This paper describes a fast algorithm for estimating formation permeability from Stoneley wave logs. The procedure uses a simplified Biot-Rosenbaum model formulation. The input to the inversion is the Stoneley wave spectral amplitudes at each depth and receiver, the borehole fluid properties (velocity and density), the borehole caliper log, the formation density and porosity (from log data), and the compressional and shear velocities for the interval of interest. The model uses the borehole caliper and elastic properties to compute the Stoneley wave excitation (that is, predicted amplitude without permeability effects) as a function of frequency, and the porosity and permeability to compute the fluid flow amplitude reduction. This method also uses a reference depth of known permeability and compares amplitude variations at other depths relative to the reference depth. The permeability value obtained from the inversion represents the best fit over all receivers and all relevant frequencies. A processing example is shown to demonstrate the ability of this technique to extract formation permeability from Stoneley wave logs.

INTRODUCTION

Because acoustic logging is fast and relatively inexpensive to perform compared to other borehole permeability measurement techniques (e.g., borehole pump test, core measurement, etc.), estimating permeability from Stoneley wave logs provides an effective and economic means to obtain formation fluid transport properties. It has long been recognized that the borehole Stoneley wave amplitude is correlated to formation permeability
Theoretical models based on Biot (1962) theory have been developed (Rosenbaum, 1974; Schmitt et al., 1988) and have been validated by laboratory measurements (Winkler et al., 1988). These models have also been used to formulate inversion procedures for estimating permeability from Stoneley wave data (Burns, 1990; Cheng and Cheng, 1991). However, the inversion based on full Biot theory is computationally very time consuming and therefore is not suitable for processing field Stoneley wave data of large volume. Recently, Tang et al. (1991) developed a simplified Biot model for the borehole Stoneley wave propagation. This model has been expanded to include tool effects and is also applicable to slow formation situations (see the appendix). This model yields results that are consistent with full Biot theory, but the formulation and computation are much simplified. Especially for the inversion procedure where intensive calculation of the forward model is required, the simplified model is by orders of magnitude faster than the full Biot model. In this study, we use the simplified model to formulate an inversion procedure for estimating permeability from Stoneley wave logs. We will also apply the inversion processing technique to a field data set from a water well in New Mexico to estimate permeability for this well.

**MODEL DESCRIPTION**

During the Stoneley wave propagation in a permeable borehole, the borehole wave excites three types of waves in the formation. They are compressional and shear waves (the "fast" waves by Biot's definition) and the slow wave. The slow wave is primarily associated with the motion of pore fluid. In the full Biot theory, the three waves are intimately coupled in their interaction with the borehole Stoneley wave (Rosenbaum, 1974). Therefore the calculation is quite involved. In the simplified theory (Tang et al., 1991), the interaction is decomposed into two parts. The first is the interaction of the Stoneley wave with the fast waves. (i.e., an equivalent elastic formation whose acoustic properties are those of the fluid-saturated rocks). The second is the interaction of the Stoneley wave with the slow wave. Because the slow wave is strongly frequency-dependent and its excitation due to the borehole Stoneley wave is a dynamic wave phenomenon, the frequency dependency of the dynamic fluid flow must be accounted for. The model takes advantage of the theory of dynamic permeability (Johnson et al., 1987) to measure the amount of Stoneley wave energy that is carried away into the formation by the slow wave. The use of dynamic permeability in the simplified model captures the frequency-dependent behavior of the slow wave and greatly simplifies the formulation and numerical computation involved. A review of the model and model update are described in the appendix.

The input parameters to the model are: bulk density and compressional and shear velocities of the saturated rocks (these parameters are used to calculate the response of the equivalent elastic formation to the Stoneley wave); formation permeability, porosity, and pore structure tortuosity (tortuosity usually equals 3 for porous media and 1 for fractures); pore fluid density, acoustic speed, and viscosity (usually pore fluid is taken as borehole fluid). The output of this model is the complex Stoneley wavenumber (see Tang et al., 1991, and the appendix for its functional form) which gives the Stoneley wave amplitude attenuation and phase velocity dispersion due to the permeable formation.
In the permeability estimation procedure, various parameters such as borehole diameter, formation density, velocities, porosity, etc. can be obtained from borehole logs. The pore fluid properties can be estimated from the type of borehole fluid used. Given these parameters, formation permeability can be estimated using the model theory.

INVERSION METHODOLOGY

The inversion procedure can be formulated for two data acquisition configurations: array and iso-offset. In the array configuration, the amplitude decay of the waveform across the array can be used in the inversion to calculate permeability. This method requires that each receiver in the array should have the same frequency response. Another effect that may significantly affect the inversion results is that the effect of intrinsic attenuation in borehole fluid and formation (particularly in the bore fluid) also contributes to the amplitude decay of Stoneley waves. To minimize this effect, the iso-offset configuration is used in the present formulation.

For the iso-offset Stoneley wave data at depth $z$, the recorded wave spectrum includes several contributions, as given in the following:

$$A(f, z) = S(f)E(f, z)P[p(z), f] \exp[ik(p)dz]Q(f)R(f)$$ (1)

where $S(f)$ is source spectrum and $R(f)$ is receiver response; $f$ denotes frequency. Because the same source and receiver are used in the iso-offset data, $S(f)$ and $R(f)$ are assumed unchanged during logging. $Q(f)$ is the amplitude reduction due to intrinsic attenuation along the wave path from source to receiver. $Q(f)$ is primarily controlled by the intrinsic loss within the borehole fluid, amplitude decay due to formation anelasticity being small compared to that due to permeability effects, especially at low frequencies (Tang and Cheng, 1993). Therefore, if the bore fluid attenuation does not vary significantly in the depth range of interest, the variations in $Q(f)$ can be neglected. In the inversion procedure, the Stoneley wave excitation function $E(f, z)$, excitation reduction function $P[p(z), f]$ due to permeability $p(z)$, and the propagation loss due to permeability along the wave path $dz$ ($dz$ is source-receiver spacing) are of primary importance and will be described below.

Excitation Function $E(f, z)$

$E(f, z)$ is defined as the Stoneley wave amplitude produced by the source of unit intensity for a borehole surrounded by an elastic formation. For a permeable formation, $E(f, z)$ is calculated with the equivalent elastic formation parameters ($P$ and $S$ velocities and density of the fluid-saturated rock) at depth $z$. In addition, $E(f, z)$ is a strong function of borehole diameter as well as the tool diameter. Therefore, a borehole caliper log is needed to compute $E(f, z)$. The procedure for computing the excitation function is described in Tang and Cheng (1993). During logging, the elastic parameters and borehole diameter change along the well bore. This results in the change of $E(f, z)$ and
therefore change of the excited Stoneley wave amplitude. This will be demonstrated later in connection with a data processing example.

Excitation Reduction due to Permeability $P[p(z), f]$

In a permeable formation, the excited Stoneley wave amplitude is lower than that predicted by $E(f, z)$ calculated using the equivalent elastic formation parameters, because of the dynamic fluid flow effects at the borehole wall. Using the full Biot theory, the complete excitation function for a permeable formation can be calculated (Schmitt et al., 1988), but the formulation and calculation are quite complicated. Based on the simplified Biot-Rosenbaum model, Tang and Cheng (1993) presented a simple method to calculate the excitation reduction due to permeability. An example of using the simple method, compared with the result from the full Biot theory, is shown in Figure 1. The two results agree fairly well. The importance of this effect is that in estimating permeability using amplitudes of low-frequency Stoneley waves, it is necessary to take into account the effects of permeability on the excitation of Stoneley wave amplitude.

Propagation Loss due to Permeability $\exp[ik(p)dz]$

The Stoneley wave amplitude decay along a permeable borehole is described by the exponential function $\exp[ik(p)dz]$, where the complex Stoneley wavenumber $k(p)$ as a function of permeability $p$ is calculated using the simplified Biot theory of Tang et al. (1991). Tang and Cheng (1993) have also incorporated the effects of a logging tool in the calculation of $k(p)$. In the simplified theory, the Stoneley wavenumber corresponding to the equivalent elastic formation is first calculated. This wavenumber is then combined with the permeability effects to give $k(p)$. Therefore, such parameters as formation density, $P$ and $S$ velocities, and borehole caliper are also needed to calculate $k(p)$.

Having established the relation between Stoneley wave amplitude and permeability (Eq. (1)) in conjunction with other borehole parameters, we can use this relation to formulate an inversion procedure to estimate formation permeability from Stoneley wave amplitude logs. For the iso-offset configuration, the absolute permeability estimation needs to identify a reference depth $z_0$ at which the permeability $p_0$ is known. For practical purposes, this depth can be found within an impermeable section (this section usually corresponds to the highest Stoneley wave amplitude), for which the reference permeability $p_0 = 0$.

For the reference depth of known permeability, the Stoneley wave amplitude at other depth $z$ is compared with that of the reference depth. This is done through an amplitude minimization procedure. The minimization is performed by finding the appropriate permeability $p$ that minimizes the difference between the amplitudes modified from $A(f, z)$ and $A(f, z_0)$ for the frequency range $F$ centered at the center frequency of the waves. Using Eq. (1), cost function for this minimization can be constructed as
Fast Inversion for Permeability

\[ W(p) = \sum_{j=1}^{N} \sum_{f} ||A_j(f, z)E(f, z_0)P(p_0, f)exp[ik(p_0)dz_j] \\
- A_j(f, z_0)E(f, z)P(p, f)exp[ik(p)dz_j]|| \]  \hspace{1cm} (2)

where \( ||.|| \) is the \( l_2 \) norm of the complex functions involved. In the case of an array tool having \( N \) receivers, the cost function \( W(p) \) sums over all the iso-offset receiver pairs with different source receiver spacing \( d_j \) \((j = 1, \ldots, N)\). In this way, the estimated permeability \( p \) is the average permeability from source to the center of the receiver array. Because the source and receiver functions \( S(f) \) and \( R(f) \) and intrinsic attenuation effect \( Q(f) \) are assumed to be the same for all depths, these functions are not included in the minimization procedure. Because of the fast and efficient way of calculating \( P(p, f) \) and \( k(p) \) using the simplified theory (Tang et al., 1991; Tang and Cheng, 1993), the inversion can be performed quickly and efficiently, allowing a large data set to be processed within a reasonable period of time.

DATA PROCESSING

The actual processing procedure for permeability estimation is illustrated in Figure 2. The waveform data are first input into the computer disk. If the compressional and/or shear velocity information is missing in the log data, the waveform data are processed to obtain the velocity logs. Accurate estimation of shear velocity is required because the Stoneley wave excitation is very sensitive to formation shear velocity, especially in the slow formation situations. For a slow formation, shear velocity can be obtained from dipole logging data or from the Stoneley wave data using a dispersive processing technique (Tang et al., 1992; Tang, 1993). This technique uses the dispersive wave analysis and therefore corrects the dispersion effects in the velocity data. After the velocity processing, the velocity logs are combined with other log data such as density, porosity, and caliper to form an input file for the permeability estimation program. The Stoneley wave event in the waveform data is first tracked for each receiver to determine the approximate arrival time of the Stoneley waves. If there are significant scattering effects in the waveform data, the data are separated as transmitted and scattered or reflected waves using the arrival time pick as a guide. The wavefield separation is based on the incoherent/coherent wavefield separation algorithm developed by Reiter and Burns (1991). The wavefield separation procedure is important when processing Stoneley waves in the vicinity of fracture zones. By this wavefield separation, the reflections generated by the fracture zones will be largely suppressed in the coherent or transmitted wavefield. The transmitted wavefield can be used to estimate the average permeability of the fracture zones. From the input borehole parameters and coherent waveform data, permeability is estimated using the above described inversion algorithm. The inversion program is fast and effective. It takes about 1 to 3 seconds to obtain permeability for one depth on a Sparc 1 workstation.
In this section, we demonstrate the permeability processing procedure using a logging data set obtained from a water well in Sandoval, New Mexico. This well was recently drilled into a very permeable formation with unconsolidated sands and gravels. Figure 3 shows the caliper, bulk density and porosity logs for the depth range of 50' to 1700' logged in the well. Of particular interest is that the borehole has an unusually large diameter (averaged about 40% – 50%).

Three acoustic logging data sets were collected. They are: high-frequency monopole, dipole, and low-frequency Stoneley waveform logs. Because the formation is acoustically very slow, no shear waves can be measured from the high frequency monopole waveforms. For this reason, dipole logging was performed to determine shear wave slowness values. However, it was disappointing to find that the dipole tool in this large diameter well functioned like a monopole tool, generating predominantly compressional waves and tube waves. We then decided to derive the shear wave slowness values from the low-frequency Stoneley wave log, because in the slow formation situation the sensitivity of the Stoneley wave to shear velocity is most significant (Cheng and Toksoz, 1983).

Figure 4 shows the Stoneley waveform log (receiver 1, decimation factor is 15 for this plot) for the depth range from 50' to 1700'. These waves were obtained by low-pass filtering (below 1 kHz) the raw data. As can be seen, there are significant variations both in amplitude and arrival time. We applied the dispersive processing technique to the Stoneley wave and obtained the shear wave slowness log shown in Figure 5, together with the compressional wave slowness obtained from processing the high frequency monopole waveforms. The validity of the shear slowness log can be verified both by its magnitude and vertical variation. The average $V_p/V_s$ ratio is around 2.5, quite consistent with the published information on unconsolidated sedimentary formations (Castagna et al., 1985; Williams, 1990). Also, there is a good correlation between the compressional and shear slowness logs. The validity of the shear slowness log will be further verified when we use this log to calculate the theoretical Stoneley wave excitation amplitude and compare it with the measured amplitude.

The Stoneley waveform data shown in Figure 4 were processed to pick the arrival time of a certain phase (first negative peak) of the Stoneley wave. The picked time is also plotted in this figure (solid line). A taper of 8 ms duration centered around the picked time is used to window the Stoneley wave. Because the waveforms are quite coherent and there are no scattering events, the wavefield separation procedure was omitted. We then applied the above described permeability inversion procedure to the Stoneley wave data, using the log parameters ($V_p$, $V_s$, caliper, density, and porosity logs shown in Figures 3 and 5) as input. For the waveform data at each depth, we calculate the theoretical Stoneley wave excitation amplitude (that is, predicted Stoneley wave amplitude without permeability effects) for the frequency range of inversion (200-800 Hz). In this frequency range, the calculated amplitude is compared with the measured amplitude for all receivers in the (array) tool. If the measured value falls below the calculated value, a permeability is estimated by using the minimization procedure described in Eq. (2).
The reference depth required for the estimation is selected by inspecting the waveform plot in Figure 4, which shows the highest Stoneley wave amplitude at 1430'. This depth is chosen as the reference depth assuming a nonpermeable formation.

Figure 6a shows the comparison between the calculated (dashed line) and measured (solid line) Stoneley wave amplitudes. The measured amplitude is the average value over the inversion frequency range and over the 8 receivers in the tool. The two amplitudes are scaled relative to their respective values at the reference depth. As can be seen from this figure, in the lower section the two amplitudes correlate with each other very well, considering the fact that they are obtained from different procedures and that the calculated amplitude is a prediction based on the elastic wave theory (Tang and Cheng, 1993). According to this theory the Stoneley wave amplitude in a slow formation is strongly dependent on the shear velocity of the formation. This dependency is evidenced by the good correlation between the calculated and measured amplitudes in the lower section of Figure 6a, which also proves the validity of our shear slowness log obtained from Stoneley wave inversion (Figure 5). In the upper section, especially in the range from the top down to 750 ft, the measured amplitude falls significantly below the predicted amplitude, indicating the effects of permeability. The permeability estimates are plotted on a logarithmic scale in Figure 6b. In the upper section, very high permeability values (on the order of 10 darcies) are estimated. The permeability decreases with increasing depth. In the lower section, the permeability values are an order of magnitude lower than in the upper section (except around 980 ft and 1300 ft intervals), varying mostly in the 1 darcy range. In addition, the lower section shows many low-permeability intervals. The permeability results were reported to the geologists working on this well. The results are consistent with their knowledge and experience about this well. The unconsolidated sands and gravels are mostly found in the upper section corresponding to the region with high permeability estimates. In the lower section, they found many shale beddings interplaced with the sands. This is also supported by the lower permeability values and low-permeability intervals in the lower section of our permeability log.

CONCLUSIONS

A fast inversion technique has been developed to estimate the fluid transport property of a well bore from the Stoneley wave data. The inversion algorithm is based on the simplified Biot-Rosenbaum model and the theory of dynamic response of a borehole to the Stoneley wave excitation. The combination of the two theories, when applied to the iso-offset Stoneley wave data, allows the variations in the Stoneley wave amplitude to be realistically modeled in terms of the elastic and flow properties of the formation. Therefore, given the elastic properties and other borehole parameters from log data, the flow property (i.e., permeability) can be estimated using the model theory. The use of the simplified Biot-Rosenbaum theory allows the inversion to be performed fast and efficiently, thus providing an effective method for assessing the fluid transport properties of a well bore. We have demonstrate the validity and effectiveness of the inversion approach using an acoustic logging data set from a water well. The results gave a good indication of permeability and its variation along the well bore.
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REFERENCES


CORRECTING SIMPLIFIED MODEL FOR SOFT FORMATIONS

Based on the theory of dynamic permeability (Johnson et al., 1987), Tang et al. (1991) obtained a simplified theory for calculating borehole Stoneley wave propagation in permeable porous boreholes. The simplified model results agree with the complete model of Biot-Rosenbaum theory in the fast formation \( V_s > V_f \) cases; in the slow (or soft) formation \( V_s < V_f \) cases the simple model overestimates the Stoneley wave attenuation at high frequencies. In this appendix, we present a correction for this discrepancy, which makes the simplified model results agree with the full model in the slow formation situations.

According to Tang et al. (1991), the Stoneley wavenumber \( k \) is given by

\[
k = \sqrt{\frac{2i\rho_f \kappa_n(\omega)}{R\mu} \sqrt{-i\omega/D} + \frac{K_1(R\sqrt{-i\omega/D} + k_e^2)}{K_0(R\sqrt{-i\omega/D} + k_e^2)}},
\]

where \( K_n (n = 0, 1) \) is the \( n \)th order modified Bessel function of the second kind; \( D = \kappa(\omega)\rho_f V_f^2 / \phi\mu(1 + \xi) \) is the pore fluid diffusivity; \( \kappa(\omega) \) is the dynamic permeability as a function of angular frequency \( \omega \), as given in Johnson et al. (1987) theory; \( \rho_f, \mu, \) and \( V_f \) are pore fluid density, velocity, and viscosity, respectively; \( \xi \) is a correction for solid matrix elasticity (Tang et al., 1991); \( k_e \) is the Stoneley wavenumber in an equivalent elastic formation whose properties are those of the fluid saturated rock; the borehole radius is \( R \). Using this simple formula, we calculate the Stoneley wave attenuation for three permeability values, 0.2, 1, and 5 darcies, respectively. The formation is a fast one with \( V_p = 4 \text{ km/s} \) and \( V_s = 2.3 \text{ km/s} \), and \( \phi = 0.25 \). The results are compared with those calculated using the full Biot theory in Figure (A1.a). The two sets of results agree with each other very well. However, Eq. (A-1) overestimates the attenuation of Stoneley wave at high frequencies in the case of a soft formation (see Tang et al., 1991). This points out that the first order perturbation solution (Eq. (A-1)) is inadequate when the borehole wall becomes compliant because of the reduced shear rigidity of the formation. Instead of searching for higher order solutions, we make a simple correction based on the consideration of the wall compliance. The borehole compliance is defined as (Tang et al., 1990)

\[
BC = f_e R \frac{I_1(f_e R)}{I_0(f_e R)},
\]

where \( I_n \ (n = 0, 1) \) is the \( n \)th order modified Bessel function of the first kind; \( f_e = \sqrt{k_e^2 - \omega^2/V_f^2} \) is the radial Stoneley wavenumber for the equivalent elastic formation. By numerical testing, we found that if we modify the second term under the square root sign in Eq. (A-1) by dividing it with \( (1 + BC^\gamma) \), where \( \gamma = (V_s/V_f)^2 \), then the Stoneley wave attenuation calculated using this equation will be consistent with that from the full Biot theory. A comparison is shown in Figure A1.b, for a soft formation with \( V_p = 3 \text{ km/s} \), \( V_s = 1.2 \text{ km/s} \), and \( \phi = 0.25 \). For the three permeability values ranging from medium (0.2 d), high (1 d), and very high (5 d), the simplified model results follow the
full theory results quite well, indicating that the simple model has about the sensitivity to formation permeability as the full theory.

With the simple correction for borehole wall compliance, the simplified theory can be used to model Stoneley wave propagation in permeable boreholes with both fast and slow formations. In particular, this simple and sufficiently accurate theory (compared to the full Biot theory) can be used to formulate an inversion procedure to extract formation permeability from Stoneley wave measurements.
Figure 1: Reduction of Stoneley wave excitation due to permeability and comparison between simple theory (solid line) and exact theory (dashed line)
Figure 2: Procedure for permeability estimation from Stoneley wave logs
Figure 3: Caliper, bulk density, and neutron porosity from the Sandoval well.
Figure 4: Stoneley waveform data (receiver 1) from the Sandoval well.
Figure 5: Compressional and shear wave slowness logs. The shear log is obtained from inversion processing of the Stoneley wave data shown in Figure 3.
Figure 6: (a) Comparison between the predicted (solid line) and measured Stoneley wave amplitudes. (b) Estimated permeability log.
Figure A-1: Comparison of Stoneley wave attenuation from the simplified model and the complete theory of the Biot-Rosenbaum model for three permeability values (0.2, 1, and 5 darcies). (a) hard formations case; (b) soft formation case after correction for the wall compliance.