Goal: Deep understanding of fundamental issues in GT, MD with applications in engineering problems with applications in engineering problems while presenting open research problems

Study of multi-person decision problems

Course structure

1. Static Games of complete information (35%)
   - Both Matrix games and cont. games
   - Solution concepts, dominance (Nash Eq.), Rationalizability, Correl Eq.
   - Existence and Uniqueness (super-modular potential)
   - Computation of Eq.
   - Learning (Myopic, Bayesian)

2. Dynamic Games of complete information (20%)
   a. Sub-game perfect eq.
   b. Simple bargaining models
   c. Nash's bargaining solution
   d. Repeated games
      i. Folk Theorems
      ii. Multiplayer DP

3. Static Games with incomplete information (15%)
   a. Bayesian Nash Eq.
   b. Simple auctions
   c. Optimal Auctions
   d. Optimal Mechanisms

4. Mechanism Design (15%)
   a. Efficient Mechanisms
   b. Dominant strategy implementations
   c. Nash Implementations

5. Network Games (15%)
   a. Utility based resource allocation
   b. Selfish network routing
   c. Network anarchy
   d. Eq. concepts (Wardrop eq.)
   e. Pricing/Price of anarchy/stability

Main text: Game Theory by Fudenberg & Tirole MIT Press 1991
Pigou's example (1920)

J Users ->
L1 (gamma 1)
-> li .) specifies delay on each link dependant on level of congestion
2/J units of flow
L2 (gamma-2)

a. Maximize total delay encountered by flows of users

Min l1 (gamma 1) gamma 1 + l2 (gamma 2) gamma 2
Xi >= 0
s.t. gamma 1= sigma [i=1 to j] Xi

Solvable using Lagrange multipliers

L1 (gamma 1) + L1' (gamma1) (gamma1) = Lambda 1
L2 (gamma 2) + (L2)' (gamma 2) (gamma 2 = Lambda 2

Lambda 1 = Lambda 2 = 2

=> Gamma 1 = 1
Gamma 2 = 1

Assumptions
○ Half-up, half-down
○ Centralized problem
○ All users are obedient
○ Everyone is symmetric

b. Selfish Users
   a. Choosing path of minimum delay

   epsilon
   2 units of traffic ----->
   ------>
   (2-epsilon)

   b. => all traffic takes top link
   c. System optimum or social optimum

C1 (gamma) = 1.1. + 2.1 = 3
C2 (gamma) = 2.2 = 4
C1 (gamma)/C2 (gamma) = 3/4 ("magic #")

There are other resource allocation mechanisms where the bound is again
3/4. hence magic #

There is some form of game theory interaction
c. Large Users

\[ L_1 (\text{Gamma}) = \frac{3}{2} \text{gamma} \]

2 Users (1 Unit of flow each)
\[ L_2 (\text{gamma}) = 2 \]

Represent possible actions & payoffs (delays by matrix)

\[
\begin{array}{c|c|c}
P_2 & U & D \\
\hline
U & 3,3 & 3/2,2 \\
D & 2,3/2 & 2,2
\end{array}
\]

No incentive to deviate unilaterally

d. Induce them to choose actions that will yield centralized solution

e. \Rightarrow "price"

\[
P_1 = (L_1)' (\text{Gamma} 1) \text{ Gamma} 1 = \text{Gamma} 1 \\
P_2 = (L_2)' (\text{Gamma} 2) \text{ Gamma} 2 = 0
\]

Choose now the link with smallest "effective cost" \( L_1 (\text{gamma}) = P' \)

If in the eq \( \text{gamma}1 > 0 \) and \( \text{gamma} 2 > 0 \) then

\[
\begin{align*}
L_1 (\text{Gamma} 1) + P_1 &= L_2/\text{Gamma} 2 + p_2 \\
L_1 (\text{Gamma} 1) &= (L_1)' (\text{Gamma} 1) \text{ Gamma} 1 = L_2 (\text{Gamma} 2) + (L_2)' (\text{Gamma} 2) \text{ Gamma} 2
\end{align*}
\]

Marginal congestion cost

2. Rate Control Problem

People have different service req.s

J/2 Type-1 users
\[
L_9(\text{gamma}) = \text{gamma Type 1 U(x)} = 3x \ x \text{ belongs } \{0, 1/j, 3/j\} \]
J/2 Type 2 users

a. Choose rates to maximize \((\text{total utility} - \text{total delay})\)

b. \(\text{DVs} = X_j \quad J = 1 \ldots J/2\)

c. \(X_j' = J = 1 \ldots J/2\)

Max \(\Sigma (J=1 \ldots J/2) U_1(X_j) + \Sigma (j=1\ldots J/2) U(X'_j) - L(\gamma) \Gamma\)

s.t. \(\gamma = \Sigma (J=1 \ldots J/2) X_j + \Sigma (j=1 \ldots J/2) X_j'\)

Will people tell the truth:

1 \(\rightarrow\) will

2 \(\rightarrow\) \(S = I \text{ payoff } 5/J - 2/J * 3/2 = 2/J\)

\(J = \Pi\)

\(3/J - 1/J * 3/2 = 3/2/J\)

Exercise: Show that if you charge saying type 1 \(1/J\) units