Econometric Analysis of the Historical Growth and Volatility Trends of Various Metals

by

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Submitted to the Department of Materials Science and Engineering
in Partial Fulfillment of the Requirements for the Degree of

Bachelor of Science

at the

Massachusetts Institute of Technology

June 2012

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ABSTRACT

Post Malthusian economics, there is growing recognition of the impact technological change and advance has on market activity. By studying historical production and price trends, boundaries of feasible growth can be determined and, dependent on a firm's goals, materials that may require added recyclability, substitution, or engineering efficiency identified. Therefore, a contextual understanding of growth and volatility can mitigate negative economic impacts.

This study involved the econometric analysis of a 16-metal survey. Various analysis techniques were used to determine historical growth and volatility trends of both industrial and precious metals. For this data set, a typical sustained annualized growth rate of production was between 0.0% and 5.0% based on 20-year CAGR data. Price growth tended to range between -2.5% and 2.5% for 20-year time frames but was much more volatile in the short-term. From 1979 to 2009, 56% of all annual value growth rates were greater than +10.0% or less than -10.0%. Additionally, several metals had coefficients of variation greater than unity thereby being classified as hyper-variant. While the premise of a commodities exchange is to heighten the predictability of value, little difference of price volatility was found between metals on (0.28) and off (0.31) open exchanges. Aside from the survey, case studies of tantalum and niobium were completed. These co-mined materials appeared to have a strong correlation with their price growth as well as their production trends.

Thesis Supervisor: Joel Phillip Clark
Title: Professor of Materials Systems and Engineering Systems

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Acknowledgments

It has been my pleasure to submit this work as a culmination of my years as a student in the Department of Materials Science and Engineering and I recognize the many individuals that helped me reach this apex.

Firstly, I'd like to acknowledge Dr. Elisa Alonso for both her accessibility and guidance through this thesis process. Her patience, experience, and clarity were invaluable during the analysis and writing stages. With that regard, I would also like to thank Professor Randolph Kirchain for his assistance both during the thesis writing stage as well as during 3.080 – Economic and Environmental Materials Selection.

Throughout my MIT experience, several peers have provided the encouragement and camaraderie that is necessary to be successful at an institution like MIT. In Course 3, Chinedum Umachi, Justin Breucop, and Yasmine Doleyres helped me get through many all-nighters and I believe we all grew an intellectual endurance from those experiences. I would also like to recognize Arathi Ramachandran – my overseas study partner – as we both attended the Cambridge-MIT Exchange program. Having someone in my major that I was academically compatible with was invaluable during such a life-changing year. Outside of my major, Amanda Valentin and Naomi Lynch have been great friends and supporters of my professional and personal growth these last four years. Since our experience in engineering design, to MIT acceptance, and entering the real world, I have been happy to recognize these women as close friends.

Finally, I would like to recognize my family. From an early age, they have ensured that I have had access to programs and resources that would allow me to learn. Their guidance ultimately drove me to an acceptance at MIT and an opportunity to realize my goals. This thesis would not be possible without their unconditional love and support and for that I thank them.

I. Introduction

A. Motivation and Background

In a world of unlimited wants and limited resources, almost all materials can be defined as scarce. Two centuries ago, Malthus formalized the idea that population growth would ultimately outrun resource availability (Malthus 1798). Since then, studies have proposed other institutional influences such as industrialization, technological improvements, and sociopolitical factors as elements that might either mitigate or exacerbate scarcity.

Barnett and Morse are usually noted as the first pair to use historical data to make a comment on scarcity. Using the 1962 work *Trends in Natural Resource Commodities* by Potter and Christy, Barnett and Morse published their 1963 book, *Scarcity and Growth (Barnett and Morse 1963)*. In this piece, they presented data to support the argument that, in fact, scarcity was becoming *less* of a problem. One of the major premises upon which they based this knowledge was the declining end-user cost for various resources.

Population growth was the major feature in classical views of scarcity but as the 20th century progressed, the United States began to develop a reputation for its over consumption and rightfully so. From 1950-1970, floor space per capita increased by 100% (~250 to 500square feet), vehicle gas consumption rose by approximately 15% (~720 to 830 gallons per vehicle), and in San Francisco, per capita water use grew by 45% (~110 to 180 gallons per capita daily) (Diamond and Moezzi 2004). Along with each of these categories of overuse were unfortunate economic as well as environmental side effects.

Growing knowledge of the environmental implications of industrialization, skyrocketing oil prices due to global political issues, and the energy crisis all became an impetus for a revisit of Barnett and Morse's findings.

More researchers departed from the 1963 Barnett and Morse study and recognized that more effort needed to be applied to reconcile the impact of availability crises. There were many governmental reactions to materials shortages. In 1951, the U.S. President's Materials Policy Commission was established and produced a study that consisted of five volumes: Foundations for Growth and Security, The Outlook for Key Commodities, The Outlook for Energy Resources, The Promise of Technology, and Selected Reports to the Commission. This culmination was entitled *Resources for Freedom*. In the 1970s, another presidential commission was established to revisit studies of availability and according to Slade, by 1982 – the year of her paper *Trends in Natural-Resource Commodity Prices* - there was still no consensus amongst researchers about the general direction of scarcity. Were natural resources becoming more or less scarce? Upon correlating a relative price index metric with time, Slade derived U-shaped curves (Slade 1982). Though she even admits that her methods were "simple and naïve", her study represents a scientific era in which commodity valuation was influenced by factors far beyond availability and many researchers were racing to develop the most fitting set of parameters.

A major undertaking many companies are confronted with is the development of indicators to define a particular input material as "scarce". That definition extends much further than a comparison of use versus production rate (or conversely supply and demand). Issues such as the localization of production and extraction, variation of end-use industries, recyclability, and volatility of price are essential (Alonso, Field et al. 2007). Alonso defines two larger categories of scarcity risk factors: institutional inefficiencies and physical constraints. While little can be done to impact the amount of a resource that is produced naturally, understanding and preparing for institutional inefficiencies is very valuable for a firm.

A first step involves prioritizing to what degree a material is considered critical. Criticality is relative depending on the country of origin and industry of interest for the firms. In a U.S. DOE publication, criticality of rare earth metals was based on a matrix with two dimensions: importance to clean energy and supply risk. In this particular case, importance to clean energy was based on demand in the clean energy sector and substitutability. Supply risk involved basic availability, competing technology demand, co-dependence on other markets, producer diversity, and socio-political factors (Bauer, Diamond et al. 2010). If a study was considering different classes of metals, the axes of this so-called "criticality matrix" could differ dramatically. In a similar report produced by the European Union in 2010, the environmental country risk and recyclability became very heavily weighted metrics for their definition of criticality (European Commision 2010).

B. Problem Statement

There is no question that defining and understanding whether materials are critical is extremely important for both firms and governments. There have been several instances in history where increasing scarcity had a strong impact on price and other economic factors. Globally, people are preparing for a movement from a fossil-fuels based economy to a renewables-based energy system (Kleijn and van der Voet 2010). Emerging technologies in particular can cause dramatic shifts in demand as their commercial use grows. Gruber writes about the demand for vehicle electrification and questions whether the existing availability of lithium can sustain this expected future demand (Gruber, Medina et al. 2011). The key takeaway from these aforementioned studies is that metals-dependent industries shall be changing drastically in the near future. Though it is extremely difficult to predict consumer or firm behavior, analysis of historical price and production trends can provide context about the feasibility of certain growth and variation trends.

The goal of this study is to understand the growth and volatility of a survey of sixteen metals and compare trends after categorization. Additionally, a case study of tantalum and niobium are carried out to comment on the quantitative effects of various historical events as well as the relationship between co-mined materials.

II. Methodology

A. Data Selection

The United States Geological Survey (USGS) provides information related to natural resources to the public. In particular, they have a large database of historical information about the worldwide supply, demand, use, and flow of various minerals and materials.

The USGS relies on national and international surveys, literature, site visits and personal contacts to obtain its information. Over the course of a year, at least 150 surveys are created to have an understanding of every stage of a minerals cycle; exploration, use, purchase, and recycling. Working with each state, they also have a cohort of mineral geologists and principal contacts who assist with the refining of results. The National Minerals Information Center (NMIC) synthesizes its source data to provide periodic publications for over 100 minerals in over 170 countries (USGS Information Sheet). They additionally produce studies advising the United States legislature about critical and strategic materials.

Drawing mainly from the USGS information, sixteen metals were selected for this study: aluminum, beryllium, chromium, copper, gold, lead, magnesium, molybdenum, niobium, pig iron, platinum group metals, rare earth metals, silver, tantalum, tin, and zinc.

For these metals, production statistics are standardized to metric tons and values are reported in 1998 dollars per metric ton. Production is measured as an estimate of mined material after refining. According to USGS, "Unit value is a measure of the price of a physical unit of apparent consumption (in this case a metric ton) in dollars" (Kelly and Matos 2011). To produce results in 1998 U.S. dollars, the CPI (consumer price index) conversion factor with base of 1998 is used.

Some of the price data USGS provides comes from Ryan's Notes reports. This company tracks, reports on and sets prices for pig iron, lead, zinc, tin, and tantalum used in this study. There are 27 materials in total that are reported in their weekly studies. Similar to the way in which USGS develops a consensus, Ryan's Notes interviews market makers – consumers, traders, and producers – and determines an appropriate price range. For each of the metals, there are specifications such as amount, quality, and method of packaging that the price is based on. This company has been reporting for over 40 years and a wide range of institutions uses its data.

i. Availability of data

A full set of data is defined as having annual prices from 1900-2009 and annual production tonnage from 1900 to 2011. Unfortunately, many of the surveyed 16 metals had some partially missing data (Table 20). USGS data sheets state the sources used and reasons for missing data. Each data sheet also defines exactly what each measurement corresponds to. These specifications may correspond to the quantity, quality, or packaging methods of the material.

Beryllium, copper, lead, magnesium, molybdenum, niobium, pig iron, rare earth metals, tantalum, and tin were all missing at least one point from the considerations that would have classified them as complete. Most of the missing data points are for values as opposed to production. A major factor for this is that not all of the metals surveyed were on open markets.

With the help of information technology as well as the establishment of more data collection bureaus, access to historical metal quotes has become more readily available since the mid 19th century.

ii. Survey and case study data

The sixteen surveyed metals were chosen to provide a diverse and well-rounded set. The first filter for selection was isolating critical metals in various industrial applications. Secondly, it was preferred that there was price and production data available for at least the last 30 years. Metals with nearly 100 years of data were appropriate and preferred for the general metals survey.

Tantalum and niobium were chosen for the case studies. Detailed measures of copper's trends were added to the appendix (Figure 18-Figure 21) for comparison. Tantalum's production and value have had dramatic rises and falls in the last 50 years. This unusual behavior led to its selection for further research into its market dynamics. In addition, niobium, a metal that is co-mined with tantalum, was selected to explore the relationship between co-mined minerals. Copper is a metal whose historical use goes back as far as 10,000 years ago (Stanczak 2005). In more recent times, its production growth has been fairly constant year-over-year which makes it an interesting material to compare with the other two metals. Also, copper is sold on a number of global markets including the London Metal Exchange (LME), the Commodity Exchange (COMEX), and the Shanghai Futures Exchange (SHFE) while tantalum and niobium are not sold in any open markets.

B. Data Manipulation Techniques

Throughout this paper, growth and volatility calculations are used to evaluate the historical trends of the production and unit value for the sixteen surveyed metals.

i. Growth

Year-Over-Year Growth

The yearly growth rate calculation (Equation 1) is the same as that for a percent difference. Because this is a value that does not scale with the absolute value of two points it is a useful tool for comparing two unlike populations.

Equation 1

$$\frac{Final - Initial}{Initial} \times 100 = YearlyGrowth$$

Compound Annual Growth Rate (CAGR)

CAGRs are interesting because they too are a standardized growth term that allows for easy comparison between different samples. If the CAGR formula is used for one year, its result is the same as a year-over-year calculation. The result of this statistical term is an approximated fixed yearly exponential growth trend over a particular time period based on an initial and final value.

Equation 2

$$\left[\left(\frac{Final}{Initial}\right)^{1/\#ofYears} - 1\right] \times 100 = CAGR$$

Because this is a two-point calculation, it cannot account for all of the interesting behaviors that take place in-between the initial and final values. Particularly for metals like tantalum, a simple CAGR could either lead one to believe that that the sample has a very high or very low yearly growth trend depending on which time period is selected (see Figure 6). In order to moderate the impact date choice makes on the solution, a 'trending CAGR' was calculated. Essentially, 5-year, 10-year, and 20-year CAGRs were calculated for every year in the series and then they were averaged for the last thirty years. Using this method, the smoothed annualized gain accounts for all fluctuating behaviors in the curves. For each metal in the survey, the CAGRs were averaged from 1979-2009 for price and 1979-2011 for production.

Using this method, it is worth noting that different data is referenced depending on the number of periods. In the 5-year averaged CAGR, data ranges from 1974-2009 (2011) while in the 20-year series, data ranges from 1959-2009 (2011).

ii. Volatility

Having an understanding of volatility informs how predictable the data is. Particularly in describing value (or price), high volatility is associated with riskiness and can influence consumer decisions.

Standard Deviation

Standard deviation is an oft-used calculation to make assumptions about volatility. This statistical term determines the dispersion from the mean for a set of data (Equation 3).

Equation 3

$$\sigma = \sqrt{\frac{\sum (x_i - m)^2}{n - 1}}$$

After the yearly growth rate, 5-year CAGRs, 10-year CAGRs, and 20-year CAGRs were determined, the standard deviation based on the last 30 years (1979-2009 for price and 1979-2011 for production) was calculated for both value and production.

To put the standard deviation of production and value into context, the coefficient of variation was utilized. This is a dimensionless number that is calculated by dividing the standard deviation by the mean. Because there are such wide-ranging statistics, this measure makes comparison between different metals more appropriate.

Auto-regressive Integrated Moving Average (ARIMA)

ARIMA is used to forecast the behavior of time-series data. This statistical technique can be manipulated in many different ways to be the most appropriate for the data being modeled. Written in the form ARIMA(p,d,q), integers (p, d, q) represent the degree of the auto-regressive (p), integrated (d), and moving average (q) character of the set.

Using this method, there ideally should be stationarity and past data should influence the following points. Highly volatile data will likely require data manipulation or smoothing techniques. If appropriate priming of data is not done, the result would be a poor t-statistic. If the data is exhibiting seasonality, that can be taken into consideration using the ARIMA method.

Auto-regression (1,0,0)

Autocorrelation determines how related a current reference point is to another a certain number of periods prior to it. The number of periods between reference points is referred to as the lag. For an auto-regressive ARIMA model with a lag of 1 – ARIMA (1,0,0) – a value at a particular time [Y(t)] is determined by its immediately preceding value [Y(t-1)] multiplied by an autocorrelation factor [A(lag=1)] plus an error term [E(t)] (Equation 4).

Equation 4 ARIMA (1,0,0) or AR(1)

$$Y(t) = A(1) \times Y(t-1) + E(t)$$

For ARIMA (2,0,0), a given point is influenced by its two preceding values and therefore second ordered (Equation 5).

Equation 5 Second Order Auto-regressive ARIMA - ARIMA (2,0,0) or AR(2)

$$Y(t) = A(1) \times Y(t-1) + A(2) \times Y(t-2) + E(t)$$

Differencing (0,1,0)

If there is a long-term growing (or declining) trend, the data is exhibiting nonstationarity. To ameliorate this, the previous period value [Y(t-1)] can be subtracted from the current one [Y(t)] giving a first differenced and more stationary data set. If there is still a noticeable growth trend, differencing should be done again and the data will be considered second differenced.

Equation 6 ARIMA (0,1,0) or I(1)

$$Y(t) = Y(t-1) + \mu$$

An ARIMA (0,1,0) is the same as a random walk model. This is most helpful for highly irregular data. If the constant term (μ) is zero, the model suggests that there is no drift in the system. When the constant is nonzero, however, there is either a net upward or downward trend and the data is said to exhibit drift.

Moving Average (0, 0, 1)

A moving average minimizes volatility thereby making high-level trends more apparent. The average from a fixed number of periods is calculated then recalculated after shifting one period. In a moving average ARIMA model, or MA(1), a particular value is related to the *errors* of previous periods via the moving average coefficient (θ). This is particularly convenient in the case where a random walk model has some overall growth or decline.

Equation 7 ARIMA (0,0,1) or MA(1)

$$Y(t) = -\theta \times E(t-1) + E(t)$$

Though the MA(1) (Equation 7) looks similar to AR(1) (Equation 4), *deviations* from expected values as opposed to the *values themselves* influence the calculation.

III. Results and Discussion

A. Metals Survey

i. Production

Table 1 Summary of statistical information related to production from 1900-2011[1]

Metal	Min (t)	Max (t)	Spread (t)	Percent Difference
Pig Iron	36,700,000	1,100,000,000	1,063,300,000	2897%
Aluminum	6,800	44,100,000	44,093,200	648429%
Copper	495,000	16,100,000	15,605,000	3153%
Zinc	464,000	12,400,000	11,936,000	2572%
Chromium	16,500	7,390,000	7,373,500	44688%
Lead	749,000	4,500,000	3,751,000	501%
Magnesium	19,600	780,000	760,400	3880%
Tin	88,400	301,000	212,600	240%
Molybdenum	10	250,000	249,990	2499900%
Rare Earths	12	137,000	136,988	1141567%
Niobium	2,480	63,000	60,520	2440%
Silver	3,970	23,800	19,830	499%
Gold	386	2,700	2,314	599%
Tantalum	215	1,430	1,215	565%
Pt-Group	2	514	512	27835%
Beryllium	15	468	453	3020%

---

[1] Not all metals had a starting value in 1900. See appendix (Table 20) for details on what data is missing for each metal.

a. Trends

The sixteen observed metals had very different levels of production. The least produced metal (beryllium) only had maximum tonnage of 468 metric tons from 1900 to 2011 whereas pig iron had maximum tonnage of 1.1 billion metric tons.

Aside from tin and molybdenum, the spread between the minimum and maximum production follows the same descending trend as the maximum. There are several factors that can attribute to the order of the metals in Table 1. Ease of extraction, end-user demand, and worldwide reserves are just a few.

Percent difference was calculated to determine which metals had increased the most in the last 110 years. Molybdenum, rare earths, and aluminum had the most significant differences between their minimum and maximum extremes. Conversely, tin, lead, and silver had the smallest percent difference between their minimum and maximum production.

Table 2 CAGR between minimum and maximum production

Metal	Min Year	Max Year	Spread CAGR
Beryllium	1937	1956	20%
RareEarths	1925	2006	12%
Tantalum	1986	2004	11%
Molybdenum	1900	2011	10%
Aluminum	1900	2011	8%
Niobium	1964	2008	8%
PtGroup	1921	2006	7%
Chromium	1900	2008	6%
Magnesium	1937	2011	5%
PigIron	1921	2011	4%
Zinc	1921	2011	4%
Copper	1900	2011	3%
Silver	1946	2011	3%
Tin	1945	2007	2%
Gold	1900	2011	2%
Lead	1900	2011	2%

To better understand growth, CAGRs were used. This introduced a temporal component allowing one to identify some of the fastest growing periods for the surveyed metals. Having either a large percent difference or a small difference between the years that the minimum and maximum production occurs can lead to a high CAGR. As such, the three metals with the greatest percent differences were also ranked in the top four highest spread CAGR metals.

Silver showed 65 years between its minimum and maximum, tin had 62, and lead had 111. Because these metals also had fairly small percent differences, their spread CAGRs all fell in the bottom 4 (3%, 2%, and 2% respectively). These metals had some of the smallest

CAGRs between minimum and maximum points because they occurred at such different time periods.

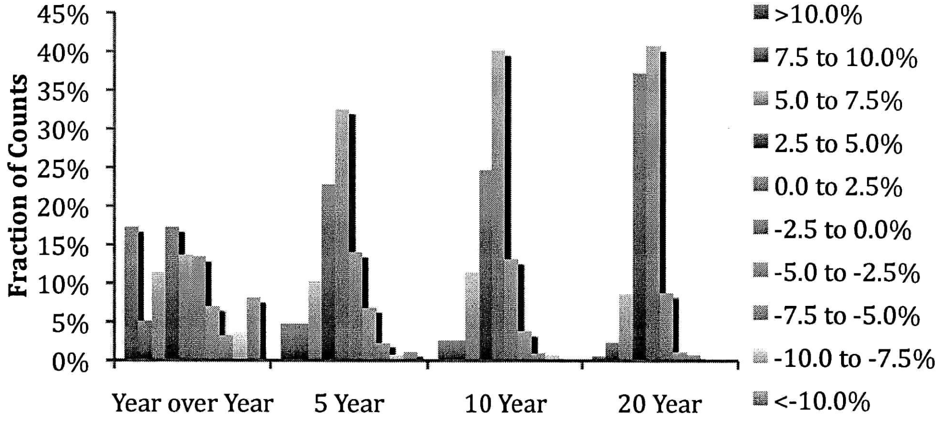


Figure 1 Histogram of Production CAGRs for Entire Metals Survey

From 1979-2011, production CAGRs were counted and presented in histogram format for all sixteen metals. The data approached a near-normal distribution with the mean just over 0.0%. Year-over-year, there were a number of instances where growth was over 10.0%. For five-year time-spans, the highest growth category fell from 17% of counts above 10.0% growth to only 5%. Observation of 20-year trending CAGRs makes it apparent that collectively the metals production has grown moderately over this observed time period. 89% of all 20-year CAGRs were positive - 78% of which were between 0.0 and 5.0%.

b. Volatility

To an extent, having an understanding of peak values and the range between them provides a context for volatility. Particularly if a spike happened in recent years, firms may choose to be more cautious with its use and even seek alternatives. Two types of analyses were carried out to quantify volatility.

Standard Deviation

Table 3 Volatility Measures of Growth Rates for Production from 1979-2011

Metal	Yearly Average St Dev	Average 5 Yr Running St Dev	Average 10 Yr Running St Dev	Average 20 Yr Running St Dev	Coefficient of Variation
Beryllium	0.53	0.13	0.07	0.04	36%
Niobium	0.26	0.06	0.04	0.02	68%
Tantalum	0.24	0.09	0.06	0.03	59%
Chromium	0.23	0.04	0.02	0.01	33%
Molybdenum	0.15	0.04	0.02	0.02	34%
RareEarths	0.13	0.04	0.02	0.02	49%
Magnesium	0.12	0.04	0.02	0.01	37%
Tin	0.08	0.03	0.02	0.01	16%
PtGroup	0.06	0.03	0.01	0.02	29%
Lead	0.05	0.02	0.01	0.01	11%
Aluminum	0.05	0.02	0.01	0.01	37%
PigIron	0.05	0.03	0.02	0.01	29%
Silver	0.04	0.02	0.01	0.00	22%
Gold	0.04	0.02	0.02	0.01	22%
Zinc	0.03	0.01	0.01	0.01	23%
Copper	0.03	0.01	0.01	0.00	28%

Using this measure for yearly growth, it appears that beryllium, chromium, niobium, and tantalum are the most volatile metals. Most of the other metals have standard deviations that fall below 0.10.

As would be expected, running averages of the CAGRs have declining variation with longer time periods. The only exception to this is platinum group metals. While its 10-year trailing CAGR was 1%, the 20-year trailing CAGR was 2%. This could be the result of the 20-year lagging CAGR taking points earlier than 1979 into consideration. From 1959 (the first initial point in the CAGR calculations) to 1979, there is steady and high growth of production (an average of 10.3% year over year growth). The introduction and adoption of the catalytic converter, which uses platinum group metals, to the automobile occurred in the US between 1974 and 1975 (Gerard and Lave 2005). Over the entire 1959-2011 time frame, however, the average yearly growth rate is only 5.5% as a result of a strong decline of production beginning around 2007.

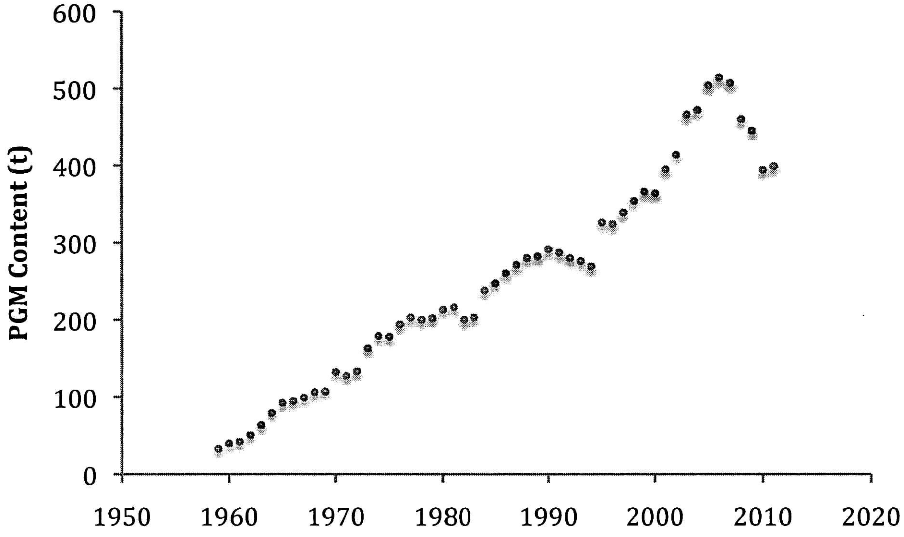


Figure 2 Time-Series Production of Platinum Group Metals

The coefficient of variation does not follow the same descending trend as the standard deviation. Niobium, tantalum, and rare earths all had very high coefficients of variation while lead and tin had very low levels.

ii. Value

Table 4 Summary of statistical information related to value from 1900-2009[2]

Metal	Min (98$/t)	Max (98$/t)	Spread (98$/t)	Percent Difference
Niobium	-	-	-	-
PigIron	-	-	-	-
Gold	4,920,000	39,000,000	34,080,000	693%
PtGroup	3,900,000	33,900,000	30,000,000	769%
Beryllium	152,000	2,960,000	2,808,000	1847%
Silver	99,900	1,312,000	1,212,100	1213%
Tantalum	55,300	562,000	506,700	916%
Tin	5,770	36,900	31,130	540%
Molybdenum	4,760	114,000	109,240	2295%
Magnesium	2,210	179,000	176,790	8000%
Copper	1,510	9,360	7,850	520%
Aluminum	1,300	20,000	18,700	1438%
Zinc	762	5,050	4,288	563%
Lead	638	2,690	2,052	322%
Chromium	232	2,740	2,508	1081%
RareEarths	29	145,000	144,971	499900%

a. Trends

Unfortunately, some of the metals had incomplete data sets for their values. After omitting niobium and pig iron from the survey, rare earths have by far the greatest percent difference between its minimum and maximum price. In 1933, a metric ton of rare earth oxide would cost $145,000, however, in 1939, the price plummeted to $29. Other metals with high percent differences included magnesium (8000% price drop) and molybdenum (2295% price increase).

---

[2] Niobium and Pig Iron had a significant number of missing values so they were disregarded from further analysis

Table 5 CAGR between minimum and maximum values

Metal	Min Year	Max Year	Spread CAGR
Niobium	-	-	-
PigIron	-	-	-
Gold	1970	1980	23%
PtGroup	1900	1917	14%
Molybdenum	1922	1979	6%
Silver	1931	1980	5%
Tin	1932	1980	4%
Chromium	1921	2008	3%
Copper	2002	1916	-2%
Aluminum	2002	1916	-3%
Lead	1985	1948	-4%
Beryllium	2001	1935	-4%
Magnesium	2003	1915	-5%
Tantalum	2009	1980	-8%
Zinc	1932	1915	-11%
RareEarths	1939	1933	-76%

Though values, on average, do not have as great of a percent difference between their minimum and maximum values as production, the CAGR of spreads tend to vary much more. Platinum-group metals (14%), gold (23%), rare earths (-76%), and zinc (-11%) all had the greatest smoothed growth trends and copper (-2%), chromium (3%), and aluminum (-3%) had the smallest.

In this case, the years at which minimum and maximum values occurred plays an important role. For production, the average minimum/maximum spread of years was 81. Also, the year of the maximum always happened later than the year the minimum occurred and most maximums for production occurred in the 21st century.

For unit value, however, the average spread of years was 49 and approximately half of the CAGR spreads are negative as a result of the maximum value occurring before the minimum yielding price drops.

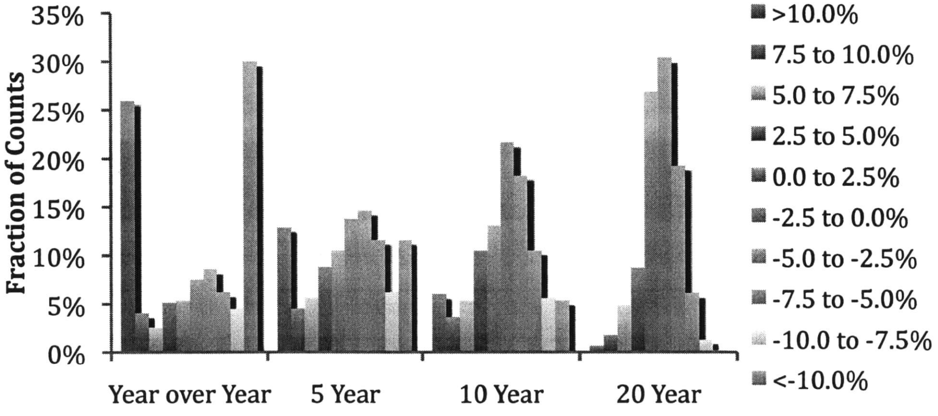


Figure 3 Histogram of Value CAGRs for the Entire Metals Survey

Though production growth appeared to be generally positive, value growths tended to be more negative. Yearly, growth rates were much more negative. 30% of values of rates from 1979-2009 were less than -10.0%. Also, compared to the production histogram, yearly value rates tended to fall more at the extremes. There were almost twice as many counts ±10.0% (56% versus 25%). Between five and ten year CAGRs, there was a shift from -2.5 to -5.0% growth occurring most frequently to -2.5 to 0.0%.

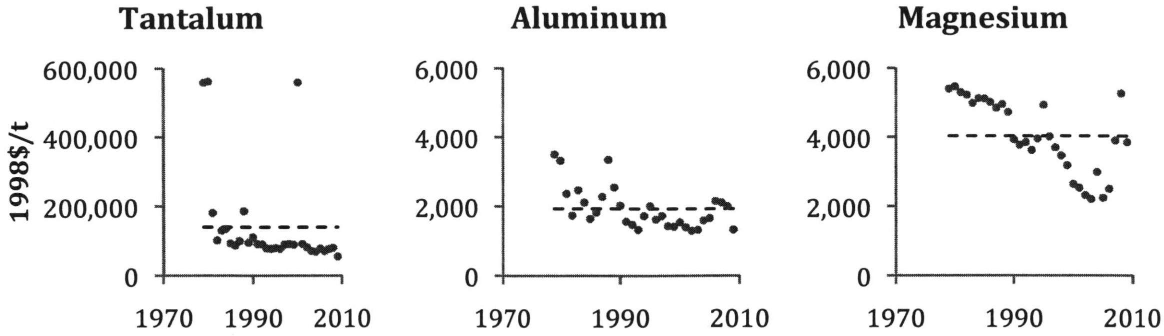
b. Volatility

Standard Deviation

Table 6 Volatility Measures of Growth Rates for Real Value from 1979-2009

Metal	Yearly Average St Dev	Average 5 Yr Running St Dev	Average10 Yr Running St Dev	Average 20 Yr Running St Dev	Coefficient of Variation
Niobium	-	-	-	-	-
PigIron	-	-	-	-	-
Tantalum	1.02	0.16	0.09	0.03	102%
Molybdenum	0.57	0.23	0.11	0.06	112%
RareEarths	0.52	0.11	0.08	0.03	40%
Zinc	0.30	0.10	0.04	0.02	33%
Silver	0.29	0.11	0.08	0.05	80%
Chromium	0.27	0.07	0.05	0.03	34%
PtGroup	0.24	0.08	0.04	0.02	27%
Copper	0.24	0.11	0.05	0.02	39%
Lead	0.23	0.09	0.04	0.02	39%
Aluminum	0.21	0.07	0.03	0.01	31%
Gold	0.21	0.09	0.07	0.05	40%
Tin	0.21	0.07	0.06	0.04	61%
Beryllium	0.21	0.09	0.07	0.03	43%
Magnesium	0.17	0.05	0.03	0.02	26%

Among the 16 metals, tantalum's standard deviation of value growth was significantly higher (1.02 versus the 0.28 average of the others). Second to tantalum is molybdenum (0.57) and rare earths (0.52). The time-range has a great impact on the ranking of these metals. Only the most recent 30 years were considered. Tantalum is the only metal that has both its minimum and maximum unit values within this time frame. Though magnesium had one of the greatest percent differences between its minimum and maximum values, when constricted to the latest thirty years, its standard deviation of yearly growth is actually the smallest at 17%.



Percent Difference:

Mean/Max: 301% Mean/Max: 81% Mean/Max: 35%
Mean/Min: -61% Mean/Min: -33% Mean/Min: -45%

Figure 4 Value Versus Time Plots for Metals with Differing Standard Deviations

Above are value versus time plots for three metals with standard deviations that were ranked first, middle-rung, and last. The percent difference between the mean (also denoted with dashed lines) and maximum as well as between mean and minimum were calculated. Magnesium had the lowest standard deviation and also has a much smaller percent difference (between mean and maximum) than either tantalum or aluminum.

IV. Analysis of Metals

In addition to interpreting results by relating a single metal to the full set, there are several ways to break up this survey of sixteen to make assumptions about broader trends. Those considered included whether a metal was on an open exchange or not, the rate of production, if the metal was base or precious, and the relationship between co-mined materials.

A. Metals Exchange

There are hundreds of commodities exchanges that exist worldwide. The four largest and most recognized for metals are LME, NYMEX, COMEX, and SHFE.

The London Metals Exchange (LME) is the largest market for options and future contracts on particular metals. The contracts can be temporally manipulated in a number of ways (from daily to multi-year expiry dates). Futures and options contracts are available for *aluminum*, *copper*, *tin*, nickel, *zinc*, *lead*, aluminum alloy, steel billet, cobalt, and *molybdenum*[3].

Though the official commencement of this exchange was in 1877, the concept of metals exchanges was formalized in London in 1571 under Queen Elizabeth I's reign (Hart 2007). Forums were organized to bring together classes of people from financiers to producers. During the Industrial Revolution, the market became globalized, causing a great deal of uncertainty regarding how much one would be able to sell ore brought from distant countries for. When the telegraph entered commercial use, it became possible for miners to notify those involved in the market of their cargo. After futures contracts became more controlled, buyers no longer had to worry about surprising increases in prices and sellers did not have to worry about drops in price (Hart 2007).

As London was growing and establishing its official exchange, merchants met in Manhattan to discuss the exchange of butter and cheese. With time, the variety of commodities grew and eventually the New York Mercantile Exchange, or NYMEX, was

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[3] Italicized metals are referenced in the metals survey above

established. Chicago became the other American hub for commodity exchanges and in 1933 COMEX (Commodity Exchange, Inc – a division of NYMEX) was formed (Goodman 2011).

A century later and halfway around the world, the Shanghai Metals Exchange (SHME) was established in 1992 and had markets for the non-ferrous metals *copper*, *aluminum*, *lead*, *zinc*, *tin*, and nickel. The sheer size of China attributed to the growth of its commodities exchanges. Particularly with the countries recent technological advance, China has become one of the world's largest producers as well as consumers of particular products. In 1999, SHME along with two other commodities exchanges combined to form SHFE (Shanghai Futures Exchange) in which *copper*, *aluminum*, *zinc*, *lead*, natural rubber, fuel oil, steel wire rod, steel rebar, and *gold* are currently traded. SHFE alone has become the third largest non-ferrous metals exchange in the world.

For the following analysis the sixteen surveyed metals were broken up into their respective categories. All analysis was done for the years 1979-2009 (for values) or 1979-2011 (for production). It is worth noting that not all metals have been on open exchanges during that entire time period.

Table 7 Metals Exchange Categories

LME	COMEX	SHFE	No Metals Exchange
Aluminum, Copper, Lead, Molybdenum, Tin, Zinc	Aluminum, Copper, Gold, Silver	Aluminum, Copper, Gold, Lead, Zinc	Beryllium, Chromium, Magnesium, Niobium, Pig Iron, Rare Earths, Tantalum, Platinum-group metals [4]

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[4] Pt-group metals appear in NYMEX, however, they are the only metal from the survey in that exchange. As a result, these metals were classified as not being in an exchange.

Metals on an open exchange tend to be pricier than those that are not. This is likely because these metals are widely considered critical. From 1979-2009, all metals on an exchange had an average price of approximately $3.3M while those not on an exchange were over 1000 times cheaper (Table 8).

Table 8 Metals Exchange Value Statistics Averaged from 1979-2009

Classification	Min (98$/t)	Max (98$/t)	Average (98$/t)	2009 Value (98$/t)
Metals Exchange	1,765,981	6,875,280	3,318,023	3,665,282
COMEX	2,045,453	10,080,298	4,123,786	6,041,093
LME	2,472	27,587	7,001	6,423
SHFE	2,013,396	9,753,005	4,047,087	5,951,670
No Metals Exchange	31,799	219,782	106,859	46,734

There were two surprising results from this study. Firstly, the price volatility of all of the metals categories are not widely different (Table 9) and secondly, the standard deviation of production growth has a strong bias towards metals not on open exchanges.

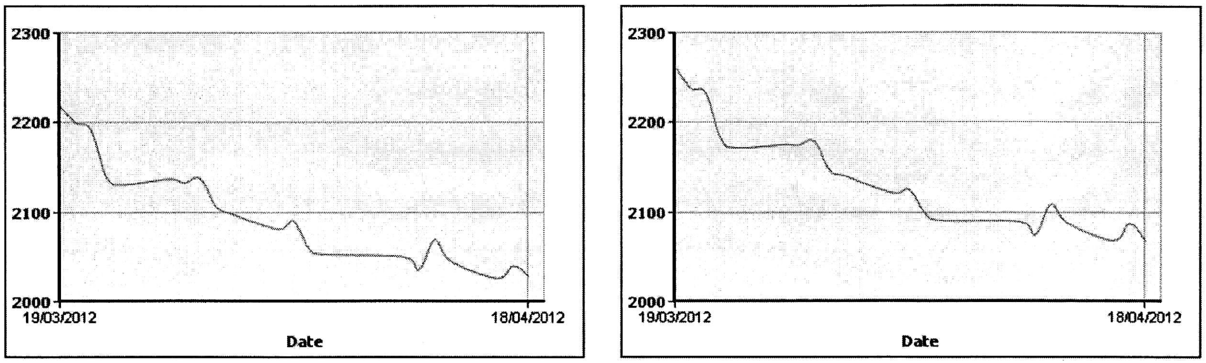
Table 9 Volatility of Value Growth from 1979-2011

Classification	Yearly St Dev	5 Yr Running St Dev	10 Yr Running St Dev	20 Yr Running St Dev
Metals Exchange	28%	11%	6%	3%
COMEX	24%	9%	6%	3%
LME	29%	11%	6%	3%
SHFE	24%	9%	5%	3%
No Metals Exchange	31%	7%	5%	2%

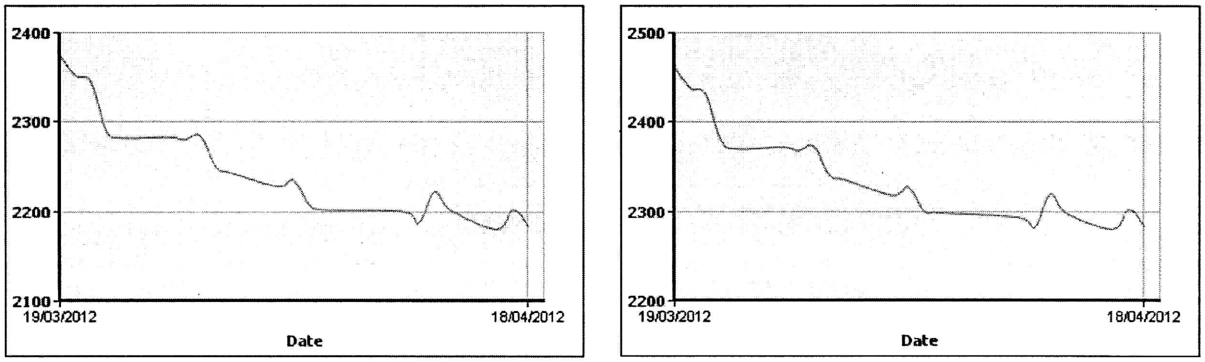
Table 10 Metals Exchange Production: Volatility of Growth from 1979-2011

Classification	Yearly St Dev	5 Yr Running St Dev	10 Yr Running St Dev	20 Yr Running St Dev	Coefficient of Variation
Metals Exchange	6%	2%	1%	1%	0.25
COMEX	4%	2%	1%	1%	0.27
LME	7%	2%	1%	1%	0.25
SHFE	4%	2%	1%	1%	0.28
No Metals Exchange	22%	6%	4%	2%	0.44

Regarding production, metals on an open exchange appear to be significantly more stable than those not on a metals exchange. The standard deviation of yearly growth for non-market metals is many times greater than those on exchanges. Also, the coefficient of variation is nearly twice as great. This is possibly due to the nature of futures contracts: they are put in place so that sharp changes in availability do not occur and therefore significantly impact the market.



Cash buyer 3-months buyer



15-months buyer 27-months buyer

Figure 5 LME Aluminum Futures Contract Rates for Varying Time Periods[5]

Above is a sample of futures contract price charts for aluminum. As with most purchases, it is advantageous to pay up front and be a cash purchaser. Even still, day-to-day prices can differ substantially. Though the trends for these four time periods are all similar, the scale of the y-axis changes quite a lot. A cash buyer would have paid approximately $2030 on 4/18/2012, $2090 for 3 months, $2190 for 15 months, and $2300 for 27 months ahead.

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[5] y-axes are in units of 2012 U.S. dollars per ton

B. Production Rate

Economic theory would suggest that materials with high levels of production would also have fairly low values and vise versa. The sixteen metals were classified into three groups dependent on their levels of production as follows:

Table 11 Levels of Production

Lowest Production (<75,000t)	Medium Production (<7,500,000t)	Greatest Production (>7,500,000t)
Beryllium, Gold, Niobium, Pt-group Metals, Silver, Tantalum	Chromium, Lead, Magnesium, Molybdenum, Rare Earths, Tin	Aluminum, Copper, Pig Iron

After segmentation and observation of their production statistics, it is apparent that these metals differ by orders of magnitude.

Table 12 Production Statistics from 1979-2011 for Differing Production Levels

Classification	Min (t)	Max (t)	Average (t)	2011 Production (t)
Greatest Production	159,200,000	386,733,333	219,418,788	386,733,333
Medium Production	936,233	2,226,333	1,411,010	2,200,500
Low Production	3,500	15,305	7,811	15,155

In 2011, the average of production for the highest category was over 25,000 times greater than those in the lowest production category.

Table 13 Production Volatility Statistics from 1979-2011

Classification	Yearly St Dev	5 Yr Running St Dev	10 Yr Running St Dev	20 Yr Running St Dev	Coefficient of Variation
Greatest Production	4%	2%	1%	1%	0.31
Medium Production	13%	4%	2%	1%	0.30
Low Production	20%	6%	4%	2%	0.39

While the production is quite different from metal to metal, so is the degree of dispersion. Metals with low levels of production tend to be much more volatile. In terms of standard deviation, the amount of possible change from year to year could be as high as 20%. Furthermore, the coefficient of variation is the highest.

Volatility results for production were sensible. Difficulty of extraction or low levels of natural reserves make the predictability of production hard to determine year-over-year. With this in mind, it presents the question of how difficulty of extraction is reflected in pricing.

Table 14 Production Value Statistics from 1979-2009

Classification	Min (98$/t)	Max (98$/t)	Average (98$/t)	2009 Value (98$/t)
Greatest Production	1,037	3,222	1,769	2,685
Medium Production	2,870	28,900	7,938	7,617
Low Production	2,682,553	10,538,350	5,092,636	6,652,460

The average value of low-production metals is significantly higher than those with high levels of production. Referring to the table of metals values from 1900-2009 (Table 4), gold, platinum-group metals, beryllium, silver, and tantalum are at the top of the list[6]. Medium and high production level metals were in mixed positions following the top five. Perhaps after a particular threshold of production level, other factors become weighted more heavily in the valuation of a material.

Table 15 Value Volatility Statistics from 1979-2009

Classification	Yearly St Dev	5 Yr Running St Dev	10 Yr Running St Dev	20 Yr Running St Dev	Coefficient of Variation
Greatest Production	23%	9%	4%	2%	0.35
Medium Production	33%	10%	6%	3%	0.52
Low Production	39%	11%	7%	4%	0.58

While the value of a metal may be high, regardless of its production level the variation is fairly similar. Low production metals have a yearly growth standard deviation of 39% compared with their high growth, high production counterparts that have a dispersion of 23%. Expectedly, if a metals level of production has consistent growth, prices are much easier to set and will tend to vary less. For the coefficient of variation, low production metals had an average 0.58 but with a wide spread. Platinum-group metals had a coefficient of variation of only 0.27 while tantalum's was 1.02. The other two categories did not have as much difference between all of the metals in their categories.

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[6] Niobium had some missing value data and was, therefore, excluded from the survey

C. Base and Precious Metals

Base metals tend to be abundant in nature and therefore in high use across various industries. Unfortunately, they also tend to corrode and oxidize easily particularly in a moist environment. According to the US Customs and Border Protection Agency, of the surveyed metals, eleven are classified as base metals (Table 16). Conversely, precious metals are found in low concentrations in Earth's crust and naturally occur in a non-oxidized state.

Table 16 Base and Precious Metals

Base	Precious
Aluminum, Beryllium, Chromium, Copper, Lead, Magnesium, Molybdenum, Niobium, Pig Iron, Rare Earths, Tantalum, Tin, Zinc	Gold, Pt-group Metals, Silver

Table 17 Production Statistics for Base and Precious Metals (1979-2011)

Classification	Min (t)	Max (t)	Average (t)	2011 Production (t)
Base	40,762,474	98,823,818	56,235,216	98,811,419
Precious	4,037	9,005	6,317	8,966

The production of base metals is approximately four orders of magnitude greater than that of precious metals.

Table 18 Production Volatility for Base and Precious Metals (1979-2011)

Classification	Yearly St Dev	5 Yr Running St Dev	10 Yr Running St Dev	20 Yr Running St Dev	Coefficient of Variation
Base	15%	4%	3%	2%	0.34
Precious	5%	2%	2%	1%	0.25

Gold and silver's wide use for currency exchanges adds another factor for its demand. Because of both their difficulty in extraction and use for monetary systems, precious metals tend to be much more expensive than their base metal counterparts. In 2009, precious metals on average cost about $10.9M per metric ton (1998 dollars) while base metals cost $35,723 per metric ton.

Table 19 Price Statistics for Base and Precious Metals from 1979-2009

Classification	Min (98$/t)	Max (98$/t)	Average (98$/t)	2009 Value (98$/t)
Base	19,530	141,025	65,263	35,723
Precious	5,293,000	20,570,667	9,940,069	10,983,000

D. Case Studies

In the following sections, tantalum and niobium were researched further because of their distinct historical production and value trends. Additionally, these two metals are co-mined (with tin in some ore bodies). To compare their statistics to an industrial metal, a more detailed account of copper's statistics can be found in the appendix (Figure 18-Figure 21).

i. Tantalum

Tantalum showed multiple instances of extreme year-over-year change in the past 30 years that inspired a closer look at its activity.

There are three different forms in which tantalum can be bought: tantalum ore/concentrate, tantalum oxide/salts, and capacitor-grade tantalum powder. Capacitor-grade powders typically make up about 25% of end-use. The material's high melting point and resistance to corrosion make it an ideal material for use in extreme environments. Electronic applications tend to dominate use. Over time this metal has also been used as a substitute for platinum.

The defense industry also makes heavy use of tantalum and (at least in America) it is classified as a critical and strategic metal. Aluminum, titanium, tungsten, and zirconium can all be used as a substitute in the defense industry but each of these options has different penalties: either they are more expensive than tantalum or the performance is not as great.

Value

Though tantalum is not traded openly on any metals exchanges, Ryan's Notes supplies value data to USGS by surveying market makers. Consumers, traders, and producers are all questioned before a price point is established. The level of specification and use case are some of the largest factors affecting the price of tantalum.

From the mid-twentieth century, tantalum's value has grown and fallen leading to an extreme spread in a short period of time. In the 1960s, tantalum's use in applications specific to the defense industry was first discovered. Increased demand caused inventory stockpiling through the early 70s and affected pricing as a result. Stockpiling continued through the decade, but in 1979 and 1980, demand outpaced the amount produced thereby skyrocketing prices to the highest point seen in this metals history. These high costs were then passed on to end users further down the supply chain so consumers discovered opportunities for substitution and use declined. In many electronics, tantalum was replaced with aluminum.

By 1986 these prices were quite low as a result of demand-reduction and the stockpiling that occurred in the early 1980s. Stockpiling also explains the rise and fall of prices without a corresponding rise and fall of production. Also, though the electronics sector was growing during this time (cell phones, video cameras), miniaturization of these products meant less tantalum used per unit so there was not a corresponding growth of its demand.

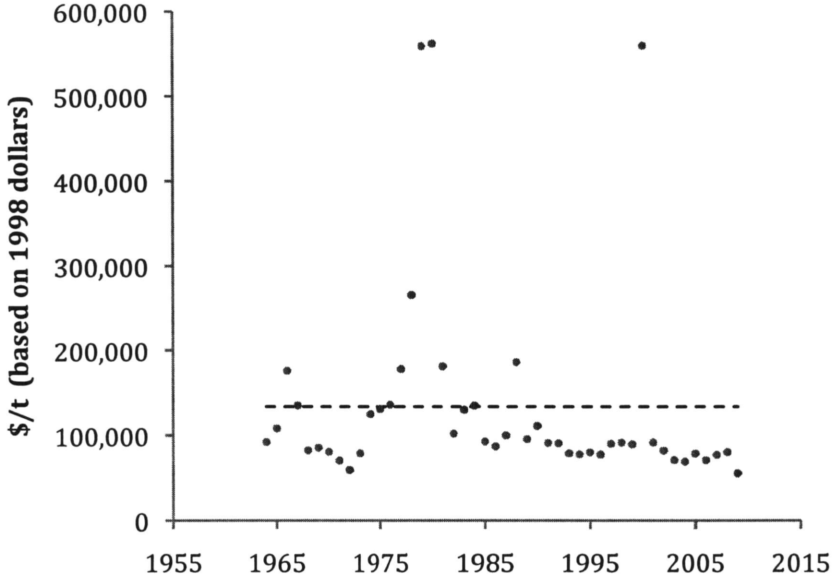

Figure 6 Bivariate Fit of Tantalum Value by Year

Production

Mines in South America and Australia have dominated the production of tantalum. Referencing the different factors that can influence scarcity (Alonso, Gregory et al. 2007), three very different situations changed the rate of tantalum's production.

In 2008 and 2009, some of the largest tantalum mines in Australia, Canada, and Africa were closed for economic reasons. As a result, in 2009, the Democratic Republic of Congo and Rwanda were responsible for 50% of the global tantalum production. Historically, political issues have lead to halted production and civil wars have caused significant price jumps. In the United States in accordance with the Conflict Minerals Law, electronics companies are required to disclose the sources of their raw materials. In recent history, more countries have advertised tantalum as a critical material encouraging the development of institutional changes that might moderate tantalum trading, pricing, and production levels.

Several countries have openly admitted their desire to find alternatives to tantalum. China – a country that purchased 80% of Brazil's tantalum supply in 2008 – has also put a significant amount of money and effort into the discovery and establishment of new mines in Africa (Tanquintic-Misa 2012).

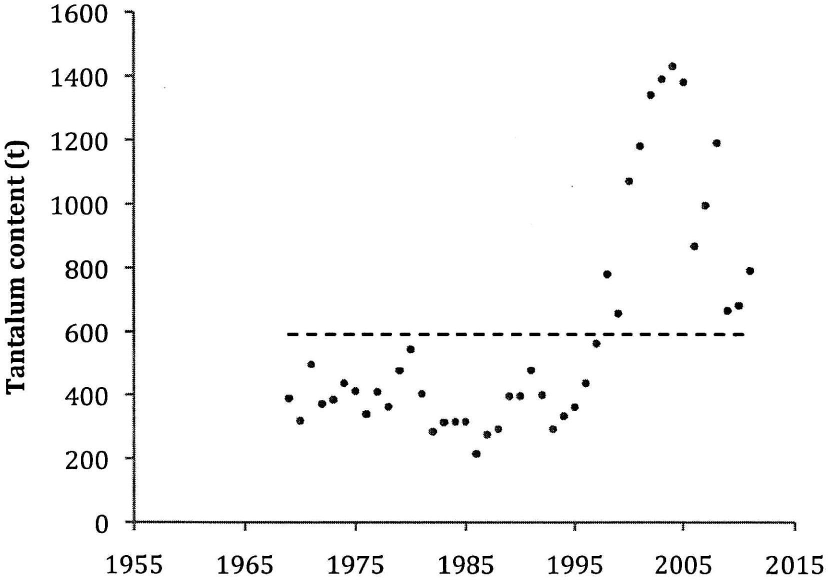

Figure 7 Bivariate fit of tantalum production by year

1979, 1980, and 2000 are all years with outlying prices. From research of Tantalum's temporal trends, stockpiling and end-use demand were the main factors that resulted in these effects. Though there does not appear to be any strong direct correlation between values and production, after comparing the year over year growth in value and production, there are some more noticeable behaviors. Upon fitting a line of best fit, the slope is greater than one suggesting that a marginal deviation in the rate of production has a large effect on the yearly growth of the value.

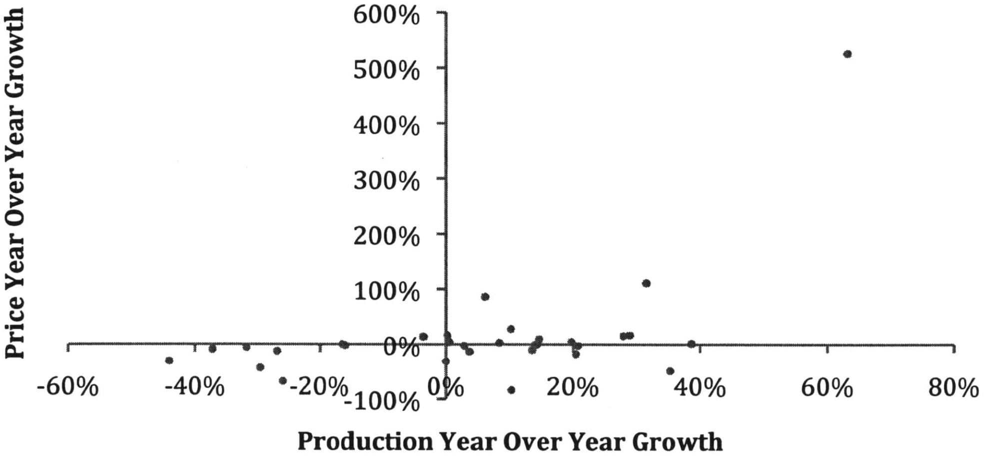

Figure 8 Price growth versus production growth for tantalum

There is, however, a high likelihood that there is a time lag between these supply and demand factors. It is difficult to determine if availability causes changes in price (standard economic view) or if some end-use demand further down the supply chain most heavily impacts the relationship in Figure 8.

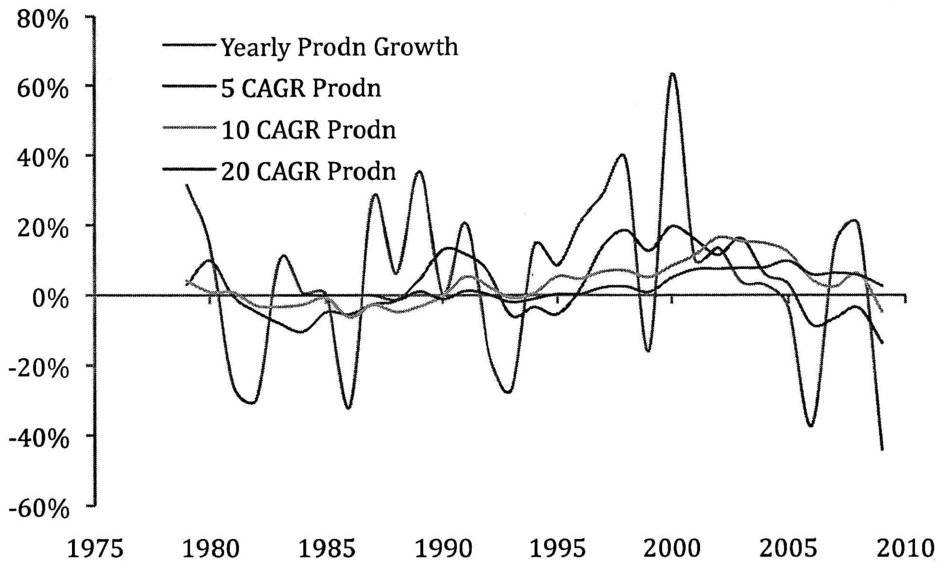

Figure 9 Trending production CAGR for 1, 5, 10, and 20 years for tantalum

In the last thirty years, production growth has fluctuated quite a lot. Tantalum's standard deviation of growth was in fact the third highest in the metals survey. When longer periods of time are considered, the volatility is lessened and it becomes easier to make assumptions about trends. In observation of the 20-year trending CAGR, world wide production growth has been increasing over time. There are very few instances of negative growth. Since the beginning of the 21st century, the growth rate seems to have plateaued at approximately 10%.

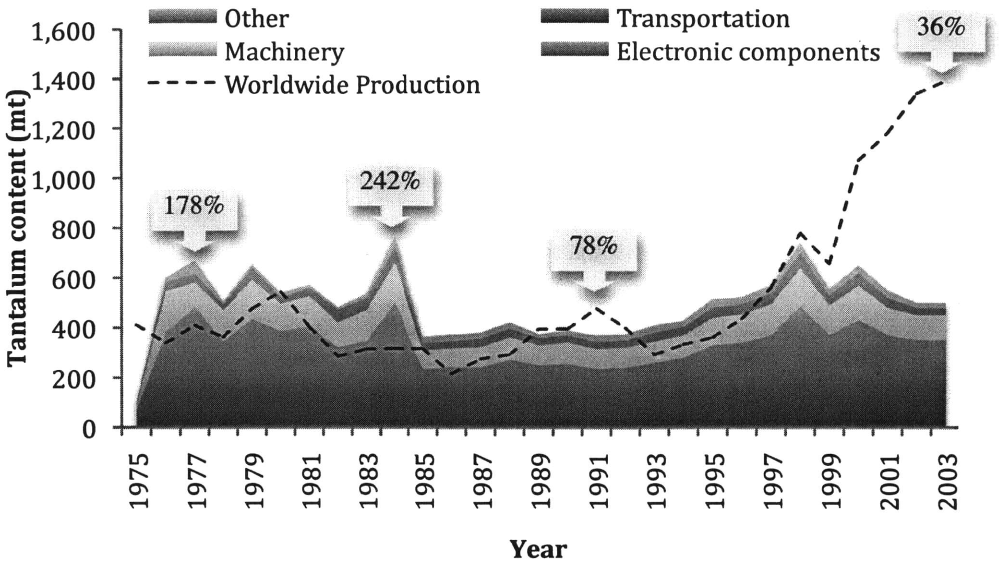

Figure 10 U.S. Tantalum End-Use Statistics and World Production Data

Above, the U.S. use of tantalum is charted and broken up into four industrial categories: electronic components, transportation, machinery, and other. End-use is difficult to approximate and the USGS derives these statistics by applying estimated end-use percentages based on Mineral Commodity Summary publications (U.S.G.S. 2011).

Until 1989, U.S. end-use almost always outpaced world production. While U.S. use levels are high, the country has not had a significant amount of production since 1959 because the ore grade was very low (Cunningham). The yellow boxes highlight the ratio between U.S. end-use and world production. In 1984, use was nearly 2.5 times greater than the amount produced that same year (or any year from 1975 to 1984). As discussed earlier, high demand and high prices lead firms to seek substitution opportunities and the following year use was only 15% greater than the amount produced. In the 1990s, production took off without a matching increase in demand or use.

Though there was anticipated growth of the electronics sector, the data was not representative of that. During the entire surveyed period of time, electronics had an average percentage of total use of approximately 66%±3%, machinery had an average percentage of 21%±2%, transportation's average was 8%±1%, and the other category had an average of 5%±2%. Though the time period from 1975 to 2003 had entry of many new technologies, the use mix remained fairly constant.

ii. Niobium

Niobium derives its name from the Greek mythological figure Niobe – the daughter of Tantalus. For quite some time, scientists thought that niobium and tantalum were actually the same metal. This was primarily assumed because of the difficulty of isolating the two similar metals. Though the mineral was initially discovered in 1734, it would not be until 1864 that Wilhelm Blomstrand would be able to isolate niobium and it became identified as its own element (Britannica 2012).

Value

The significance of this metal is its resistance to corrosion and heat conductivity. While tantalum price was heavily driven by end-use demand, niobium's price is affected by the amount of columbium mineral available. According to USGS specialists, when production is affected, there are stronger price effects than may be the case for other metals.

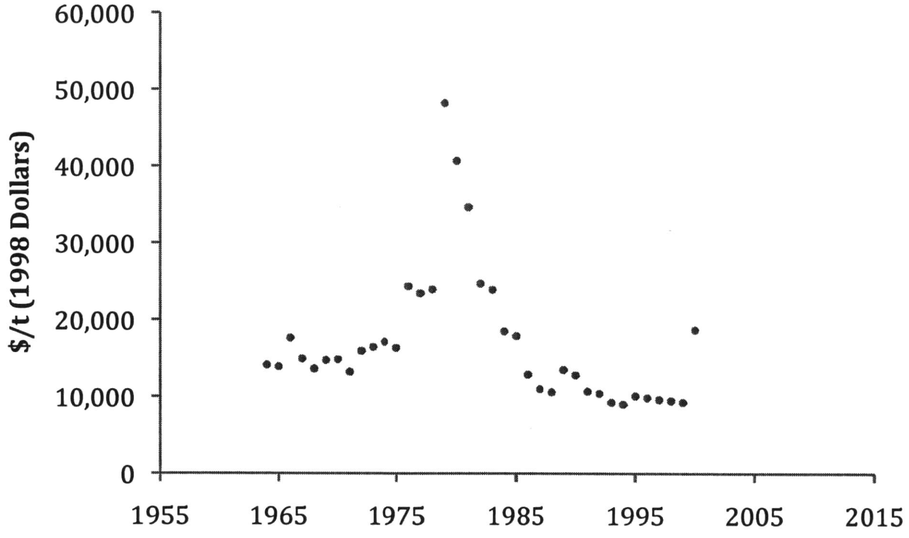

Figure 11 Bivariate fit of niobium price by year

Contrary to that claim, in comparing yearly growth of price with yearly production growth, the data appears to be scattered and random. It is likely that it takes some time before price effects are realized from production level changes.

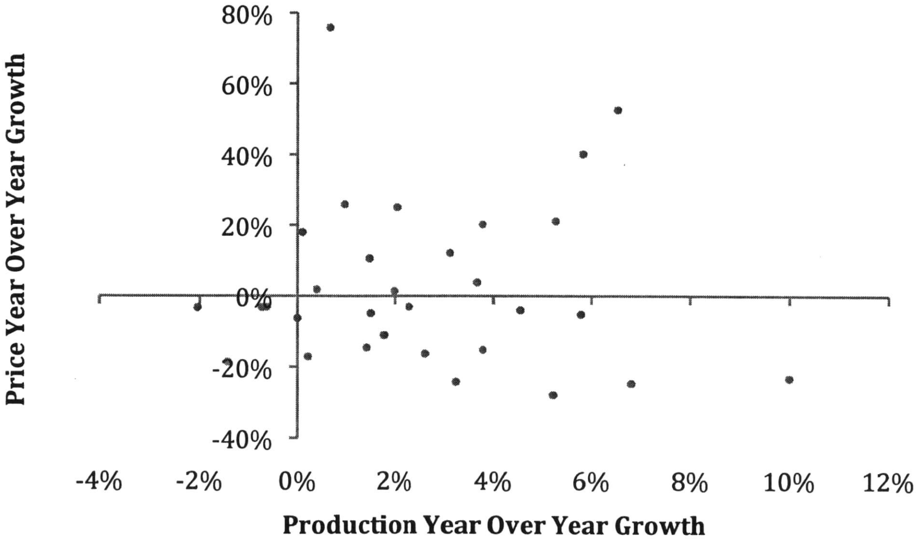

Figure 12 Price growth versus production growth for niobium

Production

Niobium also has several applications in the defense industry. It is therefore classed as a strategic metal and in the late 1950s the United States initiated a program to increase worldwide discovery and production of tantalum and niobium ore. The result of this was the discovery that the majority of domestic deposits are of low grade and this ultimately led to the termination of this program. As a result, prices fell and columbium (one of niobium's feed minerals) exploration decreased as well.

Unfortunately, recycling is not a significant source of niobium so consumers are very reliant on primary production. The discovery of the steel strengthening effects of small amounts of columbium in the 1960s had a direct impact on both the value and production of the metal. Pyrochlore deposits were established in Brazil and Canada around this time to sustain the growing demand for columbium.

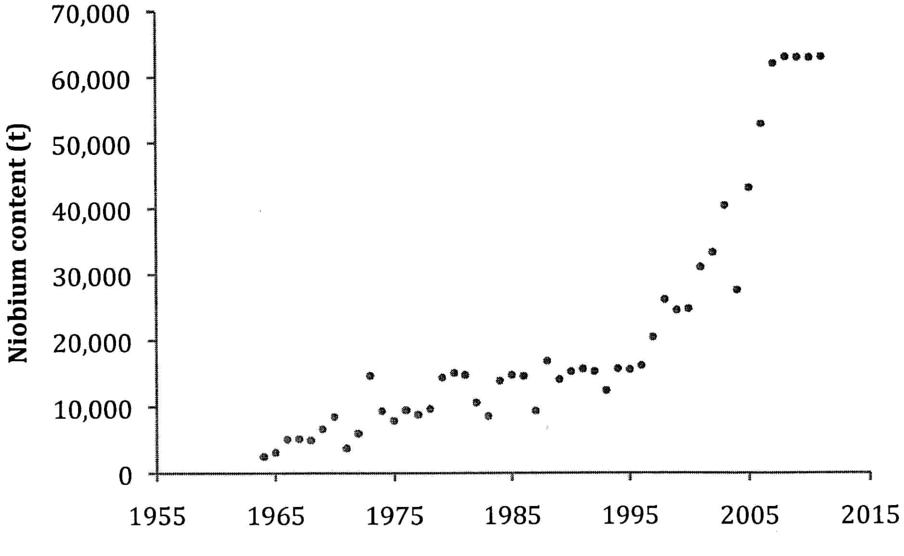

Figure 13 Bivariate fit of niobium production by year

Niobium's production growth has been more consistently positive than tantalum. Its peaks of growth, however, are not nearly as extreme. The 20-year production CAGRs from 1979-2009 were nearly flat around 3%. Only one 5-year CAGR falls below 0% and four yearly growth rates were negative.

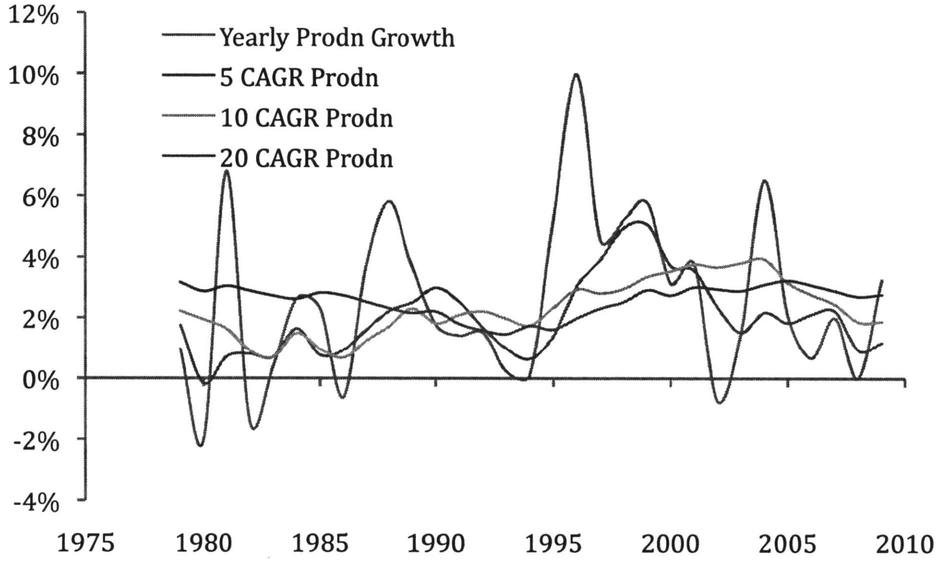

Figure 14 Trending production CAGR for 1, 5, 10, and 20 years for niobium

Similarly to tantalum, the mix of end-use for niobium was also fairly consistent. Microalloyed steels made up approximately 64%±5% of the U.S. end-use, stainless steels were approximately 14%±2%, superalloys were around 20%±5%, other uses only contributed to 1%±0% annually. There is also very little variation in the total amount used.

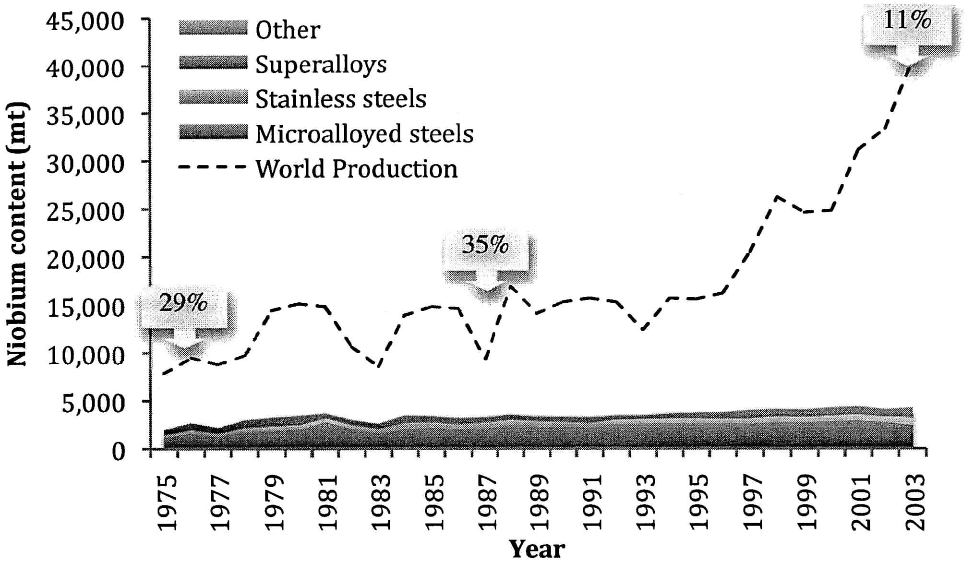

Figure 15 U.S. End-Use and World Production of Niobium

Worldwide production of niobium always outpaced the amount used in the U.S. from 1975 to 2003. Additionally, the difference between apparent consumption and world production has been growing.

iii. Co-mined Metals

As aforementioned, the fact that tantalum and niobium are co-mined made them interesting materials to study. In accordance with the data already obtained, price and production growth trends were compared for the two metals.

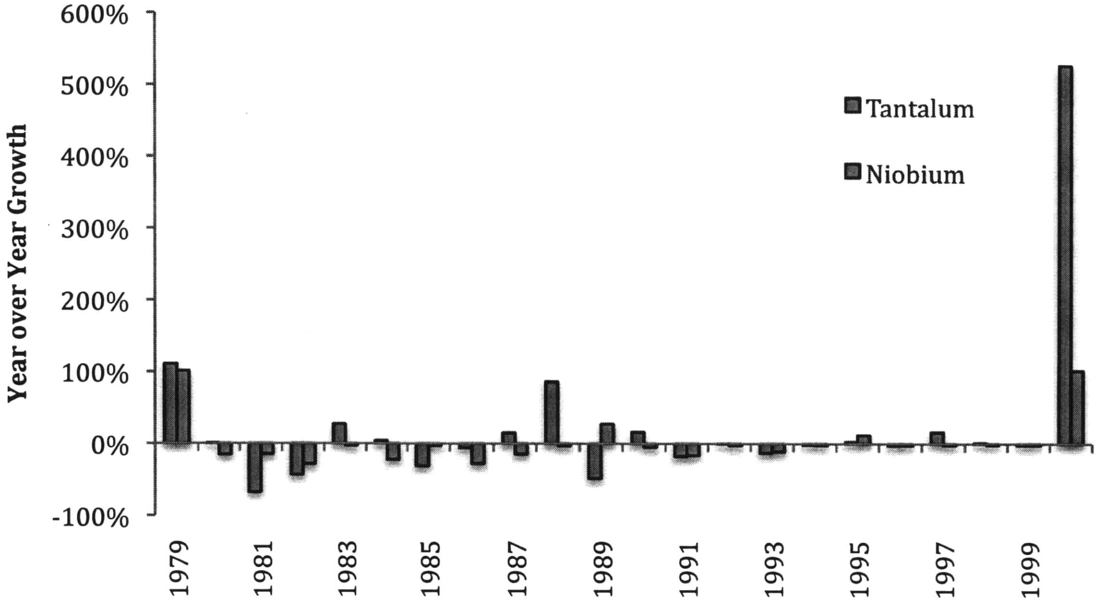

Figure 16 Yearly Growth Trends (Value)

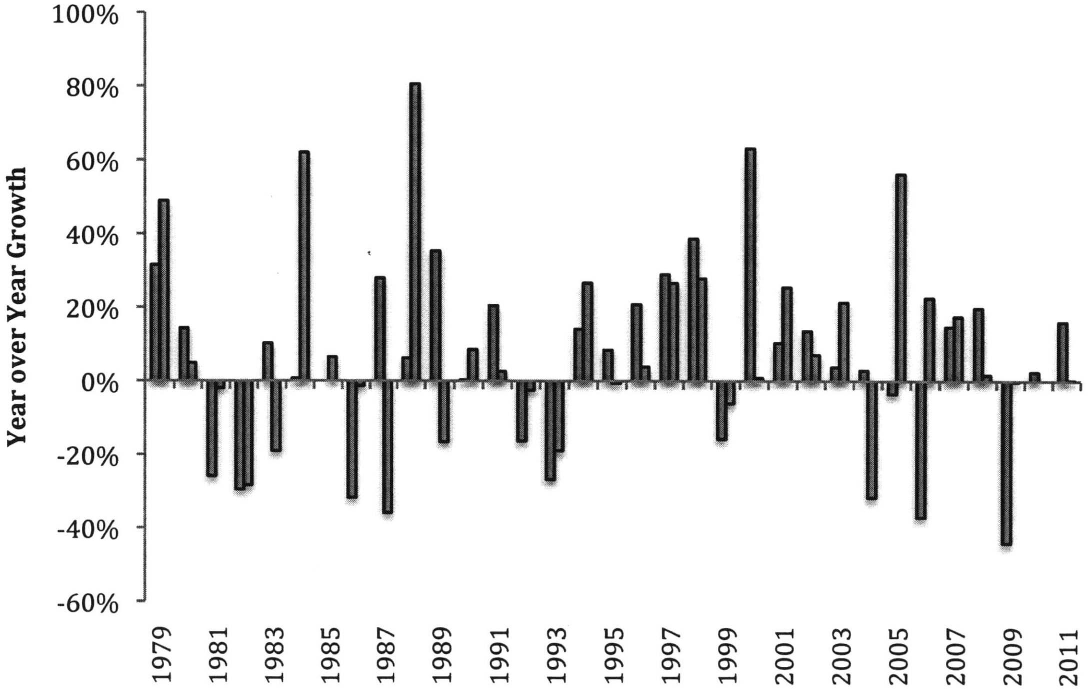

Figure 17 Yearly growth trends (Production)

As niobium and tantalum are co-mined, it would be expected that their production levels would move together. Though the year-over-year growth trends are typically in the same direction (either both positive or both negative), their magnitudes are quite different. One possible factor affecting this is that different mines produce these metals at different ratios and there has been volatility in individual mine production (Crockett and Sutphin 1993).

Value tends to trend in a similar way, which is surprising because tantalum and niobium are not necessarily used for the same purposes or bought by similar consumers. Therefore, it appears that production and material availability has some effect on the valuation of both of these metals.

Initially, ARIMA was a desired method to be used to determine how well correlated the two data sets are. ARIMA was considered for production trends because of the influence past production rates have on the future. Having information about both growth and volatility could potentially provide a basis to make short-term forecasts. Unfortunately, due to the poor results achieved in other tests, it was not a pursued method. Other options included granger causality or cross correlation statistical methods.

The mixed-model, ARMA (1,1), was predicted to yield the best response. Referring to the discussion in Auto-regressive Integrated Moving Average (ARIMA) (page 16), the AR component is used because there is a relationship between past and current production. The MA(1) component accounts for the regular growth trends. There is no differencing component because the trends are not seasonal.

Equation 8 ARMA (1,1) Model

$$A(1) \times Y(t-1) = -\theta \times E(t-1)$$

After using JMP software, the hypothesis that an ARMA (1,1) model would be an appropriate approximation was untrue. The obtained t-statistic concluded that the fit was not good.

Several modifications were made to better the results of this calculation. First, log(production) was used as the raw data rather than the actual production. Second, different ARIMA models were applied (even including an I(1) term) but the t-statistics remained below the threshold to be considered "good" data. It was ruled that ARIMA is particularly unhelpful for the study of metals with sporadic production.

V. Conclusions and Future Work

There were a few key insights that were gained from this study and could be potentially helpful to individuals or firms interested in understanding the range of growth and volatility for different metals. In terms of production, a typical sustained growth rate was between 0.0 and 5.0%. 78% of all 20-year growth rates for this data set fell within this range and 89% of all production rates were positive. Conversely, there is more unpredictability with values and most 20-year points fell within the range of -2.5% to 2.5%. Most firms, however, care about yearly trends when observing price trends. In this survey, nearly 60% of yearly price growth rates were either less than -10.0% or greater than +10.0%. In addition to the extremity of the rates, there is much volatility with value data. Coefficients of variation ranged from 26% to 112%. Coefficients greater than unity are considered hyper-variant and therefore exhibit an extreme level of dispersion. Surprisingly, there was little price volatility difference between metals on (0.28) and off (0.31) open exchanges. Studies of tantalum and niobium showed an interesting correspondence with production rates (as would be expected) as well as price.

In the future, this study could be modified in a number of ways to develop a greater understanding of both the growth and volatility of various metals and minerals. An essential first step would involve expanding the survey. Analyzing only 16 materials leaves little room to make significant generalizations about different classifications of metals. A larger survey would also allow for the expansion of the types of categorizations that could be made.

Moreover, a correlation with end-market demand could be studied. As an example platinum tends to trend very similarly to automotive demand. Trends of other metals with a significant percentage of end-use in particular industrial sectors could be taken into consideration. A second manipulation could be related to the geographic range of a material's availability. For metals that are only mined in one or two countries, it would be expected that their volatility would be much greater as source nations have a near-monopoly on the market. The tantalum case study provided some context regarding the impact of civil war in African nations that provide a significant supply of the metal. A quantitative assessment of the impact of a metal's supply being largely derived from conflict countries could be of particular interest to firms especially with the recent emergence of the Conflict Minerals Law.

VI. Appendix

Table 20 Missing Values

Value	Production
Beryllium starts at 1935	Beryllium starts at 1935
Copper starts at 1935	Copper starts at 1935
Magnesium starts at 1915	Magnesium starts at 1937
Tantalum starts at 1964	Tantalum starts at 1969
Pig iron ends at 1989	Pig iron starts at 1910
Molybdenum starts at 1912	
Niobium ends at 2000	
Rare earth metals start at 1922	
	Tin starts at 1905
	Lead missing 1901-05, 14-18, 26, 37, 40-44

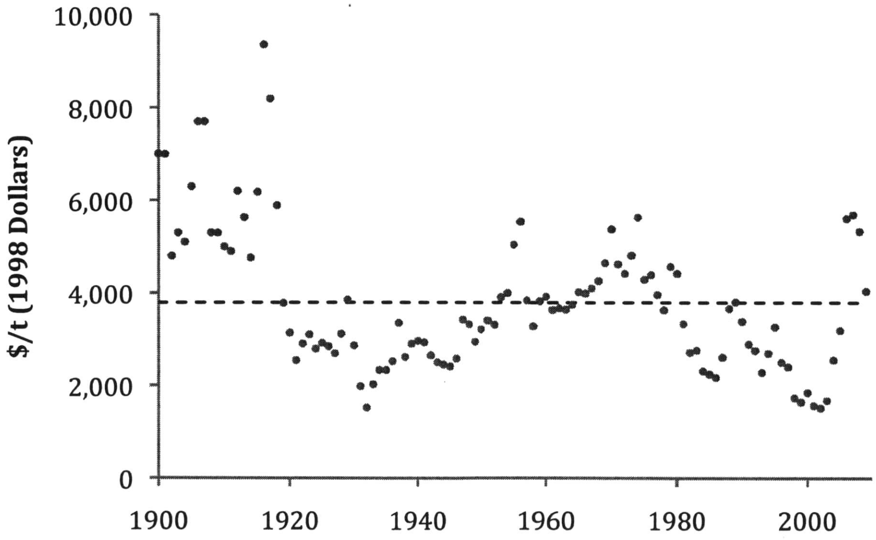

Figure 18 Bivariate fit of copper value by year

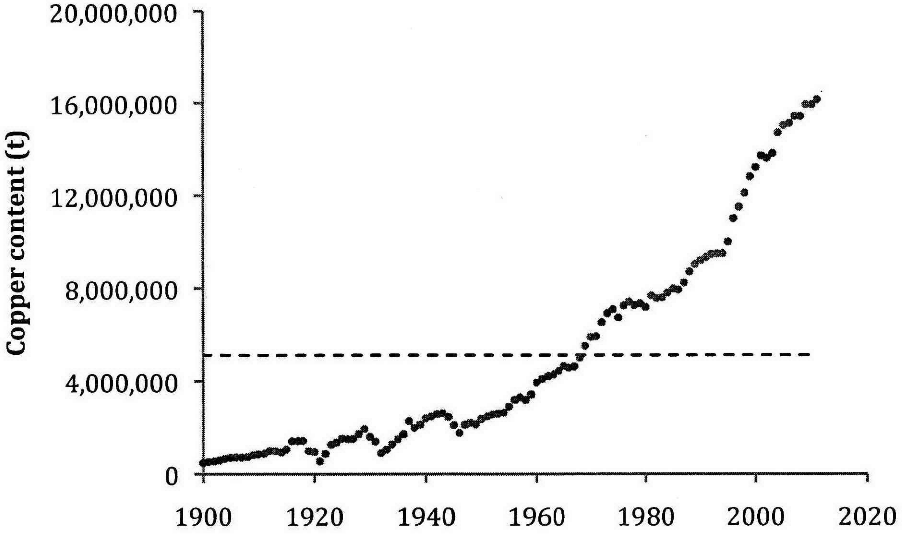

Figure 19 Bivariate fit of copper production by year

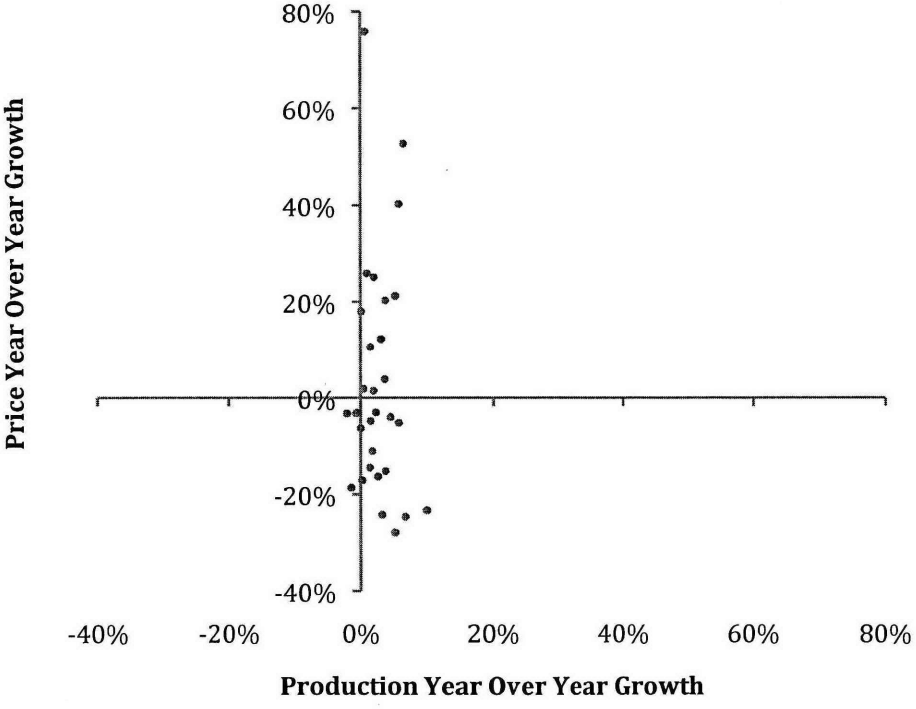

Figure 20 Growth of price versus growth of production

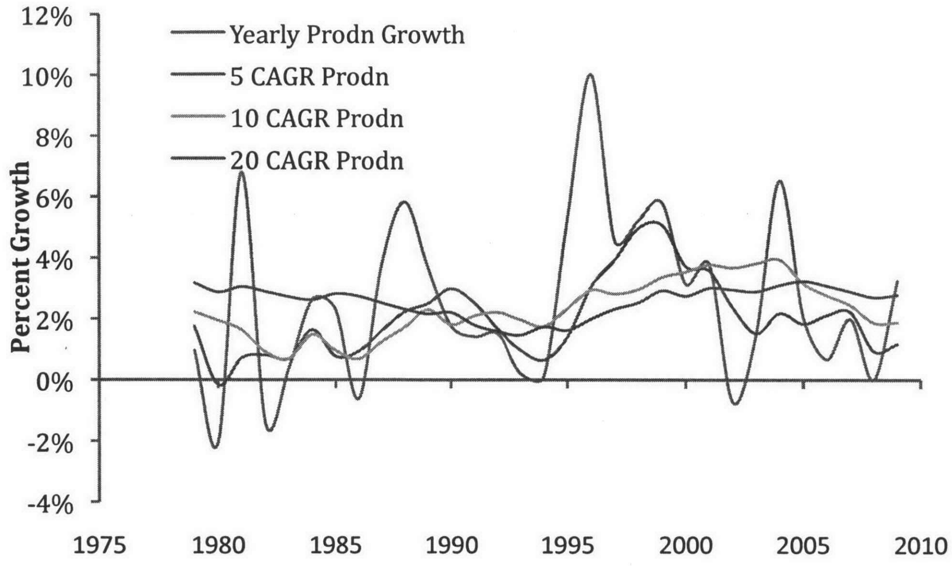

Figure 21 Copper's trending CAGRs for 1, 5, 10, and 20 years

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