LABORATORY STUDY OF FREQUENCY
DEPENDENT STREAMING POTENTIALS

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ABSTRACT

Frequency dependent streaming potentials were measured on a glass capillary, porous
filter, and a sample of Boise sandstone. The pore diameters for these three samples
range from 1 millimeter to 34 micrometers. The frequencies used in these experiments
range from 0–600 Hz with the critical frequencies being 6.8 Hz, 90 Hz, and 400 Hz
for the three specimens. The fluid was moved relative to the sample with the pressure
measured by hydrophones and the streaming potential measured using silver silver­
chloride electrodes. Both Packard’s (1953) and Pride’s (1994) models satisfactorily
predict the streaming potential behavior for these frequencies, and the measured critical
frequencies are directly related to the sample pore diameters.

INTRODUCTION

The frequency dependent electrokinetic properties of rocks and soils are currently un­
known, even though these properties are vital to our understanding of the electromag­
netic signals generated by seismic waves propagating in the earth. It is believed that
these electromagnetic signals are generated by as a seismic wave passes through rock
and causes fluid to flow rocks relative to the mineral matrix. This relative flow induces
a streaming current, which oscillates as an electric dipole at the same frequency as the
seismic wave that is exciting the medium. The frequency dependent streaming current
flows through the conductive portion of the rock and develops a frequency dependent
streaming potential. Frequency dependent streaming potentials induced by a seismic
wave will be referred to as the seismoelectric effect. Full utilization of the seismoelec­
tric effect cannot be achieved until the physics of the processes are verified and model
parameters are obtained for use during modeling and interpretation.
Packard (1953) proposed that the frequency dependent streaming potential coupling coefficient remains constant at its DC value until it approaches a critical frequency when higher than the critical frequency, the streaming potential coupling coefficient decays with increasing frequency. Packard’s experimentation was performed on a limited number of capillary samples of large radii and with no changes in solution chemistry. He was able to achieve a maximum measuring frequency of 200 Hz. No one to date has satisfactorily fit frequency dependent streaming potential data to theoretical curves for porous media or for capillaries with diameters less than 155 micrometers.

Pride (1994) proposed a generalized theory for frequency dependent streaming potentials in porous media. No experimental work has been performed to validate this theory. Validation of Pride’s theory for a range of rock types and solution chemistries is needed. The porous media theory of Pride can then be compared with the capillary theory of Packard, and in turn will provide valuable information, which can be used in the interpretation of seismoelectric signals.

THEORY

Streaming potentials are a subset of electrokinetic phenomena. Also included in this subset are electroosmosis, electrophoresis, and sedimentation potentials. Electrokinetic phenomena are a consequence of a mobile space charge region that exists at the interfacial boundary of two different phases. The mobile space charge region occurs when two different phases are brought into contact with each other. This region is commonly referred to as the electrical double layer (EDL). The most simplified approximations of the EDL can be represented by a parallel plate capacitor (Helmholtz model), or a charge distribution that decays exponentially away from the surface (Guoy-Chapman model). The plane closest to the surface at which fluid motion can take place is called the slipping plane. The slipping plane has a potential defined as the zeta potential (ζ), which is characteristic of the solid and liquid that comprise the interface. The diffuse layer extends from approximately the slipping plane into the bulk of the liquid phase. Streaming potentials occur when relative motion between the two phases displaces ions tangentially along the slipping plane by viscous effects in the water. This displacement of ions is referred to as convection current ($I_{\text{conv}}$) and has properties similar to an ideal current source. $I_{\text{conv}}$ is given in equation (1) where $\varepsilon$ is the dielectric constant, “$a$” is the radius of the capillary, $\Delta P$ is the pressure across the sample, $\eta$ is the viscosity of the fluid, and $\zeta$ is the zeta potential described above.

$$I_{\text{conv}} = \frac{-\pi \varepsilon a^2 \zeta \Delta P}{\eta}. \quad (1)$$

In steady state equilibrium the convection current must be balanced by a conduction current ($I_{\text{cond}}$), where $\sigma$ is the fluid conductivity and $\Delta V$ is the voltage measured across the sample. Hence by ohms law,

$$I_{\text{cond}} = \pi \sigma a^2 \Delta V. \quad (2)$$
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Equating the convection and conduction currents, which must be equal at equilib­rium, leads to the Helmholtz-Smoluchowski equation.

\[ \Delta V = -\frac{\varepsilon \zeta}{\eta \sigma} \Delta P. \quad (3) \]

Details of the derivation of this classic expression can be found in texts such as Colloid Science (Kruyt, 1952). It should be noted when viewing this equation that it is absent of geometry terms for the specimen. The ratio \( \Delta V/\Delta P \) will be referred to as the cross coupling coefficient or simply the coupling coefficient for the remainder of the paper.

AC streaming potentials have the same basic electrokinetic principles as DC stream­ing potentials. However, instead of constant pressure being applied across the sample, the pressure is varied sinusoidally. As the frequency is increased, inertial effects in the fluid start to retard the motion of the fluid in the center of the pore space, as the fluid transitions from viscous-dominated flow to inertial-dominated flow. At higher frequencies the flow through the rock becomes inefficient. Therefore, at frequencies higher than the transition from viscous to inertial flow it takes more pressure to shear the same quantity of ions from the diffuse zone than it does when strictly in the viscous flow regime. Because a time varying pressure source is applied, we rederive the Helmholtz-Schmoluchowski equation and get the results shown in equation (4), where \( a \) is the capillary radius, \( \omega \) is the angular frequency, \( I_0 \) and \( I_1 \) are Bessel functions and \( k \) is given in equation (5) where \( \tau \) is the fluid density.

\[ C_{AC}(\omega) = \frac{\Delta V(\omega)}{\Delta P(\omega)} = -\left[ \frac{\varepsilon \zeta}{\sigma \eta} \right] \frac{2}{ka} \frac{I_1(ka)}{I_0(ka)} \quad (4) \]

\[ k = \sqrt{-\frac{i \rho \omega}{\eta}}. \quad (5) \]

Pride (1994) presents a generalized model for AC streaming potentials in a porous media with his version of the AC Helmholtz-Smoluchowski equation given by,

\[ CP_{AC}(\omega) = \left[ \frac{\varepsilon \zeta}{\eta \sigma} \right] \left[ 1 - i \frac{\omega_m}{\omega_t} \right] \left( 1 - \frac{\omega}{\omega_t} \right)^2 \left( 1 - \frac{\omega^2}{\omega_t^2} \right)^2 \quad (6) \]

where \( CP_{AC} \) represents the AC coupling coefficient for porous media, \( d \) is the Debye length, and \( \Lambda \) is a typical pore radius representing a weighted volume-to-surface ratio (Pride, 1994). The dimensionless number \( m \), which is defined as

\[ m \equiv \frac{\phi}{\alpha_0 k_0} \Lambda^2 \quad (7) \]

and consists of pore-space geometry terms and reduces to 8 for a capillary. The transition frequency \( \omega_t \), which separates the low frequency viscous flow from the high frequency inertial flow, is given in equation (8) as defined by Pride (1994),

\[ \omega_t \equiv \frac{\phi \eta}{\alpha_0 k_0 \rho} \quad (8) \]

where porosity is given by \( \phi \), tortuosity by \( \alpha_0 \) and the DC permeability by \( k_0 \).
EXPERIMENTAL APPROACH

The approach used to verify the theory is to hold the capillary or porous filter stationary and while oscillating the fluid back and forth through the sample. The fluid is driven by having a sinusoidal driving pressure at one end with the other end open to the atmosphere. Silver-silver chloride electrodes are placed on either side of the sample in fluid dead legs to keep them out of possible fluid flow paths (Morgan, 1989). The frequency response of the electrodes was measured and found flat in the region used in this testing.

Data collection was achieved using two instrument preamplifiers and a 12-bit analog-to-digital board with 24.8 microvolt resolution. A Hanning window was applied to the data prior to any spectral analysis being performed. The time domain voltage and pressure were measured and the amplitude spectrum ratios were measured at the driving frequency. The raw data was saved for possible future processing. Cross-correlation analysis of the signals was performed as a verification of the amplitude spectrum measurements.

Specimens

The capillary (Capillary 1) has an inner diameter that ranges from 1.1 to 0.8 millimeters. The length of Capillary 1 is 60 centimeters. The first porous filter (Porous Filter A) has pore diameters ranging from 145 to 175 micrometers. Porous Filter A has an overall diameter of 1 centimeter and a thickness of 2 millimeters. The rock is Boise sandstone with a diameter of 18 millimeters and a thickness of 6 millimeters.

Results

Figure 1 shows data of the coupling coefficient versus frequency for Capillary 1. Each data point is the mean of 6 measurements with the standard deviation shown. The best fit Packard theoretical curve is plotted through the data points. The curve fits the data points within experimental accuracy and provides good correlation at lower to middle frequencies. At higher frequencies the standard deviation remains small despite the data's slight divergence from the curve. The last data point has a large standard deviation and is the result of a poor signal-to-noise ratio. The measured transition frequency for Capillary 1 is 6.8 Hz and the calculated transition frequency from the manufacturer's capillary dimensions is 3-7.1 Hz.

The Porous Filter A data is plotted against Pride's and Packard's theories in Figure 2. As can be seen in the figure, both models fit the data well, although Packard's model is a slightly better fit for the last three data points. For Porous Filter A, Packard's model gives a pore radius of 97.5 micrometers. The pore radius derived from Pride's model depends on the formation factor of the sample and the permeability model used. The pore radius was determined using equation (8) and substituting the Patterson-
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Walsh-Brace (PWB) permeability model shown in equation (9), for the permeability.

\[ k = \frac{a^2}{8} \left( \frac{1}{F} \right). \]  

(9)

Porous Filter A has a pore radius of 72.5 micrometers using the measured formation factor of 1.6. The manufacture provided pore radius for the Porous Filter A is 72.5-87 micrometers.

The Boise sandstone results are shown in Figure 3. The data has correlation with the theory for the low to intermediate frequencies. The transition frequency for the Boise sample is 400 Hertz. When using a formation factor of 10 and the PWB permeability model the transition frequency corresponds to a pore radius of 17 micrometers.

DISCUSSION AND APPLICATIONS

The emphasis of this study has been the verification of Packard's (1953) and Pride's (1994) theories over a range of pore sizes. Both theories fit the data identically when using \( m = 8 \) in Pride's equation. Pride states that \( m \) lies in the range of 4 to 8. The important parameter to determine from frequency dependent streaming potential curves is the transition frequency. The transition frequency can be determined graphically from the data or by curve fitting the theory to the data and then determining the transition frequency from the theory. Packard's theory for a capillary is related to the capillary radius, the viscosity and density of the fluid. In Pride's model one additional parameter is included, if the second order parameters are neglected. These second order parameters are important where the pore sizes are small and/or the electrolyte concentration is low.

The theories of Packard and Pride appear to fit the porous medium data equally well in the low to intermediate frequencies. However, when the frequencies start to approach the transition frequency as defined by Pride, the data starts to diverge from the theory. This divergence must be understood in order to absolutely conclude that the models of Packard and Pride accurately predict frequency dependent streaming potential at all frequencies. Two possible explanations for this divergence are (1) the theory is inaccurate at high frequencies; and (2) another yet unknown phenomenon is coming into effect at high frequencies.

The data on glass filters and rocks with pore diameters much larger than the diffuse zone indicates the curves generated using nd Packard’s and Pride’s theories cannot be distinguished from each other. What is different is that Pride relates the transition frequency to the governing physical and chemical parameters of the porous medium. In this respect, Pride’s model is more complete and general. Therefore, from the data thus far collected on rocks and porous filters, it can be concluded that for low to intermediate frequencies the curves generated by Packard’s and Pride’s models can accurately fit the data.
Seismoelectric Interpretation

The use of the transition frequency in the seismoelectric effect to predict permeability, appears to be limited to very porous materials or the detection of fractures in intact rock. In seismoelectric experiments of frequencies DC-1000 Hz, the use of the critical frequency to determine permeability will give 0.1 Darcy to 10 Darcies.

Permeability Determination

The use of frequency dependent streaming potentials appears to have good potential for use in the determination of permeability in the laboratory and possibly under field conditions. Using the transition frequency to determine permeability is based on Pride's equation and knowing the fluid properties, transition frequency and the formation factor of the rock. Equation 8, shown again below in another form, with \( \alpha = F \phi \), can be used to determine the permeability of the rock.

\[
\frac{1}{k_0} = \frac{\eta}{F \omega t \rho}.
\]  

CONCLUSIONS

A comparison of streaming potential data to proposed models has been presented for various pore sizes. Although there is a slight discrepancy between the capillary model and the generalized model, as a whole both Packard's and Pride's models are able to predict the capillary data. For porous media both Pride's porous media model and Packard's capillary model predict accurately the form the data should take. The difference between the two models shows up in the model parameters used to fit the curve. As the data approaches the transition frequency as defined by Pride, the data diverges from the theory for both models.

ACKNOWLEDGMENTS

We wish to thank Prof. David Lesmes at Boston College and Dr. Laurence Jouniaux (Laboratoire de Géologie de l'Ecole Normale Supérieure) for their assistance and guidance in helping to get this project started. We would also like to thank Mr. Dereck Hirst and the Laboratory of Nuclear Science Machine Shop personnel for their invaluable help in constructing the test cell and other required equipment.

This work was supported by the Borehole Acoustics and Logging/Reservoir Delineation Consortia at the Massachusetts Institute of Technology, and by AFOSR grant #F49620-95-1-0224.
REFERENCES


Figure 1: Cross coupling response data with error bars for a 1 mm capillary plotted along with the theory for a 1 mm capillary. The determined critical frequency is 6.8 Hz.
Figure 2: Cross coupling response data and theory for Porous Filter A. The determined critical frequency is 90 Hz.
Figure 3: Cross coupling response data and theory for Boise sandstone. The determined critical frequency is 400 Hz.