Problem Solving as State Space Search

Brian C. Williams
16.410-13
Session 3

Assignments

- Remember:
  Problem Set #1: Simple Scheme and Search
  Out today, due Session 4.
- Reading:
  - Solving problems by searching: AIMA Ch. 3

Course Objective 1:
Agent Architectures

To understand the major types of agents and common architectures used to develop agents.
- Mission-oriented Agents
- Self-repairing Agents
- Mobile Agents
- Agile Agents
- Communicating Agents

Course Objective 2:
Principles of Agents

16.410/13: To learn the modeling and algorithmic building blocks for creating reasoning and learning agents:
1. To formulate reasoning problems in an appropriate formal representation.
2. To describe, analyze and demonstrate the application of reasoning algorithms to solve these problem formulations.

Agent Building Blocks

- Activity Planning
- Execution/Monitoring
- Diagnosis
- Repair
- Scheduling
- Resource Allocation
- Path Planning
- Localization
- Map Building
- Trajectory Design
- Policy Construction

⇒ Most reasoning problems, like these, may be formulated as state space search.
Course Objective 3: Implementing Agents

16.413: To appreciate the challenges of building a state of the art autonomous explorer:

Fall 03:
• Mars Exploration Rover shadow mode demonstration.
Fall 04:
• Gnu Robot competition.
Fall 05:
• Model-based autonomy toolbox
• The virtual solar system
• Stay tuned for more.

Complex missions must carefully:
• Plan complex sequences of actions
• Schedule actions
• Allocate tight resources
• Monitor and diagnose behavior
• Repair or reconfigure hardware.
⇒ Most AI problems, like these, may be formulated as state space search.

Outline

• Problem Formulation
  – Problem solving as state space search
• Mathematical Model
  – Graphs and search trees
• Reasoning Algorithms
  – Depth and breadth-first search

Problem Solving as State Space Search

• Formulate Goal
  – State
    • Astronaut, Fox, Goose & Grain across river
• Formulate Problem
  – States
    • Location of Astronaut, Fox, Goose & Grain at top or bottom river bank
  – Operators
    • Astronaut drives rover and 1 or 0 items to other bank.
• Generate Solution
  – Sequence of Operators (or States)
    • Move(astronaut), Move(astronaut), . . .

Can the astronaut get its supplies safely across the Martian canal?

Astronaut
Goose
Grain
Fox
Rover
• Astronaut + 1 item allowed in the rover.
• Goose alone eats Grain
• Fox alone eats Goose

Early AI: What are the universal problem solving methods?

Simple Trivial

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**Formulation Example: 8-Puzzle**

- States: integer location for each tile AND …
- Operators: move empty square up, down, left, right
- Goal Test: goal state as given

**Example: Planning Discrete Actions**

- Swaggett & Lovell work on Apollo 13 emergency rig lithium hydroxide unit.
  - Assembly
- Mattingly works in ground simulator to identify new sequence handling severe power limitations.
  - Planning & Resource Allocation
- Mattingly identifies novel reconfiguration, exploiting LEM batteries for power.
  - Reconfiguration and Repair

**Planning as State Space Search: STRIPS Operator Representation**

- **Available actions**
  - **Strips operators**
    - **pickup hose**
      - precondition: (clear hose)
      - effect:
        - (not (clear hose))
        - (on-table hose)
        - (empty arm)

- **precondition**
  - (clear hose)
  - (empty arm)

- **goal (partial state):**
  - (connected a b)
  - (attached b a))

- A Plan is an operator sequence from the initial state to the goal.

**STRIPS Action Assumptions**

- Atomic time.
- Agent is omniscient (no sensing necessary).
- Agent is sole cause of change.
- Actions have deterministic effects.
- No indirect effects.
Succinct Encodings:
STRIPS Action Schemata
• Instead of defining:
  pickup-hose and pickup-clamp and …
• Define a single pickup schema (with variables ?v):
  (:operator pick-up
   :parameters (hose ?ob1)
   :precondition (and (clear ?ob1)
   (on-table ?ob1)
   (empty arm))
   :effect (and (not (clear ?ob1))
   (not (on-table ?ob1))
   (not (empty arm))
   (holding arm ?ob1)))

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Problem Formulation: A Graph

Directed Graph (one-way streets)
Undirected Graph (two-way streets)

Examples of Graphs
Planning Actions (graph of possible states of the world)
Roadmap

Directed Graph (one-way streets)
Sub graph
Subset of vertices and edges between vertices in Subset

Undirected Graph (two-way streets)
Clique
A complete subgraph (All vertices are adjacent)

Strongly connected graph
Directed path between all vertices.

Connected graph
Path between all vertices.

Complete graph
All vertices are adjacent.
Formal Representation of a Graph

A Graph G is represented as an ordered pair \( \langle V, E \rangle \), where:
- \( V \) is a set of vertices \( \{v_1, \ldots\} \), \( V = \{\text{Bos, SFO, LA, Dallas, Wash DC}\} \)
- \( E \) is a set of (directed) edges \( \{e_1, \ldots\} \), \( E = \{\langle \text{SFO, Bos} \rangle, \langle \text{SFO, LA} \rangle, \langle \text{LA, Dallas} \rangle, \langle \text{Dallas, Wash DC} \rangle\} \)

An edge is a pair \( \langle v_1, v_2 \rangle \) of vertices, where:
- \( v_2 \) is the head of the edge,
- \( v_1 \) is the tail of the edge

Note: A set has no duplicate elements.

A (directed) path is a sequence of vertices \( \langle v_1, \ldots, v_n \rangle \) such that each successive pair \( \langle v_i, v_{i+1} \rangle \) is a (directed) edge in G.

A simple path has no repeated vertices.

A cycle is a subpath where start = end.

Search Trees

Search Trees
Search Trees

A tree $T$ is a directed Graph, such that there exists exactly one directed path between any pair of vertices.

Outline

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Classes of Search

<table>
<thead>
<tr>
<th>Blind</th>
<th>Depth-First</th>
<th>Systematic exploration of whole tree</th>
</tr>
</thead>
<tbody>
<tr>
<td>(uninformed)</td>
<td>Breadth-First</td>
<td>until the goal is found.</td>
</tr>
<tr>
<td></td>
<td>Iterative-Deepening</td>
<td></td>
</tr>
<tr>
<td>Heuristic (informed)</td>
<td>Hill-Climbing</td>
<td>Uses heuristic measure of goodness of a node, e.g., estimated distance to goal.</td>
</tr>
<tr>
<td></td>
<td>Best-First</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Beam</td>
<td></td>
</tr>
<tr>
<td>Optimal (informed)</td>
<td>Branch&amp;Bound</td>
<td>Uses path &quot;length&quot; measure. Finds &quot;shortest&quot; path. A* also uses heuristic</td>
</tr>
<tr>
<td></td>
<td>A*</td>
<td></td>
</tr>
</tbody>
</table>

Depth First Search (DFS)

Idea: After visiting node
• Visit children, before siblings
• Visit siblings left to right
Breadth First Search (BFS)

Idea: After visiting node
• Visit siblings, before children
• Visit relatives left to right (top to bottom)

Elements of Algorithm Design

Description: (today)
– stylized pseudo code: sufficient to analyze and implement the algorithm
  (implementation next Wednesday).

Analysis: (following Monday)
• Soundness:
  – when a solution is returned, is it guaranteed to be correct?
• Completeness:
  – is the algorithm guaranteed to find a solution when there is one?
• Time complexity:
  – how long does it take to find a solution?
• Space complexity:
  – how much memory does it need to perform search?

Outline

• Problem Formulation: State space search
• Model: Graphs and search trees
• Reasoning Algorithms: DFS and BFS
  – A generic search algorithm
  – Depth-first search example
  – Handling cycles
  – Breadth-first search example

Simple Search Algorithm

Let Q be a list of partial paths, Let S be the start node and Let G be the Goal node.

1. Initialize Q with partial path <S>
2. If Q is empty, fail. Else, pick a partial path N from Q
3. If head(N) = G, return N (goal reached!)
4. Else:
   a) Remove N from Q
   b) Find all children of head(N) and create all one-step extensions of N to each child.
   c) Add all extended paths to Q
   d) Go to step 2.
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Depth-First Search (DFS)

Idea:
- Visit children, then siblings
- Visit siblings left to right, (top to bottom).

Assuming that we pick the first element of Q,
Then where do we add path extensions to the Q?

Simple Search Algorithm

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Depth-First
Pick first element of Q; Add path extensions to front of Q

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<thead>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>C A S (D A S) (B S)</td>
<td>4</td>
<td>G D A S) (B S)</td>
</tr>
<tr>
<td></td>
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Added paths in blue

Brian Williams, Fall 05

Depth-First
Pick first element of Q; Add path extensions to front of Q

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Added paths in blue

Brian Williams, Fall 05
Depth-First

Pick first element of Q; Add path extensions to front of Q

| 1 | Q | 2 | (S) | 3 | (C D A S) (B S) | 4 | (G D A S) (B S) | 5 | (C A S) (G D A S) (B S) | 6 | (G D A S) (B S) |

Simple Search Algorithm

Let Q be a list of partial paths,
Let S be the start node and
Let G be the Goal node.

1. Initialize Q with partial path (S)
2. If Q is empty, fail. Else, pick a partial path N from Q
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  – Handling cycles
  – Breadth-first search example

Issue: Starting at S and moving top to bottom, will depth-first search ever reach G?

Depth-First

Effort can be wasted in more mild cases

How Do We Avoid Repeat Visits?

Idea:
• Keep track of nodes already visited.
• Do not place visited nodes on Q.

Does this maintain correctness?
• Any goal reachable from a node that was visited a second time would be reachable from that node the first time.

Does it always improve efficiency?
• Visits only a subset of the original paths, such that each node appears at most once at the head of a path in Q.
How Do We Modify The Simple Search Algorithm?

Let Q be a list of partial paths, 
Let S be the start node and 
Let G be the Goal node.
1. Initialize Q with partial path (S) as only entry;
2. If Q is empty, fail. Else, pick some partial path N from Q
3. If head(N) = G, return N (goal reached!)
4. Else
   a) Remove N from Q
   b) Find all children of head(N) not in Visited and create all the one-step extensions of N to each child.
   c) Add to Q all the extended paths;
   d) Add children of head(N) to Visited
   e) Go to step 2.

Simple Search Algorithm

Let Q be a list of partial paths, 
Let S be the start node and 
Let G be the Goal node.
1. Initialize Q with partial path (S) as only entry; set Visited = {} 
2. If Q is empty, fail. Else, pick some partial path N from Q
3. If head(N) = G, return N (goal reached!)
4. Else
   a) Remove N from Q
   b) Find all children of head(N) not in Visited and create all the one-step extensions of N to each child.
   c) Add to Q all the extended paths;
   d) Add children of head(N) to Visited
   e) Go to step 2.

Testing for the Goal

- This algorithm stops (in step 3) when head(N) = G.
- We could have performed this test in step 6 as each extended path is added to Q. This would catch termination earlier and be perfectly correct for all the searches we have covered so far.
- However, performing the test in step 6 will be incorrect for the optimal searches we look at later. We have chosen to leave the test in step 3 to maintain uniformity with these future searches.

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Breadth First Search (BFS)

Idea:
• Visit siblings before their children
• Visit relatives left to right

Assuming that we pick the first element of Q. Then where do we add path extensions to the Q?
Breadth-First w Visited
Pick first element of Q; Add path extensions to end of Q

<table>
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<tbody>
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We could stop here, when the first path to the goal is generated.
### Breadth-First w/ Visited

Pick first element of Q; Add path extensions to end of Q

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### Breadth-First w/ Visited

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### Breadth First Search (DFS)

Depth-first:
Add path extensions to front of Q
Pick first element of Q

### Breadth First Search (BFS)

Breadth-first:
Add path extensions to back of Q
Pick first element of Q

For each search type, where do we place the children on the queue?
What You Should Know

- Most problem solving tasks may be formulated as state space search.
- Mathematical representations for search are graphs and search trees.
- Depth-first and breadth-first search may be framed, among others, as instances of a generic search strategy.
- Cycle detection is required to achieve efficiency and completeness.

Appendix

Breadth-First (without Visited list)

Pick first element of Q; Add path extensions to end of Q

```
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Breadth-First (without Visited list)

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Added paths in blue

Revisited nodes in pink

* We could have stopped here, when the first path to the goal was generated.
Breadth-First (without Visited list)

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Pick first element of Q; Add path extensions to end of Q

<table>
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