ABSTRACT

The measurement of in-situ permeability is very important in exploration and production logging. Observed data show that tube wave attenuation in full waveform acoustic logs is correlated with formation permeability. It is postulated that attenuation is due to fluid flowing away from the borehole into the formation. In this paper we investigate the theoretical relationship between tube wave attenuation and permeability using two different models. The first is a simple model of a borehole with absorbing walls, and the second is a borehole with a Biot porous medium in the formation. Both models give qualitatively similar results. Tube wave attenuation increases with increasing permeability. Attenuation also increases with increasing frequency and porosity. We have also investigated the relative effects of intrinsic formation attenuation (anelasticity) and permeability on the attenuation of tube waves. Intrinsic attenuation was introduced into the models by means of complex velocities. It is found that in rocks with low to medium permeability (less than 100 millidarcies), intrinsic attenuation is the major contributor to tube wave attenuation. However, in high permeability (greater than 100 millidarcies) rocks, fluid flow associated with in-situ permeability is as important as intrinsic attenuation in controlling tube wave attenuation. In either case, if one can estimate the intrinsic formation attenuation from the other parts of the full waveform (such as the P wave or the pseudo-Rayleigh wave), an estimate of the permeability of the formation can be obtained. We tested the models using published data on core permeability and tube wave amplitudes. By assuming an average value of intrinsic attenuation appropriate to the formations under study, we obtained a good agreement between theory and data.

INTRODUCTION

Formation permeability is an important physical parameter that determines the efficiency of petroleum production of a reservoir. Many different approaches have been utilized to estimate reservoir permeability. For example, core samples can be taken and permeabilities measured in laboratories by permeameters (Van Golf-Racht,
However, permeability thus determined suffers from uncertainties caused by the effects of overburden and the possible effects of the existence of fracture spacings that are larger than the size of the core sample itself. Besides laboratory measurements, various schemes for in-situ measurement have also been developed. One approach is to measure formation porosity and utilize some empirical or theoretical relationship between permeability and porosity to calculate permeability. Permeability values determined on the basis of this approach are highly model dependent. Another approach, commonly known as a packer test, is to pressurize a capped well and monitor the transient pressure change. The decay of the pressure head can be theoretically related to the permeability of the rock formation. This approach gives a fairly accurate averaged permeability over the length of the capped section of the well. In order to evaluate permeability of individual stratum, different approaches are necessary.

The existence of a relationship between tube wave attenuation and the permeability of a porous formation was demonstrated by many investigators both theoretically (Rosenbaum, 1974) and observationally (Bamber and Evans; 1967, Paillet, 1983; Williams et al., 1984). Although many mechanisms have been proposed to explain seismic wave attenuation in porous formations (Toksöz and Johnston, 1981), attenuation of tube waves in a borehole involves an additional mechanism which arises due to fluid flow from the borehole into the surrounding formation by the passing wave. Recently, an approximate theory to study this particular mode of attenuation for an isolated single fracture has been developed by Mathieu and Toksöz (1984). However, the contribution of this mode of attenuation to the total attenuation of tube waves remains to be addressed.

In this paper, we first present two different approaches to calculate the attenuation of tube waves as a function of the permeability of the formation. One approach is based on the complex impedance contrast between the borehole fluid and the surrounding formation. The other approach is based on the communication between the borehole fluid and a porous Biot formation, as presented by Rosenbaum (1974). We then examine the contribution of these mechanisms to the total tube wave attenuation as compared with intrinsic attenuation. Our theoretical results are finally compared with field data presented in Williams et al. (1984).

### MODELS OF TUBE WAVE ATTENUATION

#### Absorbing wall model

A tube wave, also known as borehole Stoneley wave, is a guided wave in a borehole. Its energy is distributed along the borehole wall and within the borehole fluid. Therefore, the propagation of a tube wave within a borehole can be treated as simply a guided wave within a circular pipe. In order to further simplify the analysis, we chose to consider only the dominating pulse instead of the complete wave train as in full waveform analysis. It is an appropriate simplification since in practice, attenuation is mostly measured by comparison of the amplitudes of the dominating pulses associated with the tube waves among different seismic traces. In order to isolate the effect of the fluid flow on tube wave attenuation, we further assume that it is the only mechanism responsible for tube wave attenuation. Consequently, the problem can be treated as a guided wave propagating inside a circular pipe with an absorbing wall. Mathematical formulation of such a problem has been done by Morse
In-situ Permeability

By matching the impedance in the fluid to the impedance at the borehole wall, the attenuation of the tube wave can be obtained. The borehole wall impedance for a porous formation surrounding a cylindrical fluid cavity was given by White (1983). It is a function of the permeability of the formation as well as the borehole fluid viscosity. Given the borehole and formation properties, the formulation of Morse (1939) can be solved to give the attenuation of the tube wave.

Biot model

An alternative approach to solving the problem of tube wave attenuation in a porous formation is the use of the Biot porous solid model. Rosenbaum (1974) formulated the problem of elastic wave propagation in a borehole in a formation with the properties of a Biot porous solid. The resulting period equation is complex, as opposed to the case of an elastic solid, where the period equation is real. The root of the period equation with a phase velocity less than that of the formation shear wave velocity and the borehole fluid compressional velocity is the tube wave root. At any given frequency, we can solve the period equation for the tube wave phase velocity. In the case of a Biot formation the tube wave phase velocity from the period equation is complex. Thus the tube wave propagation is attenuated. The amount of attenuation is given by the imaginary part of the phase velocity relative to the real part of the phase velocity. The ratio of the imaginary to the real part of the phase velocity is equal to 1/Q, where Q is the quality factor. In addition, we have added formation or “intrinsic” attenuation to the Rosenbaum formulation by the use of complex velocities (Cheng et al., 1982). In this manner, we can separately evaluate the relative contributions of intrinsic attenuation and attenuation due to a Biot porous solid to the total tube wave attenuation.

RESULTS AND DISCUSSION

In this section, we will first present and discuss the theoretical results of attenuation (1/Q) as a function of the permeability and porosity of the porous formation for both the absorbing wall and the Biot porous rock model. For our models, the Biot porous rock is assumed to be in total communication with the borehole fluid. This is accomplished by setting the impedance coupling factor κ to zero in the period equation (Rosenbaum, 1974). We will also present the attenuation results as a function of the tube wave frequency. Finally, we will compare our theoretical results with data presented in Williams et al. (1984).

In Figure 1 we present the result of theoretical tube wave attenuation (1/Q) as a function of permeability for three values of porosity. The frequency is taken to be 5 kHz. Figure 1a is for the absorbing wall model and Figure 1b is for the Biot porous formation model. As expected, attenuation increases with increasing porosity and permeability. Above about 100 millidarcy, the attenuation of the tube wave becomes significant. The absorbing wall model (Figure 1a) shows higher tube wave attenuation than the Biot porous rock model. At 1 darcy, the attenuation (1/Q) for the absorbing wall model is about 0.4, while that for the Biot model is less than 0.1 for a porosity of 20 percent. Although the quantitative numbers are different, the functional dependence of both models on permeability and porosity is similar.

Figure 2 shows the theoretical tube wave attenuation as a function of frequency. Formation permeability is taken to be 100 millidarcies. In both models,
attenuation decreases with increasing frequency. This is probably related to the fact that lower frequency allows more fluid movement between the borehole and formation, hence greater dissipation of energy. Once again, the absorbing wall model (Figure 2a) predicts a higher attenuation than the Biot porous formation model (Figure 2b). For example, at 10 kHz and 20 percent porosity, the absorbing wall model predicts a tube wave attenuation of 0.08, while the Biot model predicts an attenuation of less than 0.015. In the Biot model, the attenuation approaches a constant for frequencies higher than about 10 kHz. For the absorbing wall model, the attenuation appears to decrease with frequency even past 20 kHz for all three porosity models.

It appears that the absorbing wall model predicts a tube wave attenuation that is rather high. For example, in a rock with a porosity of 20 percent, a Q of 1 is reached with a permeability of less than 10 darcies. For a permeability of 1 darcy and a porosity of 20 percent, a Q of 2.5 is predicted. For comparison, the Biot model, with a x of zero, allowing for complete communication between the borehole and the pore spaces, predicts a Q of 15 for a rock with 20 percent porosity and 1 darcy permeability (Figure 1). We have concluded that the Biot model is perhaps more realistic. Thus we will use the Biot model in the following sections to study the effects of intrinsic or formation attenuation and in comparison with data.

Using the complex velocity model with the Rosenbaum formulation, we can proceed to calculate the relative effects of intrinsic or formation attenuation and fluid flow on the attenuation of tube waves. In Figure 3 we have plotted the total tube wave attenuation as a function of frequency for a formation of 30 percent porosity and a permeability of 100 millidarcies. Formation and borehole fluid attenuations are taken to be Qf = 100, Q = 50, and Qf = 20. At frequencies above 5 kHz, the total attenuation is almost constant with frequency. Below 5 kHz, the total attenuation increases sharply as the frequency decreases. In Figure 3 we have also plotted the attenuation due to intrinsic Q and that due to the Biot fluid flow mechanism. It is clear that the intrinsic attenuation dominates tube wave attenuation in the case for frequencies above 2 kHz. Intrinsic attenuation is almost constant throughout the entire frequency range, with a slight increase at the low frequency end. This is consistent with the results from partition coefficient analysis (Cheng et al., 1982). As seen in Figure 2, the effect of attenuation due to fluid flow increases with decreasing frequency, dominating over the effect of intrinsic attenuation at frequencies below 2 kHz.

In Figure 4 we have plotted the tube wave attenuation for a formation with a porosity of 30 percent and a permeability of 300 millidarcies. Intrinsic attenuation remains the same as in Figure 3. In this case, the contribution to overall attenuation from fluid flow is higher than that from intrinsic attenuation up to a frequency of 4 kHz. Above 4 kHz, the contributions from the two sources of attenuation are comparable. Even at this high level of permeability, the contribution of intrinsic attenuation to the total attenuation of the tube wave is not negligible.

In Figures 5 and 6, we compare our theoretical results with the data given in Williams et al. (1984) in two different formations. The first formation (well 1 in Williams et al., 1984) is a relatively low permeability formation, with core permeability in the range of 1 to 10 millidarcies. The tube wave amplitude ratio between the far receiver and the near receiver ranges from 0.6 to 0.8. The separation between the two receivers is 5 ft. In Figure 6 we have plotted the observed amplitude ratios versus core permeability. We have also plotted the theoretical tube wave amplitude
ratio from the Biot model versus permeability, with and without intrinsic attenuation. In the latter case, the $Q$ values used are: $Q_p = 100$, $Q_a = 60$, $Q_f = 30$. We can see from the figure that without intrinsic attenuation, the theory does not predict the observed tube wave attenuation. With reasonable values of intrinsic attenuation, the theory agrees well with the data.

In Figure 6 we plotted the data from well 3 in Williams et al. (1984). The formation is rather permeable, with core permeability ranging from above 1 darcy to about 10 millidarcies. The tube wave amplitude ratio ranges from 0.3 to 0.6. We have also plotted three different curves from the Biot model, one without intrinsic attenuation, one with $Q_p = 100$, $Q_a = 50$, and $Q_f = 20$; and one with $Q_p = 30$, $Q_a = 15$, and $Q_f = 10$. As we can see, the low $Q$ model fits the data better at lower permeabilities, while the higher $Q$ model fits the data better at higher permeabilities. In any case, the model without intrinsic attenuation does not fit the data at all.

CONCLUSIONS

We have presented two models of tube wave attenuation as a function of in-situ permeability. One is a simple absorbing wall model for the borehole. The other is much more complex and is based on the Rosenbaum formulation of a borehole in a Biot porous formation. We have introduced intrinsic formation attenuation into the Rosenbaum formulation by the use of complex velocities. For both models, the tube wave attenuation increases as a function of formation porosity and permeability, and decreases with frequency. We found that the Biot model predicts a more reasonable tube wave attenuation as a function of permeability than the absorbing wall model. We found that at typical logging frequencies, intrinsic attenuation dominates tube wave attenuation for formation permeabilities less than about 100 millidarcies. Above 100 millidarcies, the effect of formation permeability on tube wave attenuation is comparable to or greater than that of intrinsic attenuation. We have compared the results from the Biot model with data given in Williams et al. (1984). With typical values of intrinsic formation attenuation included, we are able to obtain good agreement between theory and data.

ACKNOWLEDGEMENTS

This work is supported by the Full Waveform Acoustic Logging Consortium at M.I.T.

REFERENCES


Figure 1: Tube wave attenuation ($1/Q$) as a function of permeability for formations of different porosities calculated using (a) the absorbing wall model, and (b) the Biot model.
Figure 2: Tube wave attenuation (1/Q) as a function of frequency for formations of different porosities calculated using (a) the absorbing wall model, and (b) the Biot model.
Figure 3: Tube wave attenuation (1/Q) versus frequency for a formation with a porosity of 30 percent and a permeability of 100 millidarcies calculated using the Biot model. Intrinsic Q used are: $Q_p = 100$, $Q_s = 50$, and $Q_f = 20$. Total attenuation is the sum of intrinsic attenuation and attenuation due to fluid flow.
Figure 4: Same as Figure 3 for a formation with a permeability of 300 millidarcies. All other parameters remain the same.
Figure 5: Theoretical (Biot model) and observed amplitude ratio for two receivers 5 ft (1.53 m) apart versus permeability. The data (points) are from well 1 in Williams et al. (1984). The top line is the theoretical result without intrinsic attenuation. The bottom line is with $Q_p = 100$, $Q_s = 50$, and $Q_f = 30$. 
Figure 6: Same as Figure 5 for well 3 in Williams et al. (1984). The top line is the theoretical result without intrinsic attenuation. The middle line is with $Q_p = 100$, $Q_s = 50$, and $Q_f = 20$. The bottom line is with $Q_p = 30$, $Q_s = 15$, and $Q_f = 10$. 