinal Response for Default Method II Change of Direction

Figure 5-46: Lateral Response for Default Method II Change of Direction
Steer input vs. time: Steer Input = 60 deg then -60 deg @ 300 deg/s Brake Input = 50 N @ 100 N/s

Figure 5-47: Steer Angles for Default Method II Change of Direction

Long. Accel. vs. time: Steer Input = 60/0/-60 deg @ 100 deg/s Brake Input = 30/0/30 N @ 100 N/s

Figure 5-48: Longitudinal Response for Managed Method II Change of Direction
Figure 5-49: Lateral Response for Managed Method II Change of Direction

Figure 5-50: Steer Angles for Managed Method II Change of Direction
Some interesting things are seen in these figures. Basically what occurs with this approach is that after the initial transition into Method II, when the steering wheel angle changes sign the commanded yaw rate is set to zero until the transition occurs. This was accomplished for the simulation by simply setting the steering wheel angle equal to zero. Then the brake pedal force goes to zero, which results in both wheels straightening out as the system returns to Normal Steering Control. Then as the steering wheel angle goes in the other direction and the brake pedal is reapplied, the transition back into Method II occurs, but this time with the role of the two wheels swapped. The results are a much smoother transition than the default approach for Method II. There are some significant oscillations when the brake pedal is reapplied at 10 seconds. This is due largely to the bandwidth limitations imposed on the actuators. In the next section this same simulation will be run with the rate limits removed from the actuators.

5.7.4. Influence of Steering Actuator Bandwidth on Performance

As was described previously, a rate limit was used on the steering actuators to limit the speed with which they can turn the road wheels. Although the actual dynamics of the actuators were not modeled, these rate limits, along with the angle limits, put reasonable bounds on the performance obtained from the actuators. This bandwidth limit resulted in some lags in the vehicle response compared with the commanded response under some conditions. In this section two simulations were conducted to investigate the influence of actuator bandwidth on performance. Obviously the higher the rate capability of the actuators, the better the actual road wheel angles can follow the commanded angles.

Without limits on actuator performance, Method II could switch wheels instantaneously for left versus right turns. The next simulation conducted is exactly the same as that shown in Figure 5-45 through Figure 5-47, but with the actuator rate limit removed. This is a theoretical ideal, but it does result in a smooth transition as is seen. The results are shown in Figure 5-51 through Figure 5-53. The longitudinal response in Figure 5-51 has an unusual spike at around 6 seconds due to the instantaneous change in road wheel angles that is seen in Figure 5-53, but otherwise is as expected. As can be seen in Figure 5-52, the yaw rate has a very smooth transition and follows the commanded value very well. This is expected since the road wheel angles make the transition instantaneously. However, the lateral acceleration still shows the transition. This is largely due to
the fact that yaw rate is the controlled variable and no attempt is made to explicitly force a smooth transition in lateral acceleration.

One final thing to note from these results is that even without a rate limit on the actuators, the transition from normal steering to Method II is still apparent. The conclusion from this is that either there are further improvements that can be made in the controller, or else there is a fundamental limitation in performance due to the interaction between braking and steering. Most likely the former is the case.

![Graph showing longitudinal acceleration and velocity vs. time](image)

**Figure 5-51:** Longitudinal Response for Default Method II Change of Direction – No Rate Limit
Figure 5-52: Lateral Response for Default Method II Change of Direction – No Rate Limit

Figure 5-53: Steer Angles for Default Method II Change of Direction – No Rate Limit
The final simulation conducted was again for swapping the roles of the front wheels when making a change in direction of the vehicle using Method II. This simulation is exactly the same as those in Figure 5-48 through Figure 5-50 except that the rate limit on the road wheel actuators was eliminated. The results show that the transitions are smoother. In particular, the oscillations in yaw rate that were seen in Figure 5-49 when the brake pedal was applied at 10 seconds are greatly reduced in Figure 5-55. This is due to the road wheel actuators being able to keep up with the commanded values.

Obviously there are many other simulations that could be conducted for other conditions where actuator bandwidth may influence performance. This is an area that could use further study since the results here only give a small picture of the overall influence actuator speed has on performance. Instead of looking only at the complete elimination of rate limits on the actuators, a more complete study would involve looking at the change in performance as bandwidth increases. Ideally such a study would involve modeling the actual dynamics of the actuators instead of just using rate and angle limits as was done here.

Figure 5-54: Long. Response for Managed Method II Change of Direction – No Rate Limit
Figure 5-55: Lateral Response for Managed Method II Change of Direction – No Rate Limit

Figure 5-56: Steer Angles for Managed Method II Change of Direction – No Rate Limit
5.7.5. Estimated vs. Actual Vehicle Yaw Rate

The final simulations to be covered are related to the yaw rate that is used for the closed-loop yaw control. Obviously using the actual vehicle yaw rate will provide the best performance. This assumes that the vehicle has a yaw sensor for measuring yaw rate. Assuming that the vehicle being used has wheel speed sensors on the rear wheels, which would be the case if the vehicle had anti-lock brakes and would certainly be the case for a brake-by-wire system, an estimate of yaw rate could be used. Since in this research the assumption is made that the primary brake system has failed, there will be no brake torques applied on the rear wheels.

Since the rear wheels are free to rotate, the longitudinal slip of the tires should be minimal, with only the influence of tire inertia coming into play. As a result, during a turn the outside wheel will rotate faster than the inside wheel by a level that is related to the vehicle yaw rate. By taking the difference in speed between the two rear wheels, multiplying by the rolling radius of the tires, then dividing by the rear track width, the result is an estimate of the yaw rate of the vehicle. If found to be adequate, this estimate could be used in place of a yaw sensor, creating a potential cost savings. This estimate could also be used as a comparison with measured value from a yaw sensor to confirm that the yaw sensor is providing valid output.

Simulations were conducted to show how well the estimated yaw rate compares with the actual yaw rate. The first simulation was conducted with normal steering operation and no brake pedal input. The steering input is the same 60 degrees at 2 seconds then -60 degrees at 6 seconds that was used in the previous section. The results are shown in Figure 5-57. As can be seen, the estimated yaw rate tracks the actual yaw rate very well.

Figure 5-58 shows a similar comparison of actual and estimated yaw rate for the exact conditions that were conducted in Figure 5-46 with Method II operation. Although these are some unusual conditions with a very oscillatory response, the results show that the estimated yaw rate still provides very good results. From these simulations it is seen that for the assumption of failed brakes, the estimated yaw rate using the rear wheel speeds is a good alternative to a yaw sensor.
Figure 5-57: Actual vs. Estimated Yaw Rate for Normal Steering and No Brake Input

Figure 5-58: Actual vs. Estimated Yaw Rate for Method II Failed Brake Steering Control
5.8. Conclusions from Closed-Loop Control

This chapter has covered a lot of material related to the closed-loop performance of the vehicle under Failed Brake Steering control and the ability to provide the desired performance to the driver. As a result it is worthwhile to summarize the major conclusions.

1. The general approach for Failed Brake Steering Control is to maintain normal steering whenever there is no brake pedal input. When there is a brake pedal input, if the vehicle is going straight then both wheels are turned inward to stop. If a steering input then occurs, both wheels are turned in the same direction to steer. If the steering input occurs before the brake input then the outside wheel in the turn is used for stopping and the inside wheel is used for steering.

2. For stopping control, a simple open-loop gain is used to relate brake pedal force to toe-in angle. This gain is different depending upon which method is used.

3. To steer the vehicle, closed-loop control of vehicle yaw rate is used. The commanded yaw rate is based upon the performance of the vehicle with normal steering. The difference between commanded yaw rate and actual yaw rate is used to adjust the road wheel angle(s).

4. There is some interaction between the steering and stopping modes, but under most circumstances it is minimal. Some of this is due to bandwidth limitations of the steering actuators.

5. Simulations show that the Failed Brake Steering Control described in this chapter does a reasonable job of providing the driver with the same performance that is seen under normal steering and braking. Of course this is only true within the capabilities of each of the methods used.

6. One of the most significant areas of attention is the transitions between the various modes of operation. Managing these transitions is critical to having acceptable performance. Methods were demonstrated for making the transitions in a controlled manner.

7. In many cases the demonstrations of the various transitions were done in a simplified manner showing that the transitions can be made, but without defining all of the exact circumstances defining when a transition should occur. This is an area that needs further study.

8. A brief investigation of the influence of actuator bandwidth on performance was conducted. As expected, faster steering actuators provide better performance.

9. Finally an investigation was done comparing actual vehicle yaw rate with yaw rate estimated from the angular velocities of the rear wheels. The results show that the estimated yaw rate is a very good alternative to a yaw sensor when there is no brake torque on the rear wheels.

One final thing to note about the implementations of Failed Brake Steering Control shown in this chapter is that many of them were done in very simple and sometimes crude ways for the sake of showing that the desired outcome was feasible to achieve. In particular, many of the transitions
between modes of operation were accomplished with relays and switches that changed states depending on different vehicle characteristics. Now that these transitions have been shown to be feasible, more elegant methods should be used for any actual implementation. In particular, using some form of a state machine to handle the transitions between the various states would probably be much cleaner and work much better than the approach used here to show feasibility.

The end result of this chapter is that indeed it is possible, for low to moderate levels of lateral and longitudinal acceleration, to provide the driver with performance under conditions of a failed brake system that is similar to that under normal steering and braking. This can be done in a way that requires nothing different from the driver. The inputs are still just steering wheel angle and brake pedal force. The controller then takes care of determining the needed road wheel angles to provide the desired performance. Although the performance with failed brakes does not match perfectly with the normal performance, it is close enough that it is believed the driver can make any minor adjustments necessary. Another potential area of further work would be to look into the actual interactions between the driver and the Failed Brake Steering Controller.

5.9. Summary

In this chapter the overall approach to providing closed-loop control of the steering actuators during conditions of a failed brake system have been covered. The general approach was laid out for doing this and then the specific methods for providing steering and braking using only the two steering actuators was discussed. Next the approaches for handling the various transitions between the different operating modes of the system were covered. Simulations were conducted showing vehicle response to the different methods of combined steering and stopping using only the steering actuators as well as showing how well the transitions can be made between the different modes of operation. Finally investigations were conducted into the influence of steering actuator bandwidth on performance and the feasibility of using yaw rate estimated from the rear wheel speeds in place of the actual vehicle yaw rate measured with a yaw sensor. The chapter ended with a summary of the results and conclusions from the work conducted.
6. CONCLUSIONS AND RECOMMENDATIONS

Many different things have come out of this research. Besides pursuing the original idea of using the steering system to provide emergency braking, a new vehicle model, called NAVDyn, was generated that can be used for a wide range of vehicle handling simulations. Conclusions resulting from all the work described in this thesis are presented here along with recommendations for further work.

6.1. NAVDyn

The derivation, implementation and validation of a new vehicle model for simulating vehicle handling was presented. This model was generated to address some specific requirements for a model to be used for vehicle handling analysis with any combinations of road wheel angles and brake torques at each of the four wheels. As discussed, the model needed to be easy to use, simple to customize, competent for simulating vehicle handling and have the capability of adding in control systems as needed. The resulting model that was created is called Non-Linear Analysis of Vehicle Dynamics (NAVDyn) and it was shown to have all of these features.

The derivation of the equations that define NAVDyn was done using as few simplifications as possible for the eight degrees-of-freedom that were included. Effects of suspension and steering kinematics and compliances were also included. A tire model, STIREMOD, was selected that is capable of covering the complete non-linear operating range of a tire under numerous conditions. In addition, a method was selected for handling the tire slip angles and slip ratios that does not have the typical numerical problems when vehicle speed goes to zero. This approach applies lags to all tire outputs and more closely represents what occurs with an actual tire.

NAVDyn was implemented in a way that is easy to follow and makes future customization quite simple. The model is composed of connected subsystems that match closely with those in an actual vehicle. Customization of an individual subsystem can be done without requiring changes to the rest of the model. The flow of information in the model can be easily traced and any information internal to the model that is not a direct output can be obtained with very little difficulty.
The model as implemented was validated against both actual vehicle test data and CarSim, a high-order model. The results show that NAVDyn does an excellent job of predicting actual vehicle response for vehicle handling on a smooth road. NAVDyn and CarSim had very similar results under most circumstances, with NAVDyn having even better results in some cases. The end result is a model that is competent for vehicle handling and is easy to use for a wide range of circumstances. This is exactly what was needed for pursuing the main idea in this thesis.

6.2. Emergency Braking with the Steering System

The main idea pursued in this thesis was the feasibility of using two independent steering actuators to provide stopping capability in the event of a primary brake system failure while still maintaining directional control. Many different aspects of this topic were covered. Open-loop investigations were done to gain a fundamental understanding of how the vehicle responds to independent steer angles of the front wheels. The results of the open-loop study were then used to develop a closed-loop approach to providing desired vehicle response.

The open-loop investigations showed that it is indeed possible to stop and steer the vehicle at the same time using only two steering actuators. A maximum deceleration level of 0.3-0.4 g can be achieved with the steering system for straight-line braking. This is certainly enough to be worth considering. In addition, the tradeoffs between longitudinal and lateral response were covered. It was found that as high as 0.7 g lateral acceleration can be achieved while still providing 0.2 g deceleration. Some unusual results were found when steering the wheels in opposite directions. Under certain circumstances the vehicle will respond in the opposite direction originally expected or may start going in one direction and then switch and go in the other direction. These results were explained by the fact that whichever tire is operating closer to the point of peak lateral force will prevail and determine the direction the vehicle responds. This was one of the major conclusions, which was used to develop the closed-loop approach.

Out of the open-loop simulations came two different methods for providing combined steering and stopping capability. Method I is used when a brake pedal force is applied before a steering input. This method involves turning both wheels inward by the same angle for stopping and then turning both wheels in the same direction to steer. Method II is used when the vehicle is already
in a turn when the brake pedal is applied. This method uses the outside wheel in the turn for stopping and the inside wheel for steering. Benefits and limitations of both methods were discussed. The implementation was done so that there are no changes in the driver inputs. The driver still inputs steering wheel angle and brake pedal force and the steering controller adjusts the road wheel angles as needed to provide the desired response.

The most difficult and possibly the most important aspect of the closed-loop control was managing the transitions between the various operating modes. These transitions include going between normal steering operating and Failed Brake Steering Control, switching between Method I and Method II and changes in direction when using Method II. Approaches were shown for making each of these transitions in a controlled manner. Although further work is needed, it was shown that the transitions could be made relatively smoothly.

The end result of all of this work is that it has been shown that low to moderate levels of longitudinal and lateral vehicle response can be provided using only two independent steering actuators. With this approach, the steering system can be used to provide emergency stopping capability for the vehicle while still maintaining directional control. The benefit is that for a vehicle equipped with steer-by-wire and brake-by-wire systems, it may be possible to reduce the level of redundancy required if the primary brake system must always provide stopping capability, which in turn could reduce cost. Alternatively this approach could be used to provide an added level of backup and in turn increase the overall level of safety.

6.3. Recommendations for Further Work

In addition to the conclusions above, several potential areas of future work were also found. Many of these recommendations were discussed in previous chapters and will be summarized here along with some additional recommendations not covered previously.

1. Limited information is available for operating tires at very high slip angles. Further work needs to be conducted to evaluate tires under these extreme conditions. Besides just evaluating the normal performance characteristics for slip angles beyond those that can currently be tested, two other areas need to be investigated. The first is the relationship
between slip angle and tire wear and how this influences the tire outputs. The second is how the temperature of the tire changes over time when operating at high slip angles and how temperature affects the output of the tires.

2. For the creation of NAVDyn the pitch degree-of-freedom was neglected. For the simulations used to compare NAVDyn against CarSim and the actual vehicle test data, this did not appear to have a significant impact. The conditions most likely to be affected by pitch are combined steering and braking near the limit of adhesion. Since there was a problem with the Ford Taurus test data for these conditions, they were not covered in this thesis. This is an area that would need to be investigated if NAVDyn were to be used for these conditions.

3. During one of the Failed Brake Steering Control simulations it was noticed that there were oscillations in the vehicle response that occurred for vehicle speeds below around 20 kph. Some possible explanations were presented, but to be sure this is not significant further study is needed.

4. The control system methods used in this thesis were very basic and only intended to show the feasibility of being able to steer and stop a vehicle with only two steering actuators. Before any actual implementation, other more advanced and/or elegant control methods should be considered. In addition, more work could be done to optimize the performance provided by the controller by adjusting the various controller gains. Conducting more formal analyses of closed-loop response would be helpful as well.

5. In this thesis the methods for making the transitions between modes of operation of Failed Brake Steering Control were covered, but defining the conditions under which the transitions actually occur was not. Defining the exact conditions under which the transitions occur needs to be done. This requires making tradeoffs between lateral and longitudinal response of the vehicle and specifying the state of the vehicle necessary to cause a transition. This is work that would most likely be done in cooperation with a vehicle manufacturer.

6. Related to the transitions, in this thesis some very simple and sometimes crude approaches were used just to show that the approach for making the transitions was feasible. Further work should be done using a better approach for making these transitions. Use of a state
machine would probably work best where the state of the vehicle can be used to define when
to change modes of operation.

7. A very limited investigation of the influence of actuator bandwidth was conducted in this
thesis. A more complete investigation should be done to show the influence of actuator
bandwidth for a broader range of conditions. In addition, when using the ideas in this thesis
on an actual steer-by-wire system, the dynamics of the steering system should be added into
the analysis. In this thesis only rate and angle limits were used to place bounds on actuator
performance.

8. Estimated yaw rate was briefly covered in this thesis showing that it tracks actual yaw rate
quite well. A more comprehensive study should be done which includes the influences of
yaw sensor and wheel speed sensor resolution and accuracy to further determine how well
estimated yaw rate compares with measured yaw rate from a yaw sensor. In addition, the
significance of the errors caused by using estimated yaw rate on vehicle performance should
be addressed.

9. One of the assumptions in this thesis has been that if the Failed Brake Steering Controller can
provide performance that is within a reasonable percentage of normal vehicle performance,
the driver can make minor corrections as needed. An area that could be studied further is the
interaction of the driver with the steering system. In particular, it should be determined if the
assumption is correct that the driver can make the needed corrections or do the driver’s
corrections actually cause other problems.

10. All of the work for this thesis has assumed two independent steering actuators turning only
the front wheels. Another area that could be investigated is expanding the idea to four-wheel
steering. There may be some possible performance benefits as well as the possibility to
reduce the interaction between directional control and deceleration level.

11. This thesis has assumed the worst case scenario of a total failure in the brake system. Another
area of investigation would be looking at a partial brake system failure. Many brake system
failures may result in loss of braking on only two wheels instead of all four. For partial
failures of the brake system the steering system may only need to supplement the brake
system instead of replacing it, or may need to compensate for yaw disturbances caused by unequal braking forces on the left versus right sides of the vehicle.

12. Finally, at the very beginning of this thesis it was discussed that the original idea was to look into using the capability of another existing system to provide backup in the event of a failure of the primary system. This was pursued for using the steering to provide stopping capability if the brakes fail. The opposite scenario of using the brakes to steer the vehicle in the event of a steering system failure also warrants study.

By-wire systems are relatively new to the automotive industry. As such there is still a lot of work yet to be done to prepare these systems for use. During this initial development, as investigations are conducted, many times more new questions are created than are answered. However, as these questions continue to get answered a better understanding of such systems will emerge until they are ready for use.
APPENDIX A - DERIVATION USING EULER-LAGRANGE

As mentioned in 2.3.1.4, the main equations of motion for the vehicle model were derived initially using Newton-Euler. For the purpose of checking the resulting equations to make sure they were correct, the equations were derived a second time using Euler-Lagrange. The results from the Euler-Lagrange derivation are shown below and are compared against the Newton-Euler approach confirming that they are correct.

To obtain the equations of motion using Euler-Lagrange, generalized coordinates need to be defined, the Lagrangian computed, and then Lagrange’s equation for each of the generalized coordinates needs to be calculated. The first step is to define the generalized coordinates. This is quite straightforward for this problem since the generalized coordinates are simply the coordinates associated with each of the degrees-of-freedom for the vehicle. For these derivations we will only be concerned with the first four degrees-of-freedom, excluding the last four that cover the angular velocities of the wheels. For this derivation we also use the inertial coordinates as opposed to the chassis coordinates. The four generalized coordinates are the longitudinal and lateral positions and the roll and yaw angles, $X$, $Y$, $\phi$, and $\psi$.

Next the Lagrangian needs to be calculated, which is

$$\mathcal{L} = T^* - U$$  \hspace{1cm} (A.1)

where $T^*$ is the kinetic co-energy and $U$ is the potential energy. The kinetic co-energy for this problem is given by

$$T^* = \frac{1}{2} M_s V_s^T V_s + \frac{1}{2} M_{uf} V_{uf}^T V_{uf} + \frac{1}{2} M_{ur} V_{ur}^T V_{ur} + \frac{1}{2} \Omega_b^T I_b \Omega_b + \frac{1}{2} \Omega_b^T I_{uf} \Omega_{uf} + \frac{1}{2} \Omega_c^T I_w \Omega_c.$$  \hspace{1cm} (A.2)

The potential energy is given by

$$U = M_s \dot{gh}_s \cos\phi + M_{uf} \dot{gh}_{uf} + M_{ur} \dot{gh}_{ur} + \frac{1}{2} K_p \phi^2.$$  \hspace{1cm} (A.3)
$K_o$ and $B_o$ are the sum of the front and rear roll stiffness and damping found in (2.71). All of the velocities in the Lagrangian were calculated previously. The results from (2.14), (2.20), (2.27), (2.7), and (2.8) are repeated here for reference:

\[
\mathbf{V}_{uf} = V_{ux} \mathbf{i} + \left( V_{uy} + l_f \psi \right) \mathbf{j} \tag{A.4}
\]

\[
\mathbf{V}_{ur} = V_{ux} \mathbf{i} + \left( V_{uy} - l_r \psi \right) \mathbf{j} \tag{A.5}
\]

\[
\mathbf{V}_s = \left( V_{ox} - h \psi \sin \phi \right) \mathbf{i} + \left( V_{oy} + h \phi \cos \phi + l_{cr} \psi \right) \mathbf{j} + h \phi \cos \phi \mathbf{k} \tag{A.6}
\]

\[
\Omega = \dot{\psi} \mathbf{k} \tag{A.7}
\]

\[
\Omega = \dot{\phi} \mathbf{i} + \psi \mathbf{k} . \tag{A.8}
\]

For calculation of the kinetic co-energy, the angular velocity of the sprung mass needs to be in body-fixed coordinates, which from (2.41) is given by

\[
\Omega = \dot{\phi} \mathbf{i} + \psi \sin \phi \mathbf{j} + \psi \cos \phi \mathbf{k} . \tag{A.9}
\]

We can see that the above linear velocities are not all in terms of the generalized coordinates. To change these equations into generalized coordinates, recall from (2.5) that

\[
V_{ox} = \dot{X} \cos \psi + \dot{Y} \sin \psi \tag{A.10}
\]

\[
V_{oy} = -\dot{X} \sin \psi + \dot{Y} \cos \psi .
\]

Substituting (A.10) into (A.4), (A.5), and (A.6) gives

\[
\mathbf{V}_{uf} = \left( \dot{X} \cos \psi + \dot{Y} \sin \psi \right) \mathbf{i} + \left( -\dot{X} \sin \psi + \dot{Y} \cos \psi + l_f \psi \right) \mathbf{j} \tag{A.11}
\]

\[
\mathbf{V}_{ur} = \left( \dot{X} \cos \psi + \dot{Y} \sin \psi \right) \mathbf{i} + \left( -\dot{X} \sin \psi + \dot{Y} \cos \psi - l_r \psi \right) \mathbf{j} \tag{A.12}
\]
\[
\mathbf{V}_t = \left( \dot{X} \cos \psi + \dot{Y} \sin \psi - h \dot{\psi} \sin \phi \right) \mathbf{i} \\
+ \left( -\dot{X} \sin \psi + \dot{Y} \cos \psi + h \dot{\psi} \cos \phi + l_{cg} \dot{\psi} \right) \mathbf{j} \\
+ h \dot{\phi} \cos \phi \mathbf{k}
\]  
(A.13)

From (2.32) and (2.42) we know that

\[
\begin{bmatrix}
I_{xuf} & 0 & 0 \\
0 & I_{yuf} & 0 \\
0 & 0 & I_{zuf}
\end{bmatrix} \quad \text{(A.14)}
\]

\[
\begin{bmatrix}
I_{xur} & 0 & 0 \\
0 & I_{yur} & 0 \\
0 & 0 & I_{zur}
\end{bmatrix} \quad \text{(A.15)}
\]

\[
\begin{bmatrix}
I_{xx} & 0 & I_{xcs} \\
0 & I_{yys} & 0 \\
I_{xcs} & 0 & I_{zzs}
\end{bmatrix} \quad \text{(A.16)}
\]

With all of the above information, the Lagrangian can be calculated and is given by

\[
\mathcal{L} = \frac{1}{2} \left\{ M_s \left[ \dot{X}^2 + \dot{Y}^2 + h_s \dot{\psi}^2 + \psi^2 \left( h_s^2 \sin^2 \phi + l_{cg}^2 \right) - 2h_s \dot{X} \left( \psi \cos \psi \sin \phi + \dot{\phi} \sin \psi \cos \phi \right) \\
- 2h_s \dot{Y} \left( \psi \sin \psi \sin \phi - \dot{\phi} \cos \psi \cos \phi \right) + 2l_{cg} \dot{\psi} \left( \dot{Y} \cos \psi - \dot{X} \sin \psi + h_s \dot{\phi} \cos \phi \right) \right] \\
+ M_{af} \left[ \dot{X}^2 + \dot{Y}^2 - 2l_f \dot{\psi} \left( \dot{X} \sin \psi - \dot{Y} \cos \psi - l_f \dot{\psi} \right) \right] \\
+ M_{ur} \left[ \dot{X}^2 + \dot{Y}^2 + 2l_f \dot{\psi} \left( \dot{X} \sin \psi - \dot{Y} \cos \psi - l_f \dot{\psi} \right) \right] \\
+ I_{xx} \dot{\phi}^2 + 2I_{xcs} \dot{\phi} \dot{\psi} \cos \phi + \left( I_{yys} \sin^2 \phi + I_{zzs} \cos^2 \phi + I_{zuf} + I_{zur} \right) \dot{\psi}^2 \right\} \\
- M_s gh_s \cos \phi - M_{af} gh_{af} - M_{ur} gh_{ur} - \frac{1}{2} K \dot{\phi}^2
\]

(A.17)

With the Lagrangian calculated, the next step is to calculate the four Lagrange's equations for each of the generalized variables. Doing them in order, the longitudinal translation equation is first and given by
\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{X}} \right) - \frac{\partial L}{\partial X} = \Xi_x \tag{A.18}
\]

where \( \Xi_x \) is the generalized force in the \( X \) direction. Carrying out the calculations in (A.18) and remembering that \( M_s L_{cg} + M_{uf} I_f - M_{ur} l_r = 0 \) and \( M = M_s + M_{uf} + M_{ur} \), we get

\[
\frac{\partial L}{\partial X} = 0 \tag{A.19}
\]

\[
\frac{\partial L}{\partial X} = M \dot{X} - M_s \left( \phi \sin \psi \cos \phi + \psi \cos \psi \sin \phi \right) \tag{A.20}
\]

In order to compare the results with the Newton-Euler equations, all \( \dot{X} \) and \( \dot{Y} \) terms need to be replaced by \( V_{ax} \) and \( V_{oy} \). This can be done through the following relations

\[
\dot{X} = V_{ax} \cos \psi - V_{oy} \sin \psi \\
\dot{Y} = V_{ax} \sin \psi + V_{oy} \cos \psi \tag{A.21}
\]

Making the above substitutions and taking the time derivative of (A.20) we end up with

\[
M \left( \dot{V}_{ax} \cos \psi - \dot{V}_{oy} \sin \psi - V_{oy} \dot{\psi} \cos \psi - \dot{V}_{oy} \psi \cos \psi \right) - M_s h_s \psi \cos \psi \sin \phi - M_s h_s \phi \sin \psi \cos \phi - M_s h_s \phi \sin \psi \sin \phi - M_h \left( \psi \cos \psi \cos \phi + \phi \sin \psi \cos \phi \right) - M_h \left( \dot{\psi} \sin \psi \sin \phi + \dot{\phi} \sin \psi \sin \phi \right) = \Xi_x \tag{A.22}
\]

To make any further simplifications, the \( Y \) equation is needed. Following the same procedure, we get

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{Y}} \right) - \frac{\partial L}{\partial Y} = \Xi_y \tag{A.23}
\]

\[
M \left( \dot{V}_{ax} \sin \psi + V_{ax} \dot{\psi} \cos \psi + \dot{V}_{oy} \cos \psi - V_{oy} \psi \sin \psi \right) - M_s h_s \psi \sin \psi \sin \phi + M_s h_s \phi \cos \psi \cos \phi - M_s h_s \phi \cos \psi \sin \phi - M_h \left( \psi \cos \psi \sin \phi + \dot{\psi} \sin \psi \cos \phi \right) = \Xi_y \tag{A.24}
\]
Next equations (A.22) and (A.24) need to be separated into two equations with only $\dot{V}_{ax}$ in one and $\dot{V}_{oy}$ in the other. To get $\dot{V}_{ax}$, multiply (A.22) by $\cos \psi$ and add it to (A.24) times $\sin \psi$. Doing this and simplifying the result gives

$$M \dot{V}_{ax} - MV_{oy} \dot{\psi} - M \dot{h}_x \dot{\psi} \cos \phi - 2M \dot{h}_y \dot{\phi} \cos \phi = \Xi_x \cos \psi + \Xi_y \sin \psi.$$  \hspace{1cm} (A.25)

Solving this for $\dot{V}_{ax}$ gives

$$\dot{V}_{ax} = \frac{(\Xi_x \cos \psi + \Xi_y \sin \psi) + M \left(2h_x \dot{\phi} \cos \phi + h_y \dot{\psi} \sin \phi \right)}{M} + V_{oy} \dot{\psi}.$$  \hspace{1cm} (A.26)

From this it is easy to see that $\Xi_x \cos \psi + \Xi_y \sin \psi$ is nothing more than the sum of the forces acting along the x-direction of the chassis coordinates. So we see that comparing (A.26) with (2.61) we get the same result.

Next, to get $\dot{V}_{oy}$ multiply (A.22) by $-\sin \psi$ and add it to (A.24) times $\cos \psi$. Doing this and simplifying the result gives

$$M \dot{V}_{oy} + MV_{ax} \dot{\psi} + M \dot{h}_x \dot{\phi} \cos \phi - M \dot{h}_y \dot{\phi}^2 \sin \phi - M \dot{h}_y \dot{\phi} \sin \phi = -\Xi_x \sin \psi + \Xi_y \cos \psi.$$ \hspace{1cm} (A.27)

Solving for $\dot{V}_{oy}$ gives

$$\dot{V}_{oy} = \frac{(-\Xi_x \sin \psi + \Xi_y \cos \psi) - M \left(h_x \dot{\phi} \cos \phi - h_y \dot{\phi}^2 \sin \phi - h_y \dot{\phi} \sin \phi \right)}{M} - V_{ax} \dot{\psi}.$$  \hspace{1cm} (A.28)

Again, it is easy to see that $-\Xi_x \sin \psi + \Xi_y \cos \psi$ is nothing more than the sum of the forces acting along the y-direction of the chassis coordinates. Comparing (A.28) with (2.62) we again see that the results are the same.

Next we move on to the roll equation. Lagrange’s equation for roll is

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = \Xi_{\phi}.$$ \hspace{1cm} (A.29)
The \( \ddot{X} \) and \( \dot{Y} \) terms in the Lagrangian can be replaced by \( V_{ax} \) and \( V_{oy} \) since there are no \( \phi \) terms involved. Using (A.21) and replacing the appropriate terms in (A.17) gives

\[
\mathcal{L} = \frac{1}{2} \left\{ M_s \left[ V_{ax}^2 + V_{oy}^2 - 2h_s \left( V_{ax} \dot{\psi} \sin \phi - V_{oy} \dot{\phi} \cos \phi \right) + 2l_{cgs} V_{oy} \dot{\psi} \right]
+ 2l_{cgs} h_s \dot{\phi} \dot{\psi} \cos \phi + h_s^2 \dot{\psi}^2 + \psi^2 \left( h_s^2 \sin^2 \phi + l_{cgs}^2 \right) \right\}

+ M_{af} \left[ V_{ax}^2 + V_{oy}^2 + 2l_f V_{oy} \dot{\psi} + l_f^2 \dot{\psi}^2 \right]

+ M_{ur} \left[ V_{ax}^2 + V_{oy}^2 - 2l_r V_{oy} \dot{\psi} + l_r^2 \dot{\psi}^2 \right]

+ I_{xx} \ddot{\phi}^2 + 2I_{xx} \dot{\phi} \dot{\psi} \cos \phi + \left( I_{yy} \sin^2 \phi + I_{zz} \cos^2 \phi + I_{zorf} + I_{zaur} \right) \dot{\psi}^2 \right\}

- M_s g h_s \cos \phi - M_{af} g h_{af} - M_{ur} g h_{ur} - \frac{1}{2} K_\phi \dot{\phi}^2

(\text{A.30})

Next use (A.30) and conduct the calculations in (A.29). After simplifying and solving for \( \dot{\phi} \), remembering the terms defined in (2.56), the resulting roll equation is

\[
\dot{\phi} = \frac{M_s g h_s \sin \phi + \Xi_\phi - K_\phi \dot{\phi} - I_{xxo} \dot{\psi} \cos \phi - M_s h_s a_{oy} \cos \phi + \left( I_{yy} \sin^2 \phi + I_{zz} \cos^2 \phi + I_{zorf} + I_{zaur} \right) \dot{\psi}^2 \sin \phi \cos \phi}{I_{xxo}}

(\text{A.31})

and the only generalized force related to roll is the roll damping term given by \( -B_\phi \dot{\phi} \). Replacing \( \Xi_\phi \) with this term makes the first three terms in (A.31) exactly equal to the sum of the moments given in (2.71), neglecting the aerodynamic roll moment. With this information it can be seen that (A.31) and (2.63) are exactly the same equation.

The final equation to be calculated is for yaw. Lagrange’s equation for yaw is

\[
\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\psi}} \right) - \left( \frac{\partial \mathcal{L}}{\partial \psi} \right) = \Xi_\psi.

(\text{A.32})

Using the relations in (A.10) it is easy to see that
\[ \frac{\partial V_{ax}}{\partial \psi} = -\dot{X} \sin \psi + \dot{Y} \cos \psi = V_{oy} \]

\[ \frac{\partial V_{oy}}{\partial \psi} = -\dot{X} \cos \psi - \dot{Y} \sin \psi = -V_{ox} \]  \hspace{1cm} (A.33)

\[ \frac{\partial V_{ax}}{\partial \dot{\psi}} = 0 = \frac{\partial V_{oy}}{\partial \dot{\psi}} \]

With this information and using the Lagrangian from (A.30) the equation for yaw can be calculated. After simplification and using the term defined in (2.58), the resulting equation is

\[ \ddot{\psi} = \frac{\Xi_{\psi} - I_{xx0} \dot{\phi} \cos \phi + I_{xzo} \dot{\phi}^2 \sin \phi - 2 \left( I_{yzz} - I_{zzz} + M_z h_s^2 \right) \dot{\psi} \sin \phi \cos \phi + M_z h_s a_{ax} \sin \phi}{I_{xx0}}. \]  \hspace{1cm} (A.34)

The only generalized forces for yaw are those from (2.73). So it is clear that (A.34) and (2.64) are the same equation.

So from this alternate derivation of the equations of motion the original equations using Newton-Euler have been confirmed to be correct.
APPENDIX B – TIRE MODEL (STIREMOD)

The complete details of STIREMOD are found in [16] and [23]. Key information and the main equations are repeated here for reference. STIREMOD is a tire model designed for the full range of operating conditions under both on- and off-road surface conditions. The model inputs include longitudinal and lateral slip, camber or inclination angle, and normal load. The model produces tire forces throughout the adhesion range up through the peak coefficient of friction and throughout the saturation region to the limit slide coefficient of friction. The parameters used in the model can be adjusted to represent any reasonable tire on almost any surface.

Without giving the full detail here, STIREMOD computes a composite slip parameter, which is a function of both tire slip angle and longitudinal slip ratio. This composite slip is then used in a force saturation function. From the force saturation function tire longitudinal and lateral forces and aligning moment are computed. There are two different sets of equations used for STIREMOD. The first are the basic equations of the model and the second are equations showing how the parameters used in the model change with load. These equations are shown below.

One item to note: if the equations below are compared against those in [16], there will be some minor differences. The reason is that there were some errors in the equations in [16]. If the equations in [23] are compared against those in [16], there are some differences that should not be there. After a personal discussion with one of the authors of [16], the equations shown below were confirmed to be the correct equations.

Summary of STIREMOD Basic Equations

Composite Slip:

\[
\sigma = \frac{\pi a_p^2}{8 F_z} \left[ \frac{K_s}{\mu_y^2} \tan^2 \alpha + \frac{K_s^2}{\mu_x^2} \left( \frac{s}{1 - s} \right)^2 \right]
\]  
(B.1)
Force Saturation Function:

\[ f(\sigma) = \frac{F_c}{\mu F_z} = \frac{C_1 \sigma^3 + C_2 \sigma^2 + C_3 \sigma}{C_4 \sigma^3 + C_5 \sigma^2 + C_6 \sigma + 1} \]  

(B.2)

Normalized Side Force:

\[ \frac{F_y}{\mu_y F_z} = \frac{-f(\sigma) K_z \tan \alpha}{\sqrt{K_{iz}^2 \tan^2 \alpha + K_{iz}^2 s^2}} + Y'_y \gamma \]  

(B.3)

Normalized Longitudinal Force:

\[ \frac{F_x}{\mu_x F_z} = \frac{-f(\sigma) K'_{xz} s}{\sqrt{K_{xz}^2 \tan^2 \alpha + K_{xz}^2 s^2}} \]  

(B.4)

Aligning Moment:

\[ M_z = \frac{K_{a} a^2 \mu_x^2 \tan \alpha}{(1 + G_\sigma^2)^2 \left[ \frac{K_z}{2} - G_\sigma K_{cz} \frac{s}{1 - s} \left( 2 + \sigma^2 \right) \right]} \]  

(B.5)

Slip to Slide Transition:

\[ \mu_x = \mu_{px} \left( 1 - K_{p_x} \sqrt{\sin^2 \alpha + s^2 \cos^2 \alpha} \right) \]  

\[ \mu_y = \mu_{py} \left( 1 - K_{p_y} \sqrt{\sin^2 \alpha + s^2 \cos^2 \alpha} \right) \]  

(B.6)

Lateral/Longitudinal Stiffness Transition:

\[ K_{c}' = K_{c} + (K_z - K_{cz}) \sqrt{\sin^2 \alpha + s^2 \cos^2 \alpha} \]  

(B.7)

Camber Force Stiffness Transition:

\[ Y_{y}' = Y_y \left[ 1 - K_{y} f^2(\sigma) \right] \]  

(B.8)
Load Varying Parameter Equations

Lateral Stiffness Coefficient

\[
K_s = \frac{2}{a_{po}} \left[ A_0 + A_1 F_z - \frac{A_1}{A_2} F_z^2 + K_x \left( \frac{F_1}{F_z} \right) \right] \quad \text{(B.9)}
\]

Longitudinal Stiffness Coefficient:

\[
K_x = \frac{2}{a_{po}} F_z \left( \frac{CS}{FZ} \right) \quad \text{(B.10)}
\]

Camber Thrust Stiffness:

\[
Y_y = A_3 F_z - \frac{A_3}{A_4} F_z^2 \quad \text{(B.11)}
\]

Aligning Moment Coefficient:

\[
K_m = K_1 F_z \quad \text{(B.12)}
\]

Peak Tire/Road Coefficient of Friction:

\[
\mu_{px} = \left( B_{1x} F_z + B_{3x} + B_{4x} F_z^2 \right) \frac{SN_o}{SN_T} \quad \text{(B.13)}
\]

\[
\mu_{py} = \left( B_{1y} F_z + B_{3y} + B_{4y} F_z^2 \right) \frac{SN_o}{SN_T} \quad \text{(B.13)}
\]

Tire Contact Patch Length:

\[
a_p = a_{po} \left( 1 - K_a \frac{F_1}{F_z} \right) \quad \text{(B.14)}
\]

\[
a_{po} = \frac{0.0768 \sqrt{F_z F_{zt}}}{T_u \left( T_p + 5 \right)} \quad \text{(B.15)}
\]
The parameters used in the above equations are as follows:

- \( A_0, A_1, A_2 \) = Calspan cornering stiffness versus normal load parameters
- \( A_3, A_4 \) = Calspan camber stiffness versus normal load parameters
- \( B_{lx}, B_{ly}, B_{lx} \) = Calspan peak longitudinal force versus normal load parameters
- \( B_{ly}, B_{ly}, B_{ly} \) = Calspan peak lateral force versus normal load parameters
- \( C_1, C_2, C_3, C_4, C_5 \) = Shaping coefficients for force saturation function
- \( CS \) = Calspan coefficient for longitudinal tire force stiffness
- \( FZ \) = Estimated long. force for increasing lat. stiffness under hard braking
- \( F_{ext} \) = Tire design load
- \( F_{zt} \) = Aligning moment shaping parameters
- \( G_1, G_2 \) = Calspan coeff. for aligning moment stiffness variation with normal load
- \( K_x \) = Coefficient of elongation of tire contact patch due to longitudinal force
- \( K_y \) = Coefficient for cornering stiffness dependence on longitudinal force
- \( K_{x,x} \) = Coefficient for camber stiffness dependence on force saturation
- \( K_{u,x} \) = Coefficient of the decay in longitudinal friction with increasing slip ratio
- \( K_{u,y} \) = Coefficient of the decay in lateral friction with increasing slip angle
- \( SN_o \) = Skid number of the simulated surface
- \( SN_T \) = Skid number of the tire test
- \( T_p \) = Tire inflation pressure (psi)
- \( T_w \) = Tire contact patch width (in)

The actual values used for the above parameters are given in Appendix C. For more complete descriptions of the above parameters, see [16]. The term \( F_x \) in (B.9) and (B.14) is the longitudinal tire force. Since the actual x-direction force is not known in the current time step, an estimate is used. There are several ways that this could be done. For NAVDyn, this estimate is simply the x-direction force from the previous time step. In [16] there was a different approach used that works for low slip levels, but is not valid for high slip. Using the value from the previous time step provides more accurate results across the entire operating range.

The skid numbers used in (B.13) are equivalent to using the coefficient of friction. For the implementation of NAVDyn, instead of using the skid numbers, the coefficient of friction is used in the equation. In order to do this, the appropriate constant is added into the equation. In
Appendix C, which shows the actual parameter values used for the tires, instead of the skid numbers, the coefficients of friction, $\mu_x$ and $\mu_y$, are used.
APPENDIX C - P205/65R15 TIRE PARAMETERS

The actual values used for the tire parameters described in Appendix B are given in this appendix. These parameters represent the actual tires used on the 1994 Ford Taurus GL that was used for model validation. The actual tires used on the Taurus during all vehicle testing were General Ameritech P205/65R15 steel belted radials. The parameters for these tires were obtained from Jeffrey Christos of JPC Engineering. Jeffrey formerly worked at the National Highway Traffic Safety Administration (NHTSA) on the National Advanced Driving Simulator project. The actual Matlab M-file containing the parameters used is shown below.

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% TIRE SIMULATION PARAMETERS
% General Ameritech P205/65R15
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Jon Demerly Revised: 7 March 2000
%
% Tire parameter file based on STIREMOD model described in SAE 970559
%
% Parameter file contains the parameters for a General Ameritech P205/65R15
% tire from a 1994 Ford Taurus GL per file provided by Jeffrey Christos from
% STI/JPC Engineering
%
% Revision History:
% 6-Aug-99 Per e-mail exchange with Jeffrey Christos, the camber parameters
% are incorrect. Changed A_3 from -0.42042 to -0.0869 and A_4 from
% 984.25 to 343.31.
% 7-Mar-00 Changed values and definition of terms K_tI and Tau_x. After
% reviewing the equations used for tire lags and re-reading SAE
% 950311, K_tI should be simply the characteristic length in m for
% lateral response and Tau_x should be an equivalent longitudinal
% characteristic length. As a result K_tI is redefined as K_tIl for
% the lateral characteristic length and Tau_x is redefined as K_tIx
% for the longitudinal characteristic length.

disp('    
disp('Loading P205/65R15 Tire Parameters...')
disp('    

A_0 = 2438.1; % Quadratic coefficients for lateral stiffness coefficient
A_1 = 15.99; % Quadratic coefficients for lateral stiffness coefficient
A_2 = 3787.3; % Quadratic coefficients for lateral stiffness coefficient
A_3 = -0.0869; % Quadratic coefficients for camber stiffness coefficient
A_4 = 343.31; % Quadratic coefficients for camber stiffness coefficient
B_1x = 8.57E-4; % Quadratic coeff. for long. coeff. of friction
B_3x = 0.70402; % Quadratic coeff. for long. coeff. of friction
B_4x = -5.0229E-7; % Quadratic coeff. for long. coeff. of friction
B_1y = -1.6516E-4; % Quadratic coeff. for lat. coeff. of friction
B_3y = 1.16; % Quadratic coeff. for lat. coeff. of friction
B_4y = -2.5069E-8; % Quadratic coeff. for lat. coeff. of friction
C_1 = 0.042; % Polynomial coeff. for saturation function (Radial Tire)
c_2 = 2.104; % Polynomial coeff. for saturation function (Radial Tire)
```
\( c_3 = 2.0724; \) \{ Polynomial coeff. for saturation function (Radial Tire) \\
\( c_4 = 0.4411; \) \{ Polynomial coeff. for saturation function (Radial Tire) \\
\( c_5 = 1.2732; \) \{ Polynomial coeff. for saturation function (Radial Tire) \\
\( C_5 = 16.7535; \) \{ Coefficient for longitudinal tire force stiffness \\
\( F_{zt} = 1400; \) \{ Tire design load [lb] \\
\( G_1 = 0.9789; \) \{ Shaping coefficients for tire aligning torque \\
\( G_2 = -0.3344; \) \{ Shaping coefficients for tire aligning torque \\
\( K_1 = -1.1534E-4; \) \{ Coeff. for aligning moment dependence on vertical load \\
\( K_a = -0.103; \) \{ Coeff. for tire patch length dependence on long. force \\
\( K_x = 0; \) \{ Coeff. for cornering stiffness dependence on long. force \\
\( K_{\gamma \alpha} = 0.9; \) \{ Coeff. for camber stiffness dependence on force saturation \\
\( K_{m\mu x} = 0.3984; \) \{ Coeff. for limit slip change in long. coeff. of friction \\
\( K_{m\mu y} = 0.6077; \) \{ Coeff. for limit slip change in lat. coeff. of friction \\
\( R_t = 1.025; \) \{ Rolling radius of tire [ft] \\
\( T_p = 35; \) \{ Tire pressure [psi] \\
\( T_w = 5.8; \) \{ nominal tread width [in] \\
\( \mu_0 x = 0.85; \) \{ Nominal longitudinal coefficient of friction of road surface \\
\( \mu_0 y = 0.85; \) \{ Nominal lateral coefficient of friction of road surface \\
\% Characteristic lengths used for calculating tire lags [m] \\
\( K_{t\ell x} = 0.091; \) \{ Long. characteristic length - value taken from SAE 950311 \\
\( K_{t\ell y} = 0.91; \) \{ Lat. characteristic length - value taken from SAE 950311 \\
\) tire='P205/65R15'; \\
\) disp(' ') \\
\) disp('P205/65R15 Tire Data Loaded!') \\
\) disp(' ') \\
\% Added "tire='P205/65R15'" on 7/28/99 for automating plots to include data \\
\% on tire used.
APPENDIX D - 1994 FORD TAURUS GL PARAMETERS

This appendix gives the values for the parameters that describe a 1994 Ford Taurus GL. These parameters come from work done as part of the National Advanced Driving Simulator Project. The complete description of how the parameters were measured and the results are found in [29]. Since the default vehicle in CarSim v4.0.2 is also the 1994 Ford Taurus GL, some of the parameters shown below were taken from CarSim and some directly from [29]. The actual Matlab M-file containing the parameters used is shown below. For each parameter there is a notation, described at the end, telling the source of the information.

```
% VEHICLE SIMULATION PARAMETERS
% 1994 Ford Taurus GL
% Jon Demerly Revised: 26 Apr 2000
% This file represents the parameters for the Ford Taurus GL used as the
target vehicle for the National Advanced Driving Simulator (NADS). This
is the same vehicle used in CarSim v4.0.2. Where data plots are given,
both quadratic and linear curve fits are used where appropriate. Data
comes from both sources.
% Change Log:
% 21Jun1999 First version created
% 27Jul1999 Height of wheel hub changed to be calculated from tire
% radius given in tire parameter data set, thus tire
% parameters must be opened first.
% 28Jul1999 Added line for "vehicle='CarSimTaurus_v2'" for automating
data to include in plots. Added input line for user to
% enter NAVDyn version number.
% 29Jul1999 Added parameter K_brkp for gain from pedal force to brake
% system pressure. Adjusted other gains accordingly.
% 13Aug1999 Added mps2kph for converting back to kph.
% 18Aug1999 Added table for relating vehicle speed and steering wheel
% angle to yaw rate - for use in closed-loop control
% 26Jan2000 Added variable "set_spd" to define the target speed for the
% cruise control system
% 09Mar2000 Added deadband for brake pedal force before pressure buildup
% begins based upon NADS data for Taurus. Used 2nd order curve
% fit of pedal force vs. pressure data and picked x-intercept.
% Changed file to Taurus_GL - previously was CarSimTaurus_v2.
% Changed several brake system parameters based upon NADS data.
% Besides deadband, pedal force to pressure conversion changed
% and front and rear brake gains changed. Also changed brake
% system time constant based upon actual Taurus data.
% 26Apr2000 Recalculated values for the yaw rate vs. vehicle speed and
% steering wheel angle lookup table based upon NAVDyn v1.4.
```

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disp(' ');
disp('Loading Ford Taurus Parameters from Taurus_GL...');
disp(' ');

% GENERAL CONSTANTS
%-------------------------------------------------------------------
g = 9.8100; % Acceleration due to gravity (m/s^2)
kph2mps = 1000 / 3600; % Convert kph to m/sec (m/sec/kph)
mps2kph = 3600 / 1000; % Convert m/sec to kph (kph/m/sec)
deg2rad = pi/180; % Convert degrees to radians
rad2deg = 180/pi; % Convert radians to degrees

% INITIAL CONDITIONS:
%--------------------------------------------------------------------
V_oxi = 100 * kph2mps; % Initial forward velocity in kph (m/s)
set_spd = 100 * kph2mps; % Set speed for cruise control (m/s)

% All other initial conditions are zero

% KNOWN VEHICLE PARAMETERS:
%--------------------------------------------------------------------

% X-distance between the center of gravity of the sprung mass and the % front and rear axles (m)
l_fs = 1.01476; % 2.690 m wheelbase
l_rs = 1.67524; %

% Front and rear track (m)
t_f = 1.540; %
t_r = 1.530; %

% Distance between the ground and the static roll center (m)
h_f = 0.130; %
h_r = 0.110; %

% Height of mass CG (m)
h_cg = 0.567851; % Height of sprung mass CG above ground
h_cguf = 0.320; % Height of front unsprung mass above ground
h_cgur = 0.320; % Height of rear unsprung mass above ground

% Y-distance from the vehicle centerline to the center of gravity % of the sprung mass (m)
y_scg = 0.0; %* Assumed to be zero in model, if not, model must be changed

% Total vehicle mass (kg)
M = 1704.7; % (3750 lb.)

% Unsprung mass (kg)
M_uf = 98.1; % Both wheels
M_ur = 79.7; % Both wheels
% Moments of inertia of sprung mass about its CG (kg-m^2)

\[ I_{xxs} = 440.911 \]  \%1
\[ I_{yys} = 2498.900 \]  \%1
\[ I_{zzs} = 2619.280 \]  \%1

% Products of inertia of the sprung mass about its CG (kg-m^2)

\[ I_{xys} = 0.0 \]  \%*
\[ I_{xz} = 7.54097 \]  \%1
\[ I_{yz} = 0.0 \]  \%*

% VEHICLE PARAMETERS CALCULATED FROM KNOWN PARAMETERS:

% Wheelbase (m)

\[ L = l_{fs} + l_{rs}; \]

% Sprung mass (kg)

\[ M_s = M - M_{uf} - M_{ur}; \]

% Longitudinal distance from sprung CG to total mass CG (m)

\[ l_{cgs} = (M_{ur}l_{rs} - M_{uf}l_{fs})/M; \]
\[ l_f = l_{fs} + l_{cgs}; \]  \% Distance from total CG to front axle
\[ l_r = l_{rs} - l_{cgs}; \]  \% Distance from total CG to rear axle

% Vehicle static axle masses (kg)

\[ M_f = M \times l_r/L; \]
\[ M_r = M \times l_f/L; \]

% Heights of sprung mass CG above roll center (height of origin) (m)

\[ h_o = h_f + l_f \times (h_r - h_f)/L; \]  \% Height of origin above ground
\[ h_s = h_{cgs} - h_o; \]  \% Height of sprung mass CG above the origin
\[ h_{uf} = h_{cgu} - h_o; \]  \% Height of front unsprung mass above the origin
\[ h_{ur} = h_{cgu} - h_o; \]  \% Height of rear unsprung mass above the origin

% Height of total vehicle CG (m)

\[ h_{cg} = (M_s h_{cgs} + M_{uf} h_{cgu} + M_{ur} h_{cgu})/M; \]

% Moments of inertia of unsprung masses about their CG (kg-m^2)

\[ I_{zzuf} = M_{uf} \times (t_{f}^2/2)^2; \]  \% Inertia of front unsprung mass about its own CG
\[ I_{zjur} = M_{ur} \times (t_{r}^2/2)^2; \]  \% Inertia of rear unsprung mass about its own CG

% Calculated moments of inertia (kg-m^2)

\[ I_{xxso} = I_{xxs} + M_s \times h_s^2; \]  \% Roll inertia of sprung mass about origin
\[ I_{zz} = I_{zzs} + M_s (l_{cgs}^2 + I_{zzuf} + \ldots)
                  \quad M_{uf} l_{f}^2 + I_{zjur} + M_{ur} l_r^2; \]  \% Total yaw inertia of vehicle

% Calculated products of inertia (kg-m^2)

\[ I_{xzso} = I_{xzs} + M_s \times h_s l_{cgs}; \]  \% Inertia of sprung mass about the origin
\[ I_{xz} = I_{xzs} + M_s h_s l_{cgs} + \ldots \]
\[ M_{uf} h_{uf} l_{f} - M_{ur} h_{ur} l_{r}; \] Inertia of total vehicle

% KNOWN SUSPENSION PARAMETERS:

% Nominal camber angles - positive when wheels lean out at top (degrees)
\[
\gamma_{0f} = 0 \times \text{deg2rad}; \quad \% \text{Front camber angle (rad)}
\gamma_{0r} = 0 \times \text{deg2rad}; \quad \% \text{Rear camber angle (rad)}
\]

% Camber vs. suspension compression coefficients (deg/mm or deg/mm^2)
\[
K_{camf1} = -0.0137; \quad \% \text{Front linear coefficient from curve fit of data}
K_{camf2} = -0.0001; \quad \% \text{Front quadratic coefficient from curve fit of data}
K_{camr1} = -0.0147; \quad \% \text{Rear linear coefficient from curve fit of data}
K_{camr2} = -6e-5; \quad \% \text{Rear quadratic coefficient from curve fit of data}
\]

% Compliance camber coefficients (rad/N)
\[
K_{ccambf} = 0; \quad \% \text{Front compliance camber}
K_{ccambr} = 0; \quad \% \text{Rear compliance camber}
\]

% Ride spring rate (N/mm)
\[
K_{spf} = 27.85; \quad \% \text{Taken from curves, average for jounce and rebound}
K_{spr} = 18.16; \quad \% \text{Taken from curves, average for jounce and rebound}
\]

% Shock absorber damping coefficients for jounce and rebound (N-sec/mm)
\[
B_{shjf1} = 2.9915; \quad \% \text{Curve fit of data for jounce up to 390 mm/sec}
B_{shjf2} = -0.0039; \quad \% \text{Curve fit of data for jounce up to 390 mm/sec}
B_{shrf1} = 1.5408; \quad \% \text{Curve fit of data for jounce up to 390 mm/sec}
B_{shrf2} = 0.0022; \quad \% \text{Curve fit of data for jounce up to 390 mm/sec}
B_{shjr1} = 2.9915; \quad \% \text{Same as front}
B_{shjr2} = -0.0039; \quad \% \text{Same as front}
B_{shrr1} = 1.5408; \quad \% \text{Same as front}
B_{shrr2} = 0.0022; \quad \% \text{Same as front}
\]

% Auxiliary roll stiffness (Nm/rad)
\[
K_{rf} = 384.0*180/\pi; \quad \% \text{(384.0 Nm/deg)}
K_{rr} = 344.4*180/\pi; \quad \% \text{(344.4 Nm/deg)}
\]

% SUSPENSION PARAMETERS CALCULATED FROM KNOWN PARAMETERS:

% Total front and rear roll stiffnesses (indep susp) (Nm/rad)
\[
K_{phif} = 0.766*(K_{spf}*1000)*t_f^2/2 + K_{rf}; \quad \%
K_{phir} = 0.827*(K_{spr}*1000)*t_r^2/2 + K_{rr}; \quad \%
K_{phi} = K_{phif} + K_{phir};
\]

% Front and rear roll damping (indep susp) (Nm-sec/rad)
\[
B_{phif1} = 0.766*(B_{shjf1} + B_{shrf1})*1000*t_f^2/2 + K_{rf}; \quad \%
B_{phif2} = 0.766*(B_{shjf2} + B_{shrf2})*1000^2*t_f^3/8;
B_{phir1} = 0.827*(B_{shjr1} + B_{shrr1})*1000*l_r^2/2 + K_{rr}; \quad \%
B_{phir2} = 0.827*(B_{shjr2} + B_{shrr2})*1000^2*l_r^3/8;
\]
\[ B_{\phi 1} = B_{\phi 1f} + B_{\phi 1r}; \]
\[ B_{\phi 2} = B_{\phi 2f} + B_{\phi 2r}; \]

% Nominal camber of four wheels in vehicle coordinates (rad)
\[ \gamma_0 = [-\gamma_{0f} \gamma_{0f} -\gamma_{0r} \gamma_{0r}]; \]

% Roll camber coefficients (unitless)
\[ K_{\text{gam}1} = K_{\text{cam}1} \cdot 1000 \cdot t_f/2 \cdot \text{deg2rad}; \quad \% \text{Front roll camber - linear} \]
\[ K_{\text{gam}2} = K_{\text{cam}2} \cdot 1000^2 \cdot t_f^2/4 \cdot \text{deg2rad}; \quad \% \text{Front roll camber - quadratic} \]
\[ K_{\text{gam}1r} = K_{\text{cam}1r} \cdot 1000 \cdot t_r/2 \cdot \text{deg2rad}; \quad \% \text{Rear roll camber - linear} \]
\[ K_{\text{gam}2r} = K_{\text{cam}2r} \cdot 1000^2 \cdot t_r^2/4 \cdot \text{deg2rad}; \quad \% \text{Rear roll camber - quadratic} \]

% TIRE PARAMETERS:
\[ H_{\text{hub}} (m) \]
\[ R_{\text{wf}} = R_t \cdot 0.3048; \quad \% \text{Radius of front tire from tire data set} \]
\[ R_{\text{wr}} = R_t \cdot 0.3048; \quad \% \text{Radius of rear tire from tire data set} \]
\[ R_w = [R_{\text{wf}} R_{\text{wf}} R_{\text{wr}} R_{\text{wr}}]; \]

% Moment of inertia of the wheel about its spin axis (kg-m^2)
\[ I_{\text{wf}} = 0.990; \quad \% \]
\[ I_{\text{wr}} = 0.990; \quad \% \]

% KNOWN STEERING SUBSYSTEM PARAMETERS:

% Inverse steering gear ratio (unitless)
\[ K_{sr} = 15.97; \quad \% \text{Ratio of steering wheel angle to road wheel angle} \]

% Steering system kinematic linkage coefficient (positive increases LF)
\[ K_{sk} = -0.0023; \quad \% \text{Quadratic coefficient for steering kinematics} \]

% Nominal toe angles (toe-in is positive) (deg)
\[ \text{toe}_{f0} = -0.065 \cdot \text{deg2rad}; \quad \% \text{Nominal front toe angle (rad)} \]
\[ \text{toe}_{r0} = 0.21 \cdot \text{deg2rad}; \quad \% \text{Nominal rear toe angle (rad)} \]

% Toe vs. suspension compression coefficients (deg/mm or deg/mm^2)
\[ K_{\text{toef}1} = -0.0068; \quad \% \text{Front linear coefficient from curve fit of data} \]
\[ K_{\text{toef}2} = 0.0001; \quad \% \text{Front quadratic coefficient from curve fit of data} \]
\[ K_{\text{toer}1} = 0.0062; \quad \% \text{Rear linear coefficient from curve fit of data} \]
\[ K_{\text{toer}2} = -8e-6; \quad \% \text{Rear quadratic coefficient from curve fit of data} \]

% Suspension compliance steer coefficients
\[ \text{Eps}_{\text{Fxf}} = 0.4283e-3 \cdot \text{deg2rad}; \quad \% \text{Front longitudinal force vs. toe (rad/N)} \]
\[ \text{Eps}_{\text{Fyf}} = -0.270e-3 \cdot \text{deg2rad}; \quad \% \text{Front lateral force vs. steer (rad/N)} \]
\[ \text{Eps}_{\text{Mzf}} = 0.380e-2 \cdot \text{deg2rad}; \quad \% \text{Front aligning torque vs. steer (rad/Nm)} \]
\[ \text{Eps}_{\text{Fx}} = 0.0 \cdot \text{deg2rad}; \quad \% \text{Rear longitudinal force vs. toe (rad/N)} \]
\[ \text{Eps}_{\text{Fyr}} = -0.10e-4 \cdot \text{deg2rad}; \quad \% \text{Rear lateral force vs. steer (rad/N)} \]
\[ \text{Eps}_{\text{Mzr}} = 0.190e-2 \cdot \text{deg2rad}; \quad \% \text{Rear aligning torque vs. steer (rad/Nm)} \]
% STEERING SUBSYSTEM PARAMETERS CALCULATED FROM KNOWN PARAMETERS:

% Roll steer coefficients (unitless)

Eps_f1 = K_toef1*1000*t_f/2*deg2rad; %1 Front linear coefficient
Eps_f2 = K_toef2*(1000*t_f/2)^2*deg2rad; %1 Front quadratic coefficient
Eps_r1 = K_toer1*1000*t_r/2*deg2rad; %1 Rear linear coefficient
Eps_r2 = K_toer2*(1000*t_r/2)^2*deg2rad; %1 Rear quadratic coefficient

% BRAKING SUBSYSTEM Parameters:

% Deadband in brake pedal force before pressure buildup (N)
B_db = 7.24; %3 Computed from x-intercept of force vs. pressure curve fit

% Brake gain for converting pedal force to brake system pressure (kPa/N)
K_brkp = 34.2; %3

% Brake system lag from driver input to brake output
Tau_b = 0.25; %3 Time constant for brake system lag determined from actual
% vehicle response (also matches value from Xia's thesis)

% Proportioning curve for rear brakes
prop_break = 60*K_brkp; %1 Pressure up to which front and rear are the same
prop_ratio = 0.407; %1 Portion of brake pressure beyond break point that
% goes to the rear brakes

% Brake gains for converting brake pressure to brake torque (Nm/kPa)
K_brkf = 0.309; %3 Front brake gain from NADS report
K_brkr = 0.0926; %3 Rear brake gain from NADS report

% OTHER CALCULATED PARAMETERS:

% Table rows for 2-D lookup table relating vehicle speed and steering
% angle to yaw rate. Yawrow gives the vehicle speed each row corresponds
% to and yawcol gives the steering wheel angle each column corresponds to.

yawrow = kph2mps*[0 20 30 35 40 45 50 55 60 65 70 80 90 100 120 140 160];
yawcol = deg2rad*[0 15 30 45 60 75 90 120 150 180];

% These values were generated using NAVDyn v1.4, P205/65R15 tires and
% Taurus_GL. Generated 26Apr2000.
kph0=[0 0 0 0 0 0 0 0 0 0];
kph20=[0 1.8 3.6 5.4 7.2 9.0 10.8 14.4 18.0 21.6];
kph25=[0 2.1 4.3 6.5 8.6 10.8 13.0 17.3 21.7 26.0];
kph30=[0 2.4 4.9 7.4 9.9 12.4 14.9 19.9 24.8 29.8];
kph35=[0 2.7 5.4 8.2 11.0 13.7 16.5 22.0 27.4 32.7];
kph40=[0 2.9 5.8 8.9 11.9 14.8 17.8 23.6 29.2 34.2];
kph45=[0 3.1 6.2 9.4 12.6 15.7 18.8 24.5 30.0 33.9];
kph50=[0 3.2 6.5 9.8 13.1 16.3 19.5 25.2 29.6 32.1];
vehicle='Taurus GL';
version=input('Enter NAVDyn version number: ','s');

disp('');
disp('Ford Taurus Parameters from Taurus_GL loaded!');
disp('');
APPENDIX E – MATLAB FILES

This appendix includes various Matlab M-files that were generated to automate the process of creating plots for some of the standard outputs of interest from NAVDyn. The actual M-file text is shown below for these files.

Frequency Response

The first file shown before is a modified version of the FREQ_R.M file that was generated by Jeffrey Chrstos for use in processing the NADS data for the Ford Taurus as described in [28]. This file was changed to include processing of the simulation data from a pulse steer run and then plotting of both the NADS data and the processed data from NAVDyn on a single plot.

% FREQ_R4.M
% Compute vehicle frequency responses from pulse input runs.
% FREQ_R4 uses only every 4th data point from the simulation data.
% Simulation should be run using fixed-step of 0.0025 sec.
% Written by: Jeffrey P. Chrstos
% Modified by: Jon Demerly for use with NAVDyn
% Revision History
% Ver Date Description
% 1.00 May 23, 1995 Initial version of program
% 1.01 Jun 01, 1995 Added computation of coherence function
% 1.02 Jun 02, 1995 Added maneuver file name to graph title
% 1.03 Jun 07, 1995 Added code to skip files that do not exist
% 1.04 Sep 28, 1995 Added code to save data files
% 1.05 Oct 03, 1995 Added checking for non-existent channels
% 1.06 Apr 09, 1997 Provided a variable for the source directory (P.G.)
% 1.06a Mar 13, 2000 Modified to compute for NAVDyn output (J.D.D.)
% clear;
F_Version='1.06a';
disp('Program FREQ_R2 - Compute vehicle frequency responses from simulation')
disp([' Version: ' num2str(F_Version)])
disp(' ')
disp(' ')

249
% get data file name
% F_BaseName=input('Enter NADS data maneuver code: ','s');
FileName=['VOf' F_BaseName 'F'];
F_SourceDir = ['c:\school\thesis\nads\94taurus\filtered\'];
%
% allocate memory
% a_sy_CSD=zeros(100,1);
% a_sy_PSD=zeros(100,1);
% a_sy_coh=zeros(100,1);
% a_sy_f=zeros(100,1);
% r_CSD=zeros(100,1);
% r_PSD=zeros(100,1);
% r_coh=zeros(100,1);
% r_f=zeros(100,1);
% phidot_CSD=zeros(100,1);
% phidotPSD=zeros(100,1);
% phidotcoh=zeros(100,1);
% phidot_f=zeros(100,1);
% xlimit=20;
% DT=.01;
% fid=fopen([F_SourceDir,FileName,'.mat'],'r');
% if fid == -1
%    disp(["Skipping File 'name' - does not exist"])}
% else
%    % process file
%    fclose(fid);
%    disp(["Processing File 'FileName"])
%    eval(['load ',F_SourceDir,FileName,';'])
%    % Ay
%    if exist('a_sy')
%        A_sy=decimate(a_sy,4);
%        Delta_sw=decimate(delta_sw,4);
%        [CSD,PSD,coh,f]=tfeexpl(Delta_sw-mean(Delta_sw(1:100)),...
%                               A_sy-mean(A_sy(1:100)),DT,xlimit);
%        a_sy_CSD(1:length(CSD))=CSD;
%        a_sy_PSD(1:length(PSD))=PSD;
%        a_sy_coh(1:length(coh))=coh;
%        a_sy_f(1:length(f),1)=f;
%    end
%    % Yaw Rate
%    if exist('psidot')
%        Psidot=decimate(psidot,4);
%        [CSD,PSD,coh,f]=tfeexpl(Delta_sw-mean(Delta_sw(1:100)),...
%                                 Psidot-mean(Psidot(1:100)),DT,xlimit);
%        psidot_CSD(1:length(CSD))=CSD;
%        psidot_PSD(1:length(PSD))=PSD;
%        psidot_coh(1:length(coh))=coh;
%        psidot_f(1:length(f),1)=f;
end
% Roll Rate
% if exist('phidot')
  Phidot=decimate(phidot,4);
  [CSD,PSD,coh,f]=tfexpl(Delta_sw-mean(Delta_sw(1:100)),...
    Phidot-mean(Phidot(1:100)),DT,xlimit);
  phidot_CSD(1:length(CSD))=CSD;
  phidot_PSD(1:length(PSD))=PSD;
  phidot_coh(1:length(coh))=coh;
  phidot_f(1:length(f),1)=f;
end
% end
% if exist('a_s')
  a_s_mag=abs(a_s_CSD)./abs(a_s_PSD);
  a_s_ph=(angle(a_s_CSD)-angle(a_s_PSD));
  for i=1:length(a_s_ph)
    if a_s_ph(i) < 2*pi
      a_s_ph(i)=a_s_ph(i)+2*pi;
    end
  end
  a_s_ph=unwrap(a_s_ph,pi)*180/pi;
end
% if exist('psidot')
  psidot_mag=abs(psdot_CSD)./abs(psdot_PSD);
  psidot_ph=(angle(psdot_CSD)-angle(psdot_PSD));
  for i=1:length(psdot_ph)
    if psidot_ph(i) < 2*pi
      psidot_ph(i)=psidot_ph(i)+2*pi;
    end
  end
  psidot_ph=unwrap(psdot_ph,pi)*180/pi;
end
% if exist('phidot')
  phidot_mag=abs(phidot_CSD)./abs(phidot_PSD);
  phidot_ph=(angle(phidot_CSD)-angle(phidot_PSD));
  for i=1:length(phidot_ph)
    if phidot_ph(i) > 0
      phidot_ph(i)=phidot_ph(i)-2*pi;
    end
  end
  phidot_ph=unwrap(phidot_ph,pi)*180/pi;
end
% % plot results
% % Lateral Acceleration
% figure(20)
% subplot(3,1,1);
% semilogx(a_s_f(:,1),20*log10(a_s_mag),'*-')
% hold
% semilogx(AY_CHAS_CG_f(:,1),20*log10(AY_CHAS_CG_mag_m),k.-')
% semilogx(AY_CHAS_CG_f(:,1),20*log10(AY_CHAS_CG_mag_m+...
% AY_CHAS_CG_conf.*AY_CHAS_CG_mag_m),k.-')
semilogx(AY_CHAS.CG_f(:,1),20*log10(AY_CHAS.CG_mag_m-... 
AY_CHAS.CG_conf.*AY_CHAS.CG_mag_m),’k.-.’)
ax=axis;
axis([1 xlimit -10 30]);
grid;
ylabel(’dB mag - (m/sec^2/rad)’);
xlabel(’Frequency (rad/sec)’);
title([’Lateral Acceleration Frequency Response ’ TitleName])
hold off
% subplot(3,1,2);
semilogx(a-sy-f(:,),asy-ph,’*-’)
hold
semilogx(AYCHASCGf(:,1),AYCHASCGphm, ’k.-. ’)
semilogx(AYCHASCG_f(:,1),AYCHASCGphm+AYCHASCGconf*180/pi, ’k.-.’)
semilogx(AYCHASCG_f(:,1),AYCHASCGphm-AY_CHASCGconf*180/pi, ’k.-.’)
ax=axis;
axis([1 xlimit -180 60]);
grid;
ylabel(’phase (deg)’);
xlabel(’Frequency (rad/sec)’);
hold off
% subplot(3,1,3);
semilogx(AYCHAS.CG_f(:,1),AY_CHAS.CG_coh_m,’k’)
ax=axis;
axis([1 xlimit 0 1]);
grid;
ylabel(’Coherence’);
xlabel(’Frequency (rad/sec)’);
hold off
pause
% Yaw Rate
% figure(21)
% subplot(3,1,1);
semilogx(psidot-f(:,1),20*log10(psidot_mag),’*-’)
hold
semilogx(RVZ_CHAS-f(:,1),20*log10(RVZ_CHAS_mag_m),’k.-.’)
semilogx(RVZ_CHAS_f(:,1),20*log10(RVZ_CHAS_mag_m+... 
RVZ_CHAS_conf.*RVZ_CHAS_mag_m),’k.-.’)
semilogx(RVZ_CHAS_f(:,1),20*log10(RVZ_CHAS_mag_m-... 
RVZ_CHAS_conf.*RVZ_CHAS_mag_m),’k.-.’)
ax=axis;
axis([1 xlimit -40 0]);
grid;
ylabel(’dB mag - (rad/sec/rad)’);
xlabel(’Frequency (rad/sec)’);
title([’Yaw Rate Frequency Response ’ TitleName])
hold off
% subplot(3,1,2);
semilogx(psidot-f(:,1),psidot_ph,’*-’)
hold
semilogx(RVZ_CHAS-f(:,1),RVZ_CHAS_ph_m,’k.-.’)
semilogx(RVZ_CHAS_f(:,1),RVZ_CHAS_ph_m+RVZ_CHAS_conf*180/pi, ’k.-.’)
semilogx(RVZ_CHAS_f(:,1),RVZ_CHAS_ph_m-RVZ_CHAS_conf*180/pi, ’k.-.’)
ax=axis;
axis([1 xlimit -270 0]);
grid;
ylabel('phase (deg)');
xlabel('Frequency (rad/sec)');
hold off

% subplot(3,1,3);
semilogx(RVZ_CHAS_f(:,1),RVZ_CHAS_coh_m,'k')
ax=axis;
axis([1 xlim 0 1]);
grid;
ylabel('Coherence');
xlabel('Frequency (rad/sec)');
hold off

% pause
%

% Roll Rate
%
figure(22)
subplot(3,1,1);
semilogx(phidot_f(:,1),20*log10(phidot_mag),'*--')
hold
semilogx(RVX_CHAS_f(:,1),20*log10(RVX_CHAS_mag_m),'k--')
semilogx(RVX_CHAS_f(:,1),20*log10(RVX_CHAS_mag_m+RVX_CHAS_conf.*RVX_CHAS_mag_m),'k--')
semilogx(RVX_CHAS_f(:,1),20*log10(RVX_CHAS_mag_m-RVX_CHAS_conf.*RVX_CHAS_mag_m),'k--')
ax=axis;
axis([1 xlim -50 0]);
grid;
ylabel('dB mag - (rad/sec/rad)');
xlabel('Frequency (rad/sec)');
title(['Roll Rate Frequency Response - ' 'TitleName'])
hold off

% subplot(3,1,2);
semilogx(phidot_f(:,1),phidot_ph,'*--')
hold
semilogx(RVX_CHAS_f(:,1),RVX_CHAS_ph_m,'k--')
semilogx(RVX_CHAS_f(:,1),RVX_CHAS_ph_m+RVX_CHAS_conf*180/pi,'k--')
semilogx(RVX_CHAS_f(:,1),RVX_CHAS_ph_m-RVX_CHAS_conf*180/pi,'k--')
ax=axis;
axis([1 xlim ax(3) ax(4)]);
grid;
ylabel('phase (deg)');
xlabel('Frequency (rad/sec)');
hold off

% subplot(3,1,3);
semilogx(RVX_CHAS_f(:,1),RVX_CHAS_coh_m,'k')
ax=axis;
axis([1 xlim 0 1]);
grid;
ylabel('Coherence');
xlabel('Frequency (rad/sec)');
hold off
Plot Files

The files shown below were generated to automate the standard plots that show the output from NAVDyn simulations. Plots include longitudinal response, lateral response, roll response, vehicle trajectory/heading, vehicle and tire slip, steering inputs/outputs, brake inputs/outputs, and tire forces.

```matlab
%PLOTLONG - plots the longitudinal acceleration and velocity
if brkstat==1
    brake_status='Normal';
else
    brake_status='Failed';
end
exist_steer=exist('steer_input');
exist_brake=exist('brake_input');
if exist_steer + exist_brake < 2
    steer_input=input('Enter Steering Input: ','s');
    brake_input=input('Enter Brake Input: ','s');
elseif exist_steer + exist_brake == 2
    ans1=input(['Is current steer input: ' 'steer_input ' (y/n): '],'s');
    if ans1=='n'
        steer_input=input('Enter Current Steering Input: ','s');
    end
    ans2=input(['Is current brake input: ' 'brake_input ' (y/n): '],'s');
    if ans2=='n'
        brake_input=input('Enter Current Brake Input: ','s');
    end
else
    error('"steer_input" or "brake_input" already defined as other data type')
end

figure(1)
set(1,'Position', [1 33 1021 699])
orient portrait
subplot(2,1,1)
plot(tout,a-ox/g);grid
xlabel('Time (sec)')
ylabel('Vehicle Longitudinal Acceleration, a_o-x (g)')
title(['Long. Accel. vs. time: Steer Input = ' steer_input ...
    ' Brake Input = ' brake_input])

subplot(2,1,2)
plot(tout,Vxox/kph2mps);grid
xlabel('Time (sec)')
ylabel('Vehicle Longitudinal Velocity, V_o-x (km/hr)')
title(['Long. Velocity vs. time: Steer Input = ' steer_input ...
    ' Brake Input = ' brake_input])
text(0.5,-0.2, [date ' 'NAVDyn v' version ': Vehicle=' vehicle ...
    ' Tire= ' tire ' Brakes= ' brake_status ' Initial Speed= ' ...
    num2str(Voxi*mps2kph) 'kph'],'sc','HorizontalAlignment',...'center','FontSize',8)
```

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if brkstat==1
    brake_status='Normal';
else
    brake_status='Failed';
end

exist_steer=exist('steer_input');
extist_brake=exist('brake_input');

if exist_steer + exist_brake < 2
    steer_input=input('Enter Steering Input: ','s');
    brake_input=input('Enter Brake Input: ','s');
elseif exist_steer + exist_brake == 2
    ans1=input(['Is current steer input: ' 'steer_input' '? (y/n): '],'s');
    if ans1=='n'
        steer_input=input('Enter Current Steering Input: ','s');
    end
    ans2=input(['Is current brake input: ' 'brake_input' '? (y/n): '],'s');
    if ans2=='n'
        brake_input=input('Enter Current Brake Input: ','s');
    end
else
    error('"steer_input" or "brake_input" already defined as other data type')
end

figure(2)
set(2,'Position', [1 33 1021 699])
orient portrait
subplot(2,1,1)
plot(tout,a_oy/g);grid
xlabel('Time (sec)')
ylabel('Vehicle Lateral Acceleration, a_o_y (g)')
title(['Lat. Accel vs. time: Steer Input = ' steer_input ...
        ' Brake Input = ' brake_input])

subplot(2,1,2)
plot(tout,psidot*rad2deg);grid
xlabel('Time (sec)')
ylabel('Vehicle Yaw Rate, psidot (deg/sec)')
title(['Yaw Rate vs. time: Steer Input = ' steer_input ...
        ' Brake Input = ' brake_input])
text(0.5,-0.2,[date ' 'NAVDyn v' version ' 'Vehicle=' vehicle ...
        ' Tire=' tire ' Brakes=' brake_status ' Initial Speed=' ...
        num2str(V_oxi*mps2kph) 'kph'], 'sc','HorizontalAlignment',...
        'center','FontSize',8)
steer_input=input('Enter Steering Input: ', 's');
brake_input=input('Enter Brake Input: ', 's');
elseif exist('steer_input') + exist('brake_input') == 2
  ans1=input(['Is current steer input: ' 'steer_input'? (y/n): '], 's');
  if ans1=='n'
    steer_input=input('Enter Current Steering Input: ', 's');
  end
  ans2=input(['Is current brake input: ' 'brake_input'? (y/n): '], 's');
  if ans2=='n'
    brake_input=input('Enter Current Brake Input: ', 's');
  end
else
  error('"steer_input" or "brake_input" already defined as other data type')
end

figure(3)
set(3,'Position', [1 3 3 1021 699])
orient portrait
subplot(2,1,1)
plot(tout,phi*rad2deg);grid
xlabel('Time (sec)')
ylabel('Vehicle Body Roll Angle, phi (deg)')
title(['Roll Angle vs. time: Steer Input = ' steer_input ... '
  Brake Input = ' brake_input])

subplot(2,1,2)
plot(tout,phidot*rad2deg);grid
xlabel('Time (sec)')
ylabel('Vehicle Body Roll Rate, phidot (deg/sec)')
title(['Roll Rate vs. time: Steer Input = ' steer_input ... '
  Brake Input = ' brake_input])

text(0.5,-0.2,['NAVDyn v' version:
  Vehicle=' vehicle ... '
  Tire=' tire 
  Brakes=' brakestatus ' Initial Speed=' ...
  num2str(V_oxi*mps2kph) 'kph', 'sc', 'HorizontalAlignment', 'center', 'FontSize', 8)

%PLOTTRAJ - plots the vehicle trajectory and heading angle
if brkstat==1
  brake_status='Normal';
else
  brake_status='Failed';
end

exist_steer=exist('steer_input');
exist_brake=exist('brake_input');
if exist_steer + exist_brake < 2
  steer_input=input('Enter Steering Input: ', 's');
  brake_input=input('Enter Brake Input: ', 's');
elseif exist_steer + exist_brake == 2
  ans1=input(['Is current steer input: ' 'steer_input'? (y/n): '], 's');
  if ans1=='n'
    steer_input=input('Enter Current Steering Input: ', 's');
  end
  ans2=input(['Is current brake input: ' 'brake_input'? (y/n): '], 's');
  if ans2=='n'
    brake_input=input('Enter Current Brake Input: ', 's');
  end
else
  error('"steer_input" or "brake_input" already defined as other data type')
end

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error('"steer_input" or "brake_input" already defined as other data type')
end

figure(4)
set(4,'Position', [1 33 1021 699])
orient portrait
subplot(2,1,1)
plot(Y,X);grid
xlabel('Lateral Position, Y (m)')
ylabel('Longitudinal Position, X (m)')
title(['Vehicle Trajectory: Steer Input = ' steerinput ... ...
' Brake Input = ' brake_input])

subplot(2,1,2)
plot(tout,psi*rad2deg);grid
xlabel('Time (sec)')
ylabel('Vehicle Heading Angle, psi (deg)')
title(['Heading Angle vs. time: Steer Input = ' steer-input ... ...
' Brake Input = ' brake_input])

text(0.5,-0.2,[date ' NAVDyn v' version ': Vehicle=' vehicle ...
' Tire=' tire ' Brakes=' brake_status ' Initial Speed=' ...
' num2str(V_oxi*mps2kph) 'kph' ],'sc','HorizontalAlignment',...
' center','FontSize',8)

%PLOTSLIP - plots the vehicle sideslip angle, camber angle, ...
tire slip angles and tire slip ratios

if brkstat==1
  brake_status='Normal';
else
  brake_status='Failed';
end

exist_steer=exist('steer_input');
exist_brake=exist('brake_input');

if exist_steer + exist_brake < 2
  steer_input=input('Enter Steering Input: ','s');
  brake_input=input('Enter Brake Input: ','s');
elseif exist_steer + exist_brake == 2
  ans1=input(['Is current steer input: ' 'steer_input' '? (y/n): '],'s');
  if ans1=='n'
    steer_input=input('Enter Current Steering Input: ','s');
  end
  ans2=input(['Is current brake input: ' 'brake_input' '? (y/n): '],'s');
  if ans2=='n'
    brake_input=input('Enter Current Brake Input: ','s');
  end
else
  error('"steer_input" or "brake_input" already defined as other data type')
end

figure(5)
set(5,'Position', [1 33 1021 699])
orient portrait
subplot(2,1,1)
plot(tout,beta*rad2deg);grid
xlabel('Time (sec)')
ylabel('Vehicle Sideslip Angle, beta (deg)')
title(['Sideslip vs time: Steer Input = ' steer_input ...
' Brake Input = ' brake_input])

subplot(2,1,2)
plot(tout,gamma(:,1)*rad2deg)
hold;
plot(tout,gamma(:,2)*rad2deg,'g:','LineWidth',2)
plot(tout,gamma(:,3)*rad2deg,'r--','LineWidth',2)
plot(tout,gamma(:,4)*rad2deg,'c-.','LineWidth',2)
grid
xlabel('Time (sec)')
ylabel('Tire Inclination Angle, gamma (deg)')
title(['Camber vs. time: Steer Input = ' steer_input ...
' Brake Input = ' brake_input])
legend('Left Front','Right Front','Left Rear','Right Rear',0);
hold;

figure(6)
set(6,'Position', [1 3 1021 699])
orient portrait
subplot(2,1,1)
plot(tout,alpha(:,1)*rad2deg)
hold;
plot(tout,alpha(:,2)*rad2deg,'g:','LineWidth',2)
plot(tout,alpha(:,3)*rad2deg,'r--','LineWidth',2)
plot(tout,alpha(:,4)*rad2deg,'c-.','LineWidth',2)
grid
xlabel('Time (sec)')
ylabel('Tire Slip Angle, alpha (deg)')
title(['Slip Angle vs. time: Steer Input = ' steer_input ...
' Brake Input = ' brake_input])
legend('Left Front','Right Front','Left Rear','Right Rear',0);
hold;

subplot(2,1,2)
plot(tout,slip(:,1))
hold;
plot(tout,slip(:,2),'g:','LineWidth',2)
plot(tout,slip(:,3),'r--','LineWidth',2)
plot(tout,slip(:,4),'c-.','LineWidth',2)
grid
xlabel('Time (sec)')
ylabel('Tire Longitudinal Slip Ratio, slip (unitless)')
title(['Slip Ratio vs. time: Steer Input = ' steer_input ...
' Brake Input = ' brake_input])
hold;
text(0.5,-0.2, [date ' ' 'NAVDyn v' version ': Vehicle=' vehicle ...
' Tire=' tire ' Brakes=' brake_status ' Initial Speed=' ...
num2str(V_w*mps2kph) 'kph'], 'sc', 'HorizontalAlignment', ...
'center', 'FontSize', 8)

%PLOTSTR - plots the steering wheel input angle and the road wheel angles

if brkstat==1
    brake_status='Normal';
else

brake_status='Failed';
end

exist steer=exist('steer_input');
exist brake=exist('brake_input');

if exist steer + exist brake < 2
  steer_input=input('Enter Steering Input: ','s');
  brake_input=input('Enter Brake Input: ','s');
elseif exist steer + exist brake == 2
  ans1=input(['Is current steer input: ' 'steer_input'? (y/n): '],'s');
  if ans1=='n'
    steer_input=input('Enter Current Steering Input: ','s');
  end
  ans2=input(['Is current brake input: ' 'brake_input'? (y/n): '],'s');
  if ans2=='n'
    brake_input=input('Enter Current Brake Input: ','s');
  end
else
  error('"steer_input" or "brake_input" already defined as other data type')
end

figure(10)
set(10,'Position', [1 33 1021 699])
orient portrait
subplot(2,1,1)
plot(tout,delta_sw*rad2deg);grid
xlabel('Time (sec)')
ylabel('Steering Wheel Input Angle, delta_s_w (deg)')
title(['Steer Input vs. time: Steer Input = ' steer_input ...
      ' Brake Input = ' brake_input])

subplot(2,1,2)
plot(tout,delta_rw(:,1)*rad2deg) hold
plot(tout,delta_rw(:,2)*rad2deg,'b-.','LineWidth',2)
plot(tout,delta_rw(:,3)*rad2deg,'k--','LineWidth',2)
plot(tout,delta_rw(:,4)*rad2deg,'k:','LineWidth',2)
grid
xlabel('Time (sec)')
ylabel('Road Wheel Steer Angles, delta_r_w (deg)')
title(['Road Wheel Angle vs. time: Steer Input = ' steer_input ...
      ' Brake Input = ' brake_input])
legend('Left Front','Right Front','Left Rear','Right Rear',0);
hold
text(0.5,-0.2,[date ' ' 'NAVDyn v' version ': Vehicle= vehicle ...
              'Tire= tire ' Brakes= brake_status ' Initial Speed=
              num2str(V_xi*mps2kph) 'kph','sc','HorizontalAlignment',...
              'center','FontSize',8)

%PLOTBRK - plots the brake pedal input force and the torque generated at each wheel

if brkstat==1
  brake_status='Normal';
else
  brake_status='Failed';
end

exist steer=exist('steer_input');
exist_brake = exist('brake_input');

if exist_steer + exist_brake < 2
    steer_input = input('Enter Steering Input: ', 's');
    brake_input = input('Enter Brake Input: ', 's');
elseif exist_steer + exist_brake == 2
    ans1 = input(['Is current steer input: ', 'steer_input '], 's');
    if ans1 == 'n'
        steer_input = input('Enter Current Steering Input: ', 's');
    end
    ans2 = input(['Is current brake input: ', 'brake_input '], 's');
    if ans2 == 'n'
        brake_input = input('Enter Current Brake Input: ', 's');
    end
else
    error('"steer_input" or "brake_input" already defined as other data type')
end

figure(11)
s Orient portrait
subplot(2,1,1)
plot(tout, F_br); grid
xlabel('Time (sec)')
ylabel('Brake Pedal Input Force, F_br (N)')
title(['Brake Input vs. time: Steer Input = ', steer_input ... 
      'Brake Input = ', brake_input])

subplot(2,1,2)
plot(tout, T_w(:,1), 'b:', 'LineWidth', 2)
plot(tout, T_w(:,2), 'k--', 'LineWidth', 2)
plot(tout, T_w(:,3), 'k-.', 'LineWidth', 2)
grid
xlabel('Time (sec)')
ylabel('Brake Torque, T_w (Nm)')
title(['Brake Torque vs. time: Steer Input = ', steer_input ... 
      'Brake Input = ', brake_input])
legend('Left Front', 'Right Front', 'Left Rear', 'Right Rear')
hold

text(0.5, -0.2, date ' NAVDyn v ' version ' Vehicle= ' vehicle ... 
    'Tire= ' tire ' Brakes= ' brake_status ' Initial Speed= ' ... 
    num2str(V_oxi*mps2kph) 'kph', 'sc', 'HorizontalAlignment', ... 
    'center', 'FontSize', 8)

%PLOTFORCE - plots the longitudinal, lateral and normal force at the tires

if brkstat == 1
    brake_status = 'Normal';
else
    brake_status = 'Failed';
end

exist_steer = exist('steer_input');
exist_brake = exist('brake_input');

if exist_steer + exist_brake < 2
    steer_input = input('Enter Steering Input: ', 's');
    brake_input = input('Enter Brake Input: ', 's');
end
elseif exist steer + exist brake == 2
  ansl=input(['Is current steer input: ' ' steer_input '? (y/n): '],'s');
  if ansl=='n'
    steer_input=input('Enter Current Steering Input: ','s');
  end
  ans2=input(['Is current brake input: ' ' brake_input '? (y/n): '],'s');
  if ans2=='n'
    brake_input=input('Enter Current Brake Input: ','s');
  end
else
  error('"steer input" or "brake_input" already defined as other data type')
end

figure(7)
set(7,'Position', [1 33 1021 699])
orient portrait
plot(tout,Fz(:,1))
hold
plot(tout,Fz(:,2),'g:','LineWidth',2)
plot(tout,Fz(:,3),'r--','LineWidth',2)
plot(tout,Fz(:,4),'c-.','LineWidth',2)
grid
xlabel('Time (sec)')
ylabel('Tire Normal Force, Fz (N)')
title([['Normal Force vs. time: Steer Input = ' 'steer_input ...
  ' Brake Input = ' brake_input])
legend('Left Front','Right Front','Left Rear','Right Rear',0);
hold

figure(8)
set(8,'Position', [1 33 1021 699])
orient portrait
subplot(2,1,1)
plot(tout,Fwx(:,1))
hold
plot(tout,Fwx(:,2),'g:','LineWidth',2)
plot(tout,Fwx(:,3),'r--','LineWidth',2)
plot(tout,Fwx(:,4),'c-.','LineWidth',2)
grid
xlabel('Time (sec)')
ylabel('Tire Longitudinal Force, Fw_x (N)')
title([['Tire Long. Force vs. time: Steer Input = ' 'steer_input ...
  ' Brake Input = ' brake_input])
legend('Left Front','Right Front','Left Rear','Right Rear',0);
hold;

subplot(2,1,2)
plot(tout,Fwy(:,1))
hold
plot(tout,Fwy(:,2),'g:','LineWidth',2)
plot(tout,Fwy(:,3),'r--','LineWidth',2)
plot(tout,Fwy(:,4),'c-.','LineWidth',2)
grid
xlabel('Time (sec)')
ylabel('Tire Lateral Force, Fw_y (N)')
title([['Tire Lat. Force vs. time: Steer Input = ' 'steer_input ...
Brake Input = 'brake_input]
end;

figure(9)
set(9,'Position', [1 33 1021 699])
orient portrait
subplot(2,1,1)
plot(tout,Fx(:,1))
hold;
plot(tout,Fx(:,2),'g:','LineWidth',2)
plot(tout,Fx(:,3),'r--','LineWidth',2)
plot(tout,Fx(:,4),'c-.','LineWidth',2)
grid
xlabel('Time (sec)')
ylabel('Vehicle Longitudinal Force, \( F_x \) (N)')
title(['Veh. Long. Force vs. time: Steer Input = ' steer_input ...
' Brake Input = ' brake_input])
legend('Left Front','Right Front','Left Rear','Right Rear',0);
hold;

subplot(2,1,2)
plot(tout,Fy(:,1))
hold;
plot(tout,Fy(:,2),'g:','LineWidth',2)
plot(tout,Fy(:,3),'r--','LineWidth',2)
plot(tout,Fy(:,4),'c-.','LineWidth',2)
grid
xlabel('Time (sec)')
ylabel('Vehicle Lateral Force, \( F_y \) (N)')
title(['Veh. Lat. Force vs. time: Steer Input = ' steer_input ...
' Brake Input = ' brake_input])

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APPENDIX F - CARSIM PARAMETERS

This appendix gives the parameters that were used in CarSim for conducting comparisons with NAVDyn and the actual Ford Taurus data. The information below is the output file from CarSim showing the parameters.

PARSFILE
* 18-DOF CarSim Vehicle Model.
* Generated by AutoSim August 14, 1997.
* All rights reserved.
*

TITLE SF

* Input File: D:\CARSIM\RUNS\556.LPI
* Run was made 11:45 on Apr 10, 2000

FORMAT BINARY

INTOPT 3, Type of integration (0=Adams, 1=Gear+PEDERV, 2=Gear diff., 3=RK2) (-)
IPRINT 30, number of time steps between output printing (counts)
EPSINT .100000E-01 , Dimensionless integrator error tolerance (-)
STARTT .000000 , simulation start time (s)
STEP .250000E-02 , simulation time step (s)
STEP0 .100000E-04 , Initial integration step (s)
STEPMIN .100000E-06 , Minimum allowable integration step (s)
STOPT 9.00000 , simulation stop time (s)

* PARAMETER VALUES

ACSAERO 2.00000 , Area of vehicle cross-section (for aerodynamics) (m2)
CD AERO .343100 , Aero drag coef: Fx = -Q*A*(Cdaero + Kdaero*Beta**2) (-)
CLAERO .160000 , Aero lift coef: Fz = -Q*A*(Claero + Klaero*Beta**2) (-)
CMAERO .105450 , Aero pitch coef: My = Q*A*Lwb*(Cmaero + Kmaero*Beta**2) (-)
CSFY(1) -.270000E-03 , Compliance coef. for steer at front axle: d(steer)/d(Fy) (deg/N)
CSFY(2) -.100000E-04 , Compliance coef. for steer at rear axle: d(steer)/d(Fy) (deg/N)
CSMZ(1) -.270000E-03 , Compliance coef. for steer at front axle: d(steer)/d(Mz) (deg/N/m)
CSMZ(2) -.100000E-04 , Compliance coef. for steer at rear axle: d(steer)/d(Mz) (deg/N/m)
CTFX(1) .000000E-02 , Compliance coef. for toe at front axle: d(toe)/d(Fx) (deg/N)
CTFX(2) .428300E-03 , Compliance coef. for toe at front axle: d(toe)/d(Fx) (deg/N)
DAIR .000000 , Air density (kg/m3)
HCG 542.000 , Height of entire vehicle C.G. above the ground (mm)
HCGS 567.851 , CALC - Height of sprung-mass CG above ground (mm)
HRC(1) 130.000 , Height of front axle roll center above the ground (mm)
HRC(2) 110.000 , Height of rear axle roll center above the ground (mm)
HWC(1) 320.000 , Height of front wheel center at design load (mm)
HWC(2) 320.000 , Height of rear wheel center at design load (mm)
IW(1) .990000 , Spin moment of inertia of front wheel (kg-m2)
IW(2) .990000 , Spin moment of inertia of rear wheel (kg-m2)
IXX 555.500 , Moment of inertia of entire vehicle (kg-m2)
IXXS 440.511 , CALC - Sprung-mass IXX moment of inertia (kg-m2)
IXZ .000000 , Product of inertia of entire vehicle (kg-m2)
IXZS 7.54097 , CALC - Sprung-mass IXZ product of inertia (kg-m2)
IYY 2832.70 , Moment of inertia of entire vehicle (kg-m2)
IYYS 2498.90 , CALC - Sprung-mass IYY moment of inertia (kg-m2)
IZZ 3048.10 , Moment of inertia of entire vehicle (kg-m2)
IZZS 2619.28, CALC - Sprung-mass IZZ moment of inertia (kg-m²)
KAUX(1) 384.000, Front auxiliary stiffness, including anti-sway bar (N-m/deg)
KAUX(2) 344.400, Rear auxiliary stiffness, including anti-sway bar (N-m/deg)
KDAERO 282000E-03, Aero drag coeff: Fx = -Q*A*(Cdaero + Kdaero*Beta**2) (1/deg²)
KGAMMA(1) 0.00000, Front tire camber stiffness (N/deg)
KGAMMA(2) 0.00000, Rear tire camber stiffness (N/deg)
KLAERO 282000E-03, Aero lift coeff: Fz = -Q*A*(Claero + Klaero*Beta**2) (1/deg²)
KMAERO 186400E-02, Aero pitch coeff: My = Q*A*Lwb*(Cmaero + Kmaero*Beta**2) (1/deg²)
KNAERO 918000E-02, Aero yaw coeff: Mz = -Q*A*Lwb*Knaero*Beta (1/deg)
KRAERO 770000E-02, Aero roll coeff: Mx = -Q*A*Lwb*Kraero*Beta (1/deg)
KT(1) 220.000, Front tire vertical stiffness (for tire) (N/mm)
KT(2) 220.000, Rear tire vertical stiffness (for tire) (N/mm)
KYAERO 345000E-01, Aero side force coeff: Fy = -Q*A*Kyaero*Beta (1/deg)
LCGS 1014.76, CALC - Distance from front axle to sprung-mass CG (mm)
LCGT 1034.69, CALC - Distance from F axle to total vehicle CG (mm)
LRELAX(1) 333.660, Front tire relaxation length (mm)
LRELAX(2) 333.660, Rear tire relaxation length (mm)
LTK(1) 1540.00, Front axle track width (mm)
LTK(2) 1530.00, Rear axle track width (mm)
LWB 2690.00, Wheelbase (mm)
MF 1049.00, Vehicle mass supported by front axle (2 wheels) (kg)
MR 655.700, Vehicle mass supported by rear axle (2 wheels) (kg)
MS 1526.90, CALC - Mass of sprung mass (kg)
MT 1704.70, CALC - Total vehicle mass (kg)
MU 0.85000, Tire-road coefficient of friction (-)
MUS(1) 98.1000, Front axle unsprung mass (2 wheels) (kg)
MUS(2) 79.7000, Rear axle unsprung mass (2 wheels) (kg)
PSIWND 0.00000, Heading of wind (0 deg=tailwind) (deg)
RAP(1) 770000E-01, Ratio: wheelbase change per unit jounce at front axle (-)
RAP(2) 1.00000, Ratio: wheelbase change per unit jounce at rear axle (-)
RDAMP(1) 1.00000, Ratio: front jounce at wheel to damper stroke (-)
RDAMP(2) 1.00000, Ratio: rear jounce at wheel to damper stroke (-)
RDRIVE 0.00000, Ratio: rear drive torque to total: 0 = FWD, 0.5 = 4WD, 1 = RWD (-)
RMF 615358, CALC - Ratio: proportion of load on front axle (-)
RMR 384642, CALC - Ratio: proportion of load on rear axle (-)
ROLL_STOP 45.0000, Roll limit for stopping the simulation (deg)
RSRPRNG(1) 1.00000, Front ratio of jounce at wheel to spring jounce (-)
RSRPRNG(2) 1.00000, Rear ratio of jounce at wheel to spring jounce (-)
RSW 15.9700, Steering gear ratio (-)
RTIME .222222, CALC - Computational efficiency (sec/sim. sec) (-)
SPEED_KI 157000, Integral control gain for speed controller (rev/s²)
SPEED_KP 141000, Proportional control gain for speed controller (rev/s)
SPEED_ON_OFF .000000, Speed control switch (0.0 -> off, 1.0 -> on) (-)

STEERSW_TABLE Steer wheel angle vs. time
  0.00000  0.00000, point in table: (s, deg)
  1.00000  0.00000, point in table: (s, deg)

* Tire Data:
* Longitudinal force (N) as a function of slip rate (-) and vertical load (N).
* top row = load, left column = slip, other cells = longitudinal tire force

FX_CARPET 4, columns in table
  0.00000  2495.49  4145.04  5791.47
  .720000E-02  322.108  533.267  750.181
  .990000E-02  447.936  741.937 1042.61
  1.980000E-01  603.833 1482.52  2072.47
  5.040000E-01  1877.20  3154.92  4269.58
  .9990000E-01  2494.36  4524.81  5542.27
  .150300  2522.24  4524.07  5784.03
  .199800  2827.26  4530.24  5746.98

* Tire Data:
* Longitudinal force (N) as a function of slip rate (-) and vertical load (N).
* top row = load, left column = slip, other cells = longitudinal tire force

FX_CARPET 4, columns in table
  0.00000  2495.49  4145.04  5791.47
  .720000E-02  322.108  533.267  750.181
  .990000E-02  447.936  741.937 1042.61
  1.980000E-01  603.833 1482.52  2072.47
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  .9990000E-01  2494.36  4524.81  5542.27
  .150300  2522.24  4524.07  5784.03
  .199800  2827.26  4530.24  5746.98

264
* Lateral force (N) as a function of slip angle (deg) and vertical load (N).
  * top row = load, left column = slip, other cells = lateral tire force

<table>
<thead>
<tr>
<th>Load (N)</th>
<th>Slip Angle (deg)</th>
<th>Lateral Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>250200</td>
<td>0.25</td>
<td>2581.77</td>
</tr>
<tr>
<td>299700</td>
<td>0.29</td>
<td>2526.93</td>
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<td>0.35</td>
<td>2470.03</td>
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<tr>
<td>400500</td>
<td>0.40</td>
<td>2414.86</td>
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<tr>
<td>450000</td>
<td>0.45</td>
<td>2382.99</td>
</tr>
</tbody>
</table>

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</tr>
<tr>
<td>450000</td>
<td>0.45</td>
<td>2382.99</td>
</tr>
</tbody>
</table>

* Aligning moment (N-m) as a function of slip angle (deg) and vertical load (N).
  * top row = load, left column = slip, other cells = aligning moment

<table>
<thead>
<tr>
<th>Load (N)</th>
<th>Slip Angle (deg)</th>
<th>Aligning Moment (N-m)</th>
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<tbody>
<tr>
<td>250200</td>
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<td>2581.77</td>
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</tr>
<tr>
<td>450000</td>
<td>0.45</td>
<td>2382.99</td>
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* Longitudinal force (N) as a function of slip rate (-) and vertical load (N).
  * top row = load, left column = slip, other cells = longitudinal tire force

<table>
<thead>
<tr>
<th>Load (N)</th>
<th>Slip Ratio (-)</th>
<th>Longitudinal Force (N)</th>
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<td>299700</td>
<td>0.29</td>
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<td>2382.99</td>
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* Lateral force (N) as a function of slip angle (deg) and vertical load (N).
* top row = load, left column = slip, other cells = lateral tire force

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<tr>
<th>Load (N)</th>
<th>Slip (deg)</th>
<th>Lateral Force (N)</th>
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<tbody>
<tr>
<td>0.00000</td>
<td>0.00000</td>
<td>2495.49 4145.04 5791.47</td>
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<tr>
<td>0.99000</td>
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<td>3.69000</td>
<td>2369.26 3453.48 4236.35</td>
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<td>2668.16 4126.49 5289.08</td>
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* Aligning moment (N-m) as a function of slip angle (deg) and vertical load (N).
* top row = load, left column = slip, other cells = aligning moment

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<th>Load (N)</th>
<th>Slip (deg)</th>
<th>Aligning Moment (N-m)</th>
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</tr>
</tbody>
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| VLOW_ALPH(1) 15.0000 | Front low-speed threshold for modified tire relaxation equations (kph) |
| VLOW_ALPH(2) 15.0000 | Rear low-speed threshold for modified tire relaxation equations (kph) |
| VLOW_KAPPA(1) 8.00000 | Front low-speed threshold for modified longitudinal slip equations (kph) |
| VLOW_KAPPA(2) 8.00000 | Rear low-speed threshold for modified longitudinal slip equations (kph) |
| VLOW_SPIN(1) 10.0000 | Front low-speed threshold for modified wheel spin equations (kph) |
| VLOW_SPIN(2) 10.0000 | Rear low-speed threshold for modified wheel spin equations (kph) |
| V_STOP -1.00000 | Low-speed limit for stopping the simulation (kph) |

<table>
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<th>Brake input vs. time</th>
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<tr>
<td>6.03000</td>
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<tr>
<td>6.23000</td>
</tr>
</tbody>
</table>

* Camber angle vs. susp jounce (IAXLE: 1=F, 2=R)

IAXLE  1    , Table ID number
CAMBER_TABLE Camber angle vs. susp jounce (IAXLE: 1=F, 2=R)
-70.0000  , .430000  , point in table: (mm, deg)
-60.0000  , .440000  , point in table: (mm, deg)
-50.0000  , .420000  , point in table: (mm, deg)
-40.0000  , .380000  , point in table: (mm, deg)
-30.0000  , .310000  , point in table: (mm, deg)
-20.0000  , .230000  , point in table: (mm, deg)
-10.0000  , .130000  , point in table: (mm, deg)
  0.0000   , .000000  , point in table: (mm, deg)
  10.0000  , -.150000 , point in table: (mm, deg)
  20.0000  , -.320000 , point in table: (mm, deg)
  30.0000  , -.510000 , point in table: (mm, deg)
  40.0000  , -.720000 , point in table: (mm, deg)
  50.0000  , -.960000 , point in table: (mm, deg)
  60.0000  , -1.21000 , point in table: (mm, deg)
  70.0000  , -1.49000 , point in table: (mm, deg)

ENDTABLE
Table ID number

<table>
<thead>
<tr>
<th>Camber Table Camber angle vs. susp jounce (IAXLE: 1=F, 2=R)</th>
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<tbody>
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<tr>
<td>-70.0000, .730000</td>
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<td>-60.0000, .660000</td>
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<td>-50.0000, .580000</td>
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<tr>
<td>-40.0000, .490000</td>
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<tr>
<td>-30.0000, .380000</td>
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<tr>
<td>-20.0000, .270000</td>
</tr>
<tr>
<td>-10.0000, .140000</td>
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<tr>
<td>0.0000, 0.0000</td>
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<tr>
<td>10.0000, -.150000</td>
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<td>70.0000, -.133000</td>
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ENDTABLE

- Susp comp damping force (1 side) vs. jnc rate (iexle: 1=F, 2=R)

Table ID number

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<thead>
<tr>
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ENDTABLE

- Susp comp spring force (1 side) vs. jounce (iexle: 1=F, 2=R)

Table ID number

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>point in table: (mm, N)</td>
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ENDTABLE
<table>
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<th>FSTABLE Susp comp spring force (1 side) vs. jounce (IAXLE: 1=F,2=R)</th>
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<th>LTARG_TABLE Lateral offset (left) for driver, relative to input path</th>
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<th>MYBK_TABLE Brake torque at wheel vs. line pressure</th>
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</thead>
<tbody>
<tr>
<td>.000000 , .000000 , point in table: (kPa, N-m)</td>
</tr>
<tr>
<td>46.5000 , 376.569 , point in table: (kPa, N-m)</td>
</tr>
<tr>
<td>60.3000 , 525.300 , point in table: (kPa, N-m)</td>
</tr>
<tr>
<td>76.9000 , 749.665 , point in table: (kPa, N-m)</td>
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<tr>
<td>83.9700 , 876.199 , point in table: (kPa, N-m)</td>
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<tr>
<td>97.7000 , 1034.52 , point in table: (kPa, N-m)</td>
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<tr>
<td>101.600 , 1074.82 , point in table: (kPa, N-m)</td>
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<tr>
<td>104.900 , 1081.04 , point in table: (kPa, N-m)</td>
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<tr>
<td>110.900 , 1116.51 , point in table: (kPa, N-m)</td>
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<td>137.900 , 1406.92 , point in table: (kPa, N-m)</td>
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<td>168.700 , 1672.33 , point in table: (kPa, N-m)</td>
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<td>101.600 , 1074.82 , point in table: (kPa, N-m)</td>
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<tr>
<td>104.900 , 1081.06 , point in table: (kPa, N-m)</td>
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<tr>
<td>110.900 , 1116.51 , point in table: (kPa, N-m)</td>
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<tr>
<td>137.900 , 1406.92 , point in table: (kPa, N-m)</td>
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<tr>
<td>168.700 , 1672.33 , point in table: (kPa, N-m)</td>
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</table>
IAXLE 2
ISIDE 2
MYBK_TABLE Brake torque at wheel vs. line pressure

<table>
<thead>
<tr>
<th>pressure</th>
<th>torque</th>
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<tbody>
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<td>47.0000</td>
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<tr>
<td>60.0000</td>
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<tr>
<td>77.0000</td>
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<tr>
<td>138.000</td>
<td>182.000</td>
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<td>169.000</td>
<td>192.000</td>
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ENDTABLE

SPEED_TABLE Vehicle forward speed vs. time (speed control)

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<tbody>
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ENDTABLE

STEERRF_TABLE Rear/Front steer ratio vs. vehicle speed

<table>
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<th>ratio</th>
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<tbody>
<tr>
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<td>.00000</td>
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<tr>
<td>5.0000</td>
<td>.00000</td>
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</tbody>
</table>

ENDTABLE

* Front wheel steer vs. average angle (iside: 1=lf, 2=rf)

ISIDE  1 , Table ID number
STEERRW_TABLE Front wheel steer vs. average angle (ISIDE: 1=LF, 2=RF)

<table>
<thead>
<tr>
<th>angle</th>
<th>angle</th>
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</thead>
<tbody>
<tr>
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<td>-21.460</td>
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<tr>
<td>-19.830</td>
<td>-18.910</td>
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<tr>
<td>-17.000</td>
<td>-16.320</td>
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<td>-14.170</td>
<td>-13.700</td>
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<td>-11.330</td>
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<td>-2.7800</td>
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<td>2.7800</td>
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<tr>
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<tr>
<td>14.170</td>
<td>13.700</td>
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<tr>
<td>17.000</td>
<td>16.320</td>
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<tr>
<td>19.830</td>
<td>18.910</td>
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<tr>
<td>22.670</td>
<td>21.460</td>
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ENDTABLE

ISIDE  2 , Table ID number
STEERRW_TABLE Front wheel steer vs. average angle (ISIDE: 1=LF, 2=RF)

<table>
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<tr>
<td>2.8000</td>
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<td>5.5300</td>
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<td>8.4100</td>
<td>8.2400</td>
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270
### Toe angle vs. susp. jounce (IAXLE: 1=F, 2=R)

<table>
<thead>
<tr>
<th>IAXLE</th>
<th>Table ID number</th>
<th>TOE_TABLE Toe angle vs. susp. jounce (IAXLE: 1=F, 2=R)</th>
</tr>
</thead>
<tbody>
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<td>30.0000 , .000000E-01 , point in table: (mm, deg)</td>
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<tr>
<td></td>
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<td>70.0000 , .000000E-01 , point in table: (mm, deg)</td>
</tr>
</tbody>
</table>

### Final Conditions

- **XO**: 87.96785211158, Abs. X trans. of S0 (m)
- **YO**: 3320090796824E-16, Abs. Y trans. of S0 (m)
- **ZO**: -7159160687246E-02, Abs. Z trans. of S0 (m)
- **YAW**: .7063873143629E-14, Abs. Z rot. of Spp (deg)
- **PITCH**: .4165322388220, Y rot. of Sp rel. to Spp (deg)
- **ROLL**: .1363479207420E-16, X rot. of S rel. to Sp (deg)
- **JNC_WLFL**: 7.273448045157E-02, Trans. of USM_LF0 rel. to USM_LFJ (m)
- **JNC_WLRR**: -1.256252418218E-01, Trans. of USM_RR0 rel. to USM_RRJ (m)
- **JNC_WLFL**: 7.273448045157E-02, Trans. of USM_RF0 rel. to USM_RFJ (m)
- **JNC_WLRR**: -1.256252418218E-01, Trans. of USM_RR0 rel. to USM_RRJ (m)
ROTLF 21.9552444253, Rotation angle of LF wheel (rev)
ROTLR 43.19949149410, Rotation angle of LR wheel (rev)
ROTRF 21.9552444253, Rotation angle of RF wheel (rev)
ROTRR 43.19949149410, Rotation angle of RR wheel (rev)
TANLF -1.686733931315E-01, Slip angle tan of LF tire, with time lag (-)
TANLR 3.055379959085E-01, Slip angle tan of LR tire, with time lag (-)
TANRF -1.686733931315E-01, Slip angle tan of RF tire, with time lag (-)
TANRR -3.055379959085E-01, Slip angle tan of RR tire, with time lag (-)
IVERR 112.0281800224, Integral of velocity error for speed controller (m)
VXS 42.29574165978E-01, Vehicle forward speed (kph)
VYS 2.289406001347E-02, Abs. Y trans. speed of SCMC (kph)
VZS 7.080156671634E-15, Abs. Z rot. speed of S (deg/s)
AVX 1.92558817158E-14, Abs. X rot. speed of S (deg/s)
JNCR_WLF -1.692780587663E-02, Trans. speed of USM_LF0 rel. to S (m/s)
JNCR_WLR 1.627612108372E-02, Trans. speed of USM_LR0 rel. to S (m/s)
JNCR_WRR -1.692780587663E-02, Trans. speed of USM_RF0 rel. to S (m/s)
AVY_LF 0.000000000000000, Angular velocity of LF wheel (rev/s)
AVY_LR 0.457675929361E-42, Angular velocity of LR wheel (rev/s)
AVY_RF 0.000000000000000, Angular velocity of RF wheel (rev/s)
AVY_RR 0.457675929361E-42, Angular velocity of RR wheel (rev/s)
END
REFERENCES


