EFFECTS OF A LOGGING TOOL ON THE
STONELEY WAVE PROPAGATION IN ELASTIC AND
POROUS FORMATIONS

by


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ABSTRACT

A detailed study is carried out to investigate the effects of an acoustic logging tool on the propagation characteristics of Stoneley waves in both elastic and porous formations. In an elastic formation, the presence of the tool in the borehole reduces the Stoneley velocity and enhances the Stoneley wave excitation. When intrinsic attenuation due to formation and bore fluid anelasticity is present, the tool reduces Stoneley attenuation due to fluid and increases the attenuation due to formation. For a permeable porous formation, the simplified Biot-Rosenbaum model of Tang et al. (1990) is modified to incorporate the effects of the tool on the Stoneley propagation. The presence of the tool increases the sensitivity of the Stoneley waves to the formation flow properties. Specifically, the dispersion of Stoneley velocity due to formation permeability is increased and the attenuation of the Stoneley waves is more pronounced, compared with the results without the tool. Consequently, in the determination of formation flow properties using Stoneley wave measurements, the effects of permeability may be better estimated using a tool with large diameter.

INTRODUCTION

An acoustic logging tool is essential to perform in-situ measurements in boreholes. The presence of the tool in the borehole modifies the excitation and propagation characteristics of borehole acoustic waves. White and Zechman (1968) carried out theoretical studies on the effects of a rigid tool and computed synthetic seismograms for a borehole containing the tool in the center. Cheng and Toksöz (1981) investigated the velocity dispersion characteristics of the guided waves in the presence of an elastic tool in the

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borehole and showed that the tool reduces the Stoneley velocity and significantly modifies the dispersion characteristics of pseudo-Rayleigh waves. In both studies, however, the specific relationship between the wave excitation function and the tool radius was not presented. In fact, this relationship is important for the design and use of actual logging tools. In addition, the effects of the tool on the Stoneley wave attenuation due to fluid and formation anelasticity have not been studied in detail. In this study, these effects will be analyzed following the previous work of White and Zechman (1968) and Cheng and Toksöz (1981).

The Stoneley wave is an interface wave borne in the borehole fluid. Since this wave is effectively associated with borehole fluid pressure, the Stoneley wave tends to drive the fluid into the formation through pores that are open to the borehole wall. Because of this, this wave is especially important for the estimation of formation permeability using acoustic logging measurements (Rosenbaum, 1974; Cheng et al., 1987; Schmitt et al. 1988; Winkler et al., 1989; Tang et al., 1990). In many analyses of wave propagation in permeable boreholes, the effects of a tool were neglected for the sake of simplicity (Schmitt et al., 1988; Winkler et al., 1989). Rosenbaum (1974) considered the effects of a rigid tool in the borehole. But the tool effects were not analyzed. Tang et al. (1990) have recently developed a simple model for Stoneley propagation in a permeable borehole. This model gives fully consistent results as the analyses of Rosenbaum (1974) and Winkler et al. (1989) in the presence of a hard porous formation (see Tang et al., 1990). In addition, the formulation and calculation of the Tang et al. (1990) model are much simplified. Moreover, as indicated in the Tang et al. (1990) article, the effects of a logging tool can be easily incorporated into the simple model. In this study, we will follow the work of Tang et al. (1990) to incorporate a rigid tool into their model and analyze the resulting effects on the propagation of Stoneley waves.

In the following section, we will first analyze the effects of a tool in the presence of an elastic formation, together with some numerical examples. Based on the elastic results, the effects of a tool in a porous formation will then be studied and illustrated with numerical examples. Finally, we will point out the general rule for the change of Stoneley characteristics due to the tool.

**EFFECTS OF A TOOL IN AN ELASTIC FORMATION**

In this section, we derive expressions that can be used to calculate Stoneley wave propagation and amplitude excitation for tools of given radius in a borehole penetrating a homogeneous, isotropic elastic formation. The effects of intrinsic attenuation due to formation and bore fluid anelasticity will also be incorporated.

The geometry of the borehole and the logging tool is shown in Figure 1. The axial direction is $z$, $R$ is the radius of the borehole, and $a$ is the radius of the tool. As a good
approximation, we assume that the tool is rigid. This assumption is valid when tools are made of hard metals (such as steel) that are very incompressible compared to the borehole fluid. In the frequency domain, the borehole fluid and formation compressional and shear displacement potentials that satisfy their respective wave equations can be written as (Cheng and Toksöz, 1981)

\[
\begin{align*}
\psi_f &= [CI_0(fr) + DK_0(fr)]e^{iks}, \quad a < r < R \\
\psi_p &= AK_0(4r)e^{iks}, \quad r \geq R \\
\psi_s &= BK_1(mr)e^{iks}, \quad r \geq R
\end{align*}
\]

where \(I_n\) and \(K_n\) (n=0,1) are the first and second kind modified Bessel functions, \(k\) is the wavenumber of the Stoneley wave in the axial direction, the radial wavenumbers are

\[
l = \sqrt{k^2 - \omega^2/V_p^2}, \quad m = \sqrt{k^2 - \omega^2/V_s^2}, \quad \text{and} \quad f = \sqrt{k^2 - \omega^2/V_f^2},
\]

\(\omega\) is angular frequency, and \(V_p, V_s,\) and \(V_f\) are compressional, shear, and fluid velocities, respectively. From the potentials, the radial displacements and stresses of the bore fluid \((u_f, \sigma_f)\) and of the formation \((u_r, \sigma_{rr}, \sigma_{rz})\) are calculated using (Ewing et al., 1957)

\[
\begin{align*}
u_f &= \frac{\partial \psi_f}{\partial r} , \\
\sigma_f &= -\rho_f \omega^2 \psi_f ,
\end{align*}
\]

and

\[
\begin{align*}
u_r &= \frac{\partial \psi_r}{\partial r} - \frac{\partial \psi_s}{\partial z} , \\
\sigma_{rr} &= -\rho \omega^2 \frac{\lambda}{\lambda + 2\mu} \psi_p + 2\mu \left( \frac{\partial^2 \psi_p}{\partial r^2} - \frac{\partial \psi_p}{\partial r} \frac{\partial \psi_r}{\partial z} \right) , \\
\sigma_{rz} &= -\rho \omega^2 \psi_s + 2\mu \left( \frac{\partial^2 \psi_p}{\partial r \partial z} - \frac{\partial \psi_p}{\partial z} \frac{\partial \psi_r}{\partial z} \right) ,
\end{align*}
\]

where \(\rho_f\) and \(\rho\) are fluid and formation densities, \(\lambda\) and \(\mu\) are Lamé constants of the formation and can be calculated from the given \(V_p, V_s,\) and \(\rho\) of the formation.

In the absence of the logging tool, wave excitation is obtained by placing a point pressure source at the center of the borehole (see Tsang and Rader, 1979). In the presence of the tool, wave excitation is simulated by assigning a radial displacement distribution \(U_r(a, \omega, z)\) at a certain part of the tool (White and Zechman, 1968). In the wavenumber \(k\) domain, the spectrum of the specified displacement is \(U_r(a, \omega, k)\). If we idealize the source distribution as a delta function located at \(z = 0\), i.e., \(U_r(a, \omega, z) = U_r(a, \omega)\delta(z)\), then its spectrum in the \(k\) domain is \(U_r(a, \omega)\). This specifies the radial fluid displacement at the tool surface

\[
u_f = U_r , \quad \text{at} \quad r = a .
\]

With the given fluid displacement at the tool, we now proceed to calculate the excitation function of the Stoneley wave in connection with the boundary conditions at the borehole
At \( r = R \), these conditions are the continuity of radial displacement and stress and vanishing of shear stress:

\[
\begin{align*}
\{ & u_r = u_f, \\
& \sigma_{rr} = \sigma_f, \quad \text{at } r = R \\
& \sigma_{rs} = 0.
\end{align*}
\]

Eqs. (4) and (5) lead to a set of linear simultaneous equations for determining the constants \( A, B, C \) and \( D \) in Eqs. (1).

\[
\begin{pmatrix}
0 & 0 & h_{13} & h_{14} \\
h_{21} & h_{22} & h_{23} & h_{24} \\
h_{31} & h_{32} & 0 & 0 \\
h_{41} & h_{42} & h_{43} & h_{44}
\end{pmatrix}
\begin{pmatrix}
A \\
B \\
C \\
D
\end{pmatrix}
= 
\begin{pmatrix}
U_r \\
0 \\
0 \\
0
\end{pmatrix},
\]

where

\[
\begin{align*}
h_{13} &= f I_1(\alpha), \\
h_{14} &= -f K_1(\alpha), \\
h_{21} &= \mu K_1(l)R, \\
h_{22} &= i\mu K_1(m)R, \\
h_{23} &= f I_1(f), \\
h_{24} &= -f K_1(f), \\
h_{31} &= 2\mu^2 K_1(l)R, \\
h_{32} &= (\rho\omega^2 - 2\mu\omega^2)K_1(m)R, \\
h_{41} &= -\rho\omega^2 K_0(l)R + 2\mu((l/R)K_1(l)R + k^2 K_0(l)R), \\
h_{42} &= 2\omega^2 K_0(m)R + K_1(m)R/m, \\
h_{43} &= \rho j\omega^2 I_0(f), \\
h_{44} &= \rho j\omega^2 K_0(f).
\end{align*}
\]

The homogeneous problem obtained by setting \( U_r = 0 \) in Eq. (6) is equivalent to the statement that the radial displacement at the tool surface vanishes – the rigid tool boundary condition. For non-trivial solutions \((A, B, C, D)\) to exist in this case, the determinant of the matrix in Eq. (6) (denoted by \(\Delta\)) must vanish. This results in the borehole dispersion equation

\[
\Delta = I_0(f) + \frac{I_1(\alpha)}{K_1(\alpha)} K_0(f) - \left[ I_1(\alpha) - \frac{I_1(\alpha)}{K_1(\alpha)} K_1(f) \right] \\
\times \frac{f \rho}{\rho_f} \left\{ \frac{2V_s^2 l m}{k^2 c^3} \left[ \frac{1}{m R} + \frac{2V_s^2 K_0(m)K_1(m)}{c^2 K_1(m)} \right] - \left( \frac{2V_s^2}{c^2} - 1 \right)^2 \frac{K_0(l)R}{K_1(l)R} \right\} \\
= 0,
\]

\( I_0(f) \) and \( I_1(\alpha) \) are the modified Bessel functions of the first kind, and \( K_0(\alpha) \) and \( K_1(\alpha) \) are the modified Bessel functions of the second kind. The constants \( \rho, \mu, \omega, \) and \( c \) represent the density, shear modulus, angular frequency, and wave velocity, respectively. The variables \( f, l, m \) and \( r \) represent the frequency, length, mass, and radius, respectively.
where \( c = \omega / k \) is the phase velocity of the wave modes that exist in the presence of the rigid tool. The pressure wave time series received by a receiver at a position \( z \) (\( z = 0 \) is the source location) on the tool is given by

\[
P(a, z, t) = \int_{-\infty}^{+\infty} \rho_f \omega^2 e^{i\omega t} dw \int_{-\infty}^{+\infty} [CI_0(f \alpha) + DK_0(f \alpha)] e^{ikz} dk
\]

\[
= \int_{-\infty}^{+\infty} e^{i\omega t} U_\tau(a, \omega) dw \int_{-\infty}^{+\infty} N e^{ikz} dk , \tag{8}
\]

with \( N \) given by

\[
N(k, \omega) = K_0(f \alpha) I_0(f \tau) - I_0(f \alpha) K_0(f \tau) - \frac{I_1(f \alpha)}{K_1(f \alpha)} \frac{K_1(f \tau)}{K_1(f \alpha)} [K_1(f \tau) I_0(f \alpha) + K_0(f \alpha) I_1(f \tau)] . \tag{9}
\]

In the derivation of Eq. (8), we have used the solutions for \( C \) and \( D \) which are found by solving the inhomogeneous problem of Eq. (6). Using the discrete wavenumber summation technique (Cheng and Toksöz, 1981) to evaluate the integral over \( k \) and performing an inverse Fourier transform over \( \omega \), we can compute the full waveform synthetic seismograms for the rigid tool case. In the present study, we are particularly interested in the Stoneley wave. The contribution of this wave mode to the integration over \( k \) can be evaluated using the residue theorem (Kurkjian, 1985).

\[
\int_{-\infty}^{+\infty} \left( \frac{N}{\Delta} e^{ikz} \right)_{\text{Stoneley}} dk = 2\pi i \left( \frac{N(k, \omega) e^{ikz}}{\partial \Delta / \partial k} \right)_{\text{Stoneley}} . \tag{10}
\]

By setting \( z = 0 \) in Eq. (10), the excitation of the Stoneley waves as a function of frequency in the presence of a rigid tool in the borehole can be evaluated. Note that although Eq. (10) is obtained by specifying the displacement at the tool surface, this excitation function is independent of the given source type (pressure or displacement). In fact, with the tool radius set to zero, Eq. (10) reduces identically to the excitation function due to a point pressure source in the borehole (Kurkjian, 1985). The evaluation of Eq. (10) requires that the \( k \) values be substituted with the Stoneley wavenumber found from Eq. (7). When the fluid and the formation are perfectly elastic, the \( k \) values for the Stoneley mode are real and can be found by searching the real \( k \)-axis for each frequency. In the presence of intrinsic body-wave attenuation due to the anelasticity of the bore fluid and formation solid, the Stoneley wavenumber \( k \) becomes complex and goes off the real \( k \)-axis. The effects of intrinsic attenuation are introduced by making the velocities \( V_p, V_s, \) and \( V_f \) complex through the following transformations (Aki and
where $Q_p$, $Q_s$, and $Q_f$ are compressional, shear, and fluid quality factors which are assumed independent of frequency using the constant-Q theory of Kjartansson (1979). The anelastic body-wave dispersion can also be added as necessary. Here we neglect this minor effect.

**Numerical Root Finding for the Stoneley Mode**

Because of the intrinsic attenuation effects, the roots of Eq. (7) become complex. However, in the vicinity of $\omega = 0$ these effects are minimal and $k$ values are very close to the real $k$-axis. In the zero-frequency limit, Eq. (7) can be asymptotically solved to give the Stoneley phase velocity as

$$c_{ST}(\omega \to 0) = V_f \left(1 - \frac{a^2}{R^2}\right)^{-1/2} \left(1 - \frac{a^2}{R^2} + \frac{\rho_f V_f^2}{\rho V_s^2}\right).$$

Using the $c_{ST}$ given by Eq. (12), the Stoneley wavenumber for a small frequency increment $\delta \omega$ from $\omega = 0$ is given as $\delta \omega/c_{ST}$. Once this initial $k$ value is known, we start tracing the roots of $k$ for successively increasing frequencies in the complex $k$ plane using a procedure described by Tsang (1985). Supposing that the root at $\omega$ has been found, the following relation is used to give the approximate location of the root at $\omega + \delta \omega$:

$$k(\omega + \delta \omega) = k(\omega) - \frac{\partial \Delta}{\partial \omega} \left(\frac{\partial \Delta}{\partial k}\right) \delta \omega.$$  

The root given by Eq. (13) is refined using the complex Newton-Raphson's method which iterates from the initial value of Eq. (13) through the following equation:

$$k^{(n+1)} = k^{(n)} - \frac{\Delta(k^{(n)}, \omega + \delta \omega)}{\partial \Delta \left(k^{(n)}, \omega + \delta \omega\right)}.$$
The iteration procedure continues until the accurate location of \( k(\omega + \delta \omega) \) is found. Eqs. (13) and (14) are then used repeatedly to find the roots at \( \omega + 2\delta \omega, \omega + 3\delta \omega, \ldots \), and so forth. Because \( \partial \Delta / \partial k \) is used in the root finding in the evaluation of Eq. (10), its exact expression is obtained by analytically differentiating Eq. (7) with respect to \( k \). This expression is given in the appendix. Once the complex Stoneley wavenumber \( k \) is found, the Stoneley phase velocity and attenuation are calculated using

\[
\begin{align*}
\epsilon_{ST} &= \omega / Re(k), \\
Q^{-1} &= 2 Im(k) / Re(k).
\end{align*}
\]

Numerical Results

In the following numerical examples, we will study the effects of the radius of a rigid tool on the excitation, velocity, and attenuation of the borehole Stoneley waves. In all examples, the borehole radius is fixed at \( R=10 \text{ cm} \) and the borehole fluid has an acoustic velocity \( V_f=1.5 \text{ km/s} \) and density \( \rho_f=1 \text{ g/cm}^3 \).

We first examine the effects of the tool radius on the excitation of the Stoneley wave for both a hard and a soft formation. Figure 2 shows the amplitude of the excitation function evaluated using Eq. (10) with \( z=0 \). The formation is a hard formation with \( V_p=5 \text{ km/s}, V_s=3 \text{ km/s}, \) and \( \rho=2.65 \text{ g/cm}^3 \). The results for tool radius \( a=0.3 R, 0.6 R, \) and \( 0.8 R \) are plotted against the result without the tool, as indicated in Figure 2. We see from this figure that for all frequencies ranging from 0 to 10 kHz, the Stoneley wave excitation is systematically increased with increasing tool radius, compared with the excitation for a purely fluid-filled bore. Figure 3 shows the excitation for a soft formation. The formation properties are: \( V_p=2.3 \text{ km/s}, V_s=1.2 \text{ km/s}, \) and \( \rho=2.3 \text{ g/cm}^3 \). In a soft formation whose shear velocity is smaller than the fluid velocity, the effective Stoneley excitation is shifted to a much lower frequency range. For this reason, the results in Figure 3 are shown only in the frequency range of \([0,6]\) kHz. However, the systematic increase in the Stoneley wave excitation with increasing tool radius is still similar to the hard formation case (Figure 2). Note that increasing tool radius has effects similar to reducing the borehole radius. For a fluid-filled borehole without a tool, Stoneley excitation increases with decreasing borehole radius (Tubman et al., 1984). We also note that the effects of intrinsic attenuation do not affect the excitation of Stoneley waves significantly. The results in Figures 2 and 3 are calculated without intrinsic attenuation. We have repeated the calculations with \( Q_p=20, Q_s=10, \) and \( Q_f=10 \) (fairly strong attenuation). The resulting curves are almost identical to those shown in Figures 2 and 3. This means that the anelastic effects in the formation solid and bore fluid play a negligible role in the Stoneley wave excitation. They are important mainly in affecting the propagation characteristics of the Stoneley waves.

The effects of the tool on the Stoneley wave propagation are now studied. We use the same formation properties as those used in Figure 2. Figure 4 shows the Stoneley wave
phase velocity in the frequency range of [0,10] kHz. The results are shown for three different tool radii \( a = 3 \) cm, 6 cm, and 8 cm, against the velocity with a fluid-filled borehole. With increasing tool radius, Stoneley velocity is systematically decreased. This result is similar to that obtained by Cheng and Toksöz (1981) using an elastic tool with large elastic moduli. Note that the decrease of Stoneley velocity becomes dramatic when the tool radius approaches the borehole radius. This can be easily illustrated by inspecting the zero-frequency solution given in Eq. (12). In fact, when the tool surface is close to the borehole wall, the Stoneley wave is similar to the slow fracture wave studied by Ferrazzini and Aki (1987) and Tang and Cheng (1988), although in our case one surface of the 'fracture' is kept rigid. The effects of the tool on the Stoneley attenuation due to intrinsic effects are varied depending on the relative importance of the contribution due to bore fluid and that due to the formation. In Figure 5, we have calculated the Stoneley attenuation \( 1/Q \) versus frequency using \( Q_p = 100, Q_s = 50, \) and \( Q_f = 20 \). The density and velocities of the formation are the same as in Figure 4. According to the analysis of partition coefficients of the formation and fluid (Cheng et al., 1982), Stoneley waves are most sensitive to the borehole fluid properties. Thus the Stoneley attenuation shown in Figure 5 is primarily the contribution from \( 1/Q_f \). The introduction of the tool decreases the fluid volume and reduces fluid contribution to the Stoneley wave attenuation. This effect is very much as expected. As we see from Figure 5, the Stoneley wave attenuation is reduced with increasing tool radius. On the other hand, if the fluid is perfectly elastic \( (Q_f = \infty) \), the tool will increase the sensitivity of the Stoneley wave to the formation anelasticity. To illustrate this, we have calculated the Stoneley attenuation \( 1/Q \) with \( Q_f \) set to \( \infty \) and \( Q_p = 20 \) and \( Q_s = 10 \); other formation properties are unchanged. The results are shown in Figure 6. As expected, the attenuation is increasingly augmented with increasing tool radius. An inspection of Figure 6 shows that the attenuation due to formation is approximately in inverse proportion to the area of the fluid annulus between formation and tool. The results shown in Figure 6 have important implications to acoustic logging in permeable porous formations. As shown by Cheng et al. (1987), Stoneley wave attenuation in a porous formation is primarily due to the formation flow properties, even in the presence of intrinsic attenuation. Thus the effects of the tool are expected to increase the sensitivity of the Stoneley wave to the flow properties.

**EFFECTS OF A TOOL IN A POROUS FORMATION**

For the study of acoustic logging in a permeable porous formation with a logging tool, the problem can be formulated by modeling the formation as a Biot solid (Biot, 1956a,b), as shown in the work of Rosenbaum (1974). The model is therefore referred to as Biot-Rosenbaum model (Cheng et al., 1987). Because of effects of fluid flow in the formation, a set of coupled differential equations has to be solved in connection with the boundary conditions at the borehole wall. This makes the model somewhat complicated. Using
the theory of dynamic permeability (Johnson et al., 1987) to account for the fluid flow effects, Tang et al. (1990) have developed a much simplified version of the Biot-Rosenbaum model. This simplified model yields almost identical results as the complete theory of the Biot-Rosenbaum model in the presence of a hard porous formation (such as sandstone). According to the simple model, the Stoneley wavenumber \( k \) in a fluid-filled borehole is given by a simple, explicit formula:

\[
k = \sqrt{k_e^2 + \frac{2i\rho_0\omega\kappa(\omega)}{R\eta} \sqrt{-\frac{i\omega}{D} + k_e^2} \frac{K_1(R\sqrt{-i\omega/D + k_e^2})}{K_0(R\sqrt{-i\omega/D + k_e^2})}},
\]

with

\[
D = \frac{\kappa(\omega)K_f}{\phi\eta(1 + \xi)} ,
\]

where \( \kappa(\omega) \) is the dynamic permeability of Johnson et al. (1987), \( K_f, \rho_0, \) and \( \eta \) are pore fluid bulk modulus, density, and viscosity, respectively, \( \phi \) is porosity, and \( \xi \) is a correction for solid elasticity. Detailed derivation of Eq. (16) can be found in the article of Tang et al. (1990). In addition, \( k_e \) in Eq. (16) is the Stoneley wavenumber corresponding to an equivalent elastic formation that consists of the pore skeleton and fluid. Thus \( k_e \) is calculated from the borehole dispersion equation using the fluid-saturated properties of the porous formation.

In the presence of a logging tool in the borehole, Eq. (16) only needs two simple modifications. The first is the factor \( 2/R \) that multiplies the second term under the square root sign in Eq. (16), which is the ratio of bore perimeter to bore area. With a tool of radius \( a \) in the borehole, the area becomes that of the fluid annulus and this ratio becomes \( 2R/(R^2 - a^2) \). Another modification is calculating the elastic formation Stoneley wavenumber \( k_e \) in Eq. (16) in conjunction with the logging tool. For a rigid tool, this procedure has been described in the previous section. We need also to note that the formation properties \( V_p, V_s, \) and \( \rho \) should be those of the fluid-saturated formation. If \( V_p, V_s, \) and \( \rho \) are given for the dry rock, they can be converted to the fluid-saturated properties using a procedure described by Tang et al. (1990). We denote the elastic formation Stoneley wavenumber calculated with the tool by \( k_{ett} \). With these two modifications, the Stoneley wavenumber for a permeable borehole with a rigid tool is given by

\[
k = \sqrt{k_{ett}^2 + \frac{2Ri\rho_0\omega\kappa(\omega)}{(R^2 - a^2)\eta} \sqrt{-\frac{i\omega}{D} + k_{ett}^2} \frac{K_1(R\sqrt{-i\omega/D + k_{ett}^2})}{K_0(R\sqrt{-i\omega/D + k_{ett}^2})}}.
\]

This formula takes into account the tool effects in a simple, straightforward manner. From the Stoneley wavenumber, the Stoneley wave velocity dispersion and attenuation with the tool effects can be calculated using Eq. (15).
Numerical Results

With Eq. (18), we can now demonstrate the effects of a rigid logging tool on the propagation characteristics of the Stoneley wave in a porous formation. Although we have the solution for the Stoneley wavenumber in a permeable borehole (Eq. 18), we are unable to calculate the corresponding Stoneley wave excitation function without doing more complicated studies to find the changes of the Stoneley wave functions (Eq. 1) due to fluid flow. This is because Eq. (16) was obtained using a perturbation theory (Tang et al., 1990). According to Rayleigh’s (1877) principle, the perturbation of the eigenvalue (the Stoneley wavenumber) can be obtained without the explicit knowledge of the perturbation in the eigen functions (the Stoneley wave functions as given in Eqs. 1). On the other hand, the calculation of the wave excitation function does need to know the change in the eigen functions. In fact, the permeable borehole affects the Stoneley wave excitation, as shown by Schmitt et al. (1988). However, since the effect of the tool is primarily to reduce the fluid volume, the tool will affect the wave excitation in a porous formation in much the same way as it affects the excitation in an elastic formation, as shown in Figures 2 and 3.

For calculating the propagation characteristics using Eq. (18), we use the properties of a sandstone with dry velocities \( V_p = 3.8 \text{ km/s}, \ V_s = 2.2 \text{ km/s}, \) porosity \( \phi = 0.25, \) dry density \( \rho = 2.65 \text{ g/cm}^3 \) and permeability of 1 Darcy. We assume that the pore fluid and the borehole fluid properties are the same fluid with \( \rho_f = \rho_0 = 1 \text{ g/cm}^3 \) and \( V_f = 1.5 \text{ km/s}. \) The borehole radius is fixed at 10 cm. In order to illustrate the effects due to fluid flow, no intrinsic attenuation is included in the following examples. Figure 7 shows the calculated Stoneley phase velocity in the frequency range of \([0,10]\) kHz for three different tool radii: \( a = 3 \text{ cm}, 5 \text{ cm}, \) and \( 7 \text{ cm}. \) Because of the fluid flow effects, the Stoneley velocity is significantly decreased at low frequencies. Compared with the dispersion curve without the tool, the Stoneley velocity with the tool is further decreased. The larger the tool, the more pronounced the decrease is. Figure 8 shows the calculated Stoneley wave attenuation \((1/Q)\) in the frequency range of \([0,10]\) kHz for the same three tool radii. The attenuation increases with increasing tool radius and is approximately in inverse proportion to the area of the fluid annulus. This behavior is very similar to the behavior in the presence of intrinsic attenuation due to formation anelasticity (Figure 6). Comparing Figures 7 and 8 with Figure 10 of Schmitt et al. (1988) in which the effects of borehole radius were illustrated, we can see that the effects of increasing tool radius on Stoneley waves is very similar to the effects of decreasing the radius of a fluid-filled borehole. This is due to the fact the Stoneley waves are controlled by the effective area of the bore fluid. Increasing tool radius or decreasing borehole radius have the same effects on the fluid area.

In order to show the effects of the tool for different formation permeabilities, we calculate Eq. (18) versus permeability in the range of \([0,10]\) Darcies. The frequency is fixed at 5 kHz and other parameters are the same as those used in Figures 7 and 8.
Figure 9 shows the Stoneley velocity versus permeability for the same three tool radii as in Figures 7 and 8. The velocity minima around 1 Darcy is related to the transition from the low-frequency region to the high-frequency region of the Biot theory – a behavior of the Biot-Rosenbaum model that is also predicted by the simple model of Tang et al. (1990). For the four decades of permeability, the effects of the tool on the velocity are nearly the same. The velocity curves are systematically shifted towards lower velocities with increasing tool radius. Figure 10 shows the calculated Stoneley attenuation versus permeability in the range of [0,10] Darcies. The attenuation increases with increasing permeability and is further increased with increasing tool radius.

The effects of a logging tool in a porous formation on the propagation of Stoneley waves can be demonstrated by looking at the synthetic seismograms of these waves. As a zero-order approximation, we approximate the Stoneley wave excitation in a porous formation using the equivalent elastic formation (i.e., the fluid-saturated) properties and calculate the seismograms for the Stoneley mode using Eqs. (8) and (10). For the propagation factor $e^{ikz}$ in these equations, the solution given in Eq. (18) is used for the wavenumber $k$. Although the excitation in the equivalent elastic formation may slightly differ from that in the permeable porous formation (Schmitt et al., 1988), the propagation characteristics are unaffected as long as the effects of the formation fluid flow on the propagation constant (i.e., the wavenumber $k$) are correctly accounted for.

In addition, we use a Kelly source (Kelly et al., 1976) for the frequency-dependence of the tool displacement $U_f(a, \omega)$. In the calculations, the center frequency of the Kelly source is chosen to be 5 kHz. Figure 11 shows calculated synthetic seismograms of the Stoneley waves for tool radii of 0.5 cm (b) and 0.7 cm (c). The seismograms without the tool (a) are also calculated for comparison. The parameters used in the calculation are the same as those used in calculating Figures 7 and 8. The permeability used in the calculations is 1 Darcy. Because such a permeability represents a fairly permeable formation, the resulting Stoneley attenuation is significant (see Figure 8).

In Figure 11(a), the Stoneley waves across an array of 3 m offset with 11 equi-spaced receivers are shown (tool effects are not included). Because of the fluid flow into the formation and the resulting attenuation, the waves are gradually attenuated across the array. The wave amplitude of the last trace is much attenuated compared with that of the first trace. With a tool of radius $a=5$ cm, the wave attenuation is further increased, as shown in Figure 11(b). When the tool radius becomes $a=7$ cm, the attenuation is nearly doubled (see Figure 8 at 5 kHz), so that the waveforms of the last three traces in Figure 11(c) become very weak. Comparing the last trace of Figure 11(c) with that of Figure 11(a), we see that the attenuation is much more severe with the tool of $a=7$ cm than without the tool. In addition, the wave signal in Figure 11(c) is delayed about 0.14 ms compared with the signal in Figure 11(a), in accordance with the decrease of Stoneley velocity shown in Figure 7 around 5 kHz.
CONCLUSIONS

In this study we made a detailed investigation on the effects of a rigid tool on the Stoneley waves in elastic and porous formations. In both formations, the presence of the tool increases the Stoneley excitation and decreases Stoneley velocity. For the attenuation effects, the tool decreases the attenuation due to bore fluid and increases the attenuation due to formation (anelasticity or pore fluid flow). We have found that these variations due to effects of the tool are proportional to the change in the area of the fluid annulus between tool and formation. This study, together with previous studies on the effects of borehole radius (Tubman et al., 1984; Schmitt and Bouchon, 1985; Schmitt et al., 1988), indicates that the fluid-borne Stoneley wave is largely controlled by the effective area of the fluid in the borehole. Increasing the tool radius or decreasing the borehole radius will reduce this fluid area, thus having similar effects on the Stoneley excitation and propagation characteristics. Qualitatively speaking, having a tool in the borehole seems to make the borehole smaller, which decreases the sensitivity of the Stoneley wave to the fluid and increases the sensitivity to the formation. A convincing example has been shown in Figures 5 and 6 for the case of intrinsic attenuation.

For the porous formation case, the use of the simple Stoneley propagation theory of Tang et al. (1990) allows us to incorporate the tool effects in a simple manner. This results in a simple, useful theory that can be applied to acoustic logging in porous formations with a logging tool. As predicted by this model, the presence of a tool in the borehole increases the sensitivity of the Stoneley wave to the formation flow properties. This illustrates that the effects of formation permeability will be better determined with a large tool than with a small tool.

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In this appendix, we derive the exact expression for the derivative of Eq. (7) with respect to the wavenumber \( k = \omega/c \). We rewrite Eq. (7) as

\[
\Delta = P_0 + \frac{f \rho f}{\rho} P_1 Q ,
\]

where the notations \( P_0, P_1, \) and \( Q \) are given by

\[
\begin{align*}
P_0 &= I_0(fR) + \frac{I_1(fa)}{K_1(fa)} K_0(fR) , \\
P_1 &= I_1(fR) - \frac{I_1(fa)}{K_1(fa)} K_1(fR) , \\
Q &= \frac{1}{l} \left\{ \left( \frac{2V_s^2}{c_s^2} - 1 \right)^2 \frac{K_0(lR)}{K_1(lR)} - \frac{2V_s^2 m}{\omega^2} \left[ \frac{1}{mR} + \frac{2V_s^2}{c_s^2} \frac{K_0(mR)}{K_1(mR)} \right] \right\} .
\end{align*}
\]

Differentiating Eq. (A.1) with respect to \( k \), we get

\[
\frac{\partial \Delta}{\partial k} = \frac{\partial P_0}{\partial k} + \frac{f \rho f}{\rho} \frac{P_1 Q}{f} \left( f \frac{\partial P_1}{\partial k} + k \frac{\partial P_1}{\partial k} \right) + \frac{\partial P_1}{\partial k} + \frac{f \rho f}{\rho} \frac{P_1}{f} \frac{\partial Q}{\partial k} ,
\]

with

\[
\begin{align*}
\frac{\partial P_0}{\partial k} &= kR \frac{I_1(fR) - \frac{I_1(fa)}{K_1(fa)} K_1(fR)}{f^2 K_1^2(fa)} + k \frac{K_0(fR)}{f^2 K_1^2(fa)} , \\
\frac{\partial P_1}{\partial k} &= kR \frac{I_0(fR) - \frac{1}{fR} I_1(fR) + K_0(fR) + \frac{1}{fR} K_1(fR) \frac{I_1(fa)}{K_1(fa)}}{f^2 K_1^2(fa)} + k \frac{K_1(fR)}{f^2 K_1^2(fa)} , \\
\frac{\partial Q}{\partial k} &= \frac{kR}{l^2} \left[ \frac{2V_s^2}{c_s^2} - 1 \right] \left( \frac{6V_s^2}{c_s^2} - \frac{8V_s^2}{V_p^2} + 1 \right) \frac{K_0(lR)}{K_1(lR)} + \frac{kR}{l^2} \left( \frac{2V_s^2}{c_s^2} - 1 \right)^2 \left[ \frac{K_0^2(lR)}{K_1^2(lR)} + \frac{1}{lR K_1(lR)} - 1 \right] \\
&- \frac{4kV_s^4}{\omega^4 m} \left( 3k^2 - \frac{2\omega^2}{V_s^2} \right) K_0(mR) + \frac{4V_s^4 R}{\omega^2} \left[ \frac{K_0^2(mR)}{K_1^2(mR)} + \frac{1}{mR K_1(mR)} - 1 \right] .
\end{align*}
\]

Although the expression for \( \frac{\partial \Delta}{\partial k} \) is quite lengthy, it can be programmed with a computer to give values that are more accurate than those obtained from finite-difference approximation.
Figure 1: Diagram showing the acoustic logging in a fluid-filled borehole with a logging tool.
Figure 2: Stoneley wave excitation function versus frequency for different tool radii, as indicated in the figure. The formation is a hard formation.
Figure 3: Stoneley wave excitation function versus frequency for different tool radii. The formation is a soft formation.
Figure 4: Stoneley wave phase velocity versus frequency for different tool radii. Compared with the velocity without the tool, Stoneley velocity is decreased with increasing tool radius.
Figure 5: Stoneley attenuation ($1/Q$) versus frequency for different tool radii. The Stoneley attenuation is primarily due to the fluid attenuation ($1/Q_f=0.05$). The fluid contribution is decreased with increasing tool radius, resulting in the decrease of Stoneley attenuation.
Figure 6: Stoneley attenuation for different tool radii. The fluid attenuation is absent ($1/Q_f=0$) and only formation attenuations ($1/Q_p$ and $1/Q_s$) are present. In this case, increasing tool radius results in increasing the sensitivity of the Stoneley wave to formation attenuations.
Figure 7: Stoneley wave dispersion due to a porous formation (permeability = 1 Darcy) for different tool radii. Compared with the velocity without the tool, the dispersion is increased with increasing tool radius.
Figure 8: Stoneley wave attenuation due to a porous formation (permeability = 1 Darcy) for different tool radii. The attenuation is increased with increasing tool radius.
Figure 9: Stoneley wave velocity versus formation permeability for different tool radii.
Figure 10: Stoneley wave attenuation versus formation permeability for different tool radii.
Figure 11: Synthetic Stoneley wave seismograms calculated for a borehole ($R=10$ cm) with a porous formation (permeability = 1 Darcy). The waves are displayed at 11 equi-spaced receivers along an array of 3 m offset. (a) Stoneley waves without a tool.
Figure 11: (b) Stoneley waves with a 5 cm radius tool.
Figure 11: (c) Stoneley waves with a 7 cm radius tool.