6.01 Midterm 2 Solutions: Fall 2009

Name:	Section:

Enter all answers in the boxes provided.

During the exam you may:

- read any paper that you want to
- use a calculator

You may not

- use a computer, phone or music player

Because of this midterm, we will not have software labs this week. Instead, we will provide lecture during your software lab time slot:

section 1	November 10	11:00am
section 2	November 10	2pm

For staff use:

1.	/20
2.	/30
3.	/10
4.	/30
5.	/10
total:	/100

1 Python Classes (20 points)

Recall that a rational number is any number that can be written in the form $\frac{a}{b}$ where both a and b are integers, and $b \neq 0$. In this problem you will write a Python class `RationalNumber` that represents a rational number.

Part a. [10 points] First write a helper procedure (that is not in the class), `gcd(a,b)`, which returns the greatest common divisor of the two input integers. You can assume that both `a` and `b` are non-negative. Don't worry about efficiency for this procedure.

Recall that that you can use the modulus operator (%) to tell whether one number evenly divides another: `a%b == 0` will be `True` only if `b` divides evenly into `a`.

```
def gcd(a, b):
    if b == 0:
        return abs(a)
    else:
        return gcd(b, a%b)

or

def gcd(a,b):
    if a==0 or b == 0:
        return max(a,b)
    for i in range(min(abs(a),abs(b)),0,-1):
        if a%i==0 and b%i==0:
            return i
```

Part b. [10 points] Now write the `RationalNumber` class. **It should contain members for both the numerator and denominator.** You may use the gcd procedure from part a, even if you haven't completed it.

The class should have the following methods:

- `__init__(self, numer, denom)`
- `reduce(self)`: reduces the fraction to its lowest terms.
- `__mul__(self, other)`: returns a new `RationalNumber` instance representing the **reduced** product of `self` and `other`.
- `__add__(self, other)`: returns a new `RationalNumber` instance representing the **reduced** sum of `self` and `other`.

```
class RationalNumber:
    def __init__(self, numer, denom):
        self.numer = numer
        self.denom = denom

    def reduce(self):
        d = gcd(self.numer, self.denom)
        self.numer /= d
        self.denom /= d

    def __mul__(self, other):
        z = RationalNumber(self.numer*other.numer,
                           self.denom*other.denom)
        z.reduce()
        return z

    def __add__(self, other):
        z = RationalNumber(self.numer*other.denom+
                           other.numer*self.denom,
                           self.denom*other.denom)
        z.reduce()
        return z
```

2 System Functions (30 points)

Part a. [4 points] A system has poles $-k_1 + \sqrt{k_1^2 - 2k_2}$ and $-k_1 - \sqrt{k_1^2 - 2k_2}$. Which of the following system functions might represent the system?

$$\frac{Y}{X} = \frac{k_1 + R}{k_2 + k_1 R + 0.5R^2} \qquad \frac{Y}{X} = \frac{k_1 + k_2 R}{0.5 + k_1 R + k_2 R^2}$$

$\qquad\qquad$ (a) $\qquad\qquad\qquad\qquad\qquad$ (b)

$$\frac{Y}{X} = \frac{1 + R}{0.5 + k_1 R + k_2 R^2} \qquad \frac{Y}{X} = \frac{1 + R}{k_2 + 0.5k_1 R^2}$$

$\qquad\qquad$ (c) $\qquad\qquad\qquad\qquad\qquad$ (d)

Circle all correct answers: (a)   (b)   (c)   (d)

Part b. [6 points] Write the system as one difference equation (if you circled multiple correct systems, choose one of them for the rest of the exercise).

(a) $y[n] = \dfrac{1}{k_2}(k_1 x[n] + x[n-1] - k_1 y[n-1] - 0.5y[n-2]$

(b) $y[n] = 2(k_1 x[n] + k_2 x[n-1] - k_1 y[n-1] - k_2 y[n-2])$

(c) $y[n] = 2(x[n] + x[n-1] - k_1 y[n-1] - k_2 y[n-2])$

(d) $y[n] = \dfrac{1}{k_2}(x[n] + x[n-1] - 0.5k_1 y[n-2])$

Part c. [16 points] For this part, you will write a state machine class that represents the system function you chose in part b.

i. **[6 points]** Describe the state respresentation you will use. If you use a list or tuple for your state, say what each element of the list or tuple represents.

A tuple (or list) containing $x[n-1]$, $y[n-1]$, and $y[n-2]$.

ii. **[10 points]** Fill in the following state machine definition to represent the system function you chose in part b. Assume that the system starts at rest.

The following state machine shows answers for each of the four posssible system functions from part a.

```
class SystemSM(sm.SM):
    k1 = 0.5
    k2 = 0.25

    startState = (0.0, 0.0, 0.0);

    def getNextValues(self, state, inp):
        (xR, yR, yRR) = state
        # y = (self.k1*inp + xR - self.k1*yR - 0.5*yRR)/k2        # (a)
        # y = 2*(self.k1*inp + self.k2*xR - self.k1*yR - self.k2*yRR) # (b)
        y = 2*(inp + xR - self.k1*yR - self.k2*yRR)                 # (c)
        # y = (inp + xR - 0.5*self.k1*yRR)/k2                        # (d)
        return ((inp, y, yR), y)
```

Part d. [4 points] Will the system, with the gains used in part c ($k_1 = 0.5, k_2 = 0.25$), be

converging or diverging? converging

oscillatory or monotonic? oscillatory

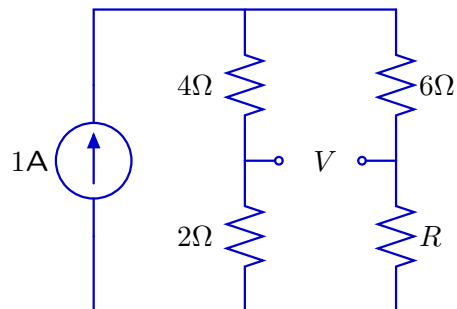# 3 Circuits (10 points)

Part a. [5 points] In the circuit below, find the voltage V. You may estimate it to within half a volt.
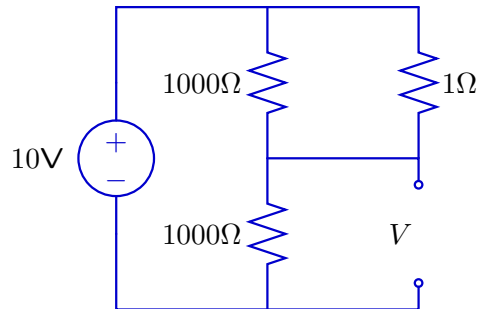

V = 10V (approximately)

Part b. [5 points] In the circuit below, find R, such that the voltage difference V is zero.

R = 3Ω

4 Motor control (30 points)

Part a. [10 points] In the following circuit diagram, assume that voltages V_L and V_R are voltages from the left and right eye sensors which increase when the light level at that eye increases. The power and ground pins of the both op-amps are connected to 10V and ground, respectively.
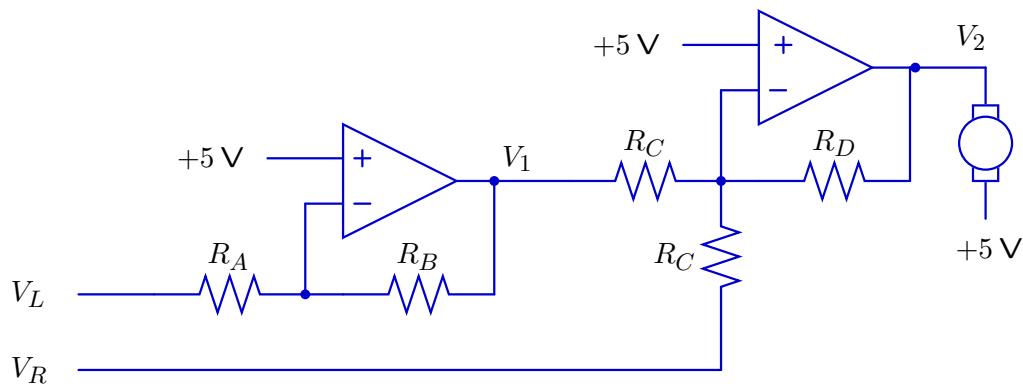
Find an expression for V_1 (the output from the first op-amp) in terms of R_A, R_B, and V_L. You should make the ideal op-amp approximation.

$$V_1 = 5 + \frac{R_B}{R_A} 5 - \frac{R_B}{R_A} V_L \quad \text{or} \quad \frac{R_A + R_B}{R_A}(5 - V_L) + V_L$$

Part b. [2 points] Choose resistor values for R_A and R_B such that $V_1 = 10 - V_L$.

$R \quad = R$
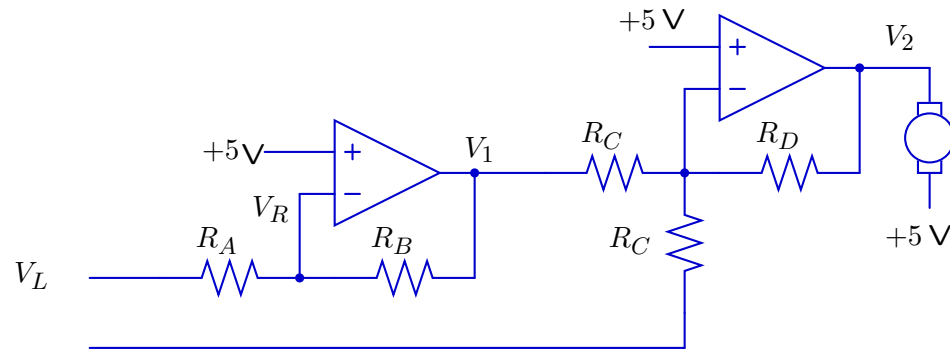
Part c. [10 points] The figure above is the same circuit as the one on the previous page. Assuming that $V_1 = 10 - V_L$, find an expression for V_2 in terms of R_C, R_D, V_L, and V_R.
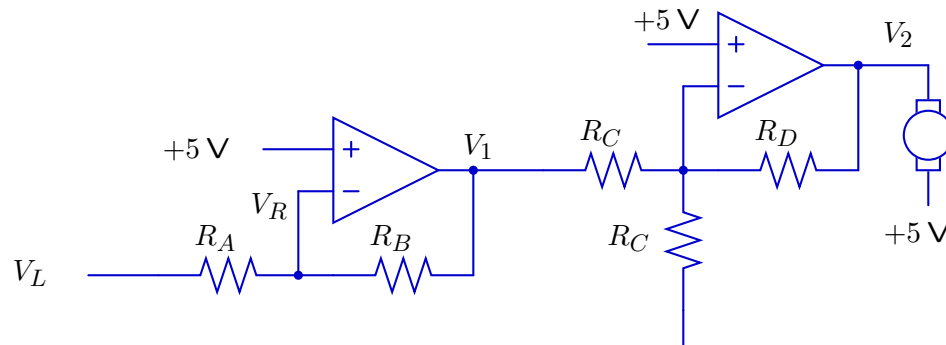
$$V_2 = \frac{R_D}{R_C}(V_L - V_R) + 5$$

Part d. [4 points] Define the gain of the circuit to be k when the voltage difference across the motor is written $V_2 - 5 = k(V_L - V_R)$. Choose values for R_C and R_D that give a gain of 2.

$R_C = R$

$R_D = 2R$

Part e. [4 points] The figure above is the same circuit as the one on the previous two pages. Circle the correct true/false answer for each of the following. In each question, the gain is as defined in part e.

i. It is not possible to have a gain of less than 1 in this circuit.   T   (F)

ii. The gain of the circuit can be changed by changing just one resistor value.   (T)   F

iii. When the gain gets large, the controller will start to oscillate.   (T)   T

iv. When the gain gets large, the controller will stop responding (as the reference design in design lab 9 did with a gain of 10).   T   (F)

5 Op-amps (10 points)

Part a. [6 points] Consider an op-amp connected as shown below. On the provided axes, plot the output voltage of the op-amp as a function of the input to the positive terminal of the op-amp, V_i. You should use the voltage-controlled voltage source model of an op-amp, as described in the course notes. Your answer should fit on the axes provided.
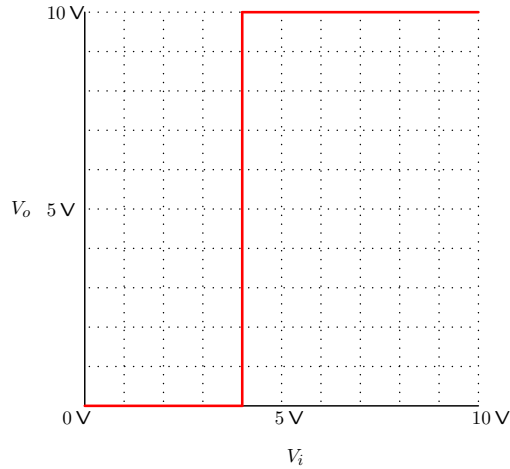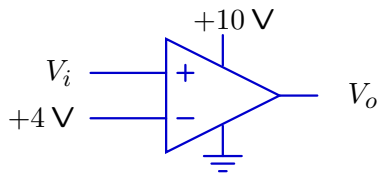
Part b. [4 points] In the circuit below, each op-amp is connected to a backlight for an LED display (shown as a light bulb). If the voltage across the light is greater than 5V, the bulb will light up, otherwise it will be off. On the provided axes, plot the number of lit backlights as a function of the input voltage, V_i. Assume that the resistor values have been chosen to provide the labeled voltages.
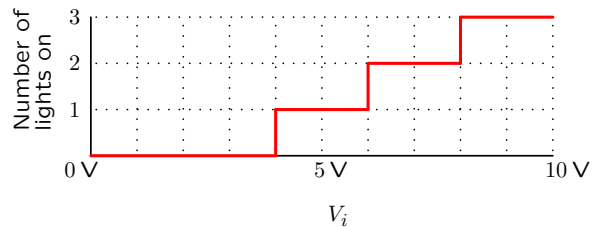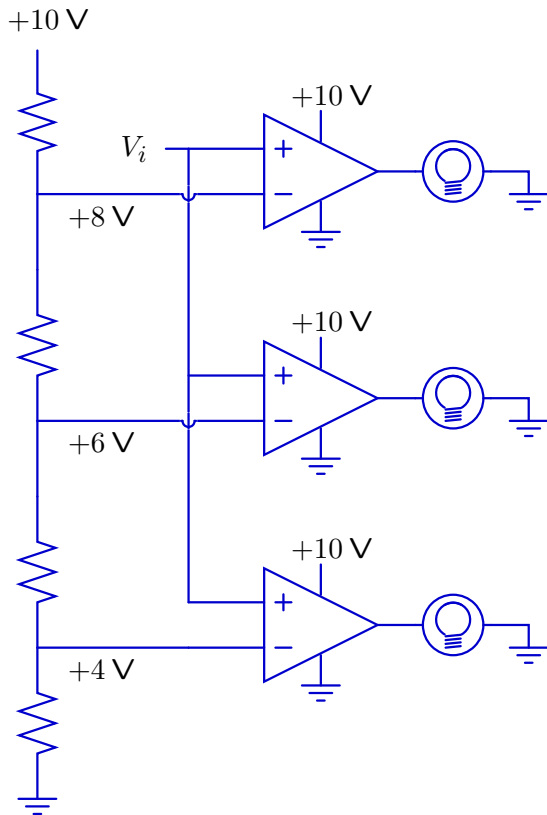
6.01 Introduction to Electrical Engineering and Computer Science I
Fall 2009