**22.351 Systems Analysis of the Nuclear Fuel Cycle**
**Fall 2005**

**Lab #4: MCNP PWR Pin Cell Model**
**Solution**

**a) U235 and U238 fission rates**

Number densities of U235 and U238 are given in MCNP input file on material definition cards

```
c     4.5 w/o UO2  [in atoms/b-cm]
m11     8016.50c 4.64149E-02
        92234.86c 8.49269E-06
        92235.54c 1.05705E-03
        92238.86c 2.21413E-02
```

Using keff=1.382 and loss to fission f-loss=5.6032E-01 from MCNP results, one can obtain average number of neutrons per fission

$$\nu = k_{eff} \frac{1}{f - loss} = \frac{1.382}{0.56032} = 2.47\,.$$

Next the normalization constant needs to be obtained. First, power generated in a modeled cell is

$$P = \frac{104.5kW}{liter} \frac{1liter}{1000cc} \frac{1000W}{kW} (1.26cm)^2 (4cm) = 663.6W \quad ,$$

where the dimensions to get the volume of the modeled cell are taken from MCNP input. The normalization constant is then

$$C = \frac{P\nu}{E_f k_{eff}} = \frac{663.6J/s\,2.47n/fiss}{200MeV/fiss\,(1.602x10^{-13}\,J/MeV)1.382} = 3.7x10^{13}\,n/s$$

The tally reaction rates are in the units of barn-n/cm$^2$ (flux multiplied by microscopic cross section). Finally, using the tallies from MCNP results, the above normalization constant and uranium number densities reaction rates can be obtained as

$$R\left[\frac{fissions}{s}\right] = tally\left[\frac{barn}{cm^2 n}\right] C\left[\frac{n}{s}\right] N\left[\frac{at}{barn-cm}\right] V_{fuel}[cm^3]\text{. Thus}$$

U235 fission – thermal: 186.4 * 3.7E13*1.057E-3*2.835 = 2.01E13 fissions/s
U235 fission – epithermal: 51.1 * 3.7E13*1.057E-3*2.835 = 5.7E12 fissions/s
U238 capture – thermal: 0.98 * 3.7E13*2.214E-2*2.835 = 2.28E12 captures/s
U238 capture – epithermal: 4.89 * 3.7E13*2.214E-2*2.835 = 1.13E13 captures /s

Spectrum indices

$$C^* = \frac{\text{U-238 captures}}{\text{U-235 fissions}} = \frac{(0.803+4)x10^{12}}{(7.29+2)x10^{12}} = 0.517$$

$$\delta_{25} = \frac{\text{epithermal U-235 fissions}}{\text{thermal U-235 fissions}} = \frac{2x10^{12}}{7.29x10^{12}} = 0.274$$

$$\rho_{28} = \frac{\text{epithermal U-238 captures}}{\text{thermal U-238 captures}} = \frac{4x10^{12}}{8.03x10^{11}} = 4.98$$

**b) Neutron spectrum**

Neutron spectrum normalized to total flux is plotted in Figure 1.

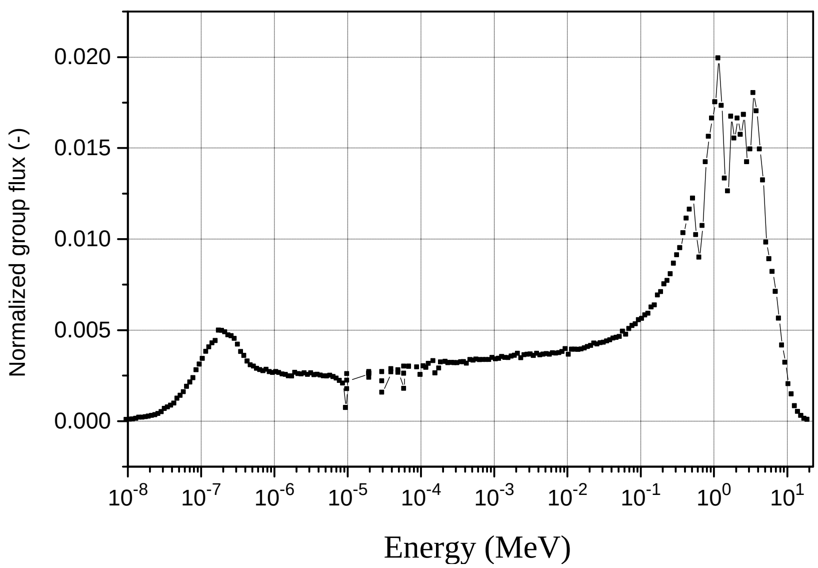


Figure 1 Neutron spectrum in PWR cell

From MCNP output, one can sum up thermal flux values in the fuel from all groups up to 0.625eV to yield 0.734n/cm$^2$-fsn. Note that this number can be later checked directly from tally in task c). Epithermal flux is thus total – thermal = 6.98-0.734=6.246 n/cm$^2$-fsn. Thus the ratio $\frac{\phi_2}{\phi_1} = \frac{0.734}{6.246} = 0.117$ .

Physical meaning of $\frac{\phi_2}{\phi_1} \approx \frac{H/HM}{X}$ : Increasing H/HM increases moderation, hence more neutrons are slowed down to thermal flux region. At the same time, less HM means less captures in U238 increasing resonance escape probability further increasing population of neutron in thermal region. On the other hand, higher enrichment increases number of fissions, i.e., the population of neutrons at high energies, shifting spectrum to the right.

### c) flux ratio in fuel and moderator

Tallies added to MCNP input to obtain neutron fluxes in moderator and fuel are shown below:

```
c  Flux in moderator
fc204 Neutron flux in coolant
f204:n    1000
sd204     3.515113
e204      0.625E-6 20.0 T
c
c  Flux in fuel
fc304 Neutron flux in fuel
f304:n    1100
sd304     2.10829
e304      0.625E-6 20.0 T
```

The results from MCNP are given in Table 1. Since we are interested in relative values, normalization constant to obtain absolute flux values is not applied.

Table 1 Epithermal and thermal flux (per source neutron) in the fuel and moderator.

| | Thermal, $\phi_2$ | Epithermal, $\phi_1$ | $\phi_2/\phi_1$ |
|---|---|---|---|
| Moderator | 8.51E-01 | 6.15E+00 | 1.38E-01 |
| Fuel | 7.36E-01 | 6.25E+00 | 1.18E-01 |
| Moder/fuel | 1.16E+00 | 9.84E-01 | |

Thermal flux in the moderator is larger than thermal flux in the fuel because thermal neutrons are born in the moderator and travel to fuel. On the other hand, fast neutrons are born in the fuel and proceed to moderator, hence epithermal flux is larger in the fuel.