22.251 Systems Analysis of the Nuclear Fuel Cycle
Fall 2005
PROBLEM SET #3

Consider three reactor types, a current large PWR, a CANDU fueled with slightly enriched U (SEU), and a small modular pebble-bed HTGR, having the following characteristics:

	PWR	PBMR	CANDU-SEU
MW(e)	1150	114	881
MW(th)	3411	265	2798
FUEL ENRICHMENT, w/o U-235	4.5	8.0	1.20
DISCHARGE BURNUP, MWd/kg	50	80	19.75
FUEL MGT	3-BATCH	CONTINUOUS ON-LINE REFUELING	

(a) Compare their uranium and separate work utilization: MWd(e)/kg U_{NAT} and MWd(e)/kg SWU for an enrichment plant tails of 0.25 w/o.

(b) Explain why the PBMR fuel cycle might be expected to be (and is or is not) superior to the PWR and/or CANDU.

PROBLEM SET #3 SOLUTION

PROBLEM 1

(a) This is a rather straightforward application of the methods previously applied in Problem Set #1.

For example, for the PWR, B_d = 50 MWd(th)/kg, thus

$$B_d = 50 \times \frac{1150}{3411} = 16.86 \text{ MWd(e)/kgP}$$

and $\frac{F}{P} = \frac{X_P - X_W}{X_F - X_W} = \frac{X_P - 0.25}{0.711 - 0.25} = 2.169Xp - 0.542$

Thus for X_P = 4.5 w/o U235, *F/P* = 9.22

and natural uranium utilization is $\frac{16.86}{9.22} = 1.83 MWd(e)/kgU_{NAT}$.

For SWU, $\frac{S}{P} = \left[V(X_P) + \frac{W}{P} V(X_W) - \frac{F}{P} V(X_F) \right]$

The following spread sheet shows detailed calculation:

	PWR	PBMR	CANDU-SEU
MWe	1150	114	881
MWth	3411	265	2798
Xp	4.5	8	1.2
Bd (MWd/kg)	50	80	19.75
Thermal eff, n	33.71%	43.02%	31.49%
Bd (Mwde/kg)	16.86	34.42	6.22
F/P	9.22	16.81	2.06
W/P	8.22	15.81	1.06
Uu (Mwde/kgUnat)	1.83	2.05	3.02
V(Xw)	5.959	5.959	5.959
V(Xf)	4.869	4.869	4.869
V(Xp)	2.780	2.052	4.305
S/P	6.871	14.419	0.592
Us (MWde/kgSWU)	2.453	2.387	10.498

(b) Some of the potential advantages/disadvantages of the subject types will become obvious later, but even now we can infer some useful generalizations from the information provided:

(1) The PBMR is small, about 1/10th the rating of the PWR & CANDU. In addition, the migration length is long in graphite compared to H_2O. Thus the PBMR probably suffers large neutron leakage losses, which will reduce its burnup potential.

(2) But its good neutron economy and the ability of its fuel to withstand high burnup are significant advantage.

(3) Also, as defined here, the utilization is based on electric output (while, burnup, of course, is based on thermal output). The PBMR has much higher thermodynamic efficiency:

$$\eta_{PWR} = \frac{1150}{3411} = 33.7\%$$

$$\eta_{PBMR} = \frac{114}{265} = 43\%$$

$$\eta_{CANDU} = \frac{881}{2798} = 31.5\%$$

this alone increases utilization by a factor of about 43/33 = 1.3, or ~30%.

(4) Compared to the PWR, both the PBMR and CANDU have the advantage of on-line refueling, hence can get more burnup for a given reload enrichment.

PROBLEM 2

I assumed 1000 kg of product as in the example in Bendict & Pigford and a tails enrichment of 0.3 w/o. Note that ^{235}U enrichment in spent LWR fuel typically is between 0.7 and 0.8 w/o [Cochran and Tsoulfanidis, pg. 225], but our spent fuel enrichment is not specified and most students used 0.711 w/o, so let's use that.

$$P = 1000kg$$
$$x_{5,w} = 0.003$$

$$F = P\frac{y_{5,P} - x_{5,W}}{z_{5,F} - x_{5,W}} = 1000\frac{0.045 - 0.003}{0.00711 - 0.003} = 10{,}219$$
$$W = F - P = 9{,}219$$

Stream	Weight Fraction ^{235}U	Weight Fraction ^{236}U	Weight Ratio ^{235}U:^{236}U, R	Mass, kg
Product	0.045	$y_{6,P}$	$R_P = \frac{0.045}{1 - 0.045 - y_{6,P}}$	1,000
Tails	0.003	$x_{6,W}$	$R_P = \frac{0.045}{1 - 0.045 - y_{6,P}}$	9,219
Feed	0.00711	$z_{6,F}=0.004$	$R_P = \frac{0.045}{1 - 0.045 - y_{6,P}}$	10,219

^{236}U is conserved according to the following equation…

$$1000 y_{6,P} + 9219 x_{6,W} - 10219 z_{6,F} = 0 \qquad (1)$$

Using Equation (12.323) in Bendict & Pigford

$$\frac{P \cdot y_{6,P}}{R_P^{1/3}} + \frac{W \cdot x_{6,W}}{R_W^{1/3}} - \frac{F \cdot z_{6,F}}{R_F^{1/3}} = 0$$

And inserting equations for R values…

$$\frac{1000y_{6,P}}{\left[\frac{0.045}{1-0.045-y_{6,P}}\right]^{1/3}}+\frac{9219x_{6,W}}{\left[\frac{0.003}{1-0.003-x_{6,W}}\right]^{1/3}}-\frac{10219z_{6,F}}{\left[\frac{0.00711}{1-0.00711-z_{6,F}}\right]^{1/3}}=0 \qquad (2)$$

If we take the ^{236}U enrichment in the feed to be 0.4 w/o as in the example…

$$z_{6,F}=0.004 \qquad (3)$$

Solving equations (1), (2), and (3) simultaneously, we get …

$$y_{6,P}=0.01705$$
$$x_{6,W}=0.002584$$
$$z_{6,F}=0.004$$

$$\frac{y_{6,P}}{y_{8,P}}=\frac{y_{6,P}}{1-y_{5,P}-y_{6,P}}=\frac{0.01705}{1-0.045-0.01705}=\boxed{0.0182}$$

Finding the reactivity penalty of ^{236}U relative to ^{235}U in the new fuel.

In the discharged fuel, we know how much mass of each nuclide we had, and we know their penalties relative to one another. So if you find a constant of proportionality *f*, we can relate mass ratio to the ratio of reactivity penalties.

$$\frac{^{236}U}{^{235}U}f=0.25$$
$$\frac{0.004}{0.00711}f=0.25$$
$$f=0.444$$

In the new fuel, we use the same constant of proportionality *f*, but with the new mass fractions.

$$\frac{^{236}U}{^{235}U}f=\frac{0.0175}{0.045}0.444=\boxed{0.1682}$$

or the reactivity penalty due to ^{236}U in the recycled fuel would be 16.7% that of ^{235}U.