LABYRINIHOS
by
GREGORY PATRICK GARVEY

## B.S. University of Wisconsin, Madison (1975)

M.F.A. University of Wisconsin, Madison (1980)

Submitted to the Department of Architecture in Partial Fulfillment of the Requirements of the Degree of

Master of Science in Visual Studies
at the
MASSACHUSEITS INSTITUTE OF TECHNOLOGY
September 1982
(C) Gregory Patrick Garvey 1982

The author hereby grants to M.I.T. permission to reproduce and to distribute copies of this thesis document in whole or in part.

Signature of Author:
Depar tynepht of Archirtectylue
Certifieã by:
Thesis Superyl'sor, Otto Piene
Accepted by:
Department of Architecture
Nicholas Negroponte, Chairman,
Rotenartmental Committee on Graduate Students

## DISCLAIMER OF QUALITY

Due to the condition of the original material, there are unavoidable flaws in this reproduction. We have made every effort possible to provide you with the best copy available. If you are dissatisfied with this product and find it unusable, please contact Document Services as soon as possible.

Thank you.

## The images contained in this document are of the best quality available.

## LABYRINTHOS

by
GREGORY PATRICK GARVEY

> Submitted to the Department of Architecture on August 6,1982 in partial fulfillment of the requirements for the Degree of Master of Science in Visual Studies.

Otto Piene, Thesis Supervisor


#### Abstract

Composition, in time and space is discussed as a general problem in graphics, music, film/video, landscape architecture and architecture. This discussion serves to introduce the primary sources which motivate the design of the labyrinth. These sources contribute to an interest in the definition of paths, pattern, and networks. This interest leads to the comparison between sequential composition and random access composition. The exploration of nonlinear composition is a primary motivation in the design and the construction of the labyrinth.


The opposition between the romantic and classical (Rubenistes and Poussinistes) representation of space and treatment of light in Western painting is examined. The rational articulation of space is contrasted to the indefinite, 'irrational' depiction: the comprehensible with the incomprehensible.

A brief survey of the present day and historical forms of labyrinths is followed by a discussion of the more salient artistic contributions, and then with a discussion of the labyrinth as a popular icon in media and in video games. The way in which the eye traces a composition in painting is compared to the path a stroller might take across a real landscape. The problem is the same for the graphic artist, the architect and the composer: how to break up space and time in order to maximize the interest in and the comprehension of the layout, environment or composition.

The nature of this perception is explored in examples taken from graphic design, architecture, painting, urban landscaping, the networks of the Sewers of Paris, the catacombs, the configuration of paths and walkways, modular design and the role of change and variation in generating intrinsic interest in each.

The Chinese Garden, recent playground design as "learning environments", the M.I.T. Architectural design studios and other examples are mentioned to show the convergence of a variety of intentions.

Change ringing and serial music show too the interest in the use of combinatorial techniques to generate unity and variety. This interest in variation is shown to be an essential part of learning and play. In literature the metaphor of the labyrinth again contrasts the opposition between the rational and the irrational.

Forms in nature, mathematical porportion and series, Artificial Intelligence, maze solving and generating algorithms are suggested as sources for design.

The Thesis Labyrinth is discussed in detail as well as the formulations that led to it. Future speculations concerning the labyrinth as a sonic installation occupy the final section and the thesis concludes with Greg Bright's Caveat.

## TABLE OF CONTENTS

ABSTRACT ..... 2
TABLE OF CONTENTS ..... 4
INTRODUCTION ..... 5
SURVEY: PAST \& PRESENT FORMS ..... 9
PAINTING \& PATHS ..... 18
PATTERN \& PERCEPTION ..... 27
THE PATH \& THE CITY ..... 36
SOURCES IN NATURE ..... 45
GARDENS, PLAY \& OTHER CURIOSITIES ..... 54
PATHS \& MUSIC ..... 63
LITERATURE \& METAPHOR ..... 69
MATH \& MODULES ..... 73
PROBLEM SOLVING \& ARTIFICIAL INTELLIGENCE ..... 89
ALGORITHMS: GENERATION \& SOLUTIONS ..... 98
LABYRINTHOS ..... 110
SPECULATIONS \& POSTSCRIPT ..... 126
BIBLIOGRAPHY ..... 133
ILLUSTRATIONS \& DIAGRAMS ..... 141


The intent of this thesis is to provide a personal account of the choices made and the factors which have influenced the design and construction of a labyrinth. The scope of this research reflects the particular interests of the author in seeking material from the past, present, and following certain conceptual proclivities as part of the design process. The focus is on an informal account of the sources and the resulting design of what is to become ultimately a large scale interactive, architectural environment.

Finally, we return to how both music and vision build things in our minds. Eye motions show us real objects; phrases show us music-objects. We learn a room with body motions; large music-sections show us "music-places". Walks and climbs move us from room to room; so do transitions between sections. Looking back in vision is like recapitulation in music; both give us time, from time to time, to revise or review conceptions of the whole. So, hearing a theme is like seeing a thing in a room. An allegro is like the room itself, and the whole sonata is like an entire building.

Marvin Minsky
. . . all of Western drama, going back 3,000 years, is very much built on the notion of sequential media, with well founded beginnings, middles, and endings. What you have seen turns that tradition around into a random access process. With these ideas, we are pulling some of the basic tenents out from under the principles of authoring and editing films. As such we had better address very soon some of the intellectual substance of some of this new kind of random access "movie".

Nicholas P. Negroponte

Free will or volition is one such notion: people are incabable of explaining how it differs stochastic caprice, but feel strongly that it does.

One of the major concerns of this thesis is to explore the implications of the concept of labyrinth and maze well beyond the aspect of a puzzle. The historical examples are rich in associations and suggestive power. The labyrinth as a metaphor for death and rebirth, sin and salvation, represents a change in a state of being and understanding and appears in many guises in many cultures.

This metaphor of transformation of understanding and comprehension indicates the fundamental nature of location specific consciousness. The labyrinth is a microcosm of the procedure of incremental search and exploration from a limited viewpoint to an understanding of the whole.

Like music, this procedure is temporal. Minsky's analogy underscores further that incremental search and observation mean actively learning a space or a musical world by constantly revising the conception of the whole as each new part is encountered.

In painting or in architecture, the space can be learned by accessing the composition or the structure in different places. In music, a definite sequence is prescribed. Because of the nature of the physical storage, the video disc suggests, like the theories of Rudolph Arnheim, that the conception of a whole can be independent of the way it is accessed. The video disc, by its nature, can be randomly
accessed. This thesis will discuss how a similar process occurs in painting, sculpture, architecture, and in the structure of change ringing.

While the temporal arts, like music or dance, suggest that a fixed sequential order determines the comprehension of the composition, random access suggests that by encountering a sufficient number of combinations of component parts (e.g., frames of the video disc, the angles of view in observing a painting or sculpture), one will reach an equally complete undertanding of the composition. In all the arts, whether temporal or randomly accessible, there have been two opposite tendencies: one seeks to prescribe precisely the order in which the work is to be perceived and the other permits an intuitive approach. The whole is either comprehended in a step-by-step, rational process or by a random, seemingly irrational process.
-9-

SURVEY:

## PAST

\&
PRESENT

FORMS

There are many paths to the top of the mountain, but the view there is always the same.

Chinese Maxim, Quoted by H.L. Mencken

Although the story of the labyrinth, built by Daedalus for King Minos of Crete, originates in Greek mythology, the pattern of the labyrinth is found in many cultures worldwide. The pattern can be seen as the outcome of the propensity to create primitive space filling patterns, the meander, the spiral etc..

Long before the first accounts of the Cretan labyrinth, labyrinthine patterns appeared on Egyptian seals and plaques. The huge temple of Amenemhet III built in the Fayoum, was described as 'the Labyrinth of Egypt' by Herodotus, Pliny and others.

The location of the original Cretan labyrinth is thought to be at the Palace of Knossos on the island of Crete. It is here, according to legend, that Theseus slew the Minotaur, and Daedalus was imprisoned for assisting Ariadne, who in turn had helped Theseus escape. As noted elsewhere the root of the word labyrinth is closely associated with the Palace. Sir Arthur Evans, who did the first archeological excavations, discredits the idea that the Palace of Knossos was itself the labyrinth and explains that the network of passages in the palace were part of a drainage system, not a labyrinth.

The first depictions of the Cretan labyrinth appear on the coins of Knossos. Throughout ancient times, the depictions on vases and Roman mosaics are of the unicursal labyrinth. In fact, Hermann Kern points out that the earliest visual formulation of the multicursal maze is not until 1550 A.D. Kern (32 p. 60).

During the Middle Ages, labyrinths were set in the floors of churches and Cathedrals. The largest and best known is in Chartres Cathedral. It is constructed of blue and white stones and is approximately 40 feet in diameter.

Instead of the seven windings of the Cretan, type there are now 11, a number that refers to sin. Its orientation--the entrance is always on the west, facing the sunset (death)-also shows the significance of the labyrinth as the image of a sinful God-alienated world. Accordingly, the path through the labyrinth can be understood as a path through the sinful world toward salvation. Hermann Kern (32 p. 62).

Turf mazes are generally found in Britain near churches, which causes some people to think that their original purpose was religious. However, Christianity adopted what was an ancient pagan labyrinth figure. The running of Turf mazes was often performed on pagan holidays such as May Day. 'Love labyrinths or the 'Rosamonds Bower' refer to erotic involvements, and the stone labyrinths found throughout Scandinavia are commonly the site of games where young boys race one another to reach a young girl at the center.

The stone labyrinths are often called Troborg or Troytown. This association of city/fortress implies that an attacker might be confused by the complexity of windings. In a related way the labyrinth is often used as a symbol of protection against evil forces or spirits.

It is only during the sixteenth and seventeenth centuries that true three-dimensional, multicursal labyrinths come into
existence in the form of topiary or hedge mazes. It had become increasingly fashionable to plant herbs, flower beds or dwarf shrubs in elaborate designs. In France, complex designs far removed from the Cretan Model were built. For example, in the labyrinth at Versailles, statues depicting Aesop's Fables were positioned throughout. Such garden mazes were intended for aesthetic contemplation rather than puzzle solving. Near London, the Hedge Maze at Hampton Court Place is one the of best-known in the world and is the oldest surviving maze in England.

Hampton Court has inspired more recent hedge mazes such as the 45 year old holly maze at the Governor's Palace in Williamsburg, Virginia. There are other examples in the United States derived from the Hampton Court Model.

Today, the maze has become a popular icon that appears on the back of cereal boxes, sunday papers, or table placemats or in 'mind teaser' type puzzle books. A three dimensional maze of mirrors, typical of amusement parks, was used for the climatic scene in Orson Welles' film Lady from Shanghai. Labyrinths and maze patterns are periodically used in graphic design and illustration as the symbol of confusion or puzzlement.

In the decade of the seventies there was a sort of 'maze craze' in the publishing industry where large numbers of maze books appeared on book racks. The self styled 'Maze King' of England, Greg Bright, published three such books and in 1971-1972 spent that year digging his Pilton trench maze in

Somerset, England.
The electronic revolution in microprocessors has proved to be a boost for the use of mazes. Video arcade games such as Pac Man, Ms. Pac Man or Berzerk use 'eat or be eaten' chases through mazes as the basic element of the games.

More sophisticated use of the maze as a principle of structuring a game is found in the computer simulated interactive fairy tale worlds of Dungeons and Dragons, or Adventure and Zoark.

Many personal computer systems offer some kind of maze game with a 3-D display in the maze as one solves it. The best example of this is on the computer at M.I.T.'s Laboratory for Computer Science. Several players sitting at different terminals attempt to hunt down the others and zap them before they get zapped. For each player, the displays shows simultaneously the plan and perspective view of that player's present position. When one player encounters a different player, the other is seen as an eyeball. The perspective views change as one moves through this multicursal maze. Players can quickly become highly skilled and the most sophisticated can change the configuration of the maze.

Through satelite transmissions, computers can be connected in transcontinental networks (e.g. the Chaos Net) where the individual can log in on several machines simultaneously. The telephone network and the growing field of telecommunications point to vast user defined interactive electronic networks where an individual can move through a virtual maze of inter-
connections.
Probably because of his prodigious tunnelling, the rodent has been forced to solve mazes in the name of science for over 100 years. A part of the popular image of the maze or labyrinth is inevitably linked to such experiments with mice and rats. An American, E.L. Thorndike, was the first to report about his experiments with a 'puzzle box', from which any animal had to figure out how to escape. The behaviurist model of learning devised all sorts of torturous mazes, with electric shock, light, food, water and other various traps and pitfalls to reinforce conditioning. Rodents must have breathed a sigh of relief with the advent of Artificial Intelligence. Live mice were replaced by mechanical ones or even simulated ones. Marvin Minsky as undergraduate at Harvard built a learning machine which simulated four rats bumping into and learning to avoid each other. McCorduck (41, p. 85).

A number of contemporary artists have used the traditional form of the Cretan labyrinth as pattern for large scale environmental works. Richard Long copies stone for stone, in his Connemara Sculpture, 1971 the typical 'troytown' stone labyrinth of Scandinavia. Alice Aycock's Maze 1972 on the Gibney Farm near New Kingston, Pennsylvania and Robert Morris' plywood Labyrinth of 1974 are full scale structures using the basic pattern with little or no modification. Richard Fleischer's Chain Link Maze 1978 in Amherst, Massachusetts casts an old form in new material. It is not only a maze but
also the creation of a visual field of overlayed chain link resulting in shifting moire patterns.

Dennis Oppenheim took as a model for his Maze in Whitewater, Wisconsin the scheme of a laboratory maze. Instead of mice he used cattle. Using bales of hay, the maze had the dimensions of $500^{\prime} \mathrm{x}$ 1000'. "In solving the maze they learned it. They digested the information that they were housed in." Kern (32 p. 65).

A different approach is taken by Ugo Dossi, who is interested in network-forming structures of association. In 1975, he made up 190 separate sheets that represented systems of conceptual relations: a "garden of paths, which branch off". Kern (32,p.66).

Terry Fox abstracts the Chartres design. Rather than building a complete structure, he associates the concept of 11 circular passages, 34 turning points and 552 steps with different components. For example, in 1976 he created two sound works titled "The labyrinth scored for the purrs of 11 cats" and "552 steps through 11 pairs of strings."

Fox recorded purring from 11 different cats. The purrs were mixed so that, through combinations of purrs, the labyrinth is traced and the goal is the simultaneous purring of all 11 cats. Terry Fox also exhibits arrangements of objects that are associated with Chartres Cathedral and the subterreanean stream which flows exactly underneath the labyrinth of the Cathedral.

Jean Tinguely, in collaboration with Daniel Spoerri and

Bernhard Luginbuhl, proposed in 1970 their Dynamic Labyrinth which was a monumental scafold of moving parts. With Luginbuhl, Tinguely began working on his most ambitious project the Gigantoleum which was to feature among almost every nonfunctional activity a tactile labyrinth.

Julio Le Parc with his insistence of linstabilite (like Tinguely's assertion that movement is static because movement always is), sought to create a work of art where the permanent form was impossible to discern. By himself, Le Parc created Kinetic Light Sculptures. The Groupe de Recherche d'Art Visuel (GRAV), of which Le Parc was a member, wanted to actively involve the spectator with the surrounding elements of an installation. Towards this end, they did a series of Labyrinths, the first in 1963 in Paris. A network of passageways contained flashing light boxes that would be encountered along the way.

In 1968, a show titled the "Magic Theater", at the Nelson Gallery, in Kansas City, exhibited several works that agressively incorporated temporal unfolding of combinations and the interaction of spectators. Prime examples are James Seawright's Electronic Peristyle and Terry Riley's Time-Lag Accumulator. In Seawright's work, spectators, by moving, activate lights sound, and air patterns. Terry Riley used tape delay to replay instantly the sounds made by the visitors to his network of glass doors. This was a labyrinth of sound existing only by the interaction of the spectators.

Such works as these show an interest in combinations, in change, in l'instabilite.

The work of art is a learning environment where the artist defines a set of possibilities which are left to be explored by the spectator. There is no one correct point of view; there are many, but they all yield the same experience.

## PAINTING

## AND

PATHS

All art aspires to the condition of Music. Walter Pater, The Renaissance. Studies in Art and Poetry 1908

The $17^{\text {th }}$ century French critic and poet Nicolas Boileau-Despreaux argues that the aesthethic rules of the Methode Classique are morally obligatory because they're built upon the foundation of raison and bon sens. In L'Academie des Beaux Arts the moral and didactic Grand Gout dominated French academic painting from the time of Nicolas Poussin, its first great proponent, to the end of the $19^{\text {th }}$ century. So in the period of Corneille and Descartes, rationalism prescribed that painting deal with clarity, precision, and order. Precision of drawing, exemplified by the line and linear abstraction embodied the moral, lawful, and general. It evokes Winkelmann's favorite phrase "noble simplicity and calm grandeur,": an appeal to the intellect and not to the eye. In American sculpture of the late 1960's Sol LeWitt's white lattices and Carl Andre's metal tiles exhibited similiar tendencies. They demonstrated the self-righteousness and moral imperative of rationalism. Such reductive works made claim to a direct and pure perception of formal properties, intolerant of distracting details and theories of expression.

There is, of course, an equally visible and opposite impulse where the surface is painterly and line is subdued. Here, form is curvilinear rather than geometric or architectonic. It is intuitive and emotional rather than intellectual exalting the spontaneous and irrational over the logic and calculation of classicism. Alfred Barr points to the two main currents of abstract art:

Apollo, Pythagoras, and Descartes watch over the Cezanne cubist, geometrical tradition; Dionysus (an Asiatic God, Plotinus and Rousseau over the Gauguin-Expressionist-non-geometrical line.

$$
\operatorname{Barr}(5, \mathrm{p} .19)
$$

In Poussin's Landscape with the Burial of Phocion (1648, The Louvre) the space recedes step by step from foreground to background with almost mathematical exactitude. The landscape in all its clarity and order is itself the memorial to the Stoic Greek.

Rubens combines the meticulous realism of the northern tradition with the dynamic anatomy, and dramatic, unified light and color of the South. The influence of the muscular, passionate figures of the Sistine Chapel, the articulating shafts of light of Caravaggic, the veils of glazed colors of the Venetians all are evident in his first major altar piece executed in 1609-10 for Antwerp's Cathedral. In the Raising of the Cross the organizing principle is the Baroque "river of force" creating a dynamic but unstable pyramid of straining bodies, bursting the confines of the frame.

In France, 175 years later, the era of Rococco in painting and architecture was doomed along with the fall of Louis XVI. Jacques Louis David ushered in Neoclassicism with the vengeance of the Terror. David's Oath of the Horatii eschews the color and the Rubeniste brush work of Watteau and Fragonard. The edges of forms are crisp and clear. The surface is as hard and smooth as enamel. The lighting is flat and uniform, devoid of any theatrical spotlighting. Like

Rubens Raising of the Cross, the figures are arrayed in triangular shapes but these are solid stable forms. The logic of the space is so clear that it's possible to determine the exact dimensions from the patterl of the tiles.

In the Quattrocento in Italy, Brunelleschi discovered scientific perspective. Florentine painting demonstrated a geometric structure that gave spatial clarity. In the Cinquecento, the Venetians bathed their canvases in a delicate golden aerial haze where the atmosphere softened contrast, and colors glowed with a new richness. Leonardo's chiaroscuro reveals objects and form not by outline, but by the fall of light. It implies contours obscured by the aerial perspective. Caravaggio takes Leonardo's light and focuses it into a penetrating beam defining form by dramatic selection and modelling the contours. The luminosity of Tintoretto is the reversal of Caravaggio's spot light. The light uses objects, not to reveal them, but to reveal itself. Light and shade are but manifestations of the immaterial atmosphere. In the same way the Impressionists allowed the phenomenon of light and color to dissolve the architectonic structure of objects.

The lines dissolve and regroup themselves with sudden stoppages and quick revivals, in response to the consolidation and slowing down of the modelling of the groups... Everything appears to be on the point of exploding: the solidity of the matter is in danger in all instances, physical law does not exist any more in this world of suspense. It is the same law of perspective that seems reversed in fantastic succession of spaces, almost cubist; organising above all the rhythms of the lights and of the
shadows that seem to multiply and propagate the tumult of the foreground to infinity, as in labyrinth of mirrors.

This is a description given by Luigi Coletti of Tintoretto's Massacre of the $\frac{\text { Innocents }}{\text { Venice. }}$ in the Church of St. Rocco in

Coletti (16, p.14)

What is really being discussed here is composition: the putting together of the elements of a medium, in both the design and the subsequent perception. On the one hand, the classic tradition emphasizes readily perceiveable order and the intuitive tradition emphasizes a whole that is beyond logic. This whole exists because of the presence of the paradoxes, conflicts, and contradictions.

De Chiricos' "painted dreams" that inspired Andre Breton, founder of Surrealism, take the classical vocabulary and confounds its rationality. Perspective is contradicted with the presence of multiple and conflicting vanishing points. The ground in the paintings seems to tilt upwards as in cubist works, destabilizing further the sense of orientation. Lighting and shadow criss-cross in unnatural and contradicting ways. Yet the totality of these irrationalities creates the effect: a challenge to one's sense of order, the momentary suspension of rules which immediately asserts those rules. As in play, a rule is made-up and subjected to repeated testing, which reinforces or redefines it.

In several of his paintings, Vermeer beckens the viewer to enter a depicted space by encountering a succession of objects as near to distant planes parallel to the picture plane.

Often in painting from the Lowlands there are hallways or passages with partially opened doors leading from a darkened room to a brightly lit courtyard. The light elicits curiosity to explore the space.

The Family of Philip IV (Las Meninas) 1656 by Velasquez is an achievement in the creation of a network of spatial relationships. Where as Dutch interior begins its spatial exposition in the foreground and terminates at the back wall, Velasquez provides continual branching, looping from foreground to background and back again. The spatial effect is even more remarkable considering the absence of a tile with which to measure space through scientific perspective. In its place, the organizing power is light. The distribution and relative size of the figures, the black frames of the paintings on the wall are foils for the syncopation of the spaces by the patches of light. The light is orchestrated so that light spilling in on the foreground immediately focuses attention on the Infantata and the painter himself. This light is in counterpoint to the light the background emanating from the stairway leading from the open door. Both accentuate the somber wall parallel to the picture plane. The overall two-dimensional disposition of these graphic elements is based on the Golden Section.

Even as one surveys the individual faces, the dynamic of the composition thrusts one around like a pinball hitting one bumper and then another. The painting rebounds back and forth from the outside to the inside, from the exterior to the
interior rather as Birkmayer said in a search for the fourth dimension.[Buendia (13, p. 223)] Velasquez is inside the painting, painting. The King and Queen reflected in the mirror appear to be outside observing. Some scholars maintain that the reflection is logically from the painting on the easel at which Velasquez is standing.

The observer is confronted with "a dense and palpable spatial complexity." It is a simultaneous, integral presentation of the whole and component parts. Susan Langer refers to "presentational symbolism which the mind reads in a flash" (35,P. 91), in opposition to discursive reasoning of language and the phenomena of systematic explicit reasoning. Langer agrees with the gestalt theory that the mind instantly organizes the pandemonium of data from the environment into meaningful wholes. Yet it is clear that one does not see or comprehend the details of this painting in one step. In fact, this painting bears repeated viewing without exhausting its interest. More likely, peripheral vision catches the expanse of the painting in a glance, but the details are seen more slowly deliberately.

Rudolf Arnheim notes an important differences between sequential arts like music, the theater, or dance and a picture or sculpture: "We are accustomed to say that dance occurs in time whereas the picture is outside of time." (4 p.361) For Arnheim, the difference is really one of emphasis between permanence and change. Time is a function of change. Change in the case of sculpture would mean simply this
viewpoint followed by that viewpoint. In sculpture, change is distributed simultaneously in space. The temporal order of the perception of the changes (e.g. sculpture) is not legislated by the work. In dance or music, the sequence of change of action or space is prescribed by the work. When a series of events is incomprehensible or apparently is disorganized, the sequence becomes a mere succession of elements. The crucial thing for Arnheim is that meaningful relationships are grasped. The logic of the process is comprehended independent of the temporal order. The recollection of a dance presents essentially a timeless whole, a network of changes related in a certain way.

Las Meninas contains a hierarchy that is comprehensible only when all the relationships are understood to be co-existent. Areas of the picture are scanned in succession. In order to comprehend the depicted order, the path of the eye does not literally map the structure of the painting in a one to one correspondence. Indeed the experiments of Buswell reported in How People look at Pictures Arnheim (4 p. 422) shows that there is little or no connection between the order and direction of eye fixations and the compositional structure of the painting.

An alternative to Arnheim's view holds that the meaning of sensory data is derived from precisely the sequence, and the disordered and unpredicatable offer the most information while the ordered and predictable provide the least; "the more probable the message, the less information it gives. Cliches,
for example, are less illuminating than great poems," [(Nobert Weiner quoted in Meyer (42, p. 27)] A simple minded application of these ideas to minimalist art may suggest that the severe reduction of information leads to a similar reduction of interest on the part of the viewer. However, this ignores the role of redundancy in a symbolic system; it confounds meaning with information. What is uniformly disordered is perhaps more informational, but not necessarily more comprehensible.
-27-

## PATTERN

\&

## PERCEPTION

Such symmetry is not for solitude.
Lord Byron

Josef Albers devoted a good portion of his life to the production and study of the interaction of color. Homage to the Square is one result where this minimalist tendency is evident. Albers did hundreds of variations on this simple theme: a square of one color inscribed in the square of another color. He owes obviously a debt to Malevich's Suprematist composition White on White (1918). Sometimes there could be a third square. This work presages that of Carl Andre et. al. by a good twenty years.

The readily exploited effect here is simultaneous contrast. Other subtler effects, such as the fluting effect, and figure ground relationships result as well. The perception of the effect is immediate and need not be mediated by verbal justification of the "painted word". Any one example, however, does not sustain interest beyond a few moments. What is of greater interest and demands a longer attention span is the total oeuvre. The true fascination comes from observing the scope of the permutations as one example is compared to the next.

Andre Malraux in Le Musee Imaginaire (The Museum Without Walls) asserts that Modern Art begins with Manet's Portrait of Clemenceau. In this portrait, the painting is about the means, the medium. The paint and its application are the true subject. For Albers, the subject and the means is the relation of color to itself. Its complete 'composition' is the enumeration of the color combinations and the different color relationships.

Pattern is the imposition of order, a rule of induction, a hypothesis for the discovery of deviations. The basic principle of pattern is repetition. Let an element, say a spot, be repeated at regular intervals, at a standard unit length. The effect measures the space: distances are calculable, the field has been quantified and rationalized. The undifferentiated empty paper page, the field of wheat in Kansas, the absence of sound--all are homogeneous, therefore, all are without scale, measure, or time. Introduce a series of dots, a fence with fence parts spaced at equal intervals, or a recurrent sonic event alternating with silence, this redundancy creates the suspense and expectation of continuation.

The rule generating the simple pattern on fabric, on the wall, in musical wallpaper for that matter, is nearly instantly grasped and quickly fades in to the background of awareness. The mind, expecting continuation, can turn its attention elsewhere unless change is introduced and the pulse is interrupted. Interest can be further sustained by the introduction of variations thereby increasing complexity.

Elements can be arranged according to similarity and difference. A single element (in graphics) can be manipulated by the simple operation of two dimensional transformations: translation, reflection, scaling, and rotation. The impulse of doodling can be formalized into the meander which is found in many primitive cultures as basic decorative space-filling ornament. A simple form is crenellation, like the square
wave, where a unit line is repeated through incremental translation, horizontally alternating with a 90 degree rotation (vertically).


If one takes the basic motif in isolation one can apply all of the two-dimensional transformations and generate an infinite number of combined arrangements. The reflection of this basic motif about the vertical or horizontal axis yields its mirror image. Rotation and reflection and their combinations produces symmetrical equilibrium. The perpendicular rotation of the basic motif of the meander yields the swastika.


Symmetry is an important way of finding a subset of the large number of possible combinations and provides unity in variety. J.D. Bernal, in his article 'Art and the Scientist. in Circle: International review of Constructive Act (1938), [quoted in March (38, p. 40)] demonstrates that for regular figures there are 230 symmetries in three dimension and 17 for two.

Decoration of ancient Greek vases from the Geometric Period show examples of the meander, the swastika, the rectilinear and curvilinear spiral. The configuration of earliest depictions of the Cretan labyrinth on the vases comes as no surprise.

The compulsion to pattern whether due to 'horror
vacui',' amor infiniti,' or merely the formalization of nervous doodling has been put to the service of metaphysical beliefs resulting in remarkable virtuoso displays. A simple motif can be elaborated by linking it to adjacent repetitions. By changing scale, the motifs can be nested or one framed by the other. A powerful resource is interlace where a line has the appearance of passing under and over other lines.

Few patterns can rival the Book of Kells in sheer wealth and complexity of interlace. It is the display of an exuberant imagination in the filling of every area both with the tangle of interlace and the occasional unexpected appearance of naturalistic creatures and saints.

As the Celtic Illuminators took sheer delight in virtuoso display of the interlace, Gothic Architecture demonstrates how functional demands of the pointed arch determined the style of interlocking ribbed vaulting and stone tracery of the windows. It was only in late Gothic that the dictum of "form follows function" was subordinated to nonfunctional ornaments.

Arnheim draws a distinction between ornament and a work of art. An ornament is meant to interpret visually the object of which it is part.
"It defines the rank and raison d'etre of a tool, a piece of furniture, a room etc... the ornament is part of the world in which we live, the work of art is an image of that world." Arnheim ( 4 p. 135)
Where the pointed arch makes structure clear, the Islamic Arabesque dissolves the architecture as a field of ornamental patterns. The prohibition of the icon forced the Islamic
craftsman to explore geometric patterns with such results as the Stalactite dome in the Hall of the Two Sisters in the $14^{\text {th }}$ century Alhambra in Granada, Spain. In the Mohammed Ali Mosque overlooking Cairo, the inscription "Allah is Great" in gold calligraphy is interlaced across the interior of the Great Dome. Rectilinear Kufic inscriptions with invocations to Allah are linked with swastikas, creating meandering geometric patterns. The ingenuity and amazing variety results from the united efforts of many minds over several centuries.

In late $19^{\text {th }}$ century Britain, William Morris lead the Arts and Crafts Movement through his support of the medieval tradition. He sparked a revival of interest in graphic design, domestic architecture, interior design in pattern and calligraphy. Morris was a major taste maker, advocating an art of simplicity, an "art for use," where furniture should be designed according to the nature of the materials: surface decoration should be flat rather than illusionistic. The progeny of Morris' ornament was Art Nouveau. The discovery of Japanese art coupled with the line of Morris' Stained Glass Figures unleashed those dynamic, sinuous curves that dominated decorative schemes in painting, graphics, wrought iron work, furniture, jewelery, and even fashion.

Antoni Gaudi (1852-1926) adapted this style to architecture, resulting in structures that appear to be freely modeled out of clay with the avoidance of straight lines and perpendiculars. He rigorously studied organic forms and crystals and developed a modular system based on the Golden

Section and other natural occurring proportions and geometries. Notable examples in Barcelona are the Casa Mila Apartment House and the unfinished Cathedral Sagrada Familia. The impulse to create elaborate and complex patterns is evident in Op Art of the 60's and the movement dubbed Pattern Painting of the late seventies. Op Art is hard edged and tends towards mere effects of optical paradoxes and illusions. Pattern painting in its painterly approach to decorative patterns was a rebellion against the puritan minimalism of the late sixties and earlier seventies.

Today, there is the tendancy to scrutinize pattern design as works of art. The Renaissance gave artists the luxury of separating their masterpieces from their design of jewelery, clothing, fountains.

Leonardo Da Vinci, in his rapacious curiosity, published prints on the problem of continuous interlace or knots which he proudly labelled The Academy of Leonardo Da Vinci about 1496 plate. Albrecht Durer copied and modified Leonardo's engraving in a series of woodcuts.

Michelangelo's design for the Campidoglio at the top of Capitoline Hill in Rome is like a huge outdoor room or stage set where interlacing paths set in the pavement make the equestrian monument of Marcus Aurelius the focal point of the entire scheme. Wherever one enters the piazza, the arcs of the paths always refer to the focal point from which they radiate. This looping, though quite visible from a bird's eye view, is for the pedestrian spread out and just barely contained by
peripheral vision. Like the underlying structure of a painting, it offers guidelines to the randomly accessible paths of observation. Many writers in their description of the Campidogiio stress that no series of photographs can convey the spatial dynamic of being there. It is inherent in the design to see its effect from a multiplicity of vantage points. No one point of observation predominates. This is true in general for three-dimensional objects. A person's visual comprehension of an object is based on the totality of observations from a variety of observation points. In fact, recognition of an object is aided by this accumulated storehouse. So, fragmentary information, cues the observer to recall and fill in the missing parts.

A sculpture done truly in the round has no one ideal observation point. The fascination results from each different angle of view that presents an unexpected variation in the relationship of the forms. To be at least memorable, the sculpture should have some aspects that dominate and assert themselves and reinforces the major interconnections of the concept of thing or object.

Projective illusionism and photographs assert the primacy of one aspect. In primitive art, often one aspect (e.g. the silhouette or a combination of simultaneous preferred views) represents the object as a symbol. The Egyptian canon of representation of the human figures combines clearly recognized aspects of the human figure: a silhouette of the head, frontal torso, side view of legs. The logic of such
representation shows the form as it really is, not the distorted view of perspective.

Cubism is often described as the attempt to simultaneously represent multiple viewpoints of some object. What cubist paintings really depict is two-dimensional patterns of tension created by the juxtaposition of visual contradictions, not an integrated composite of multiple viewpoints.

And, finally, how is it that we can recognize the same object from a multiplicity of views. For example, how can we talk together about the "same" house despite our private differences of sense-experience? This question has engaged the minds of many philosophers. More recently it has been approached very differently by Seymour Papert and Marvin Minsky in Perceptrons. Susan Langer's view is that different depictions or views of the same object are isomorphic: there are a set of relationships that are preserved after each transformation.

Consider a photograph, a painting, a pencil sketch, an architect's elevation drawing, and a builder's diagram, all showing the front view of one and the same house. ... each one of the very different images expresses the same relation of parts, which you have fastened on in formulating your conception of the house. Some versions show more such relations than others; they are more detailed. But those which do not show certain details at least show no others in place of these and so it may be understood that the details are left out. The things shown in the simplest diagram are all contained in the more elaborate renderings.

Langer (35, p. 69)

## THE PATH

$$
\&
$$

## THE CITY

Paths in the Boston Common, for example, caused much confusion: people were uncertain which walkways to use in order to arrive at particular destinations outside the Common. Their view of these outside destinations was blocked, and the paths of the Common failed to tie to outside paths.

Kevin Lynch, Image of the City
"Space is the Place." That is the conclusion, Sun Ra reaches at any performance of his African Jazz Arkestra. Space is experienced through movement or change in position verses time. In other words, space is a function of time. A path is movement through a sequence of space and is essentially linear.

In scanning a painting or sculpture, it was mentioned that the sequence in which the components of the composition are related is not prescribed by the work. Though a painting can be randomly accessed and randomly scanned, the limit or the threshold of comprehension occurs (according to Arnheim) when the hierarchy of the relationships are grasped as being coexistent. The work is eventually seen as a constant pattern of relations independent of the sequence of presentation. Similarly a park can be entered from many different points and possibly the layout understood over long enough traversal. The path one chooses determines how it is seen and in what order. The gigantic earth drawings of the Incas at Machu Pichu in Peru were apparently not intended for the limited viewpoint of the earth bound. Perhaps, only by means of a diagram can the patterns be perceived.

Structure, enclosure, utility, and cost limit similar random traversing of space in architecture. The reduction of possibilities, however, facilitates the discernment of the pattern. The resulting conceptual map becomes the omniscient bird's eye view, where path become pattern. In landscape architecture, architecture, or city planning, paths divide and
map movement on space. Paths prescribe and limit potential channels of movement or circulation, whether they be stairways, hallways, streets, alleys, sidewalks, canals, railroads, airline routes, etc. Futhermore, a path is goal oriented; all paths have starting points and destinations. As a mapping of distances, a network of paths is a model of the environment simplified and reduced to the paths, the intersections or nodes, the terminating points and distances. Yet the comprehension of a path is much more than this and at the same time much less. When one enters a building, there may be only one goal in mind. Once having reached the goal, one returns to the exit by the same path, with either no further interest in the other possible goals or, at the very minimum, only awareness of the path taken without any relationship to the context of the layout of the rest of the building. Of immediate interest in such a goal-oriented search is a subset of the network.

Arnheim distinguishes two sequences in linear compositions. There is the inherent sequence of events as intended by the artist which leads from the beginning of the piece to its end. The other is the perceptual sequence or what he calls the "path of disclosure." In architecture, the traversed paths are the perceptual threads that link the spaces of interest in a buildings: they are paths of discovery and of search.

The configuration of a path influences how one may perceive the organizational pattern of the spaces it links.

The path can parallel and reinforce that pattern or it can contrast with the form of the spatial organization. Pattern creates a visual rule by which one can envision from a particular sample the remaining structure. Paths organized in pattern promote what Kevin Lynch calls legibility or imageability. Pattern makes the structure clear. It is easily grasped and recognized. The mapping of a pattern of paths in our minds clarifies our orientation within the spatial layout of a building. Orientation is a necessity because a pattern of paths at their nodes presents intersections which are always points of decision making.

Ching distinguishes the following patterns configurations: (15, p. 271)

1. Linear paths are either straight or curvilinear and can form loops.

2. A Radial configuration has paths radiating from a common center.

3. A spiral configuration is a single continuous path either rectilinear or curvilinear that originates from a central point, revolving at an increasing distance.

4. The Grid configuration consists of two sets of perpendicular parallel paths that intersect at regular intervals.

5. A network consists of random paths that connect established points or modes.


Ching includes under the first category branching, but I feel it is sufficiently different, primarily because it features nodes at which choices must be made. Mathematically branching has a specific meaning. A tree structure is one type of branching that would rarely occur in the design of a building circulation. Ching adds a sixth category which is a composite configuration of the five other patterns. Ching suggests differentiating the scale, form, and length of paths, thereby creating hierarchy which will "avoid the creation of a disorienting image."

Like a building, a city is a spatial construction with a vast network of paths. The perception of city is always partial, fragmentary, occuring over a long period of time. Everyday there are different sights, sounds, people, and places to encounter, and none is experenced in isolation. The general outlines may vary slowly, but the details are in continuous change.

City design is a temporal art. But unlike music, it cannot prescribe an order to events nor impose closure of form (i.e., is having a definite beginning middle and end.)

Patterns of circulation are infinitely more complex than those of architecture: vehicular, pedestrian, subterreanean, all simultaneous systems of movement, operating in parallel and forming a wealth of interconnections. The incomprehensible complexity of a great city is awe inspiring. It stimulates curiosity and the temptation to explore and seek out new experiences but it is also terrifying. There are dangerous streets, and the scale of the city reduces the significance of the individual.

It's not unusual to become temporarily frustrated in traveling to a destination. The presence of other people, maps, signs (though absent at cross streets in Massachusetts), make it almost impossible to become completely lost. It can happen in a foreign city, however, where all signs are in an unfamiliar language. The sense of disorientation, anxiety and vulnerability can be terrifying. "The very word lost in our language means much much more than simple geographical uncertainty; it carries overtones of utter disaster."(Lynch 36 p. 4)

To be able to see one's location clearly, to be oriented is to know how to respond and act. For Lynch, the ability to organize into a meaningful image sensory data from the environment is fundamental to the efficiency and the "very survival of free-moving life". A clear understanding of the

Manhattan street grid makes it possible to go quickly to the Museum of Modern Art or the local precinct station. The structure of the neighborhood as an organizer of activities can become an affirmation of one's sense of identity, literally one's place in the world.

This is the adverse of the fear that comes with disorientation: it means that the sweet sense of home is strongest where home is not only familiar but distinctive as well.

Lynch (36 p.5)

It is axiomatic that complete confusion and disorientation are never pleasurable. "It must be granted that there is some value in mystification, labyrinth, or surprise in the environment. Lynch (36 p.5)

This is so, however, only under two conditions. First, there must be no danger of losing basic form or orientation, of never coming out. The surprise must occur in an over-all framework; the confusions must be small regions in a visible whole. Futhermore, the labyrinth or mystery must in itself have some form that can be explored and in time be apprehended.

For the human mind, the incomprehensible excites both fear and fascination. Visiting the sewers of Paris allows one to flirt with this fear of being utterly lost: to experience it vicariously. With the escort of the tour guide, one can safely probe the periphery of that vast subterreanean, wet and inhospitable world: one can smell the dank air, hear the
gurgle of the sewer, and imagine quite vividly being utterly lost, coming to dead ends, seeing a distant shaft of light from the street above, or the narrowing of tunnels as the watere rises etc. It is quite fitting that a tormented soul like the Phantom of the Opera should be familiar with such environs. The catacombs in Rome also inspire wonder and the fear of becoming lost. The network of underground cemeteries and passages is 600 miles in length and 10 to 70 feet in depth, much of which is still unexplored or blocked up.

For the early Christian, it was not an inhospitable world. Though used for burials, the catacombs were also a refuge from persecution. The wall inscriptions, the primitive altars, the didactic frescoes indicate that the catacombs affirmed the beliefs of the cult, where Christian iconography was grafted onto the pagan style. Hermes, the messenger becomes the Good Shepherd, providing safety in the maze of passages for the initiate.

The danger of becoming lost and disoriented does not happen in only these claustrophobic environments. Los Angeles was described in Lynch's research as being spread out, "formless with out center." Said one subject: "It's as if you were going somewhere for a long time, and when you got there you discovered there was nothing there, after all."[Lynch (36, p.41)]

Another source of disorientation in the city occurs when it becomes difficult to relate the different paths of movement to each other. Elevated highways remove the driver from the
-44-
context of the city. In Los Angeles freeways were felt not to be "in" the city. Subways and railways are other examples where the entrances may be at strategic nodes in the city. Yet, otherwise the network is detached from the rest of the environment, and the city is invisible. The patterning of the enviroment into a coherent image is asserted to be a crucial condition for the enjoyment and use of the city. The appeal to rationality again, supplies the aesthetic, moral force.
-45-

## SOURCES

IN

## NATURE

We watch an ant make his laborious way across a wind and wave-molded beach. He moves ahead, angles to the right to ease his climb up a steep dunelet, detours around a pebble, stops for a moment to exchange information with a compatriot. Thus he makes his weaving, halting way back home.
His horizons are very close, so that he deals with each obstacle as he comes to it; he probes for ways around or over it, without much thought for future obstacles. It is easy to trap him into deep detours.

Herbert Simon, The Sciences of the Artificial

Many of the patterns that have been discussed have their counterparts in nature. Indeed many are sources or examples of a particular kind of network or system of proportion which has been abstracted into a model. These models of course, suggest many design possibilities for networks of paths.

The spiral is one of the first forms to appear in primitive decoration and there are many examples in nature that could have inspired primordial doodlers. It's connection to the genesis of labrinths was traced earlier. Patterns of growth can be modelled by the equiangular or logarithmic spiral which is generated by a succession of similar triangles, each built upon the previous. This spiral growth pattern can be observed in the shell of the chamber Nautilus or the horns of the Big Horn Sheep.

Conch shells also exhibit the growth pattern of the Fibonacci series: $1,1,2,3,5,8,13$, etc. Each term is the sum of the two preceding terms. Another interesting feature is the ratio between two consecutive terms tends to approximate the Golden Section as the series progresses. These two series will be discussed in greater detail later.

The branching process that characterizes the growth of trees can be modelled using a recursive algorithm that produces a binary tree (i.e., a tree in which each branch sprouts two more branches.) More complicated tree structures can be found in the branching of rivers, electric discharge, corrosion or patterns of fractures in crystals of metal compounds. The circulatory system is an example of a
symmetric tree structure where the branching of the arterial system and the venous system meet in the hexagonal capillary network of the alveoli in the lungs.

A vast number of substances are formed by the combinations and permutations of a small set of chemical elements. Nature, offers innumerable other examples of form and structure generated from combinations of physical and chemical components.

Snowflakes exhibit great diversity of form. All planar snow crystals have star-like forms with six corners (or subsets). In general they are (symmetrically branching) hexagons. No two snow flakes have ever been discovered that are identical. One of the most exhaustive documentation of snow flakes is Snow Crystals by W.A. Bentley and Will Humphreys (9), published in 1931 with 2,453 different examples.

The forms of snowflakes vary from simple and bold hexagonal plates through very complex filigree-like configurations to simple and delicate star like structures. The complex filigree structures suggest design strategies where the perimeter of a maze becomes as complex as its interior.

The snow flake is a modular system where certain intrinsic physical, geometric, and chemical constraints determine form by the principle of conservation of energy. The miminum component parts/maximum diversity of combinations results from the least energy interaction with extrinsic environmental
conditions of temperature, humidity, wind, atmospheric pressure etc.

The principle of conservation of energy finds its expression in two and three dimensional structures by the phemomena of closest packing. It is a structural arrangement of inherent geometric stability that can be observed in the bees' honeycomb, polyhedral cells in biological systems, as well as in packing of spherical atoms in the molecules of certain metals.

If three circles are packed as densely as possible in the plane and their centers joined, a triangle is formed. With larger numbers six circles always surround a seventh, forming a regular hexagon. When the centers of packed hexagons are joined, an array of triangles results.


It turns out that the most economical partitioning of space with no dead areas between cells is hexagonal tessellation. This is of obvious interest to the designer of a modular structure.

The divison of two dimensional space by the packing of hexagons can be considered as a network that is a connected
set of straight lines or edges joined together at nodes or vertices. The matrix of triangles produced by connecting the centers of the hexagons is called the reciprocal or dual network. Furthermore a network which is equilateral, equiangular and which all vertices are equal, is considered a regular network.

In network theory many problems deal with finding the shortest distance or minimum length in the interconnection of random points in a plane. It can be demonstrated that the solution to this problem results in a series of line segments meeting in threes at 120 degree angles within field of the point array.


The maze solver will first have to find the appropriate maze to which this information can be applied.

> For any soap bubble array, either random or uniform, the cells will be organized according to a trianglar order More specifically, in a two-dimensional froth, cells meet in threes around each vertex and consequently always define triangles, and in a three-dimensional soap froth, the cells meet in fours around each vertex and always define tetrahedra. In soap froth there is never any exception to this rule! Pearce (47, p. 7)

Soap bubble packing, can be viewed as the model of all
systems in which the most economical packing of cellular modules is desired. The hexagonal network is evidenced in the dragon fly wing, the pattern of pigmentation on the giraffe, the cracking of surfaces such as mud, and ceramic glazes. When the tensions or forces are the same in all directions, the drying or growth pattern are hexgonal. Otherwise there is a mixture of 90 degree and 120 degree configurations. Finally the amusing triangulated radiolaria have a structure that is remarkably similar to Buckminster "I am a verb" Fuller's geodesic domes.

## GEOLOGICAL STRUCTURES

On the south side of the island of Crete at Gortyna, there is a cavern of winding passages near Mount Ida. Various authors of antiquity were of the opinion that this cavern, or one of the many caves or quarries in Crete was the site of the real Labyrith of Geek legend. It's quite natural to describe a cavern as a labyrinth. Especially so given that location is so near the ruins of Knossos where, as legend has it Daedalus built the Labyrinth. Matthews quotes the amusing description of the French botanist G.P. de Tournefort who visited the caverns of Gortyna on July 1, 1700.

This famous place is a subterranean passage in manner of a street, which by a thousand intricacies and windings, as it were by mere chance, and without the least regularity, pervades the whole cavity or inside of a little hill at the foot of Mount Ida, southwards, three miles from

Gortyna. The first thing you come at is a kind of cavern exceeding rustick and gently sloping: in this there is nothing extraordinary, but as you move forward the place is perfectly surprising nothing but turnings and crooked by-ways. The principal alley, which is less perplexing than the rest, in length 1200 paces, leads to the futher end of the Labyrinth and concludes in two large beautiful apartments, where strangers rest themselves with pleasure. Tho' this alley divides itself, at its extremity, into two or three branches, yet the dangerous part of the Labyrinth is not there, but rather at its entrance, about some thirty paces from the cavern on the left hand. If a man strikes into any other path, after he has gone a good way, he is so bewildered among a thousand twistings, twinings, Sinuosities, crinkle crankles, and turn again lanes, that he could scarce ever get out again without the utmost danger of being lost.
...this marvellous maze to serve for an Asylum is the Civil Wars or to skreen themselves from the Fury of a Tyrannical Government at present it is only a Retreat for Bats and the like.

Matthews (39, p. 24)

ANIMAL STRUCTURES

Like bees a large number of other insects as well as mammals exhibit social behavior that is centered in built dwellings. The fungus-growing termites in Indo Malaya are second only to humans as architects of the gratte ceil. The mounds of some African Macrotermes reach a hight of 25 to 30 feet fitted with pinnacles, chimmeys and ridges. Inside are honey-comb like structures on which the fungus is grown. Often these nests harbour various other invertebrates as guests (e.g. butterflies millipedes) and are termed
termitophiles. The leaf-cutters of the Western Hemisphere are noted agriculturists who build the most extensive network of underground colonies which put the catacombs to shame.

The Rhinotermitidae build nests in wood located in damp soils. From the nest a diffused network of tunnels to food sources radiate into the soil or above ground in the form of covered runways. Other termites and ants build different subterranian nests with many chambers and galleries occupied according to the basic division of labor of the caste system of workers, soldiers and the agents of reproduction. Dry wood termites of the family Kalotermitidae having no worker caste wisely make the young of the colony do the work.

A large number of the ground living rodents construct underground nests having two or more exits for escape from predators. Food storage areas in special chambers off the main passage way are a distinct advantage over the storage systems of the tree squirrel. Possessing a poor memory the squirrel forgets where it stored winter provisions in little holes in the ground within its territory.

Prairie dogs, formerly abundant in the western United States were responsible for large networks of tunnels. But the unparalleled burrower is the African mole rat or blesmols. It spends its entire life underground. The snout is especially adapted so the incisors extend forward with a fold of skin closing the mouth behind them. The burrows of a single blesmol can form a crisscross of tunnels, at several
-53-
levels below ground surface that may occupy an area of over thousand or more square meters.

From the microcosmic to the macrocosmic level, living and inanimate forms exhibit regularity of organization, circulatory networks, tunnels, and a variety of geometric patterns. All invite analogy, comparison or even inspiration for design. It is more important to seek out the general patterns, to make abstractions, and observe the underlying geometry than to compile a inpenetrable catalogue of examples.

## GARDENS

## PLAY

\&
OTHER
CURIOSITIES

Geometry came alive when we connected it to its precursors in the most fundamental human experience: the experience of one's body in space. Seymour Papert (45, p. 154)

In the following pages several different built forms will be examined. Their common features and distinct qualities will illustrate the aesthetic potential of a labyrinth design and take it beyond the level of a puzzle.

Since the Egyptians first laid out gardens on the banks of the Nile, the gardens of the Western World have been based on straight lines and rectangles. The formal grid pattern of trees and flowers embodied man's desire to impose order and symmetry on nature; man yearned to dominate and contain the irrational in nature. Versailles is exemplary. Built by Louis XIV it is the image of the fixed, permanent power of the sovereign.

In China, the doctrine of the Taoists proposed a completely different view of man's relationship with nature. The Taoists believed in the fundamental unity of all things and that man was an inseparable part of the universe. Instead of subduing the world, they sought harmony with it.

In 1749 , the first complete description of a Chinese garden was published in Paris. It was written by Pere Attiret, a Jesuit, employed as a painter by the Chiien-lung Emperor. This letter gradually started a revolution in garden design. In fact, in England Alexander Pope declared that one must consult the "Genius of the Place," before even starting a garden.

Maggie Keswick, in The Chinese Garden, creates a sense of the aesthetic of the Chinese garden in her description of the Wang Shih Yuan garden in Suchow:

The door is very ordinary. Once inside,
the visitor finds himself in a simple white corridor open to the sky, twenty feet or so in length, with a blank wall at the far end. It is first like the alleyway outside only painted white and more elegant in its proportions. Serving as a kind of decompression chamber, this passageway separates the noises of the city outside from the enclosed world of the garden towards which the visitor now progresses. The whole garden is in a sense a composition of courtyards. Some wind round corners out of sight. Others are half open-ended. Some are cut off like cul-de sacs, or fit into each other like pieces of a puzzle.

It is as if the designer were constantly holding back, enclosing each view, yet always suggesting new delights just beyond the further wall. Indeed, the succession of different courtyards has been building up an increasing sense of anticipation as our visitor begins to feel, however unconsciously, the rhythm of the design. By the time our visitor finally gets to the pavilion over the water, he has seen it from several different vantage points, while often it has been completely hidden.

Here he will at last feel that he has reached the farthest point. There is no way on, only back--through the main hall, along a galley, left through the 'Pavilion of the Accumulated Void' out into the courtyards to the north of the library, and finally back to the entrance, narrow, white alley leading to the world outside.

At the end, the visitor will have no more idea of the plan of the garden than he did when he started.

Keswick (33, pp. 18-21)

A Chinese garden should ultimately be a labyrinth, but should not obviously look like one. The Jesuits built a

Western styled maze for the Ch'ien-King Emperor, and the Chinese considered it a barbarian novelty. The maze with its regularity and blatant dead ends went against the sense of a Chinese Garden, which should always seem spontaneous and
uncontrived.
The Chinese Garden is a microcosm of a world in dynamic equilibrium like the Ying Yang symbol of the I-Ching. It is a peaceful place designed for contemplation, the writing of poetry, practicing calligraphy, exchanging the conversation of the cultivated scholar. The asymetrical distribution, and the variety of detail make for an environment that is somewhat the inverse of gestalt theory; the summation of the multiplicity of views approaching the infinite is larger than the whole.

Playground design of the last decade reflects a similar desire to maximize diversity and opportunities of play: to do more with less. The large, wooden framework of interconnected platforms, ranges, tunnels, bridges, ladders, matrices of tires, macrame nets, and slides offer mutiple point of access and exit.

I think that playgrounds should be renamed research environments! This is what the children are doing so vigorously. They are not playing, they are finding out how the universe works. Playgrounds provide children with experience fortified gratification of physical research. Thus, their intuitive assumptions of can do are proven: they are thereafter confident of their own capabilities for sensing and employing the principle operative in nature. (Buckminister Fuller) Quoted in Hewes (27, p. 1)

As children are growing, they are engaged in a process of self-discovery. They explore the potential of their physical selves in relation to the environment. Such environments promote spontaneous play, involving the whole body. The child moves rapidly through space, testing and developing
skill, strength, balance, coordination and bodily awareness. Play is the testing of a hypothesis of how things work. This learning is twofold according to Piaget. The child learns both from experience and must have cxperience to learn. These playgrounds have a diversity of textures, shapes, and materials that provide a rich and varied source of environmental stimuli. Research has shown this diversity to be necessary for the full development of perceptual ability.

The play structure could be described as a stairway with different activites at each step or a corridor of movement with various events along it. The most fundamental organization is a linear progression. However, such a design gives a child little choice, but to go straight through, get off, and start again. Greater potential is achieved from a parallel design, which allows interconnections in different directions. Circular, spiral, staggered, or zigzag layouts of the posts and platforms create loops of activity with more access points.

John Hiuzinga postulates one of the more famous theories of play. In Homo Ludens, (27, p. 70), he states that a "civilization arises and unfolds in and as play." Play is not merely an element of culture, but is fundamental and prior to culture. Hiuzinga indicates that play has many characteristics of religious ritual: it is often done in seriousness; it occurs in a separate and distinct place set apart by real or imagined boundaries. And as in religion where spiritual abstractions become reality, play immerses its
participants in a make-believe world.
The M.I.T. Department of Architecture has built a number of ad hoc design studios for its students. Each consists of a private work area located on multi-level, randomly interconnected platforms. These platforms are accessible by stairs, steps, ladders, or from the floor level. Huge timbers lay side by side with $2 x 4 s$ or metal pipes jutting through cement block walls. Windows and doors from demolished buildings form partitions of territoriality, yet allow access, visibility, and communication among the students. It is as if a community playground, growing both horizontally and vertically, were housed on the third floor of M.I.T.'s building 7 .

A certain attitude is encouraged here: the available material fits the need. Its expansive, rambling, but pragmatic, sense of design is unconstricted by fashion or style. Like a multicursal maze, these studios present an intriguing, unpredictable division of space. In having no discernable plan or regular pattern, they are at once fascinating and mysterious.

The unicursal labyrinth, on the other hand, derives its sense of intrigue from its observed goal. However, there is something more to it. Traveling to the center of a unicursal labyrinth and back again necessitates a 180 degree change in directions. Symbolically, it represents complete disassociation from the past, an about face that redefines one's place in the world. This change causes one not only to
abandon previous experience (i.e. the view point of the way one entered), but also to assume a new understanding through exiting from the labyrinth and re-entering into the world. For many cultures, the spiral and the unicursal labyrinth were maps of the underworld, and symbolic of death. The movement to the center and the return indicate death and rebirth: damnation and salvation.

In Meford, Massachusetts, a large wooden tower illustrates this process of transformation. But here it is one of perspective and not of spiritual belief. When one first notices the tower, one is struck with surpise. Why is a lookout tower located in a treeless marsh, bordered by a highway?

One naturally investigates the tower to satisfy curiosity. However, as one approaches, one becomes more intrigued by the tower itself. One cannot help but notice the care taken in the design and construction. Every edge and surface is beveled and finished. Corners are fitted, bracketed, and bolted. Beams and planks are at least twice the thickness required. The entire structure stands on cement moorings and rises three stories.

Upon reaching the top, one discovers there is a view after all. The distant skyline of Boston, punctuated by the Prudential and Hancock buildings, assumes a different character than when seen from the ground. The river, invisible at ground level stretches into the distance. The silhouettes of the buildings blend into the shoreline, fused
together by the aerial perspective of the dense haze. But the real surprise is to see how children have trampeled a network of paths through the 6 foot tall reeds below and are engaged in hide-and-seek or simply running down the paths for the pure pleasure of it. The exploration of this unicursal, vertical labyrinth easily represents, like rites of passage, the transformation of viewpoint from limited perspective to overview. Though the return path is the same, the conception of the way things are has changed.

The multicursal maze of the city presents a real opportunity to becoming disoriented if not hopelessly lost. An architectural maze is less formidable because it is a finite structure. Yet there are individuals whose monomanias have created seemingly endless constructions.

In Woodstock, New York, Clarence Schmidt constructed his obsession the "House of Mirrors." Built of discarded lumber, old windows, and door frames, it became a jumble of windows, doors, and floors reaching seven stories. The interior was covered with aluminum paint and foil (thus the "House of Mirrors"). This structure burnt down and so too did its successor Mark II. Had they stood, Clarence Schmidt might have gone on with his additions.

Sarah L. Winchester, the heiress to the Winchester Rifle fortune, believed it was necessary to add continuously to her house in San Jose, California new rooms in order to confuse the ghosts of the victims of the Winchester Rifle. For a period of 38 years, workmen built with out interruption so
that at the time of her death in 1922 there were some 160 rooms. Originally the house had nine rooms, and all the subsequent additions were designed to confuse and trap the ghosts. Like a surrealistic painting, the network of blank walls behind doors, doors opening into voids, stairways leading to the ceiling challenges ones common sense view of the world.
-63-

## PATHS

\&

MUSIC

Or say that the end precedes the beginning, and the end and the beginning were always there, Before the beginning, and after the end. And all is always now.
T.S. Eliot, Four Quartets

Repetition and change through variation is fundamental to the generation of pattern in the 'presentational' form of graphic design or the temporal unfolding of architecture and music. Change-ringing combines the sheer pleasure of generating variety through permutations with a trance inducing repetition, which requires a great deal of concentration for the performer. Like graphic pattern, it fills the void, articulating time by repetition. The repetition creates a continuous texture, seemingly without beginning or end. A set of bells with fixed pitches is chosen. The basic principle involved in ringing changes on these bells is called the plain hunt. The 'hunt' can be described as the path a bell takes among the other bells in the list of the permutations. The following example gives the listing for three, four, and six bells:

Bell


The number of 'changes' obtainable for a given number of bells is the factorial of the total number of bells $3!=6,4!=24$, $6!=720$, and $12!=479,001,600$.

Because of this repetition the listener can access a change randomly. Like looking at the corner of a wallpaper,
the organizing principle is instantly grasped and is extrapolated to the remaining wall. The stroller on Michelangelo's Campidoglio from any vantage point, sees a view of the pattern which affirms the organizing principle. So the listener, can follow a change at any moment and the same order will be perceived. The multiplicity of changes or the multiplicity of views (i.e., the iteration of the combinations point to the nexus of relations) is the perceiveable order.

For the composer Arnold Schoenberg (1874-1951), the notion of Grundegestalt or basic shape was fundamental to the generation of a composition. He postulated a kind of genetic code which would determine both the microscopic relationship of pitches and the macro-structure of the entire composition. In the 1950's these ideas became codified into the dogma of serial music, where a strictly ordered sequence of notes can be transformed into different combinations and permutations according to fixed rules.

Much like the graphic transformations of reflection, rotation, the serial rules of inversion, retrograde, and retrograde-inversion offer a powerful technique for generating theoretical unity in variety. Every sequence of notes in a serial composition should be reducible by these rules to the basic sequence.

The composer can use these rules to generate from the initial sequence of 12 notes a 'precompositional' matrix of possibilities. In variations of the 12-tone theory, smaller groups of notes are used as the basic units of organization.

Like the 'hunt' of change ringing, the composition can be considered a path moving through this matrix of possibilities, and any path is as good as any other.

However, serial music allows for a far greater complexity in numbers of combinations of pitches. One can also seralize transformations of register, rhythm, tempo, orchestration, and timbre. In the simple repetition of change-ringing, the order of the pitches is readily comprehensible, since all other factors are constant.

Serial music ignores the traditional organizing power of tonality and the hierarchy of tones around which they gravitate. In its place, serial music substitutes the order of the tone row.

Though in serial music, the piece may be totally ordered for the composer, the listener, may find its complexity arbitrary and incomprehensible.

The enormous complexity prevents the audience from following the intertwining of the lines and has as a macroscopic effect the irrational and fortuitous dispersion of sounds over the whole extent of the sonic spectrum. ...When linear combinations no longer operate, what will count will be the statistical mean of isolated states and of transformations at any given moment. The macroscopic effect can be controlled by the mean of the movements of the elements we select. The result is the introduction of the notion of probability, which implies the combinatorial calculus.

Iannis Xenakis Formalized Music

Implicit in the use of stochastic compositional procedures is the recognition of utility of systematizing the variables of
music and that order as such is an arbitrary result of the embedded intentions of the composer.

The use of stochastic control structures simplifies the management of large amounts of data. A composer can utilize the large number of combinations that can be generated by a computer. On a small scale the fortuitous and the unimagined can be directly incorporated into the compositional structure, while the overall structure is determined more directly by the composer.

VARIATIONS, COMBINATIONS, AND PLAY

In Music, Mind and Meaning, Marvin Minsky (1981) makes the same observation as Buckminster Fuller: play is learning. A child playing with blocks, arranges them in a variety of configurations. By trying different variations and combinations, and by noticing differences and making changes, a child is learning about space.

By slowly walking around a sculpture slight shifts of viewpoint are the procedure of spatial learning. The new juxtaposed with the familiar is comprehensible by its difference. Repetition and change by variation is the procedure of learning and play. By combining the known into new combinations, a new structure is created which can be understood in terms of the combination of its components.

The search for unity in variety of pattern or the

Grundegestalt of serial music is really a procedure for learning about space and time.

> Many adults retain that play-like fascination with making large structures out of smaller things-and one way to understand music involves building large mind-structures out of smaller music-things. so that drive to build music-structures might be the same one that makes us try to understand the world.

Marvin Minsky, Mind, Music and Meaning (55, p. 5)

An interactive sonic labyrinth is a self-contained world for building a large mind-structure of space and time. It is done incrementally. The cycles of paths, the changing views, the delight in the unexpected contained in a comprehensible whole is the process of learning. The desire to make the world comprehensible is the motivation of repetition and variation in the design of the labyrinth and in its perception.

## LITERATURE

## \&

## METAPHOR

"Well," said the priest, reaching out his hand to K., "then go." "But I can't find my way alone in this darkness." said K. "Turn left to the wall, "said the priest, "then follow the wall without leaving it and you'll come to a door."

The word labyrinth used as a metaphor refers to a difficult or confusing situation. It is closely related to the literary motif of the maze as a system of tangled paths that lead to dead ends and confusion. In general, maze has the connotation of the multicursal network while labyrinth can mean this and more.

Employed both literally and metaphorically, labyrinth and maze can be used blatantly or with some subtlety of association (e.g. Labyrinths of Iron: A History of the Subways; Labyrinth of Solitude: Life and thought in Mexico; Labyrinths, Selected Stories and other Writings of Jorge Luis Borges. ${ }^{1}$ The term can be also used as a concept of a process, as in Arnold Pacey's Maze of Ingenuity: Ideas and Idealism in the Development of Technology. ${ }^{2}$

The word itself is a tangle of associations. Scholars suggest that labyrinth is derived from labrys, meaning "the place of double axes." The Palace of Minos in Knossos on Crete is adorned with many examples of the symbol of the double ax as well as the famous frescos of bull vaulting. According to legend, this is where Daedalus built the labyrinth for the Minotaur.

Other writers speculate that the labyrinth was never an actual construction. Rather the myth was commemorated by a ritual dance depicted on vases which traced the spiral of the labyrinth. It is thought that the Troy Dance was performed When a city was founded to trace symbolically the walls of the new city.
...the Labyrinth was part of founding a town, cutting off a portion of space and transforming it from chaos into cosmos. The windings which laid the foundation also protected it from entry of all but those with knowledge, the knowledge of the way.

Jill Purce quoted by Bord (10, p. 83)

Stone and Turf mazes in some locales were called Troy Town, walls of Troy, Trojeborg or even Jericho. Indeed, the Welsh troi means 'to turn, to revolve' as does the Celtic root 'tro.' In the Middle Ages, labyrinths became the 'Chemin de Jerusalem' representing the pilgrimage to the Holy Land and Salvation.

But perhaps the richest association with labyrinths arises form its contrasted metaphoric use in both Dante's Divine Comedy and Kafka's two major novels The Castle and The Trial. Donald Pearce, in his article titled "The Castle: Kafkas's Divine Comedy," says:

Dante's path, in the Inferno and the Purgatorio... leads through a rational and conscious world in which the difficulties are moral ones, ...his task virtuous conduct. He can provide ...a guide (reason in the person of Virgil) because the world ...is subject to logical necessity. But Kafka's path leads into an irrational and unconscious world where the difficulties ...are fortuitous, his problem is thus not ethical but esthetic, his task suffering, endurance.

Flores (19, p. 167)

Here again, as in the previous discussion of the Rubenistes and the Poussinistes, the rational and the irrational become two opposing artistic representations of the
world. The rational world of Dante offers a single path of salvation with a logical passage of sin, death, and rebirth through life after death.

In contrast, Kafka's world is a confusing maze of pathways and dead ends. The protagonist is lost in an irrational world, where every path leads only to another and never to the goal. He seeks integration of his experiences, whereby he may know himself. Dante, on the other hand, seeks the omniscience of God so that in the flash of revelation, he sees the order of the universe.

1. Bolnick, D. Ed. Labyrinths of Iron: A History of the Subway, Newsweek New York, 1981.
Borges, Jorge Luis Labyrinths, Selected Stories and other writings of Jorge Luis Borges
New Directions Norton, New York, 1969.
Paz, Octavio $\quad \frac{\text { Labyrinth of Solitude: Life \& Thought }}{\text { in Mexico Grove, New York, 1962. }}$
2. Pacey, Arnold

## MATH

## \&

## MODULES

We start with the simplest vocabulary of images, with "left" and "right" and "one, two, three," and before we know how it happened the words and the numbers have conspired to make a match with nature: we catch in them the pattern of mind and matter as one."

Jacob Bronowski, The Reach of Imagination

The design of maze lends itself rather easily to imposition of a pattern. The symmetries which are produced in a geometric design simplify the co-ordination of the elements. The use of a modular system simplifies the actual physical layout, joining of parts, cutting of materials, and basic measurements. The modularity can indicate the underlying grid from which one can measure distance, thereby building expectation: in short allow for a rational foothold.

Kevin Lynch argues that indentifying and structuring cues from the external world into a coherent pattern is a vital ability for the efficiency and survival of "free moving life". The repetition of elements create rhythms that are quickly recognizable and easily organized into a pattern. The attributes of the imagining of the cityscape are clarity and legibility of the allover pattern. Once grasped, the pattern is a framework for choice, action and perhaps aesthetic enjoyment. For the designer whether artist, architect, or mathematician, there can be immense aesthetic satisfaction in discovering or inventing an underlying pattern out of obstensibly disordered and unrelated phenomena. A good patterning will make explicit the structure and rather than restricting thought or action it will on the contrary permit purposeful and articulated movement and speculation.

The advent of mass production of building products has led to the standardization of a consistent set of relationships between parts. Modular systems allow for a flexibility of arrangement and fitting in various combinations. A system
based on multiples of a single modular size give a notational grid underlying a design. The organizing power of the grid establishes a clearly legible pattern which is marked by a constant set or field of reference of points, lines, and volumes.

The unit measure of the module is affected by such factors as material sizes, structural characteristics, transporation, construction and dimensions of the human body. Once a preferred dimension is selected, then larger components can be generated by taking all multiples of the basic module. The term Bemis Set is used to describe all those dimensions x such that x is some positive integer multiple of the basic modular dimension. Because the Bemis set is closed under addition it is always possible to match some larger size components. In fact, there may be established a hierarchy of modules at different scales, all which are related proportionally. It's quite possible to use two different basic modular sizes, each used on a different axis to generate a much larger set of different modular sizes. The determination of this ratio creates a proportional system which establishes a consistent set of relationships between the elements. An example of this is the Blue and Red series of Le Corbusier's Modular.
P.H. Scholfield in his book The Theory of Proportion in Architecture expresses the view that what is common among most proportional architectural systems is an attempt,
at the creation of order apparent to the eye by the repetition of similiar figures, and that this is accompanied by the generation of patterns of relationships of mathematical proportion between the linear dimensions of the design.

Quoted in March (38, p.222)
What distinguishes modular co-ordination from many of the historical proportional systems is that the former is concerned with the utility for implementation and the latter is concerned with the aesthetic effect embedded in a metaphysical system.

With the discovery of the simple consonant ratios of the mono chord (e.g., 1:2 1:3 2:3 3:4) the Pythagoreans believed they found the ratios of the mysterious harmony that pervaded the universe: the so called "Harmony of the Spheres". The architects of the Renaissance revived the Greek proportional systems with the suggestion that what was pleasing to the ear would in reflecting the order of the universe, would be pleasing to the eye.

Andrea Palladio, one of the most influential architects of the Italian Renaissance employed an esentially additive precedure: beginning first with ideal whole number ratios for individual rooms (e.g., l:l (square), 1:2 (doublesquare)), then he would combine these as needed, so that the overall plan itself was somewhat a fortuitous shape. Earlier architects worked the other way around starting with the overall plan and subdividing it according to the ideal commensurable ratios. Such additive or successive subdividing
procedures of the whole tended to give rise to geometric progressions.

Beauty will result from the form and correspondence of the whole, with respect to the several parts, of the parts with regard to each other, and of these again to the whole: that the structure may appear an entire and complete body, wherein each member agrees with the other, and all necessary to compose what you intend to form.

Andrea Palladio, The Four Books on Architecture, Book I, Chapter 1.
Quoted in Ching (15, p.315)

When the diagonals of the two different rectangles are either parallel or perpendicular to one another, they indicate that the two rectangles are similiar. In fact, since the ratio of two successive terms in a geometric series is the same the similiar rectangles are part of that series. The art historian Heinrich Wolfflin has demonstrated the conscious use of geometric systems in paintings and architecture of the Renaissance. Through using a net of diagonal lines or what Le Corbuseir calls the trace' regulateur (regulating lines), the analysis reveals the repetition of similiar rectangles in the design's underlying structure.


The regulating line brings in this tangible form of mathematics which gives the reassuring perception of order. The choice of a regulating line fixes the fundamental geometry of the work...Le Corbuseir, Towards a New Architecture.

Quoted in Ching (15, p.304)

The Golden number $\rho$, the basis for the 'golden section' and the 'golden rectangle' is another irrational number. Its use dates back to antiquity as a proportional system for the human body (e.g. the sculptor Polyclitus) and continues to be used today surrounded by a certain mystique. It is the solution to the equation $x^{2}-x-1=0$. Its approximate value is 1.618 and is expressed by the ratio: $A: B=B: A+B$


From this a rectangle can be formed and from this a series of nested 'Golden Rectangles' can be generated producing a pattern like whirling squares. The conch shell exhibited this pattern and the inscribed spiral certainly has a similiarity to the unicursal spiral labyrinth. The way the golden rectangles nest in a pattern is an example of the Fibonacci Series where each successive term is the sum of the preceding two... $1+1=2, \quad 2+1=3$ etc.

In 1948 Le Corbusier published the Modular: A Harmonius Measure to Human Scale Universally Applicable to Architecture and Mechanics. Le Corbusier based his system on two Fibonacci series: one called the Red uses as the key dimension the
height of "those good looking men, such as policemen, in English detective novels, who are always six feet tall." The Blue series, the trademark of the Modular, is based on the height a man can reach with his arm stretched out. The series are related in such a way that each of the Blue sizes is twice some corresponding Red dimension. The Modular will acording to Le Corbusier,

> maintain the human scale everywhere and lends itself to an infinity of combinations: it insures unity with diversity... the miracle of numbers.

Quoted in Ching (15 p. 317)
The question remains whether or not a proportional system is inherently pleasing to the eye. Pyschological experiments have shown that people have been unable to distinguish between a golden rectangle and one that differs by $3 \%$ or $4 \%$. Much like appeal to the actual inherent virtue of different musical scales the claim to innate appeal of golden section is tenuous.

Christopher Alexander in an article entitled Perception and Modular Coordination, suggests, a proportional system may be appreciated visually by the way the system limits the number of possible dimensions: that the use of "repetition of similar shapes, lengths, simple whole number ratios-we must, in abroad sense, call 'ordered'." Quoted in March (38, p. 238)

The repetition of some element gives the definition of measurement a method of comparison whether it is human scale, or scaled to other forms e.g., the spires of a cathredral. Where one can reach out a touch, one can measure. To remove this is to force one to rely on visual clues rather than tactile cues to get a sense of the scale of a space. Elements such as doors or windows are scaled to human dimensions and provide orientation not only in scale but in direction and movement.

In designing and in solving a maze, one looks for a rule that generates the pattern. For example one might be interested in maximizing the length of the path the solver might take, where the solver would wish to minimizing the journey. Both the designer and the solver are interested in determining the length of that journey or in the conditional probabilites of getting there from here.

A graph can show the essential structure of a set of relationships. An early puzzle proven to have no solution by Euler related to circulation and city planning is the Konigsberg bridge problem. The problem arises from the layout of the city Konigsberg (now Kaliningrad) in Prussia. The river Nagel flows through center of the city and where it forks is located a island. Here the different parts of the city are joined by seven bridges. The problem is this: Is it possible to walk through the city starting from any point and, crossing all the bridges, each only once, to arrive back home at the same place? Quoted March (38, p.242)


In order to arrive back home at the starting point one must arrive in each vertex of the city ( $A, B, C, D$ ) the same number of times as one leaves it. The question is if it is possible that one can cross each bridge only once or equivalently to traverse each edge of the graph once. So in order for there to exist a possible route, the number of edges joined or incident with each vertex must be even. None of the four vertices have an ever number of edges so the problem has no solution.

This type of graph shows clearly the connections between the points of interest. Contrast this with a U.S. Geological Survey Map which aims at a one to one correspondence with the abundant detail of the environment. So much so that the reading is difficult. With chosen restrictions graphs are a very powerful means to show structure (e.g., of decision making or a hierarchy of an organization, electrical networks, or stochastic networks.) In graphic design and city planning and layout, the adjacency graph is useful in revealing unsuspected implications of a plan.

An adjacency problem of significance to architects, cartographers and mazeographers is the problem of contiguous regions. For example, consider the problem of designing a
maze, where you have a modular cubicle and you want the floor areas of 5 cubicles to be adjacent to one another.

Unfortunately four is the largest number of areas all of which can be mutually adjacent. By taking tinc dual of the graph it can be shown that each vertex can be connected to all the others without intersection. The graph on five vertices shows that it is impossible to connect all the edges without intersecting one another.

W.H. Matthews in Mazes and Labyrinths mentions the straight line diagram, where the paths of a labyrinth are represented by straight lines, with branches drawn perpendicular to the main axis. The main axis represents the shortest path from the entrance to the goal. The diagram ignores curves and changes in direction of the actual path. What Matthews calls a unicursal labyrinth is represented by a simple straight line. Below is a straight line diagram of the Hampton Court Maze.

-Maze at Hampton Court. Plan. (W. H. M.)

In general, a network signifies a directed graph in which each arc is assigned a numerical value. This definition should be compared with Ching's definition of a network as random paths that connect points in space. (15 p. 271) Such graphs provides an ideal model for system of routes, whether they be road maps, patterns of circulation in buildings, volume of traffic flows or the random wanderings of a mouse in a maze.

A special type of graph is a tree which itself can be defined as a connected graph with no cycles. A connected graph is a graph in which each vertex is joined to all others by chains - i.e. consecutive series of linked edges. A tree with $n$ vertices has $(n-1)$ edges. (e.g.,! $\mathbf{2}^{2} 2-1=1$ edge.) A family tree is literally a tree graph with a one to many mapping. Similarily a hierarchy of command or a causal chain of events like successive locations in a maze can be represented by a tree.


This source information can also be represented by a zeroone matrix where a 1 in the $i^{\text {th }}$ row, $j^{\text {th }}$ column signifies the event to which the row corresponds, which is the immediate predecessor of the event to which the column corresponds. When there is no such correspondence, a zero is entered.
$\left.\quad \begin{array}{llll}a & e & c & d \\ \text { a } \\ \mathrm{e} \\ \mathrm{c} \\ \mathrm{d} & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]$

When multiplied by itself this resulting matrix will reveal the predecessor one step removed.
$\left.\left.\begin{array}{cccc}a & e & c & d \\ a \\ e \\ c \\ d & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right] \quad \begin{array}{cccc}a & e & c & d \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]$

By using this technique, successive powers of the matrix of a family tree will show parents, grandparents, etc.

It is assumed in circulation studies that the subject always chose the shortest routes. A type of maze might seek to maximize the distance traveled. The procedure which determines successively the shortest routes will eventually compile the longest.

A zero-one matrix of the kind described earlier can be used to find the shortest routes between vertices. With a given matrix $G$ the element $g_{i j}$ takes the value of 1 to represent the presence of an edge in the network joining the two vertices $i$ and $j$. If there is no edge joining then the entry is zero.


Let there be a mouse in a maze. This bears some relationship to the final maze design in as far as the cubicle $C$ represents the hallway of Building $\mathrm{W}-11$. The adjacency graph looks like this:


The entries in the table of shortest values show for example that the shortest distance between $b, a$ is one while the matrix $G$ shows how many occurrences there are. The Matrix $G$ lists below the number of occurrences of the distances of 1 edge.

G
$a$
$b$
$c$
$d$
$d$
$d$$\left[\begin{array}{lllll}a & b & c & d & e \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0\end{array}\right]$
The corresponding en-
tries in the table
of shortest distances
are:

$$
\left.\begin{array}{lllll} 
\\
& \begin{array}{c}
a \\
a \\
b
\end{array} & b & c & d
\end{array}\right]
$$

This technique follows the Prihar Algorithm. Then by taking the square of matrix $G$ one finds the number of different chains, two edges long, which join a pair of vertices.
$G^{2}$


Table of Shortest Distances
The element $\left[g_{2} c\right.$ a] has value $1:$ i.e. there is one two-edged chain from $C$ to $A(c, b, a)$. Between $e$ and a there are two, either ( $e, b, a$ ) or ( $e, d, a)$ therefore $\left[g_{2} e a\right]$ takes the value 2. Because those chains contain the same edge more than once they are termed "composite chains". The diagonal of the matrix represents the number of two edged chains from a vertex to itself. e.g., $\left[g_{2} a, a\right]$ equals 2 , meaning ( $a, b, a$ ) or ( $a, d, a$ ). Taking the cube $G^{3}$ of the original matrix yields the 3-edged chains between vertices. Only one element $\left[g_{3} d c\right]$ has value >0 for which the corresponding element $\left[g_{1} d c\right]$ and $\left[g_{2} d c\right]$ in $G$ and $G 2$ were both zero. So $d$ and $\underline{c}$ are the vertices left not joined by a one edged chain or a two edged chain. Therefore, the shortest distance between $c$ and $d$ is a 3 edged chain.


Table of Shortest Values
This procedure of taking increasing powers of the original matrix could be used with larger networks. When all elements take on non-zero values the shortest distance table would be complete.

## RANDOM WALKS

He calmly rode on leaving it to his horse's discretion to go which way it pleased, firmly believing that in this consisted the very essence of adventures. Don Quixote vol. I chapter 2 Religious Observers

The game is "How do I get there form here?" Perhaps the getting there is what it is all about. Certainly one aspect of the interest in mazes is it's a problem to solved. There also is another which is a delight in following the mysterious path, with unexpected turns and dead ends.

Martin Gardner, in an introduction to random walks cites Jorge Luis Borges' essay A New Refutation of Time where the author describes a random walk through the streets of Barracas:

[^0]Gardner also reports of G.K. Chesterton's description of his second honeymoon as a "random journey into the void" where he and his wife took a passing bus, left it when they came to a railroad station, took the first train wherever it went and got off at the end of the line and randomly walked til they found a hotel. [Gardner (23 p. 66)]

A random walk is an example of a Markov chain where roughly speaking the future location of the stroller depends on his present location, but not his past locations. [Madsen (37, p.14)].

The properties of Markov chains can be used to not only characterize such movement, but also these principles can be applied to the design process. The proposed interactive labyrinth, in which everytime a door is opened the configuration of the locked and unlocked doors change, seeks to imbed the Markov property into the design. The very act of solving it, causes the configuration to change in a probabilistic way and therefore the solver, suffering from this principle of uncertainty follows a random path. Thus this labyrinth, though solvable, always presents a new, unsolved pattern.

## PROBLEM SOLVING

## \&

## ARTIFICIAL INTELLIGENCE

"Wonderful, but not incomprehensible." Herbert A. Simon (50, p. 4)

The interest in a maze can be increased by careful design of what is to the maze solver, a problem. In the field of Artificial Intelligence (A.I.) methods of problem solving have been investigated and formalized into a variety of techniques that in their procedure resemble the problem of a maze: a search for a goal in a network.

In an article entitled "How the Mind Works," which appeared in the January 24th (1982) issue of the New York Times Magazine, author Morton Hunt reports on the latest developments in Cognitive Science. Researchers recently have used "protocol analysis" where a subject who is attempting to solve a problem reports out loud everything that he or she is thinking while working through the solution. The transcript of this description, the "protocol," is analyzed step-by-step to show how a subject generated various hypothesis, "made tentative forces in this or that direction, backed away from unpromising avenues or blind alleys of thought." This description invites the observation that problem solving is essentially a search. However, to call the mind a labyrinth aims at metaphor rather than analysis.

Herbert A. Simon in Sciences of the Artifical, (50, p. 66), explores straightforwardly the metaphor.

Problem solving is often described as a search through a vast maze of possibilities, a maze that describes the environment. Successful problem solving involves searching the maze selectively and reducing it to manageable proportions.

The first notable successes of $A . I$. were programs that could solve puzzles and play games like chess. Fundamental to this kind of problem solving are the techniques of search and problem reduction (e.g., in chess, searching through thousand of moves in the game). At the present time computers can play at expert level, but not at master level chess. The reason suggested by Avron Barr and Edward A. Feigenbaum, [McCorduck (41, p. 183)] is that "finally the computer does a trial and error search instead of immediately sorting out the essential problem inessential."

A considerable amount has been learned in the past thirty years about the nature of mazes that represent common human problem-solving tasks - proving theorems, solving puzzles, playing chess, making investments, balancing assembly lines, to mention a few. All that we have learned about these mazes point to the same conclusion: that human problem solving, from the most blundering to the most insightful, involves nothing more than varying mixtures of trial and error and selectivity. The selectivity derives from various rules of thumb, or heuristics, that suggest which paths should be tried first and which roads are promising.
Simon (50, p. 207)

This in a nut shell is the approach of Artifical Intelligence. In fact it points to many of the early successes in modeling human problem solving. Yet Simon goes further in asserting that our interaction with the environment and one another is an artifical domain of human artifacts called symbols that are emitted or received and operate according to laws of our own making. For Allen Newell this is . . . . . .
"An essential condition for intelligent action of any generality is the capacity for the creation and manipulation of symbol structures," [McCorduck (41, p. 332.)]

The model of mind is considered as an information processing system. The view of intelligence has given Cognitive Science new and powerful tools for analyzing and modeling the mind. On one hand, such an approach yields quite impressive results and on the other, it nevertheless serves to underscore the immense complexity of the human mind. Hans Berliner of Carnegie-Mellon University emphasizes this point. "the problem of having a computer find its way around in its memory, as a human being does, is pretty close to hopeless."

This view is reinforced by Richard Bellman in Introduction to Artifical Intelligence Can Computers Think?

$$
\begin{aligned}
& \text { We have shown that many examples of } \\
& \text { thinking by a computer can be regarded as } \\
& \text { tracing a path through a network. } \\
& \text { Obviously other methods of human thinking } \\
& \text { exist. } \\
& \text { Bellman ( } 8, \mathrm{p} . \mathrm{x})
\end{aligned}
$$

Perhaps in the final analysis, what is of interest is the efficacy of such techniques to produce results rather than modelling exactly the way humans solve puzzles.

In fact if one attempts to generate every possible outcome of a problem the result can be combinational explosion. Shannon estimated in 1950 that an average game of chess has some $10^{120}$ different possibilities. Some policy or criterion must be employed to choose meaningful subsets that may lead to victory. In pattern recognition problems generate numbers on
the order of $2^{100}$ or $2^{1000}$. Thus pattern recognition remains one of the more troublesome areas of A. I. It is something humans do with ease, but, it is difficult to explain. Simon came up with a figure of 50,000 patterned positions that the chess masters has learned and stand in memory.

In examining the traveling salesman problem, one can quickly see the problems involved with combinatorial explosion. Consider a set of cities.


What is the shortest path, or what path takes the least time to go from city one to city N? One path is to go from the initial city directly to the terminal city. A further restriction might be that certain (or all) numbers of cities must be visited first. Another path is then:


How many paths are there? The paths that go through every possible city are found by considering going through the first
city to any of the remaining $N-1$ cities and to the remaining N-2 cities and so on. The number of the possible paths is factorial of $\mathrm{N}-2$. If there were 12 cities the number of combinations would 12-2! (i.e. 10! or 3,628,800).

The three major components of search systems are first the database, which describes both the current task domain and the goal. The second component is a set of operators that are used to manipulate that data. The third component is a control strategy for deciding what course of action to follow next, in particular what operator to apply and where to apply it.

The application of the operators can follow different strategies. Reasoning forward means, for example, in chess where one attempts to take the problem state (i.e. the initial configuration of the board) to a state satisfying the goal of checkmate. The operators are the rules of chess.

Reasoning backward involves starting at the goal and identifying a subgoal perhaps easier to solve, and again find a subgoal of the subgoal until the task domain is reduced to a set of trivial problems.

Humans often use combinations of forward and backward reasoning in problem solving. This has led to what is called in A.I. 'means end analysis`. The current goal is compared with the task domain and the difference applied to test and change the current operators. The modified situation then becomes a new point of departure for the original goals. In "How the Mind Works", Professor Paul E. Johnson of the

University of Minnesota gives this description.
What he can't tell as the subject of protocal analysis but what we're finding is that he's doing top-down and bottom-up thinking at the same time. He's seeing things in a general way but also in a highly specific way, lising his tremendous network of experiential associations and relying on his intuitive judgment.

Forward reasoning in a problem solving system produces a series of new states in the data representation while backward reasoning, by dividing the problem into a series of subproblems essentially is problem reduction. The interactive maze mentioned in another section where the act of moving through the maze changes the configuration of locked and unlocked doors is an example where the maze state-space changes according to the operation of trying to solve it.

Tree structures and graphs are used for state-space representations. The root node of the tree represents the initial problem or state. The subsequent nodes represent the new states that are produced from the initial state by application of the operator.

The state space of the traveling salesman can be represented as a tree. Instead of the shortest path, or shortest time this version of the problem requires finding a path of minimum cost between an initial node or city and a goal. This problem in other words is to find a minimum mileage trip beginning and ending at city A visiting each of the other cities once and only once. Nodes after the initial node have the list of cities visited up to that point

A B C D
A 4610
B -610
C - 5
D - - -


The best solution by inspection is $A-B-C$ with an optimal mileage of 10 .

A grab bag of devices used to expedite a search are called heuristic search techniques. Bellman uses the term in the sense that it is an approximate policy a rule that tells us what transformation to make in terms of a given state.

Bellman(8, p. 56)
Heuristic is a buzzword in A.I. but there isn't necessarily agreement in its denotative meaning. Feigenbaum and Feldman (1963) give this definition:

```
a heuristic is a rule of thumb, strategy,
trick, simplification, or any other king of
device which drastically limits search for
solutions in large problem space.
Heuristics do not guarantee optimal
solutions in fact, they do not guarantee
any solution at all. Quoted in Barr (5, p.
29)
```

Nils Nilsson draws a distinction between heuristic search and blind search. If the order in which potential solution paths are tested is arbitrary the search is a blind search. If information specific to the problem is introduced and limits the search it is then a heuristic search.

The general problem solver (G.P.S.) first developed by

Newell, Shaw and Simon in 1957, sought to separate problem solving techniques from knowledge specific to a particular task domain. The task dependent knowledge and operators were collected into data structure forming this task enviroment. The problem solving part of the system employed means-end analysis where a task is presented as an initial object and a goal object is to be transformed. The operators themselves are categorized according to the transformation they produce. The program is a succesive goal directed strategy.

Problems like the traveling salesman suggest methods determining the shortest path though a maze. However it is unlikely that most people or rodents will be disposed to make the appropriate calculations or diagrams in situ. For the designer such techniques provide a cut and dry way to determine the maxinum and minimum distances. More importantly game-trees and graphs literally provide a model for a maze while search methods such as blind and/or graphs heuristic state-space search and game-tree search provide conceptual distinctions that could be used in design. Thus, one can insure that the maze solver must, like in a blind search arbitrarily attempt every path or the solver can utilize a heuristic; like "Keep one hand or the wall and you'll eventually get out."

## ALGORITHMS:

## GENERATION

\&

## SOLUTIONS

Any path is only a path...

In his exhaustive, Mazes and Labyrinths There History and Development, W.H. Matthews (39), in the chapter on Labyrinth design and the solution of mazes devotes several paragraphs to the presentation of a working definition of labrinth. He then informs the reader that it should be taken for granted that no one else will accept this definition. In the discussion of the definition of a labyrinth, Matthews briefly mentions the "well known bridge" problems, but quickly sidesteps network theory and enumerates the essentials. He divides mazes and labyrinths into two major kinds namely unicursal and multicursal or "as some say, into non-puzzle and puzzle" or "single course or once run" types respectively. Unicursal refers to labyrinths of a single path with no alterative branches, multicursal being the natural antonym.


Mazes may be of two or three dimensions (e.g. "The complexity of a garden maze may be greatly increased if desired, by introducing tunnels and bridges." (39, p.185)). Labyrinths may be grouped into compact or diffused types, the compact being one in which all paths and barriers are contiguous, the other having spaces which are not paths enclosed by the barriers. The branching of mazes are categorized as simple or subdivided having loops and whether the goal is situated with in a loop. Finally, mazes can be
separated by noting if there are several paths to the goal, several goals, or even no goal whatsoever.

A good maze Matthew suggests offers "a fair amount of puzzledom without imposing undue fatigue," and it is well to relieve the monotony" by the introduction of arbors, statues, etc;"

Interestingly enough Matthews reports of two solution methods that are quite similar to some present day network theory and computer algorithms.

The first involves by system of marks whereby every branch will be traversed and any branch is traversed only twice: once in each direction. So that in any finite maze the solver will eventually reach the goal. In fact the solver will visit every part of the maze if he/she follows the rules assiduously.

A single path terminating in a dead end, or the entrance, or the goal is is called an odd node of the lowest order one. Two paths meeting at a point are merely a change in direction. Three paths meeting at a node represent a true branching. Thus, four paths are an even node of the lowest order. If the entrance and the goal are odd nodes, it is possible to traverse every branch once. Either the maze is unicursal or the branches form loops returning to the main route.

The working system is as follows: "Supposing without danger of their (the marks removal is our absence." At a node with no marks (meaning one you have not been to, mark the arrival path by three marks. If you see marks on the other
paths leading to a node mark the arrival path with only one mark. If the result is that there are no unmarked paths at the current node you must turn around retracing your steps on the arrival path. When there are one or more unmarked paths leading from the node, chose one and mark its entry with two marks. The guarantee of success is that upon arrival at a node, a path with three marks is never taken unless there are no paths left unmarked or with one mark only. If forced to enter a path with one mark, the solver should make two marks when leaving such a node which is, of course, a path with three marks.

In a paper by E.J. Anderson of Cambridge, England titled "Mazes: Search Games on Unknown Networks" published in Networks Vol. 11 (1981) (pp.393-397), a similar approach to puzzledom is termed the 'normal strategy'. A maze is defined as a connected finite network with two special vertices or nodes, the entrance and the exit. The normal strategy is as follows: when at a node with untraversed edges or arcs choose one of the arcs at random. When at a node having no untraversed arcs, choose the most recently traversed arc among those which have been traversed only once.


In the figure above, there are 8 routes that are determined by the 'normal strategy.' ( $1,2,4,5,7,6,4,6,7,8$ ) is
one such route. Using the normal strategy the solver explores the maze in an orderly fashion. In this way, if we are given an arc that has been traversed twice, any arc which has been traversed between these two traversals has also been traversed twice. The normal strategy prohibits choosing a doubly traversed arc so the solution proceeds by effectively removing the doubly traversed arcs of the maze as they are encountered.

The author states a lemma which says that there are no untraversed arcs from any nodes visited between the two traversals of a doubly traversed arc. So when the situation arises where all the arcs at the entrance to a maze have been doubly traversed then from the above lemma, there are no untraversed arcs from the visited nodes. Since all arcs in a maze are connected the exit must have been visited.

The normal strategy is nearly identical to Mathew's marking method but has the added expedient of systematically removing doubly traversed arcs from the set of arcs which constitutes the maze. Anderson shows that for any maze, the predicted or expected (E) length (L) of the solution path (P) is less than or equal to the entire length (L) of the maze (M).

$$
E\{L(P)\} \leq L(M),(1)
$$

Furthermore, no other strategy will have a lower expected length of the solution path.

A maze is called simple if it has just one solution route (R). Matthew's unicursal labyrinth is trivally a simple maze.

If maze (M) is just a simple path then the above (1) is trivially equal. The solution path ( $P$ ) is equal to the route $(R)$ and is the length (L) of the maze. Moreover, solving a maze that has branching only leading to dead ends will have one route after removing the doubly traversed arcs. These branches from any node will be called components (C): of the route (R). They are entered one at a time in a random order. The probability of entering a particular dead end component is 1/2. Thus:

$$
E\{L(P)\}=L(R)+\sum_{i=1}^{n}(1 / 2)\left(2 L\left(C_{i}\right)\right)=L(M)
$$

## Where the expected length of the solution path

 equals the length (L) of the route (R) plus the excursions along the component branches each doubly traversed (2L) with a probability of $1 / 2$. For a maze that has more than one correct path, Anderson shows that the equality (1) is still true. Essentially this is because when there are several correct solution paths, their prescence can only shorten the expected search for the exit. The author uses the normal strategy as an approach to the class of search games on unknown networks, where the searcher only has information about that part of the network he has traversed. Here, the problem corresponds to a problem of finding the exit to a maze. The problem can be considered as a two person game between one who sets the length of a maze and the position of the exit and a solver. A number of papers have dealt with the problem of finding animmobile or mobile hider in a mimimal expected time. A detailed review of such search games can be found in "Search

Games with Mobile and Immobile Hider," by S. Gal, SIAM J.
Contr. Optimization 17, 99-122 (:979).
When it is impractical to place marks, or even to use, like Theseus, a clue of thread, it is still possible in the majority of cases to make certain of finding the goal by the single expedient of placing one hand on the hedge on entering the maze, and consistently following the hedge around, keeping contact all the time with the same hand. Blind turnings present no difficulty as they will ony be traversed first in one direction and then in the other. The traveller being guided by his contact with the hedge alone is relieved of all necessity for making a choice. (39,p. 119) Mathews

Unfortunately, this simple minded approach breaks down when the goal is situated in a loop. The person will return to the entrance without even having visited the goal.

The Pledge Algorithm uses a procedure much like placing one's hand on the wall of the obstacle, but with some ingenious steps, the algorithm can get out the traps of loops. This universal maze-solving algorithm is named after John Pledge of Exeter, England who devised it at the early age of twelve.

The Pledge Algorithm is discussed in Turtle Geometry (1982) by Harold Abelson and Andrea diSessa. A detailed discussion of the proof is given in pages 191-193. The Algorithm is as follows:

1. Select an arbitrary initial direction, call it "north" and face that way.
2. Walk "northward" until you hit an obstacle.
3. Turn left until that obstacle is on your
right.
4. Follow the obstacle around, keeping it on your right until the total turning (including the initial turn in Step 3) is equal to zero.
5. Go back to step 2.

The basis of this algorithm is the simple-closed path theorum which states that the total turning in a single closed path is equal to +360 degrees (or +2 if measured in radians). The solver, or the turtle for that matter, observes locally the total turns it makes. A loop in a square box consists of four turns of 90 degrees in the same direction. Continued looping would result in total turnings of some multiple of 360 degrees. However if the turtle knows that repeating turns in the same direction are a multiple of 360 degrees, all it has to do to get out of a loop is to undo those turnings by having total turnings equal to zero. That is, if the turtle turned left 90 degrees four times where the total equals 360 degrees, he would then turn right 90 degrees four times effectively cancelling the turnings and avoiding the loop.


The turtle exemplifies a cybernetic approach with a feedback-regulatory system. This was the early approach to modelling intelligence and it produced a menagerie of mechanical mice, rats, and other beastial cybernauts. A more sophisticated approach is the goal seeking control of Allen Newell-Herbert Simon's early work and the later work of The General Problem Solver (GPS).

These represent two basic approaches to goal directed problem solving. In the General Problem Solver the problem is represented explicitly. It is modeled typically by the use of a description list data structure. As described earlier, the search is a sequence of operators that transform the data structures of the desired model into the desired goal.

The Logic Theorist or the "Advice Taker" of McCarthy is a system of modal logic like First Order Predicate Calculus, having a set of axioms and premises, rules of inference (e.g., Modus Ponens and Substitution) that allows derivations of new facts. Another way of saying it, is the modal approach embodies such concepts as 'cause' and 'can'. The program solves the problem by searching for a deduction that can prove the existence of a sequence of possible actions which will cause the desired goal.

Pople's Goal-Oriented Language (GOL) brings together in one language the capabilities of both the modal approach and the model approach. Pople gives examples of some of the classical problem-solving programs that can be, represented
directly in the GOL language. Among, the illustrated programs (e.g., Logic Theorist, the General Problem Solver, Chess Mating) there is a program for solving mazes. [Simon and Siklossy (51, pp. 329-413)]

MODALMAZE exhibits the operation of both the model approach and the modal approach. It is well beyond the scope of this paper to explain the syntax of the GOL language. So the following example has an almost literal English translation of the code.

MODALMAZE: (LAMDA (GOAL X)
the goal of a maze is attainable, starting from point $X$, (GOAL)
if either the goal is satisfied in the present state (EXISTS (Y A) (AND
or there exists some point $Y$ and action $A$, (CONNECTED X Y)
such that $Y$ is connected to $X$, and
(MOVE Y A)
$A$ is the action if moving to $Y$, and
(GOLDING (MODALMAZE (GOAL Y)
the goal is attainable, starting from point $Y$,
(TRANS A STATE)
in the new state derived from taking action $A . "$
The command to evaluate MODALMAZE is GOLDIG ((MODALMAZE (AT M EXIT) START) EXT).

When the assertion (AT M EXIT) which is when the mouse $M$ is AT the EXIT the desirable transformation has been reached
provided of course if it is possible to get there.
The 'normal strategy' for solving a maze as described previously proceeds by removing the doubly transversed branches of the simpie maze till the solution path is the single route from entrance to exit.

The spanning tree algorithum is nearly the reverse of the "normal strategy" procedure. It generates a simple maze that has one starting point and one finishing point. All branches are reachable from the entrance and there is by definition one and only one solution path from start to finish.

The spanning tree has the following property given a rectangular array of walled cells for any two cells of the array, there is only one path between them. The perimeter of the array is opened in two places to provide an entrance and exit. Since there is a unique path between any two cells, there is a unique solution from entrance to exit.

a

b

C

d

The algorithm follows:

1. Choose any cell of the array and call it the spanning tree. The four cells contiguous to it (fewer if it is on the edge or in a corner) thus becomes frontier cells (Figure A).
2. Randomly choose a frontier cell and connect it to one cell of the current spanning tree by erasing one barrier (Figure B). If it is adjacent to more than one cell of the spanning tree (it could be adjacent to as many as four), randomly choose one of them to connect it to and erase the appropriate barrier.
3. Check the cells adjacent to the cell just added to the spanning tree. Any such cells that are not part of the spanning tree and have not previously been marked as frontier cells must be marked as frontier cells.
4. If any frontier cells remain go back to Step 2.
5. Choose start and finish cells.

Taken from "How to Build a Maze", David Matuszek, Department of Computer Science, University of Tennessee, BYTE Magazine, December 1981.

## LABYRINTHOS

In the period from 1975 to 1980 , I worked with projected light as a means to compose immaterial form and pattern in space. By projecting on stretcned fabric, light revealed form, much like Carravagio's spotlight.

As in Velasquez's work, light was also used as spacedefining , by contrasting blotches of light and dark ranging from one location to another. In a painting, this alternation ranges from foreground to background. In these light environments, the viewer can step into the space and, by doing so, defines his or her relationship to the pattern of space-defining light.

Time is an integral part of these installations in two ways. First, there is a changing sequence of light that defines space and form progressively in time by change of angle, color, and quality. Second, there is the 'path of disclosure' (i.e., the path the viewer takes while observing the forms from differnt angles and locations). The cycle of light changes is repeated so eventually the comprehension of the compositon becomes independent of the particular sequence in which it was originally learned.

In another installation, I sought to further define and structure the path a viewer might follow in exploring a space. By using twenty-five 3 foot by 8 foot stage platforms, a matrix of alternating black and white platforms of various height were arranged like ascending and descending terraces on a hill. Light patterns were projected on the white platforms. Instead
of having the top to bottom relationship of easel painting, the horizontal placement emphasized that these paintings of light could be accessed from any angle of view. The black platforms were the walkways. Because the entrance to the room which housed the environment was located at one end, the viewer tended to follow the layout serially. While movement was further defined by handrails, there were multiple access and exit points which allowed forward, backward, and perpendicular movement across this three-dimensional 'flat on the floor' painting.

An installation of pattern, form, architecture, sound, light, and color all composed together in the configuration of a labyrinth is much more than a puzzle. Likewise, a multilevel structure of platforms and stairways can frame the perception of an environment. A more complex design than the singular intention of the tower can be devised so to exploit the fascination with observing the surrounding world from the novel overhead viewpoint. Pathways through the environment can provide multiple viewing stations of different heights and, at the same time, provide a means for getting from here to there. The labyrinth as landscape architecture was the motivation for the design of a network of paths set on some large field of grass. The paths would be designated by the basic module of an arch made of a thin strip of aluminum or plexiglass (clear or colored), anchored at either end in the ground. These arches would be flexible enough so that they
could gently sway in the wind and vibrate, thereby producing an audible sound. A series of arches separated by a short interval would form tunnels. The tunnels would cast shadows that could be colored and would change with the angle of the sun. The interval between arches and their height could vary. A dead end could be created by having the arches become progressively shorter so that only a child might get through. The interval between arches could change in such a way that at different times the participant could elect to play the game or not.

Order and disorder could be contrasted by paths whose clear articulation would gradually dissolve into randomly placed arches,like a croquet field with only the slight suggestion of where to go. Overall, the network of arches could resemble the configuration of a symmetric tree structure. Some paths would terminate in dead ends, while others might dissolve into a disorder that could be as confusing as a highly ordered maze. This previously passive space is activated into a set of relations that encourages entry. At a distance, one would hear the vibration of the structure as a whole. The arches, as they sway in and out of phase with one another, would scatter reflected light against the foil of the pattern of shadows. The whole effect is a dynamic environment of light, shadow, sound, movement, and participation.

The first partition labyrinth design took a minimalist
approach. The intent was to isolate and distill the principle of the unicursal labyrinth as a patterned, environmental sculpture. It is seen as a single path that winds back around itself dividing and filling space.


Constructed in an empty room, it transforms the way one can move through the space. Instead of entering the room and taking it all in at a single glance, like the presentational logic of a painting, the perception of the space and volume is incremental in time like music. The experience is mapped in time by the architecture of the labyrinth. The shape and size of the room can only be guessed at by the fragmentary views. Like the unicursal labyrinth, it can be walked through in one direction and then reversed. There is no goal as such. The interest is in the movement itself, revealing through the variations in viewpoint the overall layout.

As one enters the labyrinth after each turn, the corridors increase in length, like the logarithmic spiral of the nautilus shell. A feeling of suspense is created as one encounters each subsequent corridor. It must be traversed to confirm the sense of the expected pattern. The path is like the appoggiatura in music: anticipation of the exit is
augmented by each new turn, revealing yet another path to explore. The pattern establishes the anticipation and expectation of the completion of an action. Like the edit in film, suspense can be increased by delaying the expected consequence.

If the path is traversed in the opposite direction, the corridors become progressively shorter, inducing the expectation of the entrance. This movement heightens the experience of space and duration. The traversing of the path is the measurement and, therefore, the comprehension of the space mapped in time.

The effect of changing viewpoints can be increased by the placement of doors along the path at equal intervals. The doors will tighten the knot of enclosure further, limiting the participant's viewpoint. The comprehension of the layout is delayed.

Time and space can be measured by the forceful repetition of the doors. When the labyrinth is very large, the uniformity of
repetition can be so disorienting that arrival at the exit comes as a complete surprise (and relief).

It can be seen that interest is generated in the absence of a puzzle. The unicursal labyrinth, by its space dividing, space filling, and temporal nature, is fundamentally intriguing to the mind that must orient itself in the environment. What is missing is the necessity of choice which is the basis of the
multicursal maze. When forced to make a decision between two equally valid choices, one has a dilemma. When the solver is confronted with making a choice, the labyrinth becomes the puzzle of the maze.

A different design incorporates choice and decision-making as the organizing principle. The basic module is a cubicle of four doors, which can be locked and unlocked in different combinations. In order to open freely in either direction, the doors would be saloon doors. No directional bias would be built-in. Again, the matrix of cubicles are space dividing and space filling. A square or rectangular matix can fit in a variety of different sized rooms. However, the perimeter of this maze is a wall of doors. In place of one entrance or one exit, there are many. Its placement in the host room should allow passage between the walls of the enclosing room and the outer perimeter of doors.

This maze preserves the interest in the discovery and traversing of the path for its own sake. The locked doors simply interrupt the movement in one direction, and the remaining unlocked doors encourage movement in another direction.

The configuraton of locked and unlocked doors can be set by a computer which controls the locking mechanism. A feedback or movement sensing device such as an electric eye or a floor pressure pad can report to the computer the movement and location of a solver.

At the start, all the doors are unlocked. When someone steps inside the maze, the door immediately in front and behind would lock. The solver would then have to choose between the two doors perpendicular to the locked doors. If the solver entered at a corner, the choice is between trivially exiting the maze or continuing. The resulting configuration becomes a history of the solver's individual movement. Moreover, a person can in this way become a victim of his or her own past. If the maze is small, the choices one makes now will more or less determine one's future. If the maze is sufficiently large, the solver's future has a probability distribution that converges towards the certainty of being trapped the longer he or she takes to solve it. A panic button is installed that would immediately unlock at least one door in an enclosed cubicle. Periodically, the entire maze could be reset to its initial configuration of unlocked doors.

The idea of having all the doors initially unlocked before someone enters comes from the notion of the precompositional l2-tone matrix. After generating this matrix of possibilities, the composer then chooses subsets of the matix and arranges them in some imagined best sequence. This sequence when played in the concert hall is the composition.

The maze with all the doors unlocked is like the precompositional matix. The solver plays the role of the composer, choosing and selecting the sequence of cubicles. The solution path taken in its entirety is the composition.

An objection could be made that it is a mistake to compare the relation between pitches to the relation of uniform, identical cubicles. This is not theonly difference. The composer chooses a sequence of pitches based on skill and experience, while the maze solver randomly encounters identical cubicles without much thought or control over the resulting sequence

Twelve-tone theory as mentioned earlier treats the tone row as a set of ordered pitches from which different combinations are generated according to a set of rules. In a similar, way a maze can be generated by a set of rules that specifies different combinations of turns and straightaways. Both are isomorphic to the concept of a search in a network. The rules of transformation determine what order the cubicles or the pitches, in general, the order in which the events, are encountered. In both, a starting node is determined, and a goal, the terminal node, is estab- lished. The designer of the maze simultaneously presents all the possibilities and determines the rule of transformation that the solver must follow. However, it allows for stochastic caprice. Such a maze is interactive. Choice is embedded in the compositon, and the solver simply enumerates possible combinations in a random order. This order, nevertheless, results in the same conception of the maze independent of the order.

The concept of the isomorphism between the order of pitches and the order of turnings in a maze was the basis of
the Markov Music Maze Input/Output Graphic Display. The work was done at the Architecture Machine Group as a 4.201 Project. The composer uses a tablet to direct a cursor displayed on the Ramtek screen. A maze is created by knocking down walls in a grid. The user would then input a sequence of twelve notes. By using the tablet, a solution path to the maze would be drawn. The movement of the cursor up, down, left, and right would be translated into the rules for transforming the tone row. The new combinations of tones would be displayed a page at a time, color-coded for easy identification with the solution path. The term Markov is used because the musical transformations of inversion, retrograde, and retrograde inversion are applied in relation to the relative direction of the previous note (i.e., the direction of the cursor in the solution path). So the Nth note is dependent on only the $\mathrm{N}-1$ note.

The graphic display on the Ramtek shows the plan of the maze and, simultaneously, the sequence of notes. There is no puzzle to be solved per se. But there is the kinesthetic procedure of path traversal producing music notation as output.

The intent of the labyrinth of arches and the unicursal path labyrinth is to focus interest on the process or procedure of movement through change and variation in viewpoint and location. The delicate, but complex, filigree of snow crystals suggests a design strategy where the perimeter of a maze could be as confusing as the interior. As in penetrating a fjord,
the further one explores the channel, the more one is 'inside,' although the channel itself is continous with the exterior.

A recursive space dividing and space filling curve discovered by the l9th century mathematician Hilbert displays not only a complex articulation of its outer perimeter, but it has the interesting feature where the inside of the pattern becomes the outside. For the pedestrian without the benefit of the bird's eye view, it is a puzzle. As in Mathew's simple solution, it can readily be solved by placing one's hand upon the wall and following the curve till the exterior is reached.

never trapped. Further complexity can be introduced by placing doors at intervals along the curve. This way one can communicate directly from the interior to the exterior. As before, the locks on the doors can be controlled by a microcomputer which again will change the configuration of
locked and unlocked doors triggered by the solver. It can be confusing. But its 'puzzledom' does not lead to the unpleasant state of being trapped. In addition, it becomes simultaneously a unicursal labyrinth, where one follows the shape of the curve. Yet it is a multicursal maze where one can move through an interactive network of paths.

The Dragon Curve is another space dividing/filling method of partitioning that likewise is not a puzzle, but can confuse by its complexity.

In mathematical magic show, Martin Gardner lists several techniques for generating the curves. Below are some.

1. Sea Dragons of order 0 to 6 with binary formulas.


2. Geometric method

The experience of such space filling/dividing curves (e.g., Hilbert's Curve and the Dragon Curve) is essentially temporal. The movement which comes from following these curves intensifies awareness of seeing the same environment from multiple location-specific viewpoints. In a sense, one learns the spatial layout by the variation in viewpoints. Those variations become the measure by which the total expanse of enclosed space is comprehended.

To shift interest further away from mere 'puzzledom,' these different designs can become the architectural guides to a sonic labyrinth. Different combinations of sonic events can be triggered by the order in which doors are opened. A set of pitches produced by a synthesized bell could ring changes
following the interactive 'hunt' of the participant. The musical texture would be randomly accessible and intelligible in action and in perception. The sound environment itself is the goal. Like Joseph Albers' Homage to the Square series, the 'goal' is the unfolding and perception of the combinations. The interaction of the solver's choices defines the "path of disclosure." The totality of each network solution is the composition. Composer skill is measured by observing his/her chosen set of sounds and by the choice of the operators which govern the possible combinations of sonic events. These operators are embedded in the way the computer controls the configuration of locked doors. It remains a random access composition of incremental sonic events and viewpoints. Closure of form (i.e., having a beginning and an end) is simply a result of entering the maze and solving it. It is a stochastic network where the probabilites imply equally likely and likable outcomes. More than a puzzle, this labyrinth is a spatially and acoustically intriguing experience that maps onto the architectural path the comprehension of a time-dependent sound sculpture.

Because of the necessity of rational procedures in construction, a built structure is almost always visually intelligible in form. Especially when confronted with a modular design, the viewer can, as with pattern, quickly extrapolate the whole from a fragment. The thesis labyrinth,
in its modular design, exploits this expectation of repetition. However, where the structure itself is incrementally comprehensible, the conscious awareness of the solution is not. By the very act of attempting to solve the labyrinth, the configuration of the locked doors change, and there is no calculable progress. Yet the probability distribution insures that one will eventually exit. But it is a solution inspite of one's efforts. As in life, one can as often be a victim of one's past as a beneficiary. So much so that one can have the feeling of being buffeted about by forces beyond one's control and comprehension.

Upon opening the door of the exhibition room of Building W-ll, one steps into a cubicle that has two doors perpendicular to one another. Each door has a window through which can be seen two identical doors. Beyond those windows can be seen a white wall. At this point, one may surmise that there are two perpendicular sequences of three doors parallel to one another. One can also see over the top of the doors that the entire structure is enclosed by a far larger room. How much larger is difficult to tell. The perception of the exhibition room has been transformed. Previously, its rectangular dimensions could be understood in a single glance. No one steps into an enclosed space that shows a partial view of a far larger space.

This partial view arouses a curiousity to explore and
understand the layout of this structure. One tries a door and it is locked. The other is opened. As one steps into the next cubicle, one hears the click of the changing configuration of locks. From this position, one can see that there is an additional cubical. Surrounding all the cubicles is a corridor formed of windowed partitions. Perhaps at this point, one decides to leave. But, unfortunately, the door by which one came is now locked, and one is inadvertantly committed to finding a solution. So one proceeds through the door to the corridor (fortunately unlocked). But the door leading to the room behind the partition is locked. So poor and hapless, one marches down the corridor, turns, and follows it to the door at the other end of the corridor. It is open. Stepping into the roorn, one quickly realizes that the labyrinth is to be traversed again in order to leave the exhibition room.

This is a possible scenario of movement through the labyrinth. The main part of the computer algorithm locks and unlocks doors in pairs. In any given pair, one door is locked and the other is unlocked. Therefore, in any given cubicle, there will be at least one door locked and one door unlocked. The configuration of locked doors in adjacent cubicles is such that the combinations always allow passage to another cubicle. A person can thereby never become trapped. Several people moving through the labyrinth will simply cause the configuration to change state more often.

The hardware consists of an Ohio Scientific Instruments C2-4P MODII. It is a 6502 based microcomputer having Basic-in-Rom and 32 K of memory. The $\mathrm{I} / \mathrm{O}$ (input/output) parallel ports use the TTL 74LS373 Octal D-type Transparent Latch as the data output. The input is the TTLT4LS244 Octal Buffer. The address decoding and enable pulses are controlled by a series of hex inverters and nand gates driving a 1383 to 8 line decoder/multiplexer which in turn drives the buffers and the latches. (see circuit diagram).

Each door has a solenoid which when energized prevents the retraction of the bolt of the lockset. The claustrophobic can override the locked door by pushing a red panic button located just above the handle. When pushed the solenoid is de-energenized and retracts and the door can be opened. Along the doorjam of each door is mounted a pressure switch. When the door is opened, it sends $a+5$ volt pulse to the input port of the computer. The computer associates this pulse with a particular address and can distinguish one door from another (see diagram).

## SPECULATIONS

## \&

POSTSCRIPT
(The labyrinth) is at once the cosmos, the world, the individual life, the temple, the town, man, the womb or intestines of the Mother (earth), the convolutions of the brain, the consciousness, the heart, the pilgrimage, the journey, and the Way.

Jill Purce, The Mystic Spiral

When a labyrinth is no longer a puzzle "it aspires," as Walter Pater says, "to the condition of music." What is this condition? It is a one devoid of ostensible subject matter. It fulfills no functional purpose as does architecture. Nor does it imitate nature like representational painting. It seems to exist in its own universe of relations. Listening to music sets into motion hierarchical relations, hetararchical (McCorduck, p.261), concurrent, and nested processes involving everything from the lower brain stem to the cerebral cortex. To say "I have understood this music" is an elusive concept. Music theorists reduce their object of study to pitch (today: sonic event) and duration. It is trivial to say that without the dimension of time, there is no such thing as sound. So, too, there is no perception of space without time.

The analogy throughout has been that the perception of sound is isomorphic to the perception of space, in-so-far as the perception of both is a function to time. Time is the perception of difference and change. Difference implies comparison between one state and another, between one sonic event or one visual aspect and another. Repetition of the beat or of the graphic blob or the tile in the pavement allows the comparison to be continually updated. Pattern in space, on the page, or in music is an immediate difference detector.

It is difficult to say much more than this (i.e., how memory might affect the relation of repetition and difference). Perhaps the isomorphism should imply nothing more
than a sequence of events whether they be noise, lines, fenceposts, squares, or even words.

The complexity of spatial awareness involves the dynamics of binocular vision, binaural hearing, kinesthetic orientation, the sense of touch, all operating simultaneously and in parallel. The phenomenon of synesthesia suggests that somehow two unrelated sense perceptions are associated into one recallable, nameable concept. It may be that the power of music lies in the fact that it activates so many processes of mind and body and their simultaneous occurrence is the compelling factor.

Learning appears to be the process of the interaction of memory and the ongoing enumeration of difference, variation, and repetition. The child playing with two blocks isolates them momentarily from the surrounding world. Through the memory of the enumeration of many possible combinations, the relationship of the blocks to each other is understood. Moreover, this relationship is abstracted and the mind seems to take special pleasure in the recognition of this rectangular pattern elsewhere. From this comprehension of spatial relationships comes the power to act, to impose a similar understanding in other situations. As Kevin Lynch suggests, comprehension of a pattern provides an orientation from which to choose, from which to act.

The artist creates his own self-contained set of relationships. He creates his own comprehension and his own
action. In this respect, art is play. It is the enumerationn of those relationhips, arbitrarily set apart from everyday life. The labyrinth, rational in its pattern, structure, and division of space, unfortunately encloses in its perception that which cannot be articulated, enumerated, or measured. The design of this thesis labyrinth confronts the problem-solving intellect with stochastic capriciousness, which can be irrational in its perception.

## SPECULATIONS

Video Maze, consisting only of video terminals, cameras and a computer controlled switcher which changes the displayed image after receiving a pulse from a movement sensor.

Telecommunications Maze, a maze of virtual space where the stroller can move through rooms in different parts of the world from one location.

## Greg Bright's Caveat

My third maze book. When it was the last thing I wanted to do, it became the next thing I did.

Since digging the Pilton Maze in l971, I have been trying to abandon mazes. I dislike the fact that I am pressed to do them and the 'light-hearted' interest that they attract.

In 1970, I began to be drawn back to mazes because of 'the thrill of being lost' and 'the strangenesses of routing' and their 'mystery'--an aspect that now particularly disgusts me. Having used the maze form to channel my energy into one earth statement, I should have left mazes, had I the power. The subsequent period has been expended organising my worldly position and exploiting the consequences of my initial achievement. Always keenly aware of the suck of success that tends to subjugate the successful to its current, I have constantly attempted to oppose the sirenic forces of maze commitments.

Frivolity is a major gangrene of the psyche, exonerated by humanity on the grounds that weaknesses of this kind are an essential 'human' component. I have never been complicit with the 'fun and games' angle.

While uninterested in the glee of children or adults, my involvement in this book nevertheless had its compensations, particularly with the Hole Maze, namely an increasingly direct concern with the prospects of signification of raw line. Here,
the analogy with music is strong; I see the Hole Maze as symphonic. The other mazes had been commissioned during the period 1977-78. Their appearance in the newspapers, on posters or as book covers is to my taste; forms of display with pitfalls different to those of gallery viewing.

Solving the mazes is of little significance. They are patterns that may be more richly interpreted by exploration, the labour of which is dynamized by the directive of start and end.

1. Abelson, Harold and Andrea diSessa

Turltle Geometry
The Computer as a Medium for Exploring Geometry The M.I.T. Press
Cambridge, Massachusetts
London, England 1981
2. Adams, William Howard

The French Garden 1500-1800
George Braziller
New York 1979
3. Anderson, E.J.

Mazes; Search Games on Unknown Networks NETWORKS, Vol. II (1981) pp. 393-397
John Wiley \& Sons, Inc.
New York 1981
4. Arnheim, Rudolf

Art and Visual Perception
A Pyschology of the Creative Eye University of California Press Berkeley, Los Angelos, London 1971
5. Barr Jr., Alfred H.

Cubism and Abstract Art The Museum of Modern Art New York 1936, 1974
6. Barr, Avron \& Edward A. Feigenbaum, Editors

The Handbook of Artificial Intelligence Vol. I HeurisTech Press Stanford, California
William Kaufman, Inc. 1981
7. Beardsley, Jon

Probing the Earth: Contemporary Land Projects Smithsonian Institution Press Washington D.C. 1977
8. Bellman, Richard

An Introduction to Artificial Intelligence
Can Computers Think ?
Boyd \& Fraser Publishing Company
San Francisco 1978
9. Bently, W.A. and W.J. Humphreys

Snow Crystals
Dover Publications, Inc. New York 1931, 1962
10. Bord, Janet

Mazes and Labyrinths of the World
Dutton Paperback
E.P. Dutton \& Co., Inc.

New York 1975
1l. Bright, Greg
The Great Maze Book
Pantheon Books
A Division of Random House New York 1973
12. Bright, Greg

The Hole Maze Book
Pantheon Books
A Division of Random House New York 1979
13. Buendia, J. Rogelio

A Basic Guide to the Prado
By Silex
Printed in Spain 1973, 1974, 1975
14. Canaday, John

Mainstreams of Modern Art
Holt, Rinehart and Winston
New York 1959
15. Ching, Francis D.K.

Architecture: Form, Space, \& Order
Van Nostrand Reinhold Company
New York, Cincinnati, Atlanta, Dallas
San Francisco, London, Toronto 1979
16. Coletti, Luigi

Il Tintoretto
Istituto Italiano di Arti Grafiche, Bergamo, 1944
From the guide to The Great School and the Church of St. Rocco in Venice
Stamperia Di Venezia-Venezia 1975
17. Converse, A.O.

Optimization
Holt, Rinehart, and Winston, Inc New York
18. Davis, Douglas

Art and the Future
Praeger Publishers
New York, Washington 1973
19. Flores, Angel \& Homer Swander

Franz Kafka Today
University of Wisconsin Press
Madison, Wisconsin 1958, 1962, 1969
20. Gal, S.

Search Games with Mobile and Immobile Hider SIAM J. Contr. Optimization
l7, 99-122 (1979)
21. Garđner, Martin

Mathematical Magic Show
Vintage Books
A Division of Random House New York 1977
22. Gardner, Martin

Mathematical Carnival
Vintage Books
A Division of Random House New York 1977
23. Gardner, Martin

Mathematical Circus
Vintage Books
A Division of Random House New York 1981
24. Gombrich, E.H.

The Sense of Order
Cornell University Press Ithica, New York 1977
25. Green, Earl L. Ed.

Biology of the Laboratory Mouse
Second Edition
The Blakiston Division
McGraw-Hill Book Company
New York Toronto Sydney London 1965
26. Hamilton, Edith

Mythology: Timeless Tales of Gods and Heroes
A Mentor Book
New American Library
New York and Scarborough, Ontario 1940, 1969
27. Hewes, Jeremy Joan

Build Your Own Playground
A Source Book of Play Sculptures
A San Francisco Book Company
Houghton Mifflin Book
Boston 1974
28. Higgins, R. \& Wolf Vostell

Fantastic Architecture
Something Else Press
Kleins Druck-und Verlagsanstalt, Lengerich
Germany 1969
29. Janson, H.W.

History of Art
Prentice-Hall, Inc.
Englewood Cliffs, New Jersey
\& Harry Abrams, Inc.
New York 1962-1970
30. Kafka, Franz

The Castle
Vintage Books
A Division of Random House
New York 1930, 1941, 1958, 1968
The Trial
Vintage Books
A Division of Random House
New York 1937, 1956, 1964
31. Kepes, Gyorgy Ed.

Structure in Art and in Science
George Braziller, New York 1965
32. Kern, Hermann

Labyrinths: Tradition and Contemporary Works
ARTFORUM pp. 60-68
Volume XIX No. 9 May 1981
33. Keswick, Maggie

The Chinese Garden
History, Art \& Architecture
Rizzoli International Publications, Inc.
New York 1978
34. Koosis, Donald J.

Probability
John Wiley \& Sons, Inc.
New York, Chichester, Brisbane, Toronto 1973
35. Langer, Susanne K.

Philosophy in a New Key
A Mentor Book
New American Library, Inc.
New York, Ontario 1942, 1951
36. Lynch, Kevin

The Image of the City
The M.I.T. Press
Cambridge, Massachusetts
London, England 1960
37. Madsen, Richard W. \& D.L. Isaacson

Markov Chains: Theory and Applications
John Wiley \& Sons, Inc
New York, London, Sidney, Toronto
1976
38. March, Louis \& Philip Steadman

The Geometry of Environment
The M.I.T. Press
Cambridge, Massachusetts 1974
39. Matthews, W.H.

Mazes \& Labyrinths Their History and Development
Dover Publications, Inc.
New York 1970
of the original edition:
Longmans, Green and Co.
London, 1922
40. Matuszek, David

How to Build a Maze
BYTE Magazine pp. 190-196 December 1981
BYTE Publications Inc.
41. McCorduck, Pamela

Machines Who Think
W. H. Freeman and Company

San Francisco 1979
42. Meyer, Leonard B.

Music, the Arts, and Ideas
The University of Chicago Press
Chicago \& London 1967
43. Negroponte, Nicholas P.

Media Technology: Interactive Television
and Electronic Publishing
Proceedings of a Symposium for Senior Executives Industrial Liason Program of the Massachusetts Institute of Technology 1982
44. Nilsson, Nils

Problem-Solving Methods in
Artificial Intelligence
McGraw-Hill Book Company
New York, San Francisco, London 1971
45. Papert, Seymour

Mindstorms
Children, Computers \& Powerful Ideas
Basic Books Inc.
New York 1980
46. Pearce, Donald

Kafka's Divine Comedy
from
Franz Kafka Today (19) pp. 165-172
47. Pearce, Peter

Structure in Nature is a Strategy for Design
The M.I.T. Press
Cambridge Massachusetts
London, England 1978
48. Pople.Jr., Harry E.

A Goal-Oriented Language for the Computer from Representation and Meaning (51) pp. 331-413
49. Schuyt, Michael; Joost Elffers; George P. Collins

Fantastic Architecture
Harry N. Abrams, Inc.
Publishers, New York 1980
50. Simon, Herbert A.

The Sciences of the Artificial
The M.I.T. Press
Cambridge, Massachusetts
London, England 1969, 1981
51. Simon, Herbert A. \& Laurent Siklossy

Representations and Meaning:
Experiments with Information Processing Systems
. Prentice-Hall, Inc.
Englewood Cliffs, New Jersey 1972
52. Taylor, John F.A.

Design and Expression in the Visual Arts
Dover Publications, Inc.
New York 1964
53. Volbach, Wolfgang Fritz

Early Christain Art
Harry Abrams, Inc.
New York 1966
54. Wilson, Wilford G.

Change Ringing
Faber and Faber
24 Russel Square
London 1965
55. Minsky, Marvin

Music, Mind, and Meaning
M.I.T. Artificial Intelligence Laboratory
A.I. Memo No. 616

February, 1981
56. Xenakis, Iannis

Formalized Music
Thought and Mathematics in Composition Indiana University Press Bloomington 1971


定量


The Egyptian Labyrinth (Matthews)


Knossos. Maze pattern on Wall of Palace. (Matthews after Evans).


Labyrinth in Chartres Cathedral (Matthews)


Stone Labyrinth on Wier Island, Gulf of Finland (Matthews)


Leonardo Da Vinci's "knot" and Durer's Copy.


Labyrinth of Versailles (Matthews).


PAC-MAN


Twelve Order Dragon Curve (Gardner).

Terry Riley. Time-Lag Accumulator, 1968. Glass, aluminum, and mylar containing microphone and tape units. For The Magic Theater. A labyrinth of sound, made by visitors, whose voices are recorded and replayed an instant after they are heard. The artist's plan accompanies the photograph. Courtesy Museum of Contemporary Crafts, New York, and American Foundation on Automation and Employment. Left: Photo by Larry B. Nicholson. Right: Courtesy Nelson Gallery of Art, Kansas City, Mo.


DOOR CONTROL GIRCULT DIAGRAM.





[^0]:    "I tried to attain a maximum lattitude of probabilities in order not to fatigue my expectation with the necessary foresight of any one of them."

    Gardner (23 p. 66)

