LABOR SPECIALIZATION, AGGLOMERATION ECONOMIES, AND REGIONAL RESOURCE ALLOCATION

by

SUNWOONG KIM
B.S. in Arch., M.C.P., Seoul National University (1976, 1978)
M.C.R.P., Harvard University (1980)

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Signature of Author
Interdisciplinary Program in Economics and Urban Studies May 10, 1985

Certified by ___________________________ William C. Wheaton
Thesis Supervisor

Certified by ___________________________ Peter A. Diamond
Thesis Supervisor

Certified by ___________________________ Jerome Rothenberg
Third Reader

Accepted by ___________________________ Lawrence Susskind, Chairman
Ph.D. Committee, Department of Urban Studies and Planning

Accepted by ___________________________ Richard S. Eckaus, Chairman
Committee for Graduate Students, Department of Economics

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Submitted in partial fulfillment of the requirements of the degree of Doctor of Philosophy in Economics and Urban and Regional Planning

Abstract

The modern economy is characterized by the diverse and specialized labor market. After the literature survey in chapter I, two models, signalling model and bargaining model, of labor market are presented and analyzed in chapter II. With increasing returns to scale, heterogeneous labor and technology, we show that average productivity increases with the size of the market. The larger the size of the labor market, the better the matches between workers and firms resulting higher productivity. The adverse comparative static results of "kinked equilibrium" in the signalling model (a variant of Salop's[1979] monopolistic competition model) disappears in the bargaining model, because the change of regime from monopsony (in which a worker has only one viable employment opportunity) to competition (in which a worker has more than one opportunities) occurs more gradually in the bargaining model.

In chapter III, we allow workers to choose the depth and breadth of their human capital by extending the bargaining model in chapter II. In the competition case, workers want to have more specialized (more intensive and less extensive) human capital. In the monopsony-competition case, however, it is ambiguous whether workers want more specialized human capital or not, because the choice of human capital changes the level of competition, which in turn changes the human capital choice.

In chapter IV, externalities of labor market are discussed more explicitly. Although the external scale economy increases with the size of the market, average productivity is bounded by technologies. Then, negative externalities are introduced to examine the characteristics of optimal city sizes. The optimal city size is larger when the minimum efficient scale of production is larger and when it is more costly to train workers for different jobs. Analysis of the efficiency characteristics of city sizes suggests that large cities tend to be too large and small cities tend to be too small than the socially optimal configuration.

Thesis Advisors: William C. Wheaton, Associate Professor of Urban Studies and Economics
Peter A. Diamond, Professor of Economics
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CHAPTER I

Introduction and Literature Survey

I. Introduction

Theories of urban sizes have both positive and normative aspects. On one hand, they try to explain the reasons why a city exists in a certain system of cities. On the other hand, they try to exploit the efficiency gains of city sizes in national urban and regional policies. There have been a great deal of literature since Christaller's central place theory in economic geography, regional science, and urban economics on this topic. Nonetheless, the questions such as, why a city exists, why a big city exists, is there efficient size of a city, are the market forces result in the efficient city size with the presence of both positive and negative externalities, are not answered in full.

In the neoclassical paradigm of constant or decreasing returns to scale, we should have observed that economic activities are physically decentralized with the same factor payments among different regions. But the empirical findings suggest that large cities, despite their higher capital labor ratios, have higher wages than smaller cities, ceteris paribus. If we are willing to accept either free trade of products or the availability of the same production technologies, those observations would contradict the neoclassical predictions, such as factor price equalization.

There have been four major explanations to explain this obvious
contradiction. The first argument is that the higher cost of living in larger cities will drive up the nominal wages to equate the real wages among regions. This argument directly addresses the wage differentials among regions, particularly substantial differences between large cities and small cities. There are some problems in this argument. There is no convincing evidence that suggest the price level is higher in the larger cities. It seems to be true that, housing price which is a major chunk of the cost of living index, is higher in larger cities. But my casual impression is there are also some goods and services cheaper in larger cities. The major problem of the cost of living argument is, however, not whether the cost of living is higher in larger cities. Suppose the nominal wages are higher in big cities to equate the real wages among regions, large cities should have higher capital labor ratio implying that nominal return on capital is lower in larger cities. But we know that capital market is very foot-loose and any differentials in returns on capital will quickly disappear such that the differentials cannot be sustained even in the short run.

The second argument is based on the differences of environmental qualities. The difference of weather condition, pollution level, and congestion level will be capitalized such that larger cities, which is presumed to have lower overall environmental quality are expected to have higher nominal wages. If the wages differentials are capitalized value of the environmental quality, then we would expect that large cities will have higher nominal wages. If so, rental prices of capital as well as capital labor ratios should be equalized among regions. So we would expect that all regions will grow in a more or less balanced path such that every city grows with same growth rate. Obviously what we observe
in most of the countries is, on the contrary, some cities particularly large cities, grow faster than smaller cities (Renauld [1981]). Also we would expect that the wage differentials are smaller in large vs. small cities than northern vs. southern cities, which does not seem to be the case.

The third explanation known as "city lights effect", argues that people prefer large cities to small cities. This argument has two elements. The first element is that larger cities generally have larger government income transfer which attracts low income people, particularly from rural areas where the role of government is quite limited. The second element is based on the product differentiation of consumers. In large cities, where the market size is larger, a greater number of products are available which would enhance the utility of the big city residents. Although this explanation is based on the coherent economic theories and gives a rationale for the existence of large cities, it does not help to explain the higher wages and higher capital labor ratios in larger cities.

The fourth argument, which I will mainly focus on is that larger cities have higher production efficiency over smaller cities. The aggregate production function of a city shifts as the city grows. We can think of a scale augmented production function just like the Hicksian neutral technical changes. As the city size grows, the marginal cost will decrease with the increasing return to scale, which enables to have higher capital output ratio as well as the higher wages with competitive capital rentals. The economic rationale for the scale economies are quite diverse, and may differ substantially from industry to industry. These points will be discussed in more detail later on.
In this dissertation, I develop a logically consistent framework for studies of optimal city sizes, interregional movements of labor and capital, and interregional factor price differentials. The major emphasis of the research is to develop a theory based on the microeconomic optimization behavior of the agents. The motivation to build such a micro theory is to analyze the widely held hypotheses more precisely, such as increased productivity by specialization of labor (ever since the pin factory example of Smith [1776]), cumulative causation and backlash/spread effect due to Myrdal [1968] and Hirshman [1958], the inverse-U shaped size and regional income distribution hypotheses by Kuznets [1965] and Williamson [1965], scale economies of city sizes due to Sveikauskas [1975], Segel [1976], and Moomaw [1976, 1980], the infant industry argument for protection in the trade theory (for example, Ethier [1982] and Johnson [1970]), and unemployment theory based on the scale economy by Weitzman [1982].

All the theories and hypothesis mentioned above are loosely related to the notion of scale economy. With increasing returns to scale marginal cost is lower than average cost, so the competitive marginal cost pricing yields negative profits. Since the scale economies are not consistent with competition in general and thus more difficult to model, the advancement of the theories have been slow. The major approach to deal with the scale economy has been the monopolistic competition models suggested by Chamberlin [1962] and Robinson [1933] by recognizing the differentiation of the products. Since a firm has only a local monopoly in the product market, the scale economy is compatible with competition. To illustrate the idea, let us suppose a number of firms with symmetrical technologies with increasing returns to scale to produce differentiated
products, and consumers are uniformly distributed along the circle in the product attribute space. Each firm has a local monopoly power in the sense that if it were to lower the price it will attract more customers away from the firms producing close substitutes to its product. So each firm faces a downward sloping demand curve. This local monopoly power enables the firm to charge above its marginal cost. However with free entry and competition in prices all firms will charge the average cost, because new entry will eliminate any excess profits. The profit maximization condition ensures that marginal cost equals to marginal revenue. In a Nash equilibrium with perfect adjustment among firms after each entry, there will only be a finite number of firms in the market, since the number of firms will be determined by the zero profit condition.

An alternative approach which has become more popular particularly in the trade literature is the external scale economy. Since the scale economy is external to the agents in this framework, each firm perceives its own technology as having constant or decreasing returns to scale with zero or positive profits. The increased productivity with the larger size of the economy is regarded as a technological innovation which shifts the production functions out for each firm. Unfortunately, there has been no rigorous micro justification for this approach, as most models of this type appeal to the vague notion of shared infrastructure, information, and labor pool.

External scale economy, also known as agglomeration economy, has traditionally been thought of as a result of shared infrastructure and/or savings in transportation cost by locating close to one another. I find neither arguments to be terribly convincing. First, the cost of public
service and infrastructure such as water, transportation networks, etc. are much higher in large metropolitan areas than in small towns (Linn [1982], Walzer [1972], Will [1965]). My impression is that agglomeration happens in spite of the increasing marginal cost, not as a result of the decreasing marginal cost of the public service. With regards to the transportation cost savings argument, it will be more than sufficient to point out that transportation cost element is a small fraction of the total cost (less than 2 per cent) in most commodities. Thus 50 per cent reductions in transportation cost will result in only minor changes in total cost, and would not affect the locational decision of the firm.

I will concentrate on the notion of labor specialization to explain the seemingly apparent phenomenon of external scale economy, because I think it is one of the most important sources of the external scale economies, if not the most important one. The argument basically goes like the following. As the size of the economy increases there is more room for the specialization. Specialization enables workers to increase their productivity and thus earn higher wages given the same cost of acquiring the human capital given that there exist a labor market for the specialized labor. Thus the extent of the existence of a specialized labor market will determine the level of specialization.

The competitive labor market with a homogeneous workforce has been regarded as a poor theoretical paradigm for explaining the labor market. In addition to the wage stickiness and the cyclical behavior of the economy, there are two major reasons for unemployment. First, it takes time to sell or buy labor. This aspect has been swept under the rug as part of frictional unemployment. Second, more importantly there are mismatches between the available workers and the job positions.
Since the demand for output and the production technology is constantly changing and there are substantial human capital investment are required to adjust to the new labor demand, the workers' decision to invest in human capital should take into account the uncertainty of the future labor market. The same type of argument applies to the firms decision to hire workers. Firms are faced with fluctuating demands for output. Since hiring and training costs are substantial and the quality of the new workers are uncertain, firms' hiring decision should take into account the nonhomogeneity of the labor pool.

To make the exposition clear and simple, let us suppose a well defined market in which the outputs are either homogeneous or very close substitutes for each other. Also consider two groups of agents in the market, namely workers and firms. Workers have to accumulate human capital and sell their labor to firms in the factor market. The worker chooses how much he wants to invest in human capital, and the level of specialization. The choice among occupations is not allowed since it is assumed that there is a single market in which all the workers have the same occupation. It is common knowledge (in the game theoretic sense) that more specialized workers who can perform narrower range of tasks with higher productivity than more general workers given the same human capital investment. The level of specialization that a worker chooses will depend on the wage differentials between the generalist and specialist and the relative probability of being employed. For example, a worker wants to be a specialist if the probability of being employed as a specialist is same as a generalist.

Labor demand is also determined by the two market parameters: the wage differentials and the relative probability to find desired
workers. If the wage differentials are small, and specialists are widely available, that is, the probability to get a specialist is not very much lower than that of getting a generalist, then firms will want to have more specialists. Higher demand for specialists will in turn raise the wage differentials and lower the probability of finding a specialist. In a single good market, the size of the market can be measured unambiguously by either the number of workers or the level of output. In equilibrium, we would expect a higher specialization in a larger market, which is roughly consistent with the casual empiricism. For example, there are various medical specialists in a large metropolis, while all the doctors in a small town tend to be general practitioners. I would like to emphasize that the gains in the production efficiency is external to the individual agents of the market, i.e. individual agents can influence the market by only a small amount.

II. Literature Survey

We can classify the literature on urban size and distribution into the three major categories. A first approach, which is popular in the regional science and economic geography, focuses mainly on the distribution of city sizes. Central place theory, first proposed by Christaller and then developed further by Losch, tries to examine the economic and geographic hierarchy of different size of the cities. The theory envisions a system of cities as a hierarchy, with a small number of large cities and a large number of small cities forming a multi-level hierarchy. On the contrary, the studies on the statistical regularity of distribution of cities view the size of cities as a continuum, and try to
find statistical laws which govern the observed distribution of city sizes. The first major work in this thrust is the so-called "rank-order rule", popularized by Zipf in the early 1940's. There are numerous studies in this framework, using more general functional forms of Pareto distributions or log-normal distributions. There have been some efforts to relate the central place theory to the statistical regularities. The pioneering efforts in this area are Beckman [1968] and Simon [1955].

The second approach, which is mostly pursued by the economists, emphasizes the productive efficiencies of city sizes. The basic hypothesis in this line is that there is a productivity gain, at least to a certain level, in larger cities. The theories propose various reasons for the scale economies. Earlier theories emphasized the savings in transportation cost from locating firms closer to each other. Recent theories emphasize cost saving in social indirect infrastructure, labor pools, and information. The major tool in empirical studies is, however, the aggregate production function using the cross-sectional data sets across the different urban areas.

The third approach asks more policy-oriented questions, namely, whether there is a most efficient size of city, what are the sources of efficiency gains and the losses of urban concentrations, and how does the optimal city relate to national economic development strategies. These questions are more often raised among the practitioners of economic development and urban and regional planning. I will follow the order of these three major approaches in reviewing and evaluating the strengths and weaknesses of the relevant literature.

1. Theories of distribution of city sizes
In his classic study of Southern Germany, Christaller [1966: English translated version] assumes a homogeneous plain over which resources are uniformly distributed. A city's prime economic function is assumed to be service to the surrounding hinterland, including, except in the case of the smallest size cities, the lower level (smaller) cities. The market area thresholds for the various goods and services are different for several reasons. The structure of transportation costs is different. There are different levels of scale economies in production. The size and the pattern of demand would also vary among the different products and services. The smaller that the threshold level of a particular good or service is, the smaller is the size of the city needed to perform the distributional role for that good or service. As the larger city always includes similar functions to those lower level cities, the equilibrium will be characterized geometrical networks of city hierarchies.

This model predicts that cities with the same hierarchy level will have the same population. This is not observed in reality. There is a continuum of city sizes rather than discrete levels of city sizes. The "rank-size rule" approximately characterizes this feature in a particular way. It states that for the cities within a country the product of the city population and the rank of its population is approximately equal to a constant, which is the population of the prime city. This regularity has been remarkably confirmed in many countries in spite of the differences of definition of cities, level of economic developments, and so on among countries.

The "rank-size" rule is only a particular case of \( a = 1 \) in a more general form of Pareto distribution.
where \( R \) is the rank of the city, \( S \) is the population of the city, and \( A \) and \( a \) are constants. Although values of the coefficient \( a \), commonly called the Pareto coefficient, differs substantially with alternative definitions of cities, the regularities are remarkable in many countries (Rosen and Resnik [1980]). As Simon [1955] has shown, the Pareto distribution is an equilibrium state of the stochastic process in which the growth rates of population are uncorrelated with the city sizes. The underlying stochastic process of uncorrelated growth rates and city size is similar to a notion known as Gibrat's law in firm size distribution studies.

There seems to be secondary regularities observed in the in the studies of firm size and city size distributions, namely upward or downward concavities. The Pareto distribution should be plotted as a straight line in a log-log graph. However, many developing countries and some developed countries such as France show an upward concavity (that is, the second derivative is positive), in which the largest cities have more population and the medium cities have less population than was predicted by the Pareto distribution. Ijiri and Simon [1974] have shown that if the growth rate is auto-correlated (that is, current values are correlated with past values), then the curvature appears in the steady state. For example, if growth rate is positively auto correlated, then the distribution will show the upward concavity. Vining [1976] has shown that the curvature may result from the correlation between the growth rate and population size. This has an interesting behavioral implication
in the distribution of city size, namely, if there exists a scale economy of city sizes, i.e., if larger cities are more efficient than smaller cities, then the larger city will attract more population than the smaller city, ceteris paribus. So the growth rates of the large cities will be larger than the small cities, which means growth rate and population are positively correlated. With positive correlation, we would expect the upward curvature. One might want to use this characteristic to test the existence of the scale economy of cities. But city size distribution may well also depend on administrative characteristics, political power distribution, and other aspects as well as on economic factors. Thus, the mechanical application of the stochastic process theory to the derivation of efficient city size would not be appropriate.

Beckman [1958] has published an earlier effort to relate the central place hierarchical model with the Pareto distribution of city sizes. The key assumption he makes besides the central place theory is that the population of the city is proportional to the population of the market area it serves including the city itself. With the two assumptions, he derives the size of the population and the population served increase exponentially with the level of city in the hierarchy. Solving eq.(2.1) for $S$ gives population of the "rank-level" as an inverse exponential function of the "rank-level". Beckman then approximates the step function to the continuous function by choosing a mean at each rank-level to show the rank size rule. There is a criticism by Parr [1969] on the ground that the population served is not exactly treated in Beckman [1958], and the correct formulation does not yield an exact "rank-size rule". But I think the major criticism should concern the
approximation of the step function. If the number of lower level cities for each higher level city is 3, which was suggested by the original work of Christaller, the number of cities of 6th highest level is 256. Choosing one city out of 256 to approximate the step function to a continuous function seems quite crude. Besides, the approximated continuous function has 6 or 7 points (number of levels). This makes the approximation argument difficult. In my opinion, the effort to link the two different theories of city size distribution seems not been successful. Besides, the pure game of stochastic process does not illuminate very much in the urban concentration and efficiency questions because of the lack of behavioral grounds of the theory.

2. Scale economies of city sizes

It has been widely claimed that there exists a scale economy of city sizes. Similar notions such as external economy, localization economy, backward and forward linkage, and agglomeration economy have constantly attracted the research interests of urban economists, regional economists, urban geographers, and city planners. However, there seems to have been either some confusion or ambiguity over why and how the production efficiency is improved (at least to certain extent) with the size of the city. Carlino [1978, 1980] has provided three useful distinctions in the concept of scale economies of cities, namely, internal returns to scale at the plant or firm level, localization economy, and agglomeration economy. I will arrange the discussion following his framework.

First, it is conceivable that there is a scale economy in the plant level, i.e., decreasing average cost with respect to the quantity
produced. This notion is widely recognized, although highly controversial, in the production economics area. Although there are a number of industries which in reality seem to exhibit increasing returns to scale over a reasonable range of production (for example, public utilities), the linkages between the internal returns to scale and the existence and the growth of cities are not very clear. Henderson [1974] provides a theoretical explanation of the size distribution of cities along this notion. Under the assumption of complete specialization and increasing returns to scale, he manages to demonstrate a size distribution of cities determined by the level of scale economy. But complete specialization to a single industry for a reasonably large city is obviously a very strong assumption. The largest single industry defined by the two-digit industrial classification, in highly specialized U.S. cities such as Detroit and Cleveland has less than 30 per cent of the total employment of the metropolitan area.

A similar idea has been investigated in the local public sector, namely, whether there are scale economies of the municipal services. Not surprisingly the results are quite problematic. For example, Hirsh [1959] claims that expenditure per capita did not vary significantly, while Schamndt and Stephens [1960] suggested that the service index is positively correlated with population size although per capita expenditure was not significantly correlated with city size. Walzer [1972] claims, on the contrary, that a negative relationship between the service indices and the city size was found while per capita expenditure was not correlated with population. For one thing, the right index for the municipal services are not obvious in many cases. Many studies used per capita expenditure as a proxy for the level of service provided.
There are many factors to determine the per capita expenditure. Demand for public service might be substantially different from one community to another. The cost of providing the same service level may be quite different depending on the natural conditions (climate and geography), social environment (income, age, religion, race distribution). So the comparison of expenditures seems to be close to be meaningless. Some studies have used a custom made index for the service level, which could be quite controversial. In summary, the existence of scale economies in government service seems quite inconclusive.

The localization economy, Carlino's second notion of increasing returns to scale in urban areas, is due to the horizontal and vertical linkages among industries. Many firms in the same industry can share the cost of infrastructure (roads, ports, electricity, etc), information, and the specialized labor pool. While traditional location analysis (both theories and empirical studies) emphasize cost savings of the physical inputs, some recent studies focus on the importance of information and availability of specialized labor (Carlton [1969]). Vertical linkages are mainly discussed in a planning context to take advantage of transportation cost in the industrial complex development (Richardson [1977]). The notion of localization economies has some appeal as an explanation of the existence of large cities, because it can demonstrate the scale economy of a city even with the decreasing returns to scale in each plant.

The third notion, agglomeration economy or urbanization economy, is an extension of the localization economy into a more general form. In his old but still insightful book, Vernon [1960] provided plausible causes of the agglomeration economy of New York metropolitan region;
"enormous amount of rental space, extremely diversified labor force, varied group of suppliers of industrial material and services, and extensive transportation facilities ... (which) had been the consequences of its earlier growth of a century or so..." The agglomeration economy is a genuine form of externality which is mainly due to the spatial accessibilities to the diversified and specialized urban resources.

In the mid 1970's, there were some efforts to test the existence of scale economies of city sizes by using aggregated production function. These studies were stimulated by Fuch's [1967] finding that workers in large cities are paid more than the workers in small cities ceteris paribus. One possible explanation for this is that there is a efficiency gain in the larger cities so firms in the large cities can outbid the wages in the smaller cities. Sveikauskas [1975] estimated that doubling the city size will yield the 6 percent increase of productivity. Segel [1976] tried to distinguish the agglomeration effect and the increasing returns to scale (a very similar notion to that of localization economy) by having the estimation equation with both shift parameters and the sums of parameters in the Cobb-Douglas aggregated production function not equal to one. And he claimed that the sum of the parameters are not significantly different from one, while the shift parameter of population over 2 million is significant. He concludes, on the basis of this, that agglomeration is more evident while scale economy is not strong. A critical review and reestimation of the production function by Moomaw [1981] suggested that the Sveikauskas's result was overestimated because of the overestimation of capital stock in the older cities. His estimate is that 2.5 percent of increased productive efficiency is achieved by
doubling the population. But he agrees with both earlier writers that there is an efficiency gain.

The only serious effort, to my knowledge, to identify the localization economy and agglomeration economy is done by Henderson [1983]. He defines the localization economy as a productivity increase due to the number of employees in the same industry defined in two digit SIC categories, and agglomeration economies as due to the total employment in the metropolitan area. His results are basically for the localization economies. Also he finds that the localization economies level off quite soon. But he confined his study in the manufacturing industry only, in which industrial linkages and availability of workers pool tend to be more important. Presumably, service industries, including retailing and wholesaling, require more face to face contact and fast digestion of continuous information flows. Thus his conclusions for localization economies are exaggerated while those agglomeration economies are underestimated if we include all types of economic activities in cities.

Another promising line of research is the search based agglomeration (e.g. Pascal and McCall [1980] and Stuart [1979]). The idea here is the agglomeration occurs because of the search cost savings in buying or selling product and in hiring and purchasing specialized labor and input. The most notable search based agglomeration occurs in the retailing clusters such as shopping malls. The imperfect information, which can only be reduced through the search in the market may be the micro foundations for the urban agglomeration.

So far, I have mainly focused on the production efficiency of larger cities. Besides the production efficiency, consumption diversity
could be another source of agglomeration economy. For example, suppose there are two cities of different sizes with the same productivity, i.e., there are no productivity gains in the larger city. And if a producer produces different varieties of the same good, then the larger city can produce a larger number of different varieties. If consumers care about the variety of goods as well as the price, then the consumers in the large city can attain higher levels of utility with the same income. This will attract immigrants from the smaller city. This type of consumption efficiency gain never has attracted any serious analysis in the past literature, however.

3. Optimum city size and decentralization policies

In any optimal city model, a trade off is postulated between some sort of efficiency gain and an increase of social cost. It is very curious, however, that what kind measure of city size we are talking about. In most studies, population is regarded as the city size. But other alternative measures might be important and interesting as well. For example, can we make any sense by comparing a city in India with population of one million with a city in U.S.A. with the same population. Regional output could be an interesting measure of a city's size. In some studies, physical size such as the diameter of a city has been discussed as a measure of city size (e.g. Henderson [1975]). As the population density varies a great deal among different countries, the discussion of physical size should also be useful.

In the previous section, I have discuss the possible sources and analytical and empirical studies on the economies of scale. There are certainly diseconomies of scale associated with the city size. Let me
list the possible candidates, then elaborate one by one; higher land rents, congestion, pollution, worsening of social environment such as crime, social infrastructure and provision and financing of local public services. In an open system of cities, any desirable or undesirable characteristics of a site will be capitalized into the land value. These will include non-market goods such as pollution as well as the marketable attributes such as cost savings of transportation. If the economic agents and factors are perfectly mobile, then the utility level and the return on factors will be equalized everywhere in the economy. In this case, land value capitalization would not affect the efficiency. If some things are immobile in the economy, the land rent will ensure efficient allocation when all externalities are internalized. With the presence of the externality and impossibility of marginal pricing of externality and public goods, the land value capitalization would not lead to an optimal city size. The direction of market force is ambiguous.

Pollution and congestion cases seem more clear. Assuming the level of pollution or congestion is an increasing function of the number of people in the city, and the cities market price is short of the social marginal cost, a city's population will be likely to be overconcentrated. As the price that the individual perceives does not include the social cost imposed by the individual, the individual acts according to the equalization of average cost and average benefit, which in turn will lead into the overconcentration. The provision of public services in urban area has been regarded as another reason for this overconcentration. As transfer payment or public services to low income groups of people in urban areas and more likely to exceed those in rural area, there is a incentive to move into the city, particularly in the case of low income
groups. The effect of the fragmented local government may work in the other way. Thus, competition among governments may prevent local governments from engaging in income redistribution programs, and this will weaken the previous claim.

The question of whether the competitive market system will lead to too large or too small populations (when compared with the optimal size) is problematic. If there exists only scale economies not diseconomies (such as congestion and pollution), then it is not difficult to see the equilibrium city size is too small. If there exist only negative externalities, the opposite will be true. But with the two forces both existing, one has to look at the relative strength of the positive and negative externalities. Another complication has something to do with the financing of the public goods in the local government sector and with the land value capitalization. The first question is, then, whether the optimal city size exists. If agglomeration economies always outweigh negative externalities, then the optimal size will be infinite and every city is too small. But this case is unlikely because the plausible gains will be outweighed by agglomeration losses. Marginal social benefit will be eventually level off, since, the advantages of both production efficiency and consumption efficiency tend to go away when the city becomes "large enough". Marginal savings of transportation cost, search cost, and input costs are achieved by sharing the common facility or labor pool are not likely to decline substantially after the city reaches certain size. On the contrary, marginal social cost will more likely be an increasing function of the city sizes. As an analogy, the highway congestion level increases drastically when the traffic volume increases beyond physical capacity. Costs of pollution seems to
follow the same pattern. So it is likely that an optimal city size exists. This does not imply that there is a universal, optimal city size among all urban areas at all times. Cities are historical products. And they are in different geographic and economic settings. So the optimal city size of a given city may be drastically different from another. Besides, the optimal size of a given city is subject to change depending on the technology. For example, levels of pollution and congestion are not completely exogenous. With technological improvement and public investment, the marginal social cost curve may shift.

Finally, I will briefly comment on the observed degree of urban concentration with the level of economic development. Wheaton and Shishido [1981] have found that the degree of urban concentration is an inverse of the U-curve, namely, the urban concentration becomes higher in the earlier stage of economic development and lower in the later stages. This is analogous to the Kuznets hypothesis of income distribution and the Williamson's hypothesis of regional income disparity, namely, income distribution (size distribution in Kuznets and regional disparity in Williamson) becomes more skewed in the earlier stage of the economic development and less skewed in the later stage. This can be justified by many stories, one of which is the Hirshman's "spread and backlash" effects. Rosen and Resnik [1980] also support the inverse U-curve hypothesis in their city size distribution study, although they did not say this explicitly.

However, these studies suffer from the usual criticisms of cross national comparisons. First, the countries are not a homogeneous group, their definitions of cities are different and their physical structures of cities are different. Second, cross country comparisons do not imply
time series changes. That is to say, we cannot infer from the empirical observation that if a country becomes more developed, then it will be more decentralized and so on. Third, the real outcomes of city distributions are also affected by the urban and regional policy and general economic development strategy. Namely, many countries which have been successful in growing faster in the last decades are industry-promoting countries which push toward manufacturing, which is mainly urban based. Therefore, although the observation that middle income countries are more spatially concentrated than the lower and upper income countries seems to be true, it should be interpreted with caution.

III. Thesis Outline

In this section, I will briefly sketch the outline of the dissertation. Footnotes and figures appear at the end of each chapter. In chapter II, two models of agglomeration economies will be presented and analyzed. The external economies are generated mainly from the fact that with larger market size, labor specialization can occur more fully to exploit the productive efficiency of individual worker's skill which comparatively more efficient than others. Individual firms are assumed to have constant marginal productivity technologies with some minimum efficient scale so that they exhibit increasing returns to scale. However, its ability to hire such workers will be limited by the size of the local labor market. The main results of chapter II is that average labor productivity will rise as the size of the market increase given the minimum efficient scale, the marginal productivity, and the loss of productivity due to the mismatch between the worker and the job.

In chapter III, workers are allowed to choose the level and the
extent of his human capital. The former will be called intensive human
capital, and the latter will be called extensive human capital
respectively. An extension of the bargaining model in chapter II will be
analyzed. In a larger market, firms will have technologies which require
more specialized labor, since the labor pool is large and diverse enough
to support such specialized production. By the same token, workers will
be more specialized (i.e., they will have more intensive and less
extensive human capital), since there is a higher probability to get the
better matching job when the jobs in the market are more diverse.

The analyses in Chapter II and III suggest that regions will
follow the divergent growth paths. If the scale economies prevails, then
the real wages will be higher in the larger cities, which in turn,
attract more people with same or higher return on capital. If a city has
a slightly larger endowment then agglomeration of the larger city will
occur. Larger cities become larger and smaller cities become smaller.
In reality, the extreme case of agglomeration will not occur because of
the following stabilizing forces.

First of all, the external scale economy may peter out after a
certain level. In this assumption, a city will grow up to a point after
which small cities will grow faster to exploit the efficiency gain of the
growth. Second, spatial concentration will result in the higher rents on
land, which will jack up the living cost as well as the production cost.
Also the cost to provide urban services, such as water, sewer,
electricity may increase rapidly with the growth. So workers will demand
higher wages, and relative advantage to locate in larger cities become
less attractive. Third, spatial concentration will also lead into the
non-pecuniary externality. In larger cities, the external diseconomies
of scale such as congestion and pollution may be rapidly rising with the city sizes, which will lead into the efficiency loss in larger cities. So steady state will have large cities as well as the small cities.

In chapter IV, negative externalities will be incorporated into the model in order to discuss optimal city sizes, efficiency characteristics of systems of cities, and possible policy roles. The presence of externalities, in general, result in an inefficient market solution, and there is a room for the public action to improve the efficiency. Policy alternatives are quite diverse ranging from tax incentives to forced decentralization, and social consequences of the policies may be far reaching. Rapid urbanization in many developing countries in the past three decades create a great deal of tension in the traditional social structure and life style. It is hoped that the model would be a useful guideline to evaluate such alternative policy measures.
CHAPTER II

Labor Specialization and Agglomeration Economies

I. Introduction

One of the major characteristics of a modern economy is that diverse and specialized economic activities are concentrated in small geographical areas. The process of concentration of economic activities is loosely referred as urbanization, in which the emphasis is placed on the concentration of people. Since human activities are limited by distance, the major portion of the activities of an urban man occurs within the metropolitan area in which he resides. Moreover, a typical urban man is engaged in a very specialized production activity. Only a small fraction of his output will be consumed by himself. Most of his consumption needs are satisfied by goods and services produced by others.

In fact, it is not very difficult to recognize the close inter-relationship between the concentration of economic activities and specialization. Let us take an example for illustration. Think of a small island and its sole resident, say Robinson Crusoe. He must produce various goods and services in order to survive. He must raise crops, cook food, make clothing, build a house, and so on. His energy should be devoted to many productive activities, and he needs learn how to do all of those things.

Suppose, for some reason, a group of people arrived at the island. Moreover, let us assume that some people are naturally good at
hunting, while others are good at baking, and so on. Since there is a cost involved in learning how to do anything, it would be beneficial to specialize in certain productive activities and to trade the various outputs among the village residents. Our Robinson Crusoe decided to specialize in baking. Now since he has only one production activity to worry about, he can bake more bread, more quickly, than in the previous self-sufficient situation. In other words, his average product of baking increased as a result of specialization. But notice that he could not specialize in baking before the other people arrived. He can only specialize when there are other people around to provide various goods and services other than bread. What I have described is an illustration of Adam Smith's doctrine that "the division of labor is limited by the extent of the market" (see Stigler [1959] for more discussion).

Urban agglomeration has long been explained by the cost savings in transportation and/or in sharing the urban infrastructure. A typical theory is that producers can save in transportation cost of inputs and/or outputs by locating at the points near to where they purchase inputs or sell outputs. In general, economic agents can reduce the cost of transportation or social infrastructure by locating close together. However, the transportation cost typically comprises only an insignificant fraction of the total cost of most of the goods and services produced in an urban area. Also providing a given level of public service in a large metropolitan area costs many times more than in small or medium cities (Linn[1982]). This chapter presents an alternative argument that specialization of production activities is the major source of urban agglomeration.

Although geographical proximity plays an important role in the
process of urban agglomeration, the argument is that geographical
proximity enables producers to specialize, and thus to increase
productivity. Given the usual social practice that workers commute back
and forth between their residence and workplace on a daily basis,
specialization would be limited by the size of the urban area in which
daily commuting is possible. Although average productivity will be
increased by adopting more specialized and roundabout technologies, such
technologies can only be adopted when the market is large enough so that
they can be supported by the activities of other agents in the market.

Some studies have tested the existence of scale economies of city
size by using aggregate production functions. These studies were
stimulated by the Fuch's [1967] finding that workers in large cities are
paid more than workers in small cities ceteris paribus. All the studies
which I am aware of conclude that there do exist scale economies in city
size (Sveikauskas [1975], Segel [1976], Moomaw [1981], and Henderson
[1983]).

This chapter presents models of agglomeration economies.
Localization economies and urbanization economies are not distinguished
(see Carlino [1979] for such distinction). The literature about product
differentiation emphasizes the availability of the wide variety of
products in the modern economy. The utility of consumers will be
increased either by having more variety (Dixit and Stiglitz [1979] and
Spence [1976]) or by having the variety which is more similar to the
ideal variety (Lancaster [1979]). The utility gain through the
consumption of diverse products will not be discussed in our models.
Rather we will focus on the production side of the economy by assuming a
competitive market of homogeneous output.
Another important point which our model does not address is that the worker's human capital investment decision will be made on the basis of the availability of jobs which require such specialized skills. In the highly specialized modern economy, the choice of the extent of specialization is as important as the level of human capital investment to the worker's decision, since the stream of future earnings will depend upon the extent of his specialization as well as upon his skill level. The models presented in this chapter excludes the possibility of endogenous human capital investment decisions.

We shall abstract the urban labor market from the spatial setting. The urban land market and other consequences of concentration of economic activities (e.g. congestion and pollution) will be ignored. The movement within the city is assumed to be costless. Movements between the urban market is prohibited. In short, we shall analyze the aspatial closed labor market.

Two models will be presented in this chapter. The first model is a signalling equilibrium model, and second one is a bargaining equilibrium model. The set-ups are quite similar. The major difference is how the wage is determined. In the signalling equilibrium model, wage is determined by firms on a take-it-or-leave-it basis. In the bargaining equilibrium model, wage will be determined by an axiomatic bargaining solution between workers and firms. The signalling equilibrium will be analyzed in section II. The bargaining equilibrium will be analyzed in section III. Conclusions are offered in section IV.

II. Signalling Equilibrium
1. Assumptions

Let us consider a closed economy of a continuum of workers-cum-consumers with aggregate size $N$. Workers are indexed on a circle of a unit length with uniform density. Since the circle has the unit length, the density is also $N$. The index represents the worker's skill characteristic. Sometimes we will call the index location and the difference between two indices distance. Notice that terms like "location" and "distance" do not have any geographical meanings. There is no a priori superiority or inferiority among workers' skills. The size of the difference between the indices of any two workers represents how different they are. Obviously the difference ranges from zero to one half. Every worker supplies one unit of labor provided that the net wage offer is greater than or equal to his reservation wage.

We assume that all the workers in the economy have the same reservation wage $w_0$. The reservation wage reflects either the utility of leisure or the domestic productivity of a worker. In the following discussion, $w_0$ is interpreted as domestic productivity which a worker gets when he works for himself. We call this situation self-sufficient autonomy.

There are also firms in the economy. Since we do not allow multi-plant firms, we can identify firms without any confusion. Firms are assumed to produce homogeneous goods, which are sold in the competitive output market. The output price is normalized to one. Technologies are also indexed on the unit circle. The index of the technology represents the most productive skill characteristic. The firms can choose their technologies in the long run, but not in the short run (long run and short run will be defined later). Since the
technologies only differ by their most productive skill characteristics, we can unambiguously identify the firm with its most productive skill characteristic. We shall call the characteristic the firm's location.

The critical assumption in the signalling equilibrium is that the firm cannot identify workers' location, while they know the firm's location. Thus, we assume that there is a unique firm-specific signal associated with its most desirable skill characteristic. The firm will hire any workers if they have its signal. If a worker wants to work for a particular firm, he has to invest in order to acquire the firm-specific signal (see Spence[1974] for more discussion on signalling). The cost of acquiring the signal is assumed to be a monotonically increasing function of the difference between the worker's index of skill characteristic and the firm's index of most desirable skill characteristic. As we analyze the behavior of a representative firm, and workers who have skill characteristics similar to the firm's most desirable characteristic, we will choose the firm's index as zero without loss of generality. Then we could denote the difference as \( t, 0 \leq t \leq .5 \). In particular, we will assume that the cost of acquiring the signal \( c_1(t) \) has the following properties:\(^2\):

\[
\begin{align*}
c_1(0) &= 0 \quad (2.1.a) \\
c_1'(t) &> 0, \text{ for } 0 \leq t \leq .5 \quad (2.1.b) \\
c_1''(t) &> 0, \text{ for } 0 \leq t \leq .5. \quad (2.1.c)
\end{align*}
\]

To avoid the complication of substitution between productive factors, we shall assume that labor is the only productive factor. The firm has, what we call, roundabout technology, with the minimum efficient
scale (M) and the constant marginal product (b). It is clear that the technology has an increasing returns to scale. More specifically, we assume the production function has the form of:

\[
Y = \begin{cases} 
0 & \text{if } X < M \\
(2.2.a) \\
b(X - M), & \text{if } X > M, \\
(2.2.b)
\end{cases}
\]

where \(Y\) is output, and \(X\) is the labor input normalized to the equivalent labor with the firm's most desirable skill characteristic. The firm hires only workers who possess the firm's required signal. Since the firm cannot distinguish the workers in terms of their skill characteristic endowment, the wage offer will be the same for workers with the same signal. But the possession of the signal does not increase the worker's productivity. We assume that the productivity is a decreasing function of the difference between the worker's skill characteristic and the firm's most desirable characteristic. More specifically,

\[
x = x(t) (1 - c_2(t)/b) \]  
(2.3.a) \\
c_2(0) = 0 \]  
(2.3.b) \\
c_2'(t) > 0, \]  
(2.3.c)
\]

where \(x(t)\) is the amount of labor with the difference \(t\), and \(x\) is the normalized labor unit. \(c_2(t)\) is the value of the lost product due to the difference. Total labor input \((X)\) is just the sum of the normalized labor \((x)\) of the workers.
We will call the situation short run when there is a fixed number of firms \((m)\). As we have indicated, firms do not change their location in the short run. Wage offer is the only short run decision variable of the firm. We shall concentrate only on the symmetric equilibrium. By symmetry, we mean that all the firms offer the same wage and the distances between any two neighboring firms is the same. Thus, we have:

\[ 2mH = 1, \quad (2.4) \]

where \(2H\) is the distance between any two neighboring firms. The short run profit of the firm will be:

\[ P(d) = b[2Nd - M] - 2N[wd + \int_0^d c_z(t)dt]. \quad (2.5) \]

The variable \(d\) will be called the market area of the firm. All the workers who have the characteristic difference less than \(d\) will work for the firm. If there is no gap between the neighboring market areas, then we have:

\[ d = H. \quad (2.6) \]

If there is a positive short run profit, then entry will occur. If short run profits are negative, firms will exit. Assuming that there are no costs of relocating firms, competition among firms will result in that all the firms get zero profit. The situation that the number of firms \((m)\), and thus, the distance between the neighboring firms \((2H)\) are determined endogenously by the zero profit condition will be called long.
run.

We are mainly interested in the long run symmetric Nash equilibrium. A firm will choose the location and wage offer. A worker will choose the firm he will work for by maximizing his net wage (wage offer minus his cost of acquiring the firm-specific signal), provided that it is greater than or equal to the reservation wage \( w_0 \). The firm makes its wage offer by assuming that other firms' wage offers will be held constant. In the game theory language, firms will play Stackelberg leader towards workers and play a Nash strategy to the other firms. Workers are Stackelberg followers to the firms.

2. Types of equilibria

Let us choose a representative firm \( i \) (1<i<m), and choose its most desirable skill characteristic as the origin without loss of generality. Since the situation is symmetric with respect to the two neighboring firms, we will focus our attention on one side. In the short run, our representative firm will choose wage offer \( w \) given that the neighboring firm's (firm \( j \), 1<j<m) wage offer is \( \bar{w} \). As we see in Fig. 1, workers between firm \( i \) and firm \( j \) (0<t<2H) have three options: to work for firm \( i \), to work for firm \( j \), or not to participate in the labor market and to retreat to the self-sufficient autonomy. More specifically, the behavior of the worker with skill characteristic \( t \) (0<t<2H) is:

1. work for firm \( i \), if \( w-c_1(t) > \max \{\bar{w}-c_1(2H-t), w_0\} \) (2.7.a)
2. work for firm \( j \), if \( \bar{w}-c_1(2H-t) > \max \{w-c_1(t), w_0\} \) (2.7.b)
3. not to participate in the labor market, otherwise. (2.7.c)
If \( b \leq w_0 \) (the marginal productivity of the roundabout technology is lower than that of the domestic technology), then there will be no labor market since workers will not work for firms which in turn cannot afford to pay a wage higher than \( w_0 \). Every worker in the economy will stay in self-sufficient autonomy. If \( b = w_0 \), the firms will get negative profit since \( M > 0 \). Thus, we shall assume that \( b > w_0 \) hereafter.

Depending on whether the difference between the net wage and the reservation wage of the marginal worker at the equilibrium is negative, positive, or zero, we will have three different types of equilibria, which will be referred to as, monopsony, monopolistic competition, and kinked equilibrium following the tradition of Salop [1979]. As we can see in Fig. 2, the three cases occur when the following conditions are satisfied respectively:

1. Monopsony Case, if \( w - c_1(H) < w_0 \)  \hspace{1cm} (2.8.a)
2. Monopolistic Competition Case, if \( w - c_1(H) > w_0 \)  \hspace{1cm} (2.8.b)
3. Kinked Case, if \( w - c_1(H) = w_0 \), \hspace{1cm} (2.8.c)

where \( 2H \) is the equilibrium distance. If \( w - c_1(H) < w_o \) (monopsony case), then some workers will not participate in the labor market at equilibrium. Thus, the firm will act as a monopsonist. Suppose that the representative firm raise its wage offer. Then some workers who did not participate in the labor market before will work for the firm, if the net wage offer exceeds the reservation wage. In the monopolistic competition case \( (w-c_1(H) > w_0) \), if the firm raises its wage offer with small amount, then it will attract more workers away from the neighboring firm. Thus, the labor supply to the neighboring firms will decrease whereas it would
not have changed in the first case. The labor supply curves of the two cases would be different. The kinked equilibrium \((w - c_1(H) = w_o)\) occurs, because the labor supply curve will not be differentiable at the kink. Since number of firms \((m)\) and the distance between the neighboring firms \((2H)\) are endogenous in the long run, the conditions for the three cases must be satisfied with equilibrium \(H's\).

3. Monopsony case

In the short run, the representative firm maximizes its profit (eq.(2.5)) by choosing its wage offer. However the firm's market area \((d)\) will be determined by:

\[
w - c_1(d) = w_o. \quad (2.9)
\]

That is to say, the marginal worker is indifferent between working for the firm and retreating to autonomy. Since by choosing its wage offer \((w)\), the firm chooses its market area \((d)\) uniquely, we could regard the firm's profit maximization problem as the choice of \(d\). By solving eq.(2.9) in terms of \(w\), and substituting it into eq.(2.5), and differentiating it with respect to \(d\), we get the first order condition for the profit maximization problem:

\[
b - c_2(d) = w + c_1'(d)d. \quad (2.10)
\]

This is the familiar profit maximization condition for a monopsonist. The left side of eq.(2.10) represents the marginal value product (recall that the output price is normalized to unity), and the right hand side is
the marginal outlay by hiring one more unit of labor. At equilibrium, these two must be the same. In order to attract more workers to the firm it is necessary to pay higher wage to all workers, since it is impossible to distinguish among them. The second term in eq.(2.10) represents such premium.

Since there may be gaps between the market areas of neighboring firms, (i.e., some people may stay out of the labor market), the number of firms will not be uniquely determined. The maximum number of firms \( m^* \) is, however, \( 1/2d \).

By imposing the zero profit condition, we get:

\[
\frac{bm}{2N} = c_1'd^2 + c_2d - \int_0^d c_2(t)dt \quad (2.11)
\]

Notice that the monopsony equilibrium is very unlikely to occur, because eq.(2.9), eq.(2.10), and eq.(2.11) must be satisfied when there are only two endogenous variables \( (w \text{ and } d) \). It can only happen on the knife edge of parameter values such that an additional condition must be satisfied among them. For example, the monopsony equilibrium only occurs at a point in one parameter, say \( w_0 \), family of economies.

It would be useful to solve the model explicitly by assuming the functional forms of \( c_1(t) \) and \( c_2(t) \). We will choose linear specifications:

\[
c_1(t) = k_1 t \quad (2.12.a)
\]
\[
c_2(t) = k_2 t. \quad (2.12.b)
\]

The parameter \( k_1 \) tells you how expensive the signal acquiring
activity is, and the parameter \( k_2 \) tells you how much the technology requires specific labor. High \( k_1 \) implies that signal acquiring is expensive. High \( k_2 \) means that jobs require highly specific labor. With eq.(2.12), the zero profit condition becomes:

\[
bM/N = (2k_1 + k_2) d^2. \tag{2.13}
\]

Rearranging the terms, we get:

\[
d = \sqrt{\frac{bM}{2k_1 + k_2}} N \tag{2.14}
\]

\[
m' = \left[ \frac{1}{2} \right] \frac{(2k_1 + k_2)}{2k_1 + k_2} M/N \tag{2.15}
\]

\[
w = b - \left[ \frac{(k_1 + k_2)}{2k_1 + k_2} \right] \frac{bM/N}{2}. \tag{2.16}
\]

I have mentioned the extra constraint for the long run equilibrium. To get this, we substitute eq.(2.14) and eq.(2.16) into eq.(2.9). By doing so, we get:

\[
(b - w_o)^2/b = (2k_1 + k_2) M/N. \tag{2.17}
\]

In other words, if eq.(2.17) is satisfied, then we get the monopsony equilibrium determined by eq.(2.14), and eq.(2.16). Comparative static exercise cannot be performed since it is impossible to change one parameter without changing others. For example, suppose that we change the reservation wage\( (w_o) \), then we have to change one more parameter\( (b, k_1, k_2, M, \text{ or } N) \) in order to satisfy eq.(2.17).

4. Monopolistic competition case
In the monopolistic competition case, the market area will be determined by the following equation rather than eq.(2.9):

$$w - c_1(d) = \bar{w} - c_1(2H-d).$$  \hspace{1cm} (2.18)

That is to say, the marginal worker will be indifferent between working for the representative firm (firm i) offering wage $w$ and working for the neighboring firm (firm j) offering wage $\bar{w}$ provided that the net wages are equal. Solving eq.(2.18) in terms of $w$ and substituting into eq.(2.5), differentiating it with respect to $d$, and evaluating it at $w = \bar{w}$ (or $d = H$), we get the first order condition for profit maximization:

$$b - c_2(H) = w + 2c_1'(H)H.$$  \hspace{1cm} (2.19)

Eq.(2.19) says that marginal value product should be equal to the marginal outlay at equilibrium. It is very similar to eq.(2.10) except that the second term on the right hand side is twice as great as that of eq.(2.10). If the additional workers are already working for the neighboring firm, then it is necessary to pay more in the monopolistic competition case than in the monopsony case, because they currently receive a wage higher than $w_0$. The premium of the monopolistic competition case is twice larger than that of the monopsony case because of the symmetry of the net wage functions.

Imposing the zero profit condition, we get:

$$\frac{bM}{2N} = 2c_1'(H)H^2 + c_2(H)H - \int_0^H c_2(t)dt.$$  \hspace{1cm} (2.20)
Differentiating eq.(2.20), we get the basic comparative static results:

\[
dH/dN < 0, \quad dH/db > 0, \quad dH/dM > 0. \tag{2.21}
\]

Since there will be no gaps in the monopolistic competition equilibrium, we get:

\[
dm/dN > 0, \quad dm/db < 0, \quad dm/dM < 0. \tag{2.22}
\]

By using eq.(2.19) and eq.(2.21), we get:

\[
dw/dN > 0, \quad dw/db > 0, \quad dw/dM < 0. \tag{2.23}
\]

In words, the equilibrium number of firms is greater if the size of the market \((N)\) is larger, the marginal productivity \((b)\) is lower, and the minimum efficient scale of the production \((M)\) is smaller. Also, the equilibrium wage will be higher, when the size of the market is larger, the marginal productivity is higher, and the minimum efficient scale is smaller. Our main interest lies on the comparative static results of the market size. If the size of the market increases, i.e., there are more workers in the market, there will be a greater number of firms, each of which has a smaller market size. Since there are more firms around, the average cost of acquiring signals and productivity losses due to the mismatch of jobs and workers will diminish. Thus, real productivity will increase with the size of the market.

Assuming linearity of \(c_1(t)\) and \(c_2(t)\) (eq.(2.12)), we can solve
the model explicitly:

\[ H = \frac{bM}{(4k_1+k_2)N} \quad (2.24) \]
\[ m = (1/2) \left[ \frac{1}{(4k_1+k_2)N/bM} \right] \quad (2.25) \]
\[ w = b - \frac{[(2k_1+k_2)/(4k_1+k_2)]}{\overline{bM/N}}. \quad (2.26) \]

By respectively differentiating the above equations, we get the comparative static results with regards to \( k_1 \) and \( k_2 \):

\[ \frac{dH}{dk_1} < 0, \quad \frac{dH}{dk_2} < 0 \quad (2.27) \]
\[ \frac{dm}{dk_1} > 0, \quad \frac{dm}{dk_2} > 0 \quad (2.28) \]
\[ \frac{dw}{dk_1} < 0, \quad \frac{dw}{dk_2} < 0 \quad (2.29) \]

Thus, if signal acquiring is expensive (\( k_1 \) is high), then the number of firms will be small, and the wage will be low at equilibrium. If the jobs requires more specific labor (\( k_2 \) is high), then there will be smaller number of firms and the wage will be low at equilibrium. By substituting eq.(2.24) and eq.(2.26) into eq.(2.8.b), we get the condition for the monopolistic competition equilibrium which can be expressed as:

\[ \frac{(b-w_0)^2}{b} > \left[ \frac{(3k_1+k_2)^2}{(4k_1+k_2)} \right] \frac{M}{N}. \quad (2.30) \]

5. Kinked case

In the kinked equilibrium eq.(2.8.c) must be satisfied. Imposing the zero profit condition, we get:

\[ bM/2N = (b-w_0) - c_1(H)H - \int_0^H c_2(t)dt. \quad (2.31) \]
By differentiating eq. (2.31) we get:

\[ \frac{dH}{dN} < 0, \frac{dH}{db} < 0, \frac{dH}{dM} > 0. \]  

(2.32)

By recognizing eq. (8.c), it is straightforward to see:

\[ \frac{dw}{dN} < 0, \frac{dw}{db} < 0, \frac{dw}{dM} > 0. \]  

(2.33)

It is interesting to note that the comparative static results are all perverse. The intuition for the perverse effects is roughly as follows. If there are more workers in the market, then the firms can specialize more (i.e. there are more firms in the market). The output per firm decrease as the number of firms increase in the kinked equilibrium. Since we have increasing returns to scale, the average product will be reduced if the output level is reduced. In the kinked equilibrium, the productivity loss due to the lower output outweighs the productivity gain due to the better matching.

The condition for the kinked equilibrium can be expressed alternatively:

\[ w_0 + c_1(H)H + c_1 \leq b - c_2(H) \leq w_0 + 2c_1'(H)H + c_1(H). \]  

(2.34)

This condition can be obtained by comparing eq. (2.10) and eq. (2.19). Alternatively, it can be seen graphically from Fig. 3. The kinked equilibrium will occur when the marginal value curve intersects the marginal outlay curve at the jump such as, point C.

With linearity assumption of \( c_1(t) \) and \( c_2(t) \), we can solve the
model explicitly. Substituting eq.(2.8.c) into eq.(2.31), we get a quadratic equation in terms of \( H \). Using the quadratic equation formulae, we get:

\[
H = \left[ \frac{1}{(2k_1+k_2)} \right] \left[ (b-w_0) - \sqrt{(b-w_0)^2 - (2k_1+k_2) bM/N} \right] \tag{2.35}
\]

\[
w = w_0 + \left[ \frac{k_1}{(2k_1+k_2)} \right] \left[ (b-w_0) - \sqrt{(b-w_0)^2 - (2k_1+k_2) bM/N} \right]. \tag{2.36}
\]

The other root cannot be a solution, because it violates the first inequality of (2.34). The relevant range of the parameter values for the kinked equilibrium is:

\[
(2k_1+k_2) \frac{M}{N} < (b-w_0)^2/b \leq (3k_1+k_2)^2/(4k_1+k_2) \frac{M}{N}. \tag{2.37}
\]

The first inequality is obtained by observing that the terms in the square root of eq.(2.35) must be positive. The second inequality is obtained by substituting eq.(2.35) and eq.(2.36) into the second inequality of eq.(2.34).

6. Synthesis

It would be useful to compare the three possible equilibria in a broader context. As we can see in Fig. 3, the labor supply curve of the representative firm is upward sloping implying that if the firm offer higher wage it will attract more workers. However the curve is kinked, because there is a change of regime from a monopsonistic market structure (where the firm is the only possible employer) to monopolistic competition (where the firm has to bid against the neighboring firms). The slope is steeper (labor supply is less elastic) in the monopolistic
competition case than the monopsony case. The representative firm has to pay more for the marginal worker in the former case, not because the productivity of the marginal worker is higher in the former (in fact, it is same as in the latter, i.e., $b-c_2(d)$), but because, his alternative wage is higher in the former case (i.e., $\bar{w}-c_1(2H-d) > w_0$). The marginal outlay curve represent the firm's marginal wage bill in order to have an additional worker. As the supply curve is kinked, the marginal outlay curve will have a jump at the kink. In fact, the size of the jump is $c'_1(d)d$. Depending on where the marginal value product cuts through the marginal outlay curve, we have three different cases of equilibrium. If the curves intersect in the monopsony regime (point A in Fig. 3), we will have the monopsony equilibrium, and so on. The kinked equilibrium can occur only when there is a jump in the marginal outlay. In other words, whenever the worker has to pay the signal acquiring cost, there is a possibility of the kinked equilibrium. Thus, the kinked equilibrium and its perverse comparative static will not occur if $c'_1 = 0$.

We plot the equilibrium wage with the size of the market in Fig. 4. If the size of the market ($N$) is smaller than $N_1$, the economy will stay in the self-sufficient autonomy, where every worker gets $w_0$. If $N = N_1$ ($= bM/[(2k_1+k_2)(b-w_0)^2]$), then the monopsony equilibrium will occur, where the wage will be $w_1$ ($= b - (k_1+k_2)(b-w_0)$) which is greater than $w_0$. If $N_1 < N < N_2$ ($= (4k_1+k_2)bM/[(3k_1+k_2)^2(b-w_0)^2]$), then the economy stays in the kinked equilibrium, where the wage decreases with the increase of the market size. The kinked equilibrium occurs only in a limited range of parameter values. The range will be small if $M$ is small, $k_1$ or $k_2$ is large, and $w_0$ is large. If $N \geq N_2$, then the economy will have the monopolistic competition equilibrium where the wage increases.
monotonically. However, the wage will be bounded by \( b \). In other words, the productivity increase through the better matching and specialization will not exceed the technological upper limit.

III. Bargaining Equilibrium

1. Assumptions

Many assumptions in the bargaining model are very similar to those in the signalling equilibrium model. Assumptions regarding the workers are identical. Production technologies are the same. However, we drop the assumption that firms cannot identify the characteristics of workers. Thus, there is no need for signalling. The cost of mismatch, which we shall denote \( c(t) \) where \( t \) is the distance between the worker and the representative firm, is divided between the worker and the firm through a negotiation. We shall assume that:

\[
\begin{align*}
    c(0) &= 0 & (3.1.a) \\
    c'(t) &> 0. & (3.1.b)
\end{align*}
\]

For convenience, we assume that the cost will initially be borne by the firm in the form of on-the-job-training.

The wage will be determined by the bargaining between workers and firms. This wage determination rule drastically differs from the signalling equilibrium where the wage is offered by the firm on a take-it-or-leave-it basis. Since the firm can identify the workers, the equilibrium wage \( w(t) \) will depend on the skill characteristic differences between what the firm wants and what the workers have.
The key assumption that we adopt in this model is that all the parties have equal bargaining power. More specifically, each party knows exactly how much it will gain by having the employment contract, and the bargaining outcome (i.e., wage) will be determined at the mid-point, where the worker's surplus of having the employment contract over his second best alternative is the same as the firm's marginal profit of having the worker. Negotiation is costless, and collective bargaining by workers or coalition formation by firms is not allowed. Since we shall maintain the static framework, we also assume that no pair of agents will miss a potentially beneficial bargaining opportunity.

Short run and long run are defined identically as in the case of signalling equilibrium. In the short run, there is a fixed number of firms, equally spaced. All firms earn zero profits in the long run. Thus, the equilibrium distance and the number of firms will be determined. Our major interest is, again, the long run symmetric Nash bargaining equilibrium. By symmetry, we mean equal distance between any two neighboring firms and identical wage equation for all firms.

The short run profit of the representative firm is:

$$p(d) = D[2Nd - M] - 2N \left[ \int_0^d w(t)dt + \int_0^d c(t)dt \right], \quad (3.2)$$

where $d$ is the market area of the representative firm. The only difference between eq.(3.2) and eq.(2.5) is that the wage offer is a function of the distance between the worker and the firm, whereas it is constant in signalling model. This follows from the assumption that the firm can discriminate workers.

The viability of any employment contracts will be determined by
whether the productivity of the employment is greater than the reservation wage. The size of the difference between the productivity and the reservation wage will not change the qualitative characteristics of the equilibrium, since it does not affect the employment decision of workers. However, the model will behave differently depending on the number of potential employers the worker has. For example, if the worker has two potential employers, then he can use one as a leverage for the other in wage bargaining.

Monopsony case occurs then all workers in the economy has at most one potential firm such that the marginal productivity of the roundabout technology (net of training cost) is greater than the reservation wage. If the former is smaller than the latter for all workers \( b \leq w_o \), then every agent will stay in the self-sufficient autonomy so that no labor market can be established. This case will not be discussed further. Competition case occurs when all workers have more than two potential firms. Monopsony-competition case occurs when some workers have two firms and the others have only one. Referring to Fig. 5, we can identify the three cases. We will name them as follows:

1. Monopsony if \( b - c(H) \leq w_o < b \) \hspace{1cm} (3.3.a)
2. Monopsony-competition if \( b - c(2H) < w_o < b - c(H) \) \hspace{1cm} (3.3.b)
3. Competition if \( w_o \leq b - c(2H) \). \hspace{1cm} (3.3.c)

Of course, we will restate the conditions in terms of all exogenous parameters later, as \( H \) is endogenous in the long run.

2. Monopsony case
In the monopsony case, some workers have one potential employer while the others do not have any viable employment opportunity. Thus, the latter will stay out of the labor market voluntarily. Thus, every worker in the economy has at most one potential employer such that the productivity of employment is greater than the reservation wage.

The equal bargaining power implies that:

\[ b - c(t) - w(t) = w(t) - w_0, \quad 0 \leq t \leq d, \]  

for the workers whose productivity is higher than the reservation wage. The left hand side represents the firms marginal profit net of the wage payment, and the right hand side is the worker's additional wage earned by working for the firm rather than working for himself. From eq.(3.4) and individual rationality, the bargaining equilibrium wage will be:

\[ w(t) = \frac{w_0 + b - c(t)}{2}, \quad 0 \leq t \leq d. \]  

The market area will be determined by:

\[ d = c^{-1}(b-w_0), \]  

since \( w(d) = w_0 \). Let us assume that \( c(t) \) is linear, that is:

\[ c(t) = kt. \]  

Then eq.(3.6) implies that:
\[ d = \frac{(b-w_o)}{k}. \quad (3.8) \]

Imposing the zero profit condition, we get:

\[ \frac{(b-w_o)^2}{b} = \frac{2kM}{N}. \quad (3.9) \]

If the left hand side of eq.(3.9) is smaller than the right hand side, then firms will earn negative profits. There will be no labor market equilibrium in the long run, since the firm cannot fully recover its fixed cost of the roundabout technology. On the other hand, if the left hand side is larger, then more firms will enter so that situation becomes either the monopsony-competition case or the competition case.

As we indicated in the monopsony case of the signalling equilibrium there may be gaps between neighboring firms (i.e. some workers will not seek a bargaining opportunity in the labor market). Thus, the number of firms in the long-run is not unique. However, the maximum number of the firms \( m^* \) will be:

\[ m^* = \frac{k}{2(b-w_o)}. \quad (3.10) \]

It appears that the equilibrium market area (eq.(3.8)) and the maximum number of firms (eq.(3.10)) are independent of \( M \) and \( N \), unlike in the case of the signalling equilibrium. This is not case, because the extra constraint (eq.(3.9)) must be satisfied. Just as the monopsony equilibrium in the signalling model, the monopsony equilibrium in the bargaining model can only occur on the knife edge value of parameter. If we substitute (eq.(3.9)) into eq.(3.8) and eq.(3.10), then we get very
similar results as to the signalling equilibrium case.

The average wage of the workers in the economy will be:

\[ W = w_o + \frac{(b - w_o)^2}{4kH}, \]  
\[ (3.11.a) \]

which will be reduced to:

\[ W = \frac{b + 3w_o}{4}, \]  
\[ (3.11.b) \]

when there is no gap (\( d = H \)). It is clear that \( W > w_o \).

3. Competition case

The competition case occurs when every worker in the economy can potentially work for at least two firms whose marginal product is higher than the worker's reservation wage. We assume that the bargaining occurs only among the three agents, that is, the representative worker and the two firms which have the least and the second least training cost for the worker. Since the theory of three person bargaining games has not been developed fully, the choice of the threat point of the worker is not evident. In the monopsony case, there are only two parties involved, and their threat points are exogenous and known to the bargainers. Thus the outcome is well defined with the equal bargaining power assumption. However, in the competition case, the worker's threat point is the outcome of the bargaining between the worker and the other firm, which in turn, is the outcome of the bargaining with the first firm. This point creates a non-trivial theoretical difficulty.

One plausible bargaining outcome in this case is:
\[ w(t) = \frac{b - c(t)}{2} + \frac{b - c(2H - t)}{2}. \quad (3.12) \]

In this formulation, the worker's alternative wage is assumed to be the highest possible wage in the negotiation with the other firm. Although this specification seems to overstate the bargaining power of the worker, it is compatible with our assumptions. To see this, notice the difference of the bargaining positions between workers and firms in the competition case. While the worker can only work for one firm, the firm can hire many workers. Thus, the worker is not substitutable to the firm, while the firm is substitutable to the worker. In other words, the firm has only one potential bargaining opportunity with the worker. If the worker does not accept the employment contract, then the firm loses its production opportunity with the worker for ever. This is not case for the worker. If one firm does not accept the employment contract, then he has another chance with the other firm.

With the linearity of the training cost assumption, we get the equilibrium wage function,

\[ w(t) = b - KH, \quad (3.13) \]

which, interestingly enough, is independent of \( t \). There are two elements in determining the wage of a worker. The worker who has a good match with one firm can demand higher wage because he has high productivity. However, the fact that he has a good match with one firm necessarily implies that he has a poor match with other firms. Since he has poor match with the other firm, his bargaining position will be weakened.
With the assumption of linear training costs, these two effects will exactly offset each other resulting in a flat wage schedule. If \( c(t) \) is a convex function, then \( w(t) \) will be an increasing function, and if \( c(t) \) is concave, \( w(t) \) will be a decreasing function.

Substituting eq.(3.13) into the zero profit condition, we get:

\[
H = \frac{b \mu}{kN} \quad (3.14)
\]

\[
m = \frac{1}{2} \frac{kN/bM}{1} \quad (3.15)
\]

\[
w = b - \frac{b kM}{N} \quad (3.16)
\]

It is clear that the comparative static results are identical to the case of the signalling equilibrium (compare with eq.(2.21)-eq.(2.23) and eq.(2.27)-eq.(2.29)). Notice also that the equilibrium wage is independent of \( w_0 \). The equilibrium wage will be determined completely with the parameters in the labor market and the roundabout production technology. Eq.(3.13) says that the equilibrium wage will be the productivity of the marginal worker. As the size of the market gets bigger, there will be more firms in the market, and the average match becomes better.

The condition for the competitive equilibrium (eq.(3.3.c)) can be expressed:

\[
(b - w_0)^2 / b \geq 4kM / N. \quad (3.17)
\]

Thus, given \( k \) and \( M \), if either the marginal productivity of the roundabout technologies \( b \) or the size of market \( N \) is large enough, then the economy eventually becomes the competitive case.
4. Monopsony – competition case

The monopsony-competition equilibrium occurs, if the parameter values are such that there are two groups of people whereby one group belongs to the monopsony case and the other group belongs to the competition case. The two groups have different bargaining leverage. If the worker belongs to the first group, then he has only one firm with which he can make an employment bargaining. The worker in the second group has two (or more) such firms. Thus, for the former group, the bargaining outcome will be determined exactly as in the monopsony case, while it will be like the competition case for the latter group. That is to say:

\[
\begin{align*}
    w(t) &= \frac{[w_0 + b - c(t)]}{2}, \quad 0 \leq t < L \quad (3.18.a) \\
    w(t) &= \frac{[b - c(t)]}{2} + \frac{[b - c(2H - t)]}{2}, \quad L \leq t < H \quad (3.18.b)
\end{align*}
\]

where \( L \), the boundary between the two regimes, is determined by:

\[
    w_0 = b - c(2H - L). \quad (3.19)
\]

Notice that eq.(3.19) implies that \( w(t) \) is continuous at \( t = L \). The linearity assumption implies that:

\[
    L = 2H - \frac{(b - w_0)}{k}. \quad (3.20)
\]

Substituting eq.(3.18) and eq.(3.20) into eq.(3.2), we get:
\[ p(H) = -N \left[ kH^2 - 2(b-w_0)H + \left( (b-w_0)^2/2k + bM/N \right) \right]. \tag{3.21} \]

Setting eq. (3.21) equal to zero, and solving it with respect to \( H \), we get:

\[ H = \left( \frac{1}{k} \right) \left[ (b-w_0) - \sqrt{(b-w_0)^2/2 - bM/N} \right]. \tag{3.22} \]

The other root of the zero profit condition cannot be a solution, because it violates the first inequality of eq. (3.3.b).

The ratio of the length of monopsony region to the total market area \( (L/H) \) can be regarded as an index for the degree of monopsony. It is clear that \( 0 \leq L/H \leq 1 \), where the higher the index, the more monopsonistic the market is. From eq. (3.20) and eq. (3.22), it can be shown that:

\[ \frac{d(L/H)}{dM} > 0, \frac{d(L/H)}{dN} < 0, \frac{d(L/H)}{dk} > 0, \frac{d(L/H)}{db} < 0. \tag{3.23} \]

In words, the market becomes more monopsonistic when the minimum efficient scale is large, the size of the market is small, labor is less substitutable, and the roundabout technology is inefficient.

By differentiating eq. (3.22), we get:

\[ \frac{dH}{dN} < 0, \frac{dH}{dM} > 0, \frac{dH}{db} \geq 0, \frac{dH}{dk} \geq 0. \tag{3.24} \]

Since there are no gaps in the monopsony-competition case, it follows that:
\[ \frac{dm}{dN} > 0, \frac{dm}{dM} < 0, \frac{dm}{db} > 0, \frac{dm}{db} > 0. \quad (3.25) \]

The equilibrium number of firms will be large if the size of the market is large and if the minimum efficient scale \( M \) is small. The intuition for these results is quite clear. If the size of the market is large (i.e., the density of the market is high), the firm does not have to hire the workers whose skill characteristics are quite different from its ideal one in order to recover the fixed cost generated by the minimum efficient scale. By the same token, if the minimum efficient scale is large, then the firm has to hire many workers whose skill characteristics are not so ideal.

The effect of the change of marginal productivity is ambiguous. If the economy is close to the monopsony case (i.e., \( L \) is close to \( H \)), then the higher marginal productivity will increase output while the wage function will not be affected significantly. Thus, firms are more profitable. Consequently, more firms will enter the market. On the contrary, if the economy is similar to the competition case (i.e., \( L \) is close to zero), then the firm has to pay a higher wage, which is determined by the new marginal worker, to most of the workers. Thus, the increase in the wage bill would outweigh the increase of the productivity. Firms will exit, because they yield negative profits.\(^8\) The effect of the change of the training cost is also ambiguous. If the economy is close to the monopsony case, then \( \frac{dH}{dk} > 0 \), and vice versa. The intuition is roughly the same as the marginal productivity case.\(^9\)

The average wage will be:

\[ W = (b - kH) + (2kH - (b - w_0))^2 / 4kH, \]
\[ w = w_o + \frac{\left(b - w_o\right)^2}{4K}. \] 

(3.26)

It is clear that:

\[ \frac{dW}{dH} < 0. \] 

(3.27)

Therefore,

\[ \frac{dW}{dN} > 0. \] 

(3.28)

The range of the parameter values of the monopsony-competition case is:

\[ 2kM/N < \frac{\left(b - w_o\right)^2}{b} < 4kM/N. \] 

(3.29)

5. Synthesis

Let us examine the characteristics of the equilibrium by varying the size of the market. We assume that the market adjust instantly to the long-run equilibrium with a change in the size of the market. As I have indicated before (eq.(3.9)), if \( N < N_1 \) (= \( \frac{2bK}{(b - w_o)^2} \)), then the economy stays in the self-sufficient autonomy, because firms get negative profits. Since every worker will work for himself in this case, the average wage will be \( w_o \). If \( N = N_1 \), the monopsony equilibrium will occur, where the average wage will jump up to \( w_1 \) (= \( \frac{(b + 3w_o)}{4} > w_o \)). For a limited range of the size of the market \( (N_1 < N < N_2 = \frac{4bkM}{(b - w_o)^2}) \), the monopsony-competition equilibrium will occur. If the roundabout technology is very efficient (i.e. small \( K \), small \( M \), and large \( b \)), then
the range where the monopsony-competition equilibrium occurs is small.

Although the effects of changing $b$ and $k$ are ambiguous in this case, there is no perverse effect of the size of the market in the bargaining equilibrium. The market area continually declines with the increase of the size of the market (refer to Fig. 6). Similarly, the average wage rises in the same range. The rise of the average wage can be attributed to two factors. The first element is that the average match gets better by having smaller market area (eq.(3.23)). The second element, which is unique to the bargaining model, is that more and more workers have better bargaining position, as their productivity with the other firm exceeds the reservation wage.

One may wonder why the perverse effect of the kinked model of the signalling equilibrium does not occur in the bargaining model. The fundamental reason for the perverse effects of the kinked equilibrium is that the labor supply is less elastic in the monopolistic competition case than the monopsony case. This is counter-intuitive, since one would associate competition with high elasticity. This is a peculiar result of the original set-up of Salop [1979] in which the firm has to pay the higher wage to all workers when the marginal worker has an alternative firm to work for. Thus, there is a discontinuity in the wage function between the monopsony and the monopolistic regime.

In the bargaining model, the change of regime happens in a more gradual way. To see this, let us use Fig. 7 for illustration. If $N = N_1$, then the equilibrium distance will be $H_1$, and the equilibrium wage function will be line $AC'$. If $N$ increases, then $H$ decreases. Let us choose $N$ such that $N_1 < N < N_2$, then monopsony-competition equilibrium will occur, and the equilibrium distance ($H_3$) will lie between $H_1$ and $H_2$. 
The wage function is now line BB'C. Workers between 0 and \( H_4 \) have only one employment opportunity, while workers between \( H_4 \) and \( H_3 \) have two opportunities. Thus their wage schedule is BB' rather than AB'. If \( N > N_2 \), then \( H < H_2 \). Thus competition equilibrium occurs. Let us choose a point \( H_5 \). The equilibrium wage schedule is flat in the competition case. As \( N \) approaches infinity, the wage approaches \( b \) and \( H \) approaches zero.

IV. Conclusion.

We have developed two simple models in which labor is not homogeneous, and the roundabout technology has increasing returns to scale, in order to analyze the effects of labor specialization in a modern economy. The major point is that in a highly concentrated modern urban economy, the geographical proximity among workers and firms facilitates a specialized labor market. Since specialization reduces the training cost (or the loss of the productivity due to the mismatch between workers and firms), the average productivity increases with the size of the market except the kinked equilibrium case in the signalling model where the average productivity decreases with the size of the market.

The perverse effect of the kinked equilibrium of the signalling model will not occur in the bargaining model, because the transition from the monopsony regime to the monopolistic competition region is smooth in the latter. Even in the signalling model, if the market size is big enough, then the economy will have the monopolistic competition equilibrium in which the average productivity will increase with the size of the market. In that regime, the wage will be higher, if the
roundabout technology has high marginal productivity, low minimum efficient scale, and low training cost.
Figure 1. The Representative Firm and the Workers Close to It

Figure 2. Three Possible Cases in Signalling Model
Figure 3. Three Possible Equilibria in Signalling Model
Figure 4. Size of the Market and Equilibrium Wage in Signalling Model
Figure 5. Three Possible Cases in Bargaining Model
Figure 6. Growth of Market Size and Equilibrium Market Area and Average Wage in Bargaining Model
Figure 7. Change of Wage Equation and the Growth of Market in Bargaining Model
Footnotes

1. This model has a similar basic structure with Salop's [1979]. The major differences are; first, the focus is the labor market rather than the output market; second, sellers as well as buyers must incur some costs if the characteristic of the buyer does not match exactly with the characteristic of the seller. Namely, $c_2(t) = 0$ in Salop.

2. The condition that $c_1 > 0$ is more than necessary. This can be relaxed further in many cases.

3. There has been a criticism for the use of Nash equilibrium concept in this type of circle model (Hart [1983]). The argument is that the Nash assumption is not very convincing, since each agent interacts only with two neighboring firms directly. It is possible to get around this criticism by modifying the model into two dimensions such that there are many neighboring firms for each firm. Consider the following short run set-up. There are $m$ firm indexed $1$ to $m$. There is a continuum of workers indexed $(i,j,t)$ with aggregate size $N$. Worker $(i,j,t)$ can work only firm $i$ or $j$. All ordered pairs $(i,j)$ have equal population weight. For each $(i,j)$ t is uniformly distributed on the unit interval. The behavior of the worker $(i,j,t)$ is identical to eq.(2.7). It turns out that the mathematics of this set-up is identical to the circle set-up of the text. We will stick to the circle set-up because of its intuitive appeal.

4. Notice that we impose symmetry after differentiation.

5. One can easily verify that the second order condition is also satisfied.

6. Instead of using the wage offer, one can use average net wage to yield the same comparative static results.

7. It is interesting that some plausible wage functions have serious defects. One would think that the following is a reasonable wage function:

$$w(t) = \frac{(b - c(t))}{2} + \bar{w}(2H-t)/2,$$

(A.1)

where $w(.) = \bar{w}(.)$ at equilibrium. The problem of this formulation is that $\bar{w}(2H-t)/2$ is not a credible threat point, because $\bar{w}(2H-t) > b - c(2H-t)$ if $t < H$. In other words, the
other firm will not engage in the employment contract which yield negative return. The other plausible wage function is:

\[ w(t) = \frac{(b-c(t))}{2} + \frac{((w_0+b-c(2H-t))/2)}{2}. \quad (A.2) \]

This can not be an equilibrium wage function, because it is not stable at the neighborhood of the marginal workers where the wage is lower than the marginal productivity with the other neighboring firm. Let us choose the worker at the midpoint \( t = H \) for illustration. Since \( w(H) < b-c(H) \), he can make a mutually beneficial employment contract with the other firm with the wage higher that \( w(H) \). After he gets the higher wage, he can open a new employment negotiation with the original firm and demand even higher wage.

8. For example, let \( w_0 = 0 \). Then it can be shown that:

\[
    \frac{dH}{db} < 0, \text{ if } 2kM/N < b < \left(\frac{2+1}{2}\right)kM/N \quad (A.3.a)
\]
\[
    \frac{dH}{db} > 0, \text{ if } \left(\frac{2+1}{2}\right)kM/N \leq 4kM/N. \quad (A.3.b)
\]

9. It can be shown that:

\[
    \frac{dH}{dk} > 0, \text{ if } 2kM/N < \frac{(b-w_0)^2/b}{(3+2\sqrt{2})/\left(\frac{2+1}{2}\right)} \quad (A.4.a)
\]
\[
    \frac{dH}{dk} \leq 0, \text{ if } \frac{(3+2\sqrt{2})/\left(\frac{2+1}{2}\right)}{kM/N} \leq \frac{(b-w_0)^2/b}{4kM/N}. \quad (A.4.b)
\]
CHAPTER III

Labor Specialization Decisions and the Extent of the Market

I. Introduction

In this chapter, we analyze an extension of the model presented in the previous chapter. Namely, we shall allow workers to choose their human capital. In the two models of the previous chapter, workers' skill characteristics are exogenously given. Since we shall consider the extension of the bargaining model only, the "previous model" refers to the bargaining model in Chapter II.

The human capital investment decision of the worker has two distinct dimensions. First, he decides on how much he will accumulate his human capital on certain skill characteristics. The usual terminology of "skilled labor" and "unskilled labor" stems from this consideration. However, the treatment of human capital as a scalar is not very satisfactory, once one recognizes the heterogeneity of labor. For example, a computer programmer would be regarded as a "skilled labor" for data processing industry while he is a "unskilled labor" for construction industry. The second dimension the worker decides on is the breadth of his human capital. A computer programmer may train himself for a wide range of software and hardware. Alternatively, he may learn only a specific programming language on a specific hardware. We shall call them intensive and extensive human capital, respectively.

In the modern economy where labor demand is also highly
specialized, extensive human capital is as important as intensive human capital, because there is an important trade-off between them. Although a specialist will be more productive than a generalist in a limited range of tasks which require the specialist's skill, the probability of getting such a good-matching job will be smaller for the specialist.

To characterize the worker's human capital in this way has a number of advantages over the approach in which it is characterized by a vector of skills. First, we can parameterize it completely in two variables (intensive human capital and extensive human capital) rather than a vector with a higher dimension. Second, it is not very clear how one defines a wide range of skills in a finite vector. Third, the former approach can incorporate the natural difference of human potentials in a more reasonable way. This point will be made clear when we discuss the structure of the model.

With the given cost of acquiring human capital, a worker may invest more intensively and less extensively, or vice versa. We say that worker A is more specialized than worker B, if worker A has more intensive and less extensive human capital than worker B. Sometimes we refer the former a specialist, and the latter a generalist. In our model, the choice of skills to specialize (say, the choice of occupation) is determined by the worker's potential ability endowed with his birth.

In this chapter, we will analyze the relationship between the worker's human capital investment decision and the size of the market by using a similar model described in the previous chapter. The major difference between the model in this chapter and in the previous one is that workers can choose the marginal productivity of the ideal worker in the production technology (b) and the loss of productivity due to the
mismatch between the job and the worker \( c(t) \). We shall only focus our attention on the bargaining model because it has the simpler analytical structure than the signalling model.

As in the previous model, three cases can be distinguished based on the number of viable jobs a worker has. A viable job for a worker is the job which pays higher wage than the worker’s reservation wage. The case in which all workers in the economy have at least two viable jobs will be called competition case. If all workers have at most one job opportunity, then it is called monopsony case. The third case in which some of the workers have only one viable job opportunity whereas the others have two will be called monopsony-competition case. Notice that the competition case occurs when every worker has human capital general enough to work for at least two firms which require different skill characteristics.

In section II, assumptions of the model will be described. Many assumptions are the same as in the previous model. Thus, only the assumptions different from those of the previous one will be spelled out in detail. In section III, the competition case will be analyzed. The monopsony case will be analyzed in section IV. The monopsony-competition case will be analyzed in section V. Conclusions will be given in section VI.

II. Assumptions

Consider a closed economy of a continuum of workers-cum-consumers with aggregate size \( N \). Workers are indexed on a circle of a unit length with uniform density. Since the circle has the unit length, the density
is also $N$. The index represents the worker's potential ability. The same index represented the worker's actual skill characteristic rather than his potential in the previous model. As in the previous chapter, we will call the index location and the difference between two indices distance.

Since we assume that the worker knows his potential ability, he will invest in his human capital around it. It acts as an anchor for his human capital investment decision. In this model, the worker's actual skill characteristic is determined by his human capital investment as well as his potential ability. Notice, however, that human capital investment does not change the worker's index. Thus, we can identify the worker with his potential ability, even after his human capital accumulation is finished. In other words, even though we allow worker to choose his human capital, we do not allow them to choose their best skill characteristics. Thus, the choice of skill characteristic (say, occupation) is excluded in the model. For example, a computer programmer is born with the potential ability to be a computer programmer, and he knows that his potential ability is strongest in computer programming. Thus, he will be a computer programmer. The only decisions he makes are: 1) How much he will invest in computer programming (say, get a Master's degree or go to a technical school); and 2) How wide he will train himself (e.g., only learn one programming language in one machine or learn a variety of different languages and machines).

In particular, we shall assume that the representative worker chooses two variables $(b, K)$ to maximize his lifetime expected utility. We assume risk neutrality of workers. Since our model is a static one, it follows that the worker maximizes the expected wage net of the cost of
acquiring the human capital. We normalize the location of the representative worker to the origin (zero) without loss of generality. We shall call the two variables intensive human capital and extensive human capital. The variable $b$ (intensive human capital) represents the marginal productivity of the worker when he works for the firm which requires the same skill characteristic. In the previous model, $b$ was parameterized as the marginal productivity of the "ideal" worker to a given firm, and was given exogenously. In this model, we shall treat $b$ as a worker's decision variable.

The worker also makes a decision on $K$ (extensive human capital). Generally, the more extensive human capital, the lower the expected cost of mismatch between the job and the worker, given the intensive human capital and labor demand. Specifically, we assume that the cost of mismatch between the job and the worker is assume to be:

$$c(t) = \frac{t}{K}$$

where $t$ is the distance between the worker and the firm. Notice that the cost of mismatch is independent of $b$. One can view the above mismatch function as a reparameterization of the linear cost function of the mismatch (eq.(3.7) in Chapter II). From the viewpoint of the previous model, the worker is now allowed to choose the marginal productivity of the roundabout technology ($b$) and the inverse of the slope of the linear mismatch cost function ($k = 1/K$).

Although the model is presented in a static setting, one can conceptually divide it into two stages. In the first stage, workers make their human capital investment decisions (i.e., choose $b$ and $K$). They
know the aggregate size of the market and the wage determination rules. Thus, given $b$ and $K$, they can calculate their expected wage and the cost of acquiring human capital. We impose the symmetry that every worker invests to the same level of $b$ and $K$. In the second stage, the same decision process in chapter II will be replicated. Firms will be established, and they choose technologies (location). Given the location of firms, workers make decisions whether to work or not, and if they decide to work, they will choose the firm which they will work for. We impose another kind of symmetry that all firms have same wage function, and that the distance between any two neighboring firms are identical. Equilibrium can only be realized when all firms get zero profits.

The parameters $(b, K)$ are choice variables for the workers rather than exogenous parameters. However, since we shall assume that workers make human capital investment decisions before entering the labor market, and that we restrict ourselves to the case where all workers have same $(b, K)$, firms will make production decision as if those are exogenous parameters. Thus, the results of the previous model can be carried over directly to the model in this chapter.

Firms (or plants) are assumed to produce homogeneous output, which are sold in a competitive market. The price of output is normalized to one. The firm's technology is defined by the minimum efficient scale ($M^*$), which is exogenous and same for all firms, and its ideal skill characteristic. The firm is also indexed on the unit circle, where the index represents the its ideal skill characteristic. The firm's ideal skill characteristic is the characteristic of the worker who has the highest marginal productivity with the firm given the same intensive human capital.
For the purpose of illustration, let us consider the computer software industry. Firms produce homogeneous output called software. One firm produces software by using one programming language. Other firms produce same product by using different languages. Thus, they want to hire workers who are good at the other kind of languages. If a firm hires a worker who is not good at the language the firm uses, then the cost of mismatching must incur in order to train the worker.

The production function of the representative firm is given by:

\[
Y = \begin{cases} 
0, & \text{if } X < M'/b \\ 
bX - M', & \text{if } X > M'/b 
\end{cases} \quad (2.2.a) \\

\text{where} \\
X = N \int (1 - t/K) \, dt. \quad (2.2.b) \\

Y \text{ denotes the output, } X \text{ denotes the normalized input, } M' \text{ denotes the minimum efficient scale (which is given exogenously and same for all firms), } t \text{ denotes the distance between the firm and the workers, and the integral is taken over the market area of the firm. This production function is same as in the previous model except that the minimum efficient scale is reparameterized to:}

\[ M' = bM. \quad (2.3) \]

We assume that the cost of acquiring human capital \( g(b,K) \) is assumed to be convex in \( b \) and \( K \). Namely,

\[ g_b > 0, \; g_K > 0 \quad (2.4.a) \]
\[ g_{bb} > 0, \ g_{kk} > 0, \ g_{bk} > 0 \]  \hspace{1cm} (2.4.b)

\[ g_{kk}^2 - g_{bb}g_{kk} < 0, \]  \hspace{1cm} (2.4.c)

where the subscripts refer to partial derivatives. The first set of assumptions refer that the cost of acquiring human capital is an increasing function of the level of both intensive and extensive human capital. The second set of assumptions say that the marginal costs are also increasing functions. The third assumption insure that the Hessian of \( g(b,K) \) is negative definite.

Workers and firms know exactly how much they will gain by having employment contracts, and the wage will be determined at the mid-point through bargaining between the worker and the firm, where the worker's surplus of having the employment contract over his second best alternative is same as the firm's marginal profit of having the worker. Negotiation is costless, and collective bargaining is not allowed. Since the model is static, we also assume that no pair of agents will miss a potentially beneficial bargaining opportunity.

We will limit our analysis to the long run symmetric Nash bargaining equilibrium. Long run is defined same as in the previous model. All firms get zero profit and cost of relocating (adopting a new technology) is zero. By symmetry we mean that: 1) All workers have same \((b,K)\); 2) The wage functions are identical for all firms; and 3) Firms are equally spaced on the circle. The symmetry assumption is very powerful despite its simplicity. It reduces the number of equilibria to one except the monopsony case where there may be gaps between firms' market areas. It is also a plausible assumption given that all workers and firms have symmetric endowments.
The workers are assumed to have perfect information about the size of the market and the wage determination rule so that they can calculate the wage distribution function and the equilibrium distance \(2H\) given \((b,K)\). The actual wage is a random variable, however, since the location of the firms is taken to be random. Although the worker does not know the actual location of the firms, he knows the probability distribution of the location. We assume that the probability density function is uniform on the domain of \([-H, H]\). The support of the probability density function shrinks as the number of firms in the market increases. Roughly speaking, therefore, the location of the firm is more predictable and the average distance between workers and firms is shorter when there are more firms in the market. The reduction of uncertainty does not play a role in this chapter because of the risk neutrality assumption. Workers' choices of \((b,K)\), in turn, determine the equilibrium distance \((2H)\) and the wage equation \(w(t)\) through the firms' profit maximization condition and the zero profit condition as described in chapter II.

### III. Competition Case

As in chapter II, the competition case occurs when all workers in the economy have at least two firms to work for. This condition for the existence of the competition case implies that:

\[
b - \frac{2H}{K} \geq w_0. \tag{3.1}
\]

We assume that a worker bargains only with the two firms which have the
two highest marginal productivities with the worker, and that the worker's alternative wage is the highest possible wage in the negotiation with the other firm (see Footnote 7 in chapter II for further explanations).

With the linear mismatch cost function, the long run equilibrium wage and equilibrium distance between any two neighboring firms are given by (refer eq.(3.13) and eq.(3.14)in chapter II):

\begin{align*}
w(t) &= b - \frac{H}{K} \quad (3.2.a) \\
H &= eK^{1/2} \quad (3.2.b),
\end{align*}

where \( e = \left(\frac{M^*/N}{1}\right)^{1/2} \). Since the wage is independent of the location of firms, the expected wage net of the human capital cost (W) of the representative worker is:

\[ W = b - \frac{H}{K} - g(b,K). \quad (3.3) \]

Excluding the possibilities of corner solutions, we get the two first order conditions for the worker's expected net wage maximization problem by differentiating eq.(3.3) with respect to \( b \) and \( K \):

\begin{align*}
W_b &= 1 - g_b = 0 \quad (3.4.a) \\
W_K &= HK^{-2} - g_K = 0. \quad (3.4.b)
\end{align*}

It can be easily verified that the second order conditions are satisfied by the assumptions of eq.(2.4):
\[ W_{bb} = -g_{bb} < 0 \]  
\[ W_{kk} = -2HK^{-3} - g_{kk} < 0 \]  
\[ W_{bb}W_{kk} = g_{bb}^2 - g_{bb}g_{kk} - 2HK^{-3}g_{bb} < 0. \]

By substituting eq.(3.2.b) into eq.(3.4.b), the equilibrium conditions can be re-written:

\[ g_b = 1 \]  
\[ g_k = eK^{-3/2}, \]

where all the variables are evaluated at the equilibrium. Notice that the equilibrium conditions can be characterized by single parameter e.

In order to obtain comparative static results, let us consider a small change in e. Taking total differentials, we get:

\[ g_{bb}db + g_{bk}dK = 0 \]  
\[ 3/2eK^{-5/2}dK + g_{kk}dK + g_{bk}db = K^{-3/2}de \]

Solving eq.(3.7.a) in terms of db, and substituting it into eq.(3.7.b), it can be verified that:

\[ \frac{dK}{de} = \frac{K^{-3/2}}{(3/2)eK^{-5/2} - (g_{bk}^2 - g_{bb}g_{kk}) / g_{bb}} > 0. \]  

From eq.(3.7.a) and eq.(3.8),

\[ \frac{db}{de} = -(g_{bk}/g_{bb}) \frac{dK}{de} < 0. \]
Then, it is straightforward to see:

\[
\frac{dK}{dN} < 0, \quad \frac{dK}{dM^*} > 0 \quad (3.10.a)
\]

\[
\frac{db}{dN} > 0, \quad \frac{db}{dM^*} < 0. \quad (3.10.b)
\]

Since the equilibrium number of firms in the market is (refer eq.(3.15) in chapter II),

\[
m = \frac{1}{2} e^{-2K^{-1/2}}, \quad (3.11)
\]

it is easy to see that:

\[
\frac{dm}{de} = -(1/2)e^{-2K^{-1/2}} - (1/4)e^{-4K^{-3/2}} \frac{dK}{de} < 0. \quad (3.12)
\]

Thus, it is clear that:

\[
\frac{dm}{dN} > 0, \quad \frac{dm}{dM^*} < 0. \quad (3.13)
\]

In words, the equilibrium human capital of the representative worker will be more specialized if the size of the market is larger and if the minimum efficient scale is smaller. Also, there are more firms in the market, when the size of the market is larger and the minimum efficient scale is smaller.

It will be useful to think through why workers and firms are more specialized in a larger market. First of all, given the characteristics of the labor pool (b and K), there are more firms in a larger market (eq.(3.11)). If the size of the market increases, there are more workers
whose marginal productivity is greater than the going wage. Thus, firms will get positive profits, and new firms will enter.

If there are more firms in the market, the support of the probability density function will shrink. Loosely speaking, it implies that the probability of having a good-matching job increases. The importance of extensive human capital declines relative to intensive human capital. Therefore, workers will be more specialized. More specialization implies higher $b$ and lower $K$. This feedback effect further increases the number of firms (eq. (3.11)).

IV. Monopsony Case

In the monopsony case, all workers have at most one viable job opportunity in which the worker gets higher wage than the reservation wage. Since some workers will stay out of the market in the monopsony case, the following condition must be satisfied for the workers who are exactly in middle of any neighboring firms:

$$b - H/K \leq w_0.$$  (4.1)

Given $b$ and $K$, the expected net wage for this case is determined by (refer eq. (3.11) in chapter II):

$$W = w_0 + \left( b - w_0 \right)^2 K / 4H - g(b, K),$$  (4.2)

where $H$ is any arbitrary positive real number satisfying eq. (4.1).

In order to get the first order conditions, we differentiate
eq. (4.2) by $b$ and $K$:

$$W_b = (b - w_0)K/2H - g_b = 0 \quad (4.3.a)$$
$$W_K = (b - w_0)^2/4H - g_K = 0. \quad (4.3.b)$$

The second order conditions in the monopsony case are slightly different from those of the competition case (eq.(3.5)). They are:

$$K/2H - g_{bb} < 0 \quad (4.4.a)$$
$$- g_{kk} < 0 \quad (4.4.b)$$
$$[(b-w_0)/2H - g_{bk}]^2 + (K/2H - g_{bb}) g_{kk} < 0 \quad (4.4.c)$$

The convexity assumption of the human capital acquiring cost (eq.(2.4)) is not sufficient for eq.(4.4.a) and eq.(4.4.c) to be satisfied. In order to satisfy eq.(4.4.a), a stronger condition is needed than $g_{bb} > 0$. Using the first order equilibrium conditions (eq.(4.3)), eq.(4.4.a) can be re-written as:

$$g_{bb} > g_b^2 / 2g_k. \quad (4.3.a')$$

Eq.(4.4.c) is more difficult to interpret. It is clear that the second order term in eq.(4.4.c) is negative if eq.(4.3.a) is satisfied. By using eq.(4.3.a), the first term of eq.(4.4.c) can be written as:

$$(b-w_0)/2H - g_{bk} = g_b/K - g_{bk}. \quad (4.5)$$

It is not clear whether eq.(4.5) is positive or negative. The first term
in the right hand side of eq.(4.5) reflects the marginal cost of acquiring intensive human capital averaged out by extensive human capital. The second term reflects the marginal cost rather than the average with respect to extensive human capital. Nonetheless, we shall assume that eq.(4.4) be satisfied in the monopsony case and in the monopsony-competition case, since workers take H exogenous in the latter case also.

In the long run, the zero profit condition must be satisfied. This condition does not determine H as in the cases of competition and monopsony-competition. Instead, it gives an additional constraint on (b,K):

\[(b-w_0)^2 = \frac{2M^*}{KN}.\]  

(4.6)

Thus, equilibrium b and K are overdetermined in the monopsony case, because there are three equations to determine the two endogenous equilibrium variables. Suppose that we determine the values of b and K from eq.(4.3), and that we substitute them into eq.(4.6). If the left hand side of eq.(4.6) is smaller than the right hand side, then firms yield negative profits. Thus, there will be no monopsony equilibrium in the long run, since the firms cannot fully recover the fixed cost of the roundabout technology. On the other hand, if the reverse is true, then more firms will enter the market. It will reduce H so that the situations becomes either the monopsony-competition case or the competition case.

Put differently, the monopsony equilibrium is on the knife-edge in parameters (w_0, M^*, and N). Unless the parameters are such that there
exist $b$ and $K$ satisfying the three equilibrium conditions, the monopsony equilibrium will not occur. The comparative static exercise cannot be performed, because one cannot change one parameter values holding others constant. If only one parameter is changed from an equilibrium, then the economy can no longer be at an equilibrium, because not all the equilibrium conditions can be satisfied any longer.

Notice that $H$ is not determined by the equilibrium conditions. The only restriction regarding to $H$ is the inequality constraint of eq.(4.1). Thus, there are infinite number of $H$'s which satisfy the constraint. If the equation is satisfied by the equality, then there are no gaps between the market areas of any two neighboring firms. This special case can be regarded as a limiting case of the monopsony-competition case, which will be analyzed in the next section.

V. Monopsony-Competition Case

The monopsony-competition case occurs when some workers in the economy have two viable jobs whereas the others have only one. This implies that the productivity of the marginal worker who is at the boundary of the firm's market area is greater than the reservation wage, and that the productivity of the ideal worker of the neighboring firm at the representative firm is lower than the reservation wage. Namely,

$$b - \frac{2H}{K} < w_o < b - \frac{H}{K}.$$  

(5.1)

As shown in chapter II, the expected net wage of the worker in the monopsony-competition case has the same functional form as in the
monopsony case (refer eq.(3.26) in chapter II):

\[ W = w_0 + (b - w_0)^2 K / 4H - g(b,K), \]  

(5.2)

Unlike the monopsony case, however, H is determined endogenously (refer eq.(3.22) in chapter II):

\[ H = K \left[ (b-w_o) - \frac{(b-w_o)^2}{2} - \frac{e^2}{K} \right]^{1/2}. \]  

(5.3)

Differentiating eq.(5.2), we get the first order conditions for the worker's expected net wage maximization problem:

\[ W_b = \frac{(b-w_0)K}{2H} - g_b = 0 \]  

(5.4.a)

\[ W_K = \frac{(b-w_0)^2}{4H} - g_K = 0. \]  

(5.4.b)

The second order conditions are assumed to be satisfied as in the monopsony case. Since the worker is assumed to take H given, the first order conditions are identical to those of the monopsony case even though H is determined endogenously. However, eq.(5.3) constitutes an equilibrium condition as well as eq.(5.4) to determine the equilibrium (b,K,H).

Let us denote the right hand side of eq.(5.3) as:

\[ H = h(b,K,e). \]  

(5.3')

By using the existence condition of the monopsony-competition case (eq.(5.1) or eq.(3.29) in chapter II), it can be shown that:
Intuitively speaking, if the average productivity of workers \( b \) increases \textit{ceteris paribus}, then the firm's profit increases because the firm gets a half of the productivity increases by the equal bargaining power assumption. Higher profits induce the entry of new firms. If the labor pool is more substitutable (higher \( K \)), the labor market becomes more competitive. Then the area of the competitive regime relative to the firm's market area will increase. This will raise the firm's average wage bill. Profits will decline, and firms will exit. If the size of the market \( (N) \) increases or if the fixed cost element \( (M') \) decreases, firms will get higher profit, which leads to the entry of new firms.

In order to examine the comparative static results, we take total differentials of the equilibrium conditions:

\[
\begin{align*}
K & \ - g_{bK} \ db \ + \ \frac{(b-w_o)K}{2H} \\
& = 0 \ (5.6.a)
\end{align*}
\]

\[
\begin{align*}
\frac{b-w_o}{2H} & \ db \ - \ \frac{(b-w_o)K}{2H^2} \\
& = \frac{(b-w_o)^2}{4H^2} \\
& = 0 \ (5.6.b)
\end{align*}
\]

\[
\begin{align*}
- h_b \ db \ - \ h_k \ dK \ + \ dH & = h_e \ de \ (5.6.c)
\end{align*}
\]

It is impossible to come up with an unambiguous result by applying Cramer's rule to eq.(5.6), because: 1) the sign of the expression \( (b-w_o)/2H - g_{bK} \ (eq.(4.5)) \), is not determined \textit{a priori}, and 2) some of the terms would have ambiguous signs even if the sign of that expression
is determined.

In order to get some intuition, let us simplify the analysis by assuming that one of the two human capital variables \((b,K)\) is exogenous. We will come back to the discussion of three variable case later. First, let us assume that \(K\) is exogenous. Then, eq.(5.6) becomes:

\[
\frac{K}{2H} - \frac{(b-w_o)K}{2H^2} \frac{db}{dH} - \frac{g_{bb}}{2H^2} dH = 0 \quad (5.7.a)
\]

\[
- h_b \frac{db}{dH} + dH = h_e \frac{de}{dH} \quad (5.7.b)
\]

By applying Cramer's rule, one can derive:

\[
db/de < 0, \frac{dH/de}{dH} > 0; \text{ if } K/2H - g_{bb} < h_b(b-w_o)K/2H^2 \quad (5.8.a)
\]

\[
db/dH < 0 \quad (5.8.b)
\]

The condition in eq.(5.8.a) is a stability condition. It says that entry of new firms reduces profit per firm, and vice versa. Otherwise, there will be no stable equilibrium with finite number of firms. To illustrate the idea, let us refer to Figure 1. Two curves are drawn in the plane of \((b,H)\). The curve WW' refers the worker's expected wage maximization condition (eq.(5.4.a)). Every point on the curve is the worker's optimal choice of \(b\) given \(H\). The slope of the curve is \(\{K/2H - g_{bb}\} / \{(b-w_o)K/2H^2\}\). We will denote it as \(dH/db\)\(_w\). The curve ZZ' is the zero profit condition (eq.(5.3)). The slope of the curve is \(h_b\). Firms get positive profits in the upper-right region of the ZZ' curve and negative profits in the lower-left region.

Notice that both curves are downward-sloping. The configuration
in Figure 1 (i.e., $dH/db|_w < h_b$) shows an stable equilibrium. The exact dynamic structure will be spelled out later. Suppose the size of the market ($N$) increases. This means $e$ declines. Then, the ZZ' curve will shift down, since $h_e > 0$. The initial equilibrium point $A$ is now in the region of positive profits. Thus, firms will enter. The entry will reduce $H$. Workers invest more in $b$ to take advantage of the reduction of $H$. Although profit will increase by the increase of the productivity of workers, it will be dominated by the decrease of profits due to the larger number of firms. Since profit will decrease with more entry, the economy will reach a new equilibrium point $B$ where firms get zero profits.

If the slope of the ZZ' curve is steeper than that of the WW' curve, on the contrary, then the system is not stable. For example, if $N$ increases, then firms get positive profit. More firms will enter. Workers will invest more in $b$. The increase of profit due to the higher productivity of the workers will be stronger than the dilution of profits due to the new entry. Thus, more firms will enter and the process goes on without stopping.

Notice that we have adopted a particular dynamic structure, namely, we assumed that the entry and exit of firms is the key element of adjustment. If the size of the market or the minimum efficient scale changes, then profits will rise or fall holding $(b,K)$ constant. Since non-zero profits will induce entry or exit of firms, $H$ will vary. As $H$ varies, workers make a new choice of $(b,K)$. With different $(b,K)$ $H$ will vary, and the process keeps going until it reaches another equilibrium.

Let us turn to the case where $b$ is taken exogenous and $(K,H)$ are endogenous. Then, it is straightforward to show that:
\[ \frac{dK}{de} < 0, \quad \frac{dH}{de} > 0, \quad \frac{dK}{dH} < 0 \quad (5.9) \]

In this case, the zero profit condition is upward sloping \( h_K > 0 \) on the plane of \((K,H)\), whereas the worker's expected wage maximization equation is downward sloping. If the size of the market increases, then firms get positive profits. More firms will enter, and \( H \) will decline. Workers will invest more in \( K \). Since workers become more substitutable between each other, the labor market becomes more competitive. The area of the competitive regime increase relative to the total market area. In other words, higher \( K \) necessarily implies the loss of profitability. Thus, the comparative static results are unambiguous. A similar stability condition as in the \((b,H)\) case implies that \( h_K < g_{KK} / \{(b-w_0)^2/4H^2\} \).

Notice that \( dK/de < 0 \). Namely workers will have more extensive human capital if the size of the market is large. This is the key element which makes the comparative static results of the \((b,K,H)\) system ambiguous. To see this, let us apply the analyses of the \((b,e)\) system and the \((K,e)\) system into the \((b,K,e)\) system. Suppose that the size of the market increase, then the direct effect is that \( H \) will decrease \( (h_e > 0) \). With lower \( H \), workers will choose higher \( b \) and \( K \) based on the analyses of the two variable cases. The increase in \( b \) will in turn induce lower \( H \) \( (h_K < 0) \). Thus the effect through \( b \) is all monotonic and well-behaved. However, higher \( K \) implies higher \( H \) \( (h_K > 0) \). Thus, the feedback effect on \( H \) through \( K \) is opposite to the initial effect through \( e \). Depending on which of the two is stronger, the total effects will be ambiguous. As we will see later, however, \( dH/de \) is positive unambiguously, if a stability condition is satisfied.
Let us come back to the original system in which $b$ and $K$ are both endogenous as well as $H$. One can re-write the equilibrium equations into a more manageable form by noticing that the worker makes a choice of $(b,K)$ only on the basis of $H$. There are no direct effects of $e$ on $(b,K)$. Therefore, $(b,K)$ will vary indirectly. Assuming that the Jacobian of eq.(5.4) with respect to $b$ and $K$ does not vanish, we can rewrite the equilibrium conditions as follows:

\begin{align}
    b &= f(H) \\
    K &= g(H) \\
    H &= h(b,K,e)
\end{align}

By taking total differentials and applying Cramer's rule, one can show that:

\begin{align}
    \frac{db}{de} &= \frac{hef'}{1 - hbf' - hKg'} \quad (5.11.a) \\
    \frac{dK}{de} &= \frac{heg'}{1 - hbf' - hKg'} \quad (5.11.b) \\
    \frac{dH}{de} &= \frac{he}{1 - hbf' - hKg'} \quad (5.11.c)
\end{align}

A similar stability argument as in the $(b,H)$ case implies that the denominator of eq.(5.11) is positive. The above dynamic structure can be represented in a system of difference equations as follows:
where \( t \) represents a time period. By substituting eq.(5.10.a') and eq.(5.10.b') into eq.(5.10.c') and differentiating it with respect to \( H_t \), we get:

\[
dH_{t+1}/dH_t = h_{bf'} + h_{kg'}.
\]

The stability condition implies:

\[-1 < h_{bf'} + h_{kg'} < 1.\] (5.13)

When \( h_{bf'} + h_{kg'} = 1 \), the system does not have a unique solution.

If we assume that the stability condition (eq.(5.13)) is satisfied, then the signs of eq.(5.11) are determined as follows:

\[
\begin{align*}
\text{sign}(db/de) &= \text{sign}(f') \quad (5.11.a') \\
\text{sign}(dK/de) &= \text{sign}(g') \quad (5.11.b') \\
dH/de &> 0. \quad (4.11.c')
\end{align*}
\]

Thus, the system will behave in a predictable way if \( b \) and \( K \) are monotonic in \( H \). But, unfortunately \( b \) and \( K \) are not monotonic in \( H \). This can be seen by applying Cramer's rule in the system of eq.(5.6.a) and eq.(5.6.b). Remembering the second order condition, one can derive:
It can be seen that $f'$ and $g'$ can take both positive and negative signs depending on parameter values making signs of eq.(5.9) ambiguous.

We will analyze the problem in a different perspective. It will be useful to define a new variable $z$ as follows:

$$z = \frac{(b - w_o)K}{2H}. \quad (5.15)$$

From eq.(5.1), it is clear that:

$$\frac{1}{2} < z < 1. \quad (5.16)$$

From eq.(3.20) in chapter II, one can see that:

$$z = 1 - \frac{L}{2H}, \quad (5.17)$$

where $L$ is the market area of the firm under the monopsony regime. Thus, the new variable can be regarded as an index for the degree of competition at equilibrium. Higher value of $z$ means that the market is more competitive. The monopsony case without any gaps occurs when $z = 1/2$, and competitive case occurs when $z = 1$. Then the equilibrium conditions (eq.(5.3) and eq.(5.4)) can be re-written:

\[
\text{sign}(f') = \text{sign} \left[ \frac{K}{2} \left( b - w_o \right) + \frac{b - w_o}{4} \left( b - w_o \right) \right] \]  
\[
\text{sign}(g') = \text{sign} \left[ \frac{K}{2H} \left( b - w_o \right) - \frac{b - w_o}{4} \left( b - w_o \right) \right] \]
\begin{align*}
z - g_b &= 0 \quad \text{(5.18.a)} \\
z^2 HK^{-2} - g_K &= 0 \quad \text{(5.18.b)} \\
\frac{1}{z} &= 2 - \left[ 2 - \frac{4e^2}{(b-w_0)^2 K} \right]^{1/2}. \quad \text{(5.18.c)}
\end{align*}

One would notice that the first two equilibrium conditions are similar to those of the monopsony case (eq.(3.4)) except the appearance of \(z\). If \(z = 1\), then it coincides with the competition case. Moreover, if we assume that \(z\) is constant then the comparative static results of the competition case (eq.(3.10)) will go through. In other words, holding the degree of competition \((z)\) constant workers will be more specialized if the size of the market is large, or if the minimum efficient scale is small.

In fact, however, the degree of competition varies, when the parameter values change. For example, if the size of the market increases, workers will be more specialized holding \(z\) constant. This implies higher \(b\) and lower \(K\). The new value of \((b,K)\) will change \(z\). It is not clear whether \(z\) will increase or decrease, because high \(b\) implies high \(z\) and low \(K\) implies low \(h\) (eq.(5.18.c)). However, it is more likely that \(z\) rises with the decrease of \(e\), because 1) \(z\) is more sensitive to the change in \(b\) than in \(K\); and 2) lower \(e\) further reduces \(z\) directly through eq.(5.18.c). Even if we can safely assume that \(z\) rises with the increase of the size of the market, the effects of higher \(z\) on \((b,K)\) are ambiguous (eq.(5.18.a)-(5.18.b)).

\textbf{VI. Conclusions}
An extension of the bargaining model in chapter II was analyzed to examine the relationship between the human capital investment decision and the size of the market. If the economy is competitive, i.e., all workers in the economy have at least two viable jobs at which he can earn the wage higher than his reservation wage, the worker's human capital will be more specialized in a larger market. The worker will want to have a more intensive human capital and less extensive human capital. In a larger market, the probability to have a good-matching job would be high, since there are more firms around which seek different skill characteristics.

The number of firms in the market will be determined as in chapter II. Since the firm has increasing returns to scale, average cost decline with the size of the firm. However, as the size of the firm increases, the firm cannot find the workers with skill characteristics that the firm wants to have. Put differently, the cost of mismatch between workers and jobs increases with the size of the firm. Given the size of the firm and the characteristics of the labor pool, the cost of mismatch declines with the size of the market, because the fixed cost element becomes less significant.

The monopsony case only occurs on the knife-edge values in parameters so that firms get exactly zero profit. This not a very interesting case since the probability of having such an equilibrium is zero. Moreover, the comparative statics cannot be performed because one parameter cannot be changed holding others constant. If only one parameter is changed, then the economy is no longer be at equilibrium. The monopsony equilibrium is not unique if we restrict our analysis to the symmetric equilibria, since it is possible to have gaps between the
market areas of two neighboring firms. If we restrict further to the case of no gaps, then the monopsony case is a limiting case of the monopsony-competition case.

The monopsony-competition case is more problematic, since the boundary between the monopsony regime and the competition region is determined endogenously as well as the market area. The degree of competition (an index of the ratio of the area of the competition regime to the whole market area), may increase or decrease with the changes of parameter values. If we were able to hold the degree of competition constant, the comparative statics would be the same as in the competition case.

If we treat the extensive human capital as exogenous rather than endogenous, then the increase of the market size and/or the decrease of the minimum efficient scale unambiguously raise the level of intensive human capital assuming that the stability condition is satisfied. However, holding intensive capital constant, the increase of the market size will increase the level of extensive human capital, which makes the comparative static results ambiguous.

If we assume a stability condition (that is, $1 - h_b f' - h_K g' > 0$), then the number of firms will increase with the size of the market and decrease with the minimum efficient scale unambiguously. Even with the stability condition the effects of $e$ on $(b, K)$ are still ambiguous, since $b$ and $K$ are not monotonic in $H$.

Finally, it will be useful to comment on the adjustment mechanism suggested in the models of chapter II and chapter III. In reality, it is conceivable that the adjustment of local labor market occurs in three major channels. First, labor supply may change through migration and
labor force participation rate. Second, labor demand may change through exit and entry of firms and technological innovations. Third, wage determination rule may change when market structure and bargaining mechanism changes. So far we assumed that long run labor supply is completely inelastic, whereas labor demand (supply of new firms) is completely elastic. In particular, in order to examine different wage determination rules, we analyzed the signalling model and the bargaining model in chapter II. In chapter III, the adjustment only occurs through entry and exit of firms. In chapter IV, the assumption of closed economy will be relaxed in order to examine the characteristic of systems of cities.
Figure 1. Stable Equilibrium in \((b,H)\) Case
I. Introduction

This chapter presents a theoretical model of scale economies, externalities, and efficiency of regional resource allocation. Our purpose is two-fold. First, we want to construct a model to explain the nature of agglomeration economies. In particular, we will focus on the increasing productivity due to the growth of the local labor market. A distinction will be made between internal scale economies, which will be called simply scale economies or increasing returns to scale hereafter, and external scale economies. The former is a phenomenon of increasing productivity (or decreasing average cost) due to the increasing size of the production level of an individual producer, which is under his control. The latter refers to the productivity gain due to the size of the economy, which presumably cannot be controlled by any single producer. We want to analyze the nature of the external scale economies in relation to the internal scale economies.

The second objective of the chapter is to evaluate various urbanization policy issues in rapidly urbanizing countries. Questions like optimal city sizes, decentralization policies and regional investment decisions have occupied the minds of economists and policy makers for quite a long time (see Renaud[1981] for a comprehensive survey on these topics). Although there has been a great deal of policy
discussion about them, the theory of agglomeration economies has not been well developed. It is hoped that the model presented here will shed some light on these issues.

Scale economies cause perfect competition to break down\(^2\), because the largest firm can always underbid the smaller competitors, leading eventually to a natural monopoly. Moreover, competitive marginal pricing yields negative profit to the producers since marginal cost will be lower than the average cost\(^3\). One way of modeling scale economies with competition is to assume that scale economies are completely external to firms, so that competition can be preserved (see Helpman[1983] and Chipman[1970] for examples of this approach). In this approach, it is assumed that productivity is an increasing function of the aggregated production level which is exogenous to individual firms. The typical result of this approach is that the competitive equilibrium size is smaller than the optimal size, because firms do not recognize the external economies of scale created by the increase of their own activity levels (see Shukla and Stark[1985]). This approach is unsatisfactory for our purpose, because we are particularly interested in why such external scale economies exist.

The second approach is the utilization of the notion of Chamberlinian monopolistic competition to the increasing return to scale production technology (see, for example, Krugman[1979], Weitzman[1982]). In this approach, competition can be maintained even when firms have increasing returns to scale technologies. As any firm's output is substitutable to another's, the firm faces a downward sloping demand curve. Thus, if the firm produces more in order to reduce its average cost, it also has to reduce the price to sell the increased output. The
model presented in this chapter adopts the monopolistic competition framework in labor markets to explain the external scale economy. Labor specialization is emphasized as a source of external scale economies rather than the sharing of infrastructure due to locational proximities.

Agglomeration economies have been recognized for quite a long time. Fuch[1967] has found that wages in large cities are significantly higher than wages in small cities, ceteris paribus. More recent studies, estimating aggregated production functions of cities, consistently report that large cities are more productive after controlling for capital labor ratio, industrial mix, skill level of workers, and so on (see Sveikauskas[1975], Segel[1976], Moomaw[1981], and Henderson[1983]). Also we observe regional specializations such as concentration of financial institutions in New York City and high-tech industry around Boston and the "Silicon Valley". Moreover industries which require highly skilled labor tend to concentrate more than others (Carlton[1969]). Naturally one would turn to external scale economies for an explanation for such phenomena.

Traditional theories for agglomeration economies are mainly based on cost savings due to shared infrastructure and/or reduced transportation cost accruing to firms by locating close to one another\textsuperscript{4}. However, neither of the arguments is very convincing from the perspective of a modern urban economy. First, the cost of public services and infrastructure such as water, transportation networks, etc. is much higher in large metropolitan areas than in small towns (Linn[1982], Walzer[1972]). We think that agglomeration happens in spite of the increasing marginal cost, not as a result of the decreasing marginal cost of public service. Second, with regards to the transportation cost
savings argument, it should be sufficient to point out that transportation costs are a small fraction of the total cost (less than 2 per cent) of most commodities. Thus large reductions in transportation costs will result in only minor changes in total cost, and are not likely to affect the locational decision of the firm.

In the following model, labor specialization is the key source of external scale economies. Labor specialization and the roundabout nature of modern production technologies are the basic characteristics of modern urban economies. Although geographical proximity plays an important role in the process of urban agglomeration, the argument is that geographical proximity enables producers to specialize, and thus to increase productivity. Given the usual social practice that workers commute back and forth between their residence and workplace on a daily basis, specialization would be limited by the size of the urban area in which daily commuting is possible. Although average productivity will be increased by adopting more specialized and roundabout technologies, such technologies can only be adopted when the market is large enough so that they can be supported by the activities of other agents in the market. Since an individual agent in the market can not control the size of the market, the productivity gain through the specialization is an externality.

There has been growing concern about the rapid urbanization and the concentration of population in very large cities among economists and policy makers in many developing countries. The average annual rate of increase of urban populations in less developed countries has been over 4 per cent during the past three decades. The annual growth rate of urban population in developed countries has been decreasing from 2.5 per cent
to 1.5 percent, for the same period. Since the world population has been
growing steadily by about 2 per cent annually the major thrust of
urbanization has been occurring in developing countries. Although
development planning traditionally has been carried out in sectoral
level, there has been a growing need for a national urbanization policy
in order to coordinate the planning activities in more specific spatial
context.

Nonetheless, there are conflicting views about the phenomena of
rapid urbanization. On the one hand, proponents of urbanization cite the
classical development theory by Lewis[1955] and Fei and Ranis[1964].
Those authors implicitly interpreted urbanization as a process of
releasing rural labor, which has low marginal productivity, to industrial
labor. By holding the wage at subsistence level which is lower than the
marginal productivity of the industrial sector, the economy can
accumulate capital and thus grow faster. The proponents of urbanization
sometimes use a different rationale. Planners in the industrial sector,
for example, favor the concentration of activities in order to exploit
increasing returns to scale and/or external economies. On the other
hand, congestion and pollution in high density urban areas have been
widely recognized. The emergence of the "super-metropolis" in developing
countries has put a great deal of pressure on the utilization of new
investment in urban resources. To name a few, problems in housing,
transportation, environmental degradation, and social pathology have
become severe in those cities. It seems that new investment cannot catch
up with the growth of the population in the very large cities. The
general conclusion of negative externalities is that realized activity
will be greater than optimal activity, because agents do not take into
account their marginal social cost they impose on other agents (recall
the "Tragedy of the Commons"). The higher marginal costs required to
provide a given average level of urban services (such as utilities,
housing, and transportation) have been cited as a reason for
decentralization policies.

We make no pretense of generality. Specific assumptions will be
made in order to make the argument clear and simple. In Section II, we
will develop a model of labor specialization. Then, section III
incorporates negative externalities to examine the properties of optimum
city size. In Section IV, we apply the model to evaluate the efficiency
characteristics of city growth and allocation of resources in a system of
cities. Conclusions are offered in section V.

II. The basic model

1. Assumptions

Let us consider a closed economy of a continuum of
workers-cum-consumers with aggregate size \( N \) (see Figure 1). Workers are
indexed on a circle of a unit length with uniform density. Since the
circle has the unit length, the density is also \( N \). The index represents
the worker's skill characteristic. Sometimes we will call the index
location and the difference between two indices distance. Notice that
terms like "location" and "distance" do not have any geographical
meanings. There is no a priori superiority or inferiority among workers' 
skills. The size of the difference between the indices of any two
workers represents how different they are. Obviously the difference
ranges from zero to one half. Every worker supplies one unit of labor
provided that the net wage offer is greater than or equal to his reservation wage.

We assume that all the workers in the economy have the same reservation wage $w_0$. The reservation wage reflects either the utility of leisure or the domestic productivity of a worker. In the following discussion, $w_0$ is interpreted as domestic productivity which a worker gets when he works for himself. We call this situation self-sufficient autonomy.

There are also firms in the economy. Since we do not allow multi-plant firms, we can identify firms without any confusion. Firms are assumed to produce homogeneous goods, which are sold in the competitive output market. The output price is normalized to one. Technologies are also indexed on the unit circle. The index of the technology represents the most productive skill characteristic. The firms can choose their technologies in the long run, but not in the short run (long run and short run will be defined later). Since the technologies only differ by their most productive skill characteristics, we can unambiguously identify the firm with its most productive skill characteristic. We shall call the characteristic the firm's location.

The mismatch between the location of jobs and workers (i.e. the difference between workers' skill and firm's most desirable skill) is one of the most essential parts of the model. If a worker works for a firm located at a distance $t$, then there will be a training cost of $2c(t)$ or loss of production incurred. We shall assume that the cost will be borne equally by the worker and the firm, and $c(t)$ is linear with the distance between the worker and the firm:
\[ c(t) = kt \] (2.1)

Notice that we have already normalized that \( c(0) = 0 \). The parameter \( k \) represents the degree of substitutability among workers. High \( k \) implies that the production requires more specific type of labor. The shared payment of the training cost ensures that both workers and firms have the incentive to have the better matching partners\(^8\).

To avoid the complication of substitution between productive factors, we shall assume that labor is the only productive factor. The firm has, what we call, roundabout technology, with the minimum efficient scale (\( M \)) and the constant marginal product (\( b \)). It is clear that the technology has an increasing returns to scale. More specifically, we assume the production function has the form of:

\[
Y = \begin{cases} 
0 & , \text{if } X < M \\
 b (X - M) & , \text{if } X \geq M,
\end{cases} (2.2.a)
\]

where \( Y \) is output, and \( X \) is the labor input normalized to the equivalent labor with the firm's most desirable skill characteristic. The normalization is assumed to have the following functional form:

\[
x = x(t) \left( 1 - \frac{c(t)}{b} \right) \] (2.2.c)

where \( x(t) \) is the amount of labor with the distance \( t \), and \( x \) is the normalized labor unit, and \( c(t) \) is the value of the firm's share of the training cost due to the difference. Total labor input \( (X) \) is just the
sum of the normalized labor \((x)\) of the workers.

We will call the situation short run when there is a fixed number of firms \((m)\). As we have indicated, firms do not change their location in the short run. Wage offer is the only short run decision variable of the firm. If there is a positive short run profit, then entry will occur. If short run profits are negative, firms will exit. Assuming that there are no costs of relocating firms, competition among firms will result in that all the firms get zero profit. The situation that the number of firms \((m)\), and thus, the distance between the neighboring firms \((2H)\) are determined endogenously by the zero profit condition will be called long run.

We are mainly interested in the monopolistically competitive\(^8\) long run symmetric\(^10\) Nash\(^11\) equilibrium. By monopolistic competition, we mean that every worker in the economy has at least two firms which can pay the net wage higher than the reservation wage (i.e., \(w - c(H) > w_o\)). By symmetry, we mean that all the firms offer the same wage and the distances between any two neighboring firms is the same. A firm will choose the location and wage offer. A worker will choose the firm he will work for by maximizing his net wage (wage offer minus his share of the training cost), provided that it is greater than or equal to the reservation wage \(w_o\). The firm makes its wage offer by assuming that other firms' wage offers will be held constant. In the game theory language, firms will play Stackelberg leader towards workers and play a Nash strategy vis-a-vis the other firms. Workers are Stackelberg followers to the firms.

2. Monopolistically competitive long run symmetric Nash equilibrium
There is a unique equilibrium in this model. To solve for the equilibrium, let us first consider a firm's profit maximization problem in the short run. Suppose there are m firms, and all the neighboring firms are equally spaced with distance 2H. Thus, we have:

\[ 2mH = 1 \]  

(2.3)

Since all the firms are symmetric, we can choose a representative firm. Its location can be normalized to zero without any loss of generality. Suppose that other firms wage offers are all fixed at \( \bar{w} \). The representative firm will want to choose its wage offer \( w \). Consider a worker located between the representative firm and any of its two neighboring firms. Since the situations are identical in both directions, we can consider only one side. Let us denote the distance between the worker and the representative firm with wage offer \( w \) as \( d \).

Then for the marginal worker who is indifferent between working for the representative firm and the other adjacent firm, the net wages for the two firms must be equal. That is to say:

\[ w - c(d) = \bar{w} - c(2H - d) \]  

(2.4)

where \( d \) is the location of the marginal worker. Given that the other firms' wage offers are all equal to \( \bar{w} \), the firm will choose the wage offer to maximize its profit, that is:

\[ \max \ b \left[ 2Nd - M \right] - 2N \left[ wd + \int_0^d c(t) \, dt \right] \]  

(2.5)
Since the number of workers of the representative firm is a function of its wage offer, eq.(2.4) can be regarded as a constraint to the firm's maximization problem, eq.(2.5). We can solve eq.(2.4) for \( d \), and substitute it in eq.(2.5). Then we get the first order condition of profit maximization by differentiating with respect to \( w \) and setting it equal to zero. Evaluating the first order condition at \( w = \bar{w} \) (and \( d = H \)) implied by the symmetry assumptions of the equilibrium, we get:

\[
b - kH = w + 2kH \tag{2.6}
\]

with \( c(t) \) given by eq.(2.1). Eq.(2.6) states that the marginal value product is equal to the marginal outlay at the margin. The left hand side is the marginal product of the marginal worker who produces \( b \) while incurring a training cost of \( kH \). Since the output price is one, it is the marginal value product. If the firm hires the marginal worker, then it pays the premium of \( 2kH \) over the prevailing wage, because it has to attract the worker away from the neighboring firm. The equilibrium wage offer is lower than the marginal productivity, because the firm has a local monopsony power, i.e., if it increases its wage offer then it can attract more workers from the neighboring firms.

Substituting eq.(2.6) into eq.(2.5), we get the profit for a firm (p) given the number of firms in the market:

\[
p = \frac{5kN}{4m^2} - bM \tag{2.7}
\]

It is immediately clear that if there are fewer firms in the market then the profit per firm will be higher. In the long run, however, the profit
will be driven to zero, with free entry, exit, and no adjustment cost of relocating firms.

Imposing the zero profit condition, we get:

\[ m = \frac{1}{2} \sqrt{\frac{5kN}{bM}} \quad (2.8.a) \]
\[ H = \left[ \frac{bM}{5kN} \right] \quad (2.8.b) \]
\[ w = b - 3 \left[ \frac{bkM}{5N} \right] \quad (2.8.c) \]

By examining eq.(2.8.a), the equilibrium number of firms (m) will be higher if the training cost (k) is high, if the minimum efficient scale (M) is low, if marginal productivity is low, and if the size of the market (N) is large. We call the economy more specialized if there are more firms around. Notice that the specialization in this model is the specialization of labor and production technologies rather than the variety of outputs. It would be useful to examine the extreme cases of eq.(2.8). If \( M = 0 \), i.e., production is divisible into infinitesimal units, then \( m = \infty \), and \( w = b \) suggesting that the economy degenerates to a small scale self-sufficiency type. On the other hand, if \( k = 0 \), i.e., workers are homogeneous, then economy becomes a natural monopoly situation.

It is worthwhile to note that net wages are not equal among workers. Workers who are located closer to their firms enjoy higher net wages, since they pay less for training. This follows from the assumption that workers cannot change their locations, and thus, there is no choice of specialization by workers. The most appropriate index for welfare is the average net wage payment, which will be denoted by \( W \):
Notice that $W$ can be interpreted as the average productivity per worker of the economy, since firms get zero profit.\,\(^{12}\) By differentiating eq.\,(2.9), we get the basic comparative static results:

\[
\begin{align*}
dW/db &> 0 & (2.10.a) \\
dW/dk &< 0 & (2.10.b) \\
dW/dM &< 0 & (2.10.c) \\
dW/dN &> 0 & (2.10.d)
\end{align*}
\]

Thus, the average productivity will be higher when the marginal productivity is high, training cost is low, minimum efficient scale is small and the size of the market is large. Our main interest rests on eq.\,(2.10.d) which shows that the average productivity increases with the size of the market. It is clear that the average productivity is bounded from above by the marginal productivity, which it approaches asymptotically as the size of the market goes to infinity.

From eq.\,(2.9), we can see that there exists a minimum market size in which monopolistically competitive labor market economy is viable (the market wage is greater than the reservation wage so that specialization can take place). Let us denote the minimum size of the market $N_o$. The condition that the net wage of all workers is greater than the reservation wage ($w-kH > w_o$) implies that $N > N_o$, where,

\[
N_o = \frac{5bkM}{16(b-w_o)^2} \quad (2.11)
\]
It follows that:

\[
\begin{align*}
\frac{dN_o}{db} &< 0 & \text{(2.12.a)} \\
\frac{dN_o}{dM} &> 0 & \text{(2.12.b)} \\
\frac{dN_o}{dk} &> 0 & \text{(2.12.c)} \\
\frac{dN_o}{dw_o} &> 0 & \text{(2.12.d)}
\end{align*}
\]

Thus, if the technology of the specialized production (eq.(2.2)) has a large minimum efficient scale and low marginal productivity, then the economy is only viable when there is a large market. Similarly, if the reservation wage is high, then the minimum scale of the market is high. Also, a large market is required if the economy has high \( k \) (jobs require specific skills and training is costly). The argument illustrates the stylized fact that industries which tend to concentrate are ones which have a high minimum efficient scale (such as steel and automobile industry) and require highly specialized workers (such as electronic and finance).

The higher productivity of the larger economy results from two factors. The first, what we call the internal scale economy is, that the average cost goes down with the size of the production level. The second, external scale economy, occurs because firms can hire more specialized labor in a larger economy. Notice that the number of workers in a firm as well as the number of firms increases with the size of the market. These two effects can be distinguished more precisely in this model. By differentiating eq.(2.9) with respect to \( N \) and rearranging we obtain:
\[ \frac{dW}{dN} = \left( \frac{bM}{N^2} \right) \left( 7m/10 \right) \quad (2.13) \]

The first term of the right hand side of eq.(2.13) refers to the productivity gain by increasing returns to scale. To see this, one may divide eq.(2.1.b) by \( X \) in order to get the average product of a worker; differentiate with respect to \( X \); and evaluate at \( X = N \). The second term refers to the productivity gain by external scale economy. Although the productivity gain by external scale economy increase with the number of firms (and thus, with the size of the market), the more rapid decline of the productivity gain due to the internal scale economy will drive the total effect to zero as the market size goes to infinity. The basic reason of the external scale economy in this model is that firms will be able to recruit more specialized workers to reduce the training cost. In other words, there will be better matches between jobs and workers in larger markets.

From the worker's point of view, the gain of the net wage has two components. The first is that the worker gets the higher wage offer in the larger market, because productivity is higher. The second is that the average cost of a job mismatch is reduced in the large market. On average, workers get better jobs which require lower training costs. Given the functional specification of the model the former is six times greater than the latter.

III. Congestion externality and optimal city size

The purpose of this section is to introduce negative externalities into the model developed in the previous section in order
to show the existence of the optimal city size. The basic result of the model of the previous section is that average productivity increases with the size of the market given increasing returns to scale technologies, heterogeneous labor and technologies. It implies that the city will grow indefinitely. However, such extreme agglomeration is rarely observed in reality. We observe that cities stop growing beyond certain points. A discussion of possible explanations of stabilization follows. We will discuss the points not pursued in this chapter first.

First of all, the internal scale economies may vanish. With the expansion of the organization, the cost of control and management may increase so that the production function becomes concave. Although the model does not relate increased labor specialization to lower production efficiency in our model, internal scale diseconomies may eventually lower the average productivity of workers. In the model of the previous section, the technology will approach constant returns to scale as the firm size becomes large. Although the external scale economy increases with the size of the market (thus unbounded), the average productivity is bounded by the marginal productivity. Put differently, the average productivity is bounded by the technology even with the unbounded external scale economy.

A more compelling reason for the stabilization, however, comes from the physical consequence of the concentration of economic activities. The price of fixed resources, particularly land, will rise with the increased size of the market. It is well documented that urban rent increases with the size and density of the urban area. The increase of rent will cause three effects on the welfare of the residents. The first effect is that with higher price of land, consumers will substitute
away from land to other goods to equate the marginal rate of substitution to the price ratio. With a given income, this causes decline in the welfare of the residents. Second, since land is also a productive factor, cost of production will rise ceteris paribus. Thus prices of consumer goods will go up, further lowering the utility of the residents. The third effect is that the higher rents will be distributed to land owners in the economy, and thus, increase the welfare of the owners. This inflationary consequence of urbanization through the pecuniary externality of rising rent will be deliberately avoided in the following discussion in order to maintain a single productive factor model.

More specifically, the two stabilizing forces described above are natural market outcomes. Since these effects do not change the efficiency characteristics of the economy, we will concentrate on genuine externalities prevalent in the urban economy such as congestion and pollution. As the density of economic activity goes up, the level of interference among the agents over the limited urban resources will grow up. A typical example is traffic congestion. With the inability of the market to internalize those externalities, negative externalities may be associated with the size of the market. We shall incorporate the negative externality in the model.

In particular, we will assume that the negative externality works through the firms' production function by lowering the marginal productivity of workers ($b$) as the market size increases ($N$). In other words, $b$ is to assumed to be a function of $N$ rather than a constant. We will call $b(N)$ a congestion function. The agents regard $b$ to be constant since individual agents cannot significantly influence the size of the market. Specifically, we will assume that the congestion function
satisfies the following conditions:

\[ b'(N) < 0 \quad (3.1.a) \]

\[ \lim_{N \to \infty} b(N) = b' \quad (3.1.b) \]

There exists \( C > N_0 \) such that

\[ b(N) = b_0 \text{ for } N \leq C. \quad (3.1.c) \]

Eq. (3.1.a) says that marginal productivity of workers decreases with the size of the market because of the higher level of congestion. The next condition states that the congestion externality eventually drives the marginal productivity of workers to \( b' \). The last condition assumes that there exists a non-trivial assimilative capacity of the city economy \( C \), under which there will be no negative externalities.

With the conditions of eq. (3.1), we can rewrite eq. (2.9) as follows:

\[ W(b(N), N; k, M) = b(N) - \frac{7}{2} \sqrt{b(N) k M / 5 N}. \quad (2.9') \]

Differentiating eq. (2.9'), we obtain:

\[ \frac{dW}{dN} = b' - \frac{1}{2} \left( 1 - \frac{W}{b} \right) \left( \frac{b'}{b} - \frac{b}{N} \right). \quad (3.2) \]

It is clear that:

\[ \lim_{N \to \infty} \frac{dW}{dN} = b' < 0 \quad (3.3.a) \]

\[ \frac{dW}{dN} \bigg|_{N = C} = \frac{1}{2} \left( 1 - \frac{W}{b} \right) \left( \frac{b}{N} \right) / 2 > 0 \quad (3.3.b) \]

\[ \lim_{N \to \infty} W = b' \quad (3.3.c) \]
Applying the mean-value theorem it is easily seen that there exists at least one local maximum at which \( W'(N) = 0 \). Thus it follows that there exists a global maximum. This establishes the existence of the optimal city size, defined as \( N^* \), which maximizes the average productivity (eq.(2.9')). Necessary conditions for the local optimal city size are:

\[
W_b b' + W_N = 0 \quad (3.4.a)
\]

\[
W_b b'' + W_{bb} b'^2 + W_{DN} b' + W_{NN} < 0, \quad (3.4.b)
\]

evaluated at \( N = N^* \). Subscripts represent partial derivatives. Of course a sufficient condition for the unique global optimum is that \( W(N) \) is globally concave, i.e. eq.(3.4.b) is satisfied for \( N > 0 \).

To get the comparative static results, we differentiate eq.(3.4.a) with respect to \( k \) and \( M \), and obtain:

\[
\frac{dN^*}{dk} = - \frac{[W_b k b' + W_N k]}{[W_b b'' + W_{bb} b'^2 + W_{DN} b' + W_{NN}]} > 0 \quad (3.5.a)
\]

\[
\frac{dN^*}{dM} = - \frac{[W_D M b' + W_{NM}]}{[W_b b'' + W_{bb} b'^2 + W_{DN} b' + W_{NN}]} > 0, \quad (3.5.b)
\]

because, \( W_b k < 0, W_D M < 0, W_N k > 0, \) and \( W_{NM} > 0 \). Thus we have normative rules for the optimal city size. The optimal city size will be large when there are a large minimum efficient scale and high job specific requirements. If we interpret the market as an industry, then industries which require large initial investments and highly specialized labor inputs will want to locate in a large city.

IV. Growth of cities, migration, and efficiency of system of cities
We have examined the model in the context of a closed economy. An important question is the behavior of the economy in a system of cities. In this section, we shall allow migration among regions. By region, we mean an integrated urban labor market or homogeneous rural area. We assume that the migration rate is linear with the net wages differentials between the two regions:

\[ \dot{N} = e (W - W_0), \quad (4.1) \]

where the "dot" represents a time derivative. Notice that we implicitly assume risk neutrality. Variations of eq.(4.1) have been estimated by many authors (see Todaro[1976] and Yap[1977] for survey). Most of the studies conclude that economic motivation is the major determinant of migration. In order to focus the discussion on migration, we shall assume that there is no natural growth of population. We will divide the discussion in two subsection. First, we examine the growth of the economy in an open region. The key assumption here is that there is an infinite supply of migrants at a fixed utility level. This assumption reflects the notion of "unlimited supply of labor" in the classical development theory. Thus, this subsection is more relevant for the rural-to-urban migration. In the second sub-section, we examine the closed region case, in which regional wages adjust the migration process. It is more relevant for the urban-to-urban migration.

1. Growth of cities in an open region

First, let us assume that migration is costless. Open region assumption implies that \( W_0 \) is fixed. We will call the background region
"rural" and our economy "urban". There will be migration into or
migration out of the region depends on whether $W$ is greater or smaller
than $W_o$. Let us denote the optimal wage as $W^*$ (wage level evaluated at
$N'$). If $W(N_o) < W_o$, then there will be emigration from the city to
reduce the size of the market to zero. In this case, city has no raison
d'être, and is of limited interest.

Without the natural growth of population of cities, $W(N)$ can be
regarded as the "migration demand function", because it relates the size
of the city to the wage level. Assumptions of "unlimited supply of
labor" and costless migration implies that the supply curve is
horizontal.

Depending on the shape of $W(N)$ and the size of $W_o$, we can
identify three major cases (Figure 2). First, the city may grow
indefinitely if $W(N) > W_o$, for $N > N_o$ (case A in Figure 2). Notice that
this situation is not likely to occur, because the horizontal supply
curve assumption will not be warranted for very large $N$. Second, as in
the case C, the two curves may intersect just once. Notice that
equilibrium city size is stable and always larger than the optimal city
size ($W^*$). Third, if the curves intersect more than once (case B), then
there exist at least one stable equilibrium city size which is greater
than local optimal. This is not a surprising result. It is a standard
conclusion that the competitive equilibrium is larger than the optimum
with negative externalities.

It is also interesting to examine the cause of the instability of
the other type of equilibrium at which the migration demand function
($W(N)$) cuts the supply function from below. Suppose that the city
manifests an increasing returns to scale at a equilibrium. If, for some
reason, the size of the urban market increases, then the urban wage will rise through a more extensive specialization. The higher urban wage will, in turn, attract more worker from rural area. This cumulative process of urban growth will continue even if congestion externalities occur. The growth will stop when the negative externalities are strong enough to offset the agglomeration economies such that the urban wage is equal to the rural wage.

If migration is costly (moving cost, any additional human capital investment for the new labor market, psychological cost, and so on), then the model behaves somewhat differently. First of all, the supply curve will not be horizontal. It will have a U-shape, whose minimum is at the current $N$. The increasing part of the curve reflects that more workers are willing to migrate into the urban area when the urban wage is higher. On the contrary, the decreasing part says that more workers in the urban area will migrate out of the urban area when they are compensated more for their out-migration. For the sake of simplicity, we assume that urban-to-rural migration is costless so that the migration supply curve is upward sloping. Also we assume that $W(N)$ is globally concave (Figure 3).

Depending on the shape of the two curves, there are three main possibilities. In the first case, migration may not occur at all, if $W(N)$ is smaller than the sum of $W_o$ and migration cost (case A in Figure 3). Case A is more likely to occur when the migration cost is high and/or $N_o$ is large (i.e., $b$ is small, $M$ is large, $k$ is large, and $w_o$ is large). Second, there may be a stable equilibrium in which the equilibrium city size is smaller than the optimal size (case B). Case B is more likely to happen if the migration cost is moderate and/or $N^*$ is
large (κ is large, M is large, b is large, and w₀ is small). In the third case (case C), there will be a stable equilibrium in which the equilibrium city size is greater than the optimal city size. This case will happen if the migration cost is low and N' is small (i.e., κ is small and M is small).

2. Growth of cities in a closed region

We now turn to the question of the growth of cities in a closed region, in which wages can adjust to equilibrate the migration flow. To simplify the argument, we consider the two city case only. We assume that the total number of workers is fixed at N. Maintaining the assumption of costless migration of workers, we can describe the two-city system with the following set of equations:

\[ W_1 = W_1(b_1(N_1), N_1; M_1, k_1) \]  \hspace{1cm} (4.2.a)
\[ W_2 = W_2(b_2(N_2), N_2; M_2, k_2) \]  \hspace{1cm} (4.2.b)
\[ N_1 = -N_2 = e(W_1-W_2) \]  \hspace{1cm} (4.2.c)
\[ N_1 + N_2 = N \]  \hspace{1cm} (4.2.d).

Then, one can immediately notice that there are possibilities of multiple equilibria such that:

\[ W_1(N_1) = W_2(N_2) \]  \hspace{1cm} (4.3.a)
\[ N_1 + N_2 = N. \]  \hspace{1cm} (4.3.b).

The local stability condition for the distribution of workers is:
\[ W_1'(N_1) + W_2'(N_2) < 0. \] (4.4)

Let us name \( N \) such that \( N_1 < N_2 \). Suppose that two cities are symmetric, i.e., \( W_1(.) = W_2(.) \), and \( W(.) \) is globally concave, then it is easy to see that:

\[ N_1 < N' < N_2. \] (4.5)

The significant result is that the bootstrapping of inefficient allocation of workers between the two regions can be easily maintained if the stability condition is satisfied. In effect the stability condition states that the economies of scale of the smaller city is smaller than the diseconomies of scale of the larger city. This is exactly the situation that the migration from the larger city to the smaller city is socially desirable.

This point can be illustrated by using Figure 4. Provided that \( N_1 + N_2 = N \), the situation \((W, N_1, N_2)\) is a steady state equilibrium, because \( W(N_1) = W(N_2) \). Suppose that the local stability condition (eq.(4.4)) is satisfied at the equilibrium. Moving workers from the larger city to the smaller city is a Pareto improvement, because the net wages of both cities will be higher than before. But this cannot be sustained because workers will migrate back from the small city to the large city in order to get the higher wage. Thus, it is difficult to get out of the inefficient steady state equilibrium. One can apply this dynamic analysis to the case of rapidly urbanizing countries. If the economy follows the pattern of unlimited supply of workers in the earlier stage of the urbanization process, our analysis of the closed region will
apply. In other words, there are always danger that large cities will be too large. Even when the wages of smaller regions increase to catch up with the larger city, the efficient decentralization can be easily blocked so that large city will stay larger than optimal while smaller cities will remain smaller than optimal.

V. Conclusion

With increasing returns to scale technology and heterogeneity of workers and jobs, we have constructed a model of an economy which displays external scale economies. Although the external scale economy intensifies with the size of the market, the productivity of the economy is bounded by the technology since the internal scale economy becomes insignificant with the rise of production level.

Given that there exists an assimilative capacity beyond which congestion externalities eventually drive down the marginal productivity of workers to the lower bound $b^*$, there exist an optimal city size which maximizes the average productivity of the workers. If there is an unlimited supply of workers at a given wage level outside of the economy, the stable equilibrium is always greater than the local optimal. If the wage function is globally concave with respect to the size of the market (i.e., the optimal city size is unique), then the stable equilibrium is unique, and it is always larger than the optimal city size.

Even when migration occurs to equilibrate the wages of the regions, systems of cities can be easily bootstrapped in an inefficient allocation of workers provided that the stability condition is satisfied. This may exist despite wage adjustments in the cities. Policy
interventions to control overconcentration of large cities and planned centralization such as growth pole type strategy are sometimes called for.
Figure 1. The basic model
Figure 2. Stability of equilibria city sizes in open region with costless migration
Figure 3. Stability of equilibria city sizes in open region with costly migration
Figure 4. Stable inefficient labor allocation in a two-city system.
Footnotes

1. There have been some micro justifications for the increasing returns to scale besides the traditional fixed cost argument. First, learning model of Arrow[1962] demonstrated that the productivity increases as the production level increases. Second, random delivery of services of factors (commonly known as the repairman problem) can be a source of increasing returns to scale (Arrow et. al.[1972], Rothschild and Werden[1979]) by reducing the average probability of the idling of productive factors.

2. The convexity assumption of production sets in the general equilibrium model of Arrow-Debreu-Hahn precludes the possibility of increasing returns to scale. Many of the fundamental results (such as the existence of the competitive equilibrium) cannot be obtained without the assumption.

3. This is a classic problem of financing and pricing of public goods such as roads and bridges.

4. Koopmans and Beckmann[1957] have demonstrated that the optimal assignment problem with transportation cost cannot be sustained by the decentralized system with rents on the locations. The basic cause of the market breakdown is the locational externality caused by others through the transportation cost change.

5. Jacobs[1969] provides a historical analysis of the productivity increase through the labor specialization in urban economies.

6. It is possible to interpret the model with spatial implication. Workers have to commute to their workplace and transportation costs are borne equally by workers and firms. However, the geometry of the city (circle) is not very realistic.

7. We can view this assumption in two ways. First, firms recognize a possible worker's skill individually and bargain for the wage. The bargaining outcome assumed here is that both parties share the cost of training equally. The other way of viewing this is that the worker has to pay to achieve the signal acceptable to the firm. The firms cannot distinguish the workers ex ante, but has to pay for the on-job-training cost, which cannot be attributable to any workers.
8. If all the cost of mismatch were borne by firms, workers would have no incentive to choose the right firms, and vice versa.

9. Depending on the parameter values, there will be the case of monopsony equilibrium where some of the workers would not participate in the labor market. If the size of the market is big enough, however, we will only have the monopolistically competitive equilibrium, which resembles the modern economy most. The other cases are analyzed in chapter II.

10. Of course there will be possibilities of asymmetric equilibria. Given the a priori symmetry of the assumptions of the model, we have no interest in them.

11. There has been a criticism for the use of Nash equilibrium concept in this type of circle model (Hart[1983]). The argument is that the Nash assumption is not very convincing, since each agent interacts only with two neighboring firms directly. It is possible to get around this criticism by modifying the model into two dimensions such that there are many neighboring firms for each firm. See Footnote 3 in chapter II for detail.

12. It is true that the local monopsony power of firms will be reduced in a larger market. However, since the zero profit condition is satisfied with competition, we can safely interpret the wage increase as the productivity increase.
The city is a complicated place. Various activities are performed by a variety of groups of people in the city. So called "the function of a city" is a characterization of such human activities. Depending on the interest of an investigator, the function of a city has been defined as a market, a political arena, an innovation center, a place where major decisions are made, or a built environment. Given the complex political, socio-economic and physical structure of modern cities, it would be inevitable that any disciplinary approaches do not capture the full spectrum of important and interesting academic and practical issues. Since the argument provided in this dissertation takes a very narrow viewpoint, it would be useful to examine the argument from a broader perspective. This concluding chapter is an attempt to synthesize the argument of the thesis with other important related topics.

In this dissertation, we have regarded a city as a local labor market, where workers and firms look for each other. Workers choose firms in order to maximize net wages, whereas firms hire workers to maximize profits. The intrinsic heterogeneity and diversity of workers and firms are the main ingredient of the argument. The value of one worker to a firm (or vice versa) is different from that of another, since all workers and firms are different. One would argue that an agent is willing to pay more for the better matching partner. The city is a place
in which such differences are evaluated and mediated.

The growth of a city is also a complicated phenomenon. First of all, it is not at all clear what the term "growth" means. Vaguely speaking, it is a process of enlarging the "size" of a city. The size of a city may take a variety of measures such as population, employment, output, income, capital stock, or geographical area. Nonetheless, those measures are highly correlated with one another in a given country. We have used the number of workers to measure the size of the market. Since we are mainly interested in the local labor market aspect of the city, the choice is well-suited.

The major concern of the dissertation has been the relationship between the size of the market and the productivity of urban economy. The term agglomeration economies loosely refers the stylized fact that average productivity rises with the size of the market. We argued that traditional explanations for agglomeration economies are neither rigorous nor convincing, and suggested an alternative theory for urban agglomeration.

We concentrated on labor specialization in order to explain agglomeration economies. The basic argument is as follows. Given increasing returns to scale production technologies, firms can afford to adopt more specialized technologies in a larger and more diverse labor market, since they will be able to find enough workers in order to maintain their plant at an efficient level. Thus, given the distribution of human capital characteristics, the average matching quality between workers and jobs in larger market would be higher than in smaller market. On the hand, workers can afford to specialize more, as there are more diverse and specialized firms in the market. The probability of finding
a better matching job will be higher when there are more firms in the market. Since they put their human capital in more specialized skills, their productivity will increase with given level of human capital investment cost. In short, agglomeration economies occur by two factors. First, given that the characteristics of human capital are heterogeneous, the average matching quality between the jobs and workers increases with the size of the market. Second, as the size of the market increases, workers invest their human capital in a more specialized manner so that their productivity increases with given level of human capital cost.

In order to avoid the complication of heterogeneity of outputs, we have assumed that firms produce homogeneous output. With homogeneous output, we did not have to describe the demand pattern of consumers so that we could concentrate on the labor market. However, casual observations suggest that the diversity of products is closely related to the diversity of workers. Therefore, it would be interesting to extend the model to allow firms produce different products (or different varieties) in order to examine the interaction between the labor market and goods market.

One may hardly separate between the productivity increase due to agglomeration economies and technological innovations. It is a well known fact that cities, particularly large cities, are responsible for most of the important technological innovations. One would guess that the success of innovations are somehow associated with labor specialization and the capacity to provide specialized goods and services available in large cities. The aspect of this endogenous process of innovation and agglomeration has not been addressed in this dissertation at all.
For one thing, the process of innovation is not understood very well. Questions like: How does a new innovation occur? How does it develop? What are the determinants of successful innovations? How is it transferred to others? are not answered adequately. Although, there is substantial literature about information diffusion process in geography, most of the authors are concerned about when and how the new information arrives rather than how the new innovations occur and are adopted. More work regarding the relationships between labor specialization, agglomeration and innovations is suggested.

The increase of productivity and of wage has been regarded as the driving force of the urban growth process. As the size of the city grows, firms can exploit the increasing returns to scale technology. Thus, average cost will decline. Such productivity increase will be further reinforced by agglomeration economies and labor specialization. Large cities will have higher productivity. Higher productivity will be transmitted to higher wages. Higher wage in larger city will attract more workers from smaller cities and rural areas. In short, the growth of the city is unstable. Even if two cities in the same regions are slightly different in early settlement period, the small difference in the beginning generates big difference in later stage.

The city is an artifact through which an individual communicates with other people effectively and efficiently. It seems clear that there are increasing returns to scale in information processing and dissemination. Major communication channels, public or private, go through large cities. Newspapers, TV stations, telephone switching, satellite stations, and other communication facilities are primarily located in cities. The same argument applies to transportation. Major
interregional transportation facilities such as airports, railroad stations, seaports, distribution centers are in large cities. This reflects the view of central place theory. Namely, a city is a service center for its hinterland, and every activity has a non-trivial minimum efficient scale below which the activity cannot be sustained efficiently.

It has been a powerful conceptual framework to explain systems of cities and city hierarchies. The central place theory gives a theoretical underpinning for the growth pole policy. The basic idea is that the benefits of the center will be proliferated to the hinterland. Thus, in order to promote the economic prosperity in a region, you have to promote the economic prosperity of the core region. The target core region (usually medium-sized cities) are called growth poles. Most of the practical efforts of such policies have not been very successful. Many authors pointed out that the trickling down effect is much smaller than expected. Cities turned out to be quite open. Namely, the interactions between the cities in different regions are quite strong relative to those between the cities and their hinterland.

Moreover the central place theory does not explain regional specialization. All cities at the same hierarchy level should have identical function, since the function of a city is determined by the size of its hinterland. Empirically, it is far from true. We observe that cities of similar sizes may have complete different industrial structures and functions.

The Heckscher-Ohlin (H-O) framework, the work horse model of the modern trade theory, provides an explanation of regional specialization. The fundamental result of H-O theory along with Rybczynski theorem and Stolper-Samuelson Theorem is that the region will specialize in the
industry which the it has a comparative advantage. For example, if the region is abundant in labor, it will specialize in labor intensive industry and so on.

The H-O theory has been criticized by some of the empirical findings of international trade. They are: 1) Leontief paradox (the U.S. which has the highest capital labor ratio exports labor intensive goods and imports capital intensive goods); and 2) the bulk of the international trade is intra-industry trade rather the inter-industry trade. There have been attempts to include the human capital element in order to explain the Leontief paradox. New theoretical efforts are more concerned about increasing returns to scale, and product differentiation. These elements are incorporated into our model. For example, one wonders why Switzerland specializes in watches. A layman will answer that it is because the country has a highly productive watch industry. The productivity is high, because there are so many good watch mechanics, precision metal cutters and other workers who are suitable for such industry. Then the question is why there are so many of them in Switzerland. If we accept the assumption that workers make their human capital decision based on future earning capabilities, the workers in Switzerland choose to be watch mechanics, because they command higher wage. Thus, there is an element of increasing returns to scale and instability in the argument. If a economy is specialized enough in one industry such that its productivity is high enough, then the workers invest their human capital in that industry so that it becomes more productive. Rather that the neoclassical marginalist's argument that marginal productivity will decrease with the level of production, this argument supports the argument that marginal productivity increase with
the level of production.

The major function of a city changes over time. Historically, for example, American cities before early 19th century were centers for trade. The typical U.S. city was a place for commerce, international trade, and trading crafts and agricultural products. After the industrial revolution, however, it became a place for production of various manufacturing goods. It produced textile, garment, machinery, and other types of light and heavy manufacturing goods. Since 1960's, it became a center for service industries. Employment in baking, insurance, information services and other office activities now exceed the manufacturing employment in most of the major American cities.

As the demand pattern and technologies change over time, the local industrial structure and the labor pool must adjust. We have been observing that many industrial cities in Northeast and Midwest of the U.S. have economic troubles. Unemployment rates rise and household income falls. Plant closing sometimes severely affects the local economy. Many state and local governments have tried to promote economic development by various fiscal incentives. The fiscal approach in general has not been very effective, since the firm's location decision is affected very little by the local fiscal consideration. The characteristic of local labor pool seems much more important determinant of such location decisions. From the viewpoint of the model in this dissertation, it would be worthwhile to direct incentive packages to more specific industries which have better matching with the local labor market. With a little adjustment cost of the existing labor pool, such approach can generate new vitality in the local economy.

The model presented in the dissertation would serve a framework
to study the problems of rapid urbanization in developing countries. The urbanization trend in developing countries have been quite impressive for the past three decades. Besides the debate whether urbanization is mainly by migration or natural growth, the rapid urbanization put a great deal of pressure on the direction of regional investment policies. With the population increase of more than 4 per cent a year, the largest cities in developing countries lack a great deal of social infrastructure. Housing is in great shortage. Streets are congested. Air and water quality have fallen drastically. With heavy concentration of people in a small number of prime cities, the national economy has to face the trade-off between the investment in social infrastructure in those cities and other investment in smaller cities. Since large cities are likely to stay large, a careful national urbanization policies is called for.

It may appear that we are trying to denounce the importance of transportation cost and any other frictional costs to overcome distance. This is not the case. In fact, we have adopted a extreme assumption that intra-urban transportation cost is zero while inter-urban transportation cost is infinite. If we adopt a more realistic assumption, say any transportation is neither costless nor prohibitively expensive, we might expect a global stability in a national system of cities.

The point is, however, that there are no local stabilities in systems of cities. The system may be (and we think it is) globally stable, but locally unstable. For example, we do not expect New York City will attract all the population of the U.S. (global stability). There will be substantial number of people in Chicago, Los Angeles, and other metropolitan areas. However, we expect one city or a few cities to
dominate a region (local instability). Moreover, the dominant city is a product of insignificant differences or accidents in the beginning rather than of the intrinsic difference among cities. It is not all clear why Chicago rather than Gary became the regional center. Also it is not clear why Manhattan rather than Staten Island became the Central Business District (CBD) of New York city region. They did not have any differences in their intrinsic endowments related to their locale.

Our philosophical underpinning is that once you get some minimum momentum, the process continues up to a point that it cannot be sustained globally. This viewpoint contrasts with the general equilibrium framework of Arrow-Debreu, where the global convexity is assumed. The fundamental welfare theorems of Arrow-Debreu model do not apply to the case of local instability-global stability framework. There is no fine adjustment of marginal cost (or marginal product, etc.) in a locally unstable regime. Convexity is violated. The globally stable equilibrium is a historical product. One cannot predict the equilibrium ex ante. Moreover, the equilibrium will in general not be a Pareto optimal. For example, the "qwerty" typewriter keyboard which is the most widely used English keyboard is proven to be not efficient. However, the change to the most efficient keyboard can be easily blocked. No individuals, or groups, unless they are powerful enough, can induce such change. The same kind of argument applies to the location of CBD in most of the old cities. Is Manhattan the socially optimal location for CBD? Many people may argue against it. It is an island, and one has to cross a bridge to get there and so on. But, the point is that the socially inferior equilibrium cannot be broken without significant collective action. Moreover, it is easier to set the direction in earlier stage. Since
large cities are likely to stay large, this welfare implication is very important for rapid urbanization phenomena in many developing countries. The welfare cost of very large cities in those countries may be significant because of misallocation of resources among regions.
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