18.440 Probability and Random Variables
Spring 2009

18.440, PS1

Relates to the material of Ross, Chapter 1, "Combinatorial Analysis"

1. Canadian postal codes are strings as follows: letter, digit, letter, space, digit, letter, digit. Digits are decimal from 0 to 9. Letters are in capitals. For example, B2R 7E0 is a possible code.
(a) If all 26 letters were allowed, how many possible codes would there be?
(b) What if just the two letters O and I were not allowed, leaving 24 letters?
(c) In fact none of the six letters D, F, I, O, Q, U is allowed. The code H0H 0H0 is reserved for Santa Claus. The first digit is 0 for "rural" codes and 1 through 9 for "urban" ones (towns or cities). (Thus e.g. B2R 7E0 is "urban".) With these restrictions, how many rural codes are possible other than Santa Claus'?

2. Eight people are invited to a party, consisting of four pairs to be called "couples" (they might be spouses, partners, siblings...). How many seating arrangements are there of the people around a circular table in which:

(a) In this part only, assume that each couple consists of one man and one woman and the genders must alternate around the table, so that no two people of the same gender sit side by side. How many such arrangements are there in which no two members of any couple sit side by side?
(b) Without the additional assumptions in part (a), how many arrangements are there in which the two members of each couple must sit side by side?

In each part, consider two arrangements the same if they result from rotating the table, i.e. each person has the same neighbor on the right and on the left as in the other arrangement.

3. If 16 people are to be divided into three committees of sizes 3, 6 and 7, in how many ways can this be done?

4. From a set of N widgets, of which m are defective and the rest are not, a sample of n of the N is chosen. Assume that $0 < m < N$ and $0 < n < N$.
(a) How many possible samples are there containing exactly k defectives?
(b) Under what conditions on N, m, n, and k (all integers ≥ 0) is the answer to (a) equal to 0? *Hint*: there are three different conditions. One is $k > m$. What are the other two?

5. Suppose along a straight railroad line, there are communication relay towers located one every 10 miles. Each train crew has a wireless phone with a range of 11 miles. So, if just one tower is not working, a train is still always within range of a working tower. Suppose there are 16 total towers along a 150-mile stretch and those at each end are working, but among the 14 others, 5 are not.
(a) How many possible arrangements are there of the working and non-working towers? *Hint*: You can probably see this directly. Partly for part (b), look at Section 1.6 of Ross. Let $10x_i$ be the number of miles from the ith working tower to the next one for $i = 1, ..., 10$. Do you get an answer this way, and does it agree with the one you got directly?
(b) In how many of these arrangements are train crews always within range of a working tower, and for how many are they sometimes out of range? *Hint*: To be always within range, what condition must x_i satisfy and in how many ways can that happen?