## 1 Circuits (20 points)

Consider the following circuit where the resistance $R$ is in the range $0 \leq R \leq \infty$.


Part a. Determine $I_{1}$ if $R=0 \Omega$.
$I_{1}=\square$

Part b. Determine $V_{1}$ if $R=1 \Omega$.
$V_{1}=\square$

## 3 State Machine Behaviors (20 points)

Consider the following state machine.

```
class mySystem(sm.SM):
    startState = (0,0)
    def getNextValues(self, state, inp):
        (y1,y2) = state
        y0 = inp + y1 + y2
        return ((y0,y1), y0)
```

Part a. An instance of mySystem can be represented by a block diagram that contains only adders, gains, and/or delays. Draw this block diagram in the space below.
$\square$

Part b. Determine the result of the following Python expression.
mySystem().transduce ([1, 0, 0, 0, 0, 0, 0, 0])
Enter the result in the box below.

Part c. Determine the magnitude of the dominant pole of the system represented by mySystem (), and enter it in the box below.
magnitude of dominant pole: $\square$

Part d. In the space below, write a new subclass of sm.SM called newSystem. Instances of newSystem should have a system function $H$ given by

$$
H=\frac{Y}{X}=1-\mathcal{R}^{3}
$$

```
class newSystem(sm.SM):
```


## 4 Motor Control (20 points)

The following circuit is a proportional controller that regulates the current through a motor by setting the motor voltage $V_{C}$ to

$$
V_{C}=K\left(I_{d}-I_{o}\right)
$$

where $K$ is the gain (notice that its dimensions are ohms), $I_{d}$ is the desired motor current, and $I_{o}$ is the actual current through the motor. Although $K$ and $I_{d}$ are not explicit in this circuit, their values can be determined from the resistor values below (see part b).


Part a. Consider the circuit inside the dotted rectangle. Determine an expression for $V_{1}$ as a function of $I_{o}$.
$\square$

Part b. Determine the gain $K$ and desired motor current $I_{d}$.
$K=\square$
$I_{d}=\square$

## 5 Members of the Club (20 points)

We would like to make a class to represent a club. A club has a list of members and a scoring function, which takes a member as an input and returns a numerical value. When a member is proposed for addition to the club, it is only added if its score is greater than the average score of the current members.

### 5.1 Join the club

We would like to make a club of basketball players, who are scored on their height. Here is a simple BallPlayer class.

```
class BallPlayer:
    def __init__(self, name, height):
        self.name = name
        self.height = height
    def getHeight(self):
        return self.height
    def __str__(self):
        return 'BallPlayer('+self.name+', ' + str(self.height) +')'
```

The Club class has an __init__ method that takes two arguments: an initial member, and a scoring function (that takes a a single member as input and returns a numerical value).

Write an expression below that will create a new club. The first member is person named 'Wilt' whose height is 84 inches. The scoring function for club members should return their height.
$c=$

Now, imagine that we try, successively, to add the following new players, whose names and heights are listed below, to club c:

- 'Shorty', 60
- 'Walt', 86
- 'Stilt', 90
- 'Larry', 85

List the resulting membership of club $c$.
$\square$

### 5.2 Implementation

Fill in the definition of the Club class. Use a list comprehension in the averageScore method.

```
class Club:
```

    def __init__(self, firstMember, scoreFunction):
        self.members \(=\) [firstMember]
        self.scoreFunction \(=\) scoreFunction
    \# Returns average score of current members.
    def averageScore(self):
    \# Adds member if it meets the criterion. Returns True if the
    \# member was added and False, if not.
    def proposeMember(self, member):
    
### 5.3 Histogram

Write a procedure to compute a histogram of the member scores, that is, a count of how many members' scores fall within some specified ranges. We specify ranges by a list of $N$ upper bounds. For example, the following bound list $(N=3)$ :

```
[3, 9, 12]
```

specifies the following $N+1=4$ ranges:

```
x < 3, 3 <= x < 9, 9 <= x < 12, 12 <= x
```

Given a list of scores that is already sorted from smallest to largest, such as: [1, 1, 4, $5,7,15]$ the resulting histogram would be: [2, 3, 0, 1] That is, 2 scores less than 3,3 scores between 3 and 9,0 scores between 9 and 12 and 1 score above 12 .

```
>>> histogram([1, 1, 4, 5, 7, 15], [3, 9, 12])
```

$[2,3,0,1]$
The output should always have $N+1$ values; values should be zero if there are no scores in the appropriate range.

```
def histogram(scores, bounds):
```


## 1 Short-Answer Questions (10 points)

Part a. Consider the following circuit.


Determine if the following equations and/or statements are

- Always True - i.e., true for all possible values of the resistors $R_{2}-R_{6}$ and voltage $V_{0}$ or
- NOT Always True - i.e., false for some or all resistor and voltage values.

Check the appropriate box for each of the following:
NOT
Always


Always
True


If $\frac{R_{2}}{R_{4}}=\frac{R_{3}}{R_{5}}$ then $i_{6}=0$
$\square$

$i_{2}+i_{3}=i_{4}+i_{5}$
$\square$

$i_{2}+i_{6}=i_{3}$
$\square$

$e_{1}=\frac{R_{4}}{R_{2}+R_{4}} V_{O}$
$\square$
$\square$ If $i_{6}=0$ then $\frac{R_{2}}{R_{2}+R_{4}}=\frac{R_{3}}{R_{3}+R_{5}}$

Part b. Our goal is to design a dimmer for a light bulb that can be modelled as a constant resistor of $10 \Omega$. We have a single 10 V power supply. Our first design uses a $10 \mathrm{k} \Omega$ potentiometer, with the goal of controlling the current through the lamp so that the current is proportional to the potentiometer setting, with a maximum current of 1 A .


Briefly (using fewer than 50 words) describe problems with this circuit.
$\square$
Suggest a better circuit. Draw it in the following box.
$\square$
Explain briefly why your circuit is better.
$\square$

## 6 Circuits (20 points)

Consider the following circuit.


Part a. Find $i_{C}$ if $I_{A}=0$.
$i_{C}=\square$

Part b. Find $i_{B}$ if $I_{A}=5 \mathrm{~A}$.
$i_{B}=\square$

Part c. Find $I_{A}$ so that $i_{C}=0$.
$I_{A}=\square$

Part d. Consider the following op-amp circuit.


Fill in the values of $R_{1}$ and $R_{2}$ required to satisfy the equations in the left column of the following table. The values must be non-negative (i.e., in the range $[0, \infty]$ ). If the equation is impossible to implement with non-negative resistors, then write "impossible" for both resistor values.

|  | $R_{1}$ | $R_{2}$ |
| :---: | :--- | :--- |
| $V_{o}=2 V_{2}-2 V_{1}$ |  |  |
| $V_{o}=2 V_{2}-V_{1}$ |  |  |
| $V_{o}=V_{2}-2 V_{1}$ |  |  |
| $V_{O}=4 V_{2}-2 V_{1}$ |  |  |

## 3. Circuits (30 / 100 points)

## Motor driver

When we built the robot head, we made the motor move in both directions by connecting one side of the head motor to an op amp circuit and the other side to a buffered voltage divider that produced +5 V . This method limited the peak speeds of the motor because the full +10 V that is available from the power supply never appeared across the motor.
Our goal is to build two circuits, one to drive each of the two motor wires. Let x represent the input voltage and let $y_{1}$ and $y_{2}$ represent the voltages applied to the two motor wires. Assume that you have a single 10 -volt power supply, so that only +10 V and 0 V are available.


Question 13: Recall the supply voltage constraints: ALL voltages (including $x, y_{1}$ and $y_{2}$ ) must be between $\mathbf{0 V}$ and $\mathbf{+ 1 0 V}$. Determine expressions for $y_{1}$ and $y_{2}$ so that the voltage across the motor is given by

$$
y_{1}-y_{2}= \begin{cases}10 & \text { if } x=10 \\ -10 & \text { if } x=0\end{cases}
$$

and both $y_{1}$ and $y_{2}$ have the form $y_{i}=m_{i} x+b_{i}$, where $m_{i}$ and $b_{i}$ are constants.

Question 14: Design circuits to implement your solutions to question 13 using resistors, op amps, and a single +10 V power supply. You can assume that $x$ is buffered (i.e., it is the output of an op amp). Draw the circuits and label them clearly.

## Analysis

A "bridge" circuit consisting of five resistors is connected to a 12 volt source as shown below.


Bridge circuit

Part 2: Op Amps. Consider the following circuit:

where all of the resistors have the same value $R=1 \mathrm{k} \Omega$.

If $V_{i}=3 \mathrm{~V}$, then $V_{o}=$

If $V_{i}=7 \mathrm{~V}$, then $V_{o}=$

If $V_{i}=9 \mathrm{~V}$, then $V_{o}=$

## Name:

Part 1: Op Amps. Assume the op-amps in the following circuit are "ideal."


Determine the current $I_{x}$ when $V_{1}=1 \mathrm{~V}$ and $V_{2}=2 \mathrm{~V}$.

$$
I_{x}=
$$

Determine the voltage $V_{A}$ when $V_{1}=1 \mathrm{~V}$ and $V_{2}=2 \mathrm{~V}$.

$$
V_{A}=
$$

Determine a general expression for $V_{A}$ in terms of $V_{1}$ and $V_{2}$.
$V_{A}=\quad * V_{1}+\quad * V_{2}$

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### 6.01 Introduction to Electrical Engineering and Computer Science I

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