Homework 5 Solutions

1. Showing that the time-averaged unexpensed value of the fuel in the core is equal to the ½ the value of a full core of fresh fuel.

We assume that a single batch receives the same amount of burnup (B_c) in each cycle. Since burnup is linear, the time-averaged unexpensed fuel is simply the average of the unexpensed fuel at the beginning and at the end of the cycle.

At the beginning of the cycle, the fresh batch still has n cycles worth of burnup left, or nB_c. The batch that has been in for one cycle already (the next freshest) has one cycle less than the fresh one remaining, or $(n-1)B_c$. This continues for all n batches down to the oldest batch which only has B_c left unexpensed. We will call the unexpensed fuel U for the purpose of this explanation.

$$U_{BOC} = nB_c + (n-1)B_c + (n-2)B_c + \cdots + B_c$$

or

$$U_{BOC} = \sum_{i=1}^{n} [n+1-i]B_c$$

At the end of cycle, every batch has one less cycle of burnup left than it did before the cycle. The end-of-cycle unexpensed fuel is then…

$$U_{EOC} = (n-1)B_c + (n-2)B_c + \cdots + B_c + 0$$

or

$$U_{EOC} = \sum_{i=1}^{n} [n-i]B_c$$

The time-averaged unexpensed fuel is then given by

$$\overline{U} = \frac{U_{BOC} + U_{EOC}}{2} = \frac{\sum_{i=1}^{n}[[n+1-i]B_c] + \sum_{i=1}^{n}[[n-i]B_c]}{2}$$

$$\overline{U} = \frac{B_c}{2}\sum_{i=1}^{n}[2n+1-2i]$$

The sum (S) of an arithmetic series is given by,

$$S = \frac{n}{2}\left(2i + (n-1)d\right)$$

Where
i = the initial term, d = the difference between successive terms and n = the number of terms in the series.

In our case,
$i = 2n - 1$
$n = n$
$d = -2$

Substituting these values into the expression for S, we get

$$S = \frac{n}{2}\left(2i + (n-1)d\right) = \frac{n}{2}\left(2(2n-1) + (n-1)(-2)\right) = n^2$$

So our expression for time averaged unexpensed fuel becomes…

$$\overline{U} = \frac{B_c}{2}\sum_{i=1}^{n}\left[2n + 1 - 2i\right] = \frac{B_c}{2}S$$

$$\overline{U} = \frac{B_c n^2}{2}$$

Now we need the value of unexpensed fuel in a full core of fresh fuel of the same type (enrichment and B_1) as above. If we have n equal batches in an equilibrium core, the burnup that each batch receives before it is discharged is nB_c. Furthermore, there are n batches in a core, so the value of a fresh core is the number of batches (n) times the discharge burnup of each (nB_c), or $\boldsymbol{n^2 B_c}$. This is indeed twice the value calculated above for time-averaged unexpensed fuel.

2.
part a)

18 month cycle

$$\left(40\frac{kW_{th}}{kgU}\right)\left(\frac{1}{1000}\frac{MW}{kW}\right)(17 months)\left(\frac{365}{12}\frac{days}{month}\right)(0.95) = 19.65\frac{MW_{th}d}{kgU}$$

Uprated Case

$$\left(50\frac{kW_{th}}{kgU}\right)\left(\frac{1}{1000}\frac{MW}{kW}\right)(17 months)\left(\frac{365}{12}\frac{days}{months}\right)(0.95) = 24.56\frac{MW_{th}d}{kgU}$$

part b) If the maximum allowable burnup B_d is 50 MWd/kg, what is value of *n*? What is it for B_d=70 MWd/kg?

$B_d = nB_c$

18 month cycle :

$$n = \frac{\left(50\frac{MW_{th}d}{kgU}\right)}{\left(19.65\frac{MW_{th}d}{kgU}\right)} = 2.54$$

$$n = \frac{\left(70\frac{MW_{th}d}{kgU}\right)}{\left(19.65\frac{MW_{th}d}{kgU}\right)} = 3.56$$

Uprated Case

$$n = \frac{\left(50\frac{MW_{th}d}{kgU}\right)}{\left(24.56\frac{MW_{th}d}{kgU}\right)} = 2.03 \qquad n = \frac{\left(70\frac{MW_{th}d}{kgU}\right)}{\left(24.56\frac{MW_{th}d}{kgU}\right)} = 2.85$$

part c) What are the values of B_1 for each case?

$$B_1 = \frac{B_d(n+1)}{2n}$$

18 month cycle

$$B_1 = \frac{\left(50\frac{MW_{th}d}{kgU}\right)(2.54+1)}{2(2.54)} = 34.84\frac{MW_{th}d}{kgU}$$

$$B_1 = \frac{\left(70\frac{MW_{th}d}{kgU}\right)(3.56+1)}{2(3.56)} = 44.83\frac{MW_{th}d}{kgU}$$

Uprated cycle

$$B_1 = \frac{\left(50\frac{MW_{th}d}{kgU}\right)(2.03+1)}{2(2.03)} = 37.32\frac{MW_{th}d}{kgU}$$

$$B_1 = \frac{\left(70\frac{MW_{th}d}{kgU}\right)(2.85+1)}{2(2.85)} = 47.28\frac{MW_{th}d}{kgU}$$

part d) What is the required enrichment, e, for the reactor?

We are given that the required enrichment as a function of B_1 is …

$$e = 0.41 + 0.115B_1 + 0.000239B_1^2$$

18 month cycle

$$e = 0.41 + 0.115\left(34.84\frac{MW_{th}d}{kgU}\right) + 0.000239\left(34.84\frac{MW_{th}d}{kgU}\right)^2 = 4.71\%$$

$$e = 0.41 + 0.115\left(44.83\frac{MW_{th}d}{kgU}\right) + 0.000239\left(44.83\frac{MW_{th}d}{kgU}\right)^2 = 6.05\%$$

Uprated cycle

$$e = 0.41 + 0.115\left(37.32\frac{MW_{th}d}{kgU}\right) + 0.000239\left(37.32\frac{MW_{th}d}{kgU}\right)^2 = 5.03\%$$

$$e = 0.41 + 0.115\left(47.28\frac{MW_{th}d}{kgU}\right) + 0.000239\left(47.28\frac{MW_{th}d}{kgU}\right)^2 = 6.38\%$$

part e) Discuss why this equation might not be as accurate for the uprated core as for the original core.

The uprated core requires additional enrichment. As a result, the assemblies will have additional reactivity at beginning of life. It is likely that this additional reactivity will be suppressed by additional burnable poisons. Thus, the core will have a larger residual reactivity penalty, which will tend to reduce burnup.

Since the uprated core runs at a higher power (and a higher average flux), it will develop higher equilibrium concentrations of fission product poisons, such as Xe and Sm. These larger concentrations impose a larger reactivity penalty, and also reduce burnup.

If the same assembly geometry is maintained, the uprated core will have to run with a higher fuel temperature. This higher fuel temperature will also impose a reactivity penalty, due to Doppler, which will also reduce burnup.

part f) Calculate fuel cost in (mills/kWhe).

These solutions will give details on how to calculate fuel cost for the 18-month cycle with B_d = 50 MW_{th}d/kgU case and the spreadsheet will show only the numbers for all four cases. Everything will be in terms of per kg of uranium loaded into the core.

First, we calculate the amount of feed required in the enrichment process. Yellowcake and conversion costs apply to pre-enriched fuel, so this mass is what we'll use first to get those components of the fuel cost.

$$\frac{F}{P} = \frac{x_p - x_w}{x_f - x_w} = \frac{4.71 - 0.3}{0.711 - 0.3} = 10.730\frac{kg\_feed}{kg\_loaded}$$

This gives kg of natural uranium feed to the enrichment process per kg of uranium loaded into the core.

The cost of this natural uranium (in the form of U_3O_8) is given by…

$$\left(10.730\frac{kg\_feed}{kg\_loaded}\right)\left(20\frac{\$}{lb\_U_3O_8}\right)\left(2.2\frac{lb}{kg}\right)\left(\frac{3(238)+8(16)}{3(238)}\frac{lb\_U_3O_8}{lb\_U}\right) = 556.76\frac{\$}{kg\_loaded}$$

The conversion cost is given by …

$$\left(8\frac{\$}{kg\_feed}\right)\left(10.730\frac{kg\_feed}{kg\_loaded}\right)=85.84\frac{\$}{kg\_loaded}$$

Next we calculate the enrichment costs per kg of uranium loaded into the core.

P = 1 kg *F = 10.73 kg* *W = F – P = 9.73 kg*

$$SWU = P\cdot V(x_p) + W\cdot V(x_w) - F\cdot V(x_f)$$

$$V(x_p)=(2x_p-1)\ln\left(\frac{x_p}{1-x_p}\right)=(2(0.0417)-1)\ln\left(\frac{0.0471}{1-0.0471}\right)=2.72$$

Assume tails enrichment to be 0.3 w/o ^{235}U.

$$V(x_w)=V(0.003)=5.77$$

$$V(x_f)=V(0.00711)=4.89$$

$$SWU = 1(2.72)+9.73(5.77)-10.73(4.89)=6.40\frac{kg\_SWU}{kg\_loaded}$$

Fabrication costs are given per kg of uranium loaded into the core and is equal to 250 $/kg loaded.

Since there is a lead-time associated with all of these cash flows, the discount rate is used to find their value at the start of irradiation. We'll set this to be the present value (PV). Usually, when an interest rate is given per year, it is implied that it is compounded monthly. In this case, it introduces very little error (~1%) to assume that 10% is an effective annual interest rate.

PV of U_3O_8 with 2-year lead-time

$$\left(556.76\frac{\$}{kg\_loaded}\right)(1+0.1)^2 = 673.68\frac{\$}{kg\_loaded}$$

PV of conversion with 1.5-year lead-time

$$\left(85.84\frac{\$}{kg\_loaded}\right)(1+0.1)^{1.5} = 99.03\frac{\$}{kg\_loaded}$$

PV of enrichment (SWU) with 1-year lead-time

$$\left(639.63\frac{\$}{kg\_loaded}\right)(1+0.1)^{1} = 703.59\frac{\$}{kg\_loaded}$$

PV of fabrication with 0.5-year lead-time

$$\left(250\frac{\$}{kg\_loaded}\right)(1+0.1)^{0.5} = 262.20\frac{\$}{kg\_loaded}$$

The sum of these is the total present value of the cash flows at the beginning of irradiation.

= 1,738.50 $/kg U loaded

The total energy in kWhe per kg of uranium is given by the burnup limit and an assumed net plant efficiency of 33%…

$$\left(50\frac{MWth\cdot d}{kg}\right)\left(0.33\frac{MWe\cdot d}{MWth\cdot d}\right)\left(1000\frac{kWe}{MWe}\right)\left(24\frac{h}{d}\right) = 3.96\times10^{5}\ \frac{kWe\cdot h}{kg\_loaded}$$

The fuel will be in the core for a time equal to a batch length times the number of batches…

T_{res}=(2.54)(18 months)=45.72 months

We must now find the monthly amount that one must pay in an annuity over 45.72 months in order to equal the PV of the investment.

The monthly revenue stream equivalent to present value PV is given by …

$$R = PV\left[\frac{i(1+i)^{N}}{(1+i)^{N}-1}\right]$$

Where *N* is equal to the number of compounding periods

Since we have the residence time in months, it is now convenient to convert the yearly interest rate to monthly.

i = 10% / 12 months = 0.00833 per month

So the monthly cash flow is equal to …

$$R = 1738.50\left[\frac{0.00833(1+0.00833)^{45.72}}{((1.0.00833)^{45.72}-1}\right] = 45.88\frac{\$}{month}$$

And the average kWe-h produced per kg per month is the total energy produced by one kg of fuel divided by the number of months it is present in the core.

$$\frac{3.96x10^5\,kWe\cdot h}{45.72months} = 8661.42\frac{kWe\cdot h}{kg\cdot month}$$

The fuel cost is now given by the monthly revenue stream divided by the electric energy produced per month.

$$\frac{\left(45.88\frac{\$}{month}\right)}{\left(8661.42\frac{kWe\cdot h}{month}\right)} = 0.005297\frac{\$}{kWe\cdot h} = 5.30\frac{mills}{kWe\cdot h}$$

Using a spreadsheet to calculate the other three cases gives …

18-month cycle	**50 MWd/kgU**	**70 MWd/kgU**
100% power	5.30 mills/kWe-h	5.26 mills/kWe-h
125% power	5.47 mills/kWe-h	5.30 mills/kWe-h

3. Using the same method as above including accounting for lead times, the incremental cost is shown in the chart below. This shows that there is a steep increase in incremental cost until higher enrichments (above 5 w/o) is reached. There, the cost increase is not as sharp.