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18.440 Probability and Random Variables
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18.440 problem set 9

1. Ross, Chap. 8, Problem 15 or 8.15.
2. Ross, Chap. 8, Problem 18(b) (7th ed.) or 8.19(b) (8th ed.).
3. (Continuation of the previous problem) (a) After n fish have been caught, what is the probability that no Type 1 fish have been caught?
(b) Using part (a) and Boole's inequality, give an upper bound for the probability that $Y > n$ for Y from the previous problem.
(c) Use the bound from part (b) to find n so that $P(Y > n) < 0.05$.
4. Ross, Chap. 8, Problem 20 (7th ed.) or 8.21 (8th ed.). *Hint:* for what values of α is $g(y) = y^\alpha$ for $y > 0$ a convex function of y ?
5. An improvement on Ross, Chap. 8, Theoretical Exercise 12: let ϕ be the standard normal density, $\phi(x) = (2\pi)^{-1/2} \exp(-x^2/2)$. Let Z be a standard normal variable.
(a) Show that for any $a > 0$, $P(Z > a) \leq \phi(a)/a$. *Hint:* show that

$$P(Z > a) \leq \int_a^{+\infty} (x/a)\phi(x)dx.$$

- (b) Show that $P(Z > a)$ is asymptotic to $\phi(a)/a$ as $a \rightarrow +\infty$. *Hint:* use L'Hospital's rule.