14.03 Fall 2004

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Due Thursday, September 23, 2004 at 5pm

Late problem sets are not accepted

1 Math tools - Part I

Consider the following function:

$$y = f(x; a) = a^2 + x - ax^2$$

where $a > 0$ is a parameter.

1. Find the first order condition for a critical point of this function.
2. Is this a maximum or a minimum or an inflection point?
3. Solve for $x^*(a)$, the maximizer of the function $f(x; a)$ .Also find $y^*(a)$, the maximized value of y as a function of a
4. Find $\frac{dx^*}{da}$ and $\frac{dy^*}{da}$
5. Now use the FOC from 1 and the implicit function theorem to find $\frac{dx^*}{da}$
6. Use the envelope theorem to find $\frac{dy^*}{da}$. Why does this theorem allow you to simplify your calculations with respect to point 4?

2 Math tools - Part II

Consider the problem

$$\begin{aligned} \max\ z &= 2x + 4x^{\frac{1}{2}}y^{\frac{1}{2}} + 2y \\ s.t.\ \ x + y &= 400 \end{aligned}$$

1. Set up the lagrangian and find the first order conditions
2. Solve explicitly for x, y, z and λ, the Lagrange multiplier.
3. Set up the dual problem of 1., i.e., minimize the primal constraint function subject to the primal objective function being equal to the value of z^* obtained in 2.
4. Solve explicitly for x, y, and λ^D, the Lagrange multiplier in the dual problem.
5. What is the relationship between λ and λ^D, and can you provide an economic explanation for this?

3 Math tools - Part III

Let

$$U = xy^2$$

1. Derive the indirect utility function as a function of p_x, p_y and M, where p_x and p_y are respectively the prices of the two goods x and y, and where M is the consumer's income.
2. Now calculate the level consumption of both goods and the level of utility achieved by this consumer if prices and income are as follows:

$$p_x = 2,\ p_y = 3\,, M = 9$$

3. Now set up the dual of this problem: minimize expenditure subject to the level of utility that you calculated in part 2 and with prices p_x and p_y. Find the expression for the expenditure function.

4 Labor market structure

You are a labor economist trying to evaluate whether the labor market for computer scientists is competitive or monopsonistic. Based on previous research, you know that the production function for computers depends only on labor input:

$$Y = -0.5L^2 + 10L,$$

where Y is the output of computers and L is the quantity of labor used. The price of a computer is $p = 2$.You also know that the labor supply, as a function of the wage, for computer scientists is the following, where w is the wage per unit of labor:

$$L = -10 + w.$$

a. Find the equilibrium wages and employment (w^C, L^C) that would prevail if the market for computer scientists were competitive. [Remember that a competitive firm takes the wage as given – that is, it assumes that the quantity of workers that it hires has no effect on the price of the next worker. (Of course, the equilibrium wage must equate demand and supply)].

b. Find the equilibrium wages and employment (w^M, L^M) that would prevail if the market was dominated by a single computer producer that acts as a monopsonist in the labor market for computer scientists.[In this case, the firm accounts for the fact that the quantity of labor it hires affects the market wage - it is not a price taker. Also, recall that the marginal revenue product of labor MRPL is the price of output (Y) times the marginal product of labor.]

c. Show the marginal cost curve for labor faced by the monopsonist and draw a graph that shows the two equilibria you found.

d. Compare the two equilibria and discuss the reasons why they differ.

e. To faciliate your study, Congress agrees to suddenly raises the minimum wage for computer scientists to $w_{\min} = 14$. What would you expect to observe after the introduction of the minimum wage under each of the two market structures (competitive and monopsonistic)? [Provide a mathematical answer.]

f. What if the minimum wage were raised to $w_{\min} = 16$? Explain. [If you weren't able to derive the numerical values for the two equilibria in the first part of the problem, give the intuition for how the results of your experiment will depend on the level of the minimum wage.]

5 RCM

You are conducting an experiment to assess the causal effect of taking low doses of aspirin (LDA) on blood pressure. Consider each of the following experimental approaches. For each, what are the assumptions necessary to draw a causal inference? How plausible are these assumptions? In aswering the following questions make use of the setup developed in the lecture on Causal Inference where T^* is the true effect of the treatment and Y is the outcome of interest, blood pressure. Y can take up two values: $Y = 0$ if blood pressure is normal and $Y = 1$ if blood pressure is high. We indicate the treatment by X, which can take two values: $X = 1$ if LDA is given to the individual and $X = 0$ if no aspirin is given. Also indicate time by $t = 1$ or $t = 2$ for 1^{st} year and 2^{nd} year respectively. The true treatment effect on the individual i at time t is the following:

$$T^*_{it} = Y_{i1,t} - Y_{i0,t}$$

For each of the following scenarios, please describe the assumptions required to obtain a valid estimate of T^*_{it} or $E[T^*_{it}]$ (the latter denotes the average treatment effect for the treated population). Describe for each question below how your estimate of T^* might be biased if your assumptions are not satisfied.

1. You choose 2,000 people at random from the entire U.S. population. You give LDA treatments to 1,000 patients and prevent the other 1,000 from using aspirin. At the end of one year, you compare the blood pressure level of treated to control patients to measure the causal effect of LDA.

2. You choose 1,000 people at random from the entire U.S. population. In the 1st year, you prevent aspirin use and measure the level of blood pressure in this population. In the 2nd year, you give LDA and measure the level of blood pressure in this population. You compare the blood pressure levels in the 1st and 2nd years to measure the causal effect.

3. You choose 2,000 people at random from the entire U.S. population. In the 1st year, you prevent aspirin use and measure the level of blood pressure. In the 2nd year, you gave LDA to 1,000 members of this population (selected at random) and continue to prevent aspirin use for the other 1,000. To measure the causal effect of LDA, you contrast the 1st to 2nd year change in blood pressure in the

LDA-treated population to the 1st to 2nd year change in blood pressure in the non-aspirin population.

4. You recruit 10 pairs of identical twins for your study. 1 of each pair receives LDA and the other of each pair is prevented from taking aspirin. After one year, you compare the level of blood pressure in these two populations to measure the causal effect.

5. You recruit 10 pairs of fraternal twins for your study, where one of each pair is female and the other is male. The male in each pair receives LDA and the female in each pair is prevented from taking aspirin. After one year, you compare the level of blood pressure between males and females to measure the causal effect.

6 Card and Krueger

1. Card and Krueger (p. 773) emphasize that one strength of their analysis is that the implementation of the New Jersey minimum wage occurred during a recession.

   (a) Consider the case where the minimum wage was implemented right before an economic expansion instead. Draw a diagram that clearly shows how an economic expansion affecting both New Jersey and Pennsylvania might have invalidated the results of the 'natural experiment.'

   (b) Someone points out to you that, according to Table 3 of C&K, fast food employment fell in PA but it barely budged in NJ. Hence, they say, if we just look at NJ, it seems pretty clear that there is no support for a monopsony scenario. How would you respond to this criticism within the RCM framework. Draw a carefully labeled diagram that illustrates your point.

2. Richard Freeman distinguishes between the short and long run impacts of minimum wage increases, implying that the long run impacts are likely to be larger (and more negative). List one or two specific economic reasons why the long run response would be larger. Consider that labor is not the only factor in fast food restaurants' production functions.