Homework #4 Solutions

Section A.

1. For the time step corresponding to a Burnup of 1 MWd/kg. The section of the CASMO output entitled “TWO GROUP DATA” is shown below…

```
* TWO GROUP DATA                                      K-INF XE, NO XE  1.33885     1.37693
    DIFF1   , DIFF2         1.4328E+00   3.4928E-01   M2     ",  "     6.1622E+01  6.1735E+01
    ABS1    , ABS2          1.1024E-02   1.2488E-01   XE2  MIC, MAC    1.1818E+06  4.8432E-03
    NUFISS1 , NUFISS2       9.6173E-03   2.1536E-01   SM2    ",  "     3.7810E+04  1.1734E-03
    REMOV1  , NU            1.3334E-02   2.4554E+00   BOR1   ",  "     8.6377E+00  4.7812E-24
    KAPPA   , XE-YIELD      3.2489E-11   6.5932E-02   BOR2   ",  "     4.0229E+02  2.2268E-22
```

The two-group formula for k_∞ is …

$$k_\infty = \frac{\nu\sum_{f1} + \dfrac{\nu\sum_{f2}\sum_{r1}}{\sum_{a2}}}{\sum_{a1} + \sum_{r1}} = \frac{.0096173 + \dfrac{(.21536)(.013334)}{(.12488)}}{(.011024) + (.013334)} = 1.33887$$

Note: All above values for macroscopic cross-section are in cm^{-1}.

This is very close to the reported value of 1.33885 at the upper right of the TWO GROUP DATA section.

2. a)

The non-leakage probability is given by

$$NLP = \frac{1}{\left(1 + M^2 B^2\right)}$$

The migration area for two groups is given by

$$M^2 = \frac{D_1}{\Sigma_{a1} + \Sigma_{r1}} + \frac{D_2}{\Sigma_{a2}}$$

$$M^2 = \frac{1.4328cm}{.011024cm^{-1} + .013334cm^{-1}} + \frac{.34928cm}{.12488cm^{-1}} = 61.6195cm^2$$

This is again very close to the value given by CASMO of 61.622cm^2

b) In the fast region, we call this value Fermi age to thermal or slowing down area and it is denoted by τ_{th}.

$$\tau_{th} = \frac{D_1}{\Sigma_{a1} + \Sigma_{r1}} = \frac{1.4328cm}{.011024cm^{-1} + .013334cm^{-1}} = 58.8226cm^2$$

$$\tau_{th} = {}^{1}\!/_{6}\,\overline{r^2_{fast}}$$

In words, this means that τ is equal to 1/6 the average of the square of the vector ("crow-flight") distance that a neutron travels from the point where it is emitted to the point where it becomes thermal.

$$r_{fast} = \sqrt{6\tau_{th}} = 18.79cm$$

This is on the order of the width of a fuel assembly.

In the thermal case, the value is called diffusion area and is denoted by L^2.

$$L^2 = {}^{1}\!/_{6}\,\overline{r^2_{therm}}$$

In words, this means that L^2 is equal to 1/6 the average of the square of the vector ("crow-flight") distance that a neutron travels from the point where it becomes thermal to the point where it is absorbed.

$$L^2 = \frac{D_2}{\Sigma_{a2}} = \frac{.34928cm}{.12488cm^{-1}} = 2.7969cm^2$$

$$r_{therm} = \sqrt{6L^2} = 4.097cm$$

This is enough distance to traverse 2 or 3 fuel rods, much smaller than one fuel assembly.

c) Now calculating non-leakage probability

$$NLP = \frac{1}{\left(1 + M^2B^2\right)} = \frac{1}{\left(1 + \left(61.6195cm^2\right)\left(.005478cm^{-2}\right)\right)} = 0.747635$$

In the NEUTRON BALANCE section of the CASMO output, the leakage is 0.25055

The non-leakage and leakage should sum to unity

NLP + leakage = 0.747635 + 0.25055 = 0.99819

So our rough calculation agrees quite well with CASMO

3. The NEUTRON BALANCE section of burnup step 1 MWd/kg is shown below

```
* NEUTRON BALANCE            GROUP 1      GROUP 2        TOTAL
  FLUX  . . . . . . .     4.8865E+00   5.1297E-01   5.3995E+00
  ABSORPTION  . . . .     3.4209E-01   4.0682E-01   7.4891E-01
  FISSION . . . . . .     1.1905E-01   2.8821E-01   4.0726E-01
  NUFISSION . . . . .     2.9844E-01   7.0156E-01   1.0000E+00
  LEAKAGE . . . . . .     2.4355E-01   7.0065E-03   2.5055E-01
  OUTSCATTER  . . . .     4.2214E-01   8.3831E-03   4.3053E-01
  K-INF  (2-GROUP). .     3.9484E-01   9.4401E-01   1.3388E+00
  ETA*F, P  . . . . .     8.7240E-01   1.7245E+00   5.4741E-01
  INV VELOCITY  . . .     5.1137E-08   2.2398E-06
  FLUX DET/CELL . . .     1.0095E+00   9.0576E-01
```

Total conservation of neutrons is given by Neutron production = Absorption + Leakage

Or in the language of CASMO

NUFISSION = ABSORPTION + LEAKAGE

1.0000 = 0.74891 + 0.25055 = 0.99946

This gives good agreement.

For the Fast Group, conservation of neutrons is given by (Neutrons produced in fission) = (Fast absorption) + (Fast Leakage) +(Outscatter to group 2) - (upscatter from group 2 to group 1)

NUFISSION = ABSORPTION1 + LEAKAGE1 + OUTSCATTER1 – OUTSCATTER2

1.0000 = 0.34209 + 0.24355 + 0.42214 – 0.0083831= 0.999397

Good agreement again

For the thermal Group, conservation of neutrons is given by (Outscatter from group 1 to group 2) = (Thermal Absorption) + (Thermal Leakage) + (Thermal Outscatter)

OUTSCATTER1= ABSORPTION2 + LEAKAGE2 + OUTSCATTER2

0.42214 = 0.40682 + 0.0070065 + 0.0083831 = 0.42221

Good agreement again

4. ANSWER: Suppose there are N identical fuel rods in the core. The specific power comes out about 10% low because the case we are running in CASMO, although it includes one Nth of all the fuel rods, does not include one Nth of the core. We have omitted the non-fuel cells which contain the guide tubes and water, the instrument channel, and also the fractionally very small inter-assemblly gap which contains water. In simple terms, there are 289 cells in a fuel assembly (17x17), but only 264 of them contain fuel rods, so we have omitted roughly 10% of the core from our calculation. Since we specified the power for CASMO per unit volume of core, we are missing about 10% of our power.

One might argue that only the fuel cells should be included in the definition of the core, but it is conventional to include the whole volume within the radial (x-y) envelope of the fuel and axially from the bottom to the top of the active (fueled) length.

Section B.

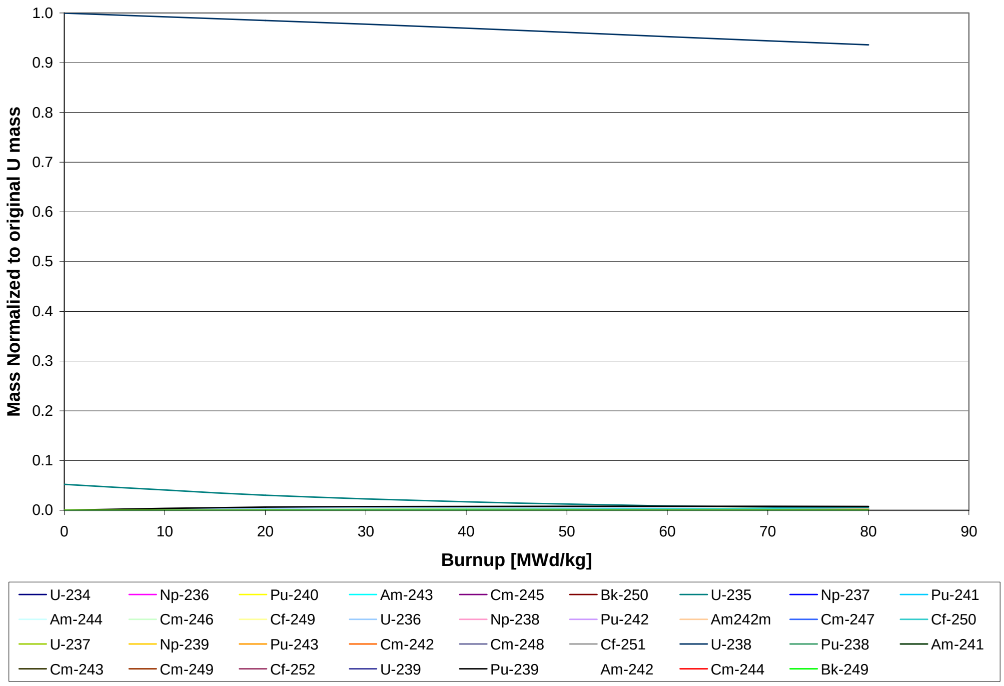
1. The initial uranium density is

$$\rho_U = \frac{N_{25}M_{25}}{N_A} + \frac{N_{28}M_{28}}{N_A} = \frac{\left(1.16321\times10^{21}\ \frac{atoms}{cm^3}\right)\left(235.04\frac{g}{mol}\right) + \left(2.1822\times10^{22}\ \frac{atoms}{cm^3}\right)\left(238.05\frac{g}{mol}\right)}{6.022\times10^{23}\ \frac{atoms}{mol}}$$

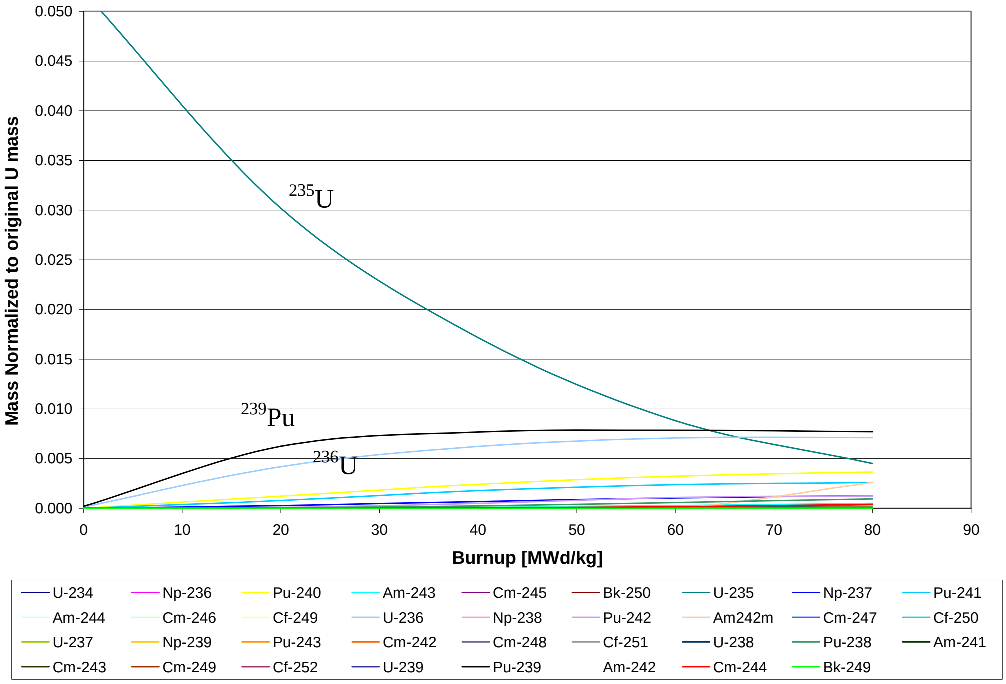
$$\rho_U = 8.626\frac{g}{cm^3}$$

Each number density must be converted to density as shown above for initial uranium and then divided by the initial uranium density.

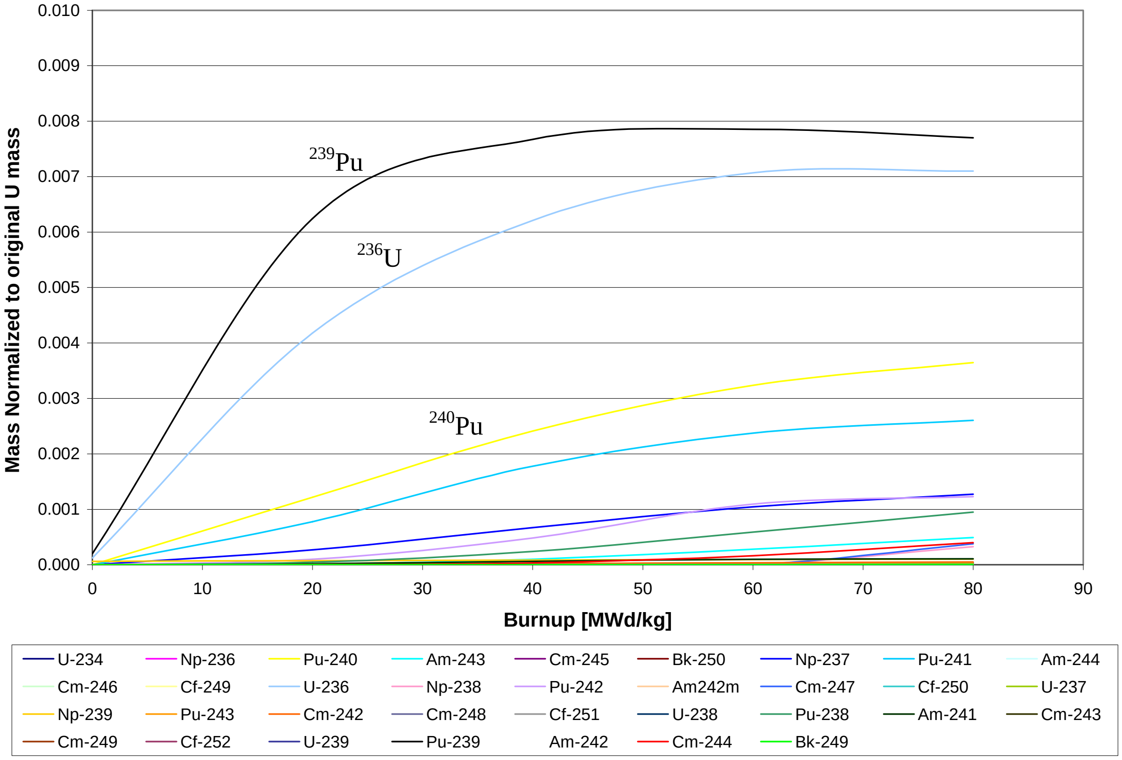
The plot should look like this…



But as you can see, it is difficult to see anything in the plot clearly except ^{238}U (at the top) and ^{235}U (the next one down). The next page shows the same plot but with a y-axis from 0 to 0.1.



And then with a scale that is even smaller



You could also plot this on a semi-log plot to get a better view of all of the actinides at once.

2. We have to multiply the mass density of the nuclides at each burnup step with the value taken from the table for spontaneous fission neutrons per gram second. This gives the number of neutrons emitted by spontaneous fission per gram of fuel-second.

	n/g-s	0 MWd/kg	20 MWd/kg	40 MWd/kg	60 MWd/kg	80 MWd/kg
U-234	5.02E-03	2.03E-15	7.15E-09	6.69E-08	2.34E-07	4.76E-07
Np-236		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Pu-240	1.02E+03	1.11E-02	1.07E+01	2.12E+01	2.85E+01	3.21E+01
Am-243	3.93	2.57E-12	3.50E-04	3.28E-03	9.44E-03	1.67E-02
Cm-245	3.87E+00	1.41E-17	1.85E-06	7.12E-05	4.73E-04	1.32E-03
Bk-250		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
U-235	2.99E-04	1.34E-04	7.79E-05	4.43E-05	2.28E-05	1.16E-05
Np-237	1.14E-04	2.43E-10	2.63E-07	6.59E-07	1.02E-06	1.25E-06
Pu-241	4.94E-02	1.08E-08	3.31E-04	7.57E-04	1.01E-03	1.11E-03
Am-244		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Cm-246	9.45E+06	4.52E-14	1.30E-01	1.18E+01	1.48E+02	6.49E+02
Cf-249		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

U-236	5.49E-03	5.79E-06	1.98E-04	2.94E-04	3.35E-04	3.36E-04
Np-238		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Pu-242	1.72E+03	7.57E-07	1.44E+00	7.18E+00	1.63E+01	1.82E+01
Am242m	1.35E+02	7.42E-12	2.78E-04	1.22E-03	1.95E-03	2.15E-03
Cm-247		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Cf-250		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
U-237		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Np-239		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Pu-243		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Cm-242	2.10E+07	5.54E-06	5.08E+02	3.19E+03	6.80E+03	9.11E+03
Cm-248		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Cf-251		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
U-238	1.36E-02	1.17E-01	1.16E-01	1.14E-01	1.12E-01	1.10E-01
Pu-238	2.59E+03	1.62E-05	1.03E+00	5.39E+00	1.32E+01	2.12E+01
Am-241	1.18E+00	1.08E-10	1.72E-04	6.37E-04	9.93E-04	1.10E-03
Cm-243	1.22E+03	1.67E-13	3.65E-04	4.58E-03	1.43E-02	2.35E-02
Cm-249		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Cf-252	2.34E+12	1.01E-09	2.61E-04	9.65E-01	1.11E+02	1.96E+03
U-239		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Pu-239	2.18E-02	3.77E-05	1.17E-03	1.44E-03	1.48E-03	1.45E-03
Am-242		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Cm-244	1.08E+07	1.98E-08	1.51E+02	3.17E+03	1.54E+04	3.71E+04
Bk-249		0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
TOTAL	n/g fuel-s	**1.29E-01**	**6.92E+02**	**6.45E+03**	**2.25E+04**	**4.90E+04**

3. At 0 burnup, this is the TWO GROUP DATA section

```
* TWO GROUP DATA                                       K-INF XE, NO XE  1.39751      1.39751
  DIFF1   , DIFF2        1.4360E+00   3.5052E-01   M2      ",  "     6.2331E+01  6.2331E+01
  ABS1    , ABS2         1.0953E-02   1.1879E-01   XE2  MIC, MAC     1.2095E+06  4.0155E-19
  NUFISS1 , NUFISS2      9.6800E-03   2.1653E-01   SM2     ",  "     3.8702E+04  1.2849E-20
  REMOV1  , NU           1.3229E-02   2.4451E+00   BOR1    ",  "     8.5733E+00  4.7456E-24
  KAPPA   , XE-YIELD     3.2456E-11   6.5790E-02   BOR2    ",  "     4.0384E+02  2.2353E-22
```

$$k_\infty = \frac{\nu\sum_{f1} + \dfrac{\nu\sum_{f2}\sum_{r1}}{\sum_{a2}}}{\sum_{a1} + \sum_{r1}}$$

If the moderator temperature decreases by 20 degrees F, the macroscopic downscatter cross-section increases by a factor of 1.015. So the new value is

$\sum_{r1,new}$ = (REMOV1)(1.015) = 0.013229(1.015) = 0.013427

So the new k_∞ is given by

$$k_{\infty,new} = \frac{\nu\sum_{f1}+\frac{\nu\sum_{f2}\sum_{r1,new}}{\sum_{a2}}}{\sum_{a1}+\sum_{r1,new}} = \frac{(0.00968)+\frac{(0.21653)(0.013427)}{0.11879}}{0.010953+0.013427} = 1.40093$$

So a 20°F decrease in moderator temperature has increased k_∞ to 1.40093 versus the value shown in CASMO of 1.39812.

The reactivity difference is

$$\Delta\rho = \frac{k_2-1}{k_2} - \frac{k_1-1}{k_1} = \frac{1.40093-1}{1.40093} - \frac{1.39812-1}{1.39812} = 0.001435$$

Divide this by 20°F to get 7.2×10^{-5} /°F

Or **-7.2 pcm/°F**

(Since our increase in reactivity corresponds to a decrease in temperature, we make it negative)

At 50 MWd/kg burnup

```
  * TWO GROUP DATA                                         K-INF XE, NO XE  0.95999     0.98467
    DIFF1   , DIFF2         1.4333E+00   3.4078E-01   M2     ",  "     5.8609E+01  5.8697E+01
    ABS1    , ABS2          1.3000E-02   1.3522E-01   XE2  MIC, MAC    1.2980E+06  4.5416E-03
    NUFISS1 , NUFISS2       6.3840E-03   1.9547E-01   SM2    ",  "     4.0766E+04  1.9842E-03
    REMOV1  , NU            1.2554E-02   2.7146E+00   BOR1   ",  "     8.4891E+00  4.6989E-24
    KAPPA   , XE-YIELD      3.3341E-11   6.9581E-02   BOR2   ",  "     4.1544E+02  2.2995E-22
```

$\sum_{r1,new}$ = (REMOV1)(1.015) = 0.012554(1.015) = 0.012742

$$k_{\infty,new} = \frac{\nu\sum_{f1}+\frac{\nu\sum_{f2}\sum_{r1,new}}{\sum_{a2}}}{\sum_{a1}+\sum_{r1,new}} = \frac{(0.006384)+\frac{(0.19547)(0.012742)}{0.13522}}{0.0130+0.012742} = 0.96354$$

CASMO said that k_∞ was 0.96045, so the reactivity difference is

$$\Delta\rho = \frac{k_2-1}{k_2} - \frac{k_1-1}{k_1} = \frac{0.96354-1}{0.96354} - \frac{0.96045-1}{0.96045} = 0.003339$$

Divide this by 20°F to get 16.7×10^{-5} /°F

Or **-16.7 pcm/°F**

(Since our increase in reactivity corresponds to a decrease in temperature, we make it negative)