3.021J / 1.021J / 10.333J / 18.361J / 22.00J Introduction to Modeling and Simulation
Spring 2008

1.021/3.021/10.333/18.361/22.00
Introduction to Modeling and Simulation
Homework assignment # 2
Handed out: 3/6/08
Due: 3/13/08

March 6, 2008

1. The exhaust tower of a nuclear plant is a hollow surface of revolution with parabolic shape with the outer radius given by $R(x) = R_0\left[1 - \frac{1}{2}\left(\frac{x}{L}\right)^2\right]$, where $R_0 = 50$m is the radius at the base $x = 0$ and $L = 70$m is the height of the tower. The wall has a uniform thickness $t = 1$ m. The tower is made of reinforced concrete with a mass density $\rho = 2700$ Kg.m^{-3} and a Young's modulus $E = 30$ GPa in compression. The purpose of this problem is to determine approximations of the displacements, strains and stresses in the tower as a result of its weight **using the weak formulation** of the problem derived previously (see e.g. Problem 1, HA1 or class notes). The governing differential equation is:

$$\frac{d}{dx}\left(EA(x)\frac{du}{dx}\right) + q(x) = 0 \quad 0 < x < L$$

The boundary conditions are:

$$u(0) = 0, \quad \left(EA(x)\frac{du}{dx}\right)\Big|_{x=L} = 0$$

Here, A(x) is the area of the cross-section of the tower and $q(x)$ is the weight per unit length along the axis of the tower.

- Propose a family of functions $\phi_j(x), j = 0, N$ that are suitable to use in the approximation function $U_N(x) = \phi_0(x) + \sum_{j=1}^{N} C_j\phi_j(x)$ of this problem. For the weight functions, use the same approximation (Galerkin weights).
- Implement in Mathematica an N-term approximation of the problem using your approximation functions.
- Compute 1 and 2-term approximations of the problem solution by hand and verify that you obtained the same displacement function $U_1(x)$ and $U_2(x)$ as Mathematica

- Plot the solution for $N = 1, 2, 3, 4$

2. Redo the previous problem using Ritz method. The potential energy of the system is:

$$\Phi(w) = \int_0^L \frac{1}{2} EA(x)\big(\frac{du}{dx}(x)\big)^2 dx - \int_0^L q(x)u(x)dx$$

3. Redo problem 3 of HA1 using the weak formulation of the problem and choosing suitable approximation functions and Galerkin weight functions.

4. Redo problem 4 of HA1 using the Ritz method. The potential energy of the beam is given by the expression:

$$\Phi(w) = \int_0^L \frac{1}{2} EI\big(\frac{d^2w}{dx^2}(x)\big)^2 dx - \int_0^L q(x)w(x)dx$$

5. (Not for grade, however this enters the test, learning-by-doing is most advisable) Derive the finite element stiffness matrix and force vector for a generic bar element with quadratic interpolation (3 nodes). Assume constant properties (area, modulus). With this information, compute a single-element solution of Problem 1).