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Lecture #7

Fuel Cycle Economics

Part 1 General Principles

- Time value of money: continuously compounded model and simple linearized version which we will use
- Busbar cost of electric energy as the sum of capital costs, operating and maintenance costs, and fuel costs
- Overall system costs and lifetime levelized costs including externalities

Part II Applied to a Batch of Nuclear Fuel

- Mass flow and SWU requirements calculations
- Sample calculation discounted to the mid-point of in-core irradiation
- cost per unit energy determined using projected burnup
- non-fuel factors which determine fuel management strategy: effect on plant capacity factor, cost of replacement energy, fuel and plant performance constraints

5.2 Time Value of Money Mechanics

The goal here is quantification of the algorithm suggested by the English proverb, "Time is money."

5.2.1 Basic Aspects*

Money deposited in a bank earns interest, and similarly for large capital-intensive endeavors, money is invested in a business via purchase of bonds or stock to earn a rate of return. Conversely borrowers pay interest or dividends as their cost of money. When compounded continuously, the future worth F of a present amount P after t years at the rate i per year is just:

$$F = P\,e^{it} \tag{5-1}$$

or equivalently the present worth now of a discrete cash flow t years in the future is:

$$P = Fe^{-it} \tag{5-2}$$

Note that i is here assumed to remain constant over the period of concern; in real life situations it may well not and thereby constitute a source of complexity and uncertainty.

Thus, for example, one dollar of cost or revenue realized 40 years in the future has a present worth of only 1.83 cents today at a discount rate, i, of 10 % per year. This explains why the far future has so little influence on conventional business planning.

Note that compounding continuously instead of at discrete intervals (for example, yearly) causes no loss in generality because of the following equivalence between continuous and discrete rates.

$$i_c = l_n\left(1 + \frac{i_d}{n}\right)^n \tag{5-3}$$

in which

i_d= discrete compounding rate per year, %/yr + 100 (hence i_d/n per period as per conventional terminology)

n = number of compounding periods per year.

Thus if i_d is 10% per year compounded annually (72 = 1), ***c*** = 9.53 %/yr.

Note that compounding at i/n for n periods is <u>not</u> the same as i for one period!

Another reason for preferring to work in terms of continuous compounding is that the present worth of any cash flow history is directly related to its Laplace transform (see Appendix 5.A) and Laplace transforms are widely tabulated.

In more ambitious texts on engineering economics it is customary to consider all four transaction combinations: discrete compounding and discrete cash flows; discrete compounding and continuous cash flows; continuous compounding and discrete cash flows; and finally, continuous compounding and continuous cash flows. Moreover, analytic expressions for the summed worth of common cash flow progressions-e .g., those representable by arithmetic or geometric series-are tabulated. For present purposes this degree of complexification is unnecessary, but Table 5.2 summarizes how some of these prescriptions are arrived at.

In the present chapter in the interest of simplicity we have succumbed to the temptation to not distinguish among "interest rate", "discount rate", "cost of money" and "rate of return."

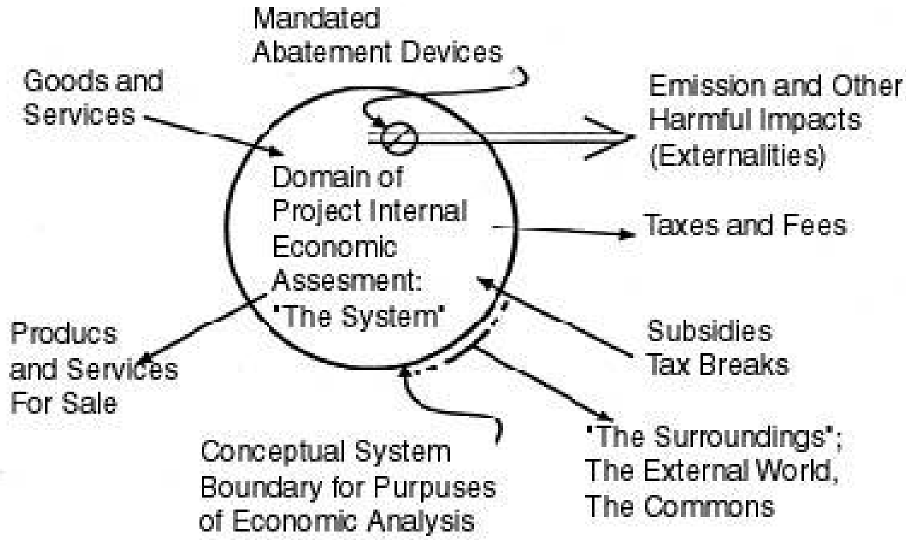


Figure 5.1. Schematic or our Point of View for Economic Analyses. (See Chapter 4 for details on adverse environmental effects.)

Courtesy of Tester, Jennifer W., Elisabeth M. Drake, Michael J. Driscoll, Michael W. Golay, and William A. Peters. *Sustainable Energy*. Boston, MA: MIT Press, 2005, 14 pages from Chapter 5. ISBN: 0262201534. Used with permission.

Table *5.2*

Present Worth of a Uniform Discrete Series of Cash Flows

The familiar geometric series encountered in algebra texts has a first term *A* at the beginning of the first interval and successive terms weighted by the ratio *R* to the power (n - 1).

It has the sum

$$S = A + AR + \ldots Ar^{n-1} = A\left[\frac{1-R^n}{1-R}\right]$$

In engineering economics we are often interested in the present worth of a series of uniform discrete cash flows starting at the end of the first interval.

Hence

$$P = S - A$$

$$N = n - 1$$

$$R = \begin{cases} e^{-1} & \text{for continuous compounding} \\ (1+x)^{-1} & \text{for periodic compounding at the end of interval} \end{cases}$$

Therefore

$$P = A\left[\frac{e^{Ni}-1}{(e^i-1)e^{ni}}\right]; \quad P = A\left[\frac{(1+x)^N - 1}{x(1+x)^N}\right]$$

which also satisfies the equivalence $i = \ln(1 + x)$.

Note that the continuous compounding result is for discrete cash flows; for continuous cash flows: of $\bar{A}$\$/yr starting at time zero one has the substitutions

$$A + \frac{\bar{A}}{i}(e^i - 1)$$

$$A = \bar{A}\left[\frac{z}{\ln(1+x)}\right]$$

Details on derivations and applications of the above expressions are found in most engineering economics textbooks, such as Ref. S-1.

Courtesy of Tester, Jennifer W., Elisabeth M. Drake, Michael J. Driscoll, Michael W. Golay, and William A. Peters. *Sustainable Energy*. Boston, MA: MIT Press, 2005, 14 pages from Chapter 5. ISBN: 0262201534. Used with permission.

Equation 5-2 allows us to consistently correct for the time value of money by expressing all costs in terms of their present worth and then computing an overall sum, ***Pt*** following which a levelized (i.e. uniform) annual rate of expenditure ***A*** $Iyr can be calculated by equating present worth over a specified T year time horizon.

$$\int_0^T \bar{A} e^{-it} dt = P_T \qquad (5\text{-}4)$$

Thus

$$\bar{A} = \left(\frac{i}{1 - e^{-iT}} \right) P_T \qquad (5\text{-}5)$$

In the limit oflargeT

$$\bar{A} \approx iP_T \qquad (5\text{-}6)$$

When P_T is an expenditure actually made at time zero (e.g., purchase of a machine) the rate A/P_T is often called the capital recovery factor or carrying charge rate. This is more familiar to us all as the rate of uniform payment on a home mortgage or car loan.

Lifetime levelized cost is a useful construct because it permits a single valued numerical comparison of alternatives having vastly different cash flow histories. For example, for generating electricity one can compare cheap machines burning expensive fuel-gas turbines, to expensive machines burning cheap fuel-nuclear fission reactors. These two cases highlight a classical tradeoff between up-front capital costs against long term continuing expenses: a task faced in virtually al] energy related case studies. To further complicate matters, the most cost-effective overall system may contain a mix of alternatives: e.g., nuclear base load plus gas-fired turbine peaking units in the case of electric power generation.

5.2.2 Application to a Typical Cash Flow Scenario

Table ***5.3*** shows the result of applying present worth concepts and levelization to an appropriate example - the generation of central station electricity (see Section ***5.2.3*** for derivations). The levelized unit cost of product, in cents per kilowatt hour, at the busbar (plant/transmission line interface) is obtained by equating levelized revenue to levelized expenditures for capital cost, operating and maintenance costs and fuel costs. The following additional embellishments are introduced.

- The cost of money ("interest paid on borrowed funds") is given by a weighted sum of specified returns on bonds and anticipated returns on stocks.
- The carrying charge rate further considers that bond interest is tax deductible (which may or many not be the case everywhere and for all time).
- Future expenses are escalated at rate y per year.
- The plant capital cost at time zero is computed from an overnight cost (i.e., hypothetical instantaneous construction), corrected for escalation and interest paid on borrowed funds over a construction period starting C years before operation.

Table 5.3

Lifetime-Levelized Busbar Cost of Electrical Energy* eb cents per kilowatt hour (0.1 times mills per kilowatt hour) is the sum of:

CAITAL- RELA TED COSTS:

$$\frac{100\phi}{8766\,L}\left(\frac{I}{K}\right)_c\left[1+\frac{x+y}{2}\right]^c$$

PLUS OPERATING AND MAINTENANCE COSTS:

$$+\frac{100}{8766\,L}\left(\frac{O}{K}\right)\left[1+\frac{yT}{2}\right]$$

PLUS FUEL COSTS:

$$+\begin{cases}\text{Nuclear} & \left[\dfrac{100}{24}\dfrac{F_0}{\eta B}\right]\left[1+\dfrac{yT}{2}\right]\\ \text{or} & \\ \text{Fossil} & \left[\dfrac{0.34 f_0}{\eta}\right]\left[1+\dfrac{yT}{2}\right]\end{cases}$$

where

	Typical LWR Value
L = plant capacity factor: actual energy output + energy if always at 100% rated power	0.80
ϕ = annual fixed charge rate (i.e., effective "mortgage" rate)	0.15/yr
= *x/(1* - t) where x is the discount rate, and r is the tax fraction (0.4)	
x = (1 - t)b $_l r_b$ + (1 - b)r_s, in which b is the fraction of capital raised selling bonds (debt fraction), and r_b is the annualized rate of return on bonds, while r_s is the return on stock (equity)	0.09/yr
$\left(\frac{1}{k}\right)_c$ = overnight specific capital cost of plant, as of the start of construction, dollars per kilowatt: cost if it could be constructed instantaneously C years before startup in dollars without inflation or escalation	1400$/KW
y = annual rate of monetary inflation (or price escalation, if different)	0.04/yr
c = time required to construct plant, years	5yrs
T = prescribed useful life of plant, years	30 yrs
$\left(\frac{O}{K}\right)_0$ = specific operating and maintenance cost as of start of operation, dollars per kilowatt per year	95$/KWe yr
η = plant thermodynamic efficiency, net kilowatts electricity produced per kilowatt of thermal energy consumed	0.33
F_0 = net unit cost of nuclear fuel, first steady-state reload batch, dollars per kilogram of uranium; including financing and waste disposal charges, as of start of plant operation	2000$/kg

B = burn up of discharged nuclear fuel, megawatt days per metric ton 45,000

f_0 = fossil fuel costs, at start of operation, cents per million British thermal units = (approximately)
dollars per barrel times 16 for residual oil;
dollars per ton times 4 for steam coal; cents per thousand standard cubic feet for natural gas;
zero for solar or fusion energy

Thus for a light water reactor (LWR) nuclear power plant, using the representative values cited above:

		Cap	+	O&M	+	Fuel		
e_b	=	4.1	+	2.2	+	0.9	=	7.2cents/kWhre

*Note that these costs represent only the cost of generating the electricity, i.e., excluding transmission and distribution. These costs are lifetime-average (i.e., "levelized") costs for a new plant starting operations today.

Three of the parameters listed in Table 5.3 deserve further comment:

The capacity factor L may vary widely among options. While all plants require maintenance outages and experience forced outages due to unexpected failures, nuclear units must typically shut down approximately one month for every 18-24 months of service for refueling. Renewable options are constrained by the diurnal and intermittent nature of sun and wind: wind turbine capacity factors are typically about 25%, and photovoltaic units as low as 15%. The capacity factor of typical auto engines is only on the order of 2%.

The rate of escalation, y, on fuel costs is in principle the sum of a component for monetary inflation p]us an allowance for scarcity-related price increases. This latter term is usually taken as a positive quantity. However, over the past century, in constant dollars, the long term average price of fuels and other mineral commodities has actually decreased (Chapter 2). In other words, economy of scale and learning curve savings also apply to resource extraction. Thus, a third, (and negative) term should really be added to allow for improvements in resource extraction and processing science and technology. This is almost never done in practice, which gives rise to false hope that renewable technologies need only outwait their fossil competition. Conversely, however, the fact that ingenuity has so-far outpaced scarcity is no guarantee of future performance.

The useful life of the plant, *T*, is rather nebulous. Power stations are refurbished as time goes by, hence there are also annual capital expenditures, which we have neglected in the model of Table 5.3. For a typical US nuclear unit in the late 1990s, these capital addition costs were on the order of 10% of the total production costs. But neither is credit taken for the fact that because of these renovations operation significantly longer than the original design life is typical. Furthermore, the net rate of actual physical deterioration is only vaguely congruent with the depreciation schedule adopted for tax purposes in the determination of carrying charges.,

The second and third terms in Table 5.3 constitute a "production cost," the sum of operating and maintenance costs plus fuel costs, which are important because they determine the rank of plants in the to-be-operated queue (capital-related costs are "sunk costs" and are in some sense irrelevant in the here and now). In 1998 US nuclear/coal/gas oil-fueled plants had production costs of 2.13/2.07/3.30/3.24 cents per kWh, respectively.

The prescription of Table 5.3 has been applied to the production of electric energy (kilowatt hours), but the general approach is easily transformed for other applications: for example building a factory' to produce automobiles and estimating the levelized cost per car, or the cost of facilities for extracting fossil fuels. Note moreover two contrasting points of view:

(a) One may estimate the likely cost of money (or the allowed rate in a highly regulated industry-as formerly in the US electric sector) and calculate the resulting cost of product, directly from the tabulated equation

(b) One may instead estimate the competitive free market allowable unit cost of product and back-calculate the achievable rate of return to see if it is attractive: the situation for most industries and increasingly the case in a deregulated electric power sector, both in the US and elsewhere.

Unlike our oversimplified example, the second approach will not in general permit a direct analytic solution.

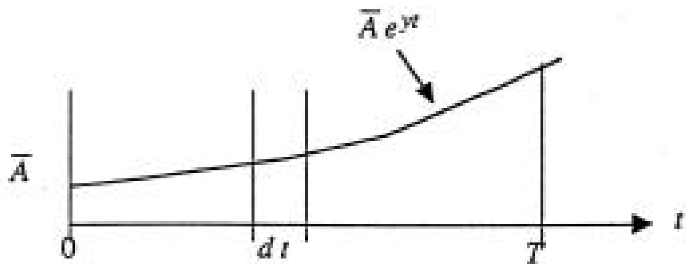
It is appropriate at this point to point out that deregulation will increase perceived investment risk, hence increase an investor's expected rate of return and thus the cost of money to the utility. This in turn will make high capital cost options less attractive than formerly. Because renewable options are generally capital intensive, it becomes doubly important to fully credit them with savings on externalities, a topic to be addressed in Section 5.7 and other chapters of this text.

Before proceeding to further interpretation and application of the prescription in Table 5.3, a brief digression to sketch out how it was derived will be instructive.

5.2.3 Derivation of Relations

The prescription set forth in Table 5-3 can be derived from no more than the preceding fundamentals plus some judicious approximations. First of all, the annualized capital cost term readily follows from Eq. 5-6, with *i* replaced by f. Other ingredients require a bit more development.

Consider a uniform annual rate of expenditure, $\bar{A}$ \$/yr, but escalated at y/yr over a period ***T,*** discounted at x/yr.



Levelizing to find the equivalent annual rate $\bar{A}_L$

$$\int_0^T \bar{A}_L\, e^{-xt}\,dt = \int_0^T \bar{A} e^{yt} e^{-xt}\,dt \qquad (5\text{-}7)$$

Then

$$\frac{\bar{A}_L}{\bar{A}} = \frac{\int_0^T e^{-(x-y)t}\,dt}{\int_0^T e^{xt}\,dt} = \left(\frac{x}{x-y}\right)\left(\frac{1-e^{-(x-y)T}}{1-e^{-xT}}\right) \qquad (5\text{-}8)$$

Expand the exponentials as Taylor series and retain terms through second order, which yields to first order:

$$\frac{\bar{A}_L}{\bar{A}} = \frac{1-\frac{(x-y)}{2}T+\ldots}{1-\frac{x}{2}T+\ldots} \approx 1+\frac{y}{2}T+\ldots \qquad (5\text{-}9)$$

which is the multiplier used on today's O&M and fuel costs in Table 5-3.

Next let's determine the capitalized cost of construction. The same diagram holds, where now A ***T*** = overnight cost of construction as of the start of construction (not plant startup as preferred by some). We want the future worth, ***F,*** as of the date of completion (startup) namely:

$$F = e^{xT}\int_0^T \bar{A}\, e^{yt} e^{-xt}\, dt \qquad (5\text{-}10)$$

Integration yields

$$F = \left(\bar{A}\,T\right) e^{xT} \left[\frac{1 - e^{yT} e^{-xT}}{(x-y)T}\right] \qquad (5.11)$$

and series expansion gives to first order

$$F = \left(\bar{A}\,T\right)\left(1 - \frac{(x-y)}{2}\,T\right)[1 + xT]$$

or (5.12)

$$F = \left(\bar{A}\,T\right)\left(1 + \frac{x+y}{2}\,T\right) = \left(\bar{A}\,T\right)\left(1 + \frac{x+y}{2}\right)^{T},$$

which is the desired relation.

For the examples in Table 5-3, we can compare the "exact" exponential relations (Eqs. 5.8 and 5.11) to their linearized versions (Eqs. 5.9 and 5.12):

350 Let ***T***= 30 yrs, y = 0.04/yr, and x = 0.09/yr
Then exact = 1.50

Whereas $\left(1 + \frac{yT}{2}\right)$ = 1.6; 6.7% high.

351 Let ***T***= ***C*** = 5 yrs, y = 0.04/yr, x = 0.09/yr
Then exact = 1.39

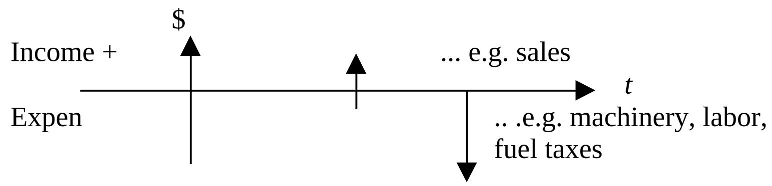
Whereas $\left(1 + \frac{x+y}{2}\right)^{C}$ = 1.37; 1.4% low.

For present purposes, the above accuracy will in general be adequate, particularly in view of the uncertainty involved: see Section 5.6

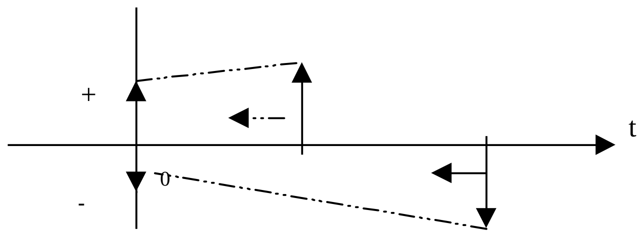
The procedure applied in this section is a specific example of the more systematic general approach outlined in Fig. 5.2, which if religiously applied should suffice to solve any of a wide range of problems.

This introduction to engineering economics has been brutally brief, covering as much in scope as is customary in a one term full subject. The following worked out example is offered as a token palliative.

(1) Draw a cash flow diagram showing all cost vectors as a function of time.

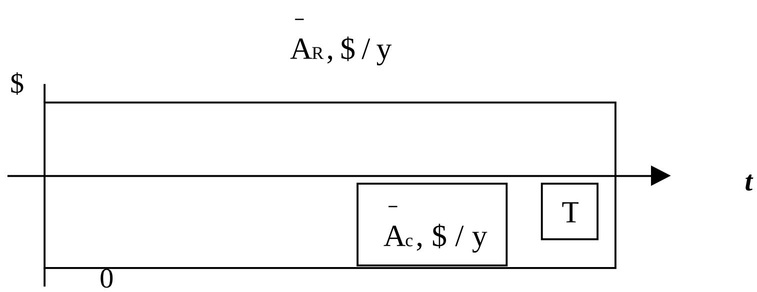


(2) Bring all cash flows back to time zero using $P = F\,e^{-it}$ (Eq. *5.2)* and (separately) sum revenues and expenses.



(3) Using Eq. 5.5, redistribute all *j* cash flows, $\sum_j Pj$ uniformly over the appropriate time horizon, *T*, with

$$\bar{A} = \frac{i \sum_j Pj}{\left(1 - e^{-iT}\right)}$$



(4) Equate levelized revenues and costs to calculate either

· cost of product at a specified rate of return

or

- rate of return for a projected viable price for the product

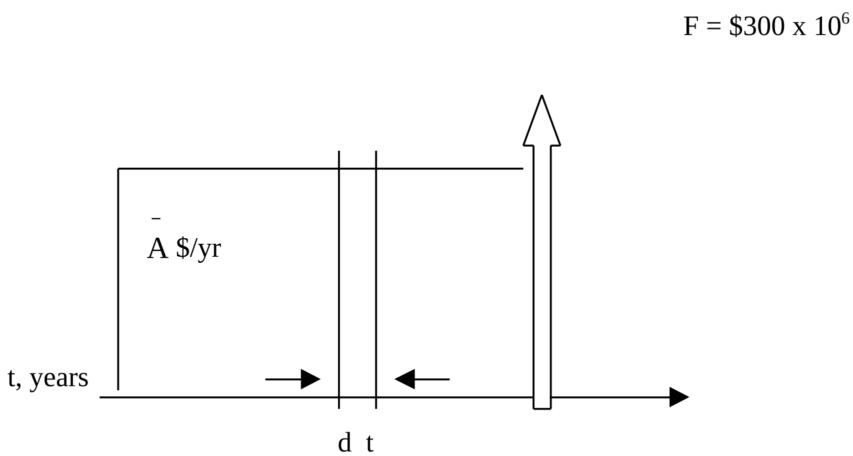
Figure 5.2. How to Calculate Lifetime Levelized Cost and/or Rate of Return

SAMPLE PROBLEM

Question: In some jurisdictions, owners must contribute to a separate interest earning account-a so-called sinking fund- to provide a future amount sufficient to decommission a nuclear power plant at the end of its useful life. Working in constant dollars, calculate the uniform rate of contributions in dollars per year at a real interest rate of 5% per year, which will total 300 million in today's dollars 30 years from now.

Specimen Solution

We have the cash flow time diagram:



Following the algorithm recommended in Fig. 5.2, we equate present worths (in millions of dollars):

$$\int_0^{30} \bar{A} e^{-0.05t}\, dt = 300e^{-0.05(30)}$$

carrying out the integration gives:

$$\frac{\bar{A}}{0.05}\left(1 - e^{0.05(30)}\right) = 300e^{-1.5}$$

or

$$\bar{A} = \frac{(300)(0.05)}{(e^{1.5} - 1)} = \underline{4.31 \text{ million dollars per year}}$$

which is only a few percent of the annual carrying charge rate on a plant costing on the order of two billion dollars.

Question: Suppose instead that the utility was required to deposit a sufficient amount as a single up-front payment.

Answer: We merely need to compute the present worth of 300 M$ @ 5% yr for 30 years.

$$P = 300 \cdot e^{-005(30)}$$

= <u>66.94 million dollars</u> (in today's, i.e. constant, dollars)

Question: What would the actual fund accrue in $t = 30$ year dollars if the rate of inflation anticipated is 3% per year.

Answer: The market interest rate would then be 5 + 3 = 8% *yr* and the future worth:

$$F = 66.94\ e^{0.08(30)}$$

$$= \underline{738 \text{ million dollars}}$$

Hence, with a large enough discount rate and a long enough plant lifetime we could easily fulfill the critics alarums that "nuclear plant decommissioning costs will exceed their initial cost of construction."

5.2.4 Pitfalls, Errors and Ambiguities

The bare-bones mathematics of engineering economics, just presented, is deceptively simple. However, there are many errors, both conceptual and specific, which can seriously compromise a too-casual analysis. The problem itself may be incorrectly posed, financial parameters incorrectly specified. system boundaries badly drawn and ancillary cash flows omitted.

At the strategic level it is important to note that the overall objective of an investor is to have all funds invested and to maximize net present worth, which may not necessarily favor an investment having the highest individual rate of return if it is small and precludes larger projects having a slightly lesser rate of return. Moreover "doing nothing" properly defined, is always an option. One can always assume a universally available option for investment at a minimum alternative rate of return (e.g., treasury' bonds, etc.). Retaining (perhaps refurbishing) a current asset may be properly considered by assuming" its fictitious sale to a hypothetical outsider at its then-current market value (not book depreciation value) who then operates it as a competitor to the other new options being considered. One must be careful to ignore sunk costs, but not tax implications of premature retirement. Finally, when it comes to interpretation of output, optimality is <u>not</u> when total costs equal benefits, but when the next increment of cost is just offset by the incremental benefit. This situation often arises with regard to the purchase of safety equipment or other abatement hardware, which incur additional health and environmental costs in their own manufacture, thus leading to diminishing returns as the standard of achievement of abatement is tightened.

Needless to say, financial parameters such as the cost of money must be selected appropriately. In particular, the analyst must be careful not to mix current and constant dollar approaches (See section 5.3): i.e., market (so-called "nominal") discount rates and escalated costs vs. deflated (so-called "real") discount rates and costs as of a reference year. Note that the market rate approach will yield a unit cost of product that is hi gher than today's market price, and therefore appear somewhat unreal to the uninitiated. And, of course, the actual year-by-year cost of product is not constant and only by chance equal to our levelized value. In addition, when comparing government-owned activities to private enterprise, note that the former usually have the considerable advantage of paying no taxes and of borrowing money at very low rates (i.e., a very low risk premium). Furthermore, tax regulations can have a significant effect: e.g., by allowing accelerated depreciation compared to actual physical deterioration. Finally, the cost of money is not the same for all aspiring competitors. Interest charged can be considered as the sum of three components: a basic growth rate, an inflation increment, and an allowance for risk. The inflation component is an allowance for <u>expected</u> monetary inflation in the future, while the risk term accounts for the <u>perceived</u> borrower's likeliness of underperformance or default in the future. Thus, these latter components may be associated with considerable uncertainty as well as bias.

Some care must be taken in constructing system boundaries in both space and time. Alternatives must be compared over the same time span. This may require sequential replication of some options, or sale at market value of others to make the future time horizons equal. Also, over a long time span neither the cost of money nor rates of inflation and cost escalation are likely to remain constant. In like vein, where beyond the busbar differences are involved (e.g., siting location) the added costs of transmission and distribution should be accounted for. In other words, all affected elements in the overall system in which the project is embedded must be evaluated.

Even when the "system" is appropriately isolated from its surrounds, it is all too easy to overlook certain cash flows. For example, by-product credits should be recognized. This is hard to overlook for a cogeneration unit, which markets both electricity and hot water, but in other instances, more creative uses may be available, such as using otherwise waste beat for climate control in greenhouses and aquaculture. Even coal-fired plants can market fly-ash as a concrete additive and scrubber sludge as feedstock for gypsum wallboard manufacture. Likewise, in real-life situations, the effect of subsidies and tax breaks on the project's cost of product must be credited. For example, wind power currently enjoys a 1.5c/kWhre federal production credit, and from time-to time, other favored technologies are awarded investment tax credits. However, subsidies *per se* are commonly attacked by free market advocates whether they may appear in policy-driven initiatives in developed countries or compassion-motivated programs in developing countries. Furthermore, there are always "cutoff" and "double-counting" questions. For example, is damage covered by paid-for insurance also reprised as an added externality? Are all options fairly burdened with future decommissioning costs? Some sectors, for example nuclear, are often required to set aside funds for that purpose, while for others this expense is left for posterity to deal with. Finally, when evaluating energy generation costs one must consider that different plant capacity factors imply unequal service. Replacement power and standby service costs should be considered.

One lesson to be drawn from this long litany of disclaimers is that economic comparisons involving the compilation of results from independent analyses in the literature are fraught with considerable modeling uncertainties, not to mention outright errors of commission or omission. Side-by-side contemporaneous comparisons of alternatives by the same experienced analyst are preferable. The Electric Power Research Institute (EPRI) has published a *Technical Assessment Guide* which describes consensus good practice for such comparisons and which is periodically updated.

5.3 Current vs. Constant-Dollar Comparisons

To overgeneralize just a bit, business people and engineers tend to work in then-current (i.e., market quoted) dollars and "nominal" interest rates, while economists prefer constant dollars and "real" interest rates (i.e., with the effect of monetary inflation removed). Constant dollars are preferable for international comparisons because the anticipated rates of monetary inflation differ from country-to-country (currently about 3%/yr in the US, but, for example, 200%/yr in Latin American in 1992-3!). In the present chapter the market (nominal) approach is applied except where explicitly noted otherwise.

While it is commonly put forth that these two approaches are compatible in that they correctly rank alternatives in the same order, a more quantitative level of assurance is needed to ascertain whether the ratio of capital to on-going costs is distorted by one's accounting convention.

A simple example will help illuminate this issue. Consider a generating facility having an initial capital cost I_0\$ and a lifetime of Tyears over which 0 & M costs, initially at the rate A_0 s/yr, escalate exponentially with time at the rate of monetary inflation, y per yr, i.e.,:

From: “Projected costs of generating electricity: update 1998, OECD 1998
Figure 9a. Levelised electricity generation costs calculated with common assumptions at 10% discount rate (Usmill/kWh)

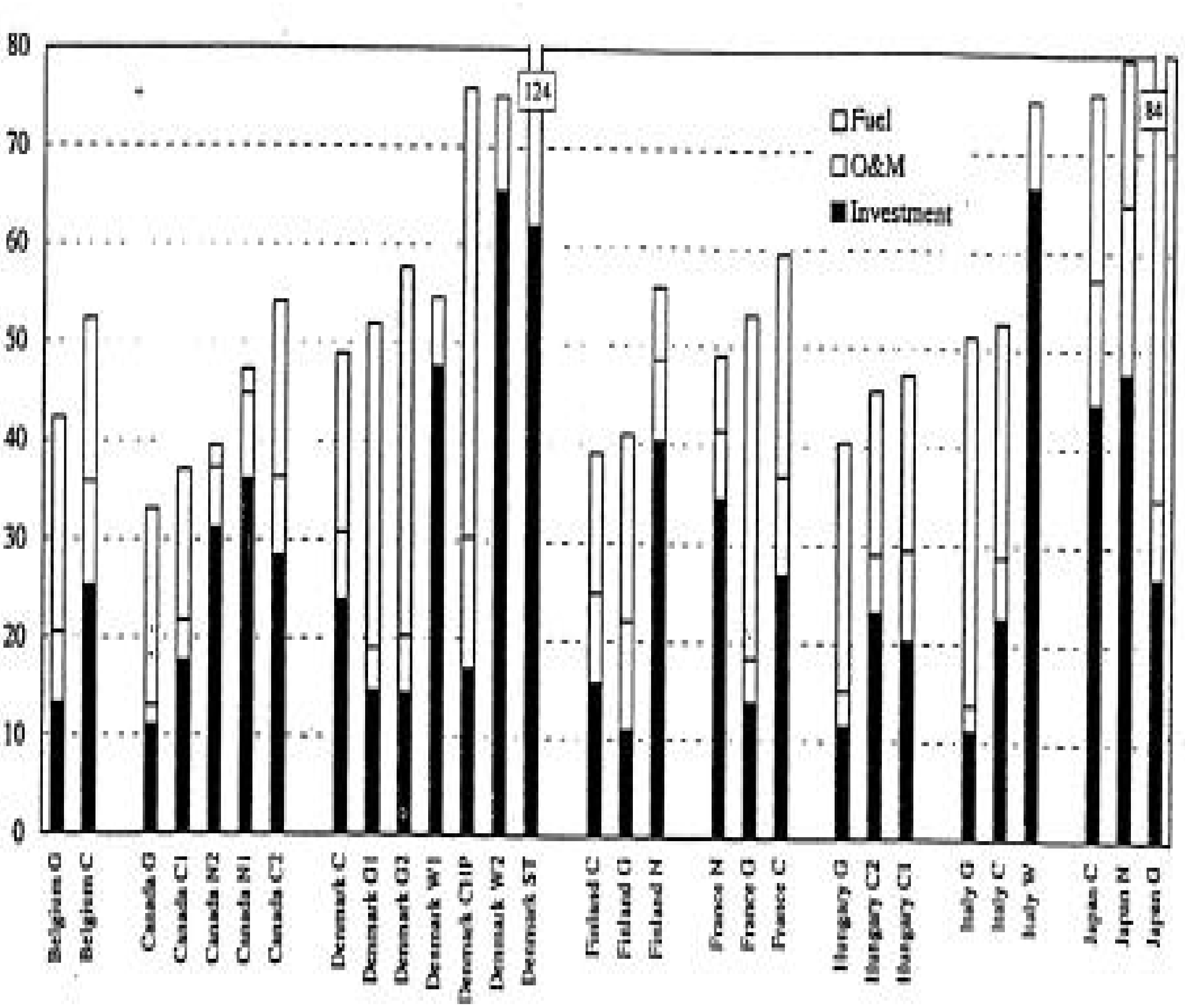
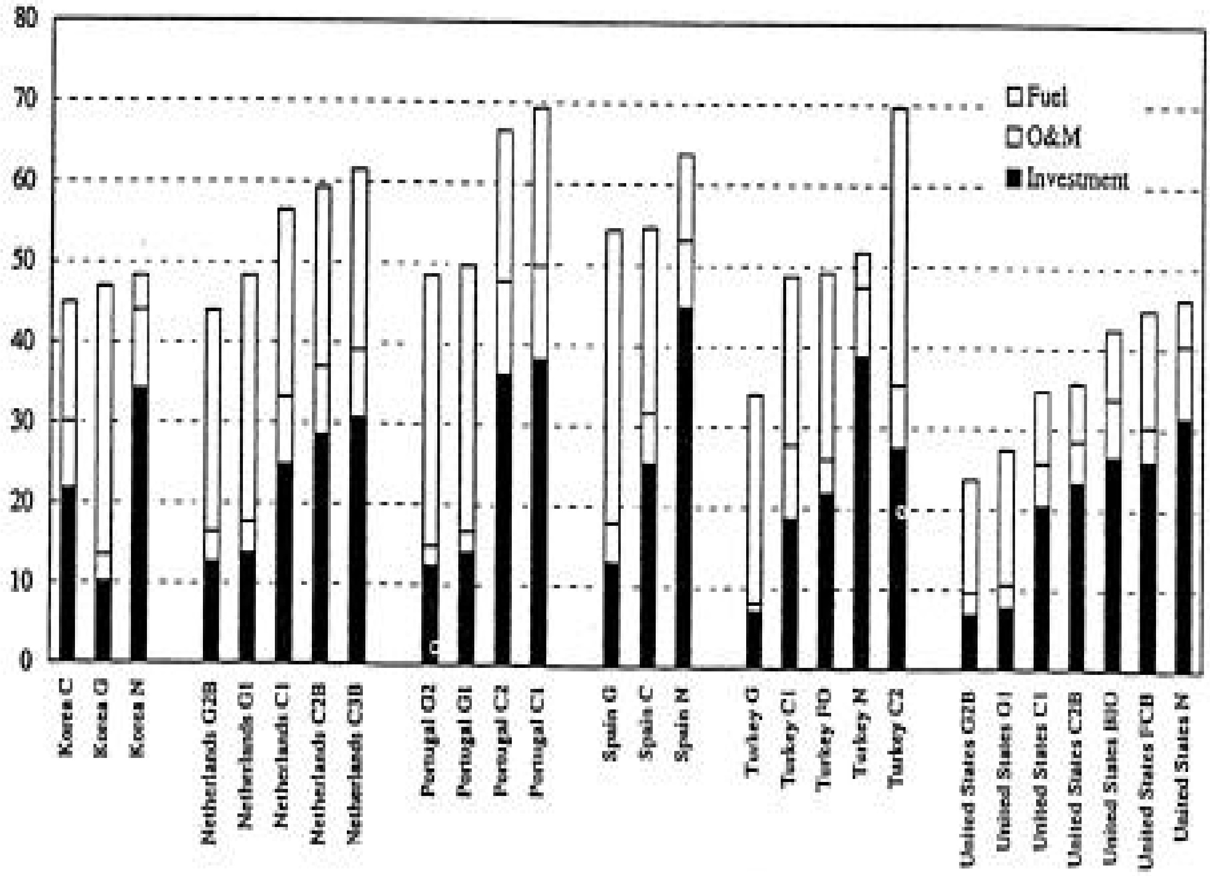


Figure 9b. Levelized electricity generation costs calculated with common assumptions at 10% discount rate (Usmil/kWh)



Note: US results refer to the Midwest region. National calculations for the Eastern and Western regions are given in Annex 2

Note: These results are in constant dollar: hence subtract the predicted rate of monetary inflation from the market rates x and y in our class notes model; also their analyses exclude taxes (set y=0).

SOME USEFUL GENERALIZATIONS / APPROXIMATIONS

Almost Everything Varies (Increases) Linearly with Everything Else:

- Amount of natural uranium vs. reload enrichment (exactly)
- SWU to enrich vs. reload enrichment
- Reactivity loss vs. current burnup
- Initial reactivity vs. enrichment or discharge burnup
- Achievable burnup vs. enrichment (initial reactivity)

* Total direct fuel front end cost vs. enrichment or burnup

But Some Things are Nonlinear:

- Enrichment plant tails: there is an optimum which minimizes the total of ore plus SWU costs
- Fuel cycle cost shows a weak minimum vs. burnup due to the accumulation of carrying charges

ESTIMATION OF RELOAD ENRICHMENT

Although detailed whole-core calculations are required to obtain reload assembly specifications, the following semi-empirical approximations can be used to illustrate the general considerations involved.

For a specified **cycle** burnup, Bc (MWD/kgU), the eventual fuel discharge burnup, Bd is related to reload batch average enrichment, Xp (w/o U-235) by:

$$Bd \approx 56\sqrt{Xp - 0.88} - 38.4 - Bc\text{, MWD/l}$$

where

Bc=O.O3O44pL Tc, MWD/kg

in which

p = <u>rated</u> core specific power, KW/kgU,

for example, 38.7 for a 4-loop Westinghouse PWR L = <u>predicted</u> cycle capacity factor, startup-to-startup (e.g., 0.85)

Tc = <u>specified</u> cycle length, months (e.g., 18)

Finally, the number of batches in core, n- or, in other words, 1/n th of the core is refueled each shutdown - is just:
n = Bd/Bc

For the parameters cited above and Xp = 4.2 w/o,

Bc = l8MWD/kg Bd = *45.6* MWD/kg n = 2.53

or approximately 40% of the assemblies are replaced each refueling. Actually one would probably push Bd to just short of the licensed/warranteed limit and solve for the resulting Xp: e.g., set Bd = 55 MWD/kg, find Xp =4.84 w/o and n = 3.06, or ~ 1/3 is refueled.