**Measurement of $B_{s}^{0} \rightarrow D_{s}(\ast)^{+}D_{s}(\ast)^{-}$ Branching Ratios**

The MIT Faculty has made this article openly available. Please share how this access benefits you. Your story matters.

<table>
<thead>
<tr>
<th><strong>Citation</strong></th>
<th>T. Aaltonen et al. (CDF Collaboration). “Measurement of $B_{s}^{0} \rightarrow D_{s}(\ast)^{+}D_{s}(\ast)^{-}$ Branching Ratios” Physical Review Letters 108, 201801 (2012). © 2012 American Physical Society</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>As Published</strong></td>
<td><a href="http://dx.doi.org/10.1103/PhysRevLett.108.201801">http://dx.doi.org/10.1103/PhysRevLett.108.201801</a></td>
</tr>
<tr>
<td><strong>Publisher</strong></td>
<td>American Physical Society</td>
</tr>
<tr>
<td><strong>Version</strong></td>
<td>Final published version</td>
</tr>
<tr>
<td><strong>Citable link</strong></td>
<td><a href="http://hdl.handle.net/1721.1/71613">http://hdl.handle.net/1721.1/71613</a></td>
</tr>
<tr>
<td><strong>Terms of Use</strong></td>
<td>Article is made available in accordance with the publisher’s policy and may be subject to US copyright law. Please refer to the publisher’s site for terms of use.</td>
</tr>
</tbody>
</table>
Measurement of $B^0_s \to D_s^{(*)+} D_s^{(*)-}$ Branching Ratios


0031-9007/12/108(20)/201801(7) 201801-1 © 2012 American Physical Society
Institute of Physics, Academia Sinica, Taipei, Taiwan 11529, Republic of China
2Argonne National Laboratory, Argonne, Illinois 60439, USA
3University of Athens, 157 71 Athens, Greece
4Institut de Fisica d’Altes Energies, ICREA, Universitat Autonoma de Barcelona, E-08193, Bellaterra (Barcelona), Spain
5Baylor University, Waco, Texas 76798, USA
6aIstituto Nazionale di Fisica Nucleare Bologna, I-40127 Bologna, Italy
6bUniversity of Bologna, I-40127 Bologna, Italy
7University of California, Davis, Davis, California 95616, USA
8University of California, Los Angeles, Los Angeles, California 90024, USA
9Instituto de Fisica de Cantabria, CSIC-University of Cantabria, 39005 Santander, Spain
10Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA
11Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637, USA
12Comenius University, 842 48 Bratislava, Slovakia; Institute of Experimental Physics, 040 01 Kosice, Slovakia
13Joint Institute for Nuclear Research, RU-141980 Dubna, Russia
14Duke University, Durham, North Carolina 27708, USA
15Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA
16University of Florida, Gainesville, Florida 32611, USA
17Laboratori Nazionali di Frascati, Istituto Nazionale di Fisica Nucleare, I-00044 Frascati, Italy
18University of Geneva, CH-1211 Geneva 4, Switzerland
19Glasgow University, Glasgow G12 8QQ, United Kingdom
20Harvard University, Cambridge, Massachusetts 02138, USA
21Division of High Energy Physics, Department of Physics, University of Helsinki and Helsinki Institute of Physics, FIN-00014, Helsinki, Finland
22University of Illinois, Urbana, Illinois 61801, USA
23The Johns Hopkins University, Baltimore, Maryland 21218, USA
24Institut für Experimentelle Kernphysik, Karlsruhe Institute of Technology, D-76131 Karlsruhe, Germany
25Center for High Energy Physics: Kyungpook National University, Daegu 702-701, Korea; Seoul National University, Seoul 151-742, Korea; Sungkyunkwan University, Suwon 440-746, Korea; Korea Institute of Science and Technology Information, Daejeon 305-806, Korea; Chonnam National University, Gwangju 500-757, Korea; Chonbuk National University, Jeonju 561-756, Korea
26Ernest Orlando Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
27University of Liverpool, Liverpool L69 7ZE, United Kingdom
28University College London, London WC1E 6BT, United Kingdom
29Centro de Investigaciones Energeticas Medioambientales y Tecnologicas, E-28040 Madrid, Spain
30Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
31Institute of Particle Physics: McGill University, Montréal, Québec, Canada H3A 2T8; Simon Fraser University, Burnaby, British Columbia, Canada V5A 1S6; University of Toronto, Toronto, Ontario, Canada M5S 1A7; and TRIUMF, Vancouver, British Columbia, Canada V6T 2A3
32University of Michigan, Ann Arbor, Michigan 48109, USA
33Michigan State University, East Lansing, Michigan 48824, USA
34Institution for Theoretical and Experimental Physics, ITEP, Moscow 117259, Russia
35University of New Mexico, Albuquerque, New Mexico 87131, USA
36The Ohio State University, Columbus, Ohio 43210, USA

(CDF Collaboration)
The dominant contribution to decays to final states of definite CP and lifetime, \( B_0 \), is a good approximation to the even and odd CP sizable decay width difference between the light and heavy under certain theoretical assumptions \([3,4]\), resulting in the relation

\[ \Gamma_{\ell} = \Gamma_{\ell}^{\text{even}} - \Gamma_{\ell}^{\text{odd}} \]

which do not yet include the latest preliminary Belle physics contributions to CP violation in the future to measure directly the lifetime of the CP eigenstate, which would complement the CP state lifetime measurement in \( B_0 \), where the uncertainties are statistical and systematic. These results are the most precise single measurements to date and provide important constraints for indirect searches for nonstandard model physics in \( B_0 \) mixing.

PACS numbers: 13.25.Hw, 12.15.Ff, 14.40.Nd

DOI: 10.1103/PhysRevLett.108.201801

A \( B_s^0 \) meson can oscillate into its antiparticle via second-order weak interaction transitions, which make its time evolution sensitive to contributions from new physics processes. Such contributions are not well constrained yet and might be responsible for the deviation from the standard model reported in Ref. [1]. The \( B_s^0 \) eigenstates with defined mass and lifetime, \( B_s^{0\ell} \) and \( B_s^{0h} \), are linear combinations of the \( B_s^0 \) and \( B_s^0 \) states and, in the standard model, correspond in good approximation to the even and odd \( CP \) eigenstates, respectively. In the absence of substantial \( CP \) violation, a sizable decay width difference between the light and heavy mass eigenstates, \( \Delta \Gamma_s = \Gamma_{\ell} - \Gamma_{\ell}^{\text{odd}} \), arises from the fact that decays to final states of definite \( CP \) are only accessible by one of the mass eigenstates. The dominant contribution to \( \Delta \Gamma_s \) is believed to come from the \( B_s^0 \to D_s^{(*)+} D_{s}^{(*)-} \) decays \([2]\), which are predominantly \( CP \)-even and saturate \( \Delta \Gamma_s \) under certain theoretical assumptions \([3,4]\), resulting in the relation

\[ 2\mathcal{B}(B_s^0 \to D_s^{(*)+} D_{s}^{(*)-}) = \frac{\Delta \Gamma_s - \Delta \Gamma_s^{/}}{\Gamma_s + \Delta \Gamma_s^{/}} = \frac{\Delta \Gamma_s^{/}}{\Gamma_s + \Delta \Gamma_s^{/}} \]

where \( \Gamma_s = (\Gamma_{\ell} + \Gamma_{\ell}^{\text{odd}}) / 2 \) \([5]\). However, three-body modes may provide a significant contribution to \( \Delta \Gamma_s \) \([6]\).

A finite value of \( \Delta \Gamma_s \) improves the experimental sensitivity to \( CP \) violation, because it allows one to distinguish the two mass eigenstates via their decay time distribution. Furthermore, the \( B_s^0 \to D_s^{(*)+} D_{s}^{(*)-} \) decays could be used in the future to measure directly the lifetime of the \( CP \)-even eigenstate, which would complement the \( CP \)-odd eigenstate lifetime measurement in \( B_s^0 \to J/\psi f_0(980) \) decays \([7]\) and provide additional information in the search for new physics contributions to \( CP \) violation in the \( B_s^0 \) system.

The \( B_s^0 \to D_s^{(*)+} D_{s}^{(*)-} \) decay modes have been previously studied by the ALEPH, CDF, D0, and Belle Collaborations \([8-11]\). The current world average branching ratios \([12]\), which do not yet include the latest preliminary Belle
results [13], are \(\mathcal{B}(B^0 \rightarrow D^+_s D^0_s) = (1.04^{+0.29}_{-0.26})\%\),
\(\mathcal{B}(B^0 \rightarrow D^{\ast+}_s D^-_s) = (2.8 \pm 1.0)\%\),
\(\mathcal{B}(B^0 \rightarrow D^{\ast+}_s D^{\ast-}_s) = (3.1 \pm 1.4)\%\), and \(\mathcal{B}(B^0 \rightarrow D^{(\ast)+}_s D^{(\ast)-}_s) = (4.5 \pm 1.4)\%\).

In a data sample corresponding to an integrated luminosity of 6.8 fb\(^{-1}\) recorded by the CDF II detector at the Tevatron \(p\bar{p}\) collider, we reconstruct \(B^0 \rightarrow D^{\ast+}_s D^{\ast-}_s\) decays with \(D^{\ast+}_s \rightarrow K^+ K^- \pi^+\). For the first time in this channel, the acceptance is calculated by using a \(D^+_s\) Dalitz model instead of a simple two-body decay model. The photon and the neutral pion from the \(D^0\) model instead of a simple two-body decay model. The mon vertex. Because the charge and mass hypothesis assignments, fitted to a comment of the tracks and the reconstructed decay vertex from tracks detected in the COT and the silicon detector [15]. The events for COT and information from the time-of-flight system located between the COT and the solenoid. The events for this analysis are the tracking systems located relevant for this analysis are the tracking systems located inside a solenoid that provides a 1.4 T magnetic field. Charged particles’ trajectories (tracks) are reconstructed inside a solenoid that provides a 1.4 T magnetic field. The extent of the COT. Kaons and pions are statistically identified by measurements of the ionization energy loss in the COT chamber (COT) with a radial extension from 40 to 137 cm. Tracks with a pseudorapidity \(|\eta| < 1.0\) pass the full radial extent of the COT. Kaons and pions are statistically identified by measurements of the ionization energy loss in the COT and information from the time-of-flight system located between the COT and the solenoid. The events for this analysis are selected on-line by identifying pairs of tracks detected in the COT and the silicon detector [15]. Minimal requirements on the momenta and the displacement of the tracks and the reconstructed decay vertex from the primary vertex are imposed.

We reconstruct \(D^+_s \rightarrow K^+ K^- \pi^+\) and \(D^+ \rightarrow K^- \pi^+ \pi^+\) decays from combinations of three tracks with appropriate charge and mass hypothesis assignments, fitted to a common vertex. Because the \(D^{\ast+}_s \rightarrow K^+ K^- \pi^+\) decay proceeds mainly via \(\phi \pi^+\) and \(K^{\ast0} K^+\), we select candidates with 1.005 < \(m(K^+ K^-)\) < 1.035 GeV/c\(^2\) and 0.837 < \(m(K^- \pi^+)< 0.947 \text{GeV/c}^2\), centered on the known \(\phi\) and \(K^{\ast0}\) masses, respectively. According to the \(D^{\ast+}_s \rightarrow K^+ K^- \pi^+\) Dalitz structure [16], this requirement has a signal acceptance of about 75% while covering only 14% of the phase space and thus increasing the signal-to-background ratio. In the following, we will denote the selected \(K^+ K^-\) and \(K^- \pi^+\) combinations as \(\phi\) and \(K^{\ast0}\), respectively, since the dominant contributions come from these resonances. However, we implicitly include contributions from other resonances and interference effects when using these terms.

Pairs of \(D^+_s \rightarrow \phi \pi^+\) or \(D^+_s \rightarrow K^{\ast0} K^+\) candidates and \(D^+ \rightarrow \phi \pi^-\) candidates are combined to form \(B^0\) candidates and fitted to a common vertex. Combinations where both charm mesons decay into a \(K^{\ast0}\) mode are not considered because of the low signal-to-background ratio. Candidate \(B^0\) mesons are reconstructed from \(D^+_s D^-\) combinations where both \(D^+_s\) decay modes are used.

To reject backgroundlike events, requirements are placed on track quality variables, \(B\) meson momentum, reconstructed \(D\) meson masses, vertex fit qualities, and vertex displacement significances. To further increase the signal purity, two artificial neural networks are used: one for candidates with a \(K^{\ast0}\) and one for candidates without. To minimize the systematic uncertainty of the relative selection efficiency, the same networks are applied to \(B^0\) and \(B^0\) candidates, and only information from the \(D^+_s\) that is common to both \(B\) meson decays is used. The networks are trained on simulated signal events, described below, and on background events from the 5.45–6.5 GeV/c\(^2\) \(B\) mass sideband. The input variables contain kinematic, lifetime, fit quality, and particle identification information. The \(B\) vertex displacement significance in the transverse plane gives the largest contribution to the discrimination power of both networks. The selection criteria on the network outputs are chosen such that they maximize the significance \(\varepsilon_{\text{MC}}/\sqrt{N_{\text{data}}}\), where \(\varepsilon_{\text{MC}}\) is the \(B^0\) selection efficiency determined from simulation and \(N_{\text{data}}\) is the number of data events in the \(B^0\) signal window from 5.343 to 5.397 GeV/c\(^2\).

About 6% of the selected \(B^0 \rightarrow D^+_s (\rightarrow \phi \pi^+) D^-\) candidates also fulfill the \(B^0\) selection requirements, where the assignment of a \(D^-\) daughter track is swapped from pion to kaon. To avoid having the same event entering the fit multiple times, we reject each event that is reconstructed as \(B^+_s\) candidates from the \(B^0\) sample. The cross-populations between the two \(B^+_s\) modes and between the two \(B^0\) modes, respectively, are negligible. The selected sample contains about 750 \(B^0\) signal events.

Simulated events are used to determine the reconstruction and selection efficiency. The \(B^0\) mesons are generated according to the momentum spectrum measured in exclusive \(B\) decays and decayed to the considered final states with the EVTGEN package [17]. For the \(B^0\) meson, we assign the lifetime of the \(B^0\) eigenstate [12] that coincides with the \(CP\)-even eigenstate in the standard model. For all the other long-lived charm and bottom mesons, the world average mean lifetimes [12] are used. The \(B^0 \rightarrow D^{\ast+}_s D^{\ast-}_s\) decay is a transition of a pseudoscalar to two vector mesons, and its angular distribution is described by three polarization amplitudes. Since these amplitudes are unknown, we take the same longitudinal polarization as measured in \(B^0 \rightarrow D^{\ast+} D^{\ast-}\) decays [18] and a vanishing \(CP\)-odd component as default values. The world average

---

**PRL 108, 201801 (2012)**  
**PHYSICAL REVIEW LETTERS**  
week ending 18 MAY 2012

---

**201801-4**
value [12] is used for the ratio of \( D_s^+ \rightarrow D_s^+ \gamma \) to \( D_s^+ \rightarrow D_s^+ \pi^0 \) decays. The dynamics of the decay \( D_s^+ \rightarrow K^+ K^- \pi^+ \) is simulated according to the Dalitz structure measured by CLEO [16]. The generated events are processed by a GEANT3 based detector simulation [19] and the same reconstruction program as applied to real data events.

The relative branching ratios times production rate are determined in a simultaneous extended unbinned maximum-likelihood fit to the \((\phi \pi^+)(\phi \pi^-), (\bar{K}^0 K^+)(\phi \pi^-), \) \((\phi \pi^+)(K^+ \pi^+ \pi^-)\), and \((\bar{K}^0 K^+)(K^+ \pi^- \pi^+)\) invariant mass distributions. By simultaneously fitting all four distributions, the normalization of the \( B^0 \) reflections in the \((\bar{K}^0 K^+)(\phi \pi^-)\) spectrum is constrained by the yields in the high-statistics \((\phi \pi^+)(K^+ \pi^- \pi^-)\) sample. The components of the fit function for each invariant mass distribution are fully and partially reconstructed signals, reflections, and background. The fully reconstructed \( B^0 \) and \( B^0 \) signals are parametrized by the sum of two Gaussians with relative normalizations and widths derived from simulation. To account for discrepancies between the data and simulation, a factor is introduced for the scale factors of the fully reconstructed signal components.

The normalizations of reflections are calculated in the same way but with the efficiencies replaced by the efficiencies determined from simulation. Equivalent expressions are used for the yields of partially reconstructed signal events and of reflections from \( B^0 \rightarrow (\phi \pi^+)(K^- \pi^- \pi^-) \) misreconstructed as \( B^0 \rightarrow (\phi \pi^+)(\bar{K}^0 K^+) \) are determined from simulation using empirical models. Background from random combinations of tracks and other \( B \) decays is described by an exponential plus a constant function with all parameters floated in the fit.

The yield of fully reconstructed \( B^0 \) mesons in the final state \( i \), \((\phi \pi^+)(K^- \pi^- \pi^-)\) or \((\bar{K}^0 K^+)(K^+ \pi^- \pi^-)\), is given by

\[
N_{\text{rec},i}^{B^0} = N_{\text{tot}}^{B^0} \mathcal{B}(B^0 \rightarrow D_s^+ D^-) \mathcal{B}(D_s^+ \rightarrow K^+ K^- \pi^+). 
\]

where \( N_{\text{tot}}^{B^0} \) is the total number of produced \( B^0 \) mesons and is a free parameter in the fit, the branching ratios are taken from Ref. [12], and the efficiency \( \epsilon_i^{B^0} \) is determined from simulation. Equivalent expressions are used for the yields of partially reconstructed \( B^0 \) decays with an additional branching ratio factor for the \( D_s^+ \) and \( D_s^+ \) decays.

The normalizations of reflections are calculated in the same way but with the efficiencies replaced by the misreconstruction fractions determined from simulation. The number of fully reconstructed \( B^0 \) mesons in the final state \( i \), \((\phi \pi^+)(\phi \pi^-)\) or \((\bar{K}^0 K^+)(\phi \pi^-)\), where the \( D_s^+ \) decays in the same mode as the \( D_s^+ \) from the \( B^0 \) decay is given by

\[
N_{\text{rec},i}^{B^0} = N_{\text{rec},f}^{B^0} f_{D_s^+ D_s^+} \mathcal{B}(D_s^+ \rightarrow K^+ K^- \pi^+) \epsilon_i^{B^0},
\]

with \( f_{D_s^+ D_s^+} \) as a free parameter and \( N_{\text{rec},f}^{B^0} \) given by Eq. (3). Equivalent equations hold for partially reconstructed \( B^0 \) decays.

Projections of the fit result are compared to the distribution of data events in Fig. 1. The statistical significance of each signal exceeds 10σ as estimated from a likelihood ratio of the fit with and without the signal component.

Systematic uncertainties on the fitted signal yields arise from the signal and background models. Because the width scale factors of the fully reconstructed signal components are allowed to float in the fit, the systematic uncertainties of these components are already included in the statistical uncertainty.
errors. To estimate the systematic effect due to the fixed shapes of the partially reconstructed signal components and reflections, we repeat the fit multiple times with shape parameters randomly varied according to the covariance matrix of the fits of the shapes to simulated data. The mean deviations with respect to the central values are assigned as systematic uncertainties. The systematic uncertainties due to the background mass model are estimated from the changes in the results caused by using a second-order polynomial instead of the sum of an exponential and a constant function. By applying the selection optimization procedure on the normalization instead of the signal mode, we verified that a possible selection bias is negligible.

Systematic effects in the relative efficiency determination can be caused by a simulation that does not describe the data accurately. One source of systematic uncertainties is the trigger simulation, which can lead to a discrepancy in the $B$ meson momentum spectrum. Although this effect cancels to first order in the ratio measurement, it is accounted for by a reweighting of the simulated events. The systematic uncertainties due to the detector simulation are estimated by the shift of the results with respect to the case in which this reweighting is not applied. The uncertainties on the world average $B^0$, $D^+$, and $D_s^+$ lifetimes are propagated by varying the lifetimes in the simulation. For the $B^0$ lifetime, we consider two cases: the $1\sigma$ lower bound of the world average short-lived eigenstate lifetime and the $1\sigma$ upper bound of the mean $B^0$ lifetime. The effects on the acceptance induced by variations of the $D_s^+ \to K^+ K^- \pi^+$ Dalitz structure are considered by generating different Dalitz model scenarios, with Dalitz model parameter values varied according to the systematic and correlated statistical uncertainties of the CLEO Dalitz fit. The uncertainties of the $D^+$ Dalitz model have a negligible effect on the result. For $B_s^0 \to D_{s+}^+ D_{s-}^-$ decays, we investigate the effects of both a longitudinal polarization fraction $f_L$ deviating from our nominal assumption and a nonzero fraction of the CP-odd component $f_{CP-}$. The fraction $f_L$ is varied in the simulation according to the uncertainty of the $f_L$ measurement in $B^0 \to D^+ D^-$. A variation of $f_{CP-}$ shows no effect on the $B_s^0 \to D_{s+}^+ D_{s-}^-$ mass line shape, fit quality, or measured branching fraction ratios. The effect of self cross feed due to a wrong assignment of kaon and pion masses is negligible.

Further systematic uncertainties arise from external input quantities. The uncertainties of intermediate and final state branching fractions, $\mathcal{B}(D_{s+}^+ \to K^+ K^- \pi^+)$, $\mathcal{B}(D^+ \to K^- \pi^+ \pi^0)$, and $\mathcal{B}(D^{++} \to D^+ \gamma/\pi^0)$, are propagated in the fit by adding Gaussian constraints to the corresponding fit parameters. The resulting uncertainties of the measured branching fraction ratios are extracted by subtracting in quadrature the statistical uncertainties of the fits with the branching fraction constrained and the one where they are fixed to the central values. When calculating the absolute branching fraction $\mathcal{B}(B_s^0 \to D_{s+}^{(*)+} D_{s-}^{(*)-})$, an additional relative uncertainty of 16% is introduced by the measurement uncertainties of $f_L/f_D$ and the branching fraction of the normalization channel $B^0 \to D_s^+ D^-$. The systematic uncertainties are summarized in Table I.

As a result, we obtain $f_{D_s D_s} = 0.183 \pm 0.021 \pm 0.017$, $f_{D^0 D_s} = 0.424 \pm 0.046 \pm 0.035$, $f_{D^0 D^0} = 0.654 \pm 0.072 \pm 0.065$, and $f_{D^0 D_s} = 1.261 \pm 0.095 \pm 0.112$, where the first uncertainties are statistical and the second systematic.

### Table I. Overview of systematic uncertainties on the measured ratios of branching fractions

<table>
<thead>
<tr>
<th>Source</th>
<th>$f_{D_s D_s}$</th>
<th>$f_{D^0 D_s}$</th>
<th>$f_{D^0 D^0}$</th>
<th>$f_{D^0 D_s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal model</td>
<td>0.003</td>
<td>0.007</td>
<td>0.009</td>
<td>0.019</td>
</tr>
<tr>
<td>Background model</td>
<td>0.001</td>
<td>0.004</td>
<td>0.030</td>
<td>0.033</td>
</tr>
<tr>
<td>Detector simulation</td>
<td>0.001</td>
<td>0.003</td>
<td>0.010</td>
<td>0.005</td>
</tr>
<tr>
<td>$B$, $D$ lifetimes</td>
<td>+0.001</td>
<td>+0.002</td>
<td>+0.003</td>
<td>+0.006</td>
</tr>
<tr>
<td>Dalitz model</td>
<td>0.011</td>
<td>0.024</td>
<td>0.038</td>
<td>0.073</td>
</tr>
<tr>
<td>Helicity model</td>
<td>0.001</td>
<td>0.005</td>
<td>0.012</td>
<td>0.008</td>
</tr>
<tr>
<td>Branching fractions</td>
<td>0.013</td>
<td>0.024</td>
<td>0.039</td>
<td>0.074</td>
</tr>
<tr>
<td>Total</td>
<td>0.017</td>
<td>0.035</td>
<td>0.065</td>
<td>0.112</td>
</tr>
</tbody>
</table>

We thank Mikhail S. Dubrovin and David Cinabro for their help in implementing the CLEO Dalitz model. We thank the Fermilab staff and the technical staffs of the participating institutions for their vital contributions. This work was supported by the U.S. Department of Energy and National Science Foundation; the Italian Istituto Nazionale di Fisica Nucleare; the Ministry of Education, Culture,
Sports, Science and Technology of Japan; the Natural Sciences and Engineering Research Council of Canada; the National Science Council of the Republic of China; the Swiss National Science Foundation; the A.P. Sloan Foundation; the Bundesministerium für Bildung und Forschung, Germany; the Korean World Class University Program, the National Research Foundation of Korea; the Forschung, Germany; the Korean World Class University Foundation; the Bundesministerium für Bildung und Forschung, Germany; the Korean World Class University Foundation; the Swiss National Science Foundation; the A. P. Sloan

\[ \sum_{i=1}^{n} a_i \]