DESIGN FOR A PRECAST CONCRETE DOME

An investigation of a precast concrete system for long span, low rise shell structures.

by Robert Einaudi
B. Arch., Cornell University, 1961

Submitted in partial fulfillment of the requirements for the degree of Master in Architecture at the
Massachusetts Institute of Technology
September, 1962

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Dear Sir:

A thesis entitled "A DESIGN FOR A PRECAST CONCRETE DOME - an investigation of a precast concrete system for long-span, low-rise shell structures," is hereby submitted in partial fulfillment of the requirements for the degree of Master in Architecture.

Respectfully submitted

Robert Einaudi
ABSTRACT

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An investigation of a precast concrete system for long-span, low-rise shell structures.

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Thesis Supervisor: Eduardo Catalano.

The author presents a system of precast concrete elements that provide all waterproofing, insulation, acoustical, lighting and mechanical requirements, and that when joined form the finished interior and exterior surface of the dome.
ACKNOWLEDGEMENTS

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INTRODUCTION

In modern society there is an ever increasing need for large uninterrupted spans. Buildings of public nature such as convention halls, sports arenas, auditoriums, airplane terminals and shopping centers all require such spaces.

For spans of 300 feet and more, a thin shell structure which transmits all loads through membrane stresses is one of the few structurally efficient solutions. To keep the total volume down, a low height span ratio is necessary.

To facilitate the investigation of long-span, low rise concrete shell structures, the author limited himself to the design of one of the shell forms - the Dome - in the belief that the knowledge gained through a thorough investigation of this one form would also be useful for the design of similar long-span, low rise structures.

Because of the wide range of application, certain variables within the structure such as support, perimeter and central skylight conditions are introduced. The author has concentrated on the non-variable system between supports, and presents a structural system of precast concrete elements that provide all waterproofing, insulation, acoustical, lighting and mechanical requirements, and that when joined form the finished surface.
DESIGN CRITERIA

A. Spatial Considerations:

1. Span and use of dome

A precast concrete spherical surface 306 feet in diameter resting on the bottom compression ring, resulted in a span of 370 feet between supports, and an overall diameter of 426 feet including the low, cantilevering perimetral areas. This was considered as a representative size for the activities most likely to take place under a dome or similar long-span shells, but the structural system evolved is not limited to these dimensions and is valid for spans anywhere between 250 feet and 500 feet.

Structures of similar size include Nervi's Palazzo dello Sport in Rome for basketball, boxing, etc., with a span of 375 feet and Harrison and Abromowitz's dome for the University of Illinois for various indoor sports and conventions, with a span of 400 feet. Both seat around 16,000 persons.

Such a span can accommodate even the indoor sports that require the greatest room, such as track (220 yard running oval) and off-season practice for baseball, football, lacrosse etc. The exact spans for such facilities depend on variables such as seating capacity, support conditions and lobbies. The areas needed for convention halls, air terminals, shopping centers or fairs will also vary greatly, and in some cases the shell roof may be used only as the central focal space of a much larger complex.

a) Height-span ratio

To cover the activities mentioned above with a shell
roof is practical only if the height-span ratio is kept to a minimum, for otherwise the shell will enclose too much unusable space, and cause heating, air-conditioning and acoustical problems. This is especially true if the overall height is increased by the use of grandstands sloping downward toward the center.

A height-span ratio of 1 to 8 above the lower compression ring was chosen as a compromise between reducing unusable space to a minimum and, leaving enough space in the event of a level floor area between supports. Another consideration was the fact that the lower the height-span ratio, the higher become the stresses in the shell, and the greater the chance for local buckling. By contrast, the higher the height-span ratio, the greater the chances for tension occurring in the circumferential ribs due to loads and wind action.

2. Auxiliary spaces

A low space around the perimeter of the dome was thought essential as a means of introduction to the large central space. Further definition of this space would depend on the requirements of the central space and the possible use of grandstands, changes in level or other space definers. It is assumed that all other spaces such as rest rooms and mechanical trenches will occur below this level and/or are integrated with grandstands or level changes.

B. The geometric subdivision of the structure
1. General considerations

It was thought desirable to use precast elements throughout the surface of the dome above the bottom compression ring, and to use cast in place of concrete below that point.

A building using both precast and cast in place concrete allows the superstructure of precast concrete elements to be cast at the factory or casting yard while the substructure is being cast on the site. This gives greater speed of construction and cuts costs.

To cast in place the dome surface, which must be highly complex and have slender proportions in order to resist buckling and keep its own weight down, is virtually impossible. On the other hand, precast concrete elements produced under factory conditions can be made to extremely close tolerances, and, because they are of a repetitive nature, their form can be complex without increasing costs. The elements can then be joined by in-situ concrete and reinforcing with a minimum use of scaffolding and time.

To cast in place the perimetral area is an easier problem since it is close to the ground and does not have the stringent statical shape requirements of the main space. Furthermore, it is the part of the structure that is most likely to vary according to the use of the building, while the dome surface represents a condition suitable to all uses. Visually, the difference between the cast in place and precast concrete creates a contrast between the low perimetral areas and the large central area, giving the latter a lighter
and lacier quality.

2. Geometry, size and physical properties of the precast concrete elements

The possible breakdown of a spherical dome surface are many, but, the final choice of the geometry and size of the element depends on the thorough investigation of the statical, constructional and architectural requirements.

Statically it was necessary to create elements that would resist handling stresses prior to their placement, and which, when joined to form the spherical surface, would carry the meridional and circumferential stresses, and give enough rigidity to the dome to prevent local buckling.

Constructionally it was necessary to find a geometry and size for the individual element that satisfied the requirements of casting, transportation, erection and joining.

Architecturally, it was necessary to convey the spatial quality of the structure both to the interior and exterior, and to integrate weatherproofing, insulation, lighting (natural and artificial), acoustics, and mechanical distribution, in each unit.

a) Statical considerations

The dome is a remarkably stable and efficient structure, and can take on extremely varied forms. Its surface is obtained by revolving a curve around a fixed axis. The most

1 Student Publications of the School of Design; North Carolina State College; see bibliography.
statically efficient dome has a parabolic profile which follows the curve of pressures of its dead weight. But, provided the plan section at all points remains a circle, the dome can take any profile desired, provided also that the circular sections can take tension as well as compression. The most common profile is circular, resulting in a spherical surface.

For a dome of low height-span ratio the circular profile approaches that of the parabolic; the spherical dome was chosen.

Due to the low rise of the dome (1:8 height-span ratio) all stresses were compressive throughout the structure, even under unsymmetrical wind and snow loads. The meridional stresses per linear foot increased very slightly towards the bottom, while the circumferential stresses decreased towards the bottom.* The angle subtended by the dome at the bottom compression ring was 28°; it would have to increase to about 51\(^\circ\) for the circumferential stresses to become tensile under uniform load. Stresses due to thermal variations are mostly taken care of by the natural "respiration" of the dome surface which can rise and fall just enough to take care of the thermal stresses (this is true of most shells).

From a purely statical viewpoint the spherical dome can be sub-divided in almost any manner desirable. From a geometric viewpoint there are fewer possibilities, and when

* see structural calculations for the exact nature of these stresses and other structural considerations.
constructional and architectural considerations are added there are still fewer possibilities. As a result, the subdivision of the dome along parallels and meridians was chosen for reasons other than statical, though of course it meets the statical requirements as well or better than other possible geometric breakdowns.

The statical requirements were solved by creating V-shaped corrugations in the direction of the meridians these gave the necessary depth against buckling and carried the meridional stresses—and by creating circumferential ribs which braced the corrugations and which in turn carried the circumferential stresses. In the meridional direction, the concrete area not required to resist handling stresses or to carry the meridional stresses in the finished dome, was removed to form openings which served to lighten the weight of the structure and to admit light, without reducing the rigidity of the structure against buckling. In the circumferential direction, only enough concrete area was provided to stiffen the precast element during handling, and to carry the circumferential stresses when joined. This produced a void along the meridions which further reduced the weight and provided space for possible mechanical and electrical installations.

To maintain a constant unit stress in the concrete, the meridional ribs become progressively larger as they diverge away from the top compression ring, while the circumferential ribs become more widely spaced since they also get larger to stay in scale with the meridians.
b. Constructional considerations

The size of the individual element must be the result of a compromise between the need for the largest possible element in order to have as few joints as possible, and the need to create a small element to facilitate casting, transportation and erection procedures.

A large element means little or no scaffolding and few joints, and therefore on-site work is reduced to a minimum. The use of more expensive lifting equipment is usually more than made up by the necessity of handling few elements.

A small piece means more joints and more scaffolding and therefore more construction time.

In Europe, where the cost of heavy equipment is greater than the cost of the extra man-hours needed to join many elements, small precast units have been used. Pier-Luigi Nervi's work is typical of this approach. For instance, in the 19½ foot diameter dome for the Palazzetto dello Sport he used 1620 elements covering an average of less than 20 square feet each and each weighing less than one ton. For the Palazzo dello Sport he increased the size of the unit, using about 1000 elements to cover a 300 foot span.

In the United States where the cost of heavy equipment is less than the extra cost of handling and joining many small elements, large precast units have been used. For instance, Paul Wiedlinger worked out a system for the New Canaan Supermarket where two normal small cranes could lift 40-foot square shells (1600 sq. ft.) that were precast on the site. (Eventually a 24-foot bay was used). For the Oakland International Airport one gantry crane was used to lift precast conoids 75 feet long and 20 feet wide weighing about 25 tons.
According to the engineer of that project, "The only limitation on the size of the shells is the size of the cranes available to handle them. Crane capacities have increased in the past 10 years and 90 ton mobile rubber tired cranes are now common. When the precast shell technique is combined with a sufficient size to minimize handling cost, a true American-style (high wage) mass production technique develops."

When the elements are cast at a factory rather than on the site, transportation problems will introduce more limitations to the element. An eight foot width is normal for a truck and trailer, and most factory mass-produced elements are limited to that size. However, elements up to ten feet wide can be transported without special permit. If the element is over twelve feet wide a special escort for the truck is required by law, and of course many other problems arise which make movement difficult if not impossible. Lin tees up to 152 feet long and weighing 47 1/2 tons each have been trucked to the building site, but their width was only 8 feet. Eighty foot trusses for a Syracuse refrigerating plant weighing 40 tons each were trucked 60 miles to the site, though it took a full 24 hours to cover that distance. This proved more economical than breaking up the truss into smaller elements.

For the above reasons it was felt that the width of the

element should be limited to 10 feet so that, if it should be cast at a factory, rather than an on-site, casting-yard, there would be no transportation problems.

To minimize joints and the problem of waterproofing them, and to minimize construction time and on-site labor, their length, and area covered, had to be as great as possible. These requirements led inevitably to a subdivision along parallels and meridians, with the maximum dimension in the direction of the meridians.

The maximum width limit meant that for a diameter of 306 feet (circumference 961 feet) 96 ten-foot elements would be needed. With this as a basis, it was decided to divide the surface in four equal parts, giving three 40-foot long units, and one 20 foot unit to the upper compression ring. Such a subdivision gave the possibilities of casting two meridional corrugations together for the third element from the bottom and four for the fourth element without ever exceeding the maximum width of ten feet. This greatly reduced the number of elements to handle without increasing their maximum size or weight, and resulted in a total of 4 different types of elements repeated respectively 96, 96, 48, and 24 times for a total of 264 elements, the heaviest weighing 10 tons, well within the lifting capacity of cranes.

The decision of whether to cast the precast elements on the site or at the factory is a decision that must ultimately be made by the contractor. But unless the contractor is very well equipped and has had previous experience in such work, it is foreseen that the better casting and curing equipment available at the factory or permanent casting yard will give greater accuracy and quality.
available for large jobs.

The meridional subdivision also took into account the visual problem of where to stop the openings created to admit light and to reduce the weight of the structure. These openings got progressively smaller towards the top and reached a practical limit in size at the joint between the third forty-foot element and the final shorter one, and as a result the latter element has no openings, providing the necessary break between the light-admitting corrugations and the top skylight. The forty foot division also represented a practical length for the members so that they could span as simply supported beams during construction with no need for scaffolding except at the joint, and no need for special ties to produce arch action.

During erection, the elements would be placed on circular rings located below each joint along the meridions. They would probably be lifted in place by two cranes; a stationary one operating from the center of the dome in the space to be occupied by the central skylight, and a movable one working along the perimeter. Both Nervi's domes were erected by means of one stationary crane at the center lifting relatively light precast concrete elements. In England, a 100-foot diameter dome was erected by using a single stationary crane at the center lifting 16 ton elements that spanned directly from the support to the central skylight.

The joining of the elements to give continuity to the structure can be accomplished by placing in-situ concrete and reinforcing in the troughs formed by the corrugations
between the elements. Sufficient continuity is provided by overlapping the projecting reinforcing rods of two adjacent elements and timing them to in-situ reinforcing and filling the space inbetween with high-strength concrete. Complete bond between the old concrete and the new is assured when the precast concrete in contact with the new concrete is sufficiently rough and has been wet and passed over with a light cement paste. Enough tensile continuity may also be achieved by inserting an inverted u-shaped reinforcing bar through corresponding loops of reinforcing projecting from the adjacent elements. The welding of reinforcing bars is unnecessary because the structure will never develop the high tensile stresses required for such a joint.

C. Architectural considerations

i) Geometry

It was felt that there should be a visual expression of the structure from both the interior and exterior. In other words, not only was the expression of the individual precast element desirable on both sides, but also the depth and spatial qualities of the structure should be expressed on both sides. (In Nervi's two precast domes the marvelous articulation and scale of the interior is lost on the exterior). This does not mean that the exterior surface should be a replica of the interior one, because each surface has a different function and is viewed from a different perspective. The exterior surface must provide only weather-proofing and possibly openings for the passage of light, while
the interior must provide acoustical treatment, both natural and artificial lighting and space for passage of mechanical ducts. Furthermore, the interior is generally seen from a static viewpoint, allowing greater investigation or awareness of detail and rhythms, while the exterior is generally seen from a moving viewpoint, and therefore requires less complexity.

These considerations led to the use of inverted V-shaped units with the bracing diaphragms on the inside. On the outside, this produced a simple corrugated surface with openings to admit light, and on the inside it produced a lower network of parallels and meridians above which one could see, in varying degrees of completeness, the facets of the diaphragms, the sloping sides of the corrugations, and the openings.

ii) Weatherproofing

To be able to express the concrete element on the exterior as well as the interior, the problem of weatherproofing had to be solved. Architects have been fighting with the problem of how to waterproof concrete shell roofs for many years without coming up with a satisfactory solution. A built-up roof hides any articulation the exterior surface may have and represents the very antithesis of a structural surface. A metal roof can be made to repeat the structural system, but is an expensive method of waterproofing and does not expose the actual structure. The many different plastic coatings such as neopreme and hypolan that are now on the market, and can be bought in almost any color desirable, have had varying degrees of success, but are once again relatively expensive to apply and maintain, and they frequently crack,
giving the impression that the structure is also cracking.

On the other hand, concrete can be and has been made impervious to water without the addition of a protective covering. When high water-cement ratios are used, much of the water does not react with the cement and eventually evaporates, leaving small inter-connected air voids throughout the concrete making it rather porous. But when only enough water is used to complete the chemical reaction with the concrete these voids are eliminated. But such a mix is very dry and needs special vibrating equipment to compact it. Experience has proved that by using a reasonably low water-cement ratio with proper vibrating and curing methods, and then by compressing the concrete by prestressing it, the surface is made impervious. When the possibility of developing tensile cracks is present in more than one direction, it is preferable to prestress the member in directions perpendicular to each other. This is a method that has been used for years to prevent tensile cracks in circular concrete tanks. In Florida, one-way prestressing of precast folded-plates has proved sufficient to make the elements completely waterproof. Architects in the North have been hesitant to take this step, because of the fear of freezing and thawing, but the Blakeslee Prestressing Company has recently prestressed in two directions the flat slab roofs for its factory in New Haven, and so far they have proved impermeable to water, snow and ice.

It was therefore decided that the precast concrete elements for the dome should be principally prestressed in
the meridional direction with normal 7-wire strands for pre-tensioning, and transversely, across the corrugations at the secondarily prestressed points uninterrupted by openings, with small diameter cables 6 inches on center. This provided enough prestress to resist all bending and prevent the occurrence of cracks in the elements during erection, and gave a maximum compressive stress in the meridional direction of 800 psi under the combined action of the prestress and the dead load in the finished dome. Transversely, along the surface of the corrugations the secondary prestressing gave an extra compression of 35 psi.

For additional resistance against freezing and thawing, the exposed surface had to be as smooth and durable as possible, both properties which can be obtained by proper vibration and steam curing, and by the use of smooth forms such as steel or fibre-glass.

Once the surface of the concrete elements was assured impervious, the next problem to overcome was the waterproofing of the joints. Since the joints are of necessity formed less carefully controlled than the concrete of the precast element and since prestressing through the joint would be both extremely expensive and ineffective,

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*If a greater assurance of impermeability is desired or required, a 3-coat application of Sodium Silicate (cost $.15 cents per 100 sq. ft.) can be made on the surface. The sodium silicate will develop an impermeable layer 1/2"-1/4" deep when the silica combines with the lime and calcium in the concrete. The substance goes on like water and penetrates into the concrete; it does not form a surface on top of it.
another method of waterproofing the joint had to be found. At first it was thought that by raising the joint above the surface of the surrounding area it would be unnecessary to waterproof it, since water would not collect there. But this proved to be impractical because of the possibility of cracks forming due to snow and ice accumulations. If the raised point were to be waterproofed by metal-flashing or by a mastic or plastic coating this problem would be solved. The geometry of the element eventually forced the joint to occur at the bottom of the corrugation. In this location, the waterproofing of the joint proved even easier and more convincing. Metal flashing was attached under the lip formed by the openings in the corrugations, forming a natural gutter where it is least obtrusive to the concrete surface. This not only provided a very effective method of waterproofing the joint, but also brought the collection of water off the concrete surface, preventing excessive abrasion. Any local tension that might occur due to some unforseen circumstance would then be taken in the unpre-stressed joint where tension cracks would be protected from the exterior by the metal flashing.

**natural lighting**

It was felt that natural lighting coming only from a central skylight and from the perimétral area would be insufficient to light a space with an overall diameter of 426 feet. Nervi tried to do so for his Palazzo dello Sport by using a very large central skylight (52 feet in diameter) and by introducing light between the supports. However, the
sharp contrast between the light sources (especially that between the supports) and the dark surfaces produces glare and prevents one from fully appreciating the structure.

Therefore it was felt that the entire surface of the dome should be able to receive either direct or indirect light. However, as little direct light as possible should be seen from below. Openings in the corrugations were necessary to reduce the dead weight of the structure; therefore it was logical to have these openings admit light.

The problem of how to modulate this light without confusing the basic structural system was eventually solved by the creation of frequent stiffening diaphragms (somewhat larger than statically necessary) that were combined with the circumferential ribs, and by closing the opening with a heat absorbing, glare reducing acrylic plastic panel 1/8 inch thick, giving a light transmittancy of about 50%. The use of a plastic panel was predicated on the need to be able to place it in position at the factory without fear of breakage during transportation and erection. The plastic panel can also be fixed only with clips and an elastic glazing compound, making it considerably cheaper than glass. From model studies, it was decided that the plastic should not diffuse the light, but allow light to penetrate directly creating light patterns on the corrugations and diaphragms. Because of the addition of the diaphragms and circumferential ribs, and because of the depth of the corrugations only about 2% of the surface will allow direct light to reach the viewer, though 33% of the dome surface is open.
iv Artificial light

It was felt that the artificial light should also emphasize the structural system, as did the natural light, and that the actual light source should be seen as little as possible. A continuous fluorescent light running through the diaphragms, and attached at their upper edge, would be seen in only 20% of the meridional ribs. Furthermore, it would be visually interrupted by each diaphragm, reducing still more the percentage seen from below. The diaphragms and corrugations would also act as reflectors, almost eliminating all contrast between them and the light source.

v Heating, air-conditioning, and ventilating system

The main problem to overcome in heating and air-conditioning a space as large as the central space under the dome is one of redistributing the heated or cooled air. Warm air will rise and collect under the bottom surface of the dome, and cool air, as it warms up, will also rise and collect there. Unless this air is brought back down and recirculated, it is impossible to work out an efficient heating or air-conditioning system.

Therefore the heated or cooled air would have to be distributed from local outlets throughout the floor area, and be redistributed through the system by ducts placed along the bottom surface of the dome. Outlets would be located at all level changes within the building, while the return air ducts would run along the meridians through the open space left by the diaphragms.

The total cross-sectional area required for the return-
air ducts (about 100 square feet), permitted the placement of one in one out of every four elements, which helped set up a secondary rhythm similar to that of the supports. The diameter of the duct varied as did the width of the corrugation, taking in air all along its length, so that it was always only about one-fifth the width of the corrugation. When it is painted white it will help diffuse the light coming from the openings.

vi Insulation and acoustical treatment

For an economical solution it was felt that the insulation should serve as form board during the casting of the element, should provide the vapor barrier and help prevent concentration of sounds. Yet the insulation should hide as little of the concrete surface as possible. This was accomplished by keeping the insulation on the corrugations, away from all edges and separated by ribs of concrete. This provided a continuous concrete rib around each opening and along the lower meridional ribs. The surface of the diaphragms, of course, remained exposed concrete, and therefore the most prominent surfaces were all concrete. Should it prove necessary, both the insulation and concrete could be painted. Because of the deep corrugations, sound is automatically diffused to provide adequate acoustical conditions; the insulation can be sound absorbing to provide better conditions.

3. The Casting Process

The width of the precast element was kept consistent with the problems of casting. Where the dimensions involved were
small, a minimum width of 2 inches was provided, gradually increasing to 3-1/4 inches for the largest element. At the location of the principal meridional prestressing a minimum thickness of 3 inches was provided, gradually increasing to 43/8 inches. Concrete placement was greatly simplified by the little space taken up by the prestressed reinforcing. The only mild steel reinforcing would be located in the circumferential rib and around the openings in the corrugations. To ease the placement and compaction by vibration of the lower portion of the element, the outer steel forms are hinged below the lower lip of the opening. When the concrete level reaches that point, the upper portion of the form is positioned, and the pouring of the element proceeds without halting. The inner steel form will take the force of the lateral prestress and will be reinforced around each diaphragm to take this force. The meridional prestress will be taken by normal concrete stress blocks.
The steel form insures a uniform, smooth concrete surface except where a wooden insert is placed at the bottom to provide a good bonding surface for the cast in place concrete. For proper compaction the form should be vibrated from without, and local internal vibrations should be used. Steam curing will provide a resistant concrete surface and will allow the elements to be cast on a daily cycle. (The ultimate strength of the concrete should be 5000 psi). This will allow the use of only one form if the elements can be cast over a period of 96 days or more. This would reduce the cost of the form per element to almost nothing. If greater casting speed is necessary 2, 3, and even 4 forms can be made.

Geometry of the supports and perimetral areas

The geometry of the supports and perimetral areas will vary greatly depending on the use and location of the dome. With prestressing it is possible to redirect the outward thrust of the dome to a vertical one, or, if enough prestress is applied, even to an inward thrust as in the Harrison and Abromowitz dome for Illinois.

However it was felt that the supports should follow the thrust of the dome in order to express the forces acting within the structure. To provide an adequate area between columns it was decided to group four elements for each column. This resulted in a total of 24 columns on about 48-foot centers. These columns provided space for a rainwater duct and for
the return air ducts. These latter ducts are redistributed within the lower compression ring to allow the return air to pass within the fan-shaped supports rather than under them.

The design of the perimetral areas joining the supports was predicted on finding a form that would not hide the dome surface or support area from the outside, and would be easy to cast in place. Hyperbolic paraboloids, the surface of which would be formed by straight tongue and groove planks, solved these requirements. They created high points between supports, accenting the entrance, and low points at the intersection between two paraboloids along the column line. This low point allows the pedestrian to see the grouping of the ribs into the support from the outside as well as the inside. Waterproofing of the perimetral areas would be accomplished by a metal roof that reflects the construction lines of the paraboloids. The stiffening edge ribs of the paraboloids, and the top of the ribs being gathered into the column would project above this surface and be left exposed.
STRUCTURAL CALCULATIONS
Precast concrete elements span between the upper compression ring (section 1.) and lower compression ring (section 8.). They are designated by the letters A, B, C and D.
TOTAL ACCUMULATED D.L. + LL. AT GIVEN SECTIONS

Ave Dead Load 75#/sq. ft.
Live Load 25#/sq. ft.
\[ w = 100#/\text{sq. ft.} \]

\[ W = w\pi r^2 \]

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<tr>
<th>SECTION</th>
<th>[ W = 100 \times 3.14 \times (19.8)^2 ]</th>
<th>[ W ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>[ 314 \times (39.7)^2 ]</td>
<td>123 K.</td>
</tr>
<tr>
<td>2.</td>
<td>[ 314 \times (59.5)^2 ]</td>
<td>496 K.</td>
</tr>
<tr>
<td>3.</td>
<td>[ 314 \times (79.3)^2 ]</td>
<td>1,100 K.</td>
</tr>
<tr>
<td>4.</td>
<td>[ 314 \times (99.2)^2 ]</td>
<td>1,970 K.</td>
</tr>
<tr>
<td>5.</td>
<td>[ 314 \times (119)^2 ]</td>
<td>3,090 K.</td>
</tr>
<tr>
<td>6.</td>
<td>[ 314 \times (139)^2 ]</td>
<td>4,450 K.</td>
</tr>
<tr>
<td>7.</td>
<td>[ 314 \times (159)^2 ]</td>
<td>6,050 K.</td>
</tr>
<tr>
<td>8.</td>
<td>[ 314 \times (159)^2 ]</td>
<td>7,900 K.</td>
</tr>
</tbody>
</table>
The diagonal lines, which are drawn tangent to the surface of the dome at a given section and intersect the line of the corresponding total accumulated weight (W), represent the total meridional stress at that section. To get the meridional stress per foot ($S_l$) divide by the circumference at the given section.

The horizontal lines, which are the horizontal projection of the difference between the meridional stresses of two succeeding sections, represent the circumferential stresses ($S_2$) at a given section. These stresses remain compressive as long as they stay to the left of the dotted curve.

GRAPHICAL ANALYSIS

This graphical method is applicable to domes of any profile whatsoever.
NUMERICAL ANALYSIS

MAX. MERIDIONAL STRESSES AT GIVEN SECTIONS DUE TO D.L. + L.L.

\[ S_1 = \frac{W}{2\pi R \sin^2 \theta} \]

<table>
<thead>
<tr>
<th>SECTION</th>
<th>[ S_1 = \frac{123,000}{2\pi (325)(.06105)^2} ]</th>
<th>16,300 #/ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>[ \frac{496,000}{20,400 (.12187)^2} ]</td>
<td>16,400 #/ft</td>
</tr>
<tr>
<td>2.</td>
<td>[ \frac{1,110,000}{20,400 (.18224)^2} ]</td>
<td>16,400 #/ft</td>
</tr>
<tr>
<td>3.</td>
<td>[ \frac{1,970,000}{20,400 (.24192)^2} ]</td>
<td>16,500 #/ft</td>
</tr>
<tr>
<td>4.</td>
<td>[ \frac{3,090,000}{20,400 (.30071)^2} ]</td>
<td>16,700 #/ft</td>
</tr>
<tr>
<td>5.</td>
<td>[ \frac{4,450,000}{20,400 (.35837)^2} ]</td>
<td>17,000 #/ft</td>
</tr>
<tr>
<td>6.</td>
<td>[ \frac{6,050,000}{20,400 (.41469)^2} ]</td>
<td>17,300 #/ft</td>
</tr>
<tr>
<td>7.</td>
<td>[ \frac{7,900,000}{20,400 (.46947)^2} ]</td>
<td>17,600 #/ft</td>
</tr>
</tbody>
</table>

Note: That the compressive stresses per linear foot increase but slightly as they go towards the lower compression ring and reach a maximum of 17,600 #/ft. These values check out with those calculated by the graphical method.
MAX. CIRCUMFERENTIAL STRESSES AT GIVEN SECTIONS DUE TO D.L. + L.L.

\[ S_2 = R_w \left( \frac{1 - \cos \theta - \cos^2 \theta}{1 + \cos \theta} \right) \]

<table>
<thead>
<tr>
<th>SECTION</th>
<th>( S_2 )</th>
<th>( \frac{1 - \cdot998 - \cdot996}{1 + \cdot998} )</th>
<th>COMRESSIVE STRESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>325 x 100</td>
<td>16,200#/1</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>32500</td>
<td>15,900#/1</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>32500</td>
<td>15,500#/1</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>32500</td>
<td>15,000#/1</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>32500</td>
<td>14,400#/1</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>32500</td>
<td>13,500#/1</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>32500</td>
<td>12,600#/1</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>32500</td>
<td>11,400#/1</td>
<td></td>
</tr>
</tbody>
</table>

Note: That the circumferential stresses steadily decrease as the slope of the shell increases. When \( \theta \leq 51^\circ - 30^\circ \) the stress reaches 0, and with \( \theta > 51^\circ - 30^\circ \) the stresses become tensile.
MAX. WIND STRESSES AT GIVEN SECTIONS

A. MERIDIONAL STRESS:

\[ S_1 = \frac{V R \cos \Psi \cos \Theta (2 - 3 \cos \Theta + \cos^3 \Theta)}{3 \sin^3 \Theta} \]

where \( \Psi = 180^\circ \), \( V = 8 \)

SECTION 2; \( \Theta = 7^\circ \)

\[ S_1 = \frac{8 \times 325 \times 1 \times 0.99255 (2 - 2.97765 + .977817)}{3 \times 0.0018101} \]

\[ = 60 \text{#/1 TENSION} \]

SECTION 8; \( \Theta = 28^\circ \)

\[ S_1 = \frac{8 \times 325 \times 1 \times 0.88295 [2 - 3(0.88295) + 0.688349]}{3 \times 0.10349} \]

\[ = 295 \text{#/1 TENSION} \]

B. CIRCUMFERENTIAL STRESS:

\[ S_2 = \frac{V R \cos \Psi (3 \sin^2 \Theta + 2 \cos^4 \Theta - 2 \cos \Theta)}{3 \sin^3 \Theta} \]

SECTION 2; \( \Theta = 7^\circ \)

\[ S_2 = \frac{8 \times 325 \times 1}{3 \times 0.0018101} \left[ 3(0.0148473) + 2(0.9705) - 2(0.99255) \right] \]

\[ = 260 \text{#/1 TENSION} \]

SECTION 8; \( \Theta = 28^\circ \)

\[ S_2 = \frac{8 \times 325 \times 0.22040}{3 \times 0.10345} \]

\[ = 885 \text{#/1 TENSION} \]

Note that the tension caused by wind is only a small fraction of the compression caused by the dead load of the dome. This insures that there will be compression in the dome at all times.

As has been previously discussed thermal stresses are
mostly taken care of by the natural "breathing" of the dome which can rise and fall just enough to throw off the thermal stresses. Any local tension that might occur by some strange coincidence would be taken up in the un-prestressed joints.

**ALLOWABLE UNIT STRESS**

Considering the geometry of the shell and the use of concrete of a compressive strength of 5000 #/sq.in, it was felt that an allowable unit stress of 800 #/sq.in. would be reasonable.

In the circumferential direction, the concrete area required for constructional and architectural reasons is such that a maximum compressive stress of only about 385 psi. is produced in the concrete due to the meridional stresses in the dome. For instance the total effective concrete area of the element and cast in place ribs joint at SECTION 7. is 392 sq. in. The element is 8' - 9" wide at that point and carries a maximum compressive stress of 17,300 p.s.i. Therefore the unit stress in the concrete is:

\[ \frac{8.75 \times 17,300}{392} = 385 \text{ psi.} \]

At Section 3. the unit stress is:

\[ \frac{3.75 \times 16,400}{162} = 380 \text{ psi.} \]

This leaves a possible additional compressive force induced by meridional prestressing of 420 #/sq.in. This is actually a very conservative estimate because the prestress force, instead of increasing the tendency of local buckling proportional to its prestress, will instead give the concrete
extra strength to resist local bending. This is because any tendency of the number to buckle is resisted by the prestress force which remains axial through the member at all times instead of producing an ever increasing lever arm due to local bending which would lead to buckling.

CHECK FOR ERECTION STRESSES IN TYPICAL PRECASE ELEMENT

During erection, element C is the critical element because of its relatively shallow depth (31 inches at mid-span) and length (39' - 9").

For approximate analysis it is assumed that the element is of constant cross-section, and works as a simply supported beam between supports 38 feet on center during transportation and erection.

\[ l = 38' - 0" \]

\[
\begin{align*}
\text{WG} & : 10,000 \# \\
\text{DL} & : 1,000 \# \text{ (cast in place concrete)} \\
\text{LL} & : 3,500 \#
\end{align*}
\]

GIRDER MOMENT \( M_G = \frac{Wg1}{8} \cdot \frac{10,000 \times 38}{8} = 47.5 \text{ K} \)

TOTAL MOMENT \( M_T = \frac{114500 \times 38}{8} = 69 \text{ K} \)

MIN. PRESTRESS NECESSARY TO TAKE CARE OF BENDING

\[
F = \frac{M_T}{0.65} = \frac{65 \times 12}{0.65 \times 31} = 41 \text{ K.}
\]

MAXIMUM PRESTRESS PERMISSIBLE

From the previous discussion we know that a prestress
force of 420 pounds per square inch of concrete area can be applied without fear of overstressing the concrete.

The concrete area at the minimum section is 120 sq. in.

\[ F = 420 \times 120 = 50.5 \text{ K} \]

Therefore it is possible to take care of all bending stresses due to handling by the use of prestressing alone.

**STEEL AREA**

Assuming \( F = 50K \), then:

\[ A_s = \frac{F}{E_s} = \frac{50}{140} = 0.357 \text{ sq. in.} \]

= four 3/8" 7-wire uncounted strands for pre-tensioning

A prestress force coinciding with the center of gravity of the concrete of the element (16.5 inches from the bottom fibre) can be obtained by placing half the prestressing steel 1-3/4 inches from the top surface of the corrugations and half 3-3/4 inches from the bottom. Because of the slight curvature of each element, this can be accomplished by holding up the prestressing steel at its third points.

**TRANSVERSE PRESTRESS**

The meridional prestress is of course essential to make the concrete waterproof, as has been discussed previously. To insure against possible appearance of cracks in the transverse direction, another secondary prestressing force should be applied. This can be furnished by placing 0.182 inch cables at 6 inch centers transversely across the corrugations at the point uninterrupted by the openings, and sufficiently stressed to produce a transverse compressive force of 35
pounds per square inch of concrete. This prestressing steel is enough to take care of any vierendeel action that might be caused by the geometry of the element.

CIRCUMFERENTIAL STRESSES

The circumferential stresses in the finished dome will be taken by the stiffening ribs provided in the precast element. For instance at Section 7, we have a circumferential stress of 12,600 pounds per foot. Since the circumferential ribs are spaced 7' - 6" o.c. at that point, the rib must resist:

\[ 12,600 \times 7.5 = 94,500 \text{ # compression.} \]

Assuming column action we need 94.5 sq.in. of concrete and 4 #5 bars tied 8 inches o.c. This provides for all the circumferential stresses through the horizontal stiffening rib, though in effect some would also be taken by the continuous surface of the corrugations between openings. Therefore, this design is on the safe side.

The concrete area provided for each circumferential rib can be calculated in the same manner.

To provide enough tensile continuity for the dome to take care of possible tension caused when the temporary supports are taken away during construction, or for other unforeseen reasons, the circumferential steel reinforcing of each element is overlapped with the corresponding steel of the adjacent element, and when the distance between circumferential ribs becomes greater than 4' - 0" o.c., an additional tie is added at the center.
DESIGN OF TOP AND BOTTOM COMPRESSION RINGS

Stress to be resisted by top compression ring:

\[ S_t \times r = 16,300 \times 20 = 326,000\# \]

Using \( f_c^t = 5000 \text{ psi} \), \( f_s^t = 50,000 \text{ psi} \).

Concrete and steel areas:

\[ 326,000 = 900A_c + 16,000A_s \]

\( A_c = 238 \text{ sq. in.} \)

\( A_s = 7 \text{ sq. in.} \)

Stress to be resisted in bottom compression ring:

\[ 17,600 \times 153 = 2,700,000\# \]

Concrete and steel areas:

\[ 2,700,000 = 900A_c + 16,000A_s \]

\( A_c = 2,000 \text{ sq. in.} \)

\( A_s = 50 \text{ sq. in.} \)

DESIGN OF COLUMNS:

Assuming each column takes the thrust of 4 ribs, it must resist:

\[ 17,600 \times 40 = 704,000\# \]

In addition it must take the load from the perimetal areas which is 246,000#.

\[ 950,000 = 900A_c + 16,000A_s \]

\( A_c = 880 \text{ sq. in.} \)

\( A_s = 10 \text{ sq. in.} \)

But because the columns must house ventilation ducts and water drainage, the concrete area provided was considerably greater. The final design, calling for 1200 sq. in. of concrete and 16 #8 bars giving 12.84 sq. inches is shown here:
The design of the hyperbolic-paraboloids has been limited to an estimate of the concrete and steel areas needed, based on current practice. Because of the relatively conservative nature of the spans a three and a half inch concrete thickness reinforced with two layers of 6 x 6 - 6/6 wire mesh should be sufficient to resist all stresses except along the edges. Along the exterior edge a rib 18 inches deep is considered sufficient for stiffness. At the inter-
section of two paraboloids a naturally effective beam is formed by the V of the joining surfaces, but an extra thickening of the concrete to 1'-0" is anticipated.

APPROXIMATE DESIGN OF FOOTINGS

(Design would vary greatly depending on soil conditions)

Dead + Live Load of the dome, supports + perimetral area = 14,000 kips

\[ f_c = 1200 \text{ psi} \]

\[ \theta = 45^\circ \]

It is assumed that a continuous prestressed footing will be used to redirect the diagonal forces perpendicularly in the ground. To take care of bending and concentrated stresses at the support extra concrete area will be provided.

Prestress required:

\[ F = \frac{W}{2\pi \cot\theta} = \frac{14,000}{6.28} = 2,230 \text{ kips} \]

Total prestress after transfer:

\[ F_0 = \frac{F}{0.8} = \frac{2,230}{0.8} = 2,787.5 \text{ kips} \]

\[ A_s = \frac{3800}{165} = 23 \text{ sq. in.} \]

\[ A_c = \frac{F_0}{f_c} = \frac{3800}{1200} = 3.160 \text{ sq. in.} \]

\[ = 2' - 6" \times 9' - 0" \]

Check:

Assuming average bearing capacity of soil is 5000 psf.

and average bearing area is 3 feet around circumference:

\[ 3 \text{ ft} \times 1200 \text{ ft} = 3,600 \text{ sq. ft.} \]

\[ 3,600 \times 5 = 18,000 \text{ bearing capacity } \text{O.K.} \]
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