The Impact of Out-of-Theater Supply Flow Visibility on In-Theater Logistics

By

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Submitted to the Department of Civil and Environmental Engineering in Partial Fulfillment of the Requirements for the Degree of

Master of Science in Transportation at the Massachusetts Institute of Technology
June 2012

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ABSTRACT

The United States Army’s end-to-end logistical network during times of conflict is made up of two separate networks. One network, managed by the Department of Defense, controls the shipment of supplies from the manufacturing facility to the theater of conflict. The other network managed by the Army, receives these supplies and distributes them to the units within the theater of operation. The synchronization between these two networks impacts the ability of Army logistical planners to efficiently manage the in-theater supply chain. Past operations demonstrate that Army planners have minimal visibility into the supplies entering the theater of operation. This causes units to become dangerously low on supplies, compromising their ability to successfully complete missions.

This thesis evaluates the impact this lack of visibility has on the performance of the in-theater supply chain. A logistical planner is developed, modeled after current operating procedures, to maximize a unit’s satisfaction, by keeping their supply levels within an acceptable range. Using mixed linear programming the planner determines the amount of each supply and the numbers of vehicles required to transport the supplies. Results confirm that increasing the visibility of incoming supplies improves the performance of the planner. The amount of improvement is dependent on the selection of specific parameters. Additionally, the variation in the amount of supplies entering a theater has an effect on the planner’s performance. We conclude that there is an operational benefit in having future knowledge of incoming supplies.

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Acknowledgments

This thesis would not have been possible without the support and assistance of two groups of people, and I owe them my deepest gratitude:

First, from a professional aspect, I will be forever indebted to Richard Hildebrant of Draper Laboratory, my technical advisor, for his time and infinite patience. His experience and willingness to advise me made this thesis possible, his whiteboard sessions were always enlightening, and his red pens and yellow highlighters succeeded in making me a better writer.

Thank you to Professor Steve Graves at Massachusetts Institute of Technology for his valuable insights as he advised me on this thesis.

Thank you to Stephan Kolitz at Draper Laboratory for his expertise and guidance along the way.

I am eternally grateful to the United States Military Academy Department of Mathematical Sciences for the opportunity to study these last two years at MIT.

I would like to thank my military peers Brian, Jane, Dave, and Matt. The four of you helped to make this experience much more enjoyable. Special thanks to Andy. It was tremendous to work side by side with a friend like you over the past two years. There are many others not mentioned here who contributed in some way to my research and publishing of this thesis. Thank you!

Second, from a personal aspect, there are many people who provided on-going support to my family allowing me the time to work on my thesis, and providing me comfort as I knew that my family had the love and help they needed during my absences.

Thank you to our new friends in Braintree who helped to transport our kids around to their various events and were so generous with their time by caring for them whenever we needed them. Thank you to “old” friends Katy and Megan who were virtually on-call and helped in so many ways.

Thank you to my in-laws for all of your help and support. I will be forever thankful to my mother in-law for her grammatical edits of my writings which helped me immensely.

A special thanks to my parents for their constant love and support of my endeavors, as well as all the time they spent visiting during hectic times. Thank you for always setting the example.

Last but certainly not least, a huge thank you to my wife Tracy, and my four wonderful sons Andrew, Cayden, Luke, and Nathan. They supported me 100%, understood my late nights and were always there to boost my spirits when I did get home. Tracy you are amazing, you kept our home running and provided a loving environment for our children almost single handedly. I am grateful for all your support, and I love you with all my heart.

Michael A. Giordano
Major, U.S. Army

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1 Introduction

1.1 Research Motivation

The United States Army depends upon its logistics system as an integral part of supporting the fighting force during times of conflict. This is a formidable challenge considering the unique circumstances faced by Army units in current operating environments. An added complication is that the Army must rely on other branches of the Armed Services to transport supplies into the theater of operations. This creates a situation where the Army’s end-to-end logistical network is comprised of two distinct networks: an out-of-theater network and an in-theater network. Historically, Army logisticians have had limited information concerning the arrival of supplies from the out-of-theater network into a theater of operations. This causes a lack of coordination, contributing to inefficient inventory management, which ultimately affects mission success.

In July 2004, the Acting Under Secretary of Defense for Acquisition, Technology, and Logistics issued a policy requiring the implementation of Radio Frequency Identification (RFID)
system within the Department of Defense. It was believed that this new technology would provide the end-to-end visibility of supplies that was previously not possible [1], and it would provide logistical planners in theater the ability to predict the arrival of incoming supplies and to efficiently manage the in-theater supply chain. However, after seven years, Army logisticians find that their visibility of the flow of incoming supplies is still limited. At best they can estimate the delivery date, but it is based on average travel times through the out-of-theater network and has a large variance, so has limited usefulness for planning purposes. They are forced to contend with an inefficient inventory system in hostile operating environments.

1.2 Problem Description

During times of conflict the U.S. Army operates a hub and spoke logistical network to sustain the units in the theater of operation. Logistical Support Areas (LSAs) operate as hubs and are responsible for supplying the spokes, Forward Operating Bases (FOBs), within their area of responsibility. LSA’s have planners on staff that analyze multiple aspects, such as available vehicles and current supply levels, in order to keep supply levels at each FOB within an acceptable range. The nature of conflict generates uncertainty in both the supply and demand. Due to these uncertainties, planning becomes more complex and FOBs can become dangerously low on supplies, compromising a unit’s ability to successfully complete missions.

The uncertainty in demand is due to a number of factors, such as the type of military operations being conducted and enemy forces. During defensive and sustainment operations the rate of enemy attacks can create fluctuations in demand. The uncertainty in supply is generated primarily from the lack of visibility of the incoming supply flow. While it is possible to know where supplies are, there is variation in the time it will take to arrive in the theater. This is because the route that takes supplies from the U.S. to the theater is not direct. At each stop
supplies are processed, which may entail un-loading, custom inspections, and a waiting period for the next available transport, all of which create variations in travel time. These variations make it near impossible to predict the flow of incoming supplies to a theater of operation.

Currently, Army logistical planners do not specifically account for all of these uncertainties. For example, they negate the impacts related to the lack of visibility of incoming supply flow by generating plans based only on the amount of supplies currently on-hand. This results in supplies lying outside the acceptable range and prevents LSAs from satisfying their customers. This thesis will maximize customer satisfaction given limited visibility of supplies flowing into theater. To accomplish this, a planner is created which determines the amount of each supply and the numbers of vehicles required to transport the supplies.

1.3 Experimentation

Three experiments are conducted in a notional operating theater with representative Army units in order to analyze the performance of the planner. The first focuses on the value of information and compares the performance of a series of trials in which visibility into the out-of-theater network increases. The second considers how varying levels of uncertainty in the flow of incoming supplies affects in-theater performance. The final experiment is conducted to assess the speed the planner generates plans compared to the size of the theater of operation.

1.4 Thesis Organization

Chapter 2 provides an overview of military logistics, the operating environment, and the Army’s logistical planning process. It also lists complicating factors which generate uncertainty in the logistical network. Chapter 3 discusses the modeling approach and formulation that addresses the operational problem. This chapter introduces variables, parameters, constraints,
and the objective function of the logistical planner. Chapter 4 provides results and analysis of
the experiments, and chapter 5 provides concluding thoughts on the research and results. It also
references future research that could enhance and improve the model.
2 Operational Overview

This chapter provides an overview of military logistics in the context of the Army. First it describes the logistical structure of the military and the Army. Next, some basic Army logistical concepts which provide background into the modeling approach are discussed. Furthermore, it explains some of the complicating factors in the military logistics network. This chapter ends by describing the Army’s logistical planning process.

2.1 Strategic Overview

2.1.1 Levels of War

Figure 2-1 [4] depicts the three levels of war: strategic, operational and tactical. The levels are used to define specific responsibilities and actions performed by Army organizations. National policy is created and national objectives are defined at the strategic level when war commences. The operational level links strategic objectives with tactical actions. Operational objectives are accomplished through a series of tactical actions that work over time to
accomplish the strategic end state. At the tactical level, friendly forces attempt to defeat enemy forces [4].

![Levels of War](image)

**Figure 2-1 Levels of War**

The levels shown in Figure 2-1 are also present within logistical operations. Organizations at the strategic level focus on filling the distribution pipeline with materiel resources. The pipeline refers to the network of air, sea and land routes into an area of operations. Operational level organizations coordinate support from the strategic level to meet requirements at the tactical level. The tactical level is where demand is generated and commodities are consumed. Maintaining adequate levels of commodities at the tactical level is imperative for mission success [13].
2.1.2 Department of Defense

The United States Department of Defense (DOD) is responsible for the U.S. Armed Forces (Army, Navy, Air Force, and Marines). One way DOD exercises authority and control over the Armed Forces is through the establishment of unified combatant commands, which are joint commands comprised of two or more military departments operating at the strategic level. Currently there are nine unified combatant commands. Five are geographical in nature (i.e. Central Command (CENTCOM) which is responsible for the central area of the globe) and four are functional in nature. One of the functional unified combatant commands is the United States Transportation Command (TRANSCOM) [11].

TRANSCOM operates at the strategic level of war and is responsible for providing end-to-end distribution for all of DOD. It is compromised of organizations from the Army, Navy and Air Force. It is, in effect, the FEDEX for DOD and it employs assets from each of the military branches to coordinate the movement of supplies and personnel via air, sea and ground.

With respect to the Army’s logistic network, TRANSCOM manages logistics at the strategic level by delivering supplies to the theater of conflict. Army logistical units at the operational level are responsible for delivering supplies within the theater. This distinction in the military’s end-to-end logistics network produces two separate logistic systems. For the purposes of this research the logistic system managed by TRANSCOM is referred to as the out-of-theater distribution network and the system managed by the Army is referred to as the in-theater network.

2.1.3 Army

Within a theater of conflict the Army’s logistical organization structure is shown in Figure 2-2 [8]. The Theater Sustainment Command (TSC) at the top is a support headquarters
that functions at the operational level of war and serves as the senior Army sustainment unit for the theater. It ensures that personnel, equipment, and commodities move to their destination with a minimum number of intervening stops and transfers.

Subordinate to the TSC are Sustainment Brigades (SUST). SUSTs are assigned to an area under the TSC’s control and provide logistical support to units in their area. These units provide the link between the operational and tactical levels of war [6]. SUSTs are comprised of Combat Sustainment Support Battalions (CSSBs). A CSSB controls the Army transportation assets that move supplies within the theater.

The only sustainment unit not controlled by the TSC within the theater is the Brigade Support Battalion (BSB). This unit is part of a Brigade Combat Team (BCT), which operates within the tactical level of war. The linkage between the BSB and the theater occurs at the
CSSB. The CSSBs receive the sustainment requirements from the BSB and pass those up the chain to planners who generate the plans to fulfill the demands.

2.2 Army Logistics

2.2.1 Classes of Supply

The Army organizes supplies into 10 major classes identified by Roman numerals. The chart in Table 2-1 details special handling requirements for each class and the type of truck required to move the supplies in that class. There are numerous commodities within each class of supply.

<table>
<thead>
<tr>
<th>Category</th>
<th>Classes of Supply</th>
<th>Special Handling Requirements</th>
<th>Types of Trucks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petroleum</td>
<td>III</td>
<td></td>
<td>Tanker</td>
</tr>
<tr>
<td>Ammunition</td>
<td>V</td>
<td>Cannot be transported with mixed cargo.</td>
<td>Tractor-trailer, PLS, tactical truck</td>
</tr>
<tr>
<td>Water</td>
<td>I (water)</td>
<td></td>
<td>Water tanker, PLS</td>
</tr>
<tr>
<td>General Cargo</td>
<td>I, II, IV, VI, VIII, IX, X</td>
<td>Can be shipped either break-bulk or in containers</td>
<td>Tractor-trailer, PLS, tactical truck</td>
</tr>
<tr>
<td>Combat Vehicles</td>
<td>VII</td>
<td></td>
<td>HETT</td>
</tr>
</tbody>
</table>

*Table 2-1 Army Classes of Supply*

2.2.2 Supply Targets

The Army uses a color scale, depicted in Figure 2-3, to communicate a unit’s on-hand supply level to senior leadership. This visual method allows leaders to quickly access supply levels for subordinate units. The scale ranges from ideal levels, green, to dangerously low levels, black. Units falling within the green and amber ranges are considered to have enough supplies on hand to conduct normal operations as set forth by mission requirements, including the ability
to perform ad hoc missions when necessary. However, units in the red and black range may be unable to conduct normal operations and their ability to perform ad hoc missions may be severely diminished due to their levels of supply.

![Example Stock Level Color Scale](image)

**Figure 2-3 Example Stock Level Color Scale**

### 2.3 Complicating Factors

This section will outline some of the areas that generate uncertainties in the Army’s logistics system. These factors can occur at any level within the system.

#### 2.3.1 Contemporary Operating Environment

The Vietnam War marked a change in the operating environment that the U.S. fights wars and conflicts. Prior to that war, a theater of conflict was characterized by comparably sized forces fighting across distinct battlefronts. The units at the rear provided medical, logistical and administrative services to the soldiers at the battlefront. Because enemy units were always beyond the battlefront, Army logistical units were able to deliver supplies with little risk of attack.
Today’s wars and conflicts are characterized by irregular warfare. Smaller forces employing unconventional methods to counter the advantages of larger forces characterize this type of warfare. These methods include terrorism and guerilla warfare [4], and there are no distinct battlefronts. Enemy forces tend to blend into the population, thus eliminating the element of safety for the units at the rear, as logistic units must deliver supplies through enemy territory. The most recent wars in Iraq and Afghanistan have shown that logistical units are easy targets for enemy forces and they are under constant threat of attack.

This environment creates uncertainty in two areas of the Army’s logistic network. First, commodities in a convoy are at risk for partial and/or entire loss. This creates uncertainty in the amount of a particular commodity that will reach the tactical unit. Second, these attacks may slow the progress of convoys. This leads to a disconcerting lack of confidence about the length of time it will take a convoy to reach its destination.

2.3.2 Infrastructure

A country’s infrastructure and physical environment are vital components in the Army’s logistical network. The physical environment consists of the types of terrain and climate. Infrastructure is the combination of road networks, airports, railways, ports and other facilities.

Current operations in countries such as Afghanistan provide insight to the potential hazards that inferior infrastructure can have on the Army’s logistic system. Poor roads, rough terrain, and extreme weather can cause damage or destruction of equipment transported through the country. Also, the limited number of airfields that can handle large aircraft creates delays in delivery times. Large aircraft must land at one of the few larger airfields, and cargo is then unloaded and reloaded onto a smaller aircraft. This extra handling increases delivery time.
2.3.3 Host Nation Trucking Services

There are many reasons why the Army may decide to contract services to host nation companies. First, during stability operations, the Army wants to work with host-nation agencies and other civilian organizations to enhance the host nation government’s legitimacy [20]. Another reason stems from the limited force protection assets available to the Army. The current operating environment places logistic convoys at risk of attack and require force protection assets to secure them. By contracting out logistic movements to the host nation the Army can better secure the selected convoys [3]. The last reason presented is due to host nation agreements that require the Army to use their services to transport U.S. supplies and equipment through their county.

The use of host nation trucking does not come without drawbacks. One is the variation in delivery time. Contracted trucks may have little incentive to provide reliable service because of the high demand for their business. Also, vehicles in poor condition and/or drivers’ selection of varying routes to the same destination can result in slower deliveries. For example, in Afghanistan, a planner can assume a convoy of military vehicles will travel on the prescribed route, which takes 7-hours to reach its destination. Yet, the same convoy operated by a contracted company may require planners to allocate as much as 4 days to reach the destination due to these drawbacks [20].

Another drawback to host-nation trucking involves in-transit visibility of vehicles. Real time global positioning systems are preferred, but contractors are not always willing to install them. In countries like Pakistan and Afghanistan, drivers fear that these systems can be used by hostile forces to find their location. Since the supplies they carry support U.S. troops, enemy forces view them as targets for attack [20]. An alternate method is to read the radio frequency
identification (RFID) tags placed on the cargo at specific points along the route. This method raises two issues. First, some countries do not allow the U.S. to install RFID tag readers along routes. They must rely on reporting procedures by the contracted companies to pass along location information. Second, host nation drivers cannot be forced to take a prescribed route, which means that they may not pass the RFID tag readers.

Finally, there are drawbacks in dealing with host nation policies, including specific regulations about clearances, customs and the number of vehicles that can cross their borders. Certain border crossings into Afghanistan, for example, are regulated, which leads to an inevitable backup of vehicles. All of these variables lead to instability in delivery times for supplies [20].

2.4 Out-of-Theater Network Overview

In the civilian world, a customer hires a company such as FEDEX to pick up and deliver a product, and the company is able to provide a good estimate on delivery time because they have control of the end-to-end movement of the cargo. However, as discussed in section 2.1, when a tactical unit requests supplies there is no single entity that controls the end-to-end transportation of supplies. This makes it difficult to estimate with any certainty when those supplies arrive. This section discusses the aspects that generate uncertainty in the out-of-theater network.

The out-of-theater logistical system is a multi-commodity, multi-modal network operated on a daily basis. It is a complex network that uses military and commercial modes of transportation to deliver commodities through multiple transshipment points into a theater of operation. Transshipment points are ports where the cargo is unloaded and queued to wait for subsequent movement. Cargo at these locations is subject to a dwell time. The dwell time for
any piece of cargo varies because of priority, local customs regulations, and other internal and external factors.

In addition to dwell time, another source of uncertainty comes from the travel time between points in the network. The nodes of the network are connected by air, sea, and ground routes, eventually terminating at logistical hubs in-theater. Each of these links has an associated travel time which can vary due to the maximum speed of the chosen mode, as well as exogenous factors such as weather. Figure 2-4 depicts a simple example of an out-of-theater network transporting cargo from the U.S (red node on the left), into a theater (green node on the right).

![Figure 2-4 Out-of-Theater Network Example](image)

Each theater of war has its own set of constraints which makes devising an out-of-theater logistical plan unique to that conflict. There are theaters of conflict without sea points, such as Afghanistan. In these cases more uncertainty is generated from the agreements that must be made with other countries for use of their seaports and road or rail systems to deliver...
commodities into theater. Each country also has its own set of transportation procedures and guideline which can add even more variation in delivery time.

2.5 In-Theater System Overview

Army doctrine is not always indicative of what occurs during military operations. Commanders must consider mission, enemy, terrain, troops available, time, and civilian considerations (METT-TC) when making decisions, which can create deviations from doctrine. In regard to these differences, the planner is based on the doctrine described by the distribution management process in *The Process of Military Distribution Management* by Lieutenant Colonel James H. Henderson, United States Army (Retired) [14]. Henderson has 24 years of experience as a Quartermaster Officer, including recent tours in support of Operation Iraqi Freedom.

The Distribution Management Process (DMP) as described by Henderson is cyclical in nature, whose time frame can range from 72-96 hours. This variation in time depends on the operational situation and organizational staff. Henderson describes a 72-hour process, where planners are matching available transportation assets with demand requests to support an area of operations.

![Figure 2-5: Distribution Management Process](image-url)
The timeframe shown in Figure 2-5, is divided into four phases: (1) Planning and Allocation, (2) Coordination and De-confliction, (3) Validation, and (4) Tracking [14]. Over time the DMP occurs continuously, such that on any given day each of the 4 phases is being performed. As shown in Figure 2-6, on Monday the process generates Thursday’s plan (Phase 1), de-conflicts Wednesday’s plan (Phase 2), validates Tuesday’s plan (Phase 3), and executes and begins tracking Monday’s plan. On Tuesday it generates Friday’s plan (Phase 1), de-conflicts Thursday’s plan (Phase 2), validates Wednesday’s plan (Phase 3), and executes and begins tracking Tuesday’s plan and so on.
Figure 2-6 also shows the evolution of a plan through the 4 Phases. For example Thursday's plan is in Phase 1 on Monday, Phase 2 on Tuesday, Phase 3 on Wednesday and Phase 4 Thursday. As new information becomes available planners are able to modify plans in Phases 1 through 3.

![Figure 2-6: Cyclical Nature of DMP](image)

A so called distribution matrix, in spreadsheet form, is used to manage the logistic plans, also known as convoy plans. Convoys are a set of vehicles that travel together from an origin to a destination. This matrix, a portion of which is shown in Figure 2-7, includes convoy plans from all phases of the DMP and is updated throughout the process. What follows is a brief description of each phase of the DMP.
2.5.1 Phase I: Planning and Allocation

The planning and allocation phase occurs 48-72 hours prior to the convoy leaving a logistic distribution center, known as a Logistical Support Area (LSA). This phase identifies the requirements of each customer and allocates transportation resources to distribute the supplies within the necessary time. The output of this phase is an initial set of convoy plans that are added to the distribution matrix. Currently, this is a manual process involving multiple spreadsheets and systems in which planners look at routes, vehicle and crew availability, and supplies on hand.

2.5.2 Phase II: Coordination and De-confliction

A theater of war is very busy. Staff at the operational level need to be attentive to where units are and what they are doing, in order to coordinate movement across the battlefield, and most importantly, to prevent fratricide. Due to the current operating environment, logistical movements traverse the same areas where offensive and defensive military operations occur. So, it is extremely important that operational staffs have good situation awareness.
The distribution matrix is analyzed through a series of daily meetings in order to coordinate movements with other forces, including the Air Force, Marines, other Army units, and foreign forces. The goal of this phase is to identify any issues that may prevent the authorization of a convoy. If a conflict is found, planners must assess the situation and create a new plan. The output of this phase is an updated, coordinated distribution matrix containing approved convoys.

2.5.3 Phase III: Validation
The approved distribution matrix now becomes the convoy plan that is used to deliver supplies to the customers. Even though it is 12 hours away from execution, the distribution matrix is continuously monitored and updates can occur. For instance, changes in demand levels could add vehicles to a convoy, or high priority orders that have been received could be added. This phase also locks in transportation assets, loads these assets with the appropriate supplies in the proper configuration, and assures that in-transit visibility devices have been tested. At the end of this phase, vehicles are loaded and lined up ready to depart.

2.5.4 Phase IV: Tracking
Once the convoy departs the LSA the tracking phase begins, using automated tools and Army standard operating procedures. The customer can also track this information. Events that occur along the route are handled using the Rules of Engagement that are in place, and they are reported back to those tracking the convoy. This provides planners the information they need to help them determine if changes are required to compensate for lost supplies or vehicles. Completion of this phase occurs when the convoy has delivered its cargo and returned to the LSA.
3 Modeling Approach and Formulation

This chapter describes the mathematical formulation of the planner. It begins with an explanation of the approach used to model the Army’s logistical distribution system. Next it discusses the input parameters, variables, and constraints of the formulation. It ends by presenting the complete formulation to the problem.

3.1 Modeling Approach

Chapter 2 described the Army’s end-to-end logistical network as two separate logistical networks. Each network is a multi-commodity, multi-modal network flow problem. From the Army’s standpoint, the out-of-theater network supply flow is just one of many inputs that planners use to develop in-theater convoy plans. In order to assess the impact this input has on in-theater inventory levels it is necessary to develop an in-theater logistical planner, as shown in Figure 3-1.
**Figure 3-1 In-Theater Modeling Approach**

The in-theater planner, termed the Army Theater Level Logistical Allocation Planner (ATLLAP), uses a mixed-integer linear program (MILP) to maintain supply levels at each base within an acceptable range at all times. The commander determines the range based on diverse factors, such as capacity restrictions, a unit’s operational posture, and the level of enemy activity. Each day the ATLLAP is executed it uses the inputs from Figure 3-1 to create convoy plans in order to maintain inventory levels within the acceptable range on the days it delivers supplies to their destination.
The planning process is modeled on the Distribution Management Process (DMP) as described in section 2.5, which outlines a four phase approach to the Army’s logistical planning process.

<table>
<thead>
<tr>
<th>Monday’s Plan</th>
<th>Tuesday’s Plan</th>
<th>Wednesday’s Plan</th>
<th>Thursday’s Plan</th>
<th>Friday’s Plan</th>
<th>Saturday’s Plan</th>
<th>Sunday’s Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 4</td>
<td>Phase 3</td>
<td>Phase 2</td>
<td>Phase 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Execute</td>
<td>Generate</td>
<td>Generate</td>
<td>Generate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plan For</td>
<td>Plan for</td>
<td>Plan for</td>
<td>Plan For</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monday</td>
<td>Tuesday</td>
<td>Wednesday</td>
<td>Thursday</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 3-2 Modeled DMP Cycle**

The ATLLAP does not model all of the procedures involved in the de-confliction and validation of plans in Phases 2 and 3, but it does allow the ability to modify the plans in these phases, by generating new plans. As shown in Figure 3-2, on Monday the ATLLAP produces plans for Tuesday, Wednesday and Thursday and executes Monday’s plan which was generated on Sunday. On Tuesday it produces new plans for Wednesday, Thursday, and Friday and executes the Tuesday plan produced on Monday.

The ATLLAP manages Phase 4, tracking the flow of supplies, by assuming that vehicles always leave on the morning of the day of departure. When a convoy reaches its destination, supplies are unloaded, stored and inventoried. As a result, supplies are not available for
consumption at the destination until the day after they arrive. The same applies to vehicles returning upon completion of a convoy mission. Maintenance and refueling for vehicles, as well as rest and downtime for drivers require scheduling of successive convoys a day after the current mission is completed.

3.2 Supply, Transport, and Storage Network

An example logistical network is shown in Figure 3-3 for illustrative purposes. The thick line bisecting the map divides the network into two sections. Each section represents an area of responsibility for the Logistics Support Area (LSA). The green numbered circles represent the bases that the LSAs support, which are the Forward Operating Bases (FOBs). The Army employs a hub-and-spoke network design in which supplies are transported from an LSA to a single FOB. The arcs connecting the LSAs and FOBs are segmented so that each segment represents one day of travel.

![Figure 3-3 Example Network](image)
The network is defined as follows:

**Bases:** $B, b \in B$

The set of all bases, $B$, is comprised of all the LSAs and FOBs in the network.

**FOBs:** $F, f \in F$

The set of all FOBs, $F$, is a subset of $B$. Each FOB, $f$, consumes supplies, but is unable to distribute supplies to other bases. Individual combat brigades and battalions reside at each FOB. The storage capacity at each FOB varies because of numerous criteria including mission requirements and space available.

**LSAs:** $L, l \in L$

The set of all LSAs, $L$, is a subset of $B$. The sustainment brigades operate out of the LSAs, which act as distribution hubs. All supplies enter the theater of operation through the LSAs, which then distribute these supplies via convoys to bases within their area of responsibility.

**Routes:** $R, r \in R$

Each route connects an LSA, $l$, to another base, $b$, in the network. For convention a route, $r$, is comprised of an ordered pair $(l, b)$, such that $r = (l, b) \in R$ where $l \neq b$. Routes connecting LSAs to bases are assumed to be directed routes for the purposes of delivering supplies. Convoys travel back to their LSA upon completion of delivery and do not transport supplies.

**Travel Time of Routes:** $TD_r$

Each route, $r$, has an associated travel time, $TD_r$, measured in days. Although it is possible for the travel time to vary, due to weather conditions, vehicle reliability and other variables, travel time is considered constant in the formulation.

**Maximum Number of Travel Days:** $W = \max(TD_r) \quad \forall r$

This value represents the number of days corresponding to the longest travel time within the network.
3.3 General Parameters

There are six general parameters that define the supply classes, vehicle modes, and time ranges used in the formulation.

Supply Classes: \(SupClass, c \in SupClass\)

The Army defines ten classes of supply, which are consolidated into five types of supply classes for the purposes of this thesis: Fuel, Water, Food, Major End Items (vehicles and repair parts), and General Cargo. Due to the complex policies and procedures that govern the transportation of ammunition, it is not modeled in this thesis. The ATLLAP is more appropriate for low hostility operations such as stability or civil support operations.

Fuel and Water are measured in gallons and each requires a specific fleet of vehicles to transport them. The remaining three supply classes can be transported on the same type of vehicles and can be intermixed as needed. These supplies are measured in pounds.

The set, \(SupClass\), is categorized into two sets on how the supply class is measured.

Supply Classes Measured in Gallons: \(SupGal, g \in SupGal = \{\text{Fuel, Water}\}\).

Supply Classes Measured in Pounds: \(SupLbs, p \in SupLbs = \{\text{Food, Major End Items, General Cargo}\}\)

Vehicles: \(Veh, v \in Veh\)

The types of vehicles used by the ATLLAP represent a sample of those vehicles found in a standard Army sustainment brigade and incorporate various capacities for each supply class. During recent operations the Army has contracted out the delivery of supplies to local transportation companies. Rather than represent all of these modes separately, it’s assumed their carrying capacity corresponds roughly to that of the Army’s vehicles, so the number of vehicles assigned to each LSA is supplemented to take into account the number of contracted vehicles.
Supply Class Vehicle Combinations: $M, m \in M$

$M$ is the set of possible supply class / vehicle combinations. A supply class / vehicle combination, $m$, is a pair $(c, v)$, such that $m = (c, v) \in M$.

This combination restricts the supply classes that can be transported on each vehicle. For example, a fuel tanker cannot carry food. It also allows for multiple supply classes, such as spare parts and food items, to be transported on the same vehicle.

Last planning day: $T$

The planner utilizes two ranges of time. The first range spans the planning horizon. The second includes enough additional time beyond the planning horizon to allow for completion of any convoy missions that might be in progress at the end of the planning horizon.

Planning Horizon Range: $S2T \in \{2 \ldots T\}$

Extended Time Range: $STW \in \{1 \ldots (T+W)\}$

### 3.4 Base Attributes

The following six parameters characterize the bases in the network.

**Daily Consumption Rate: $DC_{ct}$**

The number of pounds or gallons of a supply class, $c$, consumed daily at each base, $b$, on day $t$. The consumption rates used in the planner come from the Army’s Operational Logistics (OPLOG) Planner. This tool is maintained by Army Combined Arms Support Command (CASCOM) and estimates standard mission requirements for all classes of supply. To simulate the daily variation in a base’s consumption rate, the standard consumption rate is multiplied by a random number generated from a normal distribution, where the mean and variance are chosen from input by Army logistic analysts at the U.S. Army Training and Doctrine Command at Fort Lee, VA (TRAC-LEE).
Storage Capacity of Fuel and Water: $CapFW^g_f$

The storage capacity for fuel or water is the number of gallons of supply class, $g$, which can be stored at FOB, $f$.

Capacity of Standard Pallet: $CapPal^c$

The capacity of a standard pallet is the number of pounds of supply class, $c$, which it can hold, according to the Army’s OPLOG Planner.

Minimum Flow of a Supply Class: $MinFlow^c$

The minimum flow is the number of pounds or gallons of supply class, $c$, which can be shipped from an LSA.

Storage Capacity of General Warehouse: $CapWare_f$

The warehouse storage capacity is the number of standard sized pallets that can fit into the storage facility at each FOB, $f$. Each pallet can only hold one supply class. However, the warehouse can contain any combination of food, major end items or general cargo pallets.

Maximum Desired Stock Level: $SO^c_f$

Commanders at each FOB determine the maximum number of pounds or gallons of each supply class they wish to have on hand. This stock objective is below the storage capacity, which provides logistical planners some leeway in managing uncertainties such as travel times and consumption rates. Without this parameter FOBs could become inundated with supplies beyond their capacity.

Minimum Desired Stock Level $SM^c_b$

The minimum number of pounds or gallons of stock desired at base, $b$, is the lower bound on an inventories acceptable range. This value is chosen by a base commander to be a level below which mission success may be compromised.

The acceptable ranges for LSAs and FOBs are modeled differently as shown in Figure 3-4. LSAs receive supplies from the out-of-theater network which they have minimal control over. This can create a situation where an LSA receives supplies beyond its capacity. (This overstock
is moved to strategic reserve storage facilities in real world operations.) The ATLLAP models this by assuming an LSA has infinite capacity. Since the ATLLAP manages the supplies it ships out from an LSA, it can then manage to keep LSAs above a minimum stock level. The FOBs acceptable range is defined by the stock objective and stock minimum which the ATLLAP manages by the amount of supplies it transports to the FOB from an LSA.

![Figure 3-4 Inventory Range](image)

### 3.5 Vehicle Attributes

The following attributes relate to vehicles.

**Vehicle Capacity: \( VW^v \)**

Each vehicle, \( v \), has an associated capacity. For modes that carry fuel and water this capacity is measured in gallons, while remaining modes are measured in pounds.

**Vehicle Pallet Carrying Capacity: \( VPal^v \)**

Each vehicle, \( v \), can carry a given number of pallets.

**Minimum and Maximum vehicles in a convoy: \( VMin, VMax \)**

Commanders place restrictions on the size of a convoy to provide command and control over the convoy. Theater specific conditions such as terrain, enemy forces, and force protection,
alter a unit’s ability to manage a convoy. For example, highway driving normally provides good visibility and communications, while driving in a city with tall buildings and multiple turns may require shorter convoys. Although $V_{max}$ and $V_{min}$ may vary to meet mission-specific requirements or changing operational conditions, they are held constant in the formulation. These values do not include force protection vehicles that accompany convoys.

### 3.6 In-Theater Input

The out-of-theater network is a multi-commodity, multi-modal logistics network. The intent of this research is to analyze the impact of the out-of-theater supply flow visibility on in-theater performance. From the standpoint of in-theater planners this lack of visibility means they do not know what supplies will arrive tomorrow and beyond.

Based on conversations with Army logistic professionals, representative arrival rates, $ArriveRate^c$, for the five aggregate supply classes treated in this thesis were determined, as shown in Table 3-1 [12]. Due to a lack of available data showing when supply classes arrive into theater, a probability distribution defining the arrival rates cannot be derived. Thus, the rates in Table 3-1 are used to determine when supplies will arrive.

<table>
<thead>
<tr>
<th>Supply Class</th>
<th>Arrival Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel</td>
<td>Daily</td>
</tr>
<tr>
<td>Water</td>
<td>Daily</td>
</tr>
<tr>
<td>Food</td>
<td>Every 3 Days</td>
</tr>
<tr>
<td>Major End Items</td>
<td>Every 2 Days</td>
</tr>
<tr>
<td>General Cargo</td>
<td>Every 2 Days</td>
</tr>
</tbody>
</table>

*Table 3-1 Out-of-Theater Input Frequency of Supplies*

Supply managers at organizations such as the Defense Logistics Agency (DLA), determine shipment quantities from the daily consumption projections of military organizations. The information flow begins at tactical level organizations where projected future consumption
rates are produced. The LSAs use this information to determine projected consumption rates for their area of responsibility. This process of combining subordinate unit’s consumption rates rolls up the Army chain of command until a theater level consumption rate is sent to the appropriate commodity manager, who uses these rates to coordinate supply flow into theater.

From the standard consumption rates defined in section 3.4, the theater level consumption rate for each supply class, $TLC_t^c$ is the sum of all the base’s consumption rates:

$$TLC_t^c = \sum_b DC_{bt}^c$$  \hspace{1cm} (3.1)

The expected number of pounds or gallons of supplies, $ExpArrive^c$, which arrive in each shipment is:

$$ExpArrive^c = TLC_t^c * ArriveRate^c$$ \hspace{1cm} (3.2)

Supply Class Input Flow: $InFlow_{bt}^c$

This is the number of pounds or gallons of a supply class, $ExpArrive^c$, arriving from the out-of-theater network available for use at base, $b$, on day $t$. The only bases that receive supplies from out-of-theater are LSAs under current operating procedures.

### 3.7 Previous Plan

Logistical planners use information concerning convoys currently on the road to determine the number of pounds or gallons to transport to each base. These previous plans are obtained from the distribution matrix. The distribution matrix contains information about convoys that are either being planned or are in some state of execution, Table 3-2. The data in the table is an example based on a portion of the network in Figure 3-3. The first row shows a vehicle that is away from the LSA conducting a convoy mission, as indicated by the value of the
Day Left column being less than 1. This vehicle has delivered its supplies, since the Delivery
Day value is less than 1, and returns to the LSA on day 2, as specified by the Return Day.

The second and third rows show two vehicles that are approved to begin delivery on day
1 (Day Left column). The last row of the table shows a vehicle that is scheduled to leave on day
2 (Day Left column). Since planners are still able to make changes to convoy plans leaving after
day 1, they are not included in the formulation.

<table>
<thead>
<tr>
<th>Origin</th>
<th>Destination</th>
<th>Day Left</th>
<th>Delivery Day</th>
<th>Return Day</th>
<th>Supply Class</th>
<th>Vehicle</th>
<th>Qty</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSA_A</td>
<td>FOB 2</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>FUEL</td>
<td>5K-TANKER</td>
<td>5000</td>
</tr>
<tr>
<td>LSA_A</td>
<td>FOB 2</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>FUEL</td>
<td>HEMMT-2500</td>
<td>2500</td>
</tr>
<tr>
<td>LSA_A</td>
<td>FOB 2</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>FUEL</td>
<td>5K-TANKER</td>
<td>5000</td>
</tr>
<tr>
<td>LSA_A</td>
<td>FOB 2</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>FUEL</td>
<td>5K-TANKER</td>
<td>5000</td>
</tr>
</tbody>
</table>

Table 3-2 Distribution Matrix

Previous Plan Supply Flow: $PC_{bt}^c$

The supply flow of the previous plan is the number of pounds or gallons of supply class, $c$, available for consumption at base, $b$, at the beginning of day $t$. The second and third rows of Table 3-2 shows this convoy is delivering a total of 7500 gallons of Fuel to FOB 2 on day 2.

Previous Plan Vehicle Flow: $PV_{lt}^v$

The vehicle flow of the previous plan is the number of vehicles, $v$, added to the operational vehicle inventory at LSA, $l$, on the beginning of day $t$. The first row of Table 3-2 shows one 5K-TANKER is added to the operational inventory of LSA_A on day 2.

An important difference between supplies and vehicles is where their flow ends. Supplies are only tracked to the end of the route, since that is where they are consumed. However, vehicles are tracked back to the LSA. Both are simple calculations since the travel days are constant and are computed from the departure day when the previous plan is generated.
3.8 Initial Conditions

The ATLLAP needs initial levels of supply classes and vehicles within the network. The formulation begins planning on day 2, so it requires the inventory for supplies and vehicles on day 1:

Number of gallons or pounds of Supply Class, $c$, at base, $b$, at the end of Day 1: $SI_b^c$

Two inputs are introduced in the computation of $SI_b^c$, Equation (3.3).

$$SI_b^c = Day0_b^c + PC_{b1}^c + \text{InFlow}_{b1}^c - SLeave_{b1}^c - DC_{b1}^c$$

(3.3)

First, $Day0_b^c$, is the number of pounds or gallons of each supply class at the end of the previous day (day 0) at each base. Second, $SLeave_{b1}^c$, is the number of pounds or gallons of supplies leaving a base on day 1, which comes from the distribution matrix. So, $SI_b^c$ is the supply level at the end of day 0, plus the supplies arriving on day 1 from the previous plan, plus the supplies arriving on day 1 from the out-of-theater network, minus the supplies leaving on day 1, minus the consumption rate for day 1.

Number of Vehicles, $v$, on Hand at LSA, $l$, at the end of Day 1: $VI_l^v$

Three inputs are introduced in the computation of $VI_l^v$, Equation (3.4).

$$VI_l^v = VTotal_l^v + PV_{l1}^v - VRoad_{l1}^v - VLeave_{l1}^v$$

(3.4)

First, $VTotal_l^v$, is the total number of each vehicle type assigned to an LSA. Second, $VRoad_{l1}^v$, is the number of vehicles from an LSA, currently on the road conducting missions. Third, $VLeave_{l1}^v$, is the number of vehicles scheduled to depart an LSA on day 1. These last two inputs come from the distribution matrix. So, $VI_l^v$ is the sum of the number of vehicles assigned to the LSA and the number of vehicles returning to the LSA.
on day 1, minus the number of vehicles from the LSA on the road and the number of vehicles leaving the LSA on day 1.

### 3.9 Variables

#### 3.9.1 Decision Variables

There are three decision variables, all are non-negative.

1. $X^m_{rt}$ The number of pounds or gallons of supply class, $c$, leaving on day $t$, traveling on route $r$, in vehicle $v$, where $m$ is the pair $(c,v)$.

2. $BinX^m_{rt} = \begin{cases} 
0 & \text{if } X^m_{rt} = 0 \\
1 & \text{if } X^m_{rt} > 0
\end{cases}$

$BinX^m_{rt}$ is used to restrict the flow of supplies, $X^m_{rt}$, above a minimum flow or to no flow.

3. $Convoy_{rt} = \begin{cases} 
0 & \text{if no convoy leaves an LSA along route } r \\
1 & \text{if a convoy leaves an LSA along route } r
\end{cases}$

$Convoy_{rt}$, allows the planner to choose to not send a convoy along a certain route on a given day.

#### 3.9.2 Derived Variables

There are nine derived variables which take on values for a given set of inputs and decision variable values. All are non-negative.

1. $Y^m_{rt}$ The number of pounds or gallons of supply class, $c$, available for consumption at the destination associated with route $r$, on day $t$, in vehicle $v$, where $m$ is the pair $(c,v)$.

$$Y^m_{rt} = \begin{cases} 
X^m_{r(t-TDr)} & \text{if } 1 \leq t - TD_r \leq T \quad \forall \ m, r, t \in STW \\
0 & \text{Otherwise}
\end{cases}$$

Equation (3.5) correlates the day that supplies leave an LSA with the day they arrive at their intended destination. Therefore, every $X^m_{rt}$ has a corresponding $Y^m_{rt}$ with the only difference being the time they represent. To prevent the ATLLAP from generating values for $Y^m_{rt}$ that don’t
correspond to an $X_{rt}^m$, Equation (3.5) forces $Y_{rt}^m$ to a zero quantity. The information in Table 3-3 shows an example of the situations that are forced to zero. It displays the day that supplies become available by summing the day they left the LSA and the number of travel days. For example, the area highlighted in red refers to supplies traveling to a base 2 days away. If the ATLLAP ships out supplies on day 2, day 4 is the day they become available. Thus, the variable is forced to a zero quantity for any day prior to day 4, because it is impossible for the ATLLAP to produce a plan that has supplies arriving any earlier.

<table>
<thead>
<tr>
<th>Day Left LSA</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

*Table 3-3 Supply Class Availability Computation*

The following three integer variables manage vehicle flow and the availability of vehicles to be scheduled for convoy missions.

2. $VC_{lt}^v$ The number of vehicles of type, $v$, available for convoy missions at LSA, $l$, at the end of day $t$.

3. $VX_{rt}^v$ The number of vehicles of type, $v$, traveling on route, $r$, leaving on day $t$.

4. $VR_{rt}^v$ The number of vehicles of type, $v$, traveling along route, $r$, that have returned and are added to vehicle inventory levels on day $t$.

$$VR_{rt}^v = \begin{cases} 
VX_{r(t-2*TD_r)}^v & \text{if } 1 \leq t - 2 \cdot TD_r \leq T \\
0 & \text{Otherwise}
\end{cases} \quad \forall v, r, t \in \mathcal{S2T}$$ (3.6)
Equation (3.6) manages vehicles returning to their LSA once they have completed their delivery. As in Equation (3.5), this variable is forced to zero in situations where it is impossible for a vehicle to return to an LSA. The logic from Table 3-3 is applied. However, now it must be determined when a vehicle returns and is available for another convoy mission. As previously discussed, this is the day after it returns to the LSA. For example, if a vehicle leaves on day 3 to a base that is 2 travel days away, it arrives on day 4. (It travels on days 3 and 4, arriving sometime on day 4). It would leave its destination base on day 5 and because travel times are constant, it returns to its LSA on day 6, and can be utilized in another convoy mission on day 7.

5. \( SL_{bt} \) The number of pounds or gallons of a supply class, \( c \), on hand at base, \( b \), at the end of the day \( t \).

The following two variables measure the number of pounds or gallons that the inventory level lays outside of the acceptable range.

6. \( OS_{ft} \) The number of pounds or gallons of a supply class, \( c \), over stocked at FOB \( f \), at the end of the day \( t \).

\[
OS_{ft}^c \geq SL_{ft}^c - SO_f^c \\
\forall c, f, t \in STW
\]  
(3.7)

\( OS_{ft}^c \) is calculated only for FOBs because LSAs do not have an associated stock objective. When a supply class is over stocked, this non-negative variable becomes greater than zero since \( SL_{ft}^c \) becomes greater than \( SO_f^c \). If \( SL_{ft}^c \) is less than \( SO_f^c \), then \( OS_{ft}^c \) is constrained to be zero.

7. \( US_{bt} \) The number of pounds or gallons of a supply class, \( c \), under stocked at base \( b \), at the end of the day \( t \).

\[
US_{bt}^c \geq SM_b^c - SL_{bt}^c \\
\forall c, b, t \in STW
\]  
(3.8)
When a supply class is under stocked this non-negative variable becomes greater than zero since \( SL_{bt}^c \leq SM_{bt}^c \), in which case \( US_{bt}^c \) is constrained to be zero.

The following two variables are integers and manage the number of pallets.

8. \( PalX_{rt}^m \) The number of pallets carrying supply class, \( p \), leaving on day \( t \), traveling on route \( r \), in vehicle \( v \), where \( m = (p,v) \).

9. \( PalSL_{ft}^c \) The number of pallets carrying supply class, \( p \), at FOB \( f \), on day \( t \).

3.10 Constraints

3.10.1 Flow Constraints

The equations in Figure 3-5 show the derived variables \( SL_{bt}^c \) and \( VC_{lt}^v \) as defined in the previous section. These keep track of the supply and vehicle inventory level at the end of each day. They also manage the conservation of supply and vehicle flow at all bases. Since the process of defining supply and vehicle flow is similar, they are presented together.

\[
SL_{bt}^c = \begin{cases} 
SL_{bt}^c & \text{if } t = 1 \\
SL_{bt}^{c(t-1)} + PC_{bt}^c + \sum_{I: (l,b) \in R} \sum_{V: (c,v) \in M} y_{(l,b)}^{(c,v)}_{t} + \text{InFlow}_{bt}^c - \sum_{b' \in B} \sum_{(b,b') \in R} \sum_{V: (c,v) \in M} X_{(b,b')}^{(c,v)}_{(t,b)} - DC_{bt}^c & \forall c, b, t \in S2T \quad(3.9)
\end{cases}
\]

3.10.2 Vehicle Flow Constraints

\[
VC_{lt}^v = \begin{cases} 
VL_{lt}^v & \text{if } t = 1 \\
VC_{lt}^{v(t-1)} + PV_{lt}^v + \sum_{b: (l,b) \in R} VR_{(l,b)}^v_t - \sum_{b: (l,b) \in R} VX_{(l,b)}^v_t & \forall v, l, t \in S2T \quad(3.10)
\end{cases}
\]

*Figure 3-5 Conservation of Flow Constraints*
$SL_{bt}$ is defined for three separate time periods, represented by the three equations in (3.9). The first represents the supply level at the end of day 1, which is simply the input parameter $S_l^b$. The second time period corresponds to the planning range $S2T$. The equation takes the inventory level from the end of the previous day, adds to it the three possible daily inputs and subtracts the two daily outputs. The first input is the supplies from the previous plan, $PC_{bt}$, the second input is the supply flow into the base from an LSA, $\Sigma_{l:b,b'} \in R \Sigma_{v:(c,v) \in M} Y_{(l,b)_{t'}}^{(c,v)}$. The third input is the of supply flow from out-of-theater, $InFlow_{bt}$. Next, the two possible outputs from a base are subtracted. The first is the flow of supplies out of a base, $\Sigma_{b':b,b} \in B \Sigma_{v:(c,v) \in M} X_{(b,b')_{t'}}^{(c,v)}$. The routes are such that no route leaves an FOB, thus the planner cannot flow any supplies out from an FOB. The second is the daily consumption of supplies at a base, $DC_{bt}$. The last time period, $t > T$, begins when the last convoy leaves an LSA and continues until that convoy reaches its destination. The computation for this time period is almost the same as the second. Since the planner is no longer dispatching convoys, it can no longer output supplies from LSAs. Thus, the flow of supplies out of a base, $\Sigma_{b':b,b} \in B \Sigma_{v:(c,v) \in M} X_{(b,b')_{t'}}^{(c,v)}$, is omitted.

The second set of equations (3.10) is similar to the first, except it manages vehicles instead of supplies. Here, there are only two time periods; the third time period does not apply, because once the planner stops dispatching convoys there are no new missions for returning vehicles. The first time period is the number of vehicles available to be assigned to a convoy on day 1, $V_I^{b}$. The second time period adds the number of returning vehicles to the number of vehicles available on the previous day and subtracts the vehicles leaving on that day. There are two situations in which a vehicle can return to an LSA, those from the previous plan, $PV_{It}$, and
those from current plans, $\sum_{b: (l,b) \in R} VR(l,b)_t$. The vehicle output from an LSA is the number of vehicles that the planner sends out of the LSA to all other bases, $\sum_{b: (l,b) \in R} VX(l,b)_t$.

### 3.10.2 Pallet Capacity Constraints

The constraints listed in Figure 3-6 impose restrictions on pallets.

\[
X^{(c,v)}_{rt} \geq \text{MinFlow}^c \ast \text{Bin}X^{(c,v)}_{rt} \quad \forall \ r, t \in S2T, c, v: (c, v) \in M \tag{3.11}
\]
\[
PalX^{(p,v)}_{rt} \geq \frac{X^{(p,v)}_{rt}}{\text{CapPal}^p} \quad \forall \ r, t \in S2T, v, p: (p, v) \in M \tag{3.12}
\]

**Figure 3-6 Pallet Capacity Constraints**

Constraint (3.11) constrains the number of pounds or gallons transported to be above a minimum amount. This allows the ATLLAP to send partial pallets of supplies and prevents it from transporting extremely small unrealistic amounts of supplies.

Constraint (3.12) rounds the number of pallets needed to accommodate the flow, $X^{(p,v)}_{rt}$, of each supply to the next highest integer value, since $PalX^{(p,v)}_{rt}$ is an integer. Different supply classes may not share a pallet.

### 3.10.3 Storage Capacity Constraints

Storage constraints are shown in Figure 3-7.

\[
SL^g_{ft} \leq \text{CapFW}^g_f \quad \forall \ f, g, t \in STW \tag{3.13}
\]
\[
PalSL^p_{ft} \geq \frac{SL^p_{ft}}{\text{CapPal}^p} \quad \forall \ f, p, t \in STW \tag{3.14}
\]
\[
\sum_p PalSL^p_{ft} \leq \text{CapWare}_f \quad \forall \ f, t \in STW \tag{3.15}
\]

**Figure 3-7 Storage Capacity Constraints**
Constraint (3.13) restricts the amount of water and fuel at an FOB to its maximum capacity. The remaining supply classes are stored together in a warehouse whose capacity is measured by the number of pallets it can accommodate.

Constraint (3.14) rounds the number of pallets, PalSL_{ft}, (an integer), required to accommodate the amount of each supply class on hand at the end of the day to the next highest integer value.

Constraint (3.15) restricts the total number of pallets, \( \sum_p PalSL_{ft} \), to be no greater than the warehouse's capacity.

### 3.10.4 Convoy Constraints

The constraints below restrict the size and the number of vehicles in a convoy.

\[
\begin{align*}
VX^{p}_{rt} &= \frac{X^{(g,v)}_{rt}}{VW^v} \quad \forall r, t \in S2T, v, g: (g, v) \in M \\
VX^{p}_{rt} &\geq \frac{\sum_{p \in LBS| (p, v) \in M} X^{(p,v)}_{rt}}{VW^v} \quad \forall r, t \in S2T, v, p: (p, v) \in M \\
VX^{p}_{rt} &\geq \frac{\sum_{p \in LBS| (p, v) \in M} PalX^{(p,v)}_{rt}}{VPal^v} \quad \forall r, t \in S2T, v, p: (p, v) \in M \\
\sum_{m} VX^{m}_{rt} &\leq VMax \times Convoy_{rt} \quad \forall r, t \\
\sum_{m} VX^{m}_{rt} &\leq VMin \times Convoy_{rt} \quad \forall r, t
\end{align*}
\]

*Figure 3-8 Convoy Constraints*
The vehicle capacity constraints determine the number of vehicles, \( V_{Xt} \), required to carry their supplies along a route on each day. These constraints also link the flow of supplies with the movement of vehicles.

Constraint (3.16) manages the supply classes measured in gallons (fuel and water). It forces these classes to be transported in full truckload quantities. In actual operations these classes are almost always shipped in full vehicle quantities.

The next two constraints restrict the number of vehicles carrying supplies measured in pounds. These supply classes can be intermixed on vehicles to best utilize their carrying capability. Since multiple supplies can share vehicles the weight and number of pallets is summed to determine the number of vehicles required.

Constraint (3.17) uses the summed weight of the supplies, \( \sum_{p \in LBS} (p,v) \in M X_{rt}^{(p,v)} \), to ensure the number of vehicles have an aggregated weight capacity that can accommodate the total weight.

Constraint (3.18) uses the sum of the number of pallets, \( \sum_{p \in LBS} (p,v) \in M PalX_{rt}^{(p,v)} \), to ensure the number of vehicles have enough capacity to hold the total number of pallets.

Constraints (3.19) and (3.20) bound the number of vehicles in a convoy.

### 3.10.5 Non-Negativity Constraints

This section discusses the non-negative constraints placed on, \( SL_{lt} \), as shown below.

\[
\begin{align*}
SL_{ft}^c & \geq 0 & \forall \ c, l, f : (l, f) & \in R, t \in \{2 \ldots (T + TD(l,f))\} \\
SL_{lt}^c & \geq 0 & \forall \ c, l, t & \in S2T
\end{align*}
\]

Figure 3-9 Non-Negativity Constraints
Constraint (3.21) constrains the stock level, $SL_{ft}$, at an FOB to be greater than or equal to zero only on the days the planner has the ability to deliver supplies to that FOB. This constraint is necessary to prevent the planner from over-stocking FOBs in order to keep their inventory levels within the acceptable range beyond the day the last convoy arrives. To understand this concept an example based on Figure 3-3 is provided in Table 3-4. In this example the planner is run on Monday, which means it creates plans for Tuesday, Wednesday and Thursday. The table shows that Tuesday’s planned convoys arrive at FOB 1 on Wednesday and FOB 2 on Thursday, due to the difference in travel days. In addition, Thursday’s planned convoys arrive at FOB 1 on Friday and at FOB 2 on Saturday. Thus, when planning on Monday the ATLLAP does not plan any convoys that reach FOB 1 on Saturday. It is important to realize that when the ATLLAP is run on Tuesday it plans a convoy that arrives to FOB 1 on Saturday and brings their levels above zero if necessary. To account for this, the formulation allows the stock level, $SL_{ft}$, on days the ATLLAP cannot deliver supplies to go below zero.

<table>
<thead>
<tr>
<th>FOB 1 (1 Day Travel)</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tuesday Plan Arrives</td>
<td>Wednesday Plan Arrives</td>
<td>Thursday Plan Arrives</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FOB 2 (2 Days Travel)</td>
<td>Tuesday Plan Arrives</td>
<td>Wednesday Plan Arrives</td>
<td>Thursday Plan Arrives</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3-4 Impact Capability of ATLLAP Plans

For the same reasons, Constraint (3.22) forces the stock level, $SL_{ft}$, at an LSA to be greater than or equal to zero only on the days which the planner generates plans.
3.11 Cost Function

The in-theater planner has two goals. First, maintain the inventory level at each base to be within the acceptable range during the planning timeframe. Second, reduce the exposure of soldiers on the road.

The first goal is achieved by penalizing supply levels that lie outside the acceptable range. Commanders may view supply levels above the acceptable range differently than levels below that range. To account for this, the following two cost parameters are introduced and are set by commanders.

Cost Factor Associated with being Above or Below the Acceptable Range: CostOver, CostUnder

To compute the cost of stock levels lying outside the acceptable range the ATLLAP only considers the days it is able to impact an FOB’s stock level. The same logic used to address the non-negativity constraints in Figure 3-9 is employed here. The computation is comprised of three summations. First, the weighted over-stocked amount is summed for all FOBs over all days the ATLLAP can impact the FOB.

\[
FOB_{over} = \sum_{f: (l_f) \in R} \sum_c \sum_{t \in STW: t \in \{2, \ldots, T + TD(l_f)\}} (CostOver \times OS_{ft}^c) \quad (3.23)
\]

Second, the weighted under-stocked amount is summed for all FOBs over all days the ATLLAP can impact the FOB.

\[
FOB_{under} = \sum_{f: (l_f) \in R} \sum_c \sum_{t \in STW: t \in \{2, \ldots, T + TD(l_f)\}} (WU \times US_{ft}^c) \quad (3.24)
\]
Third, the weighted under-stocked amount is summed for all LSAs over all days the ATLLAP in the planning horizon.

\[ LSA_{\text{under}} = \sum_i \sum_j \sum t \in S_2^T \left( WU \ast US^c_{it} \right) \]  

(3.25)

The second goal mentioned above is achieved by penalizing the number of convoys sent each day and the number of vehicles in a convoy. Penalizing each route forces the ATLLAP to only send a convoy when the costs of a FOB’s supplies lying outside the acceptable range are lower with a convoy being sent as compared to a convoy not being sent. To generate a further reduction each vehicle is penalized causing the ATLLAP to send the fewest number of vehicles in each convoy. To accomplish this, the following two cost parameters are introduced and are set by commanders.

**Cost factor Associated with Each Route and Vehicle:** \( CostR_r, CostV_v \)

The computation to reduce the cost of soldier exposure on the road is comprised of two summations. First, the weighted number of convoys is summed for all routes over all of the days in the planning horizon.

\[ ConvoyCost = \sum \sum r \in S_2^T \left( CostR_r \ast Convoy_{rt} \right) \]  

(3.26)

Second, the weighted number of vehicles is summed for all vehicle types along all routes over all of the days in the planning horizon.

\[ VehicleCost = \sum \sum \sum v \times r \in S_2^T \left( VX_{rt}^v \ast CostV_v \right) \]  

(3.27)
The sum of these five summations, (3.23) – (3.27), produces the cost objective function, which is to be minimized.

\[ FOB_{\text{over}} + FOB_{\text{under}} + LSA_{\text{under}} + ConvoyCost + VehicleCost \] (3.28)

### 3.12 The Problem Formulation

Below is the entire mathematical formulation. A master list of variables and parameters is found in Appendix B. The formulation is a deterministic and mixed-integer program containing binary variables.

Minimize

\[ FOB_{\text{over}} + FOB_{\text{under}} + LSA_{\text{under}} + ConvoyCost + VehicleCost \] (3.28)

Subject to:

**Conservation of Commodity Flow:**

\[
SL_{bt}^c = \begin{cases} 
SL_{bt}^c & \text{if } t = 1 \\
SL_{bt}^c(t-1) + PC_{bt}^c + \sum_{(l,b) \in \mathcal{R}} \sum_{v \in \mathcal{V}} \sum_{(c,v) \in \mathcal{M}} y_{(c,v);(l,b)t}^c + \text{InFlow}_{bt}^c & \text{if } 1 < t \leq T \\
SL_{bt}^c(t-1) + PC_{bt}^c + \sum_{l:(l,b) \in \mathcal{R}} \sum_{v \in \mathcal{V}} \sum_{(c,v) \in \mathcal{M}} y_{(c,v);(l,b)t}^c + \text{InFlow}_{bt}^c - DC_{bt}^c & \text{if } t > T
\end{cases}
\] (3.9)

**Conservation of Vehicle Flow:**

\[
VC_{lt}^v = \begin{cases} 
VL_{lt}^v & \text{if } t = 1 \\
VC_{lt}^v(t-1) + PV_{lt}^v + \sum_{b:(l,b) \in \mathcal{R}} VR_{l(b)t}^v - \sum_{b:(l,b) \in \mathcal{R}} VX_{l(b)t}^v & \text{if } t > 1
\end{cases}
\] (3.10)
Derived Variable Definitions:

\[
Y^m_{rt} = \begin{cases} 
X^m_{r(t-TDr)} & \text{if } 1 \leq t - TDr \leq T \\
0 & \text{Otherwise} 
\end{cases} \quad \forall m, r, t \in STW \tag{3.5}
\]

\[
VR^v_{rt} = \begin{cases} 
VX^v_{r(t-2*TD_r)} & \text{if } 1 \leq t - 2*TD_r \leq T \\
0 & \text{Otherwise} 
\end{cases} \quad \forall v, r, t \in S2T \tag{3.6}
\]

\[
OS^c_{ft} \geq SL^c_{ft} - SO^c_f \quad \forall c, f, t \in STW \tag{3.7}
\]

\[
US^c_{bt} \geq SM^c_b - SL^c_{bt} \quad \forall c, b, t \in STW \tag{3.8}
\]

Pallet Capacity Constraints:

\[
X^{(c,v)}_{rt} \geq \text{MinFlow}^c * \text{Bin}X^{(c,v)}_{rt} \quad \forall r, t \in S2T, c, v: (c,v) \in M \tag{3.11}
\]

\[
PalX^{(p,v)}_{rt} \geq \frac{X^{(p,v)}_{rt}}{\text{CapPal}^p} \quad \forall r, t \in S2T, v, p: (p,v) \in M \tag{3.12}
\]

Storage Capacity Constraints:

\[
SL^g_{ft} \leq \text{CapFW}^g_f \quad \forall f, g, t \in STW \tag{3.13}
\]

\[
PalSL^P_{ft} \geq \frac{SL^P_{ft}}{\text{CapPal}^p} \quad \forall f, p, t \in STW \tag{3.14}
\]

\[
\sum_p PalSL^P_{ft} \leq \text{CapWare}_f \quad \forall f, t \in STW \tag{3.15}
\]

Convoy Size Constraints:

\[
VX^v_{rt} = \frac{X^{(g,v)}_{rt}}{VW^v} \quad \forall r, t \in S2T, v, g: (g,v) \in M \tag{3.16}
\]

\[
VX^v_{rt} \geq \frac{\sum_{p\in\text{LBS}} (p,v) \in M} {VW^v} X^{(p,v)}_{rt} \quad \forall r, t \in S2T, v, p: (p,v) \in M \tag{3.17}
\]

\[
VX^v_{rt} \geq \frac{\sum_{p\in\text{LBS}} (p,v) \in M} {VPal^v} PalX^{(p,v)}_{rt} \quad \forall r, t \in S2T, v, p: (p,v) \in M \tag{3.18}
\]

\[
\sum_V VX^v_{rt} \leq VMax * \text{Convoy}_{rt} \quad \forall r, t \in S2T, \tag{3.19}
\]
\[
\sum_{\nu} VX_{rt}^\nu \geq VMin \cdot Convoy_{rt} \quad \forall r, t \in S2T \tag{3.20}
\]

**Non-Negativity Constraints:**

\[
SL_{ft}^c \geq 0 \quad \forall c,l,f: (l,f) \in R, t \in \{2 \ldots (T + TD_{(l,f)})\} \tag{3.21}
\]

\[
SL_{lt}^c \geq 0 \quad \forall c,l,t \in S2T \tag{3.22}
\]

\[
X_{rt}^m, VR_{rt}^\nu, VX_{rt}^\nu, PalX_{rt}^m \geq 0 \quad \forall v, m, l, r, t \in S2T
\]

\[
Y_{rt}^m, OS_{rt}^m, US_{bt}^c, VC_{lt}^v, PalSL_{lt}^b \geq 0 \quad \forall v, c, m, l, r, b, p, t \in STW
\]

\[
BinX_{rt}^m \in \{0,1\} \quad \forall m, r, t \in S2T
\]

\[
Convoy_{rt} \in \{0,1\} \quad \forall r, t \in S2T
\]
4 Experimentation: Design, Results, and Analysis

This chapter describes the design and implementation of three experiments. The first two are aimed at demonstrating the impact that visibility of the in-theater input flow schedule has on the performance of in-theater logistical support. The third is designed to evaluate the run time capabilities and limitations of the ATLLAP. The chapter begins with an explanation of the scenario developed for experiments 1 and 2. In all of the experiments the ATLLAP formulation is run using IBM ILOG CPLEX Optimization Studio, Version 12.3.

4.1 Scenario

All relevant data on the Army’s previous conflicts in Iraq and Afghanistan are classified. However, we have obtained representative, unclassified data from the U.S. Army Training and Doctrine Command at Fort Lee, VA (TRAC-LEE) for testing purposes. This data is used to create a theater of operation as shown in Figure 4-1. It is located in Krasnovia, a fictional country used by the Army in war games since the 1980s. It is an area of operation composed of
mountainous terrain representative of Southwestern Asia, and under this scenario military forces are conducting stability operations.

![Figure 4-1 Krasnovia](image)

The network in Figure 4-1 consists of two LSAs and six FOBs. Due to the hub-and-spoke network design, convoys travel from an LSA to a single FOB and back. The thick black line bisecting the map breaks the network into two sections. Each section represents an area of responsibility for the LSAs. The numbered circles represent the FOBs that the LSAs support. The number of personnel at each base comes from the data provided by TRAC-LEE and is used to compute the expected consumption rates for each supply class. The arcs connecting the LSAs and FOBs are segmented, where each segment represents one day of travel.
Table 4-1 shows the network and scenario parameters and their values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bases</td>
<td>8</td>
</tr>
<tr>
<td>LSAs</td>
<td>2</td>
</tr>
<tr>
<td>FOBs</td>
<td>6</td>
</tr>
<tr>
<td>Routes</td>
<td>8</td>
</tr>
<tr>
<td>Supply Classes</td>
<td>5</td>
</tr>
<tr>
<td>Vehicle Types</td>
<td>12</td>
</tr>
<tr>
<td>Supply / Vehicle Combinations</td>
<td>24</td>
</tr>
</tbody>
</table>

*Table 4-1 Scenario Parameter Quantities*

The remaining parameters and attributes specific to this scenario are discussed below and are representative of conditions similar to recent military operations:

- The last planning day, $T$, is set at 4 days into the future, consistent with the Distribution Management Process (DMP).

- The longest travel distance to any FOB, $W$, is 3 days, as shown in Figure 4-1.

- The storage capacity at each FOB for Fuel and Water, $CapFW_f$, is 7 days of expected daily consumption.

- The storage capacity of each FOB’s warehouse, $CapWare_f$, is the aggregate number of pallets holding 7 days of expected daily consumption of Food, Major End Items, and General Cargo.

- The minimum flow, $MinFlow^c$, for supplies measured in gallons, is the smallest available truckload capacity. For supplies measured in pounds, it is one quarter of the pallet capacity, for a given supply class.
The stock objective for each supply class at an FOB, $SO_f$, is 5 days of expected daily consumption.

The desired stock minimums for each base, $SM_k$, are computed as a percent of the stock objective. These percentages, shown in Table 4-2, come from the data provided by TRAC-LEE.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Minimum Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel</td>
<td>0.75</td>
</tr>
<tr>
<td>Water</td>
<td>0.74</td>
</tr>
<tr>
<td>Food</td>
<td>0.74</td>
</tr>
<tr>
<td>Major End Items</td>
<td>0.80</td>
</tr>
<tr>
<td>General Cargo</td>
<td>0.70</td>
</tr>
</tbody>
</table>

*Table 4-2 Minimum Stock Percentages*

The vehicle attributes associated with each vehicle type are listed in Table 4-3.

<table>
<thead>
<tr>
<th>Vehicles</th>
<th>Supply Classes Carried</th>
<th>Capacity (in 1000s)</th>
<th>Pallet Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRAILER-FLAT-22.5TON</td>
<td>Food, Cargo, Major</td>
<td>45 (lbs)</td>
<td>80</td>
</tr>
<tr>
<td>PLS</td>
<td>Food, Cargo, Major</td>
<td>33 (lbs)</td>
<td>64</td>
</tr>
<tr>
<td>LHS</td>
<td>Food, Cargo, Major</td>
<td>22 (lbs)</td>
<td>48</td>
</tr>
<tr>
<td>MTV-CARGO</td>
<td>Food, Cargo, Major</td>
<td>10 (lbs)</td>
<td>16</td>
</tr>
<tr>
<td>LMTV-CARGO</td>
<td>Food, Cargo, Major</td>
<td>5 (lbs)</td>
<td>10</td>
</tr>
<tr>
<td>HMMWV</td>
<td>Food, Cargo, Major</td>
<td>2.5 (lbs)</td>
<td>4</td>
</tr>
<tr>
<td>5K-TANKER</td>
<td>Fuel</td>
<td>5 (gal)</td>
<td>N/A</td>
</tr>
<tr>
<td>HEMMT-2500</td>
<td>Fuel</td>
<td>2.5 (gal)</td>
<td>N/A</td>
</tr>
<tr>
<td>MTV-FUEL-1200</td>
<td>Fuel</td>
<td>1.2 (gal)</td>
<td>N/A</td>
</tr>
<tr>
<td>MTV-FUEL-600</td>
<td>Fuel</td>
<td>0.6 (gal)</td>
<td>N/A</td>
</tr>
<tr>
<td>HIPPO-2000</td>
<td>Water</td>
<td>2 (gal)</td>
<td>N/A</td>
</tr>
<tr>
<td>CAMEL-900</td>
<td>Water</td>
<td>0.9 (gal)</td>
<td>N/A</td>
</tr>
</tbody>
</table>

*Table 4-3 Scenario Vehicle Attributes*
The maximum and minimum convoy size, $V_{Max}$ and $V_{Min}$, are set to 25 vehicles and 3 vehicles respectively.

The parameters, $PC_t^C$ and $PV_t^P$, associated with the previous plan, are generated such that each FOB receives as close to their expected consumption rate as allowed by the vehicle capacity constraints.

Table 4-4 shows the cost parameters used in computing the cost function. Since both LSAs receive supplies from the out-of-theater network, sending supplies between LSAs should only be done in emergency situations. To denote this, a higher value is placed on LSA to LSA routes.

<table>
<thead>
<tr>
<th>Cost Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CostOver</td>
<td>1</td>
</tr>
<tr>
<td>CostUnder</td>
<td>3</td>
</tr>
<tr>
<td>$CostR_{r_{LSA to LSA}}$</td>
<td>0.05</td>
</tr>
<tr>
<td>$CostR_{r_{LSA to FOB}}$</td>
<td>0.01</td>
</tr>
<tr>
<td>$CostV_{v}$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 4-4 Cost Parameters

To create an in-theater input flow schedule, all supply classes are assumed to arrive on day 1 and then follow the arrival rates in Table 4-5.

<table>
<thead>
<tr>
<th>Supply Class</th>
<th>Arrival Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel</td>
<td>Daily</td>
</tr>
<tr>
<td>Water</td>
<td>Daily</td>
</tr>
<tr>
<td>Food</td>
<td>Every 3 Days</td>
</tr>
<tr>
<td>Major End Items</td>
<td>Every 2 Days</td>
</tr>
<tr>
<td>General Cargo</td>
<td>Every 2 Days</td>
</tr>
</tbody>
</table>

Table 4-5 Out-of-Theater Input Frequency of Supplies

The number of pounds or gallons of supplies that arrive is sampled from a beta distribution parameterized with a minimum and maximum value, and two positive shape
parameters, $\alpha$ and $\beta$. The distribution is symmetric around the expected number of pounds or gallons of supplies that arrive, $\text{ExpArrive}^c$. Thus, the minimum and maximum values, $a$ and $b$ respectively, are selected such that they are an equal distance away from, $\text{ExpArrive}^c$, as shown in Equations (4.1) and (4.2), where $0 \leq \omega \leq 1$.

\begin{align*}
a &= (1 - \omega) \times \text{ExpArrive}^c \quad (4.1) \\
b &= (1 + \omega) \times \text{ExpArrive}^c \quad (4.2)
\end{align*}

The shape parameters, $\alpha$ and $\beta$, are set equal to make the probability distribution function, shown in Equation (4.3) and plotted in Figure 4-2.

\begin{equation}
f(x; \alpha, \beta, a, b) = \frac{(x - a)^{\alpha-1}(b - x)^{\beta-1}}{B(\alpha, \beta)(b - a)^{\alpha+\beta-1}} \quad (4.3)
\end{equation}

Figure 4-2 In-Theater Flow Distribution
To evaluate each experiment the ATLLAP operates over multiple days, as indicated in Figure 4-3. The input data falls into two categories, static and dynamic. The static input, such as network parameters and base attributes, are the same over the 30 day horizon while the dynamic input, such as the flow of supplies into theater, changes each day. The ATLLAP takes these inputs and generates convoy plans to be executed the following day. These plans along with the dynamic data are then advanced ahead to the next day. The ATLLAP is run again with the updated dynamic input. This loop iterates over 30 days.

*Figure 4-3 Rolling Horizon Flow Chart*
4.2 Experiment 1: Value of Information

The purpose of this experiment is to determine if there is value in a system which can provide visibility into the in-theater input schedule. The motivation for this comes from the failure of the existing RFID system established by the Department of Defense. Knowing the incoming supply schedule better should improve the performance of the in-theater supply chain. However, it is expected that there is a point beyond which increased visibility provides no additional performance value.

4.2.1 Design

Seven trials are created, with increasing visibility into the in-theater supply flow. In the first trial, representative of current Army operating procedures, the ATLLAP has no visibility into the in-theater input schedule. It is only provided with current on-hand supply levels. Each successive trial gains an additional day of visibility into the input flow schedule.

The ATLLAP assumes no supplies are expected to arrive for the days beyond those that are visible. This is due to the length of time needed to develop the experience to make assumptions concerning incoming supplies. The frequent turnover of Army personnel hinders their ability to build this experience.

For this experiment 10 random in-theater input flow schedules are created and run for each of the 7 trials. Since each schedule is run over 30 days with a maximum visibility of 6 days, each schedule consists of the number of pounds or gallons of each supply class that arrives each day over the next 36 days. The amount of each supply class that arrives is sampled from a beta distribution with maximum and minimum values of, $1.9 \times \text{ExpArrive}^e$ and $0.1 \times \text{ExpArrive}^e$, respectively. The ATLLAP’s performance for each random schedule is
determined from the results associated with the second and third week of the four week trial, to avoid transient effects at the beginning and end of the run.

### 4.2.2 Results and Analysis

Figure 4-4 shows the average daily value of the cost function for the 10 random schedules at each trial.

![Average Daily Cost Function Value of Schedules](image)

**Figure 4-4 Impact of Visibility on Average Daily Cost Function Value**

The large range of values for the first trial (no visibility) is attributed to the effect of the variation in the flow of incoming supplies. The collapse at trial 2 (1 day of visibility) by all of the schedules is an unexpected result. This implies that having 1 day of visibility provides by far the greatest added value and visibility beyond this only provides minimal improvement that is probably not worth the cost to implement. One would expect the values of the cost function at
trial 2 to be more dispersed. The reason these values plunge is likely due to the width of the acceptable range.

A wider range makes it easier for the ATLLAP to maintain acceptable supply levels, which incur no penalty. To test this theory an experiment is run with a smaller acceptable range. The range used in the original scenario is reduced by 60% on average, for each supply class. Figure 4-5 compares the average daily value of the cost function for a single random schedule run, Schedule 2, using the original and smaller range.

![Acceptable Range Comparison](image)

**Figure 4-5 Compare Varying Acceptable Ranges**

The higher values for the narrower range are expected because it is more difficult for the ATLLAP to keep supply classes within this acceptable range. The plot of the narrower range suggests that there is value in knowing the input schedule 3 days into the future, but little value beyond that. This contrasts with the wider range, which implies there is little additional value
added beyond 1 day of visibility. The difference indicates that the size of the acceptable range impacts the visibility needed to maximize in-theater performance.

Another useful measure is the number of supply classes that lie outside the acceptable range. Figure 4-6 compares the average daily number of supply classes that lie outside their acceptable range for the schedule plotted in Figure 4-5. These plots are consistent with Figure 4-5 in that the smaller range generates higher values and requires more visibility to reach optimality. It is important to realize that twelve supply classes, in the narrower range in Figure 4-6, which lie outside their range at trial 1 represent 30% of the supply classes in the scenario. This high percentage could have a significant effect on the operational capabilities of a unit. Operationally, these results suggest that visibility into the in-theater input schedule provides benefit to the units and depends on the acceptable range.

![Average Daily Number of Supply Classes Outside Acceptable Range](image)

*Figure 4-6 Average Number of Supply Classes Outside Acceptable Range*
Two additional measures are used to analyze the performance of the ATLLAP. Figure 4-7 shows the average number of convoys deployed each day and Figure 4-8 shows the average number of vehicles sent out each day, over all schedules.
These figures show a decrease in the number of convoys and vehicles as visibility into the in-theater input schedule increases. Although modest, the decrease does have operational significance. An overall reduction of approximately 1.25 convoys per day translates to less time needed to prepare and load convoys for movement, which can be used for other operational tasks. The 2.5 vehicle reduction translates to approximately 5 fewer soldiers, as there are always 2 soldiers per vehicle, departing from LSAs each day. If one considers the length of recent operations, this is significant.

All of the previous results suggest that 3 days visibility into the future provides almost all of the information needed to optimize performance, at least for this scenario. The premise is that this occurs because the planning horizon is 4 days. This means that the ATLLAP looks 4 days ahead and generates plans over the next 3 days. Even though the ATLLAP is provided information beyond 4 days it does not add additional value in this case. To test this hypothesis the planning horizon is changed to 5 days, which should result in a curve with a more gradual downward slope that flattens out after trial 5.
Figure 4-9 compares the results of one representative schedule, Schedule 8, for both the 4 and 5 day planning horizon. As expected, the schedule with the 5 day planning horizon declines more gradually and does not flatten out until trial 5. This indicates that there is a correlation between the planning horizon and the visibility required to achieve better performance.

![Planning Horizon Comparison](image)

**Figure 4-9 Planning Horizon Comparison**

### 4.2.3 Summary

Experiment 1 shows that visibility into the in-theater input flow schedule can benefit the performance of the in-theater supply chain. There are also operational benefits such as reducing the number of convoys utilized. The question of how much visibility is needed, however, is not straightforward. Under current operating procedures there appears to be no additional gain in knowing what arrives more than 3 days into the future. However, two factors were shown to have an impact on the needed visibility and its effect on performance; the acceptable range and the planning horizon. Ultimately, selecting these parameters is dictated by the types of...
operations being conducted. Criterion that make each theater unique, such as enemy forces, terrain and infrastructure, have a definite effect on the visibility required to optimize performance.

4.3 Experiment 2: Uncertainty in the In-Theater Input Flow Schedule

This experiment examines the effects that uncertainty in the in-theater input flow schedule has on the performance of the ATLLAP. This uncertainty can vary during the course of operations due to changes in customs policies, out-of-theater route selection, and other factors. Recent events in Afghanistan provide an example, where political tensions between the U.S. and Pakistan resulted in closure of the main supply route into Afghanistan which went through Pakistan. Supplies had to be transported via alternate routes, which created a change in the level of uncertainty in the in-theater input flow schedule. Since the Army has almost no control over these factors it is important to understand how the ATTLAP performs under various levels of uncertainty in the in-theater input flow schedule. The question is: how much does the performance of ATLLAP degrade as uncertainty in the in-theater input flow schedule increases.

4.3.1 Design

In order to generate uncertainty in the in-theater input flow, the number of pounds or gallons of supplies that are scheduled to arrive each day is varied by sampling from a beta probability distribution.
To accomplish this, different values of $\omega$, are used to vary its minimum and maximum values, as shown in Table 4-6.

<table>
<thead>
<tr>
<th>Trial</th>
<th>$\omega$</th>
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<tr>
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<tr>
<td>2</td>
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<td>3</td>
<td>0.25</td>
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<td>4</td>
<td>0.5</td>
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<tr>
<td>5</td>
<td>0.75</td>
</tr>
<tr>
<td>6</td>
<td>0.9</td>
</tr>
</tbody>
</table>

*Table 4-6 Level of Uncertainty*

By increasing $\omega$, the standard deviation of the distribution shown in Equation (4.4) increases, resulting in more dispersed beta distributions as shown in Figure 4-10.

$$\sigma = \sqrt{\frac{\alpha \beta (-2 \omega \text{ExpArrive}_c)^2}{(\alpha + \beta)^2(\alpha + \beta + 1)}}$$

*Figure 4-10 Comparison of Beta Distributions*

For each trial, 10 random in-theater input flow schedules are generated from their corresponding beta distribution. The schedules are run assuming that the ATLLAP has no
visibility into the future, consistent with current Army operations. The ATLLAP’s performance on each random schedule is determined from the results associated with the second and third week of the four week trial to avoid beginning and end effects.

4.3.2 Results and Analysis

Figure 4-11 shows the average value of the cost function over the 10 random schedules for each trial ($\omega = 0, 0.1, 0.25, 0.5, 0.75, 0.9$). As expected, as the variance in the input flow increases, the cost function also increases, somewhat exponentially. This result is not surprising since increasing the variance could cause the supply levels at the LSAs to become lower, resulting in supply levels at the FOBs lying further outside the acceptable range. Eventually this could lead to an infeasible situation where the LSAs do not have enough supplies to meet the demand at the FOBs.

![Average Daily Cost Function Value](image)

*Figure 4-11 Impact of Variability on Daily Average Cost Function Value*
Figure 4-12 shows the average daily number of supply classes that lie outside the acceptable range for each trial. These results increase as the variation increases, as well, although this plot does not suggest an exponential relationship as in Figure 4-11.

![Average Daily Number of Supply Classes Outside Acceptable Range](image)

**Figure 4-12 Average Number of Supply Classes Outside Acceptable Range**

To assess how uncertainty in the in-theater input flow effects the time that soldiers are on the road, two performance measures are presented. First, the average number of convoys deployed each day is shown in Figure 4-13, where a close examination of the data suggests that this increase in convoys is primarily due to additional convoys transporting supplies between LSAs. Since the LSAs receive supplies from the out-of-theater network this cross leveling of supplies suggests inefficiency in inventory management caused by variability in the in-theater input flow.
Second, Figure 4-14 shows the average number of vehicles sent out per day. Though small, the slight increase in vehicles does have an operational significance because more soldiers are exposed on the road transporting supplies.
4.3.3 Summary

Experiment 2 explored the impact that uncertainty in the in-theater input flow schedule has on the performance of the ATLLAP. The results of this experiment support the hypothesis that ATLLAP’s performance degrades as the uncertainty in the flow of supplies increases. Overall, increased uncertainty leads to a steady increase in the average number of supply classes which lie outside acceptable ranges, and also pushes them further outside the range.

4.4 Experiment 3: Capabilities and Limitations of Planner

The purpose of this experiment is to test the computational performance of the ATLLAP. This provides insight into its ability to perform in realistic operational situations.

4.4.1 Design

To test the computational performance of the ATLLAP, nine scenarios are created with a different numbers of bases, as shown in Table 4-7. From an operational perspective, one LSA would rarely service more than 20 FOBs; if it did they would most likely be small FOBs closely grouped together, which can occur in cities.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>LSAs</th>
<th>FOBs</th>
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<tr>
<td>9</td>
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</tbody>
</table>

*Table 4-7 Experiment 3 Scenarios*

To minimize the impact that base attribute data may have on the ATLLAP’s performance, 5 FOBs are chosen from the TRAC-LEE data. These FOBs are distributed equally
in each scenario. A planning horizon of 4 days is used, consistent with current Army operations. The in-theater input flow schedule for each scenario assumes every LSA receives their expected arrival amount of supplies. The other parameters in each scenario are the same as discussed in section 4.1.

4.4.2 Results and Analysis

The results from each scenario are shown in Table 4-8.

<table>
<thead>
<tr>
<th>Scenario Inputs</th>
<th>LSAs</th>
<th>FOBs</th>
<th>Supply Classes</th>
<th>Vehicle Types</th>
<th>Routes</th>
<th>Max Travel Distance</th>
<th>Planning Horizon</th>
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* DNC (Does Not Converge) - This scenario does not reach the prescribed gap tolerance before the CPLEX runs out of memory.

Table 4-8 Experiment 3: Inputs and Results

Gap tolerances, the percent difference between the MILP solution and the relaxed Linear Program (LP) solution, of 1%, 2%, 4% and 10% are used to compare the run time (seconds) of each scenario.
Figure 4-15 shows that the run time increases as the number of bases increases and as the gap tolerance decreases.

![Run Time Performance](image)

**Figure 4-15 ATLLAP Run Time Performance**

4.4.3 Summary

Experiment 3 examined the computational performance of the ATLLAP. The results show that as the number of bases increases there are tradeoffs between the speed the ATLLAP generates a solution and the optimality of the solution. The operating environment would dictate how these tradeoffs are handled. The ability of personnel to set these parameters would allow them to handle different situations. Even if the gap is large after a specified time it can still provide a starting point to human planners.
5 Future Work and Conclusions

The first section of this chapter discusses aspects of the planner that require further exploration before implementation as a logistical tool in the Army. The second section of the chapter highlights the major conclusions of the research.

5.1 Future Work

The results and analysis of the experiments from Chapter 4 demonstrate the benefits of accurate information for the future arrival of supplies. However, the accuracy and realism of the ATLLAP developed in this thesis can be improved in a number of ways. This chapter discusses four focus areas for future research and development.

5.1.1 Planner Fidelity

The planner in its current form does not take into consideration the full set of parameters necessary for deriving real-world convoy plans. Many of these were omitted to keep the magnitude of the problem within computational limits. Three areas that would benefit from additional research are identified in this section.
The first area, supply classes, represents only a small subset of Army commodities. Future research should increase the number of commodities to create a more accurate representation of supplies that actually flow into a theater of operation. This will significantly increase the scope of the problem in two ways. First, additional commodities will increase the number supply class / vehicle combinations in the planner. Second, some of the commodities require the addition of new constraints; for example, ammunition has specific transportation requirements.

Second, a more realistic packaging system should be developed. Most commodities come in multiple standard package quantities which are transported in various pallet types or containers. Expanding the parameters associated with packaging commodities will also require generating load plans for vehicles. This will allow better optimization of vehicles and could result in a decrease in the number of vehicles used to transport supplies.

The third area is route selection. This research focused on a single route using only ground transportation assets. The Army uses multiple routes and different transportation options including helicopters, airplanes, and airdrop. These asset types offer faster service times and come from both military and civilian organizations, and have different constraints. Logistical units do not have organic aircraft to conduct missions, so aircraft availability is contingent on other theater activities. Cost, speed, risk, and availability are all reasonable measures that could be used to compare routes. Further research into this could generate significant reformulation of the planner.

Increasing the fidelity in these areas leads to an increase in the size of the formulation. One approach to managing the increased problem size is through the use of heuristics, where the problem could be broken up into manageable phases. For example, optimizing the load plans to
minimize the vehicles used, or finding the best route to take could be addressed individually in a heuristic.

5.1.2 Analysis of Planning Horizon

Experiment 1 showed a correlation between the planning horizon and the visibility required to achieve better performance. This result suggests there is potential that changing the planning horizon could lead to more efficient management of the in-theater supply chain. This experiment compared the current planning horizon, developed from current policies and procedures, to an increased planning horizon. However, modifying the value of the planning horizon in the planner is not enough to evaluate the impacts which result from such a change. Future research must consider the modifications to the policies and procedures required to establish a new planning horizon, which affect all military units in theater.

To demonstrate the need to change procedures consider the current Distribution Management Process (DMP). The de-confliction phase of this process occurs at most 48 hours before the convoy is set to leave an LSA and is a crucial step to prevent fratricide. Now consider a longer planning horizon, such as 6 days, with no change to the process. This would lead to the de-confliction phase occurring 96 hours before a convoy leaves. This increases the chance that a new mission requirement could conflict with these convoy plans. When this occurs commanders must weigh the operational value of the convoy to the mission. If the mission provides more value, then the convoy will be canceled. This could potentially generate significant changes to other convoy plans when the ATLLAP is run again. The second and third order effects of resulting from a change to the planning horizon must be developed and modeled correctly to get an accurate evaluation of the impacts.
5.1.3 Robust Planning

Any planner integrated in U.S. Army doctrine must generate plans that hold up under real world uncertainties. The planner built for this thesis models the world with static parameters that are actually stochastic in nature. Future research could enhance the model by incorporating the uncertainty with either stochastic programming or robust optimization.

To illustrate the concept of robust planning, consider the travel time between an LSA and an FOB. The ATLLAP assumes these times to be constant when in reality there are many factors that affect them, such as weather, enemy forces, and terrain. Assume now that given specific conditions a probability distribution exists that describes the travel time between an LSA and an FOB. Simulations could be run to find the most likely travel time. While this value could be used in the initial plan there would be instances when conditions change while supplies are being delivered which would impact the travel time of the vehicles currently on the road. To truly be robust these effects would need to be taken into consideration when developing future plans.

Incorporating uncertainty into the planner would take considerable reformulation. One would need an in depth understanding of the theater of operations which would involve extensive research with subject matter experts and assistance from military agencies.

5.1.4 Validate

Many aspects of the ATLLAP use representative data, but actual operational data would undoubtedly confirm assumptions and/or allow for appropriate adjustments. Unfortunately, most data maintained by the military is classified as secret, however validation of the ATLLAP could be accomplished by comparing results to human planners using the same scenarios. This would quantify gains in efficiency and solution speed provided by the ATTLAP, and human planners
would provide additional insight into current operating procedures which could make the ATLLAP more realistic. The ancillary benefit would be that military planners could promote the ATLLAP as a useful tool for planners.

5.2 Conclusions

At the beginning of this research, the notion that the Army’s end-to-end logistical network is actually two distinct networks was introduced. It was established that the lack of real time information and predictable scheduling within the out-of-theater network provides poor visibility for Army logistical planners in theater, and leads to inefficient management of the in-theater supply chain, which adversely affects the performance of combat soldiers. Successful support of these soldiers ultimately determines whether battles are won or lost. The purpose of this thesis is to create a planner, the ATLLAP, whose performance is measured by how well it supplies combat soldiers.

The ATLLAP considers basic building blocks in generating convoy plans: vehicle types, packaging requirements and storage capacities. It uses an array of operational inputs to model a theater of operation. Mixed Integer Linear Programming is used to develop convoy plans which optimize the value of a cost function. The results confirm that increasing visibility in the in-theater input flow indeed improves the performance of the ATLLAP. The amount of improvement depends on parameters, such as the planning horizon and the width of the acceptable range.

We conclude that there are benefits in pursuing a system or process that can provide reliable information about supplies entering a theater of operation. A cost analysis should be conducted comparing the cost of a new system to the benefits derived from better visibility. There are financial benefits associated with using fewer vehicles such as, fuel and maintenance.
costs as well as the cost connected with reducing the number of contracted vehicles. Other
benefits are not as easily quantified, such as, fewer soldiers exposed on the roads and time saved
in generating plans.

More detailed parameter modeling must be conducted to make the ATLLAP a useful tool
for in-theater logistical planners. Extending the model to include uncertainties would allow for
more robust planning. The planner requires validation using operational data and comparison to
human planners to understand the full scope of the benefits it can provide.
# Appendix A: Abbreviations and Acronyms

<table>
<thead>
<tr>
<th>Abbreviation/Acronym</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATLLAP</td>
<td>Army Theater Level Logistical Allocation Planner</td>
</tr>
<tr>
<td>BCT</td>
<td>Brigade Combat Team</td>
</tr>
<tr>
<td>BSB</td>
<td>Brigade Support Battalion</td>
</tr>
<tr>
<td>CASCOM</td>
<td>Combined Arms Support Command</td>
</tr>
<tr>
<td>CENTCOM</td>
<td>United States Central Command</td>
</tr>
<tr>
<td>CSSB</td>
<td>Combat Sustainment Support Battalion</td>
</tr>
<tr>
<td>DMP</td>
<td>Distribution Management Process</td>
</tr>
<tr>
<td>DOD</td>
<td>Department of Defense</td>
</tr>
<tr>
<td>FOB</td>
<td>Forward Operating Base</td>
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<tr>
<td>HEMMT</td>
<td>Heavy Expanded Mobility Tactical Truck</td>
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<tr>
<td>HETT</td>
<td>Heavy Equipment Truck Transport</td>
</tr>
<tr>
<td>HMMWV</td>
<td>High Mobility Multipurpose Wheeled Vehicle</td>
</tr>
<tr>
<td>LHS</td>
<td>Load Handling System</td>
</tr>
<tr>
<td>LMTV</td>
<td>Light Medium Tactical Vehicle</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Program</td>
</tr>
<tr>
<td>LSA</td>
<td>Logistical Support Area</td>
</tr>
<tr>
<td>METT-TC</td>
<td>Mission, Enemy, Terrain, Troops available, Time, and Civilian</td>
</tr>
<tr>
<td>MILP</td>
<td>Mixed-Integer Linear Program</td>
</tr>
<tr>
<td>MTV</td>
<td>Medium Tactical Vehicle</td>
</tr>
<tr>
<td>OPLOG</td>
<td>Operational Logistics</td>
</tr>
<tr>
<td>PLS</td>
<td>Palletized Load System</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>RFID</td>
<td>Radio Frequency Identification</td>
</tr>
<tr>
<td>SUST</td>
<td>Sustainment Brigade</td>
</tr>
<tr>
<td>TRAC-LEE</td>
<td>U.S. Army Training and Doctrine Command at Fort Lee, VA</td>
</tr>
<tr>
<td>TRANSCOM</td>
<td>United States Transportation Command</td>
</tr>
<tr>
<td>TSC</td>
<td>Theater Sustainment Command</td>
</tr>
<tr>
<td>U.S.</td>
<td>United States of America</td>
</tr>
</tbody>
</table>
Appendix B: Master List of Variables and Parameters

The following are input variables to the formulation:

\( L \)  The set of all LSAs, \( l \in L \)

\( F \)  The set of all FOBs, \( f \in F \)

\( B \)  The set of all bases, \( b \in B \), where \( L \cup F = B \)

\( SupGal \)  The set of supply classes measured in gallons, \( g \in Gal \), where \( Gal = \{\text{Water, Fuel}\} \)

\( SupLbs \)  the set of supply classes measured in pounds, \( p \in Lbs \), where \( Lbs = \{\text{Food, Major, Cargo}\} \)

\( SupClass \)  the set of all supply classes, \( c \in SupClass \), where \( SupClass = SupGal \cup SupLbs \)

\( Veh \)  the set of vehicle types, \( v \in Veh \), where \( Veh = \{5\text{KTanker, HMMWV, Camel900}...\} \)

\( M \)  the set of pairs, \((c,v)\), denoting the possible supply class/vehicle combinations, where supply class, \( c \), can be transported by vehicle, \( v \), \( m \in M \)

\( R \)  the set of ordered pairs, \((l,b)\), representing sanctioned route from \( l \) to \( b \) where, \( r \in R \)

\( T \)  the last day plans are generated

\( W \)  maximum number of travel days across all routes in the network

\( S2T \)  the range of days over which plans are generated, \( \{2...T\} \)

\( STW \)  the range of days over which supplies may be delivered, \( \{1... (T+W)\} \)

\( CapFW_f^g \)  the storage capacity in gallons of a supply class, \( g \), at FOB, \( f \)

\( CapPal_f^p \)  the number of pounds of a supply class, \( p \), that fits on a standard sized pallet

\( CapWare_f \)  the number of standard sized pallets that the warehouse at FOB \( f \) can hold

\( CostR_r \)  the cost associated with a convoy traveling along a given route, \( r \)

\( CostV_v \)  the cost associated with using vehicle \( v \) to transport supplies

\( DC_{bt}^c \)  the number of pounds or gallons of a supply class, \( c \), consumed daily at each base, \( b \), on day \( t \), such that \( t \in STW \)
\( \text{InFlow}_{bt}^c \) = the number of pounds or gallons of a supply class, \( c \), added to the inventory level of base, \( b \), from the out-of-theater network, on day \( t \), such that \( t \in STW \)

\( \text{MinFlow}^c \) = the minimum number of pounds or gallons of a supply class, \( c \), that can be transported

\( \text{PC}_{bt}^c \) = based on the previous plan, the amount of a supply class, \( c \), added to the inventory level of a base, \( b \), on day \( t \), such that \( t \in STW \), due to inbound deliveries

\( \text{PV}_{lt}^v \) = based on the previous plan, the number of vehicles, \( v \), added to the vehicle inventory level of LSA, \( l \), on day \( t \), such that \( t \in STW \), due to returning convoys

\( SI_b^c \) = the number of pounds or gallons of a supply class, \( c \), on hand at the end of day 1, at base \( b \)

\( SO_f^c \) = the stock objective of a supply class, \( c \), at each FOB, \( f \)

\( SM_b^c \) = the lower bound of the acceptable level of a supply class, \( c \), at base \( b \)

\( TD_r \) = the number of travel days for a given route, \( r \)

\( VI_l^v \) = the number of vehicles, \( v \), on hand at the end of day 1, at LSA \( l \)

\( VMax \) = the maximum number of vehicles allowed in a convoy

\( VMin \) = the minimum number of vehicles allowed in a convoy

\( VPal^v \) = the number of pallets that can fit on vehicle \( v \)

\( VW^v \) = the weight capacity of vehicle \( v \)

\( WO \) = weighting factor associated with inventory levels being above \( SO_b^c \) on any day

\( WU \) = weighting factor associated with inventory levels being below \( SM_b^c \) on any day

The following decision variables are used in the formulation and are non-negative.

\( X_{rt}^m \) = the flow of supply class, \( c \), leaving on day \( t \), such that \( t \in S2T \), traveling on route \( r \), in vehicle \( v \), where \( m = (c,v) \)

\[ \text{Bin}_{rt}^m = \begin{cases} 0 & \text{if } X_{rt}^m = 0 \\ 1 & \text{if } X_{rt}^m > 0 \end{cases} \]
\[ \text{Convoy}_{rt} = \begin{cases} 0 & \text{if no convoy leaves an LSA along route } r \\ 1 & \text{if a convoy leaves an LSA along route } r \end{cases} \]

The following variables take on values for a given set of inputs and decision variable values.

- \( P_{alX}^{m}_{rt} \) = the number of pallets, containing supply class, \( p \), leaving on day \( t \), such that \( t \in S2T \), traveling on route \( r \), in vehicle \( v \), where \( m = (p,v) \) (integer)

- \( P_{alSL}^{p}_{ft} \) = the number of pallets, containing supply class, \( p \), at FOB \( f \), on day \( t \), such that \( t \in STW \) (integer)

- \( SL_{bt}^{c} \) = the number of pounds or gallons of a supply class, \( c \), on hand at the end of the day, \( t \) such that \( t \in STW \), at base \( b \)

- \( OS_{ft}^{c} \) = the number of pounds or gallons of a supply class, \( c \), over stocked at the end of the day \( t \), such that \( t \in STW \), at FOB \( f \)

- \( US_{bt}^{c} \) = the number of pounds or gallons of a supply class, \( c \), under stocked at the end of the day \( t \), such that \( t \in STW \), at base \( b \)

- \( VC_{lt}^{v} \) = the number of vehicles of type, \( v \), available for convoy missions at the end of day \( t \), such that \( t \in STW \), at LSA \( l \) (integer)

- \( VR_{rt}^{v} \) = the number of vehicles of type, \( v \), traveling along route, \( r \), that have returned and will be added to vehicle inventory levels on day \( t \), such that \( t \in S2T \) (integer)

- \( VX_{rt}^{v} \) = the number of vehicles of type, \( v \), leaving on day \( t \), such that \( t \in S2T \), traveling on route \( r \) (integer)

- \( Y_{rt}^{m} \) = the number of pounds or gallons of a supply class, \( c \), available for consumption at the destination on day \( t \), such that \( t \in STW \), arriving via route \( r \), in vehicle \( v \), where \( m = (c,v) \)
References


Schokley, Jonathan (Newly added so 19 was 18 and so on)
