DISCRETE VISUAL STRUCTURES:
Elements of Visual Grammar

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Abstract

Is it possible to reason by means of images? If it is, then with what kind of images can we organize thoughts? How can the rules governing the relations between images be established? Could these relations be as complex and productive as those defined within the grammar of the verbal language?

The basic construction of any language, especially a developed one, is a structure of formal rules which regulate the relations between its signs or elements. For verbal language it is a syntax which regulates all relationships between elements of a certain language: alphabet, words and sentences.

This work is an attempt to explore and establish a set of formal rules between a large and complex group of standardized visual signs which I call discrete visual structures. A fundamental characteristic of a discrete visual structure is its possibility to be visually represented. The relations between these structures depend primarily on their graphic organization and structural characteristics. Elements of each structure can be presented as finite parts of the plane surface. There are four basic types of discrete visual structures: spatial structure, qualitative structure, state of space and visual process.

I have based this presentation of discrete visual structures on two different types of signs: visual and verbal, but the visual presentation of images is the essential subject of this analysis. Verbal signs (written text) are used here as a necessary meta-language in order to communicate the basic ideas on discrete visual structures to the readers.

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Is it possible to reason by means of images? If it is, then with what kind of images can we organize thoughts? How can the rules governing the relations between images be established? Could these relations be as complex and productive as those defined within the grammar of the verbal language?

It is now more than twelve years since I began to be interested in various problems related to a rather unclear and polysemic concept of the visual language. These or similar questions were the roots of my interest which lead me to work on formal aspects of visual signs developing some manner of visual grammar.

I take language to be primarily a vehicle for thought or selfcommunication. In my opinion, it is also a window to the world. If we find ourselves in a room without windows, there is no picture of the world. One can say that there is no world at all. Initially, the window was small and sight was limited and poor. Throughout the ages the window became wider, more elements were added, many new colors appeared, and many details were altered and changed. Hence today, through the window of verbal
language we learn to perceive a very complex and sophisticated landscape of the world. However, since the window is merely a frame and the landscape is simply a picture, our vision of the world is naturally limited and distorted.

Any new language, based on premises other than verbal language, can open another window in our room, allowing us to see a different picture of the same world. These different pictures will give us a much wider and more complex vision of the universe. I believe that a highly organized and developed visual language provides such a new window.

The basic construction of any language, especially a developed one, is a structure of formal rules which regulate the relations between its signs or elements. For verbal language it is a syntax which regulates all relationships between elements of a certain language: alphabet, words and sentences.

This work is an attempt to explore and establish a set of formal rules between a large and complex group of standardized visual signs which I call discrete visual structures. A fundamental characteristic of a discrete visual structure is its possibility to be visually represented. The relations between these structures depend primarily on their graphic organization and structural characteristics. Elements of each structure can be presented as finite parts of the plane surface. There are four basic types of discrete visual structures: spatial structure, qualitative structure, state of space and visual process.

Spatial structure is defined as a structure of position. Each element of this structure is presented as a definite part of the plane with defined neighborhood relations with other elements of the same structure, presented in standard form. A position is a basic characteristic of each element within a spatial structure. Therefore, any particular spatial structure represents a specific universe of positions with its own topological integrity.

Qualitative structure is defined as a structure of content. Each element of this structure is presented as a definite part of the plane with defined neighborhood
relations with other elements of the same structure. Content is a basic characteristic of each element presented visually by various degrees of shading, from white to black. There are two different presentations of a qualitative structure: natural and conventional. A natural presentation keeps the natural neighborhood relations between qualities. This means that in a white–gray–black structure, white can be a neighbor of gray but not of black. In a conventional presentation the neighborhood relations can be presented arbitrarily.

State of space is defined as a union of both spatial and qualitative structures. Here we have a complex visual structure with defined both position and content for each element within a structure. With a state of space it is possible to analyze in more detail topological characteristics of a spatial structure and to distinguish various kinds of figures within a structure. State of space can also be very helpful in analyzing the relations between different qualities. It is shown that some qualities, like gray for example, are not independent and can be generated by uniform distribution of two basic qualities: black and white. Therefore, within a state of space we can get much more information about both spatial and qualitative structures. A matrix state is a distinct kind of state which represents a visual image of neighborhood relations of a certain spatial structure.

A visual process is defined as appearance of equal or different states of space presented in sequences. The order of appearance of the same or different states of space is a rhythm. With a given number of different states we can generate a random process. In this kind of process it is known what states can appear but the order of their appearance is not predictable. Another group of processes, generated by unar operators, are predictable, since we know the exact order of appearance of same or different states throughout the entire process, by knowing which particular operator is employed.

I have based this presentation of discrete visual structures on two different types of signs: visual and verbal, but the visual presentation of images is the essential
subject of this analysis. Verbal signs (written text) are used here as a necessary meta-language in order to communicate the basic ideas on discrete visual structures to the readers.
CHAPTER I

SPATIAL STRUCTURE

1.1 Let some finite set be $A = (a, b, c, d, e)$ and let the neighborhood relations between elements of the set be defined in the following way:

$$a*b, a*d, b*c, b*d, c*d, d*e.$$  

Neighborhood relation is symmetric, which means that $a*b = b*a$. If we presume that the elements of $A$-set can be presented as definite parts of a plane, with a defined form and size such as those shown in Fig.1.1a, for example, then the observed structure can also be presented in the following way (Fig.1.1b): We can see that the
spatial neighborhood relations of elements in the structure are equal to those given by the relation above. A structure whose elements are definite parts of a plane with defined spatial neighborhood relations we will name the spatial structure. A set of elements which can define a spatial structure is the generating set. However, with the set of elements shown in Fig.1.1a it is possible to present a given structure in a number of ways as shown in Fig.1.2. In order to avoid such polysemic presentation of a spatial structure, we will adopt the form of the structure presented in Fig.1.1b as the standard form for any spatial structure. Therefore, only the structure presented through such a standard form can be named a spatial structure. In the following text it will be presented as shown in Fig.1.3. At the same time the one-element (monoelement) spatial structure will be presented in this way as well.

1.2 According to the previous explication we have some idea about neighborhood relations between elements of the spatial structure. For example, in the structure shown in Fig.1.1b elements (a) and (b) are neighbors, but not elements (a) and (c). Two elements are in a neighborhood relation if they are presented in the spatial
structure in the following ways (Fig.1.4): Two elements are not in a neighborhood relation if they are presented in the spatial structure in the following ways (Fig.1.5):

![Fig.1.4](image1)

![Fig.1.5](image2)

Therefore, in spatial structure Sa (Fig.1.6), elements (a) and (b) are neighbors but not elements (a) and (c). In the structure Sb, elements (d) and (e), or (f) and (g) are neighbors. In the same structure elements (d) and (f) are not neighbors, nor are (e) and (g). In structure Sc element (k) is a neighbor of (h) and (i), but elements (h), (i) and (j) are not mutual neighbors. In structure Sd, for example, elements (l) and (m), and (l) and (n) are neighbors, but elements (m) and (n) are not. In the last structure (Se), element (p) is a neighbor of (o) and (q), but these elements are not mutual neighbors.

1.3 If we have two different generating sets: \( A = (a,b,c,d,e) \) and \( B = (f,g,h,i,j) \) with an equal number of elements but with a different form and size, as shown in
Fig. 1.7a, we can define two different spatial structures (Fig. 1.7b). The neighborhood relations between elements of these two spatial structures are:

\[ a*b, a*d, a*c, b*d, c*d, d*e; \]

\[ f*g, f*i, g*h, g*i, h*i, i*j. \]

Comparing these two relations we can see that they are equal, allowing us to come to the conclusion that two different spatial structures defined by two different generating sets can have equal relations of neighborhoods between elements. These kinds of structures are isomorphic spatial structures. For example, all structures presented in Fig. 1.8 are isomorphic spatial structures. It is obvious that structures with a different number of elements, by definition, could not be isomorphic. Therefore, the spatial structures shown in Fig. 1.9, for example, are not isomorphic. According to this we can say that all two-element spatial structures are isomorphic (Fig. 1.10). However, this
statement is not correct for structures with a number of elements, \( n > 2 \). For example,

![Fig. 1.10](image)

the three-element structures in Fig. 1.11 are not isomorphic. Each element in structure

![Fig. 1.11](image)

Sa has two neighbors, but in structure Sb there are two elements with one neighbor and one with two neighbors.

1.4 The five-element spatial structures shown in Fig. 1.12 are not isomorphic, since in structure Sa there are two elements with two neighbors; in structure Sb there is

![Fig. 1.12](image)

none with two neighbors. Observing carefully we can come to the conclusion that they are defined by the same generating set shown in Fig. 1.1a. All nonisomorphic structures defined by one generating set are homogeneous spatial structures. The number of homogeneous spatial structures define the potentiality of the corresponding generating set. For the generating set from Fig. 1.1a, for example, the potentiality is two \((p = 2)\). A potentiality of any two-element generating set is one \((p = 1)\), since there is only one possible nonisomorphic configuration with two elements. With a three-element generating set it is possible to define two nonisomorphic configurations (Fig. 1.11).
However, these two structures are not homogeneous since they are defined by two different generating sets. Therefore, it would be useful to define a three-element generating set whose potentiality is two ($p=2$). With the three-element generating set shown in Fig.1.13a, it is possible to define both nonisomorphic configurations (Fig.1.13b). It is not possible to define a three-element generating set whose potentiality is greater than two ($p>2$). With a four-element generating set it is possible to define six nonisomorphic spatial structures as shown in Fig.1.14. However, these structures are not homogeneous since they are defined by six different generating sets. Therefore, it would be interesting to define a four-element generating set whose potentiality is six. With the four-element generating set shown in Fig.1.15a it is
possible to define only four homogeneous spatial structures (Fig.1.15b). The potentiality of this generating set is four \((p=4)\). With another generating set (Fig.1.16a), it is possible to define five homogeneous spatial structures presented in Fig.1.16b; therefore its potentiality is five \((p=5)\). Until now the four-element generating set whose
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potentiality is six \((p=6)\) has not been found. With a five-element generating set it is generally possible to define 20 nonisomorphic spatial structures. However, until now it was possible to define a five-element generating set whose potentiality is only ten \((p=10)\). Two such examples are presented in Fig.1.17. It would be interesting to find a five-element generating set which can define all 20 nonisomorphic configurations, if such a generating set exists at all. However, we can easily find some five-element generating sets whose potentiality is 1, 2, 3 or more. The five-element generating set shown in Fig.1.18, for example, can define only one nonisomorphic spatial structure; therefore its potentiality is one \((p=1)\). Another five-element generating set whose potentiality is 2 \((p=2)\) is shown in Fig.1.19a, and finally an example of a five-element generating set whose potentiality is 3 \((p=3)\) is presented in Fig.1.19b.

1.5 In previous explications it was shown that a certain three-element generating set can define two nonisomorphic spatial structures (Fig.1.13). However, it would not be difficult to demonstrate that isomorphic spatial structures can also be defined by the
very same set (Fig. 1.20). These structures are at the same time isomorphic and homogeneous. Spatial structures that are both isomorphic and homogeneous we will name homomorphic structures. Two homomorphic structures are equal if all corresponding elements occupy the very same position in the structure (Fig. 1.21). However, if we observe element b in different structures, as shown in Fig. 1.22, we could note that its position remains unchanged in relation to the structure as a whole. We may conclude that the position of the element within a structure is not conditioned by its neighborhood relations with other elements of the structure. This means, that besides a given size and form and corresponding neighborhood relations, only when the spatial structure has been defined does the element acquire one additional characteristic and this is its position within the structure. The position of the element is typical of the spatial structure.

1.6 The configuration presented in Fig. 1.23 is a four-element spatial structure. Element (a) has one neighbor: (b), element (b) has three neighbors: (a,c,d), element
(c) has two: (b,d) and element (d) has two: (b,d) neighbors. If we are in position (a) there is only one possibility of changing position: (b). From position (c) and (d) there are two possibilities: (b,d and b,c respectively), and finally from (b) there are three possibilities of changing position: (a,c,d). A spatial structure has the dimension n if it contains at least one element with its n neighbors. Examples of some structures with different dimensions are presented in Fig.1.24. When a structure whose dimension is n contains elements with n-1, n-2, n-3, ... neighbors, then this is a limited spatial structure. Elements with n-1 neighbors are limits of the first order, elements with n-2 neighbors are limits of the second order, ..., elements with one neighbor are limits of the n-1 order. The three-element spatial structure shown in Fig.1.24a (Sa) is a limited two-dimension structure with two limits of the first order. The five-element spatial structure Sb is a limited four-dimension structure with one limit of the first order, two limits of the second order and one limit of the third order. The limited structure Sc has 13 elements and its dimension is 12, since there is one element in this structure with 12 neighbors. The other 12 elements are limits of the ninth order. The limited ten-element structure Sd is of dimension nine with nine limits of the eighth order. A one-element (monoelement) spatial structure (Fig.1.3) is dimension zero (d=0), since there is no neighborhood relation defined within this structure. All two-element spatial structures are of dimension one since they contain two elements, each with one
neighbor. Three-element spatial structures shown in Fig.1.25 are homogeneous and both two-dimension. Spatial structure $S_a$ is limited with two limits of the first order.

![Fig. 1.25](image)

However, in spatial structure $S_b$ all three elements have two neighbors. A structure whose elements all have the same number of neighbors is an unlimited spatial structure.

1.6.1 A two-element spatial structure is of dimension one. As both elements of this structure have the same number of neighbors (one each), the structure of such a type is unlimited and one-dimension. It can be said that all two-element spatial structures are unlimited (Fig.1.26). The minimal number of elements for such a structure is two, and, at the same time, this is the maximum number, since an unlimited one-dimension spatial structure with the number of elements $n>2$ does not exist.

1.6.2 A three-element spatial structure has only two nonisomorphic configurations (Fig.1.27). One is a limited two-dimension spatial structure with two limits of the first
order, as shown in Fig.1.27a, the other is an unlimited two-dimension spatial structure, as shown in Fig.1.27b.

![Fig.1.27b](image)

1.6.3 A four-element spatial structure has six nonisomorphic configurations as was shown before. Two are unlimited (dimension 2 and 3) and four are limited (dimension 2 and three). Some four-element unlimited two-dimension spatial structures are shown in Fig.1.28a. Another unlimited configuration is a three-dimension spatial structure (Fig.1.28b). There is only one configuration for a limited four-element spatial structure.

![Fig.1.28](image)
whose dimension is two. This kind of structure has two limits of the first order and some examples are shown in Fig.1.28c. The spatial structures in Fig.1.28d are limited three-dimension structures with three limits of the second order. The next four-element configurations are limited three-dimension spatial structures with two limits of the first order and one of the second order (Fig.1.28e). The last four-element configurations are also limited three-dimension structures with two limits of the first order (Fig.1.28f).

1.7 A minimal unlimited two-dimensional spatial structure contains three elements and can be realized in several ways as shown in Fig.1.29. In order to consider such cases it would be interesting to determine the procedures of generating some general examples of an unlimited two-dimension spatial structure, starting from its corresponding minimal form (Fig.1.30).
1.7.1. An unlimited three-dimension spatial structure can be realized with a minimum of four elements. Starting from the minimal three-dimension spatial structures presented in Fig.1.31, it is possible to give several examples of generating unlimited spatial structures of the same dimension but with the number of elements greater than the minimal \((n>4)\) as shown in Fig.1.32. Upon the basis of the minimal
structures shown above (Fig.1.31) it is possible to define some isomorphic configurations with changed form and position of elements while maintaining relations of neighborhood and the way in which they are realized. A number of examples for such structures is shown in Fig.1.33. While using the given procedures for generating unlimited structures shown in Fig.1.32, and the minimal structures shown in Fig.1.33, we can easily determine analogous procedures for generating unlimited three-dimension spatial structures whose number of elements is greater than minimal (Fig.1.34). The given examples demonstrate that all unlimited three-dimension spatial structures contain an even number of elements. With a four-element generating set it is possible to define only one nonisomorphic unlimited three-dimension spatial structure (Fig.1.35a).
six-element generating set it is possible to define only one unlimited three-dimension spatial structure (Fig. 1.35b). With eight-element generating sets it is possible to define three nonisomorphic three-dimension spatial structures (Fig. 1.35c). With ten-element generating sets there are nine (Fig. 1.35d), with 12-element sets there are 32 (Fig. 1.35e), and with 14-element sets there are 132 unlimited nonisomorphic three-dimension spatial structures. Examples of procedures for generating some of these nonisomorphic configurations are presented in Fig. 1.36.
Fig. 1.36
1.7.3 A minimal unlimited four-dimension spatial structure can be realized with a six-element generating set. Fig.1.37 shows two isomorphic configurations of such a structure. Some examples of unlimited four-dimension configurations with a different number of elements (n=8,9,10,11,12) are shown in Fig.1.38. These are all nonisomorphic unlimited four-dimension spatial structures with 8,9,10,11 and 12 elements. Starting
from minimal structures (Fig.1.37), the examples in Fig.1.39 show the procedures for generating corresponding unlimited four-dimension spatial structures with a number of elements greater than the minimal.

Fig.1.39

1.7.4 In the case of the unlimited five-dimension spatial structures, the minimal structure requires a 12-element generating set (Fig.1.40). Starting from the structures shown above it is possible to define two different procedures for generating unlimited five-dimension spatial structures (Fig.1.41). Some nonisomorphic unlimited five-dimension structures with 20 and 22 elements are shown in Fig.1.42.

Fig.1.40

Fig.1.41
1.8 If we compare the examples shown in Fig.1.30, 32, 39, and 41, we can see a rather distinct analogy with the ways in which these procedures are represented. In order to have a better comparison, these procedures are shown, in part, in Fig.1.43.
As is shown, we can define unlimited spatial structures of 1, 2, 3, 4, and 5 dimensions, but there is no one generating set which can define an unlimited spatial structure with dimension greater than five. In other words, there is no spatial structure possible in which all elements have six neighbors each, regardless of the number of elements. This is one of the interesting characteristics of spatial structures in general.

*) This generated set is suggested by Ranko Bon, Assistant Professor at M.I.T., who also gave a solution for a second example in Fig.1.17.
2.1 Let some three-element generating set be $C = (a,b,c)$ and let the neighborhood relations between elements of the set be defined in the following way: $a*b$, $b*c$. If we know that elements of a generating set can be presented as definite parts of a plane, then the observed set $C$ could be presented in the following way (Fig.2.1):

![Fig.2.1](image)

With this generating set and the given neighborhood relations we can define a spatial structure as shown in Fig.2.2a. However, the same generating set with the same neighborhood relations can define another type of structure. In this structure neighborhood relations are presented by the content of elements and not by their positions (Fig.2.2b). The content of an element can visually be presented by the intensity of light (from black to white) or by different colors. This content of an
element we will name the quality. A structure in which the neighborhood relations between elements are represented by content is the qualitative structure. The qualitative structure shown in Fig.2.2b can be expressed in different ways (Fig.2.3). All these qualitative structures are equal because the position between elements of this kind of structure is of no consequence. A one-element qualitative structure is defined by one quality of content. Some examples of such a structure are shown in Fig.2.4.

2.2 We know how to present the neighborhood relations of elements in a spatial structure. Also we have some idea about the differences in the representation of spatial and qualitative structures. This distinction will be expressed in the presentation of neighborhood relations between the elements of a qualitative structure. One can say that there are two basically different approaches regarding the denotation of the neighborhood relations between elements of the qualitative structure: natural and conventional.
2.2.1 The natural denotation of a five-element qualitative structure as defined by the generating set shown in Fig.2.5a, with the following relations of neighborhoods:

\[ a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \]

Fig.2.5

\( a*b, b*c, c*d, d*e, \) can be presented by white, light-gray, gray, dark-gray and black (w,l,g,d,b). In this denotation, qualities white and light-gray are neighbors; light-gray and gray are neighbors; gray and dark-gray as well as dark-gray and black are neighbors (w*l, l*g, g*d, d*b). However, white and gray or white and black, for example are not neighbors (Fig.2.5). Another three-element qualitative structure defined by relations of neighborhoods: \( k*m, m*n, \) can be expressed by natural presentation in the following ways (Fig.2.6): In Fig.2.6a the neighborhood relations between elements are presented by white, gray and black qualities (w*g, g*b). In Fig.2.6b these neighborhood relations are presented by gray, dark-gray and black qualities (g*d, d*b), and in Fig.2.6c by white, light gray and gray qualities (w*l, l*g). We know that in black, gray and white combination black is a neighbor of gray, but not of white. Also in white, light-gray and gray combination, gray is a neighbor of light-gray, but not of white. A two-element qualitative structure in this presentation can be defined, for example, by black*white, or black*gray, or white*gray combinations as shown in Fig.2.7.
2.2.2 In conventional denotation we can arbitrarily choose the neighboring qualities depending on the character of the qualitative structure. Let some five-element qualitative structure be defined by the following neighborhood relations: \(a*b, a*d, b*c, b*d, d*e\). This structure can be presented by the white, light-gray, gray, dark-gray and black qualities as shown in Fig.2.8. Because of conventional denotation in this qualitative structure, white and gray are neighbors as well as white and black. Also, light-gray and gray, light-gray and black as well as dark-gray and black are neighbors. However, in this same structure neither white and light-gray are neighbors, nor gray and dark-gray. This may seem unusual, but the reasons for such denotation will be elaborated further on. Three different qualitative structures defined by the following neighborhood relations are shown in Fig.2.9:

\[
\begin{align*}
\text{Sa.} & \quad a*b, a*c, b*c; \\
\text{Sb.} & \quad k*m, l*m, l*n, m*o, n*o; \\
\text{Sc.} & \quad r*p, r*s, r*t, p*s, p*t;
\end{align*}
\]

According to the given neighborhood relations in structure Sa, black, white and gray are mutual neighbors. In structure Sb, for example, white and gray are neighbors but not dark gray and black. In structure Sc, white is a neighbor of light-gray, while
dark-gray and black are not neighbors. One can assume that natural denotation can be a subset of conventional denotation. We may choose a conventional denotation which is as close as possible to natural denotation.

![Fig. 2.9](image)

2.3 Let two different generating sets (A and B) be defined as shown in Fig. 2.10, and let the neighborhood relations between elements of these two sets be defined in this manner:

**Sa.** a*b, a*c, b*c, c*d;

**Sb.** m*n, m*p, n*p, p*q;

![Fig. 2.10](image)

With the given sets and corresponding neighborhood relations it is possible to define two different qualitative structures shown in Fig. 2.11. Comparing these two structures we can see that they have equal neighborhood relations, allowing us to come to the conclusion that two different qualitative structures defined by two different generating
sets can satisfy the equal neighborhood relations between elements. These kinds of structures are isomorphic qualitative structures. Some isomorphic qualitative structures are shown in Fig.2.12. We can see that all corresponding elements in these qualitative structures are defined by the same qualities: white, gray and black. However, isomorphic qualitative structures can be defined by different qualitative contents as shown in Fig.2.13. The qualitative structure in Fig.2.13a is defined by: white, light-gray and gray qualities. The next qualitative structure (Fig.2.13b) is defined by: light-gray, gray and dark-gray, and the last structure (Fig.2.13c) is defined by: gray, dark-gray and black. Now we know that two qualitative structures can be isomorphic even if they are defined by different qualities. It is obvious that structures with a different number of elements could not be isomorphic (Fig.2.14). According to this we can say that all two-element qualitative structures are isomorphic (Fig.2.15). With
three-element generating sets it is possible to define only two nonisomorphic qualitative structures (Fig.2.16). The neighborhood relations between elements of these two qualitative structures are:

\[ \text{Sa. } a\ast b, a\ast c, b\ast c; \]
\[ \text{Sb. } e\ast f, f\ast g; \]

It is not difficult to come to the conclusion that these two relations define two nonisomorphic structures. We must not forget though, that these relations are not defined by relations between qualities. On the contrary, because of conventional denotation the relation between qualities are defined by all possible neighborhood relations between elements of a three-element generating set.

2.4 Let some nonisomorphic four-element qualitative structures be defined by the following neighborhood relations:

\[ \text{Sa. } a\ast b, a\ast c, b\ast d, c\ast d; \]
\[ \text{Sb. } e\ast f, f\ast g, g\ast h; \]
\[ \text{Sc. } k\ast l, k\ast m, k\ast n, l\ast m, l\ast n; \]

These three structures can be presented as shown in Fig.2.17. Obviously these
four-element qualitative structures are defined by the same generating set. All nonisomorphic qualitative structures defined by one generating set are homogeneous. The number of homogeneous qualitative structures define the potentiality of the corresponding generating set. In the case of conventional denotation of qualitative structure, the potentiality depends only on the number of elements of the generating set. Therefore, with a two-element generating set we can define only one nonisomorphic qualitative structure (Pq=1). With a three-element generating set there are two nonisomorphic qualitative structures (Pq=2). The maximum potentiality of a four-element generating set can be six (Pq=6), and for a five-element generating set this number can be Pq=21.

2.5 We know, for example, that a three-element generating set can define two nonisomorphic qualitative structures (Fig.2.16). It would not be difficult to demonstrate

![Diagram](image)

Fig.2.17

that isomorphic qualitative structures can also be defined by the very same generating set (Fig.2.18). The neighborhood relations between elements of the qualitative structures presented above are equal:

![Diagram](image)

Fig.2.18

Sa. a*b, b*c; Sb. d*e, e*f; Sc. g*h, h*i;
It is obvious that, at the same time, these three structures are isomorphic and homogeneous. A qualitative structure that is both isomorphic and homogeneous we will name homomorphic qualitative structure. However, these homomorphic qualitative structures are not equal because corresponding elements contain different qualities. Two homomorphic qualitative structures are equal only if all corresponding elements contain the very same quality (Fig.2.19). If we observe element p in the three different qualitative structures presented in Fig.2.20, we could note that this element possesses the same quality. We can assume that the quality of the element within a qualitative structure is not conditioned by its neighborhood relations with other elements of the structure. This means that, in addition to size and form and its corresponding neighborhood relations, the element acquire one additional characteristic: its quality.

2.6 The four-element qualitative structure shown in Fig.2.21 is defined by the following neighborhood relations: a*b, a*c, b*c, b*d. Element a has two neighbors (b,c), element b has three neighbors (a,c,d), element c has one (b) and element d has
one (b) neighbor. A qualitative structure has the dimension n if it contains at least one element with its n neighbors. Some qualitative structures with different dimensions are presented in Fig.2.22. These qualitative structures are defined by the following neighborhood relations:

\[ Sa. \quad a*b; \]
\[ Sb. \quad c*d, c*e; \]
\[ Sc. \quad f*g, f*h, f*i, h*i; \]

When a qualitative structure whose dimension is n contains elements with n-1, n-2, n-3,... neighbors, then this is a limited qualitative structure. Elements with n-1 neighbors are limits of the first order, elements with n-2 neighbors are limits of the second order, etc. The two-element qualitative structure, Sa, is a one-dimension structure without limits. The three-element qualitative structure, Sb, is a limited two-dimension qualitative structure with two limits of the first order (d,e). The four-element structure, Sc, is a limited three-dimension qualitative structure with two limits of the first order (h,i) and one limit of the second order (g). A one-element qualitative structure (Fig.2.4) is of dimension zero, since there is no neighborhood relation defined within this structure.
2.7 It was mentioned that with a three-element generating set it is possible to define two nonisomorphic qualitative structures (Fig.2.16). One is a limited two-dimension structure with two limits of the first order (Fig.2.16b). Another qualitative structure is also a two-dimension structure, but all three elements have two neighbors. The four-element qualitative structure shown in Fig.2.23 is of dimension three, and all four elements are mutual neighbors:

\[
\text{a*b, a*c, a*d, b*c, b*d, c*d;}
\]

A qualitative structure in which all elements have the same number of neighbors is an unlimited qualitative structure. A two-element qualitative structure is of dimension one as both elements of this structure have the same number of neighbors (one each). Therefore, all two-element qualitative structures are unlimited (Fig.2.24). With a three-element generating set it is possible to define only one unlimited qualitative structure. The dimension of such a structure, defined by the following relations: \(a*b, a*c, b*c,\) is two (Fig.2.25). With a four-element generating set it is possible to define
two unlimited qualitative structures (Fig.2.26). The qualitative structure in Fig.2.26a, defined by the following relations: \(a*b, a*d, b*c, c*d\), is an unlimited two-dimension structure. Another example shown in Fig.2.26b, defined by the following relations: \(k*l, k*m, k*n, l*m, l*n, m*n\), is also an unlimited but three-dimension qualitative structure. With a five-element generating set it is possible to define two unlimited qualitative structures. One, defined by the following neighborhood relations: \(a*b, a*e, b*c, c*d, d*e\), is the two-dimension structure shown in Fig.2.27a. Another unlimited structure, defined by the following neighborhood relations: \(k*l, k*m, k*n, k*p, l*m, l*n, l*p, m*n, m*p, n*p\), is the four-dimension qualitative structure shown in Fig.2.27b. We can now understand a major reason for conventional denotation. In this presentation it is possible to define all qualitative structures whose dimensions are \(1, 2, 3, 4, 5, \ldots, n\), both limited and unlimited. However, there are two important restrictions:

a. The possibility of distinguishing (expressing) different qualities is contraprophoronal to the number of elements of the qualitative structure. In a two-element qualitative structure we need only two different qualities, black and white, for example. However, in a twenty-element qualitative structure, for example, we will need twenty different qualities. It is obviously much harder to express and distinguish such a large number of different qualities.
b. In conventional denotation we cannot recognize a type of qualitative structure, whether it is limited or unlimited, or of what dimension it is, unless the neighborhood relations between elements are known. It is not possible to read it directly from the figure of the structure.

2.8 In natural denotation we can define qualities such as white (w), light-gray (l), gray (g), dark-gray (d) and black (b). All one-element qualitative structures defined by these five qualities are shown in Fig.2.28. We know that all two-element qualitative structures are unlimited and of one dimension. With a two-element generating set and five given qualities it is possible to define ten homomorphic qualitative structures in natural denotation (Fig.2.29). With a three-element generating set and five given qualities it is possible to define ten homomorphic qualitative structures in natural denotation (Fig.2.30). All these qualitative structures are limited two-dimension, with two limits of the first order. In natural denotation it is not possible to define an unlimited two-dimension qualitative structure, since in this denotation there is always
one lightest quality and one darkest quality as two limits of the structure. With a four-element generating set and five given qualities it is possible to define four homomorphic qualitative structures in natural denotation (Fig.2.31). And finally, with a five-element generating set and five given qualities it is possible to define only one qualitative structure (Fig.2.32). This is also a limited two-dimension qualitative structure since in natural denotation we can define only these neighborhood relations: w*l, l*g, g*d, d*b. One can come to the conclusion that in natural denotation it is possible to define only limited two-dimension spatial structures (for n>2) with two limits of the first order represented by the lightest and darkest qualities. The lightest quality in the structure has one neighbor, which is the next lightest quality in the structure. The
darkest quality in the structure has also one neighbor, which is the next darkest quality in the structure. All other elements in the structure have two neighbors, one lighter and one darker. We can extend the number of qualities in order to get a quality structure with a greater number of elements. We can define, for example, a new quality dark-white which is darker than white and lighter than light-gray. However, with this extension we can arrive again at a limited two-dimension qualitative structure since the new quality element (dark-white) has two neighbors: white and light-gray.

2.9 Let the two two-element qualitative structures be defined as shown in Fig.2.33. A qualitative structure, S_a, is defined by two qualities: black and white. We can assume that these two qualities relate one toward another as positive toward negative. The negative of white is black and negative of black is white. A qualitative structure, S_b, is defined also by two qualities: light-gray and dark-gray. We can assume that these two qualities also relate one toward another as positive toward negative. The negative of light-gray is dark-gray and vice versa. Two qualities which relate one toward another, as positive toward negative are complementary qualities. White is the complement of black (w=\^b), light-gray is the complement of dark-gray (l=\^d). A quality equal to its own complement is neutral quality. In natural denotation, neutral
quality is only and always gray (g=^g). A structure with an odd number of elements containing a neutral quality, with each quality having its complement, is a complete qualitative structure. The three-element qualitative structure shown in Fig.2.34a is complete since it contains a neutral quality (g) and two other qualities which are complementary (w=^b). A structure with an even number of elements, each with its complement, is a complete qualitative structure. The four-element qualitative structure shown in Fig.2.34b is complete since white is the complement of black and light-gray is the complement of dark-gray (w=^b, l=^d). A complete five-element qualitative structure can be defined by white, light-gray, gray, dark-gray and black qualities. In this structure (Fig.2.35a) white is the complement of black (w=^b), light-gray is the complement of dark-gray (l=^d), and gray is the neutral quality (g=^g). This complete qualitative structure is a limited two-dimension structure with two limits (white and black), defined by the following neighborhood relations: w*l, l*g, g*d, d*b. With the very same generating set and qualities we can define another limited two-dimension qualitative structure as shown in Fig.2.35b. In these two homomorphic structures, corresponding elements contain complementary qualities. The neutral quality gray is self-complementary. Two homomorphic qualitative structures are complementary if the corresponding elements contain complementary qualities. Examples of two-element complementary qualitative structures are shown in Fig.2.36. All three-element
complementary qualitative structures should have one neutral quality element (gray) as shown in Fig.2.37. Examples of four and five-element complementary qualitative structures are shown in Fig.2.38.

2.10 Let a three-element generating set be defined as shown in Fig.2.39. With this set we can define the three-element qualitative structure (w*g, g*b) shown in Fig.2.40a. By definition, this structure contains three different qualities: white, gray and black. But, what can we say about another structure defined by the same generating set as
shown in Fig.2.40b? This is a three-element structure defined by two qualities (white and black). Therefore, there is only one neighborhood relation between qualities in this structure (w*b), since the neighborhood relation between equal qualities is not defined. Two elements in this structure contain the same quality (black). However, by definition, in the qualitative structure there are no two elements with equal qualities. One can come to the conclusion that this structure doesn't correspond to the qualitative structure definition. Let some structure be defined with an n-element generating set and q different qualities. If n=q then this structure is a regular qualitative structure as defined before. However, if n>q the structure must contain at
least two elements with an equal quality. The structure defined by an n-element generating set and q qualities is a reduced qualitative structure if \( n > q \). Therefore, the structure shown in Fig.2.40b is a reduced qualitative structure. Some examples of reduced qualitative structures with two qualities (\( q = 2 \)) and a different number of elements (\( n > 2 \)) are presented in Fig.2.41. A special type of reduced qualitative structure can be defined as \( q = 1 \) and \( n > 1 \) (Fig.2.42). All these qualitative structures are one-quality structures, and the neighborhood relations between elements within the structures are not defined.

2.11 All reduced qualitative structures defined by the very same generating set are homogeneous (Fig.2.43). The difference between the number of elements and the number of qualities (\( n - q \)) of a reduced qualitative structure defines its degree of reduction (\( dr \)). For the regular qualitative structure, \( dr = 0 \). Examples of qualitative structures with \( n = 4 \) and \( q = 1, 2, 3, 4 \) are presented in Fig.2.44. The structure \( dr = 0 \) is a
one-quality structure with no neighborhood relations. The structure \( dr=2 \) is a
two-quality structure with only one neighborhood relation (\( w*b \)). The structure \( d=1 \) is
a three-quality structure with two neighborhood relations in natural presentation (\( w*l, 1*b \)). The last example, \( dr=0 \), is a regular qualitative structure with three neighborhood
relations in natural presentation (\( w*l, 1*d, d*b \)). With one quality (\( q=1 \)) we can define

only one qualitative structure without considering a number of elements, \( n \). However,
for the two-quality structure shown above (\( dr=2 \)), for example, there are three possible
reduced qualitative structures (Fig.2.45).

2.12 Two reduced qualitative structures \((n1,q1)\) and \((n2,q2)\) are isomorphic if they
have \( n1=n2, q1=q2, \) and equal neighborhood relations (Fig.2.46). These three structures

are isomorphic since \( n1=n2=n3=3 \) and \( q1=q2=q3=2 \), and the neighborhood relation
between qualities is \( w*b \) for all three structures. Both homogeneous and isomorphic
reduced qualitative structures are homomorphic (Fig. 2.45). Two homomorphic reduced qualitative structures are equal if the corresponding elements contain equal qualities (Fig. 2.47). Two homomorphic reduced qualitative structures are complementary if all corresponding elements contain complementary qualities (Fig. 2.48). It will be important to define a set whose elements are complementary reduced qualitative structures. Two complementary one-quality reduced structures define a binary set (B-set). As we can see in Fig. 2.49, for example, a B-set can be defined by white and black or by light-gray and dark-gray qualities. However, in further explication a B-set will be defined only by white and black qualities as shown in Fig. 2.50.
Fig. 2.50
3.1 Let a two-element generating set be $S = (p,q)$ as shown in Fig.3.1a. With this generating set, we know, it is possible to define both spatial and qualitative structures (Fig.3.1b). If the elements of qualitative structures occupy the positions of corresponding elements in a spatial structure we have a new type of structure. The unity of a spatial structure and qualitative structure defined by the same generating set determines
a state of space. With the spatial and qualitative structures shown in Fig.3.1b it is possible to define only one state of space. However, with the other two-element structures shown in Fig.3.3a we can define two states of space (Fig.3.3b). Both spatial and qualitative structures defined by the very same generating set are corresponding structures. Structures shown in Fig.3.1a are corresponding, as well as the structures shown in Fig.3.3a.

3.2 The three-element corresponding set shown in Fig.3.4a can define only one state of space (Fig.3.4b). With another three-element corresponding set, shown in Fig.3.5a, we can define two states of space (Fig.3.6b). And finally, with the three-element corresponding set shown in Fig.3.6a we can define six different states of space (Fig.3.6b). The number of different states of space defined by the same corresponding set is the potentiality of the corresponding set. The corresponding set
shown in Fig.3.6a, for example, has a potentiality of six (Pc=6). All different states of space defined by the very same corresponding set are homogeneous states (Fig.3.6b).

![Fig.3.6](image)

3.3 Two different states of space defined by the same corresponding set are shown in Fig.3.7. The spatial and qualitative structures of a corresponding set both contain four elements and are limited and two-dimension. The qualitative structure is defined in natural presentation ($w^1, l^d, d^b$). We know that in qualitative structures the spatial neighborhood relations between elements is not defined. However, in the state of space all elements of a qualitative structure have spatial neighborhood relations defined by the corresponding spatial structure. In the first state of space shown in Fig.3.7b, the spatial neighborhood relations between qualities are: $w^1, l^d, d^b$. We can see that they are equal to the given neighborhood relations between qualities defined by the corresponding qualitative structure. However, in another state white is the neighbor of black, and light-gray and dark-gray are not spatial neighbors. The spatial neighborhood relations between qualities in this state are: $l^w, w^b, b^d$. It is
obvious that these relations are not equal to the neighborhood relations defined by a given qualitative structure. Such a state of space in which spatial neighborhood relations between qualities are equal to the neighborhood relations between qualities in a qualitative structure we will designate the natural state. We can now conclude that the first state of space shown in Fig.3.7b is natural and the other one is not. Examples of some natural states of space are shown in Fig.3.8. The corresponding qualitative structures are defined in a natural presentation.

Fig.3.8

3.4 Let two different states of space be defined as shown in Fig.3.9. The spatial neighborhood relations between qualities in these two states are: a. w*g, g*b; and b. w*g, g*b. We can see that these spatial neighborhood relations are equal. Two states of space are isomorphic if they have equal spatial neighborhood relations between qualities. Examples of some isomorphic states of space are shown in Fig.3.10. The spatial neighborhood relations between qualities for these states of space are:

a. w*g, w*b, g*b;

b. g*w, w*b;

c. l*w, w*d, d*b;

d. w*l, l*d, d*b;

Fig.3.9
3.5 Let two different states of space be defined as shown in Fig.3.11. These two states are homogeneous since they are defined by the very same corresponding set. In addition, these two states are isomorphic because of the equal spatial neighborhood relations between qualities: w*g, g*b. Both isomorphic and homogeneous states of space are homomorphic. Examples of some homomorphic states are shown in Fig.3.12.
Two states of space are equal if equal qualities occupy equal positions. Given the current definition of the corresponding set, equal states of space are also homomorphic (Fig.3.13).

3.6 With the corresponding set defined by a regular qualitative structure we can generate only states in which different qualities occupy different positions (Fig.3.14a). In this kind of state there are no two different positions with equal qualities. Therefore, it is not possible to define, for example, a state of space as shown in Fig.3.14b. In order to generate such a state we should define the corresponding set by a reduced qualitative structure. A corresponding set defined by a reduced qualitative structure we will designate the reduced corresponding set or RC-set. The RC-set
which can define the state of space shown above (Fig.3.14b) is presented in Fig.3.15a. With this RC-set it is also possible to define some other states of space, as shown in Fig.3.15b. However, with another corresponding set, shown in Fig.3.16a, we can define such a state (Fig.3.16b) which is equal to the last state shown in the previous figure. These two structures are equal since equal qualities occupy equal positions. Two equal states are identical if they are homomorphic (Fig.3.17).
3.7 Such a corresponding set defined by one spatial structure \((n>1)\) and one-quality qualitative structure can generate only one state of space (Fig.3.18). It is important to define such a corresponding set which can generate both one-quality and two-quality states of space, as shown in Fig.3.19. All these states can be generated by the corresponding set of two-element spatial structure and the B-set shown in Fig.3.20. A corresponding set defined by one spatial structure and a B-set we will designate the B-corresponding set or BC-set. With the BC-set shown in Fig.3.21a we can generate
states of space shown in Fig.3.21b. However, these are only a few examples of the 81 different states we can generate with the BC-set shown in Fig.3.21a. With another
BC-set shown in Fig.3.22a we can generate only 16 different states (Fig.3.22b). All states defined by the BC-set are the B-states (binary states).

3.8 We can now define some new terms within the state of space regarding its spatial characteristics. Usually these new terms will be denoted by a black quality.
Each state of space element with a black content, whose spatial neighbors are all white is an isolated point. Examples of some isolated points defined in different structures are presented in Fig.3.23.

3.8.1 Two spatially neighboring elements with the same quality (black) are a couple (Fig.3.24). The two elements with the same quality (black) shown in Fig.3.25 are not a couple since both black elements are surrounded by white elements. Therefore these elements are two isolated points.

3.8.2 Three and more connected neighboring elements with the same quality (black), with at least one element with two neighbors, and no elements with three or more neighbors is the line. Examples of a three-point line, defined in different configurations, are presented in Fig.3.26. A line is open if it contains two points with one neighbor (two ends) as shown in Fig.3.26a. A line is closed if all points of a line have two neighbors. A minimal (three-point) closed line is a triplet (Fig.3.26b).
3.8.3 With four connected points we can define both open and closed lines. Some examples of four-point open and closed lines are shown in Fig.3.27. All four-point open lines are spatially isomorphic, as are all four-point closed lines, regardless of the differences between the structures in which they are defined. Two open (closed) lines with an equal number of points are isomorphic. Two isomorphic lines defined in equal spatial structures are homomorphic (Fig.3.28). Therefore, the five-point open lines shown in Fig.3.28a are homomorphic. The five-point open lines shown in Fig.3.28b are homomorphic too. In addition, all these five-point open lines are isomorphic.
3.8.4 Between two isolated points in the structure we can define many different lines, as shown in Fig.3.29. The number of line points, including the terminating elements (ends), determine the length of the line. The minimum line that can be charted between two points is the distance. Therefore, the length of the line shown in Fig.3.29a, for example, is \( l=5 \) and the distance between the line ends is \( d=3 \). According to this, the length of the couple is \( l=2 \) but the distance between the points of a couple is \( d=0 \). Also, the length of the triplet is \( l=2 \) but the distance between any two points of the triplet is \( d=0 \). Another interesting example is presented in Fig.3.30. The distance between points A and B is two \( (d=2) \) and there is only one
possible distance between these two points. However, for the distance between points A and C (d=3) there are four different possibilities, as shown in Fig.3.30b. Let an isolated point be defined as shown in Fig.3.31a, and let all the points with the same distance (d=2) from the given point be defined as shown in Fig.31b. These are all possible points with the distance two from the given point in this structure. We can easily determine that all these equidistant points are isolated. However, even when they are all isolated points, they divide the entire structure into two areas: exterior and interior. The interior is the area where the given point is located. It is not possible to define any connected line in which one end belongs to the interior and another belongs to the exterior. These are some interesting spatial characteristics for the structures shown above. Each spatial structure represents a specific universe of positions and neighborhood relations with its own topological characteristics. What is possible in one spatial structure, may not be possible in another. In a limited two-dimension structure, for example, it is not possible to define a closed line. It is also not possible to define a line whose length is greater than the number of elements of the structure (l<n).

3.8.5 Two lines with one common point define an intersection (Fig.3.32). An intersection is not a simple line since there is one point in this figure with more than
two neighbors. The intersections in Fig.3.32a are X-intersections and in Fig.3.32b are T-intersections. An intersection of more than two lines we will designate the star (Fig.3.33). In order to define an intersection or a star, however, we need a certain type of spatial structure. Therefore it is not possible to define the star in the structure shown in Fig.3.32.

3.9 Within a minimal unlimited 2D spatial structure it is possible to define an isolated point, a couple and a triplet (Fig.3.34). We can see that states a and d are complementary as well as states b and c. For each isolated point in an unlimited three-element structure, there is a couple as its complement and vice versa (Fig.3.35).

3.9.1 Within a minimal unlimited 3D spatial structure it is possible to define 16 different states of space, as shown in Fig.3.36. We can see that within this structure
it is possible to define an isolated point, a couple, a triplet and a figure which includes all elements of the structure. A complement of an isolated point is a triplet, and a complement of a couple is another couple.

3.9.2 Another four-element structure is limited 3D with two limits of the first order. In addition to an isolated point, we can define a couple, a triplet, two isolated points and a three-point open line, within this structure. Examples of these configurations are presented in Fig.3.37. A complement of an isolated point can be a
triplet \((a=\wedge d)\), or a three-point open line \((b=\wedge e)\). A complement of a couple can be another couple \((g=\wedge h)\) or two isolated points \((c=\wedge f)\).

3.9.3 Within a limited four-element 3D structure with one limit of the first order and one of the second order we can define an isolated point, two isolated points, a couple, a triplet and a three-point open line (Fig.3.38). A complement of an isolated

![Fig.3.38](image)

point in this structure can be a triplet \((a=\wedge b)\), or another isolated point and a couple \((c=\wedge d)\), or a three-point open line \((e=\wedge f)\). A complement of a couple is another couple \((g=\wedge h)\), and a complement of two isolated points is a couple \((i=\wedge j)\). In the last two structures we have not mentioned the states in which all positions are occupied by the same quality (all white or all black elements). These two configurations are trivial states.

3.10 Let two homogeneous states be defined as shown in Fig.3.39. Both of these two states contain equal amounts of complementary qualities, white and black. We can see that in the state \(p\) there is no element whose spatial neighbor contains the same quality. One can say that all white elements in this state are surrounded by black elements and vice versa. Within this state there are 24 black-white neighborhood relations. However, the distribution of black and white in another state \((q)\) is entirely different. One half of the state contains only black elements and the other half only
white elements. As a result of such black and white distribution, there are only four black–white neighborhood relations within this state. With the BC-set related to the states shown above we can also define many other states of space. Some of them are shown in Fig.3.40. Each of these states has a different number of black–white neighborhoods (a=8, b=12, c=16, d=20). However, it is possible to define different states with the same number of black–white neighborhoods as shown in Fig.3.41. In further explications any spatial black–white neighborhood relation we will designate a junction. Hence, all the states shown in Fig.3.41 have the same number of black–white junctions. One can say that a minimal number of black–white junctions represents the highest concentration of black/white qualities (Fig.3.39b), and a maximal number of black–white junctions represents the lowest concentration of black/white qualities, i.e uniform distribution (Fig.3.39a).
3.10.1 Now let us analyze the black&white distribution in some states defined by an equal number of black and white elements. The simplest examples are the states defined by a two-element spatial structure (Fig.3.42). It is obvious that for this kind of state there is only one black&white junction ($J=1$). Therefore, it is at the same time the highest and the lowest black/white concentration for this kind of state.

3.10.2 With a four-element generating set we can define six nonisomorphic spatial structures (Fig.3.43). Within these structures various distributions of black and white qualities are possible. In the states defined by the structure (a) there are three different types of black&white distributions (three different junction numbers) as shown in Fig.3.44. However, with structure (b) we can define such states which have only one type of black and white distribution with a number of black&white junctions $J=4$ as shown in Fig.3.45. With the third spatial structure (c) we can define two
different types of states regarding the black and white distribution, with black-white junctions $J=2$ and $J=4$ (Fig.3.46). In the case of states defined by structure $d$ it is possible to define only one type of black and white junction, with $J=2$ (Fig.3.47). With the next structure (e) we can get two different distributions of black and white: $J=2$ and $J=3$, as shown in Fig.3.48. And finally with the last structure (f) we can also arrive two different distributions: $J=3$ and $J=4$ (Fig.3.49). We can see that in the states defined by four-element spatial structures and an equal number of black and white elements, we can arrive at a maximum black/white concentration with $J=1$ and a minimum concentration with $J=4$. However, none of these structures can satisfy both distributions.

3.10.3 It would be interesting now to see what kind of black-white distributions we can form in the states defined by limited two-dimension spatial structures. In
order to be more specific: these spatial structures will have an even number of identical elements \( n > 2 \) (Fig. 3.50). All states of space defined by the structures shown above will contain an equal number of black and white elements. All such states with different black & white distributions are shown in Fig. 3.51. For the states defined by a four-element spatial structure, the uniform black & white distribution is with \( J = 3 \). Other states defined by a six-element spatial structure have the uniform black & white distribution with \( J = 5 \). And for the states defined by an eight-element spatial structure the uniform black & white distribution is \( J = 7 \). We can easily conclude that the states with \( n = 30 \) elements, for example, will have highly uniform black & white distribution (\( J = 29 \)). If we now have a state with a very large number of elements (\( n = 100 \), or
n=1000), we can easily define a state with the highest black & white concentration (J=1). However, in the case of the lowest black & white concentration (J=n-1), a black & white distribution will be so uniform that it will be not possible to distinguish a black element from a white one (Fig.3.52). This uniform distribution of black and white qualities can be accepted as a practical definition of another, third quality — gray.
There are some other ways in which one can generate a quality gray as a uniform distribution of black and white qualities, as we can see in Fig.3.53. This procedure will be clearer if we set aside all states with a uniform black & white distribution for \( n=4,6,8,10,\ldots, n \) element structures, as shown in Fig.3.54. Another example of generating a gray quality from a uniform distribution of black and white qualities is presented in Fig.3.55. With all procedures shown above one can come to the conclusion that quality gray can be treated not as an independent quality but as a derivative of two other qualities - black and white. A quality gray is the result of the uniform distribution of an equal number of elements with complementary qualities black and white in a state of space with a very large number of equal elements.

3.10.4 In a similar manner it is possible to generate some other qualities based on the uniform black & white distribution such as light-gray or dark-gray, for example. If we define light-gray as a uniform distribution of black and white qualities with a proportion of black:white = 1:2, then a procedure for generating a light-gray quality
can be defined as shown in Fig.3.56. Another procedure with a black and white proportion of black:white = 2:1 presented in Fig.3.57, can generate a complementary quality of light-gray - dark-gray. Two other examples for generating a light-gray quality with the same proportion of black:white = 1:2 are presented in the following figure (Fig.3.58). With a procedure similar to that of the second example shown above, we can generate again a complementary quality - dark-gray (Fig.3.59). After the preceding analyses, we can recognize one of the crucial aspects of the black and
white relations regarding a discrete and continual distribution of these two qualities. We know that, by definition, spatial and qualitative structures are discrete structures;

Fig. 3.59

thus the state of space, as the union of these two is also a discrete structure. However, in the case of a highly uniform distribution of two or more qualities, and

Fig. 3.60
when the state of space contains a very large number of very small elements, it is not practically possible to distinguish these qualities any more. Therefore, one can accept the conclusion that a continual distribution of a newly generated quality is established (Fig.3.60). The first and the last row in this figure represents a trivial case: a uniform distribution of only one quality is that very same quality. A uniform distribution of white is white, and a uniform distribution of black is black. It is obvious that, besides light-gray, gray and dark-gray qualities, it is possible in a similar manner to generate a practically unlimited number of other qualities on a black&white scale simply by varying the proportion of black to white.

3.10.5 During the previous analysis of a uniform distribution of black and white qualities, it was assumed that this distribution was equal for the entire state of space. However, besides this total uniform distribution it is possible to define a partially uniform distribution of black and white, with two or more distinct state of space areas, containing different black&white proportions. For example, some different states of space with a similar black&white distribution are presented in Fig.3.61. One half of these states has a proportion of black:white = 2:1, and the other half has a proportion of black:white = 1:2. When the number of elements \( n \) is very large, we will get one half of the state defined by continual uniform distribution of black and white as
dark-gray, and the second half as light-gray. However, the entire state appears as a
discrete distribution of two qualities: dark-gray and light-gray, with a proportion of
d:1 = 1:1. In the another example (Fig.3.62) it is shown how we can get a state of
space with four distinct areas: black, dark-gray, light-gray and white, defined by
different black&white distributions in the state of space with a very large number of
elements n. Because of the different local black&white distributions, the entire state
of space appears as a four segment structure with clear borders between black and
dark-gray, dark-gray and light-gray, and light-gray and white. Finally, it is possible
to define such a state of space with continual but not uniform black&white
distribution as shown in Fig.3.63. Beginning with the black quality on one side of the
state and gradually passing through gray in the middle, we can reach white at the
other side of the state.

3.10.6 We now have some idea of how to get various types of states with continual
black&white distribution. It would be interesting to define some reverse procedures:
starting from a state with continual but not uniform black&white distribution and
getting a state with discrete black&white distribution. Let one such state of space with
a very large number of elements and complex continual black&white distribution be
defined as shown in Fig.3.64. If we now divide this state into 36 segments (Fig.3.65a)
we can get the first discrete black–gray–white approximation (Fig.3.65b). With a further 9 segment division (Fig.3.66a), and assuming that black & gray segments can be approximated with black, and that white & gray segments can be approximated with the white segment, we will get the second, now only black & white approximation shown in Fig.3.66b. However, this procedure is not singular since it is possible to define a reverse procedure starting now from the state of space shown in Fig.3.66b. We can see how this procedure doesn’t necessarily result in the state shown in Fig.3.64 (Fig.3.67). This nonsingularity is a result of approximation logic applied in the procedures demonstrated above. How a different black–gray distribution in a segment can lead to the very same black segment and how a different white–gray distribution in a segment can lead to the very same white segment is presented in Fig.3.68. According to the approximations shown in the second row above, we can define
another procedure with an entirely different result as shown in Fig.3.69. The state of space defined by n elements is very complex regarding its black&white distribution.

Within entire states there are only three lines (gray) with a uniform and continual distribution of the same quality. However, within the same state there are three distinct segments with continual, but not uniform black&white distribution. In two of these segments a qualitative distribution is: gray-black-gray-white-gray-black with the middle segment containing a distribution in the following order: gray-white-gray-black-gray-white. Now, if we go back to Fig.3.64 and divide it into different types of segments (Fig.3.70a), we can get another kind of approximation (Fig.3.70b,c). In the next step (second approximation), following a five-segment division shown in Fig.3.71a, we can get an entirely different discrete black&white distribution (Fig.71b). As we can see for the same continual black&white distribution (Fig.3.64) there are at least two different corresponding discrete black&white distributions shown in Fig.3.66b and 3.71b. A state of space defined by a very large number of
elements can be named a macrostate, and a state defined by a small number of elements can be named a microstate. In spite of the fact that these are not very precise definitions, they may be practical for colloquial use. We can now identify two different types of macrostate–microstate relations. The one shown in Fig.3.63 is based on a discrete black&white distribution without any gray appearance, except in the last state (macrostate). Another example, shown in Fig.3.67, is based on a discrete black–gray–white distribution in all states except in the first state (microstate) which has only a discrete black&white distribution.

3.11 Let the two spatial structures with n=4 and n=16 elements be defined as shown in Fig.3.72a. With these two structures we can easily generate such states of space in which there are no two neighboring elements with the same quality (Fig.3.72b). In order to define such states of space we need only two different qualities. Another type of structure with the same characteristic are limited and two–dimension spatial structures (Fig.3.73). However, a minimal unlimited 2D spatial structure (Fig.3.74a) requires three different qualities in order to be covered in a way that no two equal qualities are spatial neighbors (Fig.3.74b). And, finally, a minimal unlimited 3D spatial structure (Fig.3.75a) requires four different qualities (Fig.3.75b).

Until now it was not possible to define such a spatial structure which will require
more than four different qualities to be covered in a way that no two equal qualities are spatial neighbors, regardless of the number of elements of a structure or its dimension. Four different qualities are necessary and sufficient for any spatial structure.

3.12 Let the two different states be defined as shown in Fig.3.76. These two states are complementary since the equal positions contain complementary qualities. A procedure which represents transformation of any state into its complement is a complementary operation. For any two complementary states there is at least one complementary operation. Examples of complementary operations for states shown above are presented in Fig.3.77.
Let the two-element spatial structure be defined as shown in Fig.3.78a. Each element of the structure is denoted with numbers 1 and 2. The neighborhood relations between elements of this structure can define another spatial structure as shown in Fig.3.78b. The existence of the neighborhood relations between two elements we will denote with white, and the nonexistence of this relation we will denote with black in the state of space defined in Fig.3.78c. For any given spatial structure $A$, with $n$ elements, it is possible to define a specific state of space, $M$, with $n \times n$ elements. For each neighborhood relation within spatial structure $A$ there is one corresponding element defined within the state of space, $M$. If the neighborhood relation between two elements of structure $A$ exists, the corresponding element of the state $M$ is white, and if the neighborhood relation doesn’t exist the corresponding element of the state $M$ will be black. A state of space $M$, which corresponds to the neighborhood relations of the spatial structure $A$, we will designate a matrix state. Let a limited
three-element spatial structure be defined as shown in Fig.3.79a. The corresponding matrix state is defined as shown in Fig.3.79b. However, for the same three element spatial structure it is possible to define three different denotations. In addition to the one shown above, the other two with corresponding matrix states are shown in Fig.3.80. We can see how different denotations of the same spatial structure can define different matrix states. For the unlimited three-element spatial structure, however, it is possible to define only one numerical denotation (Fig.3.81). Therefore, there is only one corresponding matrix state defined for this spatial structure. As was shown in our analysis of spatial structures with four elements, we can define six nonisomorphic spatial structures (Fig.3.82). With a limited two dimension spatial
structure (a) it is possible to define the 12 different matrix states shown in Fig.3.83. With another spatial structure (b) it is possible to define also the 12 different states shown in Fig.3.84. However, for the spatial structure (c) there are only six matrix states possible, as shown in Fig.3.85. Finally, with the last three structures d, e, and f we can define four, three and one matrix state, respectively (Fig.3.86). By definition, all elements of the matrix state with positions (1,1), (2,2), ..., (n,n) are black, since one
element cannot be its own neighbor. The number of white elements corresponds to the number of neighborhood relations within the spatial structure. For the same neighborhood relations there are two white elements in the matrix state, since we count each of these relations twice (n*m and m*n). Thus the matrix state represents a
specific representation of the neighborhood relations within the corresponding spatial structure. We can see how those spatial structures with more limits and small dimension have a greater number of corresponding matrix states, than those which are unlimited and have higher dimensions. Therefore, structures a and b have 12 different matrix states, but structure f has only one. If all elements of some spatial structure are mutual neighbors, the corresponding matrix state will contain black elements only along the left diagonal.
4.1 Let a two-element generating set be \( P = (p,q) \), and let the binary corresponding set (BC-set) be defined as shown in Fig.4.1. With a given BC-set, as it was shown before, it is possible to define four different states of space (Fig.4.2).

These are all different states we can generate with a given BC-set. In the state of space generating procedure we didn’t consider the order of appearance of the states.
However, it is possible to present these states in various arrangements as shown in Fig.4.3. The appearance of equal or different states of space presented in sequences we will designate the visual process. Let the three different states be defined as shown in Fig.4.4. Since these states are given as the set, the order of their appearance is not defined. Therefore, they do not represent the visual process. However, with these states we can generate a very large number of different processes. Some of them are shown in Fig.4.5. Each state of space which is included in the process is an element of the visual process. The first element of a process is the beginning and the last element is the end. In order to define a process we have to have at least two states (same or different) presented in sequences (Fig.4.5). A process can contain all states of a given set \((a, b, e)\) or only a portion of them \((c, d)\). Any element in a process has two neighboring states except the beginning and the end. Each of them
has only one neighbor. The element which appears before a given element is its preceding neighbor and the element which appears after a given element is its posterior neighbor. Therefore, the beginning has only one posterior neighbor and the end has only one preceding neighbor.

Fig. 4.5
4.2 Let the four-element set of states be defined as shown in Fig.4.6. Some of the processes we can define with this set are presented in Fig.4.7. It is obvious that all these processes have different beginnings and ends as well as a different number of elements. However, they are all generated by the very same set of states (or its subset). Two different processes are homogeneous if they are defined by the very same set of states or its subset. Therefore the processes shown above are all homogeneous. Examples of some other homogeneous processes generated by a four-element set of states (Fig.4.8a) are presented in Fig.4.8b.
Fig. 4.8
4.3 Let the two different three-element sets of states be defined as shown in Fig.4.9. We will denote elements of the first set with: a, b, c, and of the second set d, e, f. With these two sets we can generate the two different processes presented in Fig.4.10. Obviously, these two processes have an equal number of elements. If we compare the order of appearance of the states we can come to the conclusion that these processes are organized in the same manner: c, b, a, b, a and f, e, d, e, d. We can see that, generally, any process has its own structure. This structure is defined by the order of appearance of same or different states. The order of appearance of the same or different states in the process we will designate the rhythm. Two processes are isomorphic if they have the same rhythm. Therefore, the two processes shown in Fig.4.10 are isomorphic. Examples of some other isomorphic processes are presented in Fig.4.11.
4.4 Let a four-element set of states be defined as shown in Fig.4.12a. With this set we can generate two homogeneous processes as shown in Fig.4.12b. If we compare the structure of these two processes and the order of appearance of the states, we can conclude that they follow the same rhythm. Therefore, these processes are isomorphic. Two processes, both isomorphic and homogeneous, are homomorphic processes. Let the two homomorphic processes be defined as shown in Fig.4.13. In these two processes...
equal states appear in the same place in the process. The place of the state in the process we will call the moment of appearance. Each state has its own moment (or moments) of appearance in any given process. Two homomorphic processes are equal if equal states have the same moments of appearance (Fig. 4.13).

4.5 Any given process can be divided into segments of two or more elements (states). Such a segment of a process we will call the interval. Let the six-element process be defined as shown in Fig. 4.14. This process can be divided into two three-element intervals. Now, comparing these two intervals we can come to the conclusion that they are equal (a = b). Any process which can be divided in integers of equal intervals is the circular process. Such an interval of the circular process is the
basic interval (loop). Therefore, if we know a basic interval and the number of its repetition in a circular process, we can easily generate the entire process. The number of elements of the basic interval is the basic number. For the process shown in Fig.4.14 the basic number is three ($b=3$). The simplest type of circular process is the alternative process. This process has a basic interval of two elements ($b=2$). Some examples of alternative processes are presented in Fig.4.15.

4.5.1 Let two homomorphic processes be defined as shown in Fig.4.16. Comparing these two processes we can see that the beginning of the process A is the end of the process B; and the end of the process A is the beginning of the process B. For each element of A, its preceding neighbor is the posterior neighbor for the same element in process B. Process B is the reverse process of A if the beginning of B is the end of
A and the end of B is the beginning of A; and if for any other element of B, its preceding neighbor is the posterior neighbor of the same element in A. In this case A

![Diagram](Fig.4.17)

is the original process and B is its reverse (B=rA). This relation is symmetric: A=rB. Examples of some original processes and their reversals are shown in Fig.4.17.

4.5.2 Let a six-element process be defined as shown in Fig.4.18. This process can

![Diagram](Fig.4.18)

be divided into two three-element intervals (a,b). We can see that these intervals relate to one another as original to reverse. A process with an even number of elements (n>2) is symmetric if its two intervals (n/2) relate to one another as original to reverse. In the process with an odd number of elements, the middle element does not belong to any of the two intervals (Fig.4.19).
4.5.3 A process defined by only one state is the static process. This kind of a process represents no change at all (Fig. 4.20).

![Fig. 4.20](image)

4.6 With the two-element spatial and qualitative structures presented in Fig. 4.21a we can define only one state of space (Fig. 4.21b). This state represents only one possible state available for the process generation. However, with this single state we can generate processes which have a different number of elements (Fig. 4.22). These are all static processes but with different duration. The duration of a process is defined by the number of its elements. Therefore, the durations of the processes shown above are four, six and three, respectively. A duration is a single characteristic which determines the difference between these processes.

![Fig. 4.21](image)
4.6.1 With another corresponding set, shown in Fig.4.23a, it is possible to define two different states of space (Fig.4.23b). There are numerous processes which can be generated by these two states. Some of them are shown in Fig.4.24. We can see that

A is an alternative process; C is a circular process (b=3); D is a static process; E is a symmetric process; F and G are mutually-reversed processes. However, there are an unlimited number of different processes which can be generated by a set of two states of space. These processes can be different not only in duration but also in rhythm.
4.6.2 With a three-element set of states, defined by the structures shown in Fig.4.25a, we can generate various and more complex processes (Fig.4.25b). This is only a small fraction of the numerous processes which can be generated with three
different states of space. With four, five or more different states it is possible to generate a virtually unlimited number of different processes.

4.7 Let the three different states be defined as shown in Fig.4.26a. With these states we can generate the two processes presented in Fig.4.26b. Because of the different number of elements, these two processes have different durations. However, they have almost an equal rhythm except for the fact that in the second process each state appears twice in succession. In all this analysis it was implicitly assumed that the duration of each moment is equal in the same process and that the duration of moments in different processes is also the same. If the duration of a moment in the first process above is two times longer than the moment in the second process, then these two processes will be equal. We will now define one special process which can serve as a reference for all other processes. This process will be, by convention, uniform and endless, defined only by two different states generated in an alternative rhythm (Fig.4.27). The alternative, uniform and endless process, with a white state as
the beginning, we will designate the absolute process. The uniformity of the absolute process means that the duration of any moment in this process is equal to all others. The moment of the absolute process is the absolute moment. By convention, there is no duration defined between two states in absolute process. The duration of absolute

process as the union of durations of absolute moments represents the absolute time. In further explanations the absolute process will be presented as shown in Fig.4.28a. The processes shown in Fig.4.28b can also represent the absolute process, since the space between elements in a process is of no consequence. Therefore, the duration of

processes shown in this figure are the same. Furthermore, we will attach a row of integer numbers to the absolute process, starting with one at the beginning and adding a value of one for each following element. Now the absolute process will be presented as shown in Fig.4.29. By definition, the duration of the absolute moment is
a minimal. As a result, there is no possible process which contains at least one moment smaller than the absolute moment. We will assume that in the previous analysis, all processes were defined in respect to absolute process. Now, with the absolute process as a reference, we can be certain that the second process in Fig.4.26 has indeed a duration two times longer than the first one. Therefore, these two processes could not have the same rhythm. The procedure of establishing the biunivoce correspondence between elements of a given process \( P \) and the absolute process is the standardization of process \( P \). In addition to the order of appearance of the elements, defined by process \( P \), with this standardization we can get an exact and absolute duration of any element (state) as well as an absolute duration of the entire process. If we have a process defined as shown in Fig.4.30a, for example, and if we know the
duration of each moment related to the absolute moment (Am), we can easily define the standardized form of this process (Fig.4.30b). Instead of relative duration Dr=5 we can see now that the absolute duration of the given process is Da=11. In further explications all processes will be considered as standardized.

4.8 Let the three–element set of states be defined as shown in Fig.4.31. With these three states it is possible to generate various processes. It is known in advance exactly which states will appear in these processes. However, it is not known in what order they will appear, before the entire process is generated. How can we explain the order of appearance of the states in a given process? One explanation can be that the order of appearance of given states is a series of random events. For example, we can randomly pick up state (b) as the beginning of the process. Now we will have two solutions for the second element of the process: (a) or (c). If we pick up state (a), then (c) is necessarily the end of the process (Fig.4.32a). However, if we pick up
state (c), then state (a) will be the end (Fig.4.32b). If we have picked up, for example, another state as the beginning, the order of the elements in the process will be entirely different. Therefore, in this kind of random generation we can not predict the exact order of the state's appearance in the process. However, we know exactly what states will appear and how long the duration of the process will be. For a three-element set of states the duration will always be three. It is assumed that the duration of appearance of each state is one absolute moment.

4.8.1 There is another type of random process that it is possible to generate with the given three-element set of states presented in Fig.4.31. Again, there are three possibilities for the beginning: (a), (b), or (c). Let state (c) be the first state (beginning) that we randomly choose. Now we will return this state to the others and again we will have three possibilities for the second element of the process: (a), (b), or (c). Let (c) be the state we randomly choose the second time. We will again return this state to the others and for the third element we will have three solutions: (a), (b), or (c). It is clear that we can continue this procedure ad infinitum. With the described procedure it is possible to generate a process with a duration greater than three but first we have to select the criteria for the process termination. It is, generally, possible to define various kinds of criteria for the process termination. Let it be, for this example, the third appearance of any given state. Two such possible

![Fig.4.33](image-url)
processes are shown in Fig.4.33. As in the previous case, the duration of each chosen state is one absolute moment. In this kind of random process, its duration depends not only on the number of states, but also on the criteria for termination as well as on the rhythm of the process.

4.9 Let a two-element set of states be defined as shown in Fig.4.34. We already know how to generate various random processes with these two states. However, in any random generation of a process it is not possible to determine the order of appearance of the states before the entire process is generated. Therefore, in order to avoid this unpredictable characteristic of a random process we should try to define an entirely different approach. If we can define exactly a specific operation which transforms one state of the given set into another, then we will be close to the solution. First of all, we will define an operation which affects one state of the given set by transforming it into another state of the same set as a unar operation. The unar operation defined on a two-element set of states is defined by four unar operators: A1, A2, A3, and A4. These four operators are defined by the following table:

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
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<tbody>
<tr>
<td>a</td>
<td>a</td>
<td>a</td>
<td>b</td>
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<td>b</td>
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The unar operator ,A1, transforms state a again into state a, and transforms state b into state a. The unar operator ,A2, transforms state a into state b and vice versa. The operator ,A3, is neutral; it does not transform any of these two elements. Finally, the operator ,A4, transforms state a into b, and state b again into b. With operator A1 and two given states we can generate the two different processes presented in
Fig. 4.35. We will end the process after the sixth element appears. With operator $A_2$ and the same states, we can generate the other two processes presented in Fig. 4.36. With operator $A_3$ we can generate the two other processes presented in Fig. 4.37. And
finally, with operator A4 we can generate the last two processes presented in Fig.4.38. We can see that of all eight processes generated by unar operator Ai (i=1,2,3,4), 1 and 5 as well as 6 and 8 are identical processes.

Therefore, we now have six different processes:

1(5) – is a static process defined only by state a;
6(8) – is a static process defined only by state b;
3 – is an alternative process, beginning with state a;
4 – is an alternative process, beginning with state b;
2 – is an almost static process defined by state a, if the beginning b is excluded;
7 – is an almost static process defined by state b, if the beginning a is excluded.

These six processes generated by unar operator A are elementary processes for a two-element set of states. Therefore, if the generating operator is known, as well as the beginning of the process, it is possible exactly to anticipate the order of appearance of the states for the entire process. It is also necessary to define the criteria for the process termination.
4.9.1 Let the three-element set of states be defined as shown in Fig.4.39. Within a
three-element set we can define unar operator $B$ by the following table:

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| a | a | a | b | a | a | c | a | b | b | b | b | b | b | c | c | c | c | c | c | c | c | c | c | c | c | c | c | c | c |
| b | a | a | b | a | a | c | a | b | b | b | b | b | b | a | b | b | b | b | b | b | b | b | b | b | b | b | b | b | b |
| c | a | b | a | a | a | b | a | a | b | b | a | a | a | c | a | b | c | a | b | c | b | b | c | c | c | c | c | c | c |

With all these 27 unar operators it is possible to define 81 processes. However, only
33 of them are different. All these processes are presented below, with the appearance
of the sixth element as the termination criteria (Fig.4.40). All different processes
generated by the unar operator $B_i$ ($i=1,2,...,27$) are elementary processes for a
three-element set of states. Within a four-element set of states it is possible to define
the unar operator $G$ with 256 operations. With this operator we can generate 1024
processes. However, according to previous examples, a great number of these processes
will be the same. It is obvious that a main difficulty in process generation by unar
operators is a very large number of operations $k$ and therefore a much greater
number of possible processes $k$, especially for the sets defined by the number of states
$k>4$. 
4.9.2 Let a two-element set of states be defined as previously presented in Fig.4.34. We already know all elementary processes generated by the unary operator $A$. During the generation of elementary processes it was assumed that each process is generated by a single operator $A_i$ ($i=1,2,3,4$). Only one operator was effective all along the process. In order to generate various processes, other than elementary, we can attempt this with a successive combination of different $A_i$ operators. For example, the process presented in Fig.4.41 is not an elementary process. However, its generation can be
explained as a successive application of two operators: A2 and A1. Starting with state b as the beginning, the next two elements of the process are generated by operator A2. Operator A1 is effective instead of A2, from the third element, until the end of the process. The unar operator affects each element of the process except the last element (end). Examples of nonelementary processes generated by a successive combination of different operators A, and given a two-element set of states, are presented in Fig. 4.42. A similar procedure can be applied to the three-element set and
operator B. Examples of processes generated by successive combinations of operators $B_i$ and a given set (Fig. 4.39) are presented in Fig. 4.43.

![Diagram of processes generated by successive combinations of operators.]

4.9.3 In the previous approach to the generation of nonelementary processes by successive applications of two or more unar operators, it was assumed that in the moment when the new operator begins to function, the previous one no longer has an effect. It is possible now to assume that two operators can participate simultaneously in the process generation. Such an example is a process generated by a three-element set of states (Fig. 4.39) and unar operators $B_i$ ($i=1,2,...,27$). During this process two different operators will affect the same moment simultaneously as it affects the same
element of the process. In the moment when two different operators start to function simultaneously the process will split into two different branches. Therefore, this process will have one beginning but two ends, and one element with two posterior neighbors. An element with one preceding neighbor and two or more posterior neighbors is the fork. The process with one beginning and two or more ends is the divergent process. In addition to these kinds of processes it is possible to define a process with two or more beginnings and one end. This process, the reverse of the divergent process, will be named the convergent process. The convergent process shown in Fig.4.45 is the reverse of the process from Fig.4.44.
4.10 A specific type of visual processes can be generated with matrix states defined before. In these processes a corresponding spatial structure can serve as the process generator. By altering an enumeration of the spatial structure we can generate different matrix states (Fig. 4.46). Some examples of processes generated with matrix

Fig. 4.46
states are presented in Fig. 4.47. It is characteristic of these processes that one visual structure (spatial) generates another (process), but they are limited only to matrix states.
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