Full Waveform Inversion of P Waves for $V_s$ and $Q_p$

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ABSTRACT

We present an indirect method of determining shear wave velocities from full waveform acoustic logs based on the inversion of the spectral ratio of the P-wave trains at two source-receiver separations. This method simultaneously inverts for the formation shear wave velocity and compressional wave attenuation. The P-wave response is calculated by means of branch-cut integration. This method is useful in “soft” formations where the shear wave velocity is lower than the acoustic velocity of the borehole fluid and thus there are no refracted shear wave or pseudo-Rayleigh wave arrivals. The method is shown to give good estimates of formation shear wave velocity in both synthetic and field data. The inversion algorithm is sensitive to local minima; care must be taken to avoid them.

INTRODUCTION

The determination of formation S-wave velocity has long been one of the major objectives of full waveform acoustic logging. One important application of formation S-wave velocity is to derive from it the shear modulus, which is important in a number of production problems such as hydrofracing. With the renewed interest in shear wave reflection profiles, the S-wave velocity log is important in providing the ground truth to the reflection times. P- to S-wave velocity ratio, or equivalently the Poisson’s ratio, is an excellent interpretation tool for lithology and gas saturation.
In "hard" formations where the S-wave velocity is faster than the acoustic velocity of the borehole fluid, the existence of the shear head wave and the pseudo-Rayleigh wave makes the determination of the formation rather routine. However, in "soft" formations where the S-wave velocity is slower than the acoustic velocity of the borehole fluid, there is neither the refracted shear head wave nor the pseudo-Rayleigh wave, and the determination of the formation S-wave velocity from full waveform acoustic logs requires indirect means. It is possible to obtain S-wave velocity directly by the use of non-axisymmetry logging tools such as the Shear Wave Acoustic Log (SWAL) discussed in Zemanek et al. (1984), but such tools are not generally available yet. The indirect means of obtaining formation S-wave velocity in a soft formation generally involve the determination of the Stoneley wave velocity and then inverting for the S-wave velocity (Cheng and Toksöz 1983; Chen and Willen, 1984; Stevens and Day, 1986). However, recent discoveries by Williams et al. (1984) and Burns et al. (this issue) have shown that the Stoneley wave velocity is significantly affected by formation permeability, thus making the method useful only in formations with low permeability. In this paper, we present an alternative indirect method of determining formation S-wave velocity in a soft formation. This method is based on the inversion of the P-wave train.

$V_s$ and $Q_p$ both affect the amplitude and duration of the P-wave train. The amount of energy being converted into the P-wave train at the borehole-formation interface depends on the Poisson's ratio of the formation (Paillet and Cheng, 1986). Using finite-difference "snapshots", Stephen et al. (1985) demonstrated that S-wave energy is constantly radiating out from the P-wave train at the borehole-formation interface. Thus the amplitude of the P-wave train is affected by the Poisson's ratio, and hence $V_s$, of the formation. The P-wave train, of course, is also directly affected by formation P-wave attenuation $1/Q_p$. The objective, then, is to obtain information about $V_s$ as well as $Q_p$ from the P-wave train. We achieve this by a standard iterative non-linear inversion algorithm.

In the following sections, we first briefly discuss the direct problem of calculating synthetic microseismograms for the P-wave train propagating along a fluid-filled borehole. We then discuss the inverse problem of determining formation $V_s$ and $Q_p$ from the spectral ratio of P-wave trains at different receiver distances. Finally, we show an example of the method applied to synthetic data and to field data.

THE DIRECT PROBLEM

The P-wave train is modelled using the method of branch-cut integration (Peterson, 1974; Tsang and Rader, 1979; Kurkjian, 1985). The pressure response at a point located at cylindrical coordinate $(r,z)$ from the source located at $(0,0)$ in a fluid-filled
borehole can be written as:

\[ P(r, z, t) = \int_{-\infty}^{\infty} S(\omega) e^{-i\omega t} d\omega \int_{-\infty}^{\infty} A(k, \omega) I_0(f r) e^{ikz} dk \]  

\[ \text{(1)} \]

where,

\[ A(k, \omega) = \frac{g K_1(f R) - K_0(f R)}{g I_1(f R) + I_0(f R)} \]

with

\[ g = \frac{f \rho}{l \rho_f} \left\{ \left( \frac{2 V_p^2}{c^2} - 1 \right)^2 \frac{K_0(i R)}{K_1(i R)} - \frac{2 V_s^2 l m}{k^2 c^2} \left[ \frac{1}{m R} + \frac{2 V_p^2}{c^2} \frac{K_0(m R)}{K_1(m R)} \right] \right\} \]

\[ \text{(2)} \]

\[ l, m, f \] are the radial wavenumbers given by

\[ l = k(1 - \frac{c^2}{V_p^2})^{1/2}, \quad m = k(1 - \frac{c^2}{V_s^2})^{1/2}, \quad f = k(1 - \frac{c^2}{V_f^2})^{1/2}; \]

\[ k \] is the wavenumber in the \( z \) direction, \( \omega \) the angular frequency, \( R \) the borehole radius, \( V_p, V_s, V_f \) are the P- and S-wave velocity of the formation and the acoustic velocity of the borehole fluid, \( \rho \) and \( \rho_f \) are the density of the formation and the fluid, and \( S(\omega) \) the source spectrum. Attenuations, \( 1/Q_p, 1/Q_s \), and \( 1/Q_f \), appear in the imaginary part of the respective body wave velocities (Aki and Richards, 1980), i.e.,

\[ V_p \Rightarrow V_p(1 + \frac{i}{2Q_p}), \]

and similar expressions for \( V_s \) and \( V_f \).

The contribution to the P-wave train comes in through the contribution of the branch point \( l = 0 \) on the real \( k \) axis. Its magnitude can be calculated by the contour integral along a branch cut in the complex \( k \) plane originating from the branch point. A common branch cut to use (see Kurkjian, 1985) is one that is vertical to infinity in the first quadrant on the \( k \) plane. This vertical branch cut has the property of being a steepest descent path for large values of \( z/R \), and thus is computationally the most efficient branch cut. An alternative branch cut is one which is along the real \( k \) axis from the branch point to negative infinity (Paillet and Cheng, 1986). This branch cut has the advantage of coinciding with the branch cut associated with the modified Bessel functions. We use the former branch cut in this paper. The branch integral is then evaluated using a fourth order Gauss-Laguerre quadrature (Tsang and Rader, 1979).

**THE INVERSE PROBLEM**

The spectral ratio of the pressure responses at two source-receiver separations, \( z_1 \) and \( z_2 \), can be written as:

\[ \frac{a(z_2, \omega)}{a(z_1, \omega)} = \frac{S(\omega) \int_{L} A(k, \omega) e^{ikz_2} dk}{S(\omega) \int_{L} A(k, \omega) e^{ikz_1} dk} \]

\[ \text{(3)} \]
where $L$ is the contour around the above-mentioned branch cut.

The advantage of spectral ratio is seen from equation (3) in that the source terms cancel out. We are, in fact, looking at the transfer function between the receivers. Since, in practical applications, the source spectrum is never fully known, this formulation is necessary. If we let the spectral ratio be $y(z_1, z_2, \omega)$, equation (3) can be written as

$$y(z_1, z_2, \omega) = \frac{G_2(z_2, \omega)}{G_1(z_1, \omega)} = G(z_1, z_2, \omega)$$

(4)

with

$$G_n(z, \omega) = \int_L A(k, \omega) e^{ikz} dk, \quad n = 1, 2.$$

$A(k, \omega)$, and hence $G(z_1, z_2, \omega)$, is a function of the formation and borehole properties. In particular, in addition to the P-wave velocity, $G$ is sensitive to the shear wave velocity and the P-wave attenuation of the formation. Other less important factors that influence $G$ are formation density $\rho$, radius $R$, acoustic velocity $V_f$, density $\rho_f$, and quality factor $Q_f$ of the borehole fluid. Toksoz et al. (1985) and Cheng et al. (1986) have examined some of these effects by forward modelling techniques.

In our inversion, we assume that $V_p, V_f, \rho, \rho_f, Q_f$, and $R$ are known and invert for $V_s$ and $Q_p$. This is not an unrealistic assumption. $V_p$ can be obtained from the moveout between the two receivers (Willis and Toksoz, 1983); $V_f$ is usually around 1.53 km/s for water, with very slight variations for sea water. For drilling muds, variations may be somewhat larger. $\approx 1.45$ to 1.6 km/s, depending on the mud. Mud density is usually noted on the well logs. Formation density can be obtained from the well log, along with the borehole radius. Mud attenuation is a little more difficult to obtain. Laboratory experiments have shown that the $Q$ of a typical drilling mud is of the order of 30 (Tang et al., this issue). Effects of small variations of these parameters on the inversion results in one particular data set (DSDP site 613) are discussed in Meredith et al. (this issue).

Owing to the fact that we are not inverting for $V_p$, we can ignore the phase spectra of the pressure responses and just take the amplitude spectra. This reduces the size of the problem. Equation (4) is inverted for $V_s$ and $Q_p$ using a standard damped least-squares linearized inversion technique (see, e.g., Aki and Richards, 1980). $Q_s$ is assumed to be equal to $Q_p$ in the inversion. This is a well justified approximation since $Q_s$ does not really affect the P-wave propagation. In the inversion, we use $1/Q_p$ as the variable instead of $Q_p$ itself since the propagating term is exponentially linear in $1/Q_p$ instead of $Q_p$.

P-wave trains from two different source-receiver separations are windowed with the same window length and padded with zeroes up to the desired number of points. The
traces are then transformed into the frequency domain using the FFT (Fast Fourier Transform) and the ratio of the spectra are then obtained. Only the frequencies corresponding to the peaks in the spectra are used. In our examples, this frequency range is from 10 to 15 kHz in the synthetic data and 5 to 10 kHz in the field data. The spectral ratio data are weighted by the inverse of the frequency. For a constant $Q$ type of behavior, the spectral ratio behaves approximately inversely proportional to frequency. Thus, without weighting in a least-squares solution, the high frequency fit will be emphasized over the lower frequency components. With weighting, the fit throughout the entire frequency range will be roughly uniform.

As for the weighting of the parameters $V_s$ and $1/Q_p$, the latter is of the range of about 0.005 to 0.1, whereas the former is of the range of 0.5 to 1.5 (in km/s). The parameter $V_s$ is then multiplied by the radius (of the order of 0.1 meters) to bring it to the same order as $Q_p$.

**SYNTHETIC EXAMPLE**

We will first demonstrate the applicability of our algorithm using synthetic data. The microseismograms are generated using the discrete wavenumber summation method (Cheng and Toksöz, 1981). Thus, the method used to calculate the synthetic microseismograms is different from that used in the inversion. The formation is a soft formation, with $V_p = 2.5$ km/s, $V_s = 1.24$ km/s, $\rho = 2.1$ g/cm$^3$, $Q_p = Q_s = 20$. The borehole parameters are: $V_f = 1.5$ km/s, $\rho_f = 1.0$ g/cm$^3$, $Q_f = 50$, and borehole radius = 10 cm. Two source-receiver separations are calculated, at 2.44 and 3.05 m. The microseismograms are shown in Figure 1. The P-wave trains are windowed with a constant length window and padded with zeros up to 512 points (Figure 2). Since we are only dealing with the amplitude spectra, the absolute time scale is not important. The P-wave trains are then transformed into the frequency domain using the FFT. The spectra of the near and far offset synthetic P wave data are shown in Figure 3, and their ratio in Figure 4.

The results from our iterative linearized inversion for this synthetic data is $V_s = 1.30$ km/s and $Q_p = 20$ after about 10 iterations. The $Q_p$ is exactly that used in generating the synthetic data, while the $V_s$ is about 5% off from the input. This is an acceptable, though not perfect solution. If we examine the two dimensional residual squared error as a function of $V_s$ and $Q_p$ shown in Figure 5, we can see that there is a sharp minimum around the actual input value of $Q_p = 20$ but a much broader minimum around $V_s = 1.24$ km/s. In this particular case, the numerical noise has caused the actual minimum to be shifted slightly from the correct one. There is a trend toward a local minimum at lower velocities. Extra minima are characteristic of non-linear inversions. As we increase the bandwidth for the inversion to 5 to 15 kHz (Figure 6) in order to take
Figure 1: Synthetic microseismograms used for test of inversion algorithm. Starting time is arbitrary.

Figure 2: P-wave trains windowed from Figure 1 used for inversion. Starting time is arbitrary.
Figure 3: P-wave spectra from Figure 2.

Figure 4: Ratio of spectra shown in Figure 3.
advantage of the extra energy at low frequencies, we can see that the minimum around
the actual solution sharpens up, and the extra minimum at a lower velocity becomes
less significant. Further more, the shear wave velocity is much better defined. Taking
a cut in the residual squared error plot (Figure 5) along the line \( V_s = 1.24 \text{ km/s} \), and
plotting the error as a function of \( 1/Q_p \) (Figure 7), we can see the error has a minimum
at \( 1/Q_p = 0.05 \), or \( Q_p = 20 \), with a relatively "flat" bottom between the values of \( 1/Q_p = 0.04 \)
and 0.06. The shape of this minimum provides some qualitative measurement
of the variance in the solution.

FIELD EXAMPLE

To demonstrate the applicability of this technique to field data, we next apply it to
data from a hard formation, where \( V_s \) is greater than \( V_p \), and hence there is a S-wave arrival with which we can compare the \( V_s \) from the inversion. Unfortunately, no independent \( Q_p \) measurement is available. Nevertheless, with the lithology known, we
can at least see if the \( Q_p \) obtained is consistent with the lithology. The data is obtained
in a sandstone section. The complete microseismograms at 4.57 (near offset) and 6.10
m (far offset) source-receiver separations are shown in Figure 8. The P-wave train, S-wave/pseudo-Rayleigh wave packet, and the low frequency Stoneley wave are all evident
in the data. From the P- and S-wave moveouts, the formation P- and S-wave velocities
are determined to be 3.4 and 1.75 km/s, respectively. The latter was further confirmed
using a non-axisymmetric shear wave logging tool (Zemanek et al., 1984). The P-wave
trains are windowed and padded with zeros up to 512 points (Figure 9). The near
and far offset spectra are shown in Figure 10. The spectra have peaks at 6 kHz, with
significant energy between 5 and 10 kHz. There are secondary peaks at 11-12 kHz,
corresponding to a higher mode of the P leaky mode (Paillet and Cheng, 1986). The
spectral amplitude ratio is plotted in Figure 11. There is a relatively smooth decrease
in the spectral ratio with frequency between 5 and 10 kHz, where most of the power is
concentrated. One is tempted to fit a straight line through this region and obtain "\( Q_p \)"
from the slope. However, the "\( Q_p \)" thus obtained would be a lot lower than one would
expect from such a formation. In this case, the "\( Q_p \)" obtained by fitting a straight line
through the spectral ratio data is 7.5, much too low for a well consolidated sandstone.

The results from our iterative linearized inversion for this data are \( V_s = 1.75 \text{ km/s} \)
and \( Q_p = 50 \) after about 10 iterations. The \( V_s \) is exactly that obtained from the moveout
and shear wave log. \( Q_p \) is reasonable for a consolidated sandstone, although no inde­
pendent check was possible in this case. Figure 12 shows the two dimensional residual
squared error as a function of \( V_s \) and \( Q_p \). We can see that there is a distinct trough at
around \( V_s \) of 1.75 km/s, with a secondary trough at about 1.5 km/s. The trough in \( V_s \) is
significantly sharper than the synthetic example shown previously. Taking a cut in the
residual squared error plot along the line \( V_s = 1.75 \text{ km/s} \), and plotting the error as a
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Figure 5: Residual squared error plot as a function of $V_s$ and $1/Q_p$ for the synthetic data (10 to 15 kHz) used in the inversion.

Figure 6: Residual squared error plot as a function of $V_s$ and $1/Q_p$ for the synthetic data (5 to 15 kHz) used in the inversion.
Figure 7: Residual squared error plot as a function of $1/Q_p$ at $V_s = 1.24 \text{ km/s}$ for the synthetic data used in the inversion.

function of $1/Q_p$ (Figure 13), we can see the error has a well-defined minimum at $1/Q_p = 0.02$, or $Q_p = 50$. Thus, in this field example, the shear wave velocity and P-wave attenuation are well defined by this algorithm. Practical considerations in applying the technique to a noisy data set (DSDP site 613), such as how to avoid getting trapped in the local minima and the effects of uncertainties in the parameters assumed known, are discussed in Meredith et al. (this issue).

CONCLUSIONS

We have presented a method of obtaining formation S-wave velocity and P-wave Q from the P-wave train of the full waveform acoustic log in formations where no refracted shear wave or pseudo-Rayleigh wave exist. This method avoids the problems associated with the previously used one of inversion of Stoneley wave velocity to determine formation S-wave velocity such as permeability effects and poorly excited Stoneley waves. We have demonstrated that our inversion scheme can be applied to actual field data, and that it is possible to obtain $V_s$ from the P-wave train. $Q_p$ appears to be a bit more sensitive to the noise in the data.
Figure 8: Field data used in test of inversion algorithm

Figure 9: P-wave trains windowed from Figure 8 used for inversion. Starting time is arbitrary.
Figure 10: Spectra of P-wave trains from Figure 9 used for inversion

Figure 11: Ratio of spectra shown in Figure 10
Figure 12: Residual squared error plot as a function of $V_s$ and $1/Q_p$ for the field data used in the inversion.

Figure 13: Residual squared error plot as a function of $1/Q_p$ at $V_s = 1.75$ km/s for the field data used in the inversion.
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