18.440 Probability and Random Variables
Spring 2009

18.440 problem set 5

1. Let $f(x) = c(2 - x)$ for $0 \leq x \leq 2$ and $f(x) = 0$ elsewhere.
   (a) Evaluate c so that f is a probability density.
   (b) If X has density f, find the distribution function $F(x) = P(X \leq x)$ for all x.
   (c) Is $F(x) = 1$ for $x \geq 2$? If not, recheck (a) and (b).

2. For X in Problem 1, evaluate (a) EX and (b) $\mathrm{Var}(X)$.

3. For the uniform distribution $U[1, 3]$ on the interval $[1, 3]$,
   (a) What is its density function $f(x)$?
   (b) What is its (cumulative) distribution function $F(x)$?
   (c) What are the mean EX and variance $\mathrm{Var}(X)$ of a random variable with this distribution?
   (d) Let X_1, X_2,..., be independent with this distribution and $S_n = X_1 + \cdots + X_n$. Find the mean and variance of S_{30}.

4. Ross Chap. 5, Problem 12 (7th ed.) or 5.12 (8th ed.).

5. Let $G(\xi)$ be the volume of that part of the unit ball $x^2 + y^2 + z^2 \leq 1$ in 3 dimensions such that $x \leq \xi$, for $-1 \leq \xi \leq 1$.
(a) Find the derivative $g(\xi) = G'(\xi)$ for $-1 < \xi < 1$.
(b) Find a constant $c > 0$ such that if $f(x) = cg(x)$ for $-1 < x < 1$ and $f(x) = 0$ for $|x| \geq 1$ then f is a probability density.
(c) Find the distribution function $F(x)$ for this density f (not only in terms of G, but as an explicit function).
(d) Verify that F has the four properties of a distribution function listed near the beginning of the last section of Chapter 4 of Ross (§4.9 of the 7th edition, §4.10 of the 8th edition).