INVERSION OF TRAVEL TIME FOR VELOCITY

by

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ABSTRACT

Common source velocities and borehole compensated (BC) estimates have been used to obtain formation velocity estimates from full waveform acoustic logs (Willis and Toksöz, 1982). With both of these methods the receiver separation of the tool dictates the depth resolution of the velocities determined. This paper presents a method to 1) increase the depth resolution of the velocity estimates, and 2) remove the effects of a changing borehole radius upon the velocity estimates through formal inversion of arrival times and travel time moveouts. Results obtained by the inversion of full waveform acoustic log travel times appear quite promising. The velocity estimates on synthetic arrival times are slightly more noisy than those obtained using the standard BC technique. The most significant aspect of synthetic arrival times is that they generally appear to be unbiased. The BC technique is biased around formation boundaries and is especially biased for thin layers.

INTRODUCTION

A formal inversion of the arrival times and travel time moveouts is proposed to obtain the velocity estimates. This technique has not heretofore been applied to well log velocity data. Foster et al. (1982) applied an optimum sharpening filter to long spaced sonic data. They treated the conventional velocity log as a time series resulting from a running sum filter. They then attempted to remove the effect of this filter from the velocity log to sharpen the layer boundaries. The method proposed here makes use of all the travel time information and borehole radius information. It amounts to migrating the velocity information to its correct depth position. Optimal processing schemes are not presented from the standpoint of minimizing computer usage. Shortcuts such as transformations to reduce the size of sparse matrices may be introduced into the inversion method.

As the tool is pulled up the borehole a complete set of measurements is normally taken every time the tool moves a certain depth interval, \( Z_m \). The measurement interval, \( Z_m \), is usually much smaller than the receiver separation. Using standard methods, no matter how close together the measurements are taken, the velocity estimates always show the resolution of the receiver separation. Conceptually, information in the change (or the derivative) of the moveout with respect to the tool position is used in this paper.
to determine the finer structure. While these derivatives are not actually
taken, the information for which this paper is inverting is nevertheless
contained in them. The technique of formally inverting these data to obtain the
formation velocity and borehole radius amounts to migrating the velocity
information back to the correct depth location. It is assumed that the
formation can be described by a stack of horizontal layers of thickness \( d_i \) with
constant slowness, \( \eta_i \), or velocity \( u_i \) where:

\[
\eta_i = \frac{1}{u_i}
\]

The thickness of each layer is taken to be smaller than the receiver separation,
\( \Delta z \). The borehole radius at each layer is assumed to be a constant, \( r_i \). The four
step technique to calculate formation velocity is summarized as follows:

1) Estimate the formation slownesses, \( \eta_i \), and the borehole radii, \( r_i \).

2) Solve the forward problem to ray trace through the estimated velocity
structure.

3) Determine a matrix relationship of the form \( Ax = b \) for the corrections \( x_1 \)
and \( x_2 \) to the model parameters \( r_i \) and \( \eta_i \), respectively, and the difference
of the measured and model arrival times, \( b \).

4) Formally invert the relationship to find the corrected formation parameters,
\( x \), conceptually from \( x = A^{-1}b \).

Steps 2 through 4 are repeated using the improved estimates of \( r_i \) and \( \eta_i \) until
a satisfactory convergence is found.

**INVERSION METHOD**

**Step 1 - Starting Estimates of the Formation Slowness and Radii:**

The first step of the technique is to determine a beginning estimate of the
formation velocity. The arrival times and moveouts of the full waveform data are
recorded by an event detection and correlation scheme such as that proposed
by Willis (1983). A procedure for estimating formation velocity, \( V \), or slowness,
\( \eta \), can be to use the relation:

\[
V = \frac{\Delta z}{\Delta t} \quad \text{and} \quad \eta = \frac{\Delta t}{\Delta x}
\]  

(1)

where \( \Delta t \) is the moveout between two receivers spaced a distance, \( \Delta z \), apart.
This estimate is incorrect or biased when the tool is tilted or when the borehole
radius changed between the receivers. A borehole compensation technique can
be implemented to correct for tool tilt or borehole enlargement or restriction.

The borehole compensated velocity, \( V_{bc} \), and slowness, \( \eta_{bc} \), estimates are
obtained from:

\[
V_{bc} = \frac{2\Delta z}{\Delta t_u + \Delta t_d}
\]  

(2a)

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and

\[ \eta_{bc} = \frac{\Delta t_u + \Delta t_d}{2\Delta z} \]  \hspace{1cm} (2b)

where \( \Delta t_u \) is the travel time difference between receivers from the source at the bottom of the tool, and \( \Delta t_d \) is this difference for the source at the top of the tool. The velocity estimates from the borehole compensation method measure an average velocity over the receiver separation.

Thus an estimate of the formation velocity structure is obtained from the arrival times and moveouts recorded.

**Step 2 - Forward Problem**

*Ray tracing: Headwaves in the borehole can be ray traced using the simple model shown in Figure 1. The formation is a stack of horizontal layers of thickness, \( d_i \), with constant slowness, \( \eta_i \). The thickness of each layer is assumed to be smaller than the receiver separation, \( \Delta z \). The layer boundaries are described using layer definition points (LDPs). The layer boundaries are set to be the midpoints between the LDPs. The borehole radius at each layer is assumed constant, \( r_i \). For simplicity it is assumed that the radii are slowly changing so that the normal to the wall is always perpendicular to the vertical axis of the borehole. The travel time from source to receiver in Figure 1 can be obtained from the expression:

\[ t = \frac{\eta_f R_{bot}}{\cos \theta_{bot}} + \frac{\eta_f R_{top}}{\cos \theta_{top}} + \sum_{bot}^{top} \eta_i d_i \delta_i \]  \hspace{1cm} (3)

where

\[ \theta_{bot} = \sqrt{1 - \eta_{bot}^2 / \eta_f^2} \]  \hspace{1cm} (4)

and \( \eta_f \) is the slowness of the borehole drilling fluid. The subscripts \( top \) and \( bot \) refer to the path parameters at the top and bottom of the path, respectively. \( \delta_i \) is the fractional portion of the layer \( d_i \) through which the ray has traveled. Thus, for \( bot < i < top \), \( \delta_i = 1 \). At either end of the ray path through the formation layers, \( 0 \leq \delta_i \leq 1 \). Note that the changing borehole radius has been taken into account for raypaths in the mud, but it will be ignored in the raypaths through the formation layers. This approximation should be appropriate for a slowly changing borehole radius.

*Finding the Critical Refraction Point: While equation (2) appears to give the exact relationship for the travel time, the critical refraction point is often difficult to determine precisely with this calculation. Depending on the particular velocity structure, there may be many or no rays satisfying the condition of critical refraction. It is much easier, therefore, to estimate the first minimum travel time path.

Figure 2 shows a close-up schematic of a point source (or receiver, since the problem is completely symmetric) in the borehole and the formation layers. Calculation of the minimum time path from the source at point \( T \), to point \( E \) is required. The travel time for a ray \( TAE \) can be written as:
\[ t_{\text{TAE}} = t_{\text{TA}} + t_{\text{AE}}. \]

and for a ray TCE as:

\[ t_{\text{TCE}} = t_{\text{TC}} + t_{\text{CE}}. \]

The number of rays that need to be searched can be reduced by first shooting through the layer definition points (LDP's). The minimum time for the LDP's, for example, \( t_{\text{TBE}} \), is calculated. Shooting many more rays on either side of the minimum LDP, point B, provides a more accurate determination of the critical refraction point and the minimum travel time, \( t_{\text{BE}}^{\text{min}} \). It is possible that two or more ray paths take the same travel time. In this case, we will choose the first, (i.e., the path closest to the source T). In the presence of a highly attenuating mud, it is most likely that the first path will be larger in amplitude and more easily detected.

**Travel Time Determination:** The minimum travel times through the model (shown in Figure 2) need to be calculated for each source, \( T_m \), and receiver, \( R_k \), pair of the tool. The relationship

\[ t_{\text{BE}}^{\text{min}} = t_{\text{BE}}^{\text{min}} + t_{\text{EF}}^{\text{min}} \]

is used where \( t_{\text{BE}}^{\text{min}} \) and \( t_{\text{EF}}^{\text{min}} \) are determined using the method described in the previous section and \( t_{\text{EF}}^{\text{min}} \) is trivial to compute. The minimum travel times are hereafter referred to without the superscript \( \text{min} \) for brevity.

**Step 3 - Matrix Formulation**

*Set up A:* As the tool is pulled up the hole, it records a complete set of waveforms at discrete depth intervals. The subscript \( p \) is used to indicate the sequential number of the set of measurements and \( Z_p \) to indicate the depth at which each set was made. The depth \( Z_p \) will refer to some arbitrary fixed reference point on the tool. Thus for each tool position \( Z_p \), there are \((n_T)(n_R)\) raypaths, where \( n_T \) is the number of transmitters, and \( n_R \) is the number of receivers. The minimum travel time for each raypath can be written in the form of equation 3. The critical refraction points are not determined analytically as in equation 4, but by the ray tracing technique described previously. We can rewrite the travel time expression for each tool at depth \( Z_p \), source \( T_m \), and receiver, \( R_k \) as:

\[ t_{Z_p T_m R_k} = \sum_i C_{Z_p T_m R_k} \cdot \sum_i D_{Z_p T_m R_k} \cdot n_i \]

where

\[ D_{Z_p T_m R_k} = \begin{cases} d_i \delta_{Z_p T_m R_k} & \text{for each layer } i \text{ that the ray passes through} \\ 0 & \text{otherwise} \end{cases} \]

\[ C_{Z_p T_m R_k} = \begin{cases} \csc \varphi_i & i = \text{bot and top} \\ 0 & \text{otherwise} \end{cases} \]

and where
\[
\tau'_i = \tau_i \eta_f \\
\eta_f = \tau \eta_f \\
\eta_i = \tau \eta_i
\]

and \(\phi_i\) is the critical angle of refraction found from the ray tracing. Note that the dependence of \(C\) upon \(Z\), \(T\), and \(R\) shows up in the determination of the bottom, \(bot\), and the top, \(top\), of the ray path. The dependence of \(\delta\) upon \(Z\), \(T\), and \(R\) shows up in the selection of the ray path from the ray tracing. Using this same terminology, the travel times for a series of different tool positions and all of the corresponding source/receiver pairs can be expressed as:

\[ t = Cr' + Dn \]

or

\[ t = [CD] [r'] = A x \]

where the bold face upper case type indicates matrices, the bold face lower case type indicates column vectors, and the brackets indicate the concatenation of the matrices or vectors. For a given model with \(m\) layers, \(r'\) and \(n\) will have \(m\) rows and one column. \(x\) will have \(2m\) rows and one column. For a given set of observations taken at \(P\) different tool locations, \(C\) and \(D\) will have \(q = (n_T)(n_R)P\) rows and \(m\) columns. \(A\) will have \(q\) rows but will have \(2m\) columns. Equation 7 will henceforth be referred to as Set-up A.

For Set-up A there exists a minimum size block of data which can be inverted. This size is dictated by the length of the tool. There must be at least one ray path through each layer in order to effectively constrain the model parameters. Figure 3a shows a depth window which cannot be completely described by Set-up A. As the tool is moved to the extremes of the window, there remains a gap in the center which is unconstrained. The tool is simply too large for the window. While an average velocity over the gap may be estimated, the corresponding borehole radii are not constrained. The minimum depth window is therefore approximately two tool lengths.

The matrices in equation 7 can become quite large. Suppose a 100 foot (30 meter) section of a well is to be inverted using a model with 200 layers. The tool selected has two receivers and two sources. It records a complete set of waveforms 6 times per foot (20 times per meter). In this case \(A\) has 2400 rows and 400 columns. On an IBM 370 computer this takes 3.84 million bytes of storage for the \(A\) matrix alone. While it is possible to obtain this massive amount of core storage as virtual memory, it would be more practical to invert several smaller blocks of data sequentially.

**Set-ups B and C:** All of the equations thus far have dealt with total travel times. In order to reduce the size of the depth interval needed for the inversion an alternative set of equations can be considered. The relations for the common source moveout are: \(\Delta t_R\) between receivers \(R_k\) and \(R_j\) at a tool depth \(Z_p\), and the common receiver moveout \(\Delta t_T\), between sources \(T_m\) and \(T_i\), from

\[ \Delta t_R = t_{Z_p} \tau_m R_k - t_{Z_p} \tau_i R_j \]

\[ \Delta t_T = t_{Z_p} \tau_m T_m - t_{Z_p} \tau_i T_i \]

\[
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\]
\[ \Delta t_T = t_z R_a - t_z R_b \]

where the right hand terms can be found from equation 6. In a similar manner a matrix representation can be formed for equation 8 as:

\[ \Delta t = \begin{bmatrix} \bar{C} & \bar{D} \end{bmatrix} \begin{bmatrix} r' \\ n \end{bmatrix} = Ax \]  

(9)

where \( \bar{C} \) and \( \bar{D} \) are formed from the corresponding differences of the \( C \) terms and the \( D \) terms in equation 6 for the appropriate source and receiver pairs. Equation 9 will be referred to as Set-up B.

Set-up B allows the utilization of more tool positions than Set-up A when inverting for the same size depth window. This gives more overlapping data coverage, or redundancy, and thus a better constrained inversion. Figure 3b shows three representative tool positions which can be utilized in Set-up B. Set-up A can only utilize tool positions which are completely contained in the inversion depth window, as represented by the middle tool position in Figure 3b. Set-up B allows for the utilization of moveouts from tool positions which extend outside of the inversion window, as shown by the top and bottom tool positions in Figure 3b. This gives a more uniform data coverage over the inversion depth window.

If the results of equation 6 for the arrival times are combined with the results of equation 9 for the moveouts we obtain:

\[ \begin{bmatrix} \Delta t' \\ t \end{bmatrix} = \begin{bmatrix} \bar{C} & \bar{D} \\ \bar{C} & \bar{D} \end{bmatrix} \begin{bmatrix} r' \\ n \end{bmatrix} = Ax \]  

(10)

Equation 10 will be referred to as Set-up C.

Set-up C combines both the arrival times of Set-up A and the moveouts of Set-up B. On the surface it may appear that the terms in equation 6 have simply been rearranged to obtain equation 10. This is the case for tool positions where the tool is completely contained in the inversion depth window. These moveout terms are not needed and are superfluous. Set-up B allows the use of tool positions where part of the tool is outside of the inversion depth window. Thus, the arrival times from Set-up A are added to the moveouts of Set-up B to get Set-up C. Figure 3b can be used to illustrate this point.

The top and bottom tools shown represent data obtainable from tool positions which have part of their travel paths outside of the inversion window. From these positions the moveout information from sensors inside of the inversion window may be used. The middle tool in the figure represents those tools which are completely within the inversion depth window. For these positions both the arrival times and the moveout information can be used. Taken together, both the arrival times and the moveout information for the tool positions completely contained in the inversion window are redundant. Hence one or the other may be omitted.

Step 4 - Inversion

Problem Formulation: The relationship of the arrival times and moveouts to the model parameters has been presented. The general form of the equations is:
\[ Ax = b \]  

where \( x \) represents the model parameters to be estimated, \( b \) the observed arrival times and moveouts, and \( A \) the matrix relating \( x \) and \( b \), that has been determined by ray tracing. It has been tacitly assumed that this inversion problem is linear. A close examination shows that if the selection of the ray paths was correct then the problem is indeed linear. The problem becomes nonlinear if the ray paths are incorrect. The ray paths are decidedly a function of the inversion parameters. This nonlinearity shows up in the cosecant \( \theta \) and \( \delta \) terms of equations 3 and 6.

In order to treat this nonlinearity, Fermat’s principle is invoked (e.g., Aki et al., 1976). This assumes that the ray paths are stationary with respect to small changes in model parameters and thus changes in the ray path can be neglected in the inversion. A very good starting model can be found by using the event detection and correlation scheme for velocities developed by Willis (1983) and the companion caliper log. This makes the problem linear for the inversion. The \( A \) matrix is recomputed after each inversion iteration to reintroduce the nonlinearity due to the change in the ray path. \( \Delta b \) is defined as the difference between the observed times, \( b \), and the calculated times from the ray tracing, \( b_{calc} \), as

\[ \Delta b = b - b_{calc} = A(x - x_{est}) = Ax \]  

where

\[ x \] = the "true" formation parameters
\[ x_{est} \] = the current estimate of the formation parameters
\[ \Delta x \] = the correction to be applied to \( x_{est} \)

Equation 12 is then solved for \( x \) using the least squares solution (Willis, 1983: equation 3.11), or the damped solution (Willis, 1983: equation 3.17). The corrections are applied to the model, the forward problem is recalculated, and equation 12 is reformulated. The process is repeated until a satisfactory solution is obtained. This is usually taken to be when the improvement or corrections to the model are small.

**Estimation of Model Parameters:** In order to perform the forward ray tracing problem, a reasonable approximation of the formation parameters is needed. An existing sonic log may be used to estimate the formation slownesses. A more consistent approach, however, would be to utilize the borehole compensated velocity estimates from such an event detection and correlation scheme as proposed by Willis and Toksöz (1982). This approach would utilize the actual inversion data to obtain the formation slowness estimates.

The thickness of each layer must be determined at the outset of each iteration of the inversion. There must be sufficient thickness for at least one ray to pass through each layer. The greater the number of rays passing through each layer, the more constrained the model parameters. The borehole radii can be estimated from a caliper log.

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EXAMPLES ON SYNTHETIC DATA

Simple Step Model, Set-up A: A simple step model was devised to test the inversion method. The model slownesses are shown in figure 4a. The borehole radii are 0.33 ft (0.1 m) and the model has 83 layers. The tool configuration used is the SLS-TA sonde. It has two sources located at the bottom of the tool separated by 2 feet (0.61 m). At the top of the tool are two receivers with a separation of 2 feet (0.61 m). The sources and receivers are separated by 8 feet (2.44 m). Synthetic arrival times were generated moving the tool every 0.12 ft (0.037 m) using the ray tracing method. The depth section was taken large enough to eliminate any coverage gap. The model actually provides an overlap of about 2 ft (0.61 m). The initial guess of the formation slownesses is also shown in figure 4a. The common receiver slowness estimates determined from the synthetic arrival times, were used for the initial guess for depths greater than 1011 feet. Common source estimates were used for depths less than 1009 feet. The averages of the common source and common receiver slownesses were used as the initial guess in the coverage overlap interval of 1009 to 1011 feet. The correct borehole radii were used as the starting guess. Ray tracing was performed through the estimated structure using these initial guesses. Set-up A was used as the matrix formulation, and the least squares method was used for the inversion of Set-up A for the formation corrections. Figure 4a shows the corrected slowness structure from the inversion in the dashed line. The results are nearly identical to the original model parameters. The dashed line in Figure 4b is the inverted radius structure. The slownesses are resolved after only one iteration while the radii have been slightly altered.

Complicated Model, Set-up A: Figures 5a and 5b show a more complicated model used to test the inversion algorithm. Synthetic arrival times were generated for this model using the SLS-TA tool configuration described above. The smallest layer is 0.25 feet (0.078 meters) and there are 83 layers. The tool was moved every 0.12 feet (0.037 meters) for a total of 100 tool positions. Again, there is a tool coverage overlap of about 2 ft (0.61 m) in the center of the depth section.

The initial slowness guess is plotted 40 μseconds/ft (131 μseconds/m) below its actual value in Figure 5a for clarity. Where appropriate, the common source or common receiver slowness estimates for the initial guess were used. The average of the slownesses was used in the overlapping coverage section. The initial guess of the radii is plotted in Figure 5b 0.1 ft (30.5 mm) lower than the actual values. The results after two iterations using the damped least squares solution with $\epsilon^2 = 0.01$ are shown by the dashed lines in Figures 5a and 5b. A fairly good convergence to the original model is observed. The borehole radii at the edges of the section however, tend to deviate from the model parameters. It should be noted that the edges of the model are less constrained than the center due to the overlap of the source and receiver coverage.

Complicated Model, Set-up C: The same basic formation structure is used to test the inversion technique Set-up C with the exception that the change in borehole radius has been slightly sharpened at about 1001.5 feet. The model slownesses and radii are shown in Figures 6a and 6b respectively. A tool configuration which is directly borehole compensatable was used. It has one source at the bottom of the tool. 10 feet above that is a receiver. 2 feet above
that is another receiver. At the top of the tool is a source located 10 feet above
the top receiver. The moveout information from 142 tool positions and the
arrival times from the 12 foot (3.66 meter) source/receiver pair for 100 tool
positions were used for the inversion.

The initial slowness guess was derived using the conventional borehole
compensation method. Figure 6a shows the results of the application of the
inversion Set-up C to this velocity model. The initial slowness guess is offset 40
μseconds/foot (131 μseconds/meter) below its actual value. The initial radius
guess, shown in Figure 6b, is offset 0.1 foot (30.5 mm) below its actual value. A
somewhat poorer initial radius guess was used than was used for the test of
Set-up A. The second iteration using the least squares solution is shown as
dashed line in Figures 6a and 6b. Set-up C appears to converge more uniformly
to the model parameters than Set-up A.

Complicated Model with Noise: In order to test Set-up C for stability
under noisy conditions different levels of uniformly distributed noise were
added to the arrival and moveout times of the example in the previous section.
When the noise added is in the range of ± 0.1 μseconds, the results are nearly
indistinguishable from the noise free case. This level of noise does not appear
to adversely affect the inversion. A definite degradation of the slowness is
observed with the addition of ± 0.3 μseconds noise. The slowness errors were at
most ± 3% from the original model.

The dashed line in Figure 7a indicates the results of the Set-up C inversion
with the addition of ± 0.5 μseconds noise to the arrival and travel time data.
The borehole compensated slowness estimates used are also shown in Figure 7b
offset 40 μseconds/foot (131 μseconds/meter) below their actual values. The
same initial borehole radius guess was used for this inversion. The results
obtained after two iterations indicate that the noise is fairly uniform across the
slowness estimates. A third iteration was calculated but the estimates were not
significantly improved. The borehole radii terms deteriorate at the lower edge
of the window.

DISCUSSION

A method of inverting the travel times (Set-up A), moveouts (Set-up B), and
the travel times plus moveout information (Set-up C), for formation slownesses
and radii has been developed. The simple step discontinuity model (Figure 4)
converged to the model slowness after only one iteration. For the more
complicated models tested (Figures 5, 6 and 7), the slowness estimates achieved
optimal convergence after two iterations. Further iterations did not
substantially improve the discrepancies between the model parameters and the
inverted estimates.

A comparison of the examples for Set-up A and Set-up C illustrates that
Set-up C reduces the edge effects in the inversion. At the edges of Set-up A
there exists a basic non-uniqueness. A faster velocity layer can be offset by an
increase in the corresponding borehole radius. This radius change, however,
must propagate outward to the other radii terms. In so doing, it compensates
for the time loss in the fast layer since all layer parameters are coupled
together.
For synthetic data with added uniformly distributed noise Set-up C appears unaffected by levels of ± 0.1 μseconds noise, slightly affected (less than 3% maximum error) at ± 0.3 μseconds noise, and moderately affected (less than 7% maximum error) at ± 0.5 μseconds noise. It is interesting to note that while the borehole compensated slownesses used for the initial guess are hardly affected by the noise, the inverted structures become sensitive to higher levels of noise. The borehole compensated slownesses appear smooth but are biased by as much as 15% in many places (e.g., at depths 1012.5, 1010.5, and 1000. feet). The inverted results appear to be relatively rough but unbiased. Even with noise, maximum improvement occurs at two iterations.

The full waveform field data available had rather coarse sampling intervals, $Z_m$, of about 0.5 ft (0.15 m). This has generally been considered sufficiently small in light of the large volume of data generated by the full waveform tool. As a consequence, formation layers greater than 1 foot (0.3 meters) thick would have to be selected to apply this inversion technique to the field data. The velocity resolution available with the current configuration of the full waveform tool using standard techniques is 2 feet (0.61 meters). Thus not much would be gained by an inversion of these data. In fact, it is likely that significant biases would be introduced by the inversion of field data collected with such a coarse $Z_m$. If the layers are small the arbitrary selection of the location of the inversion layers (i.e., the L.D.P.'s) should not significantly affect the results. If one L.D.P. straddles a true formation bed boundary the bias should only extend over that layer. It is obvious that as the size of the straddling layer increases, the bias extends over this larger layer. Normally bed boundaries are located from sonic logs by identification of inflection points of the slowness changes. If layers are large and straddle boundaries information is actually lost. A large layer which straddles the boundary will indeed mask its true location.

It may be argued that for most applications a velocity resolution of 2 feet (0.61 meters) is sufficient. This is probably true for surface seismic work. The added resolution may be helpful for formation evaluation. Some full waveform tools have receiver separations of 5 feet (1.5 meters) or greater. In these applications this method could increase the depth resolution. The key factor in its application is the selection of the layer thicknesses. Close measurements, i.e., small $Z_m$, permits the selection of small layer thicknesses thereby reducing the effects of random errors and small borehole irregularities.

A simple two-dimensional forward model has been presented. Certainly there are sources of error in real data to contest this model. Dipping formation beds will introduce asymmetry into the ray-tracing as will a decentralized tool. Non-circular boreholes will also introduce errors, as will anisotropy. As these phenomena are more precisely understood appropriate corrections can be incorporated into the inversion calculation and their effects removed from the final velocity values.
REFERENCES


Foster, M., Hicks, W., and Nipper, J., 1962, Optimum inverse filters which shorten the spacing of velocity logs: Geophysics vol. 27, no. 3, pp. 317-326.


Figure 1. Schematic of ray tracing model. The tool is suspended in the drilling fluid. Each layer, \(i\), has a constant slowness, \(\eta_i\), and constant borehole radius, \(r_i\). The raypath from source to receiver is shown passing the fluid, through the formation layers, and then back through the fluid.
Figure 2. Right side—Model for ray tracing from point source T, to point receiver R. Left side—Close up of the searching technique to find minimum travel time path from T to R. Points A through F indicate some of the layer definition points.
Figure 3a. Schematic showing a depth window too small to be adequately analyzed by matrix Set-up A. The coverage gap is illustrated in the center of the depth window by the two extreme tool positions allowable.
Figure 3b. Schematic showing the same depth window as Figure 3a, but which is now analyzable by Set-ups B and C. The two extreme tool positions contribute $\Delta t$ terms. The intermediate position can contribute either $\Delta t$ and/or $t$ terms.
Figure 4a. Step discontinuity in slowness model. The original model is shown in a solid line. The initial guess is shown which smooths the discontinuity. The results of one iteration using Set-up A are shown in the dashed line (it is nearly indistinguishable from the original model).
Figure 4b. Borehole radius for the slowness step discontinuity model of Figure 4a. The solid line shows the original model radius structure which was also used as the initial guess. The dashed line shows the radius structure after one iteration using Set-up A.
Figure 5a. Complicated model to test matrix Set-up A. The model slownesses are labeled and plotted in a solid line. The initial guess is offset 40 μseconds/foot (131 μseconds/meter) below. The results after two iterations are shown in the dashed line.
Figure 5b. Complicated model to test matrix Set-up A. The model radii are labeled and plotted in a solid line. The initial guess is offset 0.1 ft (30.5 mm) below. The results after two iterations are shown in the dashed line.
Figure 6a. Complicated model to test matrix Set-up C. The model slownesses are labeled and plotted in a solid line. The initial guess is plotted in a solid line. The initial guess is offset 40 μseconds/foot (131 μseconds/meter) below. The results after two iterations are shown in the dashed line.
Figure 6b. Complicated model to test matrix Set-up C. The model radii are labeled and plotted in a solid line. The initial guess is offset 0.1 foot (30.5 mm) below. The results after two iterations are shown in the dashed line.
Figure 7a. Same model as in Figure 6a but ± 0.5 μseconds noise added to synthetic arrival times. The model slownesses are labeled and shown in a solid line. The initial guess is offset 40 μseconds/foot (131 μseconds/meter) below. The results after two iterations are shown in the dashed line.
Figure 7b. Same model as in Figure 6b but ± 0.5 µseconds noise added to synthetic arrival times. The model radii are labeled and shown in a solid line. The initial guess is offset 0.1 foot (30.5 mm) below. The results after two iterations are shown in the dashed line.