ESSAYS IN LOCAL PUBLIC FINANCE

by

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ABSTRACT

The purpose of this thesis is to develop a fairly general approach to metropolitan cost-benefit
analysis in a simple integrated framework. The first chapter mainly serves to set up the
discussion, by reviewing the main stories of local fiscal structure and, in particular, the
relationship between local policy and local market activity. Chapter 2 is essentially an effort
to generalize second-best policy rules to account for endogenous prices, local budget effects,
and endogenous populations. Part of the motivation also comes from an interest in the role of
local market conditions, such as the tenure mix of the community, in the property value
capitalization process.

Chapter 3 relaxes the assumption of resident homogeneity, which allows for substantial
extensions of the analysis in two directions. With resident heterogeneity, it is no longer the
presumption that households will prefer the same bundle of local policies simply because
they have located in the same town. The first part of the chapter approaches this question by
assuming that policies are chosen by a decisive voter, where the endogeneity of the tax price
and the population raise the possibility that equilibrium popular policies may be efficient
under certain conditions. The second part of the chapter abstracts from public choice issues,
as such, to characterize the features of optimally discriminatory public policies. That is, how
should tax and spending policies account for the heterogeneity of the resident pool? If the
local population is sufficiently mixed, for example, it turns out that it may be optimal to
subsidize entry into the community, via negative poll taxes, while using property taxation as
a means of distinguishing among residents on the basis of their housing consumption.
Alternatively, a homogeneous town may want to choose a negative property tax rate and
finance itself with a uniform head tax.

The final chapter reexamines these questions in an explicitly spatial model. The chapter
begins by reestablishing the key capitalization conditions, which are seen to depend in a
critical manner on the spatial characteristics of the community in addition to the factors
discussed earlier. The role of tenure turns out to be particularly important in evaluating
welfare expressions, and this aspect is explored by modeling rent flows between the
communities of the larger economic system. Finally, policy design considerations are
extended to account for communities with internal structure. Still more general second-best
public spending rules are calculated.

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Chapter 1
Issues in Local Public Finance: A Capitalization Primer

1 Introduction and thesis overview

The theory of the public sector has long posed several difficulties for economists raised in general equilibrium theory as well as for those involved in day-to-day policy making. General solutions have been hard to obtain to the problems of determining the value individuals place on public goods, the efficient financing of those goods, and the associated social choice issues that arise in the aggregation of preferences. This has likewise made it difficult to provide guidance to governments needing particular ways of dealing with particular situations, such as the choice of tax rates and spending levels. While there has been considerable progress in our understanding of the structure of these problems, useful answers to these kinds of questions remain far from routine.

Charles Tiebout (1956) suggested some time ago that the difficulty may not be as severe as it appears. To the extent that households reveal their fiscal preferences for a variety of such goods through their choice of community (voting with their feet), households will self-select into homogeneous groups and may largely resolve many of these problems. An extensive literature has grown up around this hypothesis in an effort to clarify its normative implications together with the equilibrium process it describes. Again, the generality of this work is for the most part limited to specific results for specific cases. A basic positive question has been the effect of finance on the ability and desire of each community to attract a single "type" of resident. This is generally a function of how households and communities differ, and how the number of types of the two compare. In a simple enough model with costless mobility, there is not much more to the story. Households differing only along one dimension in a two-good model will tend to sort across jurisdictions along that dimension. Public policies should be fairly easy to anticipate in this case as each community is comprised of either like households or a well-ordered distribution of households.

A separate normative issue concerns the appropriate design of policy in these
economies. The question may seem moot in open economies with free mobility, especially where communities are homogeneous. Once we accept a role for planners, however, the interesting issue becomes how to identify local demand curves for local fiscal activity. This entails defining the notion of a public service tax-price, which in turn requires a clarification of the process by which fiscal differentials may be capitalized into local prices. It has been argued, for example, that if local prices adjust fully to changes in local demand to equilibrate welfare across communities, price differentials will then tend to reflect net aggregate project benefit differentials. This might be the consequence of the locally inelastic supply of some good, such as land, leading to the generation of Ricardian rents. Both the method and the outcome of any cost-benefit analysis will then depend on the appropriateness of these rents as a welfare measure. This is complicated by the associated implication that individual household tax burdens depend on the effects of policy on the size of the tax base. Differences in towns are therefore endogenous to the extent they depend on the endogenously determined demand for both public goods and housing, and on the sense in which the financing of services is distortionary.

The purpose of this thesis is to work out a fairly general approach to each of these issues in a simple integrated framework, within which specific rules and procedures for characterizing residential economies and setting public shadow prices can be derived. Chapter 2 provides an overview of the simplest possible environment, where each residential jurisdiction is internally homogeneous and interjurisdictional externalities are ignored. Basic positive capitalization results are derived, which are then used to calculate second-best public spending rules for an arbitrary tax structure. This is essentially an effort to generalize the second-best policy rules of Atkinson and Stern (1974) to account for endogenous prices, local budget effects, and endogenous populations. Part of the motivation also comes from an interest in the role of local market conditions, such as the tenure mix of the community, in the property value capitalization process.

Chapter 3 relaxes the assumption of resident homogeneity, which allows for substantial
extensions of the analysis in two directions. With resident heterogeneity, it is no longer the
presumption that households will prefer the same bundle of local policies simply because
they have located in the same town. The first part of the chapter approaches this question by
assuming that policies are chosen by a decisive voter. Preferences are single-peaked in rents,
so strategic considerations are not important, but the endogeneity of the tax price and the
population raise the possibility that equilibrium popular policies may be efficient under
certain conditions. The second part of the chapter abstracts from public choice issues, as
such, to characterize the features of optimally discriminatory public policies. That is, how
should tax and spending policies account for the heterogeneity of the resident pool? If the
local population is sufficiently mixed, for example, it turns out that it may be optimal to
subsidize entry into the community, via negative head taxes, while using property taxation as
a means of distinguishing among residents on the basis of their housing consumption.
Alternatively, a homogeneous town may want to choose a negative property tax rate and
finance itself with a uniform head tax. Although fairly intuitive, these results are new to the
literature.

The final chapter reexamines these questions in an explicitly spatial model. As
expected, this offers several insights to the nonspatial treatment. The chapter begins by
reestablishing the key capitalization conditions, which are seen to depend in a critical
manner on the spatial characteristics of the community in addition to the factors discussed
earlier. The role of tenure turns out to be particularly important in evaluating welfare
expressions, and this aspect is explored by modeling rent flows between the communities of
the larger economic system. Finally, policy design considerations are extended to account for
communities with internal structure. Still more general second-best public spending rules
are calculated.

The present chapter mainly serves to further set up this discussion, by reviewing the
main stories of local fiscal structure and, in particular, the relationship between local policy
actions and local rents. This capitalization process is a fundamental component of the
arguments presented in each chapter, so it is useful to take a few pages here to provide some context. The chapter will proceed as follows. First, the central argument of each of several capitalization themes will be briefly presented using simple arguments. In part, this will amount to no more than a review of past work, but an effort will be made to give particular attention to two issues: differences in preferences and incomes among households, and the conceptual similarities embedded in the views. The final section will attempt to link these theoretical developments with empirical efforts in this area.

2 Markets for locational advantages: on rent theory

Interest in capitalization in urban land markets can be divided into three areas. Much of the original thought on the subject concerned the extent to which observable changes in housing markets could be used to determine the value of local public policy changes. For example, a good deal of discussion centered on how to measure the value of amenity changes, such as the abatement of air pollution (e.g., Polinsky and Shavell (1975)). Similarly, Rothenberg (1965) suggested that the benefits from local improvement projects were reflected in the change in aggregate property values. This use of capitalization as a cost-benefit tool is distinct, however, from its use in support of the so-called Tiebout (1956) hypothesis. In a well-known study, Oates (1969) attempted to empirically determine whether in fact residential patterns among towns revealed the preferences of households for different public service/tax packages. He hypothesized that Tiebout was correct if housing prices reflected local public good levels as well as other locational amenities.

These two approaches are concerned with the separate issues of local project evaluation and the efficiency of decentralized residential equilibrium, both involving the use of observable market indicators to reveal consumer preferences. A more recent application of the notion involves the capitalization of fiscal advantages into rents within a town (Hamilton (1976a)). Basically, the argument here is that differences in tax-prices, associated with local government activity, will depend both on the average property value (which determines the tax
rate) and on each household's property value. If everyone in a town values public goods similarly, high valued properties will effectively subsidize the consumption of households with low valued properties. The differences in tax-price will be capitalized into rents.

Clearly, these various themes touch on most of the major issues in local public finance to one degree or another. The first is concerned with determining the efficient level of local public services, the second questions the efficiency of community composition and the possible motives for competition among jurisdictions for different household types. At the same time, it appears capitalization is being used to tell many different stories simultaneously. Each is associated with a distinct school of models and it is not always easy to see how the different assumptions are related.

**Locational advantages**

It is easy to see how the value of property will depend on that property's locational attributes. Usually, one assumes land to be somewhat fixed in supply and indexed by location so that (static) demand completely determines the rent level. In turn, demand depends in part on the relation of each location to other consumption goods, their prices, and income. The Alonso-Mills-Muth monocentric equilibrium models, for example, mandate the absence of utility differentials across the city for like individuals. Consumption of nonland goods are assumed independent of location. Hence, rents are determined indirectly through their effect on disposable income. The story becomes more complicated when housing is produced with other inputs besides land, or land is used for other purposes besides housing, or housing is durable, or markets are in one manner or another imperfect, but the central tie between rents and location appears incontrovertible.

The introduction of public services into such a model has two obvious effects. As a consumption good, the effect on the desirability of a particular location will depend on the spatial pattern of public good provision. If, as is often assumed, the service is local to the city but uniformly distributed within it, the effect on the demand for land will depend on the
complementarity of the two goods and differences in land consumption by individual households. The net effect in this case, however, is expected to be small. A second factor is the extent to which public service levels induce migration in or out of the community. If the public good is a consumption good, as is likely, local improvements may encourage immigration, increasing the demand for land so that rents rise.

The latter case forms the basis for the first kind of capitalization addressed here. The question is, "How much is a public project worth to society, and what do the associated rent changes tell us?" A simple model will illustrate. The indirect utility function of any one household within a particular town can be represented by

$$V[p(u), y - c(u), g] = U^*$$ for all $u$

which is invariant over the city for like households, and where $U^*$ is the equilibrium utility level and

- $p =$ rent per unit of land
- $y =$ income
- $g =$ public service level
- $u =$ distance to the city center, the commuting target
- $c =$ commuting costs from $u$

and the price of the composite numeraire good is unity. Totally differentiating the utility level with respect to the public good gives us a decomposition of the welfare effect,

$$\frac{dV}{dg} = V_p \frac{dp}{dg} + V_y \frac{dy}{dg} + V_g$$

$$= V_y \left( -h \frac{dp}{dg} + \frac{dy}{dg} + \frac{V_g}{V_y} \right)$$
where the last result follows from Roy's identity, and where \( h \) is land. Hence, the household's valuation of the marginal project is just

\[
\frac{V_g}{V_y} = \frac{dV}{dg}/V_y + h \frac{dp}{dg} - \frac{dy}{dg}
\]

where the right hand side is the marginal rate of substitution between \( g \) and the numeraire. That is, the value or benefit of the project equals the value of the actual change in utility, plus the change in housing expenditure, less the change in income due to the project.

In this setting, changes in housing expenditure fully reflect, or reveal, public service preferences only to the extent utility and income do not vary. As an example, utility will not change if migration is costless, information uniformly distributed, and the city is so small relative to the urban system that immigrants disperse the utility advantage without affecting the system-wide welfare level. [As a variant of this notion, Starrett (1981a,b) assumes there exists a location within the city but distant enough from the project so that people at this location do not benefit directly. Since residential equilibrium requires no utility differential in the city, utilities rise nowhere even though the city is closed.] Income will change only if there exists no rental income, wages are exogenous to the city, or the effect on local production costs and labor supply is such that wages do not change. [In a more detailed version of this model, Polinsky and Rubinfeld (1978) explore the effect on wages. See also the second appendix to chapter 3.]

Although this model clarifies the situation somewhat, several problems emerge in the course of extending the argument to the population of the city—that is, when aggregating. One way of salvaging these results is to claim that is communities are approximately homogeneous with respect to income and that preferences are uniform as well. Support for this assertion comes from versions of the Tiebout model suggesting that under certain conditions households stratify themselves across towns in just that manner (e.g., Ellickson, 1971; Stiglitz, 1978; Burstein, 1980; Bucovetsky, 1981). One is hard pressed to reconcile this thesis with reality
as many know it, although the pattern is recognizably familiar. Without such assumptions, however, it is difficult to justify aggregating preferences without having to also assume an optimal distribution of income for some known social welfare function. (Alternatively, some form of compensation test might be designed or a revealed preference approach using nonparametric estimation methods might be employed; see, respectively, Bruce and Harris, 1981, and Varian, 1981.) In addition, it must be assumed that immigrants are responding to utility differentials corresponding to their household type, which requires there be a sufficient number of members of each group in the economy to capture those differentials. And finally, when looking at the entire city, account should be taken of changes in city size.

In the monocentric case, for example, the city will in general expand in area with an increased population so long as there exist undeveloped or developable land at the fringe. This can perhaps be seen most simply by noting that with a simple additive social welfare function, the utility response to a small project is

\[
d(VN)/dg = d[V_0 \int_{o}^{u} n(u) du]/dg\]

\[
= V_0 [\int_{o}^{u} n(u) du + n(u) du/dg] + N dV/dg
\]

with

- \(N\) = city population
- \(n\) = population at distance \(u\)
- \(u\) = distance to the fringe

In words, the change in social welfare equals the change in utility for all current residents, plus the utility of new residents, including those located in the newly developed area (if \(du/dg > 0\)). (If the city shrinks, the utility of those in the now abandoned areas is subtracted from the whole.)

It has been assumed here that all relevant considerations can be made while ignoring the use of land for nonresidential purposes. This is more reasonable for some situations than
for others. Where residential land competes with other uses and this relation is affected by public outputs, the discussion will have to be amended. For example, where public goods lower the price of other inputs (e.g., transportation, labor) in the private sector production process, one would expect some substitution away from land in production. Ignoring such possibilities, the qualitative results remain sound. The central idea is that of all commodities in the city, land is the most fixed in supply and hence those owning it will capture most of the surplus adjustments in related markets.

Except, that is, for the fact that fiscal externalities due to migration have been ignored. Even if a town accounts for the immigration that will be induced by public benefits, it is unlikely to weigh the cost to other communities of the lost population. That is, if the community plans for immigration, thus offsetting a common source of underprovision of public services, it will then be overproviding those services to the extent that it does not recognize the burdens it places on other cities. Another weak point in this argument is a description of how consumers initially distribute themselves across towns, or how any given system of rents is influenced by the mobility of households. These issues correspond to aspects of residential efficiency and less passive public sector activity for the urban system as a whole. A broader question, then, is how well each town's rents reflect relative advantages.

Interjurisdictional advantages

The theory of community formation owes much to the theory of clubs, a paradigm that emphasizes the role of congestion in determining optimal group population, and the theory of jurisdictional fragmentation, which emphasizes the role of taste and income differences in determining optimal tax/service bundles. A more complete description of town formation with a mobile population, though, would also include a description of the response of property prices as people change location. That is, a land market is needed. Rather than assume towns are sufficiently replicable so that preferences for public service combinations can be satisfied without increases in the rental value of property, it makes some sense to consider
that the excess value of any one community's public service package, net of taxes, relative to available alternatives, may be capitalized into that town’s property value. These changes in property values should in turn affect the adjustment toward a final residential equilibrium.

This theme can be explored at several levels. First, take the public good levels to be exogenous. Here, residential equilibrium in a system of otherwise similar towns should require the marginal net benefit (marginal benefit less the change in taxes) of residing in a given town to just equal the difference in rents due to changes in the public good level. That is, if the only difference between towns is the public service level, the differences in net benefits would be capitalized into rents.

Starrett (1981a,b), however, argues that systematic differences between towns will neutralize these differentials without capitalization effects. Basically, this result depends on the relation between household income and project benefits; in particular, if income does not vary by household residential location, then community stability conditions require the relation between cost changes and public good changes be such that where costs are higher so are incomes. If a household migrates, it must pay the appropriate taxes. Hence, net income changes. Sufficiently small projects will induce no migration so long as income effects are normal and perceivable.

The next step in this analysis would be to connect the capital gains or losses to the public choice mechanisms determining those service levels. This approach forms the basis for much of the recent work on local profit-maximizing monopoly governments. [See, for example, Frankena and Scheffman (1981), Hamilton (1978), Sonstelie and Portney (1978, 1980).] Probably the clearest example of the process is found in Yinger (1982), who divides households into residents and movers. Residents are immobile and vote for public services and associated financing schemes that, in turn, comprise the choices available to home-seeking movers. Movers treat each town's fiscal package parametrically as they bid for property, so that the perceived differences in the towns' bundles are capitalized into rents. Once they become residents, by buying in to the community, these former movers become
Intrajurisdictional advantages

The third and closely related variation on this theme is the capitalization of fiscal advantages (disadvantages) realized within a given town without internal structure. In Hamilton (1976), a proportional property tax is imposed based on average property values. In towns with housing stocks of mixed value, a fiscal surplus and a fiscal burden is incurred by each low-valued and high-valued property (as deviations from the mean value), respectively. These are capitalized into land values, since capital is somewhat mobile, and just reflect the differences in benefits among households for the public good (all are assumed to have similar preferences). If the system of differential land rents can be maintained (by zoning, for example, since land is supplied elastically in this model), the implication is that public goods will be supplied efficiently.

Mieszkowski (1978) and Straszheim (1981) extend the analysis to households with different tastes for public services. In part, they demonstrate that if the rich prefer more public goods than the poor, then a distortion is introduced by the tax structure. This negates the exact relation between preferences and rents in a mixed town, since the total value of the land is no longer independent of the nature of the mix. Explicitly considering a system of such cities and the role of household mobility clarifies the similarities between this and previous approaches. For example, consider the efficient allocation of dissimilar households among a system of cities, each providing a different service level. Households will be assigned to the towns requiring the least resources to maintain a given welfare level. In a decentralized world, this would occur only where competition for land generates rents that capitalize net public benefits.
3 The evidence

Many studies have attempted to test, in one sense or another, various aspects of these issues. The pioneering work of Oates (1969), for example, is aimed at testing the Tiebout hypothesis. It is well understood that this might represent a number of different environments, starting from systems of homogeneous communities. Although this line has sought to explore the relation between local fiscal variables and property values, it seems to have done little more than show that households value public services. Further, Epple, Zelenitz and Visscher (1978) and Brueckner (1979) attempt to demonstrate that this approach is no text of Tiebout's conjectures at all, for several reasons. Another perspective is offered by Edel and Sclar (1974) and Hamilton (1976b), who argue that property values are just benefit taxes in long run equilibrium, so that neither expenditures nor taxes should bear a systematic relation to market rents.

The results from these different approaches have been mixed. Oates (1969) regressed property values on a vector of public expenditure levels, property-tax rates and various other land and housing variables. He found that increased public services increased property values and that increased taxes decreased property values, other things being equal. Meadows (1976) used a somewhat more sophisticated model of simultaneous expenditure and tax determination, confirming Oates' results. However, the value of his coefficients on both fiscal variables were considerably lower (in absolute value) than Oates'. Edel and Sclar (1974) reproduced the Oates regression equation on a series of five cross sections of greater Boston area towns, for 1930 and each subsequent decade. The coefficients on the fiscal variables declined in absolute value substantially with time, consistent with the property-taxes-as-benefit-taxes story. Pollakowski (1973) was unable to reproduce Oates' results with a microdata sample of greater San Francisco area communities. Rather, his estimates indicated that changes in the property tax rate were enormously overcapitalized, and that changes in service levels did not affect rents significantly.

Brueckner (1979) attempts to test the efficiency of public service provision directly,
using Oates' original 1969 data. He develops a bid-rent model of property value determination, where rents are related in a complicated way to other fiscal variables by way of the endogenous, budget-balancing tax rate. Assuming that the sign of the coefficient relating property values to public expenditures reflects the efficiency of those expenditures (so that there should be no change in rents for small changes in an efficient level of services), he finds the estimated coefficients to be nonpositive. This implies at best that the public services are overprovided in the 'bedroom' communities examined.

There are several explanations for these discrepancies. For example, Epple, Zelenitz and Visscher (1978) make two main points in this regard. First, that a proper test should test for Pareto optimal Lindahl communities against the alternative hypothesis of heterogeneous towns. Their second point, however, is that the two models are simply not distinguishable by the econometric tests employed to date. In this vein, Epple (1980) reviews several of the assumptions being implicitly tested in this kind of analysis. This allows him to make firm statements about what capitalization tests are unable to test, and to make specific comments about particular econometric problems. More recent studies that spell out various specification, estimation and data explanations for differing results include Bloom, et al. (1980), Chaudry-Shah (1983), and Martinez-Vazquez and Ihlanfeldt (1984).

4 Summary

Underlying each approach taken, either in theoretical or in empirical work, is a model of the same basic adjustment mechanism. These stories can be somewhat integrated by considering a closed urban system with a finite number of cities, say two. If households are homogeneous, incomes are independent of location, and there exist no other locational attributes, then a state of equilibrium can be characterized by equal land consumption and equal utilities. Now say one town increases its public output by $\Delta g$, and residents value the service so that utilities rise there (ignoring taxes for the moment). People in the second town will migrate to the first, particularly if migration is costless, bidding rents up and bringing
the system utility level up to some extent in equilibrium. If residents are divided among landowners and tenants, increases in rents in the first town will represent the change in consumer's surplus of tenants captured by property owners. This is a simple story of the first group of models.

Just the opposite will have happened in the second town, however. Even though the system-wide welfare level is higher, the demand for land has fallen there, lowering equilibrium rents. The landowners' capital losses represent the fiscal externality imposed by the first town, as well as consumer surplus gains to residents of the second town. If the new public good level was an efficient one, the sum of the rent changes in each town will equal the value of the overall utility gain. The method of financing the service change may interfere with this process, but not if taxes are on property fixed in supply. The so-called Henry George theorem suggests that under certain conditions of efficiency in public goods supply and a fixed tax base, the surplus accruing to the fixed good will equal the cost of public good provision. That is, a confiscatory property tax in this system would be the efficient means of financing public outlays. Put another way, aggregate land rents will be equal to the value of the public service levels. This tells the part of the story of the second group of models that is most relevant here.

Finally, allow for different lot sizes in the first town, with one lot size half as valuable as the other. The property tax rate will be based on the average property value, so that households in the larger lots will pay exactly twice the taxes as fellow residents on smaller lots. If households are identical, their desired service levels will vary only insofar as their tax prices vary, which in turn depend on the value of the their lot. Hence, the willingness-to-pay and tax price of each household will differ when some service level is chosen if it is chosen by unanimity. The differences in tax price will be capitalized and we are back to the setting of the original scenario.

Little conclusive has surfaced from the empirical research, on the other hand, beyond the result that property values are related both to the costs and (various measures of) the
benefits of public policies. As has been discussed, this is due mainly to the quality of the
data, the diversity of modeling approaches, and ambiguity regarding the nature of the
capitalization and residential equilibrium process. Even with the large body of work
completed, several outstanding problems remain with the empirical strategies employed thus
far [see the appendix to chapter 2 for an extended discussion].

Finally, we know that in reality local jurisdictions can not be replicated in sufficient
numbers to match exactly the preferences of even somewhat large aggregates of households, a
condition some of the existence models require. Even if they could, there are additional
problems with formal concepts of equilibrium in this setting which have not been discussed
here (cf., Bewley, 1981). It is also true, that communities change in character and composition
with time, subject to adjustment frictions, suggesting that the extent and flavor of
capitalization will depend on the pattern of excess demands given distortions and various
disequilibrium tendencies. While this thesis will not address dynamic processes directly,
adjustment costs of several kinds will be emphasized as critical elements of the local public
economy.

The remainder of this thesis will review these issues in a fairly narrow but insightful
manner. The main problems with the literature are, again, the lack of a strong analytical
setting that is both simple and general enough to illustrate the nature of the capitalization
process. Such a model is developed in the next essay. Several interesting results are
established in this framework that clarify and structure the normative analyses that follow.
Chapter 2
Project Evaluation and Shadow Pricing in Local Public Finance

1 Introduction

The main concern of this chapter is how the allocative goals of public policy might best account for a mobile population in an open economy setting. The framework is a Tiebout-like model, where households freely migrate between independent jurisdictions offering various fiscal packages. This is well traveled ground in local finance theory, so it is surprising that policy design when policy tools induce distortions has been somewhat neglected. While much of the modern normative theory of the public sector has concerned itself with just this issue, there has been surprisingly little analysis of second-best decision rules in small open economies. Rather, following Tiebout (1956), the literature has typically emphasized the welfare properties of equilibrium location and price patterns in a first-best world, often with fixed prices and essentially lump-sum tax structures [e.g., Bucovetsky (1981)].

More recently, Turnbull and Niho (forthcoming), Zodrow and Mieszkowski (1986) and, in a series of papers, Wilson (1984, 1985, 1986a, 1986b), have dealt specifically with the taxation of somewhat elastic capital in an open economy context. Among other things, these papers show that the Samuelson (1954) condition for efficiency in public goods supply is unlikely to hold if the tax is distortive. In a closed economy setting, this result is not at all new. As outlined by Pigou (1947), and substantially clarified by Atkinson and Stern (1974) and Wildasin (1979), second-best tax and spending policies will depend critically on the general equilibrium characteristics of the economy. Because the distortive effect of local capital taxes, for example, will vary with the elasticity of the tax base, optimal policy rules will have to be adjusted for whatever general equilibrium relations capital may have with other market behavior. It follows that public spending rules should vary from the first-best to an extent determined by the interdependence of public services and the taxed goods, so that the second-best level of local spending may be either more or less than in the absence of
The issue is also addressed in the somewhat distinct capitalization literature discussed in chapter 1, which emphasizes the process by which local prices may reveal the market value of community fiscal and amenity differences. This research has made it clear that the use of intercommunity rent differentials as shadow prices in project evaluation depends in a critical way on the tax structure. This is particularly important in small economies where the need to raise revenues locally likely imposes distortions on at least two margins: taxes may change the relative cost of being a resident of a given town, and they may affect relative commodity prices.

Against this backdrop, the contribution of this chapter is twofold. An attempt is first made to work out a fairly general theory of cost-benefit analysis for local economies within which specific rules and procedures for setting shadow prices can be examined. On the spending side, there are information and aggregation issues to consider that essentially involve the difficulties associated with discovering what individuals want from the public sector. On the tax side, we will want to consider the distortions associated with an elastic tax base in the development of second-best policy rules. The model employed provides an overview of the simplest possible environment, where each residential jurisdiction is internally homogeneous and interjurisdictional externalities are ignored. Basic positive capitalization results are derived, which are then used to calculate second-best public spending rules for an arbitrary tax structure. This is essentially an effort to generalize the second-best policy rules of Atkinson and Stern (1974) to account for endogenous prices, local budget effects, and endogenous populations. A central feature of the present approach is our interest in working out the consequences of any particular change in public supplies as a fundamental aspect of project appraisal.

Part of the motivation for this study is an interest in the role of local market conditions, such as the tenure mix of the community, in the property value capitalization process. Consequently, the second theme of this chapter is the sensitivity of these public
shadow prices to the form of household income. Our interest concerns the apparently mixed incentives a local resident might face when price and income changes move in opposite directions in terms of their welfare effects, and how this might affect the nature of residential equilibrium. We find that in a suitably restricted setting consumer mobility is consistent with an efficient equilibrium if resident homeowners realize capital gains. However, access to capital gains and nondistortionary finance implies that all households face the same rent profile, regardless of their community of residence. This is a consequence of the interaction of policy design considerations and self-selection constraints.

These issues are presented in sequence. The next section quickly reviews a standard local finance model, with particular emphasis on the capacity of local markets to measure welfare in equilibrium. The analysis assumes that households sort across communities by income, and the implications of the conditions that give this result receive particular attention. (Analysis of these issues is extended to a setting with heterogeneous residents and a wider variety of policy tools in chapter 3.) The idea is to integrate the Tiebout story of residential sorting with the notion that local prices may contain information about the market value of fiscal differentials. In the third section, shadow pricing rules are derived that incorporate this information. In each of these sections, the role that the distribution of local rental income may play is analysed. The concluding part of the chapter summarizes these results.

2 A model of residential equilibrium

In this section, we present and extend a fairly standard static model of local finance to illustrate some useful results and establish a benchmark for the discussion that follows. It is important to note that the model is extremely simple in structure, and has been restricted in several significant respects in order to focus on the issues at hand. To begin, consider an economy of independent local jurisdictions, comprised of completely mobile households who differ only in income. Both the number of communities and the system population are fixed in
size. Preferences are assumed to be well behaved, and we ignore any nonfiscal distortions or interdependencies in the economy, as well as any interjurisdictional externalities that may arise from population movements or interjurisdictional spillovers. Our interest is in characterizing equilibrium, which is assumed to exist.

We also adopt the convention that each community is a sufficiently small part of the system so that local changes do not upset system-wide variables. This is somewhat equivalent to restricting ourselves to a single market in the partial equilibrium sense, except that we also allow a good deal of interaction among "local" markets. The argument could be made that this sort of assumption seriously undermines any attempt to deal with "general equilibrium" considerations, as well as being in some sense inconsistent with the assumption of a finite number of communities. In a way, it represents an imposition of competitive price-taking assumptions about behavior, without allowing for aggregate general equilibrium effects. That is, even though each community may behave as though it were insignificant, the interaction of communities will clearly affect the system-wide level of prices and hence welfare. This is particularly likely to be the case in the kind of metropolitan economies with which we are most familiar. A Nash equilibrium concept, for example, would illustrate the conflict more explicitly, and this approach will be explored in later work.

We now describe household behavior in this economy. Let the solution to the household problem of choosing housing consumption $h$ and other private goods consumption $x$, the numeraire, be represented by the indirect utility function

$$V(r,y,g) \equiv \max_{h,x} U(h,x,g) \quad (2.1)$$

subject to $rh + x \leq y$.

where

$r \equiv p(1+t) = \text{gross-of-tax price of housing}$

$g = \text{household public goods consumption}$

$p = \text{net-of-tax or market price of housing}$
\[ y = \text{(disposable) income} \]
\[ t = \text{ad valorem property tax rate} \]

(2.1) is also known as a direct-indirect utility function, or a conditional indirect utility function, since it does not represent household optimization over \( g \). Rather, (2.1) is the maximum utility available within a town offering a public service level \( g \) at unit rent \( r \). As such, it implicitly defines the gross-of-tax bid-rent schedule

\[
r = (1+t)p(g, t, y, U^*)
\]
\[ \equiv \max_{x, h} \frac{(y - x)}{h} \quad \text{s.t.} \quad U \geq U^* \]

where \( U^* \) is the equilibrium utility level in the system of communities. Define, for later use, the expenditure function of some household as the minimum expenditure necessary to give some level of utility, or

\[
e(r, g, V) \equiv \min_{x, h} x + rh \quad \text{s.t.} \quad U \geq U^* \]

We also have the result that

\[ y \equiv e(p(g, t, y, U^*)(1+t), g, U^*) \]

The indirect utility and expenditure functions serve to summarise household behavior, given prices, income and public services. Their main advantage is easy access to the envelope theorem, via Roy's "identity" \( h = -V_r / \lambda \), where subscripts denote partial derivatives, and \( \lambda \equiv V_y \) is the marginal utility of income and Shephard's lemma \( e_r = h(r, g, U) \).
From (2.1) it follows that the shadow price of a marginal change in local public spending is

\[ V_r r_g + V_y y_g + V_g = 0 \]

which describes an indifference curve in \((r, y, g)\) space. From the properties of the indirect utility function, this can be solved for the partial derivative of the gross-of-tax unit rent with respect to public services:

\[ r_g = \frac{(y_g + b)}{h} > 0 \]

where \( b \equiv V_y / \lambda \) is the dollar value of the project benefit. Similarly, unit bid-rents change with tax rates according to

\[ r_t = -\frac{y_t V_y}{V_r} = \frac{y_t}{h} \]

the sign of which has the sign of \( y_t \). Hence, \( r_t = 0 \) for \( y_t = 0 \), as the willingness-to-pay for taxes is just zero in the absence of real income effects. Gross bid-rents change with the income of residents according to

\[ r_y = -\frac{V_r}{V_y} = \frac{1}{h} > 0 \]

Note that \( p_g = r_g/(1 + t) \), \( p_y = r_y(1 + t) \) and \( p_t = (r_t - p)/(1 + t) \).

An important part of this story is how prices behave when households choose their community of residence, where this process can be said to represent the choice of \( r \) and \( g \) to
maximize $V$ in (2.1). Following Ellickson (1970, 1971), we recognize that households will be willing to trade off the gross price of housing against public goods at a rate given by the slope of their indifference curves in $(r,g)$ space. By Roy's identity this is given by

$$- \frac{V_r}{V_g} = \frac{b}{h} \quad \text{(2.3)}$$

(2.3) gives us a measure of relative taste for a project in terms of the cost of living, here limited to housing costs, in the community where it is available. These indifference curves have a slope equal to the project benefit per unit of housing consumption, or what we will sometimes call the "normalized" benefit.

Equation (2.3) defines a family of bid-rent functions for each household. Ellickson used this fact to establish a condition for homogeneity within towns, given a sufficient number of community types. If households differ only in income, they will self-select into homogenous towns if public spending demand is more (or less) income elastic than housing demand. This can be seen by differentiating the RHS of (2.3) with respect to income, giving

$$(b/h)_y = b_y/h - h_y/(b/h^2),$$

which is positive if $\varepsilon_{by} > \varepsilon_{hy}$ where $\varepsilon_{ij} = \partial \ln i / \partial \ln j$ = the elasticity of $i$ with respect to $j$. This is illustrated graphically in Figure 2.1, for the case of two income classes.
Put simply, the sorting assumption states that the 'demand' curves for \( g \) will steepen with income, so that if \( g \) is a normal good, demand for it is more income elastic than the income elasticity of housing demand. The following discussion provides a more detailed analysis of how these relationships are affected by the magnitude of the household differences involved. It also provides an alternative interpretation of the sorting condition, by demonstrating that households who differ only in income will self-sort across communities if the marginal utility of income is proportionately less (or more) sensitive to rent than to project size. This follows from the result that if public spending rises, the marginal value of real income increases. If rents rise, the marginal value of real income falls in a magnitude corresponding to the propensity to consume housing out of that income. Consequently, the net effect depends on how the two responses compare.

From the properties of the expenditure function, we have these relationships, in residential equilibrium for fixed \( V \):

\[
e_r = h(r, U', g) = -\frac{V_r}{\lambda}
\]
Using (2.5) and (2.6), we have \( \lambda = 1/e_V \), so that

\[
\lambda_g = -\frac{e_V g}{e_V^2} = -e_V g \lambda^2 \tag{2.7}
\]

From (2.5) we also know that \( e_g = -b(r,y,g) \), so that \( e_g e_V = -b_y e_V = -b_y/\lambda \). Hence, substituting back into (2.7) gives the result, for \( b_y > 0 \),

\[
\lambda_g = b_y \lambda > 0 \tag{2.8}
\]

If prices respond to a change (differential) in public spending so as to leave the household no worse or better off than before (elsewhere), the marginal utility of income rises if \( g \) is a normal good. Moreover, \( \lambda \) will rise more the higher its initial level and the higher the income elasticity of demand for \( g \). If rich households value public goods more than the poor, the relative value of rich households' income will be higher.

Similarly, the marginal utility of income decreases in gross housing prices as can be seen by following the same steps as above and obtaining:

\[
\lambda_r = -h_y \lambda \tag{2.9}
\]

which is more negative the larger the income elasticity of housing demand. If rents rise, but households are compensated by fiscal differentials, the marginal real value of income and
hence the marginal utility of income are reduced more strongly for those who are more heavily
inclined to spend marginal income on housing.

So, if public spending rises, the marginal value of real income increases. If rents rise, the marginal value of real income falls in a magnitude corresponding to the propensity to consume housing out of that income. Consequently, the net effect will depend on how the two responses compare. From (2.8) and (2.9) we obtain $\frac{\lambda_g}{\lambda_r} = -\frac{b_y}{h_y}$, where $\frac{b_y}{h_y} > \frac{b}{h}$ from Ellickson's condition. Hence, self-sorting requires that

$$-\frac{\varepsilon \lambda_g}{\varepsilon \lambda_r} > \frac{b}{h}$$

This implies that

$$\varepsilon \lambda_g > -\varepsilon \lambda_r$$

if we assume that gross rents in a homogeneous town are less than fully explained by fiscal benefits ($r < b_g$). This establishes the result. The Ellickson sorting condition stated that for indifference curves in $(r, g)$ space to steepen with $y$, normalized benefits must increase with income. Putting it somewhat differently, (2.10) says that the value of money must be less sensitive to rent variation than to variation in public services for sorting to occur.

We can also express changes in the real value of income to each income class in the absence of compensation. These expressions would be relevant in the short run or to a closed economy analysis, for example. Since $\lambda = \lambda_{g,r,e(g,r,V)}$, the uncompensated change in the marginal utility of income with respect to public spending is

$$\lambda_g = \lambda_g c + \lambda_y e v v_g$$

where the first term on the RHS is the compensated change in the marginal utility of income
we have been examining above. We already know from (2.5) and (2.6) that $e V g = b$, and from (2.8) that $\lambda g^c = b \lambda$. Substitution gives $\lambda g = b \lambda + b \lambda y$. The uncompensated effect of $dg$ on the real value of income will depend on the size of the benefit, weighted by $\lambda y$, in addition to the factors discussed previously. If $\lambda y$ is negative, as is sometimes assumed, then the real value of income for wealthier households will be relatively less the larger their benefit.

Given this framework, the following sections address the issue of policy design. This amounts to a determination of the shadow price of public spending in the face of various kinds of preexisting policy distortions, conditional upon the particular environment at hand. The case of exogenous household income, or a "tenant economy," is examined first, and the case of endogenous rental income follows in section 4.

3. Project evaluation in a tenant economy

The discussion in the previous section described an equilibrium which is sensitive to the differences among household types as well as the existing pattern of public policies in the economy. A basic result is that if tastes change with income, households will tend to outbid each other for the different levels of services in different towns. In particular, we will commonly assume that those groups with higher incomes will live in communities with higher public spending and higher associated gross-of-tax unit housing prices, as in Figure 1. Communities will tend to be homogenous, in the absence of constraints on the supply of communities. (See chapter 3 for the case where perfect sorting does not hold.)

We now wish to calculate the value of public policies, and policy differentials across communities, with an eye towards the derivation of useful policy rules that apply to these settings. This goal can be related to several themes emphasized in the recent theoretical literature in project evaluation [e.g., Drèze and Stern (forthcoming)]. The first is that market prices are presumed to be distorted, whether because of undesirable governmental interventions or the absence of optimal interventions, a problem that is likely more important
the smaller and more open the economy under consideration. Another theme is the assumption that the relevant public authority is faced with the problem of determining welfare maximizing shadow prices, discount rates, spending rules, etc., for use in public project evaluation. This authority has relatively unrestrained powers in the exercise of this task, but essentially no powers to influence the governmental tax policies, etc. that are responsible for, or could eliminate, the distortions in market prices. For example, local governments have limited control over the openness of their borders, the choice of tax instruments, or the level of extra-local tax and spending levels. Consequently, they must treat existing market distortions parametrically in the welfare maximizing exercise. A third theme is that the literature attempts to develop rules for guiding the public authority in its task, which consist of principles for deriving the optimal set of shadow prices from observable, or potentially observable, data.

The present section attempts to clarify the issues involved by analysing a particularly simple general equilibrium model in order to identify various shadow pricing rules and the conditions under which they hold. The procedure is one of exploring various issues of interest in local economies, such as the respective roles of alternative tax instruments, property value capitalization, and land ownership patterns. For example, while budgets may not concern the individual tenant, they do affect the nature of equilibrium.

We begin by noting that the utility effects of a small project can be expressed as

\[
\frac{dV}{dg} = V_r \frac{dr}{dg} + \lambda \frac{dy}{dg} + V_g
\]

Cross-sectional equilibrium requires here that households be indifferent between locations, so that \( \frac{dV}{dg} = 0 \) and giving

\[
b = h \frac{dr}{dg} - \frac{dy}{dg}
\]

(2.11)
from Roy’s identity. Hence, if disposable household income is independent of local policy \((dy/dg = 0)\), gross marginal spending benefits are capitalized into gross property value differentials, as a condition of residential equilibrium. This result is familiar from elementary welfare economics, and has several interpretations in this context.

A seemingly separate issue is what happens to the existing pattern of rents when there is a change in the behavior of a single local community. However, if the changed community lies along the frontier of existing communities, the cross-section pattern can be used to describe the effect of such a change in behavior. This result vaguely resembles the Modigliani-Miller (1958) theorem, and is valid only along the envelope of communities. That is, we assume that there exist a range of g-space along which \(dy = 0\). At this margin, prices and/or income will adjust to compensate system residents for the change to restore equilibrium, such that rational, forward-looking tenants will be indifferent to public policies. In the present setting, a local project generates direct benefits and hence surplus for local residents, but the community is so small that there is no net effect on the system-wide level of welfare as excess demand for residence in the community gives way to higher associated rents. Rather, property owners capture all local surplus. These are true Ricardian rents only if housing is fixed in supply, where their presence or absence has no behavioral effect on the supply side. They could be taxed away completely, perhaps in the form of rent control, and their value would just equal project benefits. This is yet another example of the so-called Henry George Theorem, implying that in this special case choosing fiscal policies to maximize these rents is a dual problem to maximizing resident welfare.

Put still another way, equilibrium in a tenant economy requires that variation in tax rates and project benefits be offset exactly by variation in the only locally variable price, house rents. Together, these results suggest that public project evaluation is a fairly straightforward exercise, as rents measure benefits, which are uniform within each town. Planners need only to keep their eyes on the rent level while they maintain the local fiscal budget. On the other hand, the rationale for fiscal policy, other than as a sorting mechanism,
is not clear. A project in any but the wealthiest community will lead to gentrification, as better off households immigrate and outbid residents, which is hardly in the interest of the tenant population. Nonresident property owners are the most visible beneficiaries of upgrading, but it is unclear what role they play in a tenant system.

We now characterize residential equilibrium in more detail, by examining the capitalization process more carefully. The idea is to separately identify the roles of local spending, local taxes and market characteristics. Using the definition of gross rents, and assuming income is exogenous, (2.11) can be rewritten in expanded form as

\[(1+t)\frac{dp}{dg} + \frac{p}{dt/dg} = \frac{b}{h}\]  

(2.12)

where the left-hand side of (2.12) (and hence the RHS) must be the same for all residents in a given community. We intend to allow for some endogeneity in house prices, and then to explore the informational content, via local general equilibrium effects, of those prices given some variation in fiscal variables. Rather than limit ourselves to the case where \(dp/dg = 0\), we assume that the market for housing is itself local in the sense that the equilibrium rent is determined by the interaction of local, and somewhat elastic, supply and demand. The process of property value capitalization is therefore essentially one of market clearing. Again, one issue is the use of market activity to measure welfare, while another is an exposition of the equilibrium process in this setting.

From the bid-rent function (2.2), let \(p = p(g,y,U,X)\), where \(X\) is a vector of nonfiscal variables that help to determine local housing demand, and hence the local price. Differentiating and substituting into (2.12) gives

\[|p(1+t) + p| \frac{dt}{dg} = \frac{b}{h} - pg(1+t)\]

(2.13)

for \(dy = dV = 0\). The right-hand side of (2.13) is the direct impact of variation in public
spending on household welfare. The left-hand side is the impact due to any changes in tax rates that accompany the spending changes. For residential equilibrium, (2.13) shows that the two impacts must be just offsetting. Yinger (1982) in particular has argued that mobile households with exogenous income will require that each side of this equation be equal to zero; i.e., that the marginal benefits and costs of public spending and taxation should independently net out. The first order conditions for choice of g and t to maximize (2.1) are, respectively,

\[ p_g (1+t) - b/h = 0 \]  \hspace{1cm} (2.14)
\[ p_t (1+t) + p = 0 \] \hspace{1cm} (2.15)

Substituting these back into (2.13) gives the result.

This is not surprising. We have given system residents an enormous amount of latitude in their choices, and they individually select the price they pay for fiscal variation. To maximize utility, which in this case amounts to minimizing the expenditure required to maintain utility, they will choose to have no variation in gross prices across tax rate differences. No value is placed on taxes, as such, so compensated demand for them is zero and gross-of-tax housing prices will show no variation at all across tax rate differences. Potentially mobile households are indifferent to the level of the property tax (and, presumably, do not bother to vote for or against property tax limitations). From (2.14), households will also equate variation in gross prices across spending differences to the associated normalized benefits (benefits per unit housing consumed) of that variation. The value of spending differences depends on the household project benefit and on the level of housing consumption. Compensating price differences will then depend on these characteristics as well. Still, utility maximizing g and t are not necessary for residential equilibrium; necessity requires only that the price effects of the fiscal changes are offsetting, not that each separately net to zero, so that households are indifferent between towns that
differ only in these respects. Residents may well be willing to pay for taxes that finance
desired services.

The next two subsections take the story one more step by analysing the price
adjustment process in detail for different finance structures. The public budget is
considered explicitly, as are supply constraints, and shadow pricing rules are derived in each
case.

**Shadow pricing rules with head taxation of tenants**

Assume for the moment that local services are financed exclusively with head taxes, so
that \( r = p \), and the local fiscal budget will be \( g = NT \), where \( N \) is the town population and \( T \) is
the per capita head tax. Housing prices do not enter the budget directly. To finance a one
dollar project, revenues must change by \( N \, dT/dg + T \, dN/dg \). Changes in taxes will affect
income, and residential equilibrium will require that marginal benefits be offset by the
associated changes in rents and income. The change in a resident's income will be just
\( dy/dg = -dT/dg \). Since household income has fallen, one might expect emigration. However, along
the envelope of communities, the cross-section pattern of rents can be used to describe the
value of a local project as in (2.11). Resident households will be indifferent to moving out or
in so long as the direct benefits of the project are exactly offset by the change in tax
liabilities plus any associated market-induced changes in housing costs. That is, \( dV/dg = 0 \)
implies here that

\[
b = h \, dp/dg + dT/dg \\
(2.11')
\]

From the budget, the needed change in taxes will be \( (1 - T \, dN/dg)/N \).

We also need to solve for the change in the unit house price induced by the project.
Equilibrium in the housing market requires that demand equal supply, or
\[ H(p,y,g) = S(p) \]

where \( H \) is aggregate housing demand and \( S(p) \) summarizes supplier behavior in the aggregate. Differentiating the equilibrium condition gives

\[
\frac{dp}{dg} = - \frac{(H_y \frac{dy}{dg} + H_g)/(H_p - S_p)}
\]

Although we will consider other possibilities shortly, take income exclusive of taxes as fixed, so that \( \frac{dy}{dg} = - \frac{dT}{dg} \). We can obtain a decomposition for \( H_g \) by recalling the identity

\[ h(p,g,y') = h^c(p,g,U') \quad (2.16) \]

where \( h^c \) is the compensated demand for housing, and \( y' = e(r,U',g) \), from (2.8); i.e., compensated and ordinary demands intersect at some \((U',y')\). Note that

\[ e_g = - h \frac{dr}{dg} \]

is the CV associated with some small project \( dg \) in a tenant economy. With costless mobility, residential equilibrium requires that households be compensated for differences between communities, and, with other prices fixed, the compensation will come in the form of housing costs.

Differentiating (2.16) with respect to \( g \):

\[ h_g + h_y e_g = h_g^c \]

where \( e_g = - b \) from (2.5). Substituting for \( e_g \) and multiplying by \( N \) to aggregate gives:
\[ H_g = H_g^c + bH_y \]

where \( H_g^c \) is the derivative of aggregate compensated housing demand with respect to \( g \). Incorporating the budget condition allows us to solve for the change in prices necessary to clear the market and balance the budget:

\[
\frac{dp}{dg} = -\left\{ (b - dT/dg)H_y + H_g^c \right\}/(H_p - S_p)
\]

Substituting from (2.11) gives

\[
\frac{dp}{dg} = -\frac{H_g^c}{(H_p^c - S_p)}
\]

(2.17)

using the fact that \( H_p^c \equiv Nh^c_p = N(h_p + hh_y) = H_p + hh_y \), from the conventional Slutsky decomposition. The RHS of (2.17) is positive so long as excess demand is decreasing in \( p \), and \( H_g^c > 0 \).

(2.17) deserves several comments, if only because is it the most straightforward expression for the change in the rent level we will see in this thesis. The total change in the unit market rent naturally depends on the relative responsiveness of the tax base and of excess demand to service and price variation. The more complementary are aggregate compensated housing demand and the project, the more unit rents will rise. On the other hand, if compensated demand falls with the project (i.e., households require less housing to maintain some welfare level as service levels rise), then unit prices will necessarily fall at a rate that is independent of the level of direct benefits \( b \). Moreover, if the aggregate compensated demand for housing is independent of \( g \), rents will not change at all. Marginal benefits will just equal marginal tax bills; i.e., the head tax will be a benefits tax if and only if aggregate compensated housing demand is independent of public services.
Substituting (2.17) into the benefit equation (2.11') gives

\[ b = \frac{dT}{dg} - h \frac{H \gamma}{(H - S_p)} \]

Multiplying by \( N \), to get a measure for aggregate benefits, and using the fiscal budget constraint:

\[ Nb = 1 - T \frac{dN}{dg} - H \frac{H \gamma}{(H - S_p)} \] (2.18)

The LHS of (2.18) is the aggregate marginal rate of substitution (MRS) for \( g \). The marginal rate of transformation (MRT) is just the marginal cost $1 in this simple case, so that if public spending is financed with a budget-balancing head tax, then second-best efficiency in the supply of local public goods requires that, in a tenant economy, a spending level be chosen such that

\[ N \text{ MRS} - \text{MRT} = - T \frac{dN}{dg} - H \frac{H \gamma}{(H - S_p)} \]

The first term on the RHS is what might be called a Tiebout-tax term. It represents the effect on tax revenues of the local policy change, so that the Samuelson (1954) efficiency criterion for public goods provision must account directly for the project-induced revenue change. As mentioned, the second term is the change in prices. This shows up in the welfare expression because local rents are going to outsiders; i.e., nonresident property owners. Although we will not show it here, it is easy enough to see that if all rents were returned to residents, these "pecuniary" externality price terms would wash out. Finally, note that the tax is not lump sum in nature if it affects migratory behavior and housing consumption.
Shadow pricing rules for tenants with property taxation

We have derived a shadow pricing rule (2.18) for public outputs, in terms of residents' willingness-to-pay, that accounted for public budget considerations as well as the characteristics of the local market. These results are now extended to consider excise taxes that distort the choice of housing consumption as well as community choice. The benefit equation becomes (2.12). Tax burdens are determined by the fiscal budget $g = tpH$, where the level of services is again assumed equal to the service expenditure. Differentiating the budget gives an expression for the change in the revenue base when spending increases by a unit, now expressed in elasticity form:

$$MC = 1$$

$$= (1 + t_{H_p})pH \frac{dt}{dg} + (1 + \varepsilon_{H_r}) tH \frac{dp}{dg}$$

This gives an expression for the associated change in the tax rate:

$$\frac{dt}{dg} = \frac{1 - (1 + \varepsilon_{H_r}) tH \frac{dp}{dg}}{(1 + t_{H_p})H}$$

Substituting this into the benefit measure and simplifying, we obtain the fairly straightforward expression

$$b = h(1/H + \frac{dp}{dg})/(1 + t_{H_p})$$

(2.19)

The marginal project is equated with the associated marginal cost at the optimum, which has two parts when accounting for tax base effects: the marginal tax price, $h/H$, and the change in expenditures on current housing due to the change in prices, $h \frac{dp}{dg}$, where each of these is deflated by one plus the net price elasticity of demand weighted by the tax rate. The second of these terms, and the denominator, have typically been overlooked in earlier studies,
although it is clear that the tax price is a good approximation to marginal benefits for small price changes if demand is not too price elastic.

Letting $p = p(t, g, X)$, as before, we have

$$\frac{dp}{dg} = \rho_g + \rho_t \frac{dt}{dg}$$

$$= \left[ (1 + \varepsilon_{Hr}) \varepsilon_{pg} + \varepsilon_{pt} \right]/(1 + \varepsilon_{Hr})$$

Substituting into (2.19) gives an expression for benefits in terms of the marginal effects of each policy tool on prices and demand:

$$b = (1 + \varepsilon_{Hr}) \varepsilon_{pg} + \varepsilon_{pt}$$  (2.20)

with $\theta = 1 + \varepsilon_{hp}$.

The marginal cost of changing prices has been decomposed into the price change components, which in turn depend on the elasticity of aggregate housing demand. (2.20) can be interpreted as a weighted tax price, where the numerator has been adjusted to account for price responses to the new service and tax levels, and the denominator, or tax base, has been adjusted to account for tax capitalization and the aggregate demand response. This is made particularly clear by rewriting (2.20) as

$$b = \left[ t + \varepsilon_{pg} + \varepsilon_{pt} \right] h/\theta H \left[ \theta + \varepsilon_{pt} \varepsilon_{Hr} \right]$$

The first term is each resident's tax bill, the second is the change in net house expenditures due to changing services, and the third is the gross-of-tax expenditure change due to changing taxes. Marginal benefits are equated to these increased costs as a share of total tax revenues $\theta H$, weighted by the associated adjustments to the tax base.

The efficiency of this outcome is easily calculated. The Samuelson efficiency criterion
requires that aggregate marginal benefits be equal to the marginal cost of providing a unit of the public service. In the present case, the distortive effects of property taxation must be accounted for in this calculation. Second-best efficiency will require that

\[ Nb = \left[ t + \epsilon_{pg} + \epsilon_{pt}(1 + t) \right]/\left[ t + \epsilon_{pt}(\theta + \epsilon_{Hp}) \right] \]  

(2.21)

This shadow pricing rule will give the same level as the Samuelson efficiency condition only if the bracketed term in (2.21) is equal to one, which requires that

\[ \epsilon_{pt}/\epsilon_{pg} + t\epsilon_{hp} = -1 \]

Hence, if public spending is financed with an arbitrary budget-balancing property tax, then:

(a) Second-best efficiency in the supply of local public goods requires that, in a tenant economy, a spending level be chosen such that

\[ N \text{ MRS} = \text{MRT} \left[ t + \epsilon_{pg} + \epsilon_{pt}(1 + t) \right]/\left[ t + \epsilon_{pt}(\theta + \epsilon_{Hp}) \right] \]

(b) A sufficient, though not necessary, condition for the Samuelson spending rule to be appropriate is that the public spending and tax capitalization elasticities are equal (in absolute value) at the margin, and aggregate housing (Marshalian) demand is inelastic with respect to the market price; i.e., if

\[ \epsilon_{pt} = -\epsilon_{pg} \text{ and } \epsilon_{Hp} = 0. \]

This result holds in both homogeneous and heterogeneous communities, so long as household income is exogenous to fiscal policy. It substantially generalizes the spending rules in Atkinson and Stern (1974) and elsewhere in the literature in that it allows for both
endogenous prices and an arbitrary tax structure.

We can also solve for \( \frac{dp}{dg} \) explicitly in terms of local market conditions as in section III. We know that \( H_g = H_g^c + bH_y \) and, substituting from the benefit expression \( b = h \frac{dr}{dg} \) and (2.19) that

\[
H_g = H_g^c + (hH_y \frac{dp}{dg})/(1 + tCHp)
\]

Differentiating the housing market equilibrium condition and substituting from the Slutsky equation gives an expression for the change in unit market prices with an arbitrary property tax:

\[
\frac{dp}{dg} = - \frac{H_g^c}{|H_r^c/(1 + tCHp) - S_p|}
\]

Substituting this back into the marginal benefit expression then gives

\[
(b/h)^m = - \frac{|H_g^c/(1 + tCHp)|}{|H_r^c/(1 + tCHp) - S_p|}
\]

(2.18')

This is very similar to (2.18), although the Tiebout tax term is now an adjustment to the net change in rents. It is just the RHS term of (2.18) except that the aggregate compensated demand derivatives are deflated by the term \( 1 + tCHp \), which represents the capitalized value of property taxes. The discussion of (2.18) applies here, with this difference. For example, note that \( dV = 0 \) and \( H_g^c = 0 \) imply that \( \frac{dp}{dg} = 0 \) and hence that \( b = ph \frac{dt}{dg} \); i.e., the change in tax payments equals marginal benefits. Moreover, comparing the change in unit prices in each case, we have

\[
\left. \frac{dp}{dg} \right|_{T-0} \geq \left. \frac{dp}{dg} \right|_{t-0}
\]

or, unit prices change by the same or less under a system of property taxation.
The process of capitalization is important as it reflects general equilibrium fiscal effects, within the community. Households will also tend to sort in the absence of capitalization; what is interesting is the result that if prices reflect the relative valuations of sorting, then benefits are directly observable. This must be a condition of equilibrium in a "smooth" economy. This also indicates that, in a tenant economy, local budget considerations are irrelevant as landowners pay all taxes. Mobile tenants escape the burden of taxation completely in this setting and, if left to their own devices, would demand ever increasing amounts of public spending if it did not encourage higher income households to outbid them for community residence.

The following section attempts to motivate this question somewhat further by including the net benefits of public activity as income to current residents. It is natural to consider the motives of an owner-occupier, for example, who seeks to both improve the quality of her environment by raising the quality of local services while at the same time increasing the asset value of her home. As her home appreciates, however, so do other homes in the community, and this may reduce her interest in maintaining high property values except when planning to move. The story is extended to allow for rents to flow to households as income, which may in turn be used to finance housing consumption. It is shown there that if market prices contain information about tax rate and various amenity differences, and the value of those differences, the (rental) income effects of those differences tend to be neutralizing.

4 Project evaluation with endogenous rental income

To start, assume that households own property only in their community of residence. We can either take the extreme view that capital gains (or losses) on that property are realized as they accumulate, as will result with perfect capital markets, or that rental income is distributed uniformly among current residents. Define household i's income as

\[ y^i = w^i + \rho^i R \]
where \( w \) is exogenous other income, \( R = ph \) is aggregate community rent and \( \sigma_i \) is \( i \)'s share. If rents are distributed evenly, so that \( \sigma_i = 1/N \) for all \( i \), then \( \sigma_i R = ph \) where \( h \) is the uniform level of housing consumption within a homogenous community. Hence, \( y_i = w + ph \) for each \( i \) within a given town. In this case, income varies with a local project according to

\[
dy/dg = p dh/dg + h dp/dg
\]

If rents are not distributed uniformly, perhaps because not all residents own their homes, but all town rents remain locally, then

\[
dy_i/dg = \sigma_i(p dh/dg + H dp/dg) + pH d\sigma_i/dg
\]

where we allow each share \( \sigma_i \) of income to vary as well. Note that the shares will sum to one if all rents stay within the community. Each possibility is examined in turn, beginning with the simple case of fully realized capital gains.

Let household income in a homogeneous community be given by \( y = w + ph \). Differentiating the household's expenditure function, we derive an expression that shows how income must change to maintain some utility level \( U^* \) at some \( (r,g) \), or

\[
e_r dr + e_y dV + e_g dg = dy
\]

This represents an isocost curve in \( (r,V,g) \) space for \( dy = 0 \). Substituting for \( y \) and \( r \), it is easy to see that the compensating variation (CV) associated with a small project \( dg \) is

\[
e_g = p dh/dg - h(p dt/dg + t dp/dg)
\]  

(2.22)
Benefits show up as only new net consumption expenditure and tax payment changes. Property taxes are (project) benefit taxes except to the extent that housing consumption changes where, by the envelope theorem, the latter is due entirely to capital gains.

First look at the simple case where housing consumption does not vary, so that $dh = 0$. This does not necessarily mean that compensated elasticities are zero, but merely that they may be offset by income effects. If market prices are in part determined by $(t,g)$, then (2.22) become

$$\frac{e_g}{h} = -tp_g - (p + tp_t) \frac{dt}{dg}$$  \hspace{1cm} (2.23)

If housing consumption does not change (e.g., with a fixed lot size model), normalized compensation is equated for all residents of any particular town. We can now show that fiscal policies maximizing (2.1), where income includes capital gains, are sufficient for residential equilibrium if housing consumption does not vary with $g$.

With $y = w + ph$, the first order conditions for household choice of $g$ and $t$ become

$$thp_g + e_g = 0 \hspace{1cm} (2.14')$$
$$tp_t + p = 0 \hspace{1cm} (2.15')$$

which together satisfy (2.22).

If housing consumption is not restricted, however, these results do not carry over. It is sufficient to treat the extreme case of no taxes ($t = 0$) and $h_g = 0$, so that we can assume that housing consumption is (initially) a function of its market price and income, or $h = h(p, y)$. Differentiating gives

$$\frac{dh}{dg} = |(h_p + hh_y) dp/dg|/(1 - ph_y)$$

and (2.22) becomes
\[ e_g = \frac{(h_p + h_h)(1 - p_h)}{1 - p_h} dp/dg - ph dt/dg \]

Solving for \( dp/dg \) gives

\[ e_g = \varepsilon p_g + (\varepsilon p_t - ph) dt/dg \]

where we simplify the expression by defining \( \varepsilon \equiv (\varepsilon_{hp} + \varepsilon_{hy})/(1 - \varepsilon_{hy}) \). (2.24) is the CV associated with a project dg in terms of demand elasticities weighted by the price change components.

Conditions (2.14') and (2.15') do not satisfy (2.24), as will be demonstrated by substitution. (2.24) can be rewritten as

\[ e_g + \theta p_g + h (\theta p_t + ph) dt/dg = -\varepsilon (p_g + p_t dt/dg) \]

Using (2.14') and (2.15') the LHS of this equation is zero, so that

\[ \varepsilon (p_g + p_t dt/dg) = 0 \]

A sufficient condition for this to hold is that both the price and income demand elasticities be unitary. These expressions suggest that the income effect of capital gains neutralizes the effect of capitalization exactly, unless housing consumption varies. Fiscally induced variation in market housing prices now matters only the sense that it affects relative tax payments and the value of income. The fiscal component of the price of housing becomes, effectively, the tax payment that falls on the house. Indeed, if the cost of providing public services has a special form consistent with income stratification, this fiscal component of the market price plays no role at all in equilibrium.
If households sort by income and realize capital gains on their homes, the budget-balancing market price of housing affects welfare only to the extent it affects housing consumption through capital gains. If public services are produced at constant unit cost, and each community is internally homogeneous, then per household housing consumption is uniform. This implies that per household public spending and tax payments are equal, or that $g \equiv t p h$. We can write indirect utility as

$$V(r,y,g) = V[p(1 + g/ph),w + ph,g]$$

(2.25)

Differentiating with respect to the market price gives $V_p = V_r + \lambda h = 0$, by Roy's identity.

This implies that households ignore the market price of housing when they evaluate each community for some fixed $h$. Compensation in terms of adjusting prices has no welfare effect since the associated changes in income and expenditure are exactly offset along the community budget. House prices are doing all the adjusting, rather than quantity. The difference between the change in expenditure and the change in income are the taxes paid, but tax rates fall as prices increase to maintain a constant level of spending. Hence, gross-of-tax and net-of-tax house prices change by the same amount and have a neutral net effect. Conditional on some $h, t$ and $g$, a change in the market price of housing is the same as a change in the gross price of housing, along the fiscal budget. The welfare effect of this price change is just (minus) current housing consumption, by the envelope theorem. The effect of a price change on income is also current housing consumption, so that the price and income impacts are offsetting. The effect of making household behavior conditional on some $h$ keeps the fiscal budget and household income linear in $p$.

The situation of course becomes more complicated when we allow housing consumption to vary as well; i.e., when we allow other behavioral responses besides migration. Tax rates continue to adjust to balance the local budget, but now the size of the tax base varies with $p$. This requires an increase in $t$ to raise the same level of funds, and hence an increase in $r$ over
and above the increase in $p$. Put another way, there are now two effects on the expenditure side. The value of the market price increase is just current housing consumption, by the envelope theorem. Because housing consumption also falls, a second effect is the tax payment required to make up for the declining tax base, $p th_p$.

The latter is not offset on the income side. If housing consumption falls, income is reduced by $p h_p$. The tax payment increase and the income loss both have negative welfare effects (where $h_p < 0$). Together the value of the change in $p$ is just equal to the gross expenditures on the change in housing consumption. Differentiating (2.25) again, but letting $h = h[p, y(p)]$ gives $V_p = \lambda h_p r$, or $r = rh_p$, which is the result we set out to show. The second step employed Roy's identity, while the third used the municipal budget identity.

It is useful to note that in the special case of nondistortionary taxes examined by Bucovetsky (1981), there will be no reason for prices to vary with fiscal policies; i.e., there will be no role for capitalization. Consider how this affects the first order conditions for the choice of local public services. Partially differentiating (2.25) with respect to services gives

$$V_r/h + V_g = 0$$

or $b = 1$. This is just the condition for fiscal efficiency in homogeneous communities in the absence of distortions. It implies that residential equilibrium is efficient, given perfect sorting, capital gains, nondistortionary finance, and a balanced public budget, in either of two cases: (i) no variation in $p$, or (ii) no variation in $h$. Each case will be shown in turn.

(i) Differentiating utility with respect to public spending gives

$$dV/dg = V_r dr/dg + \lambda dy/dg + V_g$$

Which is zero if $p h dt/dg = b$. From the local fiscal budget $p h dt/dg = 1$ and substitution gives $b = 1$, the Samuelson condition: the marginal benefits of public consumption equal the
marginal cost.

(ii) If market prices vary then equilibrium requires that \( \frac{dV}{dg} = 0 \) or

\[
hd\frac{dr}{dg} = b + h\frac{dp}{dg} + p\frac{dh}{dg}
\]

Along the fiscal budget we have

\[
h\frac{dr}{dg} = h\frac{dp}{dg} + 1 - pt\frac{dh}{dg}
\]

Solving for the LHS gives \( b + p\frac{dh}{dg} = 1 - pt\frac{dt}{dg} \) or

\[
b - 1 = r\frac{dh}{dg}
\]  
(2.26)

so that the outcome satisfies the Samuelson criterion only if the RHS of (2.26) is zero.

Alternatively, (2.26) can be read to state that if policies are chosen efficiently then housing consumption does not vary with \( g \). This implies that housing consumption does not vary by income as well. In other words, if policies are chosen efficiently then housing consumption is independent of fiscal policies, the size of the community and the income of its residents. Moreover, this means that the unit capitalization rate is the same everywhere.

We can now show the interesting result that if households sort by income and realize capital gains, fiscal policies are chosen efficiently, and local budgets are balanced, then each community faces the same unit shadow price; i.e., for all communities \( i,j \):

\[
\frac{dr_i}{dg_i} = \frac{dr}{dg}
\]

This can be seen by noting that in equilibrium we have
\[
\frac{dV}{dy} = V_r \frac{dr}{dy} + V_g \frac{dg}{dy} + \lambda (h \frac{dp}{dy} + p \frac{dh}{dy})
\]
\[
= 0
\]

which can be written, using Roy's identity as

\[
h \frac{dr}{dy} = b \frac{dg}{dy} + h \frac{dp}{dy} + p \frac{dh}{dy}
\]

(2.27)

Along the fiscal budget, however, we have

\[
\frac{dg}{dy} = h (\frac{dr}{dy} - \frac{dp}{dy}) + tp \frac{dh}{dy}
\]

Substituting into (2.27) gives \((b - 1) \frac{dt}{dy} + p(1 + t) \frac{dh}{dy} = 0\). If public good levels are set efficiently then \(b = 1\), so that \(p(1 + t) \frac{dh}{dy} = 0\), which is satisfied if housing consumption does not vary with income. Efficiency requires that each household type will locate on their respective indifference curves in \((r,g)\) space where the slope is \(\frac{dr}{dg} = K\), where \(1/K\) is the uniform level of housing consumption. (See figure 2.2 below.)

![Figure 2.2](image-url)
This result is a consequence of the efficiency and budget conditions. Tiebout-sorting implies that variation in gross expenditure on current housing and public benefits move together. For an efficient $g$, this means that house spending and public spending move together, or, for fixed $p$, that changes in tax payments, due to changing tax rates, and changes in public spending are equal and offsetting. Along the fiscal budget, however, variation in public spending must also be offset by changes in tax revenues, where the latter may be due to variation in tax rates and/or variation in housing consumption. As stated above, sorting implies that variation in public spending is entirely due to variation in tax rates (by the envelope theorem), so that to maintain a balanced local budget, housing consumption must not change with income.

**Shadow pricing rules with endogenous income**

We now evaluate public decision rules for the case of endogenous rental income. Differentiating the local budget, $g = tpH$, we have $1 = tH dp/dg + pH dt/dg + tp dH/dg$. We can solve for the change in aggregate housing consumption by differentiating the aggregate demand function $H(r,Y)$:

$$dH/dg = H_r dr/dg + H_Y dY/dg$$

(2.28)

where $Y = Nw + pH$ is aggregate community income, we have assumed for simplicity that $H_g = 0$, and $dY/dg = p dH/dg + H dp/dg$. Substituting into (2.28) gives

$$dH/dg = (p H_r dt/dg + [(1+t)H_r + HH_Y] dp/dg)/D$$

(2.29)

where $D = 1 - pH_Y$ is the income effect.

Equilibrium in the housing market is characterized by the market clearance equation
\( H(r,Y) = S(p) \), for \( S(p) \) = aggregate housing supply, as before. Differentiating both sides and substituting for the change in aggregate demand from (2.29) gives

\[
(p H_r \, dt/dg + [(1+t)H_r + HH_y] \, dp/dg)/D = S_p \, dp/dg
\]

Collecting terms, we can solve for changes in the tax rate:

\[
dt/dg = dp/dg (DS_p - [(1+t)H_r + HH_y])/pH_r
\]

(2.30)

Again, household marginal project benefits are measured by the difference between gross housing expenditures and any loss in income, or

\[
b = h \, dr/dg - dy/dg
\]

where we let \( y \equiv w + \alpha pH \). Substituting for the change in income due to rental income allows us to write this as

\[
b = dp/dg [h(1+t) - \alpha(H + pZ/D)] + p \, dt/dg (h - \alpha pH_r/D) - pH \, d\alpha/dg
\]

with \( Z \equiv (1+t)H_r + HH_y = (\varepsilon_{Hr} + \varepsilon_{Hy})H/p \). The last term on the RHS is the change in income due to any change in the share of rental income. If rents were distributed uniformly, for example, so that \( \alpha = 1/N \), then a change in population would change share size.

We can substitute for the \( dt/dg \) term from (2.30) to solve for marginal private benefits in terms of price changes and demand and supply derivatives. After some manipulation, one obtains

\[
b = \text{MRT} \ H (dp/dg [(1+t)c \ h/H - \alpha(1 + \varepsilon_{Sp})] - p \ d\alpha/dg)
\]

(2.31)
with $c = \frac{c_{SP} - c_{HR} - c_{HY}(1 + c_{SP})/c_{HR}(1 - c_{HY})}{c_{HR}}$. It may be possible to simplify this expression further, but it is fairly easy to interpret in its present form. The shadow pricing rule must account for supply and demand responses due both to the income effects of rent distribution and the (gross) price effects of capitalization. It would be useful to use the Slutsky decomposition to isolate the pure price effects, but the message is clear. Even in the absence of distortive taxation, the general equilibrium effects of fiscal policy on local prices must be taken into account when designing those policies. Note again that in a system of head taxes and local ownership, the pecuniary externalities associated with price changes would wash out within each community.

**Outside rental income**

A related set of results can also be demonstrated for a closed system of open cities, where rents flow between cities but are fully accounted for in the analysis. Households may own residential property in other communities, suggesting that some local property may be owned by nonresidents. The income of household $i$ in town $y$ is then

$$y^i \equiv w^i \cdot \int_y \alpha^i(y')p(y')H(y') \, dy'$$

where $y$ is the set of all towns (indexed by income class) and $\alpha^i(y')$ is $i$'s share of rental income in each town $y'$. A local project has the following effect on income:

$$dy^i/dg = \int_y [\alpha^i dp/dg + \alpha^i H dp/dg + pH \alpha^i dp/dg] \, dy'$$

A local project will have direct benefits for residents, but the capitalization of these benefits will indirectly affect welfare elsewhere as well as locally. If system residents migrate to the improved town, demand will fall elsewhere, lowering prices there and
generating capital losses for owners of those properties. At the same time, prices may rise locally. The net effect on welfare may be positive, negative or neutral in a closed system depending on how the magnitude of these effects compare.

For simplicity, we ignore taxes. If the project takes place in town \( y' \), the income-equivalent welfare of a household in some other town \( y'' \) varies according to

\[
\frac{(dV''/dg')}{\lambda''} = -h'' \frac{dp''}{dg'} + \int_y [o(y'',y)H \frac{dp}{dg} + p(H \frac{d\sigma(y'',y)}{dg'} + \sigma(y'',y)dH/dg)]dy
\]

where we denote \( g(y') \) by \( g' \), \( p(y') \) by \( p' \), etc., and \( \sigma(y'',y) \) is the town \( y \) profit share of persons with income \( y'' \neq y' \). There are no direct benefits, so the effects all come in the form of somewhat offsetting surplus and profit changes in the housing markets of all system communities. To simplify the expressions further, assume that neither profit shares nor housing consumption change; i.e., that \( d\sigma/dg' = 0 \), and \( dH/dg' = 0 \), so that the term in parentheses in the integrand disappears. Aggregating over the population of that town gives

\[
N''(dV''/dg')/\lambda'' = -H'' \frac{dp''}{dg'} + \int_y [o(y'',y)H \frac{dp}{dg}]dy
\]

Define \( y'' \) as the set of all communities except the project town \( y' \); i.e., \( y' \notin y'' \). Aggregating over this set gives an expression for local rent changes in the rest of the system,

\[
\int_{y''} N(y)(dV/dg')/\lambda dy = -H' \frac{dp}{dg'} dy + [N-N''] \int_y [o(y'',y)H \frac{dp}{dg}]dy (2.32)
\]

In city \( y' \), the welfare effect of the project includes direct marginal benefits, so that

\[
(dV'/dg')/\lambda' = b - h \frac{dp}{dg'} + \int_y [o(y',y)H \frac{dp}{dg}]dy (2.33)
\]

Profit shares are assumed to sum to one, which implies the condition
\[(N-N') \int_y \alpha(y',y) H \frac{dp}{dg} dy + N \int_y \alpha(y',y) H \frac{dp}{dg} dy = \int_y H \frac{dp}{dg} dy\]

Substituting (2.33) into (2.32), and incorporating this condition, gives

\[N'b = \int_y N(y)\frac{(dV/dg')}{\lambda} dy\]

Aggregate local benefits equal the aggregate income-equivalent welfare change. Neither side of this equation is directly observable, however. Solving for welfare in terms of net rents:

\[\frac{(dV'/dg')}{\lambda'} = (b - \alpha H \frac{dp}{dg}) + N'(N - N')\int_y \frac{(dV/dg')}{\lambda} + \alpha H \frac{dp}{dg} dy\] (2.34)

In a large enough system of open cities, \(dV\) and \(N'\) will go to zero and (2.34) will become

\[b - \alpha H \frac{dp}{dg} = 0\]

But this also holds if

\[\frac{(dV'/dg')}{\lambda'} = N' \int_y (dW + \alpha H \frac{dp}{dg}) dy/(N - N')\]

These results are restated as: In a system with endogenous rents, local benefits are fully capitalized into local rents if the project town is a sufficiently small part of the system (\(N'\) approaches zero), and/or the average rent and welfare change elsewhere is equal to the average welfare change in the project town.

Where rents "leak out," and the community is not an insignificant member of the system, it is still possible for benefits to show up completely in local rents if nonresidents experience those benefits as well, either in the form of rental income or welfare. Local rents
tend to reflect project benefits only when the project city is sufficiently small, or when the project benefits all residents of the system equally. This is even less likely to the extent the marginal utility of income is different between communities, as demonstrated in chapter 4 where the analysis is extended to a system of communities with internal spatial structure.

5 Concluding Remarks

This chapter has investigated the first order welfare impacts associated with arbitrary changes in local fiscal policy in an economy of open, independent and internally homogeneous jurisdictions under various assumptions about the source of household income. In part, this is essentially an attempt to generalize the Atkinson and Stern (1974) public spending rules to account for endogenous prices and local budget effects for arbitrary (e.g., nonoptimal) tax structures. This has in turn involved an examination of the market-clearing process and the various components of rent variation. In several cases, for example, the marginal cost of changing prices has been decomposed into the price change components, which in turn mainly depend on the elasticity of aggregate excess housing demand. A generalized tax price expression was derived which accounted for price responses to the new service and tax levels, for tax capitalization, and the aggregate demand response. Project benefits were shown to equal the weighted sum of each resident's tax bill, the change in net house expenditures due to changing services, and the gross-of-tax expenditure change due to changing taxes. Marginal benefits are equated to these increased costs as a share of total tax revenues, weighted by the associated adjustments to the tax base. This particular example offers both a useful way to think about the tax price concept and reassurance that more primitive tax price expressions may be a close approximation to general equilibrium expressions.

The chapter also argues that in a suitably restricted setting consumer mobility is consistent with an efficient equilibrium if resident homeowners realize capital gains. Moreover, capital gains may imply that households are indifferent to the market price of housing, and that all households, everywhere, face the same unit shadow price at efficient
service levels. A second-best project shadow price was calculated in the more general case of arbitrary rent shares and distortionary finance, which includes more conventional measures as special cases. As mentioned above, this substantially generalizes the Atkinson and Stern results as well as providing an insightful characterization of a simple local finance equilibrium.

The framework is flexible and simple enough to allow for a variety of useful extensions. As one example, the income effects of local rental income would also be generally expected to influence local labor market activity. It can be shown that if residents realize capital gains on their homes, project benefits will be capitalized into both rents and wages to an extent determined by the relative price and income elasticities of housing demand and labor supply. The basic argument is sketched rather quickly in an appendix to chapter 3. Chapter 3 also investigates several aspects of resident heterogeneity. The fact that residents may differ with respect to their individualized shadow prices has many consequences, not the least of which is the central role of politics, or how those differences are resolved by public authorities. The role of alternative tax instruments in dealing with resident mixing is also analyzed.
Appendix: Some Econometric Issues

A1 Introduction

The literature on empirical strategies for measuring fiscal effects on property values has a long and varied history, part of which is surveyed in chapter 1. While preliminary and incomplete, the results of chapter 2 can be used as the basis for several hypotheses which are closely related to the empirical questions pursued in earlier capitalization studies. It would be simple enough to test for evidence of income sorting, for example, by testing whether service levels increase with income. We have seen that equilibrium with capital gains requires that

\[ b - h_t \frac{dp}{dg} = 0 \]

or

\[ b + h(\frac{dp}{dg} - \frac{dr}{dg}) = 0 \]

Differentiating with respect to income gives

\[ g_y = \frac{b_y + h_y (\frac{dp}{dg} - \frac{dr}{dg})}{b_g dy} \]

or

\[ g_y = \frac{b h_y/h - b_y}{b_g dy} \]

where the denominator must be positive from second-order conditions for a maximum. Hence, \( g_y \) is positive when

\[ \frac{b_y}{h_y} > \frac{b}{h} \]

which is just the stratification condition discussed in chapter 2.
A test for efficiency might take several forms depending on the tenure pattern of the communities examined. Assuming that some residents are tenants, so that variable market prices have a compensation function, then chapter 2 implies that a test for efficiency would take the form of the hypothesis that

\[ r \frac{dh}{dg} = 0 \]

or whether housing consumption varies with the public service level.

Still, it might be said that much of the confusion over earlier econometric studies is the result of confusion over the basic process being measured; that is, over model specification. Our interest is with how fiscal variables impact the equilibrium per-unit price \( p \) of housing in each market. Since this is not directly observable, the dependent variable in this model is the total value of a residential structure, or \( p \) times the 'quantity' of housing services \( h \) in a given structure. A common approach is to suppose that housing quantity can be proxied by personal income \( y \) and a vector of physical structural attributes \( Z \), such as

\[ h = h(y,Z) \]  \hspace{1cm} (A2.1)

If the long-run price of housing is modeled as a function of fiscal variables, so that

\[ p = p(g,t) \]  \hspace{1cm} (A2.2)

then a cross-sectional property value equation would have the form

\[ R = ph \]

\[ = R(p(g,t),y,Z) \]  \hspace{1cm} (A2.3)
It has been noted in the literature that (A2.3) could be interpreted as a misspecified demand equation rather than as an equilibrium house-value function. It resembles demand function except for the inclusion of physical characteristics. The coefficient in a regression equation on the tax variable may be reflecting the price effect of the the tax, rather than reflecting tax capitalization in the price. Thus the coefficient may to some extent measure the price elasticity of housing demand rather than tax capitalization.

The ambiguity arises in part from having both income and housing characteristics in (A2.1), which is intended to reflect how various characteristics of a dwelling affect the quantity of “housing” in the structure. Income has often been included as a proxy for unmeasured housing attributes, but it is inappropriate here because it is not only correlated with housing characteristics but also with the quantity of housing demanded by a particular household. In principle, then, use of an equation such as (A2.3) with income deleted and a comprehensive set of housing characteristics included would appear to offer a satisfactory procedure for testing for fiscal capitalization, as an application of hedonic theory.

The premise of the hedonic approach as developed by Rosen (1974) and Epple (1984) is that there are a set of characteristics of a generic commodity. The consumer chooses the amount of each characteristic to be embodied in a unit of the commodity. Thus, if \( Z \) is a vector of housing characteristics, the consumer’s problem is modeled as the choice of \( Z \) and other goods \( x \) to

\[
\max \quad U(g,x,Z) \\
\text{subject to} \quad y \geq p(Z,g,t)(1+t) + x
\]

The residential equilibrium conditions then determine how \( p(Z,g,t) \) varies with \( g \) and \( t \). If, for example, this relationship can be approximated as

\[
p(Z,g,t) = Z^\infty g^{\hat{\eta}}/(1+t) \quad \text{(A2.4)}
\]
then a regression of the log of the market value of a dwelling against the log of Z, g, and (1+t) provides a test of capitalization.

An alternative approach to testing for capitalization follows Hamilton (1975) and Epple (1980), and offers an advantage in not requiring data on housing characteristics, and not being subject to the ambiguity of interpretation associated with (A2.3). This approach assumes that the amount of housing per dwelling can be represented by a single index h, eliminating the necessity of obtaining detailed data for housing characteristics. Since the relationship of housing price to nonfiscal characteristics is not always of central interest, this permits considerable economy in data gathering.

Suppose that the demand function for housing is a simple function of its gross price and household income:

\[ h = h(r,y) \]  
\[ R = R(r,p,y) \]

where \( r \equiv p(1+t) \). If \( p = p(g,t) \), then we have the value function

\[ R = R(g,t,y) \]

To test for capitalization, the following equation can be estimated, assuming Cobb-Douglas preferences as in (A2.4):

\[ \ln R = \beta_0 + \beta_1 \ln g + \beta_2 \ln (1+t) + \beta_3 \ln y + \epsilon \]

No data on housing characteristics is required for this test, and there is no ambiguity in the
Following the discussions in chapters 2 and 3, a test for capitalization of fiscal differentials should also be affected by the kind of assumption one makes about the role of tenure. Yinger (1982) derives a price function based on a model of consumer optimization that does not employ capital gains as income, so the market price function he constructs is consistent with equations (A2.6) and (A2.7) in chapter 2 for the Cobb-Douglas utility function. These first order conditions are reproduced here for convenience.

\[
g^* \cdot p_g (1 + t) + b = 0 \\
t^* \cdot p_t (1 + t) + p = 0
\]

If preferences have the following structure

\[
U = \beta_0 \ln x + \beta_1 \ln h + \beta_2 \ln g
\]

then Yinger shows that perfect capitalization solves, assuming consumer maximization, for the price function

\[
p = K \ g \frac{\beta_2}{\beta_1} / (1 + t)
\]

where K is some constant, so that an estimating equation might be (ignoring the discount rate)

\[
\ln R = \ln K + \left( \frac{\beta_2}{\beta_1} \right) \ln g - \ln (1 + t) + \ln y + \varepsilon
\]

(A2.9)

But if capital gains play a role then it was shown in chapter 2 that the first order conditions describing household behavior are
\[ g^* : p_g t + b = 0 \]
\[ t^* : p_t t + p = 0 \]

The market price function consistent with these is

\[ p = K' g^{1/t} \]

where \( K' \) is some constant and

\[ \mu = \Omega_2 \left[ \beta_0 + \Omega_1 (1 + t) \right] / \Omega_0 \]

giving an estimating equation

\[ \ln R = \ln K' + \mu(t) \ln g - \ln t + \ln y + \varepsilon \]  

(A2.10)

which differs critically from (A2.9) in that (A2.10) is nonlinear in \( t \), via \( \mu \).

Few localities are comprised exclusively of either tenants or owners, so we would not expect either (A2.9) or (A2.10) to apply to an entire community, or to the median or mean properties in a random sample of communities. We can, however, restrict the sample to one or the other group. If we look only at owner-occupied dwellings, for example, then (A2.10) amounts to a test of the perception of capital gains, gross of income tax considerations.

A2 Cross-sectional estimation with aggregate data

There are also a variety of estimation issues that arise in these contexts. The interesting case of cross-sectional aggregate data will be discussed here. For example, there remains the familiar problem of simultaneity in the determination of local property values, local tax rates and local spending due to the local fiscal budget identity. One strategy is to
argue that fiscal variables are effectively exogenous to the model when using subjurisdictional observations at the census tract level.

A related issue concerns whatever similarities observations within a particular jurisdiction may share. This is somewhat akin to the unobservable variables problem, where each city or town may have certain common features that cannot be measured directly. A third issue is the problem associated with the measurement of public consumption levels. Even if we assume that services are evenly provided within a community, there is the familiar difficulty of having only public spending with which to measure those service outputs. These estimation issues can be treated in an integrated manner, by adopting the approach used in the so-called siblings models of returns to schooling. Where (census tract) observations come from a particular town, we can assume they have a common error structure that conveys information about unobserved similarities. These issues are discussed in turn, and in each instance the emphasis will be on transforming the problematic model into one where OLS gives the desired results. It is also worth mentioning that the use of good longitudinal microdata would by itself avoid many of these problems, but these are not always available.

Simultaneity through the local budget

Tests of fiscal capitalization are often conducted using a single observation for each jurisdiction. Typically, median property value, median income, and a measure of accessibility are used along with the effective property tax rate and some measure of government services. The problem in this instance is that everything helps to determine everything else. The property tax rate, the property tax base, and public spending are linked by the local fiscal budget constraint. The parameters may not be identified, or they may be subject to simultaneity bias in estimation. Moreover, many models of the determination of public services within jurisdictions result in a dependence of public spending levels on the tax rate, median income, and property value (e.g., Bergstrom and Goodman (1973)).

An alternative estimation procedure is to use numerous observations from within each
jurisdiction. Since individuals take tax rates and service levels as given in making their housing purchase decisions, it would appear there is hope that OLS would yield unbiased estimates of the parameters of the capitalization equation. For example, let the true property value equation be

\[ \ln R_{ij} = \beta_0 + \beta_1 \ln (1 + t_j) + \beta_2 \ln g_j + \beta_3 \ln y_{ij} + \varepsilon_{ij} \]  \hspace{1cm} (A2.11)

where \( i \) indexes households and \( j \) indexes jurisdictions. Aggregating and taking means, (A2.11) becomes

\[ R_j = \beta_0 + \beta_1 \ln (1 + t_j) + \beta_2 \ln g_j + \beta_3 y_j + \varepsilon_j \]  \hspace{1cm} (A2.12)

where

\[ R_j = \sum_i \ln R_{ij} / n_j \]
\[ y_j = \sum_i \ln y_{ij} / n_j \]
\[ \varepsilon_j = \sum_i \varepsilon_{ij} / n_j \]

Since these variables are constructed from actual means (or medians) rather than from the logs of those means, an adjustment will have to made to avoid specification errors. That is, the average house value in a community is defined as

\[ \sum_i R_{ij} / n_j \]

Taking logs we have

\[ \ln (\sum_i R_{ij} / n_j) = \sum_i \ln(R_{ij}) - \ln(n_j) \]

so that
\[ R_j = \ln (\Sigma_i R_{ij}/n_j) + \ln (n_j)/n_j \]

That is, the logs of the means of property value and income (and other variables specific to each census tract) must be adjusted by the last term in this expression to transform them into the means of the logs. Naturally, this will also require a correction for heteroscedasticity.

The local per capita fiscal budget is given by a relationship like

\[ \ln g_j = R_j + \ln t_j \quad (A2.13) \]

and it is assumed that local spending levels are determined by the following model

\[ \ln g_j = \mu_0 + \mu_1 \ln t_j + \mu_2 R_j + \nu_j \quad (A2.14) \]

At issue is the estimation of \( \Omega_1, \Omega_2 \) and \( \Omega_3 \). It is clear by inspection of equations (A2.12) through (A2.14) that the coefficients in (A2.12) are not identified. The presence of the budget constraint results in coefficient estimates in (A2.12) that imply full capitalization, but the results are entirely spurious.

Suppose instead that (A2.11) is estimated by OLS. The acceptability of this procedure hinges on the correlation between \( \epsilon_{ij} \) and the RHS variables in (A2.11). If the \( \epsilon_{ij} \) are mutually uncorrelated, identically distributed, independent of the exogenous variables and independent of the error \( \nu_{ij} \) in (A2.14), then by solving for the reduced form of equations (A2.11) through (A2.13), the following correlations of \( \epsilon_{ij} \) to \( t_j \) and \( g_j \) can be derived.

\[ E(\epsilon_{ij}, t_j) = \sigma^2 \epsilon (1 - \mu_2)/\Delta n_j \quad (A2.15) \]
\[ E(\epsilon_{ij}, g_j) = \sigma^2 \epsilon (\mu_2 - \mu_1)/\Delta n_j \quad (A2.16) \]

where \( \Delta \equiv 1 - \mu_2(\Omega_1 + \Omega_2) - \mu_1(1 - \Omega_2) + \Omega_1 \)
\( \sigma^2_c \equiv \text{the variance of } c \)

Since the population of virtually all jurisdictions will be large relative to \( \sigma^2_c \), the correlations in (A2.15) and (A2.16) are effectively zero. Hence, OLS applied to (A2.11) yields unbiased parameter estimates.

b. Unobserved public outputs

Consider the following version of (A2.11)

\[
\ln R_{ij} = \beta_0 + \beta_1 \ln (1+t_j) + \beta_2 \ln g_j + \beta_3 \ln y_{ij} + \epsilon_{ij}
\]

but where \( R_{ij} \equiv \text{property value measured with error} \)

\[
= R^*_ij + \omega_{ij}
\]

\( g_j \equiv \text{observed public spending} \)

\[
= g^*_j + \eta_j
\]

and an '* denotes the true value. This implies that

\[
\epsilon_{ij} = \omega_{ij} - \beta_1 \eta_j
\]

The main problem is the familiar public inputs/outputs issue, where the relationship between the level of local spending and the actual output of public services is not clear. Here, the difference between the two is modeled as a random measurement error, specific to each jurisdiction yet common to each observation within a jurisdiction.

To lessen the impact of the underidentification problem of mismeasurement of \( g^*_j \), it is usually necessary to make use of some a priori information regarding the parameters \( \beta_1 \), \( \sigma^2_\omega \), \( \sigma^2_\eta \), and \( \sigma^2_g \) (see, for example, Maddala (1977), ch. 13). Another strategy is to make use of additional extraneous information in the form of repeated observations, across census tracts within each jurisdiction, of \( R_{ij} \) and \( g_j \).
In this case, the true model is

\[
\ln R_{ij} = \beta_0 + \beta_1 \ln (1 + t_j) + \beta_2 \ln g^* + \beta_3 \ln y_{ij} + \omega_{ij}
\]

\[
g_j = g^* + \eta_j
\]

From (A2.18) we know that

\[
\sigma^2_g = \sigma^2_g^* + \sigma^2 \eta
\]

and from (A2.17), that

\[
\sigma^2_R = \beta^2_1 \sigma^2_g^* + \sigma^2
\]

and from (A2.17) and (A2.18), that

\[
\sigma_{gR} = \beta_1 \sigma^2_g^*
\]

These can be used to obtain estimates of the parameters. Since the population variances \(\sigma^2_R\), \(\sigma^2_g\), and covariance \(\sigma_{Rg}\) can be estimated by their sample counterparts, and \(\sigma^2_\omega\) can be estimated from the OLS estimator for (A2.17), the three unknowns \(\sigma^2_\eta\), \(\sigma^2_g^*\), and \(\beta_1\) can be estimated using (A2.19) through (A2.21).

An alternative approach involves a reconsideration of the assumption that the \(c_{ij}\) are mutually uncorrelated. This implies directly that the equation for mean housing consumption fits perfectly. This follows from the fact that

\[
e_j \equiv \sum_i c_{ij}/n_j
\]
will be essentially zero. An alternative specification is to account for (A2.18), implying that the $c_{ij}$ have a common component. For example, suppose that

$$c_{ij} = \omega_{ij} - \beta_1 \eta_j$$

(A2.22)

where the $\eta_j$ are mutually uncorrelated, the $\omega_{ij}$ are mutually uncorrelated, and $\eta_j$ and $\omega_{ij}$ are uncorrelated. Under these assumptions, the error in the equation for the mean does not go to zero; that is, $E_j \neq -\beta_1 \eta_j$. The correlations in (A2.15) and (A2.16) become

$$E(c_{ij}, t_j) = \sigma^2 \eta (1 - \mu_2)(\sigma^2 \eta + \sigma^2 \omega/\eta_j)/\Delta$$

(A2.23)

$$E(c_{ij}, g_j) = \sigma^2 \eta (\mu_2 - \mu_1)(\sigma^2 \eta + \sigma^2 \omega/\eta_j)/\Delta$$

(A2.24)

It follows from (A2.23) and (A2.24) that the presence of a jurisdiction-specific component causes a bias in the application of OLS to (A2.11). The magnitude of the bias depends on the magnitude of $\sigma^2 \eta$. If the variance of the measurement error is small, then the bias in application of OLS will likewise be small. This result highlights the importance of including any jurisdiction-specific variables in (A2.11) that may affect the amount of housing that individuals consume, such as density, age of the population, racial composition, or actual public service consumption.

However, despite the inclusion of all relevant variables in (A2.11), the jurisdiction-specific component in (A2.22) is believed to be important. To correct for this, a two-stage procedure can be applied. The exogenous variables in (A2.11) and (A2.14) are used as instruments for $t_j$ and $g_j$ in a first stage regression. The fitted values of $t_j$ and $g_j$ could then be used in (A2.11) to obtain consistent estimates in a second stage regression.
A3 Mobility frictions

The empirical significance of mobility frictions is also worth noting. In the presence of utility differentials, perhaps due to such frictions, it is necessary to model these differences explicitly. Continuing the basic premise of the earlier stories, we assume that long run behavior is described by a process where households move between communities only if there is a net welfare payoff. Consequently, a person in town i will move to town j only if

\[ v_j > v_i \]

Now consider a situation where it requires some expenditure of resources to pick up and move. The decision rule becomes to move only if

\[ v_j > v_i + c \]

where \( c \) is the resource, or transaction cost associated with the potential move. Since the choice of locations is discrete, for our purposes, it is sensible to consider probit or logit specifications when imposing a stochastic structure on the decision process. For example,

\[ L = \alpha_0 + \alpha_1 \ln \left( \frac{v_j}{v_i} \right) + \alpha_2 \ln c \]

is a description of the choice between towns j and i, depending on relative welfare and the cost of obtaining one from the other, where \( L \) is an index measuring the likelihood a household chooses to move or not to move.

For some critical value of \( L \), say \( L^* \), the household will choose to move. Assuming that \( L^* \) is randomly distributed among the population according to a normal distribution, then for some specification of functional form for \( V \), \( L \) can be estimated by maximum likelihood. The fitted value \( L^* \) can then be computed, and the value of the cumulative normal distribution
F(L\(^{-}\)) is then the estimated probability that the household moves from i to j.

Unfortunately, most data only allow the observation of net population changes and not flows. The problem then is to formulate the question so that F(L\(^{-}\)) corresponds to these aggregate population changes in a manner that plausibly relates local population to both welfare levels and to housing prices. That is, to explain how house prices respond in turn to population changes—i.e., to changes in demand. One approach is to proxy differential welfare by population flows. Recall that the basic story is that house prices contain information about fiscal preferences because prices vary with aggregate demand, and people vote with their feet. That is, people tend to flow to communities where they are better off. The argument we now consider is that they do so subject to the frictions associated with moving.

Consider the equilibrium condition within any community that aggregate housing demand equal supply, or that

\[ \text{Nh}(p, t, y) = H(p) \]

This implicitly solves for the price function

\[ p = p(g, t, N) \]

where N is in turn a function of V. We could then just add a population term to our earlier property value equation. This cannot be estimated OLS, however, as N is also influenced by housing prices.

The strategy is to construct a simple model of location choice. An example of a condition for equilibrium in such a model is the adjustment equation used by Ravaillon (1981)

\[ N_t^i/\sum N_t^i = (N_{t-1}^i/\sum N_{t-1}^i)^{1-C} (\text{prob}_i)^C \]
where \( N^i_t \) = the population in town \( i \) at time \( t \)
\[ \sum N^i_t = \text{the total population in the system of towns at time } t \]
\[ \ln C \sim \mathcal{N}(c, \sigma^2) \]
\[ \text{prob}^i = F \left( \frac{\ln V^i - \ln V^j - c}{\sqrt{\sigma^2}} \right) \]
\[ = \text{the probability of preferring town } i \text{ to town } j \]

\( V^i = \text{expected utility of living in town } i \)
\( F = \text{the standard normal distribution} \)

and. This gives a probit model that explains differences in population growth between any two towns, \( i \) and \( j \). One use of the equation is to obtain reduced form estimates of community size for substitution into our property value equation, and to obtain estimates of the cost of the mobility parameter \( C \). These are indicative of the kinds of empirical questions posed by even the limited set of issues treated in this thesis.
Chapter 3
Policy Design for Small Mixed Economies

1 Introduction

The main concern of this chapter is how to account for both the diversity of a local population and tax distortions in an open economy setting. Both of these concerns are central to actual policymaking, but they have rarely been integrated in modeling efforts. The chapter relaxes the assumption of resident homogeneity, which allows for substantial extensions of the analysis in two directions. With resident heterogeneity, it is no longer the presumption that households will prefer the same bundle of local policies simply because they have located in the same town. The first part of the chapter approaches this question by assuming that policies are chosen by a decisive voter. Preferences are single-peaked in rents, so strategic considerations are not important, but the endogeneity of the tax price and the population raise the possibility that equilibrium popular policies may be efficient under certain conditions.

The second part of the chapter abstracts from public choice issues, as such, to characterize optimal Ramsey-Boiteux public policies. That is, how should tax and spending policies account for the heterogeneity of the resident pool? If the local population is sufficiently mixed, for example, it turns out that it may be optimal to subsidize entry into the community, via negative head taxes, while using property taxation as a means of distinguishing among residents on the basis of their housing consumption. Alternatively, a homogeneous town may want to choose a negative property tax rate and finance itself with a uniform head tax. Although these results are fairly intuitive, they are new to the literature.

These issues are presented in turn. The next section examines the role of local rents in a democratic equilibrium, where the median voter is decisive and residents have different tastes for housing and fiscal goods. The analysis is then extended to consider a more basic question: how the political and optimal choice of the tax base may be influenced by the
character of current and potential residents. The third section of the chapter concerns how best to design these policies, given the problems of diversity and fiscal distortion—where distortions affect decisions about the level of housing consumption as well as of community choice, and where the goals of authorities and the available policy tools somehow interact with the particular mix of residents found in the community. The focus is on tax policy and the interaction of available tax instruments. An interesting result is that in a system of property and head taxation, one or the other tax will likely be optimally negative. Appendices extend the discussion to consider related public choice and multimarket measurement issues that arise with heterogeneity. The concluding section summarizes these discussions.

2 Popular policies: the efficiency of democratic equilibria

It is always of interest to speculate on the motives of voters in this kind of model. Interest in local politics would seem to be fragile at best where households can easily choose among a vast spectrum of community types. However, supply constraints, mobility costs or access to profits may give residents incentives to vary each local fiscal package by voting, rather than only by moving, although the specification of objectives remains unclear. It would be extremely useful to evaluate different objective functions and income distributions in a discussion of the conditions on preferences, income and housing tenure under which property value- and utility-maximization are dual, for example.

This story is much less ambitious. To begin, consider the model of chapter 2 of an economy of independent local jurisdictions, comprised of completely mobile households who differ only in income. As before, both the number of communities and the system population are fixed in size. Preferences are assumed to be well behaved, and we ignore any nonfiscal distortions or interdependencies in the economy, as well as any interjurisdictional externalities that may arise from population movements or interjurisdictional spillovers. We also continue to adopt the convention that each community is a sufficiently small part of the
system so that local changes do not upset system-wide variables. Let the solution to the household problem of choosing housing consumption $h$ and other private goods consumption $x$, the numeraire, be represented by the indirect utility function

$$V(r,y,g) = \max_{h,x} U(h,x,g)$$

s.t. $rh + x \leq y$ (3.1)

where $r = p(1+t) = $ gross-of-tax price of housing

$p = $ net-of-tax or market price of housing

$y = $ income

$t = $ ad valorem property tax rate

To keep things simple, we begin with the problem of choosing the public goods level to maximize $V(y^m,r,g)$, where $y^m$ is the income of the median voter/resident. We assume the median voter is decisive, although this is only for concreteness--someone in the community is assumed to be indifferent to policy change in equilibrium. We first look at a tenant economy, where a small project induces the following change in the utility of a resident,

$$\frac{dV}{dg} = V_r \frac{dr}{dg} + V_g$$

for $\frac{dy}{dg} = 0$.

It is no longer assumed that equilibrium requires the absence of welfare differentials for each household across communities. This may be due to mobility frictions or supply constraints, as mentioned earlier. The median voter is able to choose his most preferred level of services just the same, however, so that $\frac{dV^m}{dg} = 0$ and

$$V^m_r \frac{dr}{dg} + V^m_g = 0$$
which, as we have seen in chapter 2, is satisfied for a tenant household by

\[ b^m = h^m \frac{dr}{dg} \quad (3.2) \]

from Roy's identity, where \( b \equiv V_g / V_y \) denotes the money value of a small public project.

Solving for net rents,

\[ b^m = h^m \left[ (1+t)\frac{dp}{dg} + p \frac{dt}{dg} \right] \quad (3.3) \]

Net marginal benefits for the median will be zero.

Do all residents approve? A household with income \( y \) gains if

\[ (b/h)^Y > (b/h)^m \]

or, given our earlier assumption that the income elasticity of public good benefits exceeds that for housing, only those households with incomes greater than the median will do better than the median, while those with lower incomes will see rent hikes that exceed their benefits. The net effect will depend on how the tax burden is distributed.

Tax burdens are determined by the fiscal budget. In the constant unit cost case:

\[ g \equiv tpH \]

where the level of services is assumed equal to the service expenditure. Differentiating the budget gives an expression for the change in the revenue base when spending increases by a unit:
\[ 1 = (1 + t_c H_p) p H \frac{dt}{dg} + (1 + c_{H_r}) t H \frac{dp}{dg} \]

where \( c_{ij} \) is the elasticity of i with respect to j. This gives an expression for the associated change in the tax rate:

\[ \frac{dt}{dg} = \frac{1 - (1 + c_{H_r}) t H \frac{dp}{dg}}{1 + t c_{H_p}} \]

Substituting this into the median's benefit measure (3.3) and simplifying, we obtain the fairly straightforward expression

\[ b_m = h^m (1/H + \frac{dp}{dg})/(1 + t c_{H_p}) \]  

(3.4)

The marginal project benefit for the decisive household is equated with the associated marginal cost at the optimum, which has two parts when accounting for tax base effects: the marginal tax price, \( h^m/H \), and the change in expenditures on current housing due to the change in prices, \( h \frac{dp}{dg} \), where each of these is deflated by one plus the net price elasticity of demand weighted by the tax rate. As in the homogeneous case of chapter 2, the second of these terms, and the denominator, have typically been overlooked in earlier studies, although it is clear that the tax price is a good approximation to marginal benefits for small price changes if demand is not too price elastic.

The unit market rent may vary with the project for any of a variety of reasons. As in chapter 2, the analysis focuses on two: capitalization of the tax rate and of the level of local services. There may also be income effects due to links between the production sector and public services, or any other consequence of the interaction of public policy and the factors that affect local housing demand and supply. Letting \( p = p(t, g, ...) \), we have

\[ \frac{dp}{dg} = p_g \cdot p_t \frac{dt}{dg} \]
Substituting into (3.4) gives an expression for benefits in terms of the marginal effects of each policy tool on prices and demand:

$$b^m = \left[ t + \frac{\partial \rho^m}{\partial \theta} \right] \left[ \gamma + \frac{\partial p^m}{\partial \theta} \left( \gamma + \frac{\partial p^m}{\partial \theta} \right) \right] h^m / \theta$$

with $\theta \equiv 1 + tc_{hp}$.

The marginal cost of changing prices has been decomposed into the price change components, which in turn depend on the elasticity of aggregate housing demand. (3.5) can be interpreted as a weighted tax price, where the numerator has been adjusted to account for price responses to the new service and tax levels, and the denominator, or tax base, has been adjusted to account for tax capitalization and the aggregate demand response. This is made particularly clear by rewriting (3.5) as

$$b^m = \left[ tp^m + \frac{\partial \rho^m}{\partial \theta} h^m + \frac{\partial \rho^m}{\partial \theta} \right] \left[ \gamma + \frac{\partial p^m}{\partial \theta} \left( \gamma + \frac{\partial p^m}{\partial \theta} \right) \right]$$

The first term is the median resident's tax bill, the second is the change in net house expenditures due to changing services, and the third is the gross-of-tax expenditure change due to changing taxes. Marginal benefits are equated to these increased costs as a share of total tax revenues $tpH$, weighted by the associated adjustments to the tax base.

No two households are the same in this community, so it is unlikely that the median voter will have chosen an efficient level of services. This depends, however, on the structure of demand and on the differential affects of the tax system. This can be seen as follows. First, note that although net marginal benefits are zero for the median, they will enjoy a fiscal
surplus if benefits exceed tax liabilities, or if

\[ b^m g > tph^m \]

which is satisfied when

\[ (b/h)^m > N/H \]

and which in turn requires that the bracketed term in (3.5) be greater than one. That is the case if and only if

\[ \varepsilon_{HP} < \varepsilon_{pg}/[t + \varepsilon_{pt}(1 + t)] \]

If aggregate housing is a normal good, this is satisfied if the RHS is positive, which requires that for \( \varepsilon_{pg} > 0 \) that

\[ t/(1 + t) > -\varepsilon_{pt} \]

For a tax rate of 5%, for example, a one percent rise in the tax rate to 5.05% would have to depress prices by less than \( t/(1 + t) \approx 4.8\% \) of their base level. In this situation, all the median requires is that the total derivative of unit price with respect to services is everywhere positive; i.e., that the net service capitalization effect dominates the net tax capitalization effect.

For some household \( j \) with an income greater than the median's, we also know from the sorting condition of chapter 2 that

\[ (b/h)^j > (b/h)^m \]
so that
\[ b^j > [t + \epsilon_{pg} + \epsilon_{pt}(1 + t)] h^j/Ht[\beta + \epsilon_{pt}(\theta + \epsilon_{Hp})] \]

for \( y^j > y^m \). If the median enjoys a fiscal surplus, then it is likely that some households with lower incomes will as well. Every resident will enjoy such a surplus if the bracketed term in (3.5) is sufficiently large. None will if the term is negative. In particular, we have that for some fraction \( \alpha \)

\[ (b/h)^j - \alpha [t + \epsilon_{pg} + \epsilon_{pt}(1 + t)] h^m/Ht[\beta + \epsilon_{pt}(\theta + \epsilon_{Hp})] \quad (3.6) \]

If \( \alpha > 0 \) the tax system is neutral in the sense that all residents approve of the spending and associated tax increase. From (3.5) and (3.6) we have an expressions for this term in terms of the ratio of normalized fiscal benefits:

\[ \alpha = (b/h)^j/(b/h)^m \quad (3.7) \]

As income rises, the marginal utility of income falls, such at the margin a dollar paid in taxes is worth less and less. At the same time, housing consumption increases with income, along with the tax price \( h/H \). If the two offset each other exactly, the tax is distributionally neutral and all consumers demand the same level of spending on \( g \). From (3.7), however, and our assumption about the relative taste for public spending by income, we see that \( \alpha > 1 \) only for residents with incomes that exceed that of the median voter. The willingness to pay of the more wealthy exceeds their change in tax cost, and they demand more \( g \) just as the more poor will vote for less.

This discussion also applies to a model of property owners, or landlords, who are residents of the city. Changes in wealth offset price effects except for tax payments, but the latter vary in the population just as tax prices vary in the tenant population, so that the
condition for tax neutrality remains unchanged in this case. If household income involves some local rental income, so that \( y^i = w^i + \sigma^i p \), where \( \sigma^i \) is household \( i \)'s share of local rents, then we have seen from chapter 2 (eq. 2.31) that \( i \)'s willingness-to-pay for the marginal project can be expressed as

\[
 b^i = H \left( \frac{dp}{dg} \left[ (1+t)c^i H/H - \sigma^i (1 + c_{SP}) \right] - p \right) d\sigma^i/dg
\]

with \( c = [c_{SP} - c_{HR} - c_{HY}(1 + c_{SP})]/c_{HR}(1 - c_{HY}) \). The median voter will then have to account for changes in the share of rents as well as offsetting changes in income as rents vary with a project. Otherwise, the results presented in this chapter carry over to this more general case without serious modification.

The Samuelson efficiency criterion requires that aggregate marginal benefits be equal to the marginal cost of providing a unit of the public service. In the present case, the distortive effects of property taxation must be accounted for in this calculation. Second best efficiency will require that

\[
 \int b(y) n(y) dy = [t + c_{pg} + c_{pt}(1 + t)]/t [t + c_{pt}(t + c_{hp})] 
\]

where \( n(y) \) is the density of households at each \( y \). This will give the same level as the Samuelson efficiency condition if the bracketed term is equal to one, which turns out to require that

\[
 c_{pt}/c_{pg} + t c_{hp} = -1
\]

The results thus far are summarized as:

(a) Second-best efficiency in the supply of local public goods in a heterogeneous economy requires that a spending level be chosen such that
The Samuelson condition will give the second-best spending rule if the public spending and tax capitalization elasticities are equal (in absolute value) at the margin, and aggregate housing (Marshalian) demand is inelastic with respect to the market price; i.e., if

\[ \epsilon_{pt} = -\epsilon_{pg} \text{ and } \epsilon_{Hp} = 0 \]

These hold in both homogeneous and heterogeneous communities, so long as household income is exogenous to fiscal policy. It differs from, and hence generalizes, the spending rules in Atkinson and Stern (1974) and elsewhere in the literature in that it allows for both endogenous prices and an arbitrary tax structure. It does have a specific application to the present situation, however, as the median voter enjoys no surplus if the Samuelson condition is satisfied. This corollary follows from the fact that the Samuelson condition requires that the bracketed term in (3.5) be equal to one, while the median obtains surplus only if that term is greater than one.

The issue we want to confront now is the success with which the median voter outcome is consistent with a second-best solution. From (3.7) we can substitute for \( b(y) \) in the decision rule to obtain

\[
\int [b(y) \ n(y) \ dy = \int \alpha(y) \ h(y) \ |t + \epsilon_{pg} + \epsilon_{pt}(1 + t)| / t|\theta + \epsilon_{pt}(\theta + \epsilon_{Hp})| H \ dy \\
= |t + \epsilon_{pg} + \epsilon_{pt}(1 + t)| / t|\theta + \epsilon_{pt}(\theta + \epsilon_{Hp})| H \int \alpha(y) \ h(y)dy
\]

so that our interest is in whether

\[
\int \alpha(y) \ h(y) \ dy > H \quad (3.8)
\]
It is useful to introduce specific forms for \( \alpha(y) \) and housing demand to obtain further results. Assume that the common utility function is given by the modified CES form:

\[
U(x,h,g) = (\eta g^{1/p} + (1-\eta)\theta h + (1-\theta)x)^{1/p}
\]

with \( 1 > \eta, \theta > 0, \) and \( p > 0 \). Maximizing this subject to an exogenous budget constraint and fixed prices gives the demand functions

\[
h(r,y) = \frac{y\theta}{p(1 + t)(1 + \theta)}
\]

\[
g(r) = \frac{p(1 + t)\theta}{(1 + \theta)}
\]

\[
x(y) = \frac{y}{(1 + \theta)}
\]

which together give the indirect utility function

\[
V(r,y,g) = (\eta g^{1/p} + (1-\eta)\theta^{2/r}(1 + \theta) + (1-\theta)y/(1 + \theta))^{1/p}
\]

\[
= (\eta g^{1/p} + (1-\eta)y^{1/p} [(1 - \theta + \theta^2/r)/(1 + \theta)]^{1/p})
\]

Roy’s identity can be used to solve for \( b \equiv V_g/V_y = \text{MRS} \). Differentiating:

\[
V_g = \rho \eta g^{1/p} (1-\eta)y^{1/p} [(1 - \theta + \theta^2/r)/(1 + \theta)]^{1/p} g^{1/p - 1} \eta/p
\]

and

\[
V_y = \rho \eta g^{1/p} (1-\eta)y^{1/p} [(1 - \theta + \theta^2/r)/(1 + \theta)]^{1/p} y^{1/p - 1} (1-\eta)/p
\]

so that

\[
b = \frac{(g/y)^{1/p - 1} \eta/(1-\eta)}{K(r/y)^{1/p - 1}}
\]
having substituted for the demand function \( g(r) \), and letting \( K = \left[ \beta/(1+\beta) \right]^{1/p} \eta/(1-\eta) \). Now, since \( h(r,y) = y/g(r) \), we have

\[
\frac{b}{h} = K \left( \frac{r}{y} \right)^{1/p}
\]

It can then follows that

\[
\alpha(y) = (y^m/y)^{1/p}
\]

which can be written as

\[
\alpha(y) = (y^m/y)^{1/p} \left( \frac{y}{y^m} \right)^{1/p}
\]

where \( y \) is the income of the household consuming the average amount of housing and \( h = H/N \).

Condition (3.8) can now be written as

\[
(y^m/y)^{1/p} \int (h/h)^{1/p} - 1 \ dy > N
\]

(3.9)

If the distribution of income is skewed to the left, for example, then \( y > y^m \) and

\[
(y^m/y)^{1/p} > 1
\]

\[
\int (h/h)^{1/p} - 1 \ dy > N
\]

where \( \int (h/h) \ dy = N \). Taken together, these imply that the aggregate efficiency condition is
not met unless tax neutrality holds.

This has implications for the choice of tax base. A tax on a luxury to finance a project will bring about a level of provision that is too high, given a normal distribution of income and simple majority voting. What residents are willing to pay for the level of \( g \) chosen by the median voter depends on their elasticity of demand. Moreover, the disparity between efficient and actual levels of \( g \) widens the more leftward skewed is the distribution of income. Hence, facing a median voter with a lower price elasticity of demand is tantamount to selecting a commodity with a lower elasticity for taxation. This not only affects the distribution of income adversely, but it also reduces the efficiency of the public sector. What residents in aggregate are willing to pay for the fiscal sector, as chosen by the median voter, exceeds total tax costs when the excess demand of the rich outweighs the desire for less \( g \) by the poor.

In this instance, the level of provision is inadequate in total. Households in aggregate are willing to pay more to finance more projects. There is therefore some fiscal structure that can make everyone better off at the margin. As in Atkinson and Stern (1974), suppose a marginal change in \( g \) is financed by lump sum taxes. The change in \( g \) costs \( 1 = \int dy \). Differentiating the indirect utility function gives

\[
\frac{dV}{dy} = V_y(dy + b dg - h dr)
\]

so each household is better off if

\[
1 < \int (b dg - h dr) dy
\]

Adopting the previous preference parameterization and assuming that income is distributed such that the median household consumes the average amount of housing (i.e., that \( h = h^m \)), this condition can be written as
which holds so long as the bracketed term is greater than one. What has been shown is that a system of individual lump sum taxes could be used to finance a small project and the lump sum payments could be distributed to make everyone better off.

Now consider the policies desired by nonmedian voters, each of whom is unlikely to be indifferent to small fiscal changes and may desire a different mix of the various fiscal variables. This question is related in a central way to the choice of tax base, and it will be approached from that perspective in what follows. It is therefore useful to introduce the head tax \( T \) as a policy tool, where it may proxy for a zoning policy, or a form of income tax. The tradeoff between fiscal variables will involve welfare as well as community budget considerations, all of which residents must account for.

More specifically, under what circumstances will a nonmedian resident household favor a decrease in property tax rates? Implicitly, this would require some combination of an increase in head taxes and/or a decrease in local spending. (We assume that the population holds no illusions about possible efficiency gains in public production, although they in fact may.) The consumption value of a fully capitalized property tax decrease to a household is zero, as gross-of-tax prices do not change. If capitalization is not anticipated in the political calculus, however, the value of a tax rate reduction will be perceived to be the value of current housing consumption, \( \phi \) (where we ignore any partial capitalization for simplicity of exposition). This benefit must be balanced against the costs associated with consequent changes in \( T \) and \( g \). The value of a unit spending change is \( b \), and the cost of an income reduction is the induced increase in head taxes. A given tenant household will then favor a reduction in property tax rates if

\[
\frac{dT}{dt} + b \frac{dg}{dt} < \phi
\]
First, look at the case where spending on public services does not vary (perhaps due to legal requirements, etc.). Using the budget constraint

\[ Ng = tpH + NT \]

we can see that with \( dg = 0 \), aggregate head tax revenues change by the amount

\[ \frac{dT}{dt} = - \frac{(tp_t + p + tp[H_t/H - N_t/N])/(N/H + tp_T + tp[H_T/H - N_T/N])} \]

Assume this is positive, although there are other possibilities. The denominator represents the income effect on prices, housing consumption, and population, while the numerator is the change in property tax payments. Substituting gives the decision rule. A household will, for any given public spending level, prefer a decrease in property tax rates so long as

\[ - (tp_t + p + tp[H_t/H - N_t/N])/(N/H + tp_T + tp[H_T/H - N_T/N]) < ph \]

The property tax cut is more popular the more elastic aggregate housing demand. In addition, from each household's perspective, the decision will depend on housing consumption and the relative bite of the head tax.

Now say that services are also allowed to vary with property tax rates. A household will favor the property tax cut only if

\[ - D(tp_t + p + tp[H_t/H - N_t/N]) + (b + D[1 - tp_g + tp(N_g/N - H_g/H)]) \frac{dg}{dt} < ph \]

(3.10)

using the budget constraint, where

\[ D = 1/(N/H + tp_T + tp[H_T/H - N_T/N]) \]
Although head tax revenues will likely need to increase by less than with fixed public expenditures, people must now also consider the value of lowered public spending—which has direct and indirect effects. A decrease in property tax rates will be more favorable the smaller the fall in service levels, the larger the value of those services, and the smaller the induced effect on housing consumption, population and housing prices.

By ignoring the value of community profits, we can obtain a simple, intuitive expression for the individual voter's decision rule: Each tenant voter prefers a balanced-budget property tax rate that maximizes net rents. This can be derived by differentiating the indirect utility function $V(r, g, y - T)$, and manipulating to get

$$b \frac{dg}{dt} + \frac{dT}{dt} = \frac{dV}{dt} \lambda + h \frac{dr}{dt}$$

so that (3.10) becomes

$$\frac{dV}{dt} \lambda + h \frac{dr}{dt} < ph$$

or

$$- (1-t) h \frac{dp}{dt} > \frac{dV}{dt} \lambda$$

This holds with strict equality at the second-best property tax rate, although the right hand side will be zero only for the median voter.

This section has identified several factors that will influence the decision by voters to shift revenue burdens from one tax base to another, or to change the level of public spending given revenue options. First, the greater is the efficiency loss from taxation, the smaller is the proportion of voters who will favor increased spending. Thus, spending growth is more likely when the price elasticities housing demand are small, and vice-versa. The relationship between the income elasticity of housing demand and the change in taxes is another factor
influencing these decisions. If tax increases are effectively progressive, or if income elasticities are low, a poor resident is more likely to favor a public project than is a richer household. On the other hand, the more wealthy households may favor a project even under an effectively progressive tax structure if income elasticities are sufficiently high or if the change in taxes is sufficiently regressive.

3 Second-best policy

This section of the chapter mostly abstracts from spending decisions, and their effects, to examine several aspects of tax policy. It is convenient to set up the social welfare maximization problem explicitly, and describe the tax structure using the associated first order conditions. This allows a straightforward evaluation of how the tax instruments interact with one another and with the tax base at the optimum, and how these relationships depend on the character of current and marginal households. While the presentation here is general enough to include flexible roles for local profits and resident welfare, the discussion is restricted to an examination of how overall community interests are affected by the characteristics of marginal and inframarginal residents.

Net public revenues are aggregate property taxes together with a head tax $T$, less the cost of public goods. As before, households differ only in income $y$. The (utilitarian) public problem is to choose a program $(t,g,T)$ to

$$\max \int V(r,y,g) \ n(y) \ dy$$
$$\text{s.t.} \quad tpH + NT - g = 0$$

Forming the Lagrangian

$$L = \int V(r,y,g) \ n(y) \ dy - \mu B$$
the first order conditions are

\[ L_g = \int \lambda_b n \, dy - \mu B_g = 0 \]
\[ L_t = - \int \lambda h_t n \, dy - \mu B_t = 0 \]
\[ L_T = - \int \lambda n \, dy - \mu B_T = 0 \]
\[ L_\mu = \int \lambda_n h(y) n(y) \, dy + T \int n(y) \, dy - g = 0 \]

where \( B = \text{tpH} + NT - g \) is the fiscal budget, \( \mu \) is the multiplier on \( B \), \( \lambda \equiv V_y \) is the marginal utility of income as before, and we ignore the welfare of those who have left or moved in (as \( y^m \) has changed). We do not want to ignore the effect of movers on the tax base, however. The first order condition for an efficient \( g \) can be rewritten in the now familiar form

\[ \text{MRT} = \lambda/\mu \, \text{MRS}(y) \, n(y) \, dy - B_g \]

so that the Samuelson efficiency condition is unlikely to give second-best efficiency unless \( B_g = 0 \) and the social cost of raising funds \( \mu \) is not inflated by tax distortions.

We now turn to the issue of tax design, which is approached by evaluating resident surplus directly. In the absence of a head tax, the optimizing behavior of each type-\( y \) household can be described by the expenditure function \( e(r,V,g) \), for \( V = V(r,y,g) \). At price \( r \) and public service level \( g \), the marginal surplus is then \( y - e(r,V,g) \), and a household will move to a town only if \( y - T \geq e(r,V,g) \). A marginal resident \( y^m \) will therefore have \( y^m - e(r,V^m,g) = T \), or \( y^m - e(r,V^m(r,y^m - T,g),g) = 0 \). The community tax problem might then be written as the choice of a property tax rate \( t \) and head tax \( T \) to

\[ \max \int [y - e(r,V,g)] \, n(y) \, dy \]
\[ \text{s.t. } NT + \text{tpH} = g^* \]
The Langrangian is

\[
L = \int [y - e(r,V,g)] n(y) \, dy - \mu (NT \cdot tpH - g)
\]

The first order condition for the choice of head tax is

\[
L_T = (ym - e^m)NT - \mu (N + TN_T + tpH_T + tH_D) = 0
\]  \hspace{1cm} (3.15)

since \( y_T = e_T(r,V,g) = 0 \). This expression can be simplified by noting that \( p_T = p_NT \) and

\[
H_T = h^mNT \cdot \int (h_T r_N NT - h_y) \, n(y) \, dy
= h^mNT - H_y + r_NT H_T
\]

where we let \( H_i = \int h_i \, n(y) \, dy \) denote the partial derivative of inframarginal housing consumption; i.e., net of migration, and where \( h^m \) is the housing consumption of a marginal household. Note that house rents vary with demand, such that \( p = p(N(T,..)) \), and recall that marginal surplus \( y^m - e^m = T \), so that we can rewrite (3.15) as

\[
TN_T - \mu (N + TN_T + tp(h^mNT - H_y + r_NT H_T) + tH_Dp_NT = 0
\]  \hspace{1cm} (3.16)

The first order condition for the choice of the property tax rate is

\[
L_t = - r_NT H + (y^m - e^m)N_T - \mu (TN_T + tpH_T + tH_Dp) = 0
\]  \hspace{1cm} (3.17)

by Shephard's lemma, where again \( y^m - e^m = T \). Making the various substitutions, and eliminating \( T \) in (3.16) and (3.17), we obtain the optimal property tax rate
where \( h = H/N \) and \( h_y = H_y/N \) are average housing consumption and the average demand-income derivative, respectively, and we use the fact that \( N_t = h^m h^m/h^m \); i.e., the income loss from a change in the property tax rate to a marginal household per unit of housing is the same as the loss resulting from a change in the head tax. This can be seen by differentiating the marginal surplus condition.

(3.18) is easier to work through if we first consider the case of fixed prices, such that \( p_N = 0 \). In that case

\[
\tau^* \mid_{\Delta p = 0} = \frac{(h^m/h - p)/p}{h_y h^m/h + 1/\mu} \tag{3.19}
\]

The denominator is positive, so the sign of the marginal tax rate is positive, zero or negative as \( h^m \) is greater than, equal to, or less than \( ph \). If the community is not diverse, in the sense that \( h^m \) is close to average housing consumption, the tax rate may be negative. The head tax will then finance not only the public good but subsidies to housing consumption as well. It is also clear from (3.19) that the tax rate will be larger the larger \( h^m \) relative to \( h \).

In the more general case of \( p_N > 0 \), (3.18) can be written as

\[
\tau^* = \frac{(h^m/h - p + \omega)/p(h_y h^m/h + 1/\mu) - \omega}{h^m h^m/h + 1/\mu} \tag{3.20}
\]

with \( \omega = p_N N_t/\mu < 0 \). It can be seen from (3.20) that the effect of tax capitalization in this case is to increase the optimal property tax rate relative to the no-capitalization case. To the extent that prices are depressed by tax increases, the burden on mobile consumers is much less at any given rate, and the property tax becomes less distortionary. Another interesting point about both (3.19) and (3.20) is that it is the income elasticity of demand that determines the optimal excise tax structure, not the price elasticity. Otherwise, (3.20)
has the familiar inverse elasticity Ramsey-Boiteux form, with additional terms reflecting the role of price changes and marginal/inframarginal interactions. The property tax is used to differentiate among residents according to their observed housing consumption, which depends in part on the overall attractiveness of the community that in turn determines the price level. (3.20) suggests that such differentiation will be more successful the more diverse the local population.

In summary, the optimal property tax rate may well be negative, it increases with resident diversity, its magnitude varies with the income-elasticity of demand rather than the price elasticity, and the process of capitalization allows for greater reliance on the tax as a screening device than otherwise.

(3.16) and (3.17) may also be used to solve for the head tax rate that maximizes resident surplus given some level of local public spending. Considerable manipulation of these conditions gives the somewhat awkward expression

\[ T^* = \left( \frac{H[-p(\mu+1)-p_N[pH_T/N_T + H_p N]-\mu N/N_T[pH_T/N_T + p_N H(1+1/\mu)]]}{(1+\mu)[p(H_T/N_T - H_T/N_T)-p_N H/\mu]} \right) \]

This is clearly possible to simplify, but it has been difficult to reduce the expression to one worth discussing in any detail. Rather, we will resort to the no-capitalization case for some intuition about the nature of the tax structure in the hope we are not seriously misled. Letting \( p_N = 0 \), we obtain

\[ T^* \big|_{p_N=0} = \left| 1 - (1 + 1/\mu) p_H H_T/H_T \right| / (N_T H_T/H_T - N_T) \]

The denominator is likely positive, so that the head tax rate is positive or negative as \((1 + 1/\mu) p_H H_T - H_T\) is positive or negative. Recalling that \( H_T = h^m N_T + H_r = N_T + H_r = h^m (H_T + H_r) + H_r \), it follows that \( T \) has the same sign as
where $H^C_T = h^m_H y + H^C_T$ is the compensated derivative of aggregate housing demand. Hence, the head tax will be larger the less diverse the community and the more elastic the compensated price elasticity of aggregate housing demand. The optimal tax may be negative, however, if the community is sufficiently diverse. That is, it may pay to subsidize immigration if the property tax can be used to effectively discriminate among residents. Of course, if the head tax could be individualized, it would be the only tax instrument employed. Nonlinear property taxes would also offer an advantage over a flat proportional rate in terms of revealing willingness-to-pay for the community. In the absence, however, of this authority, a combination of uniform head taxes and proportional property taxation will be used.

These qualitative results can also be derived in a more complicated model with two distortionary taxes and with a more general treatment of income taxes together with a linear property tax. First consider the income tax problem given an arbitrary property tax and local spending level; e.g., the problem faced by some higher government. The Pareto efficient structure will have to account for differences among households in their perception of the tax burden, and their ability to self-select by community. If we index households by their gross wage rate, and let

$$y = wL - T(wL)$$

where $T(.)$ is the income tax payment, $y$ is net of income tax income, $w$ is the wage rate, and $L$ is the quantity of labor supplied, the labor/leisure tradeoff faced by households can be represented in $(y, wL)$ space as in Figure 3.1 (e.g., see Stiglitz, 1982).
If households supply labor in a local labor market, and have their wage income taxed, the public problem becomes

\[
\max_{T,g,t} \quad \int V(r,y,g) n(y) \, dy \\
\text{s.t.} \quad tp\int h(y) n(y) \, dy + \int T(wL) \, dy = g^* 
\]

where

\[
V(r,y,g) \equiv \max_{h,x,L} U(x,L,h,g) \\
\text{s.t.} \quad wL - T(wL) \geq x + p(1 + t)h 
\]

The associated Lagrangian is

\[
L = \int V(r,y,g) n(y) \, dy + \mu B
\]
where $\mu$ is again the shadow price of tax revenues, and $B$ is the budget. Rather than solve the nonlinear income tax problem in all its glory, we confine ourselves to the consideration of shifts in the income tax structure. Let $T = T(wL, s)$, where $s$ is a shift parameter. The first order conditions for optimal $g, s, t$ are then:

\[
L_g = \int (V_g + V_r g + V_y g) \, n \, dy + \mu B_g
\]
\[
= \int (b - h r g + y g) \, n \, dy + \mu B_g = 0
\]

\[
L_s = \int (V_r y + V_y y) s \, n \, dy + \mu B_s = 0
\]

\[
L_t = \int (V_r t + V_y t) \, n \, dy + \mu B_t = 0
\]

\[
L_{\mu} = t \int h(y) \, n(y) \, dy + \int T(wL) \, n \, dy - g = 0
\]

where the budget effects are given by

\[
B_g = \int y g \, dT/dy \, n \, dy + t \int (ph_g + p_g h) n \, dy - 1 + F_g
\]

\[
B_s = \int T_s n \, dy + \int (wL_s + w_s L) dT/dy \, n \, dy + t \int (ph_s + p_s h) n \, dy + F_s
\]

\[
B_t = \int (wL_t + w_t L) dT/dy \, n \, dy + p \int h n \, dy + t \int (ph_t + p_t h) n \, dy + F_t
\]

and where the fringe (or open economy) effect on the budget, due to changes in the limits of integration as households migrate is given by

\[
F_i \equiv [T(y^-) \cdot tph(y^-)] \, n(y^-) \, y^- i - [T(y^+) \cdot tph(y^+)] \, n(y^+) \, y^+ i \quad (i = g, s, t)
\]

which is the difference in tax revenue paid by those who leave and who enter at the margins.

The project evaluation question involves the evaluation of some service change, or
project, $dg$, given an optimal tax structure. The project has a direct effect on utility, and indirect effects through prices, housing demand and labor supply. But since the individual household has chosen $h$ and $L$ optimally, the envelope theorem implies that the only welfare effects of project spending will be the associated tax revenue effects. These will have distributional consequences.

Say the income tax structure is designed to satisfy $L_s = 0$. Since this is optimal, the gain from any further shift in the structure must just equal its welfare cost. A small project has marginal benefit $b(y)$, and the procedure we now follow is to evaluate the redistributive effects of this project by comparing to an equally sized shift in the tax structure; i.e., such that $b = T_s$. That is, we evaluate a redistributive action of size $b$. From the first order conditions, the redistributive gain from a project of this size is

$$\int (\lambda - \mu) b(y) \, dy = -\mu \int (w L_s - w_s L) \, dT/dy \, dy$$

or the redistributive gain (the LHS) just equals the redistributive loss (the RHS). Manipulating gives: For an arbitrary property tax and an optimal income tax structure, the public spending decision rule is

$$MRT = \int \lambda / \mu MRS(y) n(y) \, dy + B_g - B_s$$

The shadow pricing rule amounts to a Samuelson rule only if the budget effects of the project and the income tax are offsetting i.e., only if the efficiency and distributional considerations net out. This formulation accounts for both the distributional impacts of policy design and the induced distortions of taxation. There is a clear difference between the project effects on tax revenue, which affect the level of distortions, and the distributional effects of the equivalent (in this setting) change in the income tax structure. Again, this rule generalizes the Atkinson and Stern (1974) spending rules by allowing for the effects of an
4 Concluding Comments

This chapter offers several insights into the normative character of a heterogeneous open economy. The first section indicated how popularly selected spending levels in mixed communities are affected by the distortionary effects of taxation and the capitalization effects of public policy. The previous section suggested how the tax and rent structure might deal with resident differences in an open economy, essentially by using taxes on residence and housing consumption to distinguish among residents. Tax structures were derived that accounted for both the mobility and diversity of the tax base. Among other things, the analysis suggested that the tax structure might best accommodate moderate levels of resident heterogeneity by using property taxes to encourage immigration. This is consistent with the observation, for example, that larger and more mixed communities tend to rely on more tax instruments, particularly excise taxes, to differentiate among their residents. We have also briefly considered policy rules that account explicitly for distributional as well as allocational issues. The difference, again, between this and other work in the area centers on the endogeneity of price levels and the self-selection aspects of community composition.
Appendix: A Model of Endogenous Tenure

We have seen how the capacity of rents to reflect the social value of fiscal differentials will depend in part on the character of resident heterogeneity. In general, this will also depend on the information available to prospective residents, which in turn may depend on the motives of local authorities and their cost structures. In this section, we consider how the choice of fiscal policy also depends on heterogeneity of the migrant pool and the tenure characteristics of the local housing market. The role of resident heterogeneity is discussed with attention to both political and market processes.

There are several differences between the framework used in this and earlier sections. Welfare will depend in part on tenure status, to the extent that owner-occupiers may realize capital gains on their homes. Tenure status is somewhat uncertain, ex ante. Households decide whether or not to migrate based on an expected utility hypothesis which may or may be borne out, depending as it does on general equilibrium market and political outcomes. The uncertainty follows from the possibility that fiscal policies may be chosen to obtain price levels that will not clear the market, in the sense that not all migrants who wish to purchase homes at the going price will find homes for sale. The quantity of homes that are available for purchase depends on the price level, which is determined by local fiscal policies. Those policies are in turn the ones preferred by the median voter, so that this voter in effect also determines the tenure mix and size of the community. Present and potential residents are ordered such that as the size of the community varies, so does the identity and hence type of the median household.

Policymakers maximize the expected welfare of the decisive voter, who may be either a homeowner or a tenant. The model is simple enough so that this amounts to choosing fiscal policies to generate a rent structure that in turn determines resident utility for each tenure-group. Present and potential residents are ordered by income, which acts to index the tax price of local services. Hence, the lower are rents, the higher is the level of immigration.
all things considered. The larger the city becomes, the more diverse it becomes by construction. It also becomes less attractive to marginal residents, as fiscal policies preferred by the median voter stray further from those preferred by marginal prospective residents. On the supply side, low rent levels depress the quantity of available housing for sale. This process is shown to lead to a growing divergence between actual and expected rents, as migrant households increasingly do not correctly anticipate their role in determining either the median voter's identity or the supply response. In this sense, tenure-status for some renters may be involuntary.

As before, individual household behavior can be summarized by the indirect utility function \( V(r,g,y) \), with \( r = p(1-t) \), where \( g \) is the local service level, \( y \) is household income, \( p \) is the price of housing, and \( t \) is the property tax rate. The housing price is determined indirectly by the choice of fiscal policies by local authorities who would like to

\[
\max V^M \\
\text{s.t. } g = tpH
\]

where \( V^M \) is the utility of the median voter. The median may be either a tenant or a homeowner, and his utility hangs in the balance. In this setting, we highlight this tension by imagining that public authorities account for the interests of each by maximizing the expected utility of the median. That is, authorities are unsure of the tenure status of the median voter. They choose a fiscal program \((t,g)\) to

\[
\max E(V^M) = P V(r,y,g) + (1-P) V(r,y,g) \\
\text{s.t. } tpH = g^*
\]  

(A3.1)

where \( E(.) \) is the expectations operator, \( H \) is the quantity of housing, \( N \) is the number of homes for sale, \( N \) is the town population, \( P = N/N \) is the probability of finding an
affordable home for sale, \( y \) is the income of an owner-occupier, and \( y \) is the income of a tenant. The difference between \( y \) and \( y \) will not be expanded on further, except to note that it may be due to something as simple as the tax treatment of imputed rental income. There are clearly many more significant differences between owning and renting, but this income differential captures enough of the distinction to motivate the analysis.

Assume that, aside from location and tenure status, households differ only in income. Given our assumption that the relative taste for \( g \) increases with income, this allows us to order households according to their bids for housing given \( g \). The median voter's bid will be higher the smaller the community. As the town population grows, the politically preferred rent level will fall.

Households will move to a community only when they expect to be better off there. They do not have to be certain of the rent they will face, however, to feel a move will likely be beneficial. In particular, their tenure status will not be decided until they actually arrive. The problem is one of uninformed consumers who do not become fully aware of every characteristic of the good they have purchased until after they have made the move. The decision of moving or not moving will depend on whether

\[
P V(r,g,y) + (1 - P) V(r,g,y) > V^* \quad (A3.2)
\]

or the expected utility of a community is greater than the utility \( V^* \) available elsewhere. Equation (A3.2) defines household's indifference curves between moving and not moving, where we denote the unit rent in the new community at which the household is just willing to move as \( p^m \); i.e., \( p^m \) is defined implicitly by \( E(V(p^m)) = V^* \). The determination of the equilibrium town size, on the other hand, is contingent on the rent set by public authorities to satisfy the median.

As population rises, the town becomes more mixed in character and the median voter moves down the income schedule and hence prefers a lower rent. In turn, the position of
marginal residents moves farther from the median. Let $p^M$ be the rent schedule set by political authorities to satisfy (A3.1) for each possible population level. The preferences of residents becomes more dispersed as population rises, thus dampening rent declines. Hence, the $p^M$ curve will tend to be flatter over $N$ than the rent $p^m$ required to induce the marginal household to move to the town. This is illustrated below in Figure A3.1.

![Figure A3.1](image)

The difference between $p^m$ and $p^M$ at any $N$ is the difference between the desired and actual positions. The actual is the rent the town would set if the marginal voter were to move there, changing the identity of the median, and the desired rent is that preferred by a household before it will move.

A third consideration is the rent $p^e$ that would clear the owner-occupied housing market, where we expect it to vary with migration according to $dp^e/dN > 0$. For a given home-supply schedule, if the population is large then there is a high associated $p^e$, and vice-versa. These two aspects of rent setting, the market $p^e$ and the political $p^M$ move in opposite directions as the resident population varies. The political rent falls with $N$ to
satisfy the preferences of the shifting median voter, while the market rent rises as the demand for homes rises. This suggests that for any particular town size, we have that $p^e$ may be either greater than, equal to, or less than $p^M$. The individual household looks at the smaller of $p^e$ and $p^M$ when evaluating the decision to move or stay, so that equilibrium obtains where the $p^m$ function intersects the lower of the two curves. The basic situation is illustrated in Figure A3.2 for two alternative specifications of the market equilibrium curve $p^e$.

![Figure A3.2](image)

In the case where the market curve is $p^e$, not all movers become homeowners in equilibrium. Residents obtain their rents by casting their fate with the median voter. If the lower rent were at the market curve, then rents are sufficient to put enough homes on the market to satisfy ownership demands. Otherwise, some residents must look elsewhere for housing, assuming they do not leave immediately. In this model, we assume the existence of a (secondary) market for rental housing that is much more flexible than that for
owner-occupied housing. No matter how low rents fall, some housing is always available—the
difference being that for any given rent, this sort of shelter provides less utility. This
tension between the market and political processes is a function of city size.
Chapter 4

Welfare Measurement and Public Policy in a Spatial Economy

1 Introduction

This chapter concerns fiscal considerations in communities with internal structure. Although many of the results of earlier chapters carry through in a natural way, we find there are several additional nontrivial considerations involved in the measurement of welfare in this setting. While chapter 3 dealt with intrinsic demand heterogeneity, the motivation for this chapter is whether the pattern of mixing will be important. The spatial framework also allows for different kinds of fiscal structures. For example, it is plausible that the level of public services might depend on spatial proximity. User charges would then require that the government effectively balance its budget at each site in the city, while the alternative of balancing the budget across locations is also possible. Each of these cases are discussed.

The chapter begins by reestablishing the key capitalization conditions, which are seen to depend in a critical manner on the spatial characteristics of the community in addition to the factors discussed earlier. Basic welfare measures are derived under certain efficiency conditions, and under different assumptions about service costs and financing. For example, if the public good is somewhat public in its production technology (e.g., decreasing cost), then marginal cost taxation will lead to inefficient allocations. Even if the public good has a private good character, similar kinds of distortions may be introduced by taxation, so that the city may be inefficiently large or small. These show up in rents in a simple enough model.

The role of tenure again turns out to be especially important in evaluating welfare effects, and various tenure specifications are considered. In particular, the analysis is extended to a more general urban model where all system rents accrue to residents in the system. The capacity of rents to capture the social benefits of public projects is then seen to depend partly on differences in the marginal value of money between communities. In the final part of the chapter second-best public spending rules are extended to account for
explicit spatial structure.

The chapter is organized into four sections, aside from the introduction and conclusion. First, the urban model is presented and some comparative statics and welfare measures are developed. The next section extends the analysis to include several simple fiscal structures with spatial significance. The role of the distribution of rents is then expanded to include capital gains in a closed system of open cities. The final section characterizes optimal public policies in a spatial environment with distortionary taxation.

2 An urban model: assumptions and comparative statics

A familiar monocentric static residential land use model is adapted to include a passive public sector, following Polinsky and Shavell (1976) among others. Households consume housing, a numeraire composite good, and a single public good. Each household can be represented, then, by the utility function:

\[ U = U[h(u), x(u), g(u)] \]  

(4.1)

where \( h = \) housing, \( u = \) distance from the city center, \( x = \) composite good, and \( g \) is the level of the public good. Within any one town, each household chooses \( h \) and \( x \) to maximize (4.1) subject to their budget constraint

\[ w + \sigma R \geq p(u)(1 + t)h(u) + x(u) + c(u) \]

where \( w = (\text{fixed}) \) household wage income
\( \sigma = \) share of aggregate land rents
\( R \equiv pH = \) aggregate land rents
\( p = \) price per unit of housing
\( H = \) aggregate housing consumption
c = transportation cost  

\( t = \text{ad valorem property tax rate} \)

We begin with an essentially one-person economy, so it suffices to solve this problem for any one household, at any location, to evaluate social welfare as a whole. It is convenient to represent the solution as the indirect utility function,

\[
V(u) = V[r(u), y(u), g(u)]
\]

(4.2)

where \( r = p(1+t) \), and \( y = w + sR - c \). This is the maximum utility obtainable given prices, income and the quantity of the public good, and it implicitly defines the market bid-rent schedule

\[
p = p(g, t, y, U')
\]

\[
= \max_{x, h} \frac{y - x}{(1+t)h}
\]

s.t. \( U \geq U' \)

where \( U' \) is the equilibrium utility level in the system of communities. Define, for later use, the expenditure function of some household as the minimum net-of-transport cost expenditure necessary to give some level of utility as

\[
e(r, g, U') = \min_{x, h} x + rh
\]

s.t. \( U \geq U' \)

where

\[
y - c = e(p(g, t, y, U')(1+t), g, U')
\]

Much of our attention will be focused on the implications of the assumption that household net-of-transport income may depend on location; i.e., that \( y_u - c_u = e_u \neq 0 \). We
begin by presenting some comparative statics for unit rents that emphasize the role of this assumption. First note that from the indirect utility function, we have

$$V_{rg} + V_{yg} + V_g = 0$$

which describes an indifference curve in $(r,y,g)$ space. This can be solved for the partial derivative of the gross-of-tax unit bid-rent with respect to public services:

$$r_g = (y_g + b)/h$$

$$= |b + (aR_g/(1 + cy))/h > 0$$

so long as aggregate rents $R$ do not fall too much with $g$ (see the next section), where

$$b(u) \equiv V_g/\lambda$$

$$= \text{the dollar value of the project benefit at distance } u$$

Similarly, unit bid-rents at each location change with tax rates according to

$$r_t = -y_t V_y/V_r$$

$$= aR_t/(1 + cy)/h$$

which has the same sign as $R_t$. A change in the fixed wage changes gross-of-tax bid-rents according to

$$r_w = -y_w V_y/V_r$$

$$= 1/h > 0$$
Note that gross- and net-of-tax bid-rent derivatives are related by: \( p_g = \frac{r_g}{1 + t} \), \( p_w = r_w(1 + t) \), and \( p_t = \frac{(r_t - p)}{(1 + t)} \).

The local public authority must observe its budget constraint, as in earlier chapters, so that

\[
\int_0^N g(u) - tp(u)\varnothing(u) du = 0
\]

where \( g(u) \) is the cost of providing the public service at distance \( u \), \( \varnothing \) is the amount of property at each location, the second term in the brackets is tax revenues at each location, and we are integrating over the set of possible locations. Note that the public service \( g \) is provided uniformly along any ring at any specific distance \( u \) from the city center, but the level may vary by \( u \). For the moment we will require only that the budget is balanced for the community as a whole, not necessarily at each \( u \). The framework is general enough, however, to allow for service expenditures, and costs, to vary with distance.

Residential equilibrium requires that four conditions be satisfied:

\[
\begin{align*}
n(u)h(u) &= \varnothing(u) \\
N - \int_0^u n(u)du &= 0 \\
p(u) &= p \\
dV/du &= 0
\end{align*}
\]

where

- \( n \) = population at distance \( u \)
- \( \varnothing \) = amount of residential land at \( u \)
- \( N \) - city population
- \( u \) - distance to the urban/rural boundary
- \( p \) = opportunity cost of urban land (rural rent)

The first two conditions require that residential land is fully occupied, and the city's
population is fully housed. The third is the condition that the rent at the fringe must just equal the rent for land's alternative use. Finally, (4.6) says that residents must be indifferent between all locations in the city.

To evaluate the effects of a unit change in fiscal policies, at all locations, on the city in any detail, we need to undertake a few more comparative statics. Solving the indirect utility function implicitly for the distance to the urban fringe, the equilibrium level of utility \( U^* \) requires that the tradeoff between services and city size be such that

\[
\frac{u_g}{y_u} = \frac{(r + b/h)h}{y_u} < 0
\]

where

\[
y_u = \frac{a_u R - c_y}{1 + c_y}
\]

which is negative (positive) if rental income rises more slowly (quickly) than transport expenses with distance and \( c_y > 0 \). Assume it is negative, which implies that the a small project depresses city size at the margin if unit rents rise. There are other possibilities, however.

If taxes depress unit rents, then they will encourage the city to grow for \( y_u < 0 \):

\[
u_t = h r_t/y_u
\]

If \( a_u < 0 \), for example, then \( u_t \) has the same sign as \( y_t \). We also have that

\[
u_w = h r_w/y_u
\]

\[
= - 1/y_u > 0
\]

for \( y_u < 0 \). Put another way, along an individual's indifference curve in \((u,w)\) space the wage must rise to compensate for an increase in the size of the city if rental income rises more
slowly than transport costs with distance.

We can derive expressions for each exogenous variable in terms of the others by incorporating condition (4.3) into (4.4) and defining the implicit function

\[ A(t,g,w,o,U^*) \equiv N - \int_u \bar{\varphi}/h(t,g,y,U^*) \, du \]

which must be zero for spatial equilibrium. The fiscal budget condition can be written as

\[ B(t,g,w,o,U^*) \equiv \int_u [g(u) - t p \varphi(u)] \, du \]

so that \( A = B = 0 \) give a balanced budget equilibrium. This allows us to solve for the compensation required for the property tax, which is required to calculate the associated distortion. By the implicit function theorem, for any given \( \sigma \) and \( g \) we can express this as

\[ w_t \bigg|_{A=0} = -\frac{A_t}{A_w} \]

where

\[ A_w = -\int_u \bar{\varphi}(h_r r_w + h_y)/h^2 \, du - n(u)u_w \]
\[ = \int_u \bar{\varphi} h_r c/h \, du - n(u)/y_u < 0 \]

for \( y_u \) not too large (i.e., the city not expand too much) and where \( h_r c < 0 \) is the price derivative of the compensated demand curve. Also,

\[ A_t = -\int_u \bar{\varphi}(h_r r_t + h_y y_t)/h^2 \, du - n(u)u_t \]
\[ = -\int_u \bar{\varphi} y_t (h_r/h + h_y)/h^2 \, du - n(u) y_t/y_u \]
\[ = -\int_u \bar{\varphi} h_r c/h^3 \, du - n(u) y_t/y_u > 0 \]

giving \( w_t > 0 \) (for \( A = 0 \)), as expected. Now,
\[ w_t \big|_{B=0} = -B_t/B_w \]

where
\[
B_t = -\left[ \int \rho \varnothing du + t \int \rho \varnothing_t du \right] + \left[ g(u) - \rho \varnothing(y) \right] u_t < 0
\]

so long as aggregate rents exceed the change in tax revenues by more than costs exceed revenues at the fringe, and

\[
B_w = -t \int \rho \varnothing_w du + \left[ g(u) - \rho \varnothing(y) \right] u_w < 0
\]

Together, these give \( w_t < 0 \) for \( B = 0 \). That is, to maintain the local budget at a given level of spending, an increase in tax rates will require a loss of income. Otherwise, aggregate rents will increase by more than enough to raise the appropriate level of revenue. Figure 4.1 illustrates a unique solution to these equilibrium conditions in \((w,t)\) space:
Following the same steps gives

\[ A_g = - \int u h_r c(b + y_g)/h - bh_y \, du - n(u)b(u)/y_u < 0 \]

and

\[ B_g = \int u (1 - t p_g \phi) \, du + |g(u) - t p \phi(u)|u_g < 0 \]

in the absence of a local fiscal deficit. These results imply that \( w_g > 0 \) for \( A = 0 \) and for \( B = 0 \). Figure 4.2 shows one possible equilibrium wage income and spending level equilibrium in \((w, g)\) space:

These associate a particular \((g^*, t^*)\) with each net-of-rental income group, although there is no guarantee of uniqueness. We now want to relate this to the spatial structure of the
community, with emphasis on the effects of variation in rental income. Using (4.2), condition (4.6) implies

\[
\frac{dV}{du} = V_p \frac{dp}{du} + \lambda \left[ R \frac{d\omega}{du} - \frac{dc}{du} \right] + V_g \frac{dg}{du} = 0
\]

or,

\[
\left( \frac{dp}{du} \right) V_p = \frac{dc}{du} - R \frac{d\omega}{du} - b \frac{dg}{du}
\]

Roy's identity then gives the rent gradient

\[
h \frac{dp}{du} = b \frac{dg}{du} + R \frac{d\omega}{du} - \frac{dc}{du}
\]

From equation (4.8) it can be seen that in the absence of both nominal income effects and spatial variation in public goods, changes in expenditures on land will just offset changing commuting costs as distance from the city center varies. With either differential income or public good variation, however, the possibility of a positive rent gradient enters, depending on the signs of \(\frac{d\omega}{du}\) and \(\frac{dg}{du}\).

A related question is how these relationships depend on household income. When households differ only with respect to income, their marginal rate of substitution between any two goods will generally be equated at different quantities. One consequence is that households will likely benefit by self-selecting into different 'markets,' associated with different prices for a given quantity. This is the basis for the familiar stratification result used in chapter 2. This result will carry over, with certain modifications, to the present case where communities have different local prices, due to spatial frictions (or, for example, production requirements). It may well be true that households of different incomes will live in the same community, although they will not want to pay the same rent for the same sites.

The spatial equilibrium condition (4.4) is satisfied if
\[ \frac{dp}{du} = \frac{(dy/du)}{h} \]

if \( dg/du = 0 \). It follows that the rent gradient varies with income according to

\[ \frac{d^2p}{du}dy = \left( c_{hy}/y \right) \frac{dp}{du} \]

\[ = - \left( \frac{d\lambda/du}{\lambda h} \right) \]

where \( c_{hy} \) = the income elasticity of housing demand. If income falls with distance and housing is a normal good, this is negative, so that the gradient flattens with income, at a rate equal to the proportional change in the marginal utility of disposable income per unit housing. (In section 4, we will see that in the special case of owner-occupiers the rent gradient reduces simply to the gradient of property tax payments. See section 4 for an extended discussion of the source of income, and the role that endogenous rental income may play in determining the rent gradient.)

This does not, however, necessarily suggest that households will segregate into different communities. Consider the interjurisdictional bid-rent map, in \((p,g)\) space, in figure 4.3.
In figure 4.3 we compare how much a household in each group is willing to pay to live at two reference locations: the city center and the city boundary \( u \). Each group is, by construction, willing to bid less with distance from the center. Moreover, we have assumed that the rich will bid more to live at the boundary than will the poor. Each pair of curves in the diagram corresponds to bids over the two locations over the range of city types (i.e., over the range of \( g \)). For example, \( p(y,0) \) is the bid curve of households with income \( y \) at location 0 (at each city's center). In the case illustrated, the poor are willing to pay more per unit of land in all towns in the interval \([0,g_u]\). This is because they outbid the rich for boundary locations even though they have steeper bid-rent curves in \((p,u)\) space. By the same reasoning, the rich are willing to pay more for \( g \) at every location for towns in the interval \([g_0,\infty)\]. But for public spending levels, and hence communities, between \( g_0 \) and \( g_u \), neither group outbids the other for all locations within any given town. The poor, for example, are willing to pay more to live at the center than the rich in this interval, while the rich are willing to pay more to live at the
boundary.

This can be seen more clearly by considering the bid maps in \((p,u)\) space, for a particular community in the interval \([g_u, g_0]\). Even though households in different income classes are willing to pay different amounts for any public service level, conditional on other community characteristics, both groups will occupy the same town if they live at different sites. We know from the comparative statics analysis of Wheaton (1974) that an increase in income in an open city will flatten and shift the bid curves of any individual household. The adjustment in the slope of the bid curve has a natural interpretation as the pure price effect as the price of housing changes relative to the price of commuting (since the price of commuting has no income component in this model). The magnitude of the shift in the curve depends on the magnitude of the real income change and the other equilibrium conditions.

How does fiscal policy affect this outcome? A new costless project does not add to the nominal income of residents, but it does increase their attraction to the city, within the constraints of their budget. In an open economy with no distortions this surplus has nowhere to go but to landowners, be they residents or not. The rent profile changes with the project according to

\[
d^2p/du \, dg = (dh/dg)(dc/du)/h^2
\]

\[
= h_p (dp/dg)(dc/du)/h^2 < 0
\]

and the bids will flatten, if they shift up, as the project inflates housing prices and lowers the demand for space in the process. Since income is not increased, the net effect of the project is to encourage people to value distance less relative to the community as a whole. The difference between this and the effect of an increase in income is the absence here of a positive income effect on housing demand. Prices rise because of an attraction to the community, not to space per se. Hence, bid curves flatten by more than would be the case if an income-equivalent amount of cash were provided in lieu of a project.
We have assumed that the rich value the project more, so the effect of the project will be more pronounced for this group in the sense that $g_y > 0$ implies that

$$(dp/dg)_{\text{rich}} > (dp/dg)_{\text{poor}}$$

which in turn implies that the bid curves of the wealthiest group will shift up the most. The outcome is that the boundary $0$ between the two groups moves in while the city boundary $u$ moves out and the rich crowd out the poor. Both groups continue to populate the city. Is it possible that the poor's bids could flatten enough to allow members of one group to hop over into the other part of the city? Not if the rich continue to value the project more. The rich curve will shift up and flatten at a greater rate than that of the poor, so that $d0/dg < 0$. At some point, the poor may be crowded out of the city altogether.

The robustness of the sorting result depends on of the method of financing, and its effects on the distribution of disposable income. If the tax is regressive, the poor will benefit that much less and the rich that much more than with a costless project. If the tax is sufficiently progressive, however, the progressivity in benefits will be neutralized. We will demonstrate in a later section that a property tax in an open city with capital gains is a benefit tax, so that project gains are just offset by tax losses in that case. In other environments, the outcome will be more complex.

Another issue central to the notion of residential equilibrium in this environment concerns the two extreme assumptions often made regarding the cost of migration. The so-called open, or long run, town represents a case where migration erodes away any local utility advantages or disadvantages. This is probably not a bad approximation to small communities within large metropolitan areas, even accounting for the presence of moving costs. The polar opposite, closed cities, keeps the resident population of the community fixed and solves for utility. An equilibrium solution to the model presented above normally requires than one adopt one or the other of these views: i.e., solving for either the population.
or the level of welfare in the community. The interest in this chapter, however, is sufficiently limited in scope to often allow the consideration of the entire range of migration frictions.

3 Welfare measures and project evaluation

Within this framework, the task at hand is to examine the relationship between rents and changes in fiscal policies. Several authors have claimed that full capitalization of fiscal benefits holds only in an open city (Polinsky and Shavell, 1976; Pines and Weiss, 1976). The reasoning is straightforward. A utility improving project induces immigration, increasing the demand for land. Since the supply of land is fixed at any location, rents are determined entirely by demand and rise until utility gains are captured by those rents. That rents vary strictly due to the change in demand suggests that, in the absence of distortions, they fully reflect individuals' valuation of the project. Analytically, this can be seen for the case of a representative household by holding income fixed and totally differentiating the indirect utility function (4.2), giving

\[
\frac{dV}{dg} = V_p \frac{dp}{dg} + V_g
\]  

(4.9)

where income does not vary with either public spending or the price level. Recognizing that this expression is zero in an open city, and applying Roy's identity, we have

\[
b = - \frac{(V_p)}{\lambda} \frac{dp}{dg}
\]

\[
= h \frac{dp}{dg}
\]  

(4.10a)

This is the primary capitalization result. It does not hold for a closed city unless \(dV/dg = 0\). The comparable expression for the closed city is

\[
b = h \frac{dp}{dg} + \frac{(dV/dg)}{\lambda}
\]  

(4.10b)
For a fixed population, utility gains do not (typically) disappear. On the other hand, neither do they fully reflect the value of the good to the extent that house prices change. If rents vary then housing expenditure changes and welfare changes each separately understate the public benefits. Since utility changes are not generally observable, this leaves little basis for project evaluation \textit{ex post}.

Neither version of equation (4.10) tells the entire story, however. Looking at the town as a whole, let total welfare be represented by the simple sum of utilities,

\[ NV = V \int u \, n(u) \, du \]

A fiscal change induces the following response in aggregate welfare:

\[
\frac{d(NV)}{dg} = N \frac{dV}{dg} + V \frac{dN}{dg}
\]

\[ = N \frac{dV}{dg} + V \int u \left( nV \frac{dV}{dg} \right) \, du + Vn(u) \frac{du}{dg} \]

\[ = \frac{dV}{dg} \int u \left( n + nV \right) \, du + Vn(u) \frac{du}{dg} \]

The change in aggregate welfare is more than the simple sum of individual utility changes. This has two sources: an adjustment is made for the change in density at each location, and an adjustment is made for a shift in the city border. From (4.9),

\[
\frac{dV}{dg} \int u (n + nV) \lambda \, du = \int u \left( V_p \frac{dp}{dg} + V_g \right) (n + V nV) \lambda \, du
\]

\[ = \int u \left( b - h \frac{dp}{dg} \right) (n + V nV) \, du \]

\[ = \int u \left( b (n + V nV) - h \frac{dp}{dg} (n + V nV) \right) \, du \]

The second step follows from Roy's identity. In summary, we then have that: The project induced change in aggregate welfare with fixed income is just the difference between changes
in benefits and rents at each location, or

\[ dW = dB - dR \]

where

\[ W \equiv \frac{dV}{dg} \int u (n + n_V) \lambda \, du \]

= the income-equivalent change in aggregate welfare

\[ dB \equiv \int u b(n + V n_V) \, du \]

= the fiscal benefit over \( u \)

\[ dR \equiv \int u h \frac{dp}{dg} (n + V n_V) \, du \]

= the change in rents over \( u \)

When residents realize rental income (i.e., capital gains), the analysis is less straightforward, as it is not always clear how aggregate rents will respond to a local project. This is because while unit housing prices will tend to rise with a project, housing consumption at each \( u \) may go either up or down, leaving the effect on aggregate rents ambiguous. With \( y \equiv w + \sigma R - c \), interjurisdictional equilibrium in an open economy implies that

\[ b = h \frac{dr}{dg} - \frac{dy}{dg} \]

\[ = h \frac{dr}{dg} - R \frac{d\sigma}{dg} - s \frac{dR}{dg} + \frac{dc}{dg} \]

We can simplify the exposition by assuming that shares and transport costs do not depend on the project, so that \( d\sigma/dg = dc/dg = 0 \). Our interest, then, is with \( dR/dg \), where

\[ R \equiv \int u p \otimes \, du \]
Differentiating, we obtain

\[ \frac{dR}{dg} = \int_u \left( \mathcal{R} \frac{dp}{dg} + p \frac{d\mathcal{R}}{dg} \right) du + p \mathcal{R} \frac{du}{dg} \]

Assume further that the amount of land at each distance does not change. It would be possible to vary this quantity by allocating more or less land to roads rather than residential uses, for example, or by considering nonconstant amounts of capital per house and including capital rents in our measure. Rather, let \( d\mathcal{R}/dg = 0 \), so that the sign of aggregate rents will depend on the sign of \( dp/dg \) and whether the city shrinks or expands. For a fixed tax rate, the former is

\[ \frac{dp}{dg} = r_g + r_y \frac{dy}{dg} \]  \hspace{1cm} (4.11)

Now, define the cumulative population at any distance as

\[ N(u) \equiv \int_u^\infty n(v) dv \]

Following Arnott, et al. (1985), we can define any distance \( u \) implicitly from \( N' \equiv N(u) \), so that \( u = u(N') \) and hence \( p(u) = p(N(u)) \). Moreover, we have \( u(N) = u \) and \( p(N) = p \). Changing variables from \( u \) to \( N(u) \) allows us to express aggregate rents in terms of the rents paid by each spatial cohort; i.e., such that

\[ R = \int_{N} p(N'(u)) h(N'(u)) dN' \]

Differentiating this expression gives

\[ \frac{dR}{dg} = \int_N (h \frac{dp}{dg} + p \frac{dh}{dg}) dN' + p(N)h(N) \frac{dN}{dg} \]

(4.12)

where
\[ dp/dg = p_g(N') \]

which is easier to sign than (4.11). From the household budget constraint we have that

\[ \emptyset p_{N'} + dc/du = 0 \]

or,

\[ p_{N'} = -(dc/dg)/\emptyset < 0 \] (4.13)

Differentiating (4.13) with respect to distance gives

\[ p_{N'N'} = -(dc^2/du^2)/\emptyset > 0 \]

Differentiating (4.13) with respect to services gives

\[ p_{N'N'} N'_g + p_{N'_g} = 0 \]

implying that \( p_{gN'} \) < 0. Since \( p(N) = g \) implies that \( p_g(N) = 0 \), we have \( p_g(N') > 0 \) for \( N' < N \). That is, we have shown that rents fall with distance, and that they fall more slowly the higher the level of services. Hence, it must be the case that rents rise in \( g \) at any distance shy of the urban fringe.

Now to sign the integrand in (4.12) we need to sign \( h_g(u) \), where at any \( u \)

\[ h_g = h_p p_g + h_y y_g \]

with

\[ y(u)_g = \sigma R_g + R_{g} - c_y y_g \]

\[ = (\sigma R_g + R_{g})/(1 + c_y) \]
Substituting back into (4.12) gives

\[
\frac{dR}{dg} = \int_N \left( p_g h + p h_p h_g + h_y (\sigma_{R_g} + R_{g_y})/(1 + c_y) \right) dN' + p(N) h(N) \frac{dN}{dg}
\]

\[
= \left( \int_N \left[ p_g (h + ph_p) + ph_y R_{g_y} \right] dN' + p(N) h(N) \frac{dN}{dg} \right) /[1 - s \int_N ph_y/(1 + c_y) dN']
\]

Using the Slutsky decomposition, the bracketed term in the numerator can be rewritten as

\[
p_g [h + p (h_p c - hh)] + p R_{h_y} a_g = ph_y (a_g R - p_g h) + h p_g (1 + ph_p c/h)
\]

where \( h_p c \) is the price derivative of the compensated demand curve for housing. Given \( p_g > 0 \), and assuming housing is a normal good, the second term on the RHS of this expression is less than or greater than zero as the compensated price elasticity of demand for housing is less than or greater than minus one; i.e., as compensated demand is price elastic or price inelastic. The first term on the RHS is negative so long as rental income increases by less than housing expenditures. If the direct income effect of the project is stronger than the expenditure effect, we are more likely to get ambiguous results. Hence, the numerator is negative if the compensated price elasticity of demand is elastic, and neither each share \( a \) nor \( N \) rise too much in \( g \). Alternatively, the numerator is positive if compensated demand is inelastic and prices do not increase too much in \( g \).

The denominator can be written as

\[
D = 1 - \int_N [ph \epsilon_{h_y}/(y + \epsilon_{c_y})] dN'
\]

which is negative or positive depending on the magnitude of the elasticities; where \( \epsilon_{ij} \) is defined as the elasticity of \( i \) with respect to \( j \), and we assume that \( \epsilon_{h_y} \) and \( \epsilon_{c_y} \) are both positive. It seems reasonable to assume that this "income effect" is not large enough to affect our results qualitatively, and we assume \( D > 0 \). In sum, we have: Aggregate rents are more
likely to fall (rise) with a project if the compensated demand for housing is price elastic (inelastic).

Having said this, we now look at the story at some arbitrary distance from the city center. This is much simpler and parallels much of the discussion in chapter 2. As an illustration, we will compare the effects of tax structures on the rents at some $u$. With a homogeneous population and public good consumption that varies only by town, a head tax and an income tax would have the same form:

$$\tau(u) = \frac{g(u)}{n(u)}$$

Each individual would have the indirect utility function

$$V = V[p(u), y - \tau(u) - c(u), g(u)]$$

Differentiating,

$$\frac{dV}{dg} = V_p \frac{dp}{dg} + \lambda \frac{dy}{dg} + V_g$$

$$= V_p \frac{dp}{dg} - \lambda \frac{d\tau}{dg} + V_g$$

$$= [V_p \frac{dp}{dg} - \lambda (dg/d\hat{g})/n + \frac{V_g}{1 - gnV/n^2}]$$

Using Roy's identity to solve for benefits gives

$$nb = \varnothing \frac{dp}{dg} - 1 \cdot (1 - gnV/n^2)(n \frac{dV}{dg})/\lambda \quad (4.15)$$

or, marginal benefits in each annulus vary from marginal costs by the amount of utility changes and rent changes. That is, benefits are distributed across utility, rent and cost changes. If the city is open, so that $dV/dg = 0$, then rents represent net benefits exactly. To
the extent that migration is costly, the resulting utility change absorbs part of net benefits.

Compare this with the ad valorem property tax rate $t$, where

$$V = V[p(u)(1 + t), y - c(u), g(u)]$$

For any tax rate $t$, this would revise (4.14) to read

$$(1 + t) \int_u \varnothing dp/dg \ du - \int_u n(V_g - dV/dg)/\lambda \ du$$

The change in rents equals the marginal benefit plus the welfare change.

But say the tax is somehow tied to the project size, such that $t = t(g)$. Totally differentiating utility then gives at some $u$:

$$\frac{dV/dg}{\lambda} = (1 + t)(V_r/\lambda)(dp/dg) + r(V_r/\lambda)(dt/dg) + b$$

where $r = p(1 + t)$, and $dy/dg = 0$. Aggregating, and using Roy's identity,

$$(1 + t) \int_u \varnothing (dp/dg + p dt/dg) \ du = \int_u n(V_g - dV/dg)/\lambda \ du$$

(4.16)

where the LHS term again includes the marginal (tax) cost of the public benefit.

An extremely special example is a balanced budget condition of the form

$$t \ p(u) \ \varnothing(u) = g(u)$$

(4.17)

so that taxes at each $u$ exactly cover the public good expenditure. Note that this is not the general case under examination in the bulk of this chapter, but we study it to illustrate some
otherwise more complicated points. Totally differentiating the budget and substituting the resulting expression into (4.16) gives

$$(1 + t) \int u \frac{dp}{dg} du = \int u \left( n(V_g - dV/dg)/\lambda - 1 \right) du$$

or

$$\int u \frac{dp}{dg} du = \int u \left( n(V_g - dV/dg)/\lambda - 1 \right) du$$

(4.18)

which in turn gives the result that the change in aggregate community rents is just equal to
the difference between net of surplus marginal benefits and marginal costs; i.e.,

$$\frac{dR}{dg} \equiv \int u \frac{dp}{dg} du = MB - MC$$

In this framework a head tax on like individuals is equivalent to a budget-balancing property tax in a spatial economy. Whatever benefits are not picked up by taxes will be captured by rents. In the first part of the next section we will see that a property tax is a 'benefit' tax only if residents have access to the capital gains on their homes. Head taxes are not benefit taxes in this setting, as benefits vary by location. Our assumption that benefits are positively associated with income then implies directly that property taxes are as well.

4 The effect of the distribution of rents

So far we have assumed that land rents accrue to parties within the city, implying that in a system of open cities and fixed wage rates resident consumer's surplus is completely captured (benefits offset costs, and welfare does not change). In closed cities, the willingness-to-pay of residents is just $b > h \frac{dr}{dg}$. This section pursues the question in two modified settings. First, we consider somewhat further the implications of allowing rents to be retained by residents. An important result is that if each household realizes capital gains, property taxes become Lindahl or 'benefit' taxes: tax payments equal project benefits. This.
is not the case with head taxes, even though residents have equal gross incomes, because of spatial frictions.

The second part of this section develops the story by keeping track of the rents that flow out of the city, and examining, in a preliminary manner, the effects on rents and welfare outside of the project community. The conditions under which local rents measure local benefits are much more restrictive than where these interjurisdictional flows are ignored.

One way these rents can otherwise be incorporated into the present framework is by assuming the city government collects and redistributes them lump sum to residents so that

\[ V = V[p(u)(1 + t), y + R/N - c(u), g(u)] \]

where

\[ R = \int u p \varnothing \, du \]

is aggregate land rent. Differentiating indirect utility gives the expression,

\[
\begin{align*}
\frac{dV}{dg} &= V_g/\lambda + (N \frac{dR}{dg} - R \frac{dN}{dg})/N^2 - (dV/dg)V_g \\
\frac{dV}{dg} &= V_g/\lambda + \frac{dR}{dg} - (R/N^2) \frac{dN}{dg} - (dV/dg)V_g
\end{align*}
\]

where \( dV/dg = 0 \) for the open city (in the long run), and \( dN/dg = 0 \) for the closed city (in the short run).

What becomes of per capita rent as a consequence of the project? In the closed city, rents will naturally increase per capita since demand increases and the population is fixed in size. In that case,

\[
\begin{align*}
dR &= \int u n(V_g - dV/dg)/\lambda \, du \\
&= \int u n(b - dV/\lambda) \, du
\end{align*}
\]

and the change in rent undervalues willingness-to-pay by the income-equivalent utility
change. That is, closed city rents fail to internalize benefits by the amount of net tenant surplus.

Although rents are redistributed lump sum, residents are affected differentially. Those living at \( u \) pay only

\[
y - c(u) - x = ph(u)
\]

in total rent, but will receive as a rebate \( R/N \), the same amount as households near the city center paying nearly \( y \) in rent. This will tend to flatten bid curves.

Alternatively, the problem can be reformulated so that all residents are homeowners and can realize the capital gains on their homes, such that

\[
V = V[p(u)(1 + t), y - c(u) + p(u) \varnothing(u)/n(u), g(u)]
\]

\[
= V[p(u)(1 + t), y - c(u) + p(u)h(u), g(u)]
\]

giving

\[
h [(1 + t) dp/dg + p dt/dg] = h dp/dg + b - dV/\lambda
\]

or

\[
h (t dp/dg + p dt/dg) = b - dV/\lambda
\]

or, using (4.3),

\[
\varnothing (t dp/dg + p dt/dg) = n(b - dV/\lambda) \tag{4.19}
\]

If rents, less taxes, are returned to tenants, then effectively rents are just the taxes and it is tax revenues which will reflect net benefits. In an open economy, property tax payments reflect the value of the services they finance exactly for each household; i.e., they are benefit taxes. That is, looking again at equation (4.19), and aggregating over the city,
\[ t \int_{\mathbf{u}} \phi dp/dg \, du = \int_{\mathbf{u}} n(b - dV/\lambda) \, du - \int_{\mathbf{u}} \phi \, dt/dg \, du \]

Adding the budget constraint, equation (4.17), eliminates the right hand side tax term, so that if residents realize capital gains on their homes, and the property tax balances the local fiscal budget, then the aggregate income-equivalent utility change of a project equals aggregate marginal benefits less the change in tax payments; i.e.,

\[ dW = \int_{\mathbf{u}} nb \, du - t \int_{\mathbf{u}} \phi dp/dg \, du \]

where \( dW \equiv dV/dg \int_{\mathbf{u}} n/\lambda \, du \).

One can see immediately that with a confiscatory tax rate (\( t = 1 \)), the result is full capitalization in an open economy (with \( dW = 0 \)). As the tax rate falls, however, rent changes increasingly overvalue benefits. Note also that the value of \( dW \) is spatially dependent, involving as it does the marginal utility of income, even though any utility change is spatially invariant.

This can also be usefully demonstrated for a closed system of open cities, where rents flow between cities but are fully accounted for. To consider the most simple case, assume that all rents in all cities are distributed equally to all residents. It is also helpful to maintain the assumption of perfect income segregation, so that we can index cities by income. The individual household budget constraint becomes

\[ y = w + c + \left[ \int_{\mathbf{y}} \int_{\mathbf{u}} \phi(u,y) \, p(u,y) \, du \, dy \right]/N \]

where \( \int_{\mathbf{y}} \) denotes integration over the set of communities \( \mathbf{y} \). Additional notation changes are that \( N(y) \) will now refer to the total population of town \( y \), while \( N \) is the population of the system as a whole. Similarly, \( u(y) \) will refer to distance \( u \) in city \( y \). For simplicity, the
boundary rent $p$ is set to zero in each town, and we ignore taxes.

A local project will have direct benefits, but the capitalization of these benefits will raise incomes elsewhere and hence affect welfare elsewhere. Demand will fall in other communities due to outmigration, but rise due to rising income. As we will see, the net effect may be positive, negative or neutral depending on how the magnitude of these effects compare.

If the project takes place in town $y'$, the welfare of a household in some other town varies according to

$$dV/dg(y') = \lambda \left[ -h \frac{dp}{dg}(y') + \int y_u \int \varnothing \frac{dp}{dg} dy du/N \right]$$

Aggregating over the population of that town gives

$$dW = - \int y_u \int \varnothing \frac{dp}{dg}(y') - n \int y \int u \int \varnothing \frac{dp}{dg} dy du/N] du$$

Define $y'$ as the set of all communities except $y'$; i.e., $y' \neq y'$. Aggregating over this set gives an expression for local rent changes in the rest of the system,

$$\int y' dW dy = - \int y' \int y_u \int \varnothing \frac{dp}{dg}(y') - n \int y \int y_u \int \varnothing \frac{dp}{dg} dy du/N] du dy$$

$$= - \int y' \int y_u \int \frac{dp}{dg}(y') dy du dy + \int y' \int y_u N(y) dy \int y \int y_u \int \frac{dp}{dg} dy du dy/N$$

$$= - \int y' \int y_u \int \frac{dp}{dg}(y') dy du dy + [N-N(y)]/N \int y \int y_u \int \frac{dp}{dg} dy dy$$

In city $y'$, we have

$$\frac{(dV/dg)}{\lambda} = b - h \frac{dp}{dg} + \int y \int y_u \int \frac{dp}{dg} dy du/N$$
Aggregating over the city gives

\[
\int_y \int_u \frac{\partial p}{\partial g} dy \, du = \frac{N}{N(y')} \int_u n(dV/dg)/\lambda + h \, dp/dg - b|du
\] (4.22)

Substituting (4.22) into (4.21) gives an expression for local welfare gains in terms of local project benefits, net of local rents, and external rent and welfare changes:

\[
dW(y') = \int_u (b - \varnothing dp/dg) du + \frac{N(y')}{|N - N(y')|} \int_y \left[ dW \cdot \int_u \frac{\partial p}{\partial g} du \right] dy
\] (23)

This is a generalization of the capitalization results in chapter 2. In a large enough system of open cities, \( dW \) and \( N(y') \) will go to zero and (4.23) will become

\[
\int_u (b - \varnothing dp/dg) du = 0
\] (4.24)

But (4.24) also holds if

\[
dW(y')/N(y') = \int_y \left[ dW \cdot \int_u \frac{\partial p}{\partial g} du \right] dy /|N - N(y')|
\]

These results are restated as: In a system with endogenous rents, local benefits are fully capitalized into local rents if the project town is a sufficiently small part of the system (\( N(y') \) approaches zero), and/or the average rent and welfare change elsewhere is equal to the average welfare change in the project town.

Where rents "leak out," and the community is not an insignificant member of the system, it is still possible for benefits to show up completely in local rents if nonresidents experience those benefits as well, either in the form of rental income or welfare. This is not likely to the extent the marginal utility of income is different between communities, as demonstrated below.
The condition for spatial equilibrium within a given city, you will recall, is that

\[ h \frac{dp}{du} + \frac{dc}{du} = 0 \]

where \( \frac{dc}{du} \) is the marginal commuting cost from location \( u \). Integrating by parts gives

\[ \int_u p \frac{dp}{du} \, du - \int_u \nabla \frac{dp}{du} \, du = p(u) \nabla(u) - p(0) \nabla(0) = 0 \] (4.)

where \( \nabla \equiv \frac{\partial}{(dc/du)} \). Using the spatial equilibrium condition (4.4), and substituting for \( \nabla \), the second term on the LHS is

\[ \int_u \nabla \frac{dp}{du} \, du = \int_u \left[ \frac{\partial}{(dc/du)} \right] (dp/du) \, du \\
= - \int_u n \, du \\
= - N(y) \]

Substituting this into (4.25), and differentiating the result with respect to \( g \) gives

\[ \int_u \frac{dp}{dg} \left( \frac{d\nabla}{du} \right) du - \frac{dN(y)}{dg} = 0 \] (4.26)

where \( \frac{d\nabla}{du} = \frac{\partial}{du} - \frac{\partial}{(d^2c/du^2)}/(dc/du)^2 \).

The project induces city growth if the amount of residential land increases at a faster rate with distance than does marginal transport costs; i.e., \( \frac{d\nabla}{du} > 0 \) so long as

\[ \epsilon_{\delta u} > \epsilon_{dc/du,u} \]

As Wheaton (1974) notes, this is in particular consistent with marginal transport costs that decrease with length of trip.
In city $y'$, we have from (4.22) that for any household

$$h \frac{dp}{dg} = b - \frac{dV}{dg} + \lambda - \int_y \int_u \Theta \frac{dp}{dg} du dy) / N$$

$$= b - \frac{dV}{dg} + \lambda - \frac{dP}{dg} / N$$

Substituting into (4.26) solves for system-wide rent adjustments

$$\frac{dP}{dg} / N = [\Theta - \frac{dN(y')}{dg}] \int_u (\frac{dY}{du}) / h du$$

where $\Theta = \int_u n (b - \frac{dV(y')}{\lambda} (\frac{dY}{du}) / \Theta du$ is the aggregate weighted local project benefits net of welfare gains. If benefits were spatially invariant, we could pull $b$ and $\lambda$ out of this expression and obtain

$$b - \frac{dV}{\lambda} = (\frac{dP}{dg}) / N + \frac{dN(y')}{dg} \int_u (\frac{dY}{du}) / h du$$

and net benefits differ from average system rent increases by a positive factor.

This section has explored the implications of local rents that flow out of the city but are accounted for in the general equilibrium system. In addition to the explicitly spatial adjustments to benefit measures, local rents are seen to reflect project benefits only when the city is sufficiently small, or when the project benefits all residents of the system equally.

5 Policy design

Spatial structure complicates the local project evaluation problem in several ways. This may be due to the additional considerations associated with spatially variant public service levels and costs, with the differential effects of rent redistribution or tenure mixing, or with related aspects of the general equilibrium interdependence of transport costs, residential density and local public budgets. Each of these factors has implications for the relatively
naive policy rules established in earlier parts of this thesis.

In this section, the public expenditure problem is reexamined in a simple spatial setting. The section begins by reviewing the argument that residential equilibrium is inconsistent with a utilitarian welfare criterion, since the marginal value of money will depend on location. We then consider the two second-best policy rules corresponding to two assumptions about the variability of available residential land in each annulus. Spatial efficiency problems due to the public goods nature of $g$ are also discussed.

In a homogeneous tenant economy, one public problem is to choose a program $(g, t)$ to maximize welfare subject to the fiscal budget; i.e., to

$$\max_{g, t} V(r, y, g)$$

s.t. $$B \equiv \int_u [g(u) - tp(u) \varnothing(u)] \, du = 0$$

In this kind of problem, the planner must also face the complicating fact that equation (6), the spatial equilibrium condition, imposes the constraint that an optimal income distribution is not preservable within a town. This is because it turns out that one cannot obtain both equal utilities and equal marginal utilities of income in this framework; i.e., the so-called Mirrlees (1972) problem. This can be shown by differentiating the marginal utility of income $\lambda$ with respect to distance, and making use of Roy's identity:

$$\frac{d\lambda}{du} = \lambda_p \frac{dp}{du} - \lambda_y \frac{dc}{du}$$

$$= - (h_y \lambda + h_{\lambda y}) \frac{dp}{du} - \lambda_y \frac{dc}{du} \quad (4.27)$$

Everything can be signed except for $\lambda_y$. This can be obtained by differentiating Roy's identity with respect to income, which gives

$$h_y = - (V_{py} - h_{\lambda y})/\lambda$$
If we substitute for $\lambda_y$ into (4.27), and incorporate the spatial equilibrium constraint (4.6), then we have

$$\frac{d\lambda}{du} = h \frac{h_y}{(\lambda \ dc/du)} > 0$$

This has also been shown by Wheaton (1974) and Wildasin (1984), in somewhat different contexts. It implies directly that a project with uniformly distributed services will have spatially differentiated benefits, even in a community with identical households.

Having said this, the first order condition for the choice of $g$ in this problem gives the second-best decision rule: choose the level of services, or project size, such that marginal costs equal the weighted sum of household benefits, adjusted for revenue effects.

$$\text{MRT} = \int n(u) \text{MRS}_g(u) \lambda \ du + p \frac{\varnothing u_g}{\mu} + B_g$$

where $\mu$ is the Lagrangian multiplier on the fiscal budget and $\varnothing_g = 0$. This is the now familiar Samuelson rule adjusted for the distortions and fringe effects associated with $B_g$ (see chapters 2 and 3 for nonspatial forms of this rule). Services should be over or underprovided relative to the no distortion case unless there are no budget effects ($B_g = 0$), and particularly unless the marginal cost of public funds $\mu$ equals the marginal utility of income $\lambda$. We typically expect that $\mu > \lambda$, so the presumption is generally that the level of services should be cut to account for the distortive effects of non lump sum financing.

In the absence of distortions, the solution to this problem is given by the familiar conditions for efficient provision of pure public goods:

$$nb = 1$$
for each u, where I is the cost of providing g at u. Aggregating over locations,

\[ \int u \n \mathrm{db} \, \mathrm{du} = u \]  

(28)

This condition can be related to land rents by invoking the so-called Henry George Theorem. It can be shown that under a variety of circumstances an optimally financed city's expenditures will equal differential land rents (cf., Arnott and Stiglitz (1979)). That is,

\[ \int u \, g \, \mathrm{du} = \int u \, \mathcal{O}[p(u) - p] \, \mathrm{du} \]

It is sufficient for our purposes to simply assume this conditions holds, although that is an interesting question in itself. Differentiating this condition with respect to g,

\[ u + g(u) \frac{\mathrm{d}u}{\mathrm{dg}} = \int u \, \mathcal{O} \frac{\mathrm{dp}}{\mathrm{dg}} \, \mathrm{du} + \mathcal{O}(u)[p(u) - p] \frac{\mathrm{du}}{\mathrm{dg}} \]

Using (4.5), we have

\[ \int u \, \mathcal{O} \frac{\mathrm{dp}}{\mathrm{dg}} \, \mathrm{du} = u + g(u) \frac{\mathrm{du}}{\mathrm{dg}} \]  

(4.29)

Because differential rents do not reflect the cost of \( \mathrm{dg} \), total rent changes differ from marginal cost by the cost of provision at the urban fringe. Substituting from equation (4.28), then, gives the capitalization efficiency result: If the Henry George Theorem holds, and public goods are provided efficiently, then benefits are fully capitalized in open or closed cities, or

\[ \int u \, \mathcal{O} \frac{\mathrm{dp}}{\mathrm{dg}} \, \mathrm{du} = \int u \, \text{MRS} \, \mathrm{du} \]
Note that this holds for the full range of migration cost assumptions. Note also that is a direct consequence of the assumptions employed: in the absence of distortions or unaccounted for surplus, rents must fully absorb benefits.

It is easy to complicate this story and allow for distortions which will lead to the failure of complete capitalization. Say, for example, that the cost of provision of the public service to a particular household was dependent upon both its location and the aggregate level of consumption, so that the good is somewhat public in nature. Let $g(u)$ continue to represent the level of $g$ supplied to each household at $u$, but let the aggregate public service supply be defined as

$$G = \int u \cdot n(u)g(u) \, du$$

The cost of supplying a household will now be

$$k = k(u, G)$$

so that the total cost of supplying $G$, namely

$$\int u \cdot k(u, G) \, du$$

depends not only on the level of consumption, but on its spatial pattern as well. In this case, there may arise problems efficiently financing the project.

The marginal cost of the project will be

$$\int u \cdot k_G \, du$$
If the service was financed by a marginal cost tax, total revenues would be

\[ \int_u n g \left[ \int_u k_G \, du \right] \, du \]

Integrating by parts and substituting for \( G \), this equals

\[ \int_u G k_G \, du \]

so that the difference between these revenues and total costs would then be

\[ \int_u G(k_G - k/G) \, du \]

The city in this case is a 'natural' monopoly, and marginal cost taxation will not balance the local budget, without other revenue sources. If these funds came from outside the community, via intergovernmental aid for example, then (1) the cost savings would be capitalized locally, and (2) the city would be inefficiently sized.

Another illustrative case in a spatial model involves a relaxation of the assumption that \( \varnothing_g = 0 \). Instead, say that the amount of developed land at each \( u \) is variable, and depends on the services provided there. Ignoring fringe effects, for simplicity's sake, the first order condition for an efficient level of services is

\[ \int_u \left[ \varnothing_g p_g + (\zeta - p) \varnothing_g \right] \, du - \mu (MC - B_g) = 0 \]

where \( \zeta(u) \) is the resource multiplier on land in the public optimization problem; i.e., the shadow price of land. From (4.6), we have \( dV/dg = 0 \), which together with this condition gives the spending rule.
The spatial character of the externality suggests that the shadow price of land is likely to exceed the market price for locations close to the city center, and vice versa for sites near the fringe. The optimal policy would then involve some combination of taxes and spending that made central sites more attractive, and distant sites less so, after accounting for the sensitivity of market rents to variation in each policy tool, and tax induced distortions.

6 Conclusion

This chapter has examined the sensitivity of rents as welfare measures to different assumptions about service costs, financing and rent flows. Rents can be used for purposes of project evaluation in cities, but several modifications to the nonspatial public shadow pricing rules have been suggested. In the spatial context, the most important may be the familiar result that the marginal utility of income will depend on location, and hence that the value of benefits and financing will be dependent upon location. In open economies, without distortions, these values will be exactly reflected in rents unless interjurisdictional income flows are sizable. Free mobility will (typically) ensure that land owners capture these welfare changes, but it is not necessary that homogeneous communities will result. If all residents are homeowners and the project is financed with a property tax, it follows from the normality of housing and the project that the tax structure will be progressive, and hence will offset the progressivity of the benefit structure. As an example, nondistortionary property taxation was shown to amount to benefit taxation in a model with capital gains.

The chapter began by reestablishing the key capitalization conditions, which depend in a critical manner on the spatial characteristics of the community in addition to the factors discussed earlier. Basic welfare measures were derived under certain efficiency conditions, and under different assumptions about service costs and financing. For example, if the public good is somewhat public in its production technology (e.g., decreasing cost), then marginal
cost taxation will lead to inefficient allocations. Even if the public good has a private good character, similar kinds of distortions may be introduced by taxation, so that the city may be inefficiently large or small.

The role of tenure again turned out to be especially important in evaluating welfare effects, and various tenure specifications were considered. In particular, the analysis was extended to a more general urban model where all system rents accrue to residents in the system. The capacity of rents to capture the social benefits of public projects was seen to depend partly on differences in the marginal value of money between communities. In the final part of the chapter second-best public spending rules were extended to account for explicit spatial structure.
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