1.011 Project Evaluation
Time & Money:
The Principle of Equivalence

Carl D. Martland

Engineering Economics, Chapter 3, Sections 3.1-3.10

1. Equivalence in Cash Flows
2. Discounting Cash Flows
3. Discrete Annuities

Concept of Equivalence

- ***"Economic equivalence is established, in general, when we are indifferent between a future payment, or series of future payments, and a present sum of money."*** *EE p. 72*
- Why is this critical?
  - We often have various options expressed as time streams of costs and benefits expressed in financial terms. Which is the best?
- Why does this get complex - and interesting?
  - What is equivalent for you might not be for me!
    - This is often the basis for negotiation & planning.

Using Equivalence

- If we have an appropriate discount rate, we can convert any arbitrary stream of cash flows to various equivalent (but more easily understood) cash flows:
  - P = present value
  - F = future value at time t
  - A = annuity of A per period for N periods
- To make these conversions, we first need to understand the "time value of money"

Time Value of Money

$1 today is worth more than $1 dollar next year

How much more depends upon the opportunities for using or investing that $1

If we invest in a government bond earning i% per year, then our $1 will be worth $(1+i) at the end of one year and $(1+i)^t$ at the end of t years

Likewise, earning $1 at the end of year t is worth $1/(1+i)^t$ today

Present Value

The Present Value of receiving cash C_t in a future year t is obtained by discounting the net benefits at an appropriate discount rate:

$$\text{PV of } C_t = C_t/(1+i)_t$$

The PV for a series of cash flows is obtained by summing the discounted benefits for each year:

$$\text{PV of Project} = \Sigma[C_t/(1+i)_t]$$

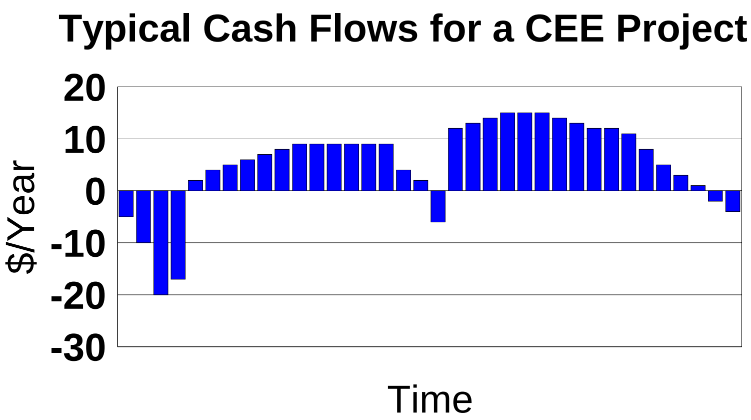
PV of $1.00 Received at Time t

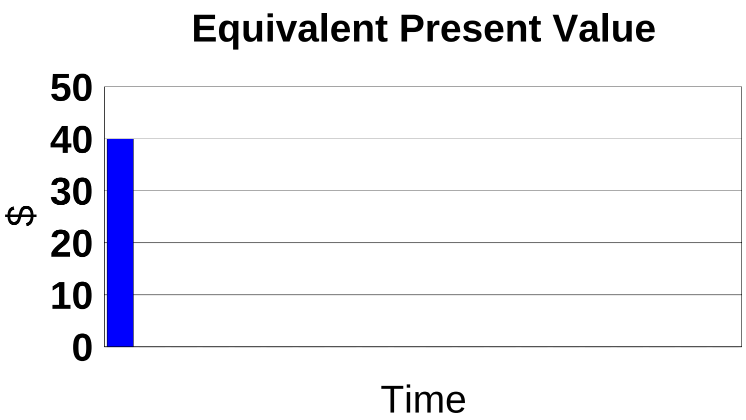
	5 Yrs	10 Yrs	20 Yrs	50 Yrs	100 Yrs
1%	0.95	.91	0.82	0.61	0.37
5%	0.78	0.61	0.38	0.088	0.0076
10%	0.62	0.038	0.15	0.0085	0.000072
20%	0.40	0.16	0.026	0.00011	0.00000001

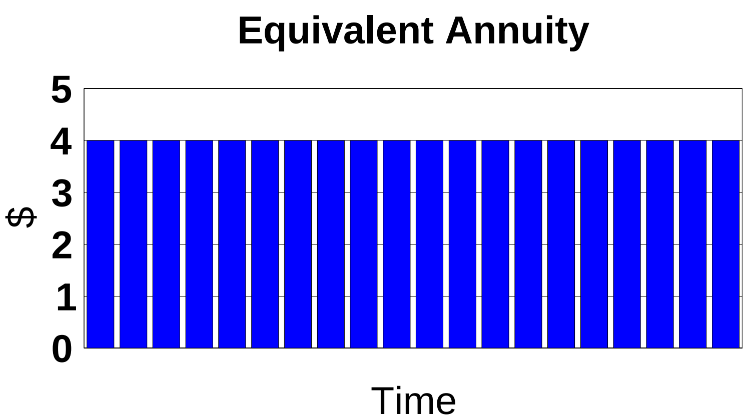
Meaning of PV of a Time Stream of Cash Flows

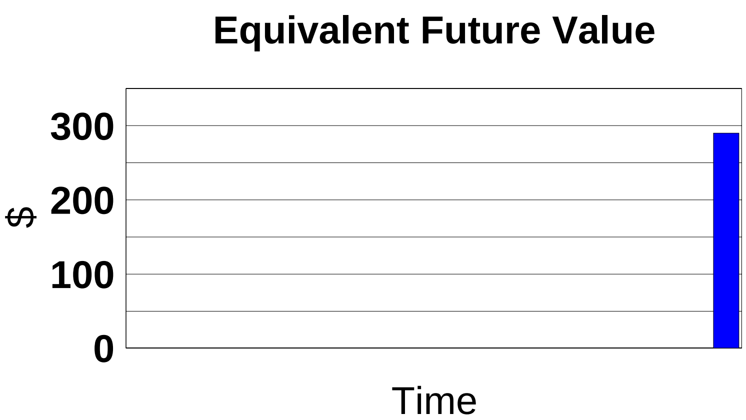
- PV > 0
  - This project is better than making an investment at i% per year for the life of the project
  - This project is worth further consideration
- PV < 0
  - This project does not provide enough financial benefits to justify investment, since alternative investments are available that will earn i% (that is the meaning of "opportunity cost")
  - The project will need additional, possibly non-cash benefits to be justified

Equivalence of Cash Flows









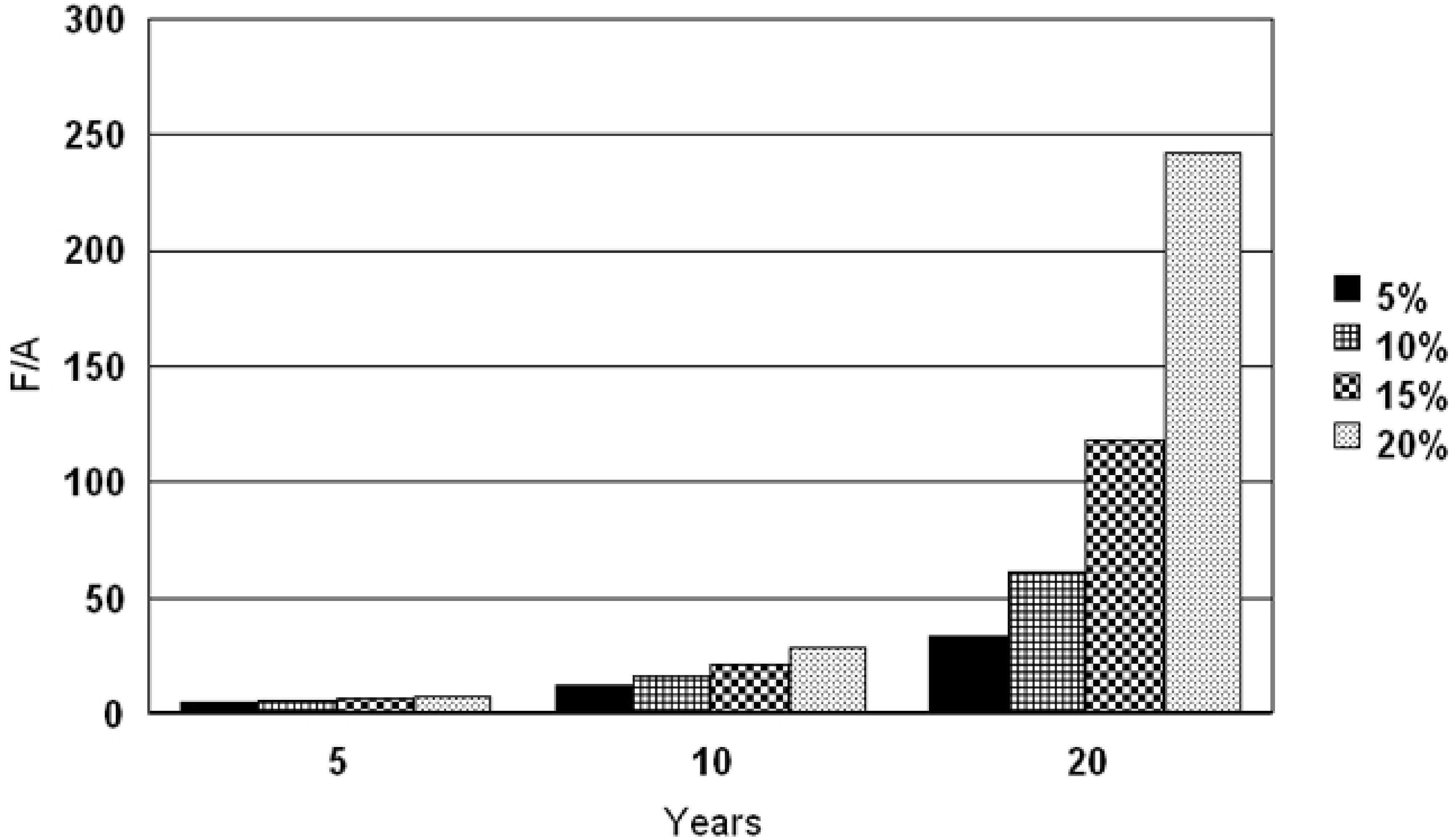
Equivalence Factors

- [F/P,i,N] = future value F after N periods given present value P and discount rate i
- [P/F,i,N] = present value given future value F, i, & N
- [F/A,i,N] = "uniform series compound amount factor"
  - How large will my IRA be after contributing $A at i% for N years?
- [A/F,i,N] = "sinking fund payment"
  - Annual savings to have a downpayment of a house in N years
- [A/P,i,N] = "capital recovery factor"
  - What will the mortgage payments be?
- [P/A,i,N] = "uniform series present worth factor"
  - My business makes $A/year - should I sell for $X?

Equivalence Factors - How Do I Get Them?

- Use the tables at the back of the book
- Use a financial calculator at a bank's web site
  - (e.g. www.boston.com to get the Boston Globe, then go to real estate and look at mortgage loans)
- Use the financial functions on a spreadsheet
- Create your own spreadsheet
- Just remember the basics:
  - $P = F/(1+i)_N$

Uniform Series, Compound Amount Factor [F/A,i,N]



Uniform Series, Capital Recovery Factor [A/P,i,N]