PROBLEM SET #8 SOLUTION

(1) Based on our assumptions, the multiplication factor (eigenvalue) of infinite homogeneous U235/moderator mixture without burnable poison at beginning of life is

$$k_\infty\left(t=0\right)=\frac{\nu\Sigma_{\mathrm{f}}^{25}}{\Sigma_{\mathrm{a}}^{25}}=\frac{\nu\sigma_{\mathrm{f}}^{25}}{\sigma_{\mathrm{a}}^{25}}>1 \text{ (supercritical)}$$

After adding some burnable poison, the supercritical clean fuel reactor will reach critical:

$$k_\infty\left(t=0\right)=\frac{\nu\Sigma_{\mathrm{f}}^{25}}{\Sigma_{\mathrm{a}}^{25}+\Sigma_{\mathrm{a}}^{\mathrm{BP}}}=\frac{\nu N^{25}\left(t=0\right)\sigma_{\mathrm{f}}^{25}}{N^{25}\left(t=0\right)\sigma_{\mathrm{a}}^{25}+N^{\mathrm{BP}}\left(t=0\right)\sigma_{\mathrm{a}}^{\mathrm{BP}}}=1$$

where

$N^{25}(t=0)$ is the initial number density of U-235 atom;

$N^{\mathrm{BP}}(t=0)$ is the initial number density of burnable poison atom.

Therefore, the macroscopic cross section of burnable poison shall be

$$\Sigma_{\mathrm{a}}^{\mathrm{BP}}=N^{\mathrm{BP}}\left(t=0\right)\sigma_{\mathrm{a}}^{\mathrm{BP}}=\nu\Sigma_{\mathrm{f}}^{25}-\Sigma_{\mathrm{a}}^{25}=N^{25}\left(t=0\right)\cdot\left(\nu\sigma_{\mathrm{f}}^{25}-\sigma_{\mathrm{a}}^{25}\right).$$

This formula says that, in principle, any material with certain absorption cross section can be added to the reactor serving as an absorber to hold down the initial reactivity as long as the required atom number density can be realized physically. However, only holding down the initial reactivity is not enough for burnable absorbers and the depletion behavior is also important. In this case, the depletion equations for U-235 and burnable poison are as follows

$$\frac{dN^{25}\left(t\right)}{dt}=-N^{25}\left(t\right)\sigma_{\mathrm{a}}^{25}\phi$$

$$\frac{dN^{\mathrm{BP}}\left(t\right)}{dt}=-N^{\mathrm{BP}}\left(t\right)\sigma_{\mathrm{a}}^{\mathrm{BP}}\phi$$

After an operation time of T seconds, the reactor is to be just critical again. The number densities of U235 and burnable absorber can be derived from above equations as

$$N^{25}\left(t=T\right)=N^{25}\left(t=0\right)e^{-\sigma_{\mathrm{a}}^{25}\phi T}$$

$$N^{\mathrm{BP}}\left(t=T\right)=N^{\mathrm{BP}}\left(t=0\right)e^{-\sigma_{\mathrm{a}}^{\mathrm{BP}}\phi T}$$

As the fuel (U-235) depletes, fission products and higher end actinides (such as U235 neutron capture) accumulate. For simplicity, we shall use FP to denote all these atoms and some appropriate absorption cross section is needed. The number density of FP atoms is

$$N^{\mathrm{FP}}\left(t=T\right)=N^{25}\left(t=0\right)\cdot\left(1-e^{-\sigma_{\mathrm{a}}^{25}\phi T}\right)\cdot\frac{2\sigma_{\mathrm{f}}^{25}}{\sigma_{\mathrm{a}}^{25}}$$

Then, the multiplication factor at time T is

$$k_\infty\left(t=T\right)=\frac{\nu N^{25}\left(t=T\right)\sigma_{\mathrm{f}}^{25}}{N^{25}\left(t=T\right)\sigma_{\mathrm{a}}^{25}+N^{\mathrm{FP}}\left(t=T\right)\sigma_{\mathrm{a}}^{\mathrm{FP}}+N^{\mathrm{BP}}\left(t=T\right)\sigma_{\mathrm{a}}^{\mathrm{BP}}}=1$$

Combining the two critical equations, we have

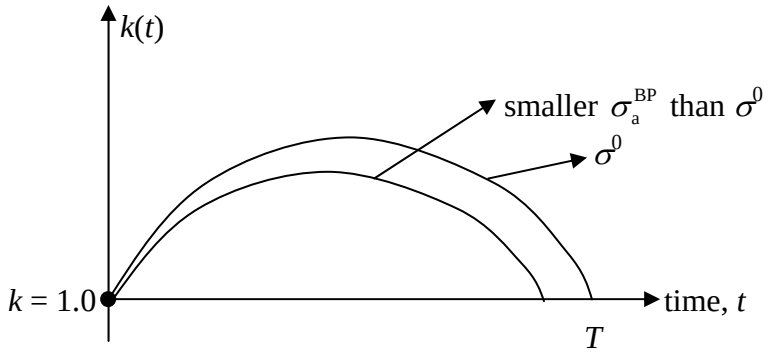
$$\sigma_a^{BP} = -\frac{1}{\phi T}\ln\left[\exp\left(-\sigma_a^{25}\phi T\right) - \frac{\sigma_a^{FP}}{\nu\sigma_f^{25} - \sigma_a^{25}}\left(1-\exp\left(-\sigma_a^{25}\phi T\right)\right)\cdot\frac{2\sigma_f^{25}}{\sigma_a^{25}}\right]$$

Therefore, the burnable poison microscopic cross section has been obtained in order to provide the desired lifetime of the reactor.

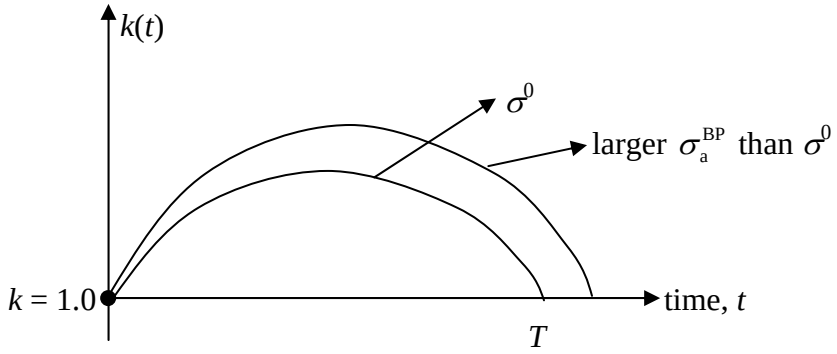
(2) In the case of smaller microscopic cross section than the desired value σ^0 in part (1), more burnable poison has to be loaded initially in order to maintain the criticality. Meanwhile, the depletion rate is proportional to the burnable poison absorption, therefore, this will end up with smaller depletion rate:

$$N^{BP}\left(t=T\right) = N^{BP}\left(t=0\right)e^{-\sigma_a^{BP}\phi T}$$

Compared to part (1), the burnup will be lower due to large residual poison. So, the amount of reactivity controlled by the external control system is plotted as follows from 0 to *T*:



(3) On the contrary, if the absorption cross section of burnable poison is greater than σ^0, the burnable poison requirement will be smaller and depletion rate will be larger. The residue of burnable poison will be smaller. Likewise, the amount of reactivity controlled by the external control system is shown in the following figure:



(4)

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See graph in course text, *The Linear Reactivity Model for Nuclear Fuel Management*.

Figure 3.3 Piecewise linear reactivity histories with burnable poison.
(from *Linear Reactivity Model for Nuclear Fuel Management*)

From traces of burnable poison A, B, and C, conceptually, $\sigma_{\mathrm{a}}^{\mathrm{BP}}(\mathrm{A}) < \sigma_{\mathrm{a}}^{\mathrm{BP}}(\mathrm{B}) < \sigma_{\mathrm{a}}^{\mathrm{BP}}(\mathrm{C})$ if about same amount of burnable poison is used.
Due to burnable poison loading, the unequal power sharing (first order) gives

$$B_{\mathrm{d}} = \frac{1}{1+\varepsilon} B_{\mathrm{e}} = \frac{1}{1+\varepsilon}\left(\frac{2n}{n+1} B_1\right).$$

Trace (A), $\quad \varepsilon = -(\theta\rho_0)\dfrac{n-1}{n(n+1)^2}$;

Trace (B), $\quad \varepsilon = -(\theta\rho_0)\dfrac{n-1}{2n(n+1)}$;

Trace (C), $\quad \varepsilon = -(\theta\rho_0)\dfrac{n-1}{(n+1)^2}$.

Therefore, Trace (A) has the most uniform power history. Trace (C) has highest discharge burnup. And Trace (B) is somewhere in the middle of the two extreme cases. Trace (C), in theory, can control the whole core reactivity entirely by burnable poison. Least external control is required. However, there are other considerations in PWRs. The activity swing will be largest for Trace (C). And the control mechanism doesn't favor fresh assemblies. (Control rod worth is smaller initially.) The actual assembly with such heavy poison loading will suffer from the reduction of power pins. The peak pin in the assembly will be questioned. Current industry practice all uses Trace (A) which has flatter power shape. The detrimental residual poison nearly cancels with unequal power sharing benefits. Thus, it is easy to analyses especially in the transients.