6.01: Introduction to EECS I

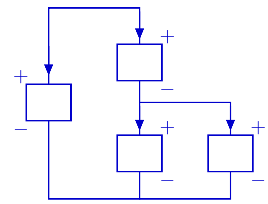
Circuits

Week 6 *October 15, 2009*

The Circuit Abstraction

Circuits represent systems as connections of component

- through which currents (through variables) flow and
- across which voltages (across variables) develop.



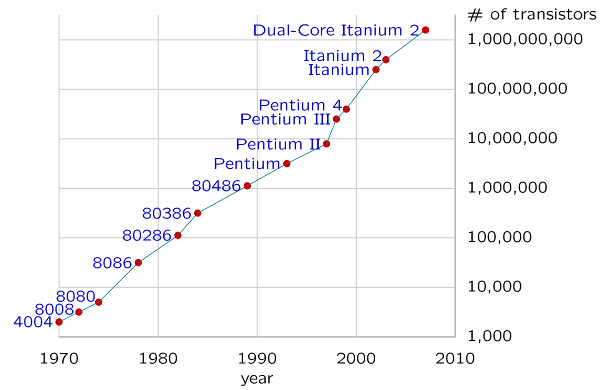
The Circuit Abstraction

Circuits are important for two very different reasons:

- as physical systems
  - power (from generators and transformers to power lines)
  - electronics (from cell phones to computers)
- as models of complex systems
  - neurons
  - brain
  - cardiovascular system
  - hearing

The Circuit Abstraction

Circuits are the basis of our enormously successful semiconductor industry.



The Circuit Abstraction

Circuits as models of complex systems: myelinated neuron.

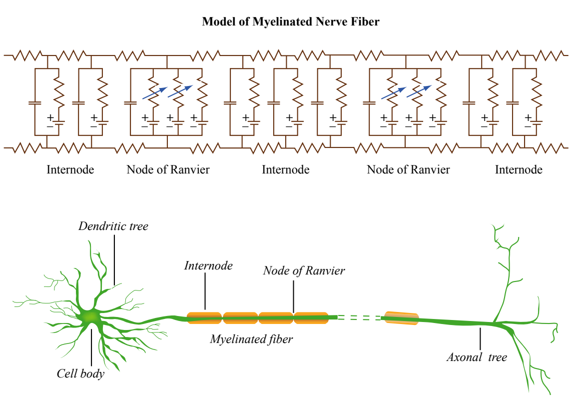
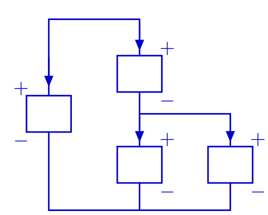


Figure by MIT OpenCourseWare.

What is a Circuit?

Circuits are connections of components

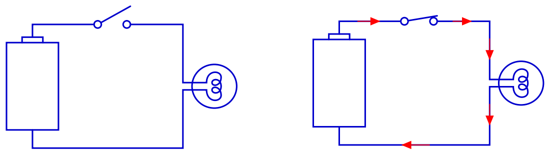
- through which currents (through variables) flow and
- across which voltages (across variables) develop.



Rules Governing Flow

Rule 1: Currents flow in loops.

Example: flow of electrical current through a flashlight



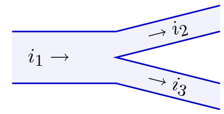
When the switch is closed, electrical current flows through the loop.

The same amount of current flows into the bulb (top path) and out of the bulb (bottom path).

Rules Governing Flow

Rule 2: Like the flow of water, the flow of electrical current (charged particles) is incompressible.

Example: flow of water through a branching point



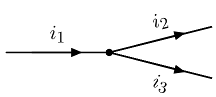
What comes in must go out.

Here $i_1 = i_2 + i_3$.

> **Kirchoff's Current Law:** the sum of the currents into a node is zero.

Rules Governing Flow

In electrical circuits, we represent current flow by arrows on lines representing connections (wires).



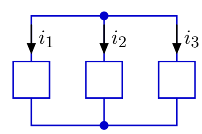
$i_1 = i_2 + i_3$.

The dot represents a "node" which represents a connection of two or more segments.

Nodes

Nodes are represented in circuit diagrams by lines that connect circuit **components**.

The following circuit has three components, each represented with a box.

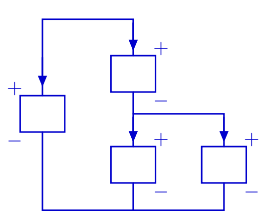


There are two nodes, each indicated by a dot. The net current into or out of each of these nodes is zero. Therefore $i_1 + i_2 + i_3 = 0$.

What is a Circuit?

Circuits are connections of components

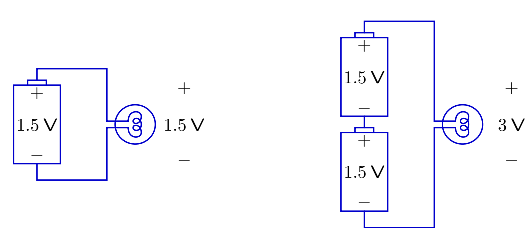
- through which currents (through variables) flow and
- across which voltages (across variables) develop.



Rules Governing Voltages

Voltages accumulate in loops.

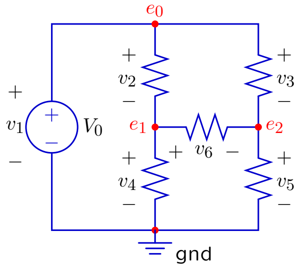
Example: the series combination of two $1.5\,\text{V}$ batteries supplies $3\,\text{V}$.



> **Kirchoff's Voltage Law:** the sum of the voltages around a closed loop is zero.

Alternative Representation: Node Voltages

Node voltages represent the voltage between each node in a circuit and an arbitrarily selected ground.



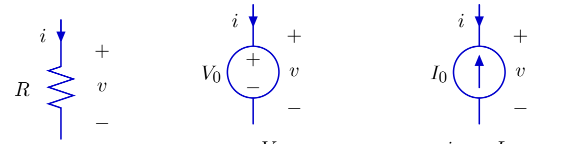
Node voltages and component voltages are different but equivalent **representations** of voltage.

- **component voltages** represent the voltages across components.
- **node voltages** represent the voltages in a circuit.

Rules Governing Components

Each component is represented by a relationship between the voltage across the component to the current through the component.

Examples:



$v = iR$

$v = V_0$ (regardless of i)

$i = -I_0$ (regardless of v)

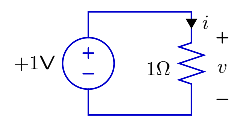
Node-Voltage-and-Component-Current (NVCC) Method

Combining KCL, node voltages, and component equations leads to the NVCC method for solving circuits:

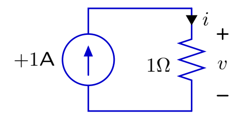
- Assign **node voltage variables** to every node except ground (whose voltage is arbitrarily taken as zero).
- Assign **component current variables** to every component in the circuit.
- Write one **constitutive relation** for each component in terms of the component current variable and the component voltage, which is the difference between the node voltages at its terminals.
- Express **KCL** at each node except ground in terms of the component currents.
- **Solve** the resulting equations.

Analyzing Simple Circuits

Analyzing simple circuits is straightforward.



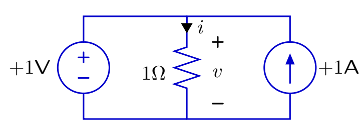
The voltage source determines the voltage across the resistor, $v = 1$V, so the current through the resistor is $i = v/R = 1/1 = 1$A.



The current source determines the current through the resistor, $i = 1$A, so the voltage across the resistor is $v = iR = 1 \times 1 = 1$V.

Check Yourself

What is the current through the resistor below?



1. 1A
2. 2A
3. 0A
4. cannot determine
5. none of the above

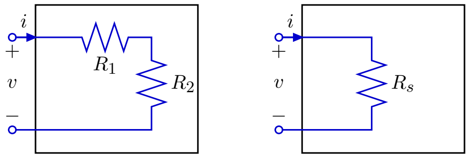
Common Patterns

There are a number of **common patterns** that facilitate design and analysis:

- series resistances
- parallel resistances
- voltage dividers
- current dividers

Series Combinations

The series combination of two resistors is equivalent to a single resistor whose resistance is the sum of the two original resistances.



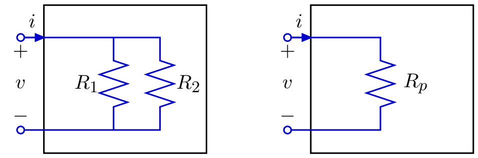
$v = R_1 i + R_2 i$ $\qquad$ $v = R_s i$

$$R_s = R_1 + R_2$$

The resistance of a series combination is always larger than either of the original resistances.

Parallel Combinations

The parallel combination of two resistors is equivalent to a single resistor whose conductance (1/resistance) is the sum of the two original conductances.



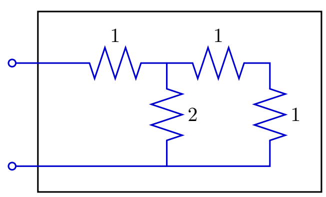
$i = \frac{v}{R_1} + \frac{v}{R_2}$ $\qquad$ $i = \frac{v}{R_p}$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \quad \rightarrow \quad R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} \equiv R_1 || R_2$$

The resistance of a parallel combination is always smaller than either of the original resistances.

Check Yourself

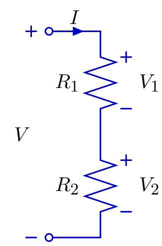
What is the equivalent resistance of the following circuit.



1. 1 $\qquad$ 2. 2 $\qquad$ 3. 0.5 $\qquad$ 4. 3 $\qquad$ 5. 5

Voltage Divider

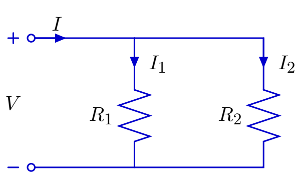
Resistors in series act as voltage dividers.



$$I = \frac{V}{R_1 + R_2}$$
$$V_1 = R_1 I = \frac{R_1}{R_1 + R_2} V$$
$$V_2 = R_2 I = \frac{R_2}{R_1 + R_2} V$$

Current Divider

Resistors in parallel act as current dividers.

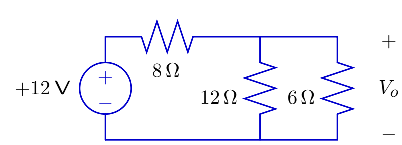


$$V = (R_1 || R_2) \, I$$

$$I_1 = \frac{V}{R_1} = \frac{R_1 || R_2}{R_1} I = \frac{1}{R_1} \frac{R_1 R_2}{R_1 + R_2} I = \frac{R_2}{R_1 + R_2} I$$
$$I_2 = \frac{V}{R_2} = \frac{R_1 || R_2}{R_2} I = \frac{1}{R_2} \frac{R_1 R_2}{R_1 + R_2} I = \frac{R_1}{R_1 + R_2} I$$

Check Yourself

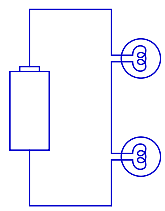
Find V_o.



Loading

Adding (or changing the value of) a component alters all of the voltages and currents in a circuit (except in degenerate cases).

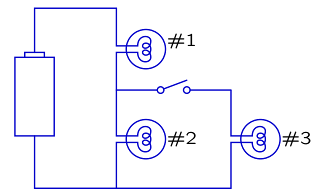
Consider identical light bulbs connected in series across a battery.



Because the same current passes through both light bulbs, they are equally bright.

Check Yourself

What happens if we add third light bulb?



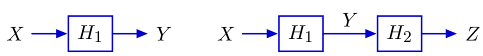
Closing the switch will make

1. bulb 1 brighter
2. bulb 2 dimmer
3. 1. and 2.
4. bulbs 1, 2, & 3 equally bright
5. none of the above

Loading

Loading did not occur in LTI systems.

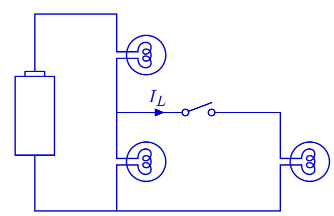
Example: adding H_2 has no effect on Y



$Y = H_1X$ regardless of H_2.

Loading

Q: What's different about a circuit?



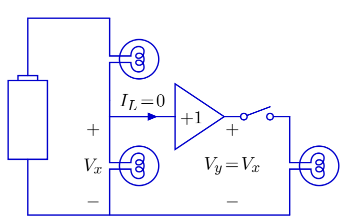
A: A new component generally alters the currents at the nodes to which it connects.

Buffering

Effects of loading can be diminished or eliminated with a buffer.

An "ideal" buffer is an amplifier that

- senses the voltage at its input **without** drawing any current, and
- sets its output voltage equal to the measured input voltage.



We will discuss how to use op-amps to make buffers in next lecture.

Summary

Circuits represent systems as connections of components

- through which currents (through variables) flow and
- across which voltages (across variables) develop.

There are a number of **common patterns** that facilitate design and analysis:

- series resistances
- parallel resistances
- voltage dividers
- current dividers

Buffers eliminate loading and thereby simplify design and analysis.

6.01 Introduction to Electrical Engineering and Computer Science I
Fall 2009