Measurement of $\Upsilon(1S)$ Spin Alignment with the CMS Detector

by

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Abstract

This thesis presents a measurement of the spin alignment of prompt $\Upsilon(1S)$ mesons produced in proton-proton collisions at $\sqrt{s} = 7$ TeV at the Large Hadron Collider using the Compact Muon Solenoid detector. Approximately 1 fb$^{-1}$ of data taken during the year 2011 is analyzed. The decay to two muons is used to identify these decays, and the angular distribution of the two muons is measured. The method is designed to measure the tensor polarization with minimal assumptions about the production mechanism involved. The decay distribution of the muons is measured in the full two dimensional angular space as a function of the transverse momentum and rapidity of the $\Upsilon$, and the analysis is repeated in the helicity and Collins-Soper frames. A frame invariant quantity is calculated in each frame from the measured decay distribution and compared. The final result disfavors large polarization, but suggests the presence of some anisotropy in the decay.

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Chapter 1

Introduction

The production of quarkonia in hadron collisions is a topic of active investigation into the nature of the strong force, governed by the theory of Quantum Chromodynamics (QCD). Measurements in proton-antiproton collisions at the Tevatron collider have proven to be a theoretical and experimental puzzle. Further experimental input from proton-proton collisions at higher energy at the Large Hadron Collider provide an excellent opportunity to shed light on this topic, and will hopefully lead to a well grounded model for the process. Measuring the spin alignment of \( \Upsilon \) mesons is an essential ingredient in the study of these processes. This measurement has been conducted using the Compact Muon Solenoid detector, one of the large general purpose detectors at the accelerator.

A successful theoretical model to explain the production of quarkonia in hadron collisions must be able to predict the cross section for the production of charmonium and bottomonium as well as their spin alignments. When the \( J/\psi \) cross section was initially measured at the Tevatron, the then current calculations at leading order proved to be off by more than an order of magnitude, and multiple order of magnitude off at high transverse momentum. The prediction for \( \Upsilon(1S) \) was also off by about an order of magnitude at high transverse momentum. Since that time there have been a number of theoretical improvements, both calculations at higher orders and the addition of production models, that have reduced the disagreement. This is also true for the heavier \( \Upsilon(1S) \) state.
However, each of these different model calculations result in different predictions for the spin alignment, or polarization, of the produced mesons [1]. This refers to the spin states of the produced spin-1 mesons at production time. Because the mesons are spin-1 they can have both vector and tensor polarizations, but not all of the components are measureable. Because of parity conservation only the tensor polarization will contribute to this production and decay process. In simplest terms, there are two "maximal" polarizations, referred to as transverse (photon-like) and longitudinal. The final spin state is expected to be a mixture of different polarization states that can be aligned along different axes. The final mixed state is governed by the production process or processes contributing. The spin alignment is experimentally accessible by studying the angular distribution of the daughter particles in decays. In the analysis presented in this thesis, the decay into a pair of muons is exploited to measure this distribution.

In the following sections, both the production and decay of Υ(1S) mesons will be discussed. Some of the different models for the production mechanism will be discussed. The connection between the spin alignment at production time and the final angular distribution of the decay products used to make the measurement will be derived. And a discussion of the current experimental results from the Tevatron experiments will be presented.

1.1 Quarkonium Hadro-Production

Quarkonia are mesons composed of a quark-antiquark pair of the same flavor together in a bound state. The term is usually reserved for the heavy quark flavors bottom and charm. The lowest mass spin 1 c ¯ c state is the J/ψ, and the lowest mass spin 1 b ¯ b state is the Υ(1S) which will be used in this analysis. The top quark decays too quickly to form hadrons, and the three light quarks are treated separately. In hadron collisions, quarkonia are produced via the strong interaction. The theory describing the strong interaction is known as Quantum Chromodynamics (QCD).

This theory describes the interaction of quarks and antiquarks with gluons in the
Standard Model. It represents the SU(3) component of the standard model [2]. All particles participating in the strong interaction are said to be “colored;” there are three types of color charge in the theory. There are six flavors of quark arranged into 3 generations. There are eight gluons corresponding to the adjoint representation of the SU(3) color group.

As far as is known, there exists no observable free, colored particle. Quarks and gluons together form into hadrons, mesons and baryons, that are in a color singlet state. But in high energy interactions, at the short distances inside of hadrons, the quarks and gluons behave like free particles. This property is known as asymptotic freedom [2]. Quantitatively this can be seen in the variation of the strong coupling constant as a function of the exchanged momentum in an interaction, shown in Section 1.1. At high energies, hard interactions of quarks and gluons can be described using standard perturbation theory. But at lower energy, long range interactions are fundamentally non-perturbative and must be modelled differently.

Figure 1-1: Evolution of the QCD coupling constant $\alpha_s$ as a function of the momentum transfer $Q$, from [3].

The production of quark bound states in hadron collisions straddles the boundary between the perturbative and non-perturbative regimes. Describing the production of quarkonium in hadron collisions is a multistep procedure, with each step corresponding to a particular regime. The initial hard scatter of the incoming partons (quarks or gluons) into a heavy quark-antiquark pair is followed up by a long-range evolution of
the pair into the final, colorless bound state, potentially interacting with other colored particles. These steps must be described differently by the theoretical framework used to make predictions. The initial hard scatter can be described by perturbative QCD, but the evolution into the bound state takes place in the non-perturbative regime.

The assumption of using different steps that can be calculated separately is called factorization. Factorization theorems divide a cross section calculation into a sum of short distance coefficients with matrix elements containing the non-perturbative physics [1]. In general they must be proven to converge. The difference between different theoretical models for quarkonium production is embodied in the calculation of the non-perturbative contributions to the production. In the following sections, two important models, the Color Singlet Model and NRQCD, will be discussed in some detail. This will not be an exhaustive review of the phenomenological landscape, but hopefully provides some insight in to the issues being examined. It is necessary to predict as many measurable parameters of the produced particle as possible because many models will produce compatible predictions for some observables. Cross section and polarization are the two most important observables to test these models.

In this measurement the “promptly” produced T(1S) mesons (the lowest energy spin 1 state in the s wave) are considered. Prompt particles are created at the interaction point, by any possible process that does not displace them. There are not expected to be non-prompt T's displaced from the interaction point, but a substantial fraction of the mesons will come from feeddown decays of other b̅b states, in both the s and p wave. Prompt production includes these production mechanisms, not only processes in which the 1S state is directly produced. A diagram of the decay chains between different b̅b states can be found in Fig. 1-1. It is very difficult to separate these feeddown decays, and essentially impossible with the current limited data set, so the measurement made here includes their contributions to the final spin alignment of the T(1S). Therefore, theory predictions need to include also the production of the higher mass states from the quark-antiquark pair and their subsequent decay, not just the directly produced T(1S), in order to be compared precisely with these experimental results. However, even without a full prediction of
the feeddown contributions it is possible to rule out some models in certain scenarios. Quarkonia from feeddown decays are predicted to have a particular polarization, so while the size of their ultimate contribution might vary, the direction it should move is more certain.

![Diagram of spectroscopy and feeddown decays of $b\bar{b}$ bound states.]

In order to better understand the range of differences between different production models, two important ones are examined here in more detail.

### 1.1.1 Color Singlet Model

The Color Singlet Model [4] (CSM) is the oldest model for hadro-production of quarkonia, originating soon after the discovery of the $J/\psi$. The defining feature of this model is that the produced $q\bar{q}$ pair shares the same quantum numbers as the final quarkonium. Thus, the pair is produced in a color-singlet state, with the same spin and angular momentum quantum numbers as the final meson. An example of a Feynman diagram in this model is found in Section 1.1.1, where the formation into a bound state is contained inside the circle.

Some parameters of this model are fixed from data on decay rates of quarkonia. After this, there are no free parameters in this model. This model was successful in predicting results from low energy collisions [5], but a problem was immediately found in explaining the cross section for $J/\psi$ production at the Tevatron. However, more recent calculations have shown that higher order interactions have very large contributions to the cross section, especially for the $c\bar{c}$ states and especially at high
transverse momentum ($p_T$) [6]. The cross section predictions change by more than an order of magnitude, as can be seen in Fig. 1-4(a). This is troubling regarding the convergence of the expansion, however it also provides for the possibility for agreement with the experimental data without changing the model. In particular, including some estimate of Next-to-Next-to-Leading-Order (NNLO) corrections for the $\Upsilon(1S)$ cross section, and applying a reasonable feeddown fraction, the prediction can come very close to reproducing the data, albeit with large theoretical uncertainties due to the corrections.

In this model, the directly produced quarkonia will be mostly longitudinally polarized, with an axis of symmetry corresponding to the quarkonium direction, when calculated at Next-to-Leading-Order (NLO) or higher, as in Fig. 1-4(b). This effect is strongest at high $p_T$. It is expected that the prompt production, including feed down, will have a diluted version of this effect. Full calculations are not currently available, but it is expected that the polarization of the feeddown component should be mostly transversely polarized [6].
1.1.2 The NRQCD Approach

The second modelling approach for quarkonium hadro-production considered here is Non-Relativistic Quantum Chromodynamics (NRQCD). This is a factorization approach that provides a full effective theory of QCD at momentum scales lower than the quarkonium mass [7]. The infinite number of long distance matrix elements are expanded in terms of the relative quark velocity \( v \). It was developed to try to bridge the gap between Tevatron cross section data and theoretical predictions. It includes the contributions from the CSM, but extends them to a full set of color octet contributions. The CSM corresponds to the color singlet production part of NRQCD at lowest order in \( v \). For example, the diagram shown in Section 1.1.2 involving fragmentation of a quasi-real gluon is included in this formulation. This model is believed by many to provide the best description for heavy quarkonium production, but it still has some shortcomings.

The factorization theorem for NRQCD has not been proven, but only demonstrated up to two-loop order. It is possible that higher order terms with very soft gluons might invalidate the expansion. Because of the double expansion in \( v \) and \( \alpha_s \), some higher order terms in the expansion may contribute to the cross section more than lower orders. However, the approach has had some phenomenological success at
Figure 1-5: Example of a lowest order diagram used in the NRQCD approach that is not contained in the Color Singlet Model. Soft gluon interactions must take place during the evolution into a bound state enclosed in the circle.

describing experimental data even at leading order [8].

Cross sections in NRQCD are predicted to have a stronger high $p_T$ tail, but this is difficult to distinguish with current data sets. Including theoretical uncertainties, the additional contributions to the cross section beyond the singlet model are allowed by, but not required to explain, the current experimental data. However, the predictions for the polarization of the quarkonia are very different than in other models, particularly at high $p_T$. While the singlet model predicts a strong longitudinal polarization, NRQCD predicts a strong transverse polarization [8].

1.1.3 Other Models

There are other models that have been developed as well to describe the production of quarkonia in hadron collisions, with varying degrees of similarity with the two just discussed. But it holds true in general that because of large uncertainties in the predictions for cross sections, those measurements alone cannot distinguish between different models. Other observables are necessary, and the spin alignment is one of the most powerful, representing an important next step to understanding this aspect of QCD. Two models that predict essentially opposite polarization have been discussed. Other models also have different predictions, in particular the Color Evaporation Model (CEM) [9] predicts a randomization of the spin direction that would create
an isotropic decay distribution. So the best model for the production is still an open question, and experimental input is key to resolving it. Before discussing the results on polarization at the Tevatron, it is essential to understand the features of the decay process of the $\Upsilon$ meson.

### 1.2 Decay of $\Upsilon$ into Muons

In order to measure the spin alignment of the produced $\Upsilon$s, the angular distribution of the decay products is examined. The decay to two muons is utilized in order to have a clean experimental signature. This decay proceeds via the electromagnetic interaction. While it only accounts for approximately 2.5% of all decays, muons are the simplest particle to identify in the detector, and the well measured trajectories of muons mean the decay is relatively simple to analyze to extract the spin alignment. This decay proceeds very quickly, and the muons are detected emerging from the primary interaction point of the proton-proton collisions.

The two types of maximal polarization (transverse and longitudinal) have different decay probabilities as a function of the angle of the daughter muons. A cartoon of the shape of the decay distribution for the muons for these two simple spin alignments is shown in Fig. 1-6. By measuring the decay distribution of the muons from $\Upsilon$ decays, one measures a certain mixture of these shapes that reflects the spin alignment of the mesons based on the mechanism that produces them.

![Figure 1-6: Shape of maximally polarized decay distributions, (a) transverse and (b) longitudinal.](image-url)
In general it is expected that the produced mesons are not in one single pure quantum state because of the nature and mixture of different production mechanisms. The produced Υ’s spin alignment is described by a general spin density matrix. The spin density matrix at production time will drive the final decay distribution that will be measured. In the following subsections, the choice of analysis coordinates and the derivation of the final decay distribution will be discussed.

1.2.1 Choice of Analysis Frame

In order to measure the decay distribution of the muons from Υ decay, it is of course necessary to choose a set of coordinate axes from which to measure the decay angles. It is not possible to know a priori along which axis the meson has its spin aligned. In this section, different possible choices of coordinates will be examined, and the effects of this choice spelled out. While the underlying physics is the same in each coordinate system, the experimental sensitivity and ease of interpretation will vary.

It is first necessary to define the laboratory coordinates used that are based on the collider and detector geometry. The line of the colliding proton beams forms the z axis, with the cylindrically shaped detector around it. Along the azimuthal direction φ the detector is very symmetric, and in general we do not consider variations along it. The more important coordinate is then the polar angle with respect to the beamline θ. But in the actual experiment when referring to particles passing through the detector, the pseudorapidity \( \eta = -\log (\tan (\theta/2)) \) is instead used, which is zero for particles travelling transversely to the beam (the “central” part of the detector) and tends to infinity for particles travelling along the beam.

This is in approximation to the rapidity of particles in hadron collisions; for massless particles they are equal. When discussing the direction of the produced Υ mesons rapidity is always used since the mass of the Υ at 9.46 GeV/c\(^2\) is substantial; it is defined:

\[
y = \frac{1}{2} \log \left( \frac{E + p_z}{E - p_z} \right)
\]

In addition to \( \eta \) or \( y \), the other important kinematic variable to consider is the
Figure 1-7: Illustration of kinematic variables describing the direction of the $\Upsilon$ in the lab frame. $p_1$ and $p_2$ represent the directions of the incoming protons. In this example the transverse momentum is entirely along $\hat{y}$ but it is of course not so in general.

component of momentum transverse to the beam line, labelled $p_T$. To describe the kinematics of a particular $\Upsilon$ decay event, one needs to specify five parameters, since there are two particles in the final state and the muon and $\Upsilon$ mass are of course fixed. In all parts of this analysis, the $\phi$ coordinates in the laboratory of the produced $\Upsilon$s are neglected; this variable is integrated out. There are therefore four parameters considered: $p_T^\Upsilon$, $y^\Upsilon$, and two angles describing the direction of the decay muons that will be discussed next. The setup of the $\Upsilon$ kinematics in the laboratory frame is depicted in Fig. 1-7.

The coordinate axes used to define the decay angles will always be defined in the rest frame of the $\Upsilon$; only the $p_T^\Upsilon$ and $y^\Upsilon$ variables used in this analysis do not refer to this frame. From the production point of view, there are three vectors describing the reaction: the boost direction from the $\Upsilon$ rest frame to the lab frame, the momentum vector of one incoming proton, and the momentum vector of the other proton. These three vectors form the production plane. The coordinate systems discussed here will choose this plane as the $xz$ plane, so that the unit vector $\hat{y}$ will be perpendicular to it. The sets of axes will be distinguished by the choice of $\hat{z}$. Once a $z$-axis has been chosen, the two angles $\theta$ and $\phi$ are defined as the standard polar and azimuthal angle in spherical coordinates. In the rest of the text, $\theta$ and $\phi$ always refer to decay angles, $y^\Upsilon$ and $\eta^\mu$ are used for laboratory frame directions and $\phi$ in the laboratory frame is always integrated out. As will be discussed in detail in Sections 1.2.2 and 1.2.3, the
decay distribution will always take the form:

\[
\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2 \theta + \lambda_{\theta\phi} \sin 2\theta \cos \phi + \lambda_\phi \sin^2 \theta \cos 2\phi
\]  \hspace{1cm} (1.1)

The \( \lambda \) coefficients are the polarization parameters that will be measured to determine the decay angular distribution. The values of these parameters depend on the choice of analysis frame, but the distribution can always be expressed in this form. The maximally polarized distributions of Fig. 1-6 are obtained with \( \hat{z} \) aligned along their axis of symmetry and setting \( \lambda_\theta = \pm 1 \) and \( \lambda_\phi = \lambda_{\theta\phi} = 0 \).

The determination of the \( \lambda \) parameters will be carried out using two sets of axes referred to as the helicity (HX) frame, and the Collins-Soper (CS) frame defined in the rest frame of the \( \Upsilon \). The helicity frame is defined with \( \hat{z} \) along the direction of the boost from the \( \Upsilon \) rest frame back to the lab frame. The CS frame is defined with \( \hat{z} \) along the bisection of the direction of one of the incoming protons and the negative of the direction of the other [?]. The z-axis in the CS frame is meant to approximate the direction of the incoming partons involved in the hard interaction. This choice is the natural choice for studying leading order Drell-Yan like processes. These two sets of axes are illustrated in Fig. 1-8. Once a set of axes are chosen, the decay angles correspond to the polar and azimuthal angles of the direction of the daughter \( \mu^+ \).

For a given underlying distribution of muons, the polarization parameters will transform based on the choice of coordinates. As an example, the same angular distribution is shown in each frame in Fig. 1-9. The polarization parameter \( \lambda_\phi^{CS} = 1 \) and the others are zero in the CS frame. The same decay distribution is shown again with the axes instead defined in the helicity frame (in this example case the \( \Upsilon \) momentum would be perpendicular to the beamline). The parameters are transformed to be \( \lambda_\phi^{HX} = -1/3, \lambda_\phi^{HX} = 1/3 \). While this is only one example of a pure state decay distribution with a fixed rotation between the two frames, the same idea applies to the general case that the value of the parameters and their difference with respect to a flat decay distribution depends on the choice of analysis frame. By analyzing in more than one frame, one can increase the potential sensitivity of the measurement.
The interpretation of the results can also be easier if for instance one measures an azimuthally symmetric distribution which indicates that the chosen analysis frame is a "natural" frame for describing the spin alignment. It is necessary to measure both the polar and azimuthal anisotropies of the distribution in order to understand the underlying process.

The measurement of the $\lambda$ parameters depends additionally on the kinematics of the events under consideration because of the experimental acceptance. This refers to the part of phase space in which it is possible to perform the measurement in the physical detector. The details of acceptance in CMS for this particular measurement will be discussed in Chapter 5, but to illustrate the point it is only necessary to think of a general case. The analysis of the decay distribution is always performed in the rest frame of the $\Upsilon$. Given the choice of analysis coordinates used, the difference between two frames will depend on the kinematics under consideration. For example, the rotation angle $\delta$ from the helicity to the CS frame varies event-by-event according to:

$$\cos \delta = \frac{m_\Upsilon p_\perp^\Upsilon}{m_T p^\Upsilon}$$  \hspace{1cm} (1.2)

where $m_T$ is the mass of the $\Upsilon$ including only the components of the momentum
Figure 1-9: The same decay distribution analyzed in different coordinate frames, with the different polarization parameters that would be measured in each. In the Collins-Soper frame, (a), the distribution is described by only 1 parameter with maximal value, but in the helicity frame, (b), the distribution is described by two non-zero parameters.

vector transverse to the \( z \) direction.

In general it cannot be expected that the analysis frame will differ from the axes of symmetry of the decay distribution by a static rotation. If this angle varies based on the event kinematics, then the measured polarization parameters necessarily depend on those kinematics as well since a set of many events are always used. An experiment will measure the average polarization parameters for the set of of events considered, which will be different from experiment to experiment. Consider the set of thought experiments in Fig. 1-10. Each experiment measures the same quantity using datasets with the same underlying polarization; the only difference is the rapidity coverage in which the \( T \) spin alignment is measured. Even assuming everything is done correctly, they will obtain different values for the polarization parameters, and an “artificial” \( p_T \) dependence is induced that varies between them. They will also measure non-zero (and varying) values for the other \( \lambda \) parameters as well. If all experiments had instead chosen to use the CS frame to analyze, all would have the same value \( (\lambda^{CS}_\theta = 1) \) for all \( p_T \).

At first glance it would seem that these results are simply incompatible with one another, but it is not really the case. There is simply not enough information to make the comparison using only \( \lambda^{HX}_\theta \). This issue can be overcome in a few ways. It is
helpful to measure in more than one frame to increase the chance to use the "correct" one. It is important to bin in rapidity as well as $p_T$; if for instance the experiment with rapidity coverage out to 1.8 had used 3 bins of rapidity each of size 0.6, then the innermost bin would produce an identical result to the experiment with coverage out only to 0.6. If a theory predicts a lack of rapidity dependence in a given coordinate frame, it is essential to measure the parameters as a function of rapidity to verify this. And by measuring the full two-dimensional angular distribution, one can study frame independent quantities, which will be discussed in Section 1.2.4.

1.2.2 Decay Distribution for a Pure State

While in general the produced $\Upsilon$ mesons will not be in a pure quantum angular momentum state, understanding how such a state decays is illustrative. By applying simple angular momentum rules, one can understand the angular distribution of the produced muons. We define the state of the spin 1 meson in terms of the basis states $| J m \rangle$ as:

$$| \Upsilon \rangle = c_1 | 1 1 \rangle + c_0 | 1 0 \rangle + c_{-1} | 1 -1 \rangle$$

(1.3)
The coefficients obey the usual normalization rule \( \Sigma_i |c_i|^2 = 1 \). The basis states are quantized along an arbitrary \( \hat{z} \) axis in the rest frame of the meson. This state undergoes an electromagnetic decay to the two muon state. The angular momentum of the two muon state is quantized along their common axis in the direction of the \( \mu^+ \), labelling this axis as \( \hat{z}' \) and the associated \( J_z \) quantum numbers as \( m' \). The two muon system must still have total angular momentum \( J = 1 \). In this decay, the mass of the muon is much smaller than the mass of the \( \Upsilon \), then given the nature of the interaction, helicity will be conserved at the vertex. This requires that \( m' \neq 0 \). The contraction of the dimuon angular momentum state with the \( \Upsilon \) state then gives the transition amplitude for the decay as a function of the decay angle of \( \hat{z}' \) with respect to \( \hat{z} \). To solve for this, we use the necessary Wigner D-matrices and the spherical angles:

\[
|J m'\rangle = \sum_{m=-J}^{J} D_{mm'}^{J}(\theta, \phi) |J m\rangle
\]  

We do not distinguish in the experiment between the two possible helicity states of the muon pair corresponding to \( m' = 1 \) and \( m' = -1 \) because they produce distributions differing by a reflection. These probabilities will add to give the final decay distribution. An example with a simple initial state is:

\[
|\Upsilon\rangle = |1 1\rangle
\]

\[
|\langle 1 m' = 1 | 1 1\rangle|^2 = |D_{11}^{1}(\theta, \phi)|^2
\]

\[
= \frac{1}{4} (1 + \cos \theta)^2
\]

\[
|\langle 1 m' = -1 | 1 1\rangle|^2 = |D_{1-1}^{1}(\theta, \phi)|^2
\]

\[
= \frac{1}{4} (1 - \cos \theta)^2
\]

\[
\frac{dN}{d\Omega} \propto \frac{1}{4} (1 + \cos \theta)^2 + \frac{1}{4} (1 - \cos \theta)^2
\]

\[
\propto 1 + \cos^2 \theta
\]

This is the decay distribution for a purely transversely polarized particle along the \( \hat{z} \) axis. The same distribution is also achieved starting from the state \( |\Upsilon\rangle = |1 -1\rangle \).
Starting from the state $|\Upsilon\rangle = |10\rangle$ gives a distribution of the form $1 - \cos^2 \theta$, which is fully longitudinally polarized. Representations of these two polarization states relative to the coordinate system in which $\lambda_\phi$ and $\lambda_{\theta\phi}$ are zero are found in Fig. 1-11.

![Figure 1-11: Maximally polarized decay distributions](image)

For a general initial pure state the decay distribution in terms of polarization parameters $\lambda$ and $A$ is given by [11]:

$$
\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2 \theta + \lambda_{\theta\phi} \sin 2\theta \cos \phi + \lambda_\phi \sin^2 \theta \cos 2\phi + \lambda_{\theta\phi}^\perp \sin 2\theta \sin \phi + \lambda_\phi^\perp \sin^2 \theta \sin 2\phi + 2A_\theta \cos \theta + 2A_\phi \sin \theta \cos \phi + 2A_\phi^\perp \sin \theta \sin \phi \quad (1.12)
$$

However, not all these terms contribute in the end. The terms with a coefficient labelled $A$ are parity violating, and not allowed for this decay. The terms with coefficient superscript $\perp$ are asymmetric about the production plane. While a particular $\Upsilon$ could be in such a spin state that these terms contribute, there will be an equal number with the opposite value of the parameter (so asymmetric in the other direction by the same amount), because the experimental setup is symmetric. Removing those terms, the reduced form of the distribution is what will be used to describe the
measured distribution in data:

\[
\frac{dN}{d\Omega} \propto \frac{1}{(3 + \lambda_\theta)} \left( 1 + \lambda_\theta \cos^2 \theta + \lambda_{\theta\phi} \sin 2\theta \cos \phi + \lambda_\phi \sin^2 \theta \cos 2\phi \right) \quad (1.13)
\]

In terms of the initial pure state coefficients, the polarization parameters are given by:

\[
\begin{align*}
\lambda_\theta &= \frac{1 - 3|c_0|^2}{1 + |c_0|^2} \\
\lambda_{\theta\phi} &= \frac{\sqrt{2} \text{Re}[c_0^*(c_1 - c_{-1})]}{1 + |c_0|^2} \\
\lambda_\phi &= \frac{2 \text{Re}(c_0^*c_{-1})}{1 + |c_0|^2}
\end{align*}
\quad (1.14)
\]

It is interesting to note that a single pure state cannot have all three of these parameters simultaneously be zero. This can only occur for certain mixed initial states. Mixed states will be discussed in the following section.

### 1.2.3 Spin Density Formalism for Mixed States

In general, the \( \Upsilon \) meson is produced in a mixed state of different pure polarization states taken in a classical average. In this section, the form of the angular decay distribution when the \( \Upsilon \) decays into two muons will be derived for the general case. This mixed state can be described in terms of a spin density matrix \( \rho \). Given a basis of spin angular momentum states \( | s s_z = m \rangle \), and given probabilities \( p_i \) of each pure state in the mixture, and where each pure state has coefficients \( c_m \) for each \( s_z = m \) basis state, the elements of the spin density matrix are given by:

\[
\rho_{mm'} = \sum_i p_i c_m^i c_{m'}^{i*}
\quad (1.15)
\]

For the spin-1 \( \Upsilon \) meson, this is a 3x3 matrix with eight independent real parameters. Alternatively, one can express the degrees of freedom in terms of complex parameters in a multipole expansion. This will be more convenient for writing out the terms of
the angular decay distribution. In this case one defines:

\[ t_{LM}^{L^*} = \sum_{m,m'} \langle sm \mid sm' ; LM \rangle \rho_{mm'} ; \quad 0 \leq L \leq 2s \text{ and } -L \leq M \leq L \]  \hspace{1cm} (1.16)

The coefficient of each \( \rho_{mm'} \) is simply the Clebsch-Gordon coefficient for the given quantum numbers. For all \( L t_{0}^{L} \) is real, and \( t_{0}^{L} = 1 \).

The spin density matrix of the parent \( \Upsilon \) and the dynamics of the decay specify the decay distribution of the daughter muons. This is a parity conserving, electromagnetic decay of a spin-1 particle to two spin-1/2 particles. The general decay distribution in the \( \Upsilon \) rest frame in terms of the spherical harmonics is given by [12]:

\[ W(\theta, \phi) = \frac{1}{4\pi} \sum_{L=0}^{2} C_{L}(0, 0; 0, 0) \sum_{m=-L}^{L} t_{LM}^{L^*} Y_{L}^{M}(\theta, \phi) \]  \hspace{1cm} (1.17)

where the \( C_{L} \) coefficients include the dynamics of the decay. Because this decay is parity conserving, \( C_{1}(0, 0; 0, 0) = 0 \). The other two \( C \) terms for a spin-1 particle decaying to two spin-1/2 particles are given by:

\[ C_{L}(0, 0; 0, 0) = \sqrt{3} [(1 - \epsilon) \langle L, 0 \mid 1, 0; 1, 0 \rangle - \epsilon \langle L, 0 \mid 1, 1; 1, -1 \rangle] \]  \hspace{1cm} (1.18)

For the electromagnetic interaction, \( \epsilon = (1 + 2m_{\mu}^{2}/m_{\Upsilon}^{2})^{-1} \) because it is approximately helicity conserving. In this analysis we approximate this by one because the \( \Upsilon \) is much more massive than the muon. This gives \( C_{0} = 1 \) and \( C_{2} = 1/\sqrt{2} \). Substituting into the general decay distribution, expanding, and combining terms, one finds:

\[ W(\theta, \phi) = \frac{1}{(4\pi)^{3/2}} \left[ 1 - \frac{1}{2} \sqrt{\frac{5}{2} t_{0}^{2}} \right] \\
+ \frac{1}{(4\pi)^{3/2}} \frac{3}{2} \sqrt{\frac{5}{2} t_{0}^{2}} \cos^{2} \theta \\
+ \frac{1}{8} \sqrt{\frac{15}{2\pi}} \sin^{2} \theta \left( \text{Re}(t_{2}^{2}) \cos 2\phi + \text{Im}(t_{2}^{2}) \sin 2\phi \right) \\
- \frac{1}{8} \sqrt{\frac{15}{2\pi}} \sin 2\theta \left( \text{Re}(t_{1}^{2}) \cos \phi + \text{Im}(t_{1}^{2}) \sin \phi \right) \]  \hspace{1cm} (1.19)
This corresponds functionally to the angular distribution already presented for the two muons. The $L = 1$ multipole parameters are not measurable because of parity conservation. The imaginary parts of the multipole parameters correspond to angular terms that are asymmetric about the production plane, and are not accessible experimentally. This leaves only three real parameters to specify the decay distribution as expected. In terms of the polarization parameters $\lambda$ defined earlier, the measured multipole parameters are given by:

$$
\begin{align*}
  t_0^2 &= \sqrt{\frac{2}{5}} \frac{2\lambda}{3 + \lambda}, \\
  \text{Re}(t_1^2) &= -\sqrt{\frac{2}{15\pi}} \frac{3}{3 + \lambda} \lambda_{\theta\phi}, \\
  \text{Re}(t_2^2) &= \sqrt{\frac{2}{15\pi}} \frac{3}{3 + \lambda} \lambda_{\phi}
\end{align*}
$$

And these multipole parameters are represented in terms of the spin density matrix elements by:

$$
\begin{align*}
  t_0^2 &= \sqrt{\frac{1}{10}} (\rho_{11} + \rho_{-1-1} - 2\rho_{00}) \\
  \text{Re}(t_1^2) &= -\sqrt{\frac{3}{10}} \text{Re} (\rho_{10} - \rho_{0-1}) \\
  \text{Re}(t_2^2) &= \sqrt{\frac{3}{5}} \text{Re} (\rho_{1-1})
\end{align*}
$$

1.2.4 Frame Invariant Observables

Measuring the three $\lambda$ parameters describing the angular distribution is not the end of the analysis. One can take a step beyond and calculate a frame invariant quantity using those parameters. Doing so offers a number of advantages. Since a different set of parameters will be measured in more than one analysis frame, comparing the two parallel results for a frame invariant quantity provides an immediate self-check. Any differences between the measurement in the two frames should be attributable only to uncertainties in the measurement. The frame invariant quantity will show no kinematically induced effects (by way of the experimental acceptance), and therefore
can be compared across bins of rapidity to test for underlying rapidity dependence of the parameters. The result can also be more easily compared between different experiments. In particular this analysis will use the quantity [13]:

\[ \tilde{\lambda} = \frac{\lambda_\theta + 3\lambda_\phi}{1 - \lambda_\phi} \] (1.22)

The frame invariance of this value can be demonstrated by considering the coordinates introduced in Section 1.2.1. All of the choices of frame are characterized by the same \(xz\) production plane. For a rotation \( \delta \) about \( \hat{y} \), the quantity \( \Lambda \) is defined:

\[ \Lambda = \frac{1}{2} (\lambda_\theta - \lambda_\phi) \sin^2 \delta - \frac{1}{2} \lambda_{\theta\phi} \sin 2\delta \] (1.23)

The three measured parameters are then transformed [13]:

\[ \lambda'_\theta = \frac{\lambda_\theta - 3\Lambda}{1 + \Lambda} \]
\[ \lambda'_\phi = \frac{\lambda_\phi + \Lambda}{1 + \Lambda} \]
\[ \lambda'_{\theta\phi} = \frac{\lambda_{\theta\phi} \cos 2\delta - \frac{1}{2} (\lambda_\theta - \lambda_\phi) \sin 2\delta}{1 + \Lambda} \] (1.24)

The first two relations can be rewritten:

\[ 3 + \lambda'_\theta = \frac{3 + \lambda_\theta}{1 + \Lambda} \]
\[ 1 - \lambda'_\phi = \frac{1 - \lambda_\phi}{1 + \Lambda} \] (1.25)
Then the ratio of these two gives:

\[
\frac{3 + \lambda'^\prime_{\theta}}{1 - \lambda'^\prime_{\phi}} = \frac{3 + \lambda_{\theta}}{1 - \lambda_{\phi}}
\]

\[
\frac{3 + \lambda_{\theta}}{1 - \lambda_{\phi}} - 3 \frac{1 - \lambda_{\theta}}{1 - \lambda_{\phi}} = \frac{3 + \lambda_{\theta}}{1 - \lambda_{\phi}} - 3 \frac{1 - \lambda_{\theta}}{1 - \lambda_{\phi}}
\]

\[
\frac{\lambda_{\theta} + 3 \lambda_{\phi}}{1 - \lambda_{\phi}} = \frac{\lambda_{\theta} + 3 \lambda_{\phi}}{1 - \lambda_{\phi}}
\]

\[
\bar{\lambda}' = \bar{\lambda}
\]  

(1.26)

There are a whole class of such rotation invariant observables, but this one is chosen in particular because of a particular physical interpretation. Take the case where the total distribution is characterized by an average of \(n\) processes labelled by \(i\), each of which separately produces a simple distribution of the form:

\[
\frac{dN}{d\Omega} = \frac{1}{3 + \lambda^i_{\theta}} \left( 1 + \lambda^i_{\theta} \cos^2 \theta^i \right)
\]  

(1.27)

with different axes (and thus different \(\theta\) variables) for each process. In this case \(\bar{\lambda}\) represents the weighted average with weights \(w^i\) over the natural polarization parameters \(\lambda_{\theta}\) given by [13]:

\[
\bar{\lambda} = \frac{\sum_{i=1}^{n} w^i \lambda^i_{\theta}}{\sum_{i=1}^{n} \frac{w^i}{3 + \lambda^i_{\theta}}}
\]  

(1.28)

. The value of \(\bar{\lambda}\) goes to +1 when all contributing processes are transverse along their axes, and to -1 when all are longitudinal.

This quantity then provides a useful final result because of its ease of comparison and potential interpretations. It is important to keep in mind, though, that measuring the frame dependent parameters is still necessary in order to fully understand the underlying dynamics. Measuring rapidity dependence or the lack thereof reveals also whether or not the axes chosen do not represent natural axes for the process under consideration.
1.3 Tevatron results

Measurements of the production of quarkonia at the Tevatron collider at Fermilab provide the bulk of the current experimental input to the study of quarkonia in hadron collisions. While the Tevatron is a proton-antiproton collider instead of a proton-proton collider like the LHC, in most models the primary production process involves gluon collisions, so the results are similar qualitatively since the main difference is then the collision energy. The measurements by the experiments CDF and D0 of the production cross sections of $\Upsilon$ and $J/\psi$ mesons [14–17], and the associated spin alignments [?, 18, 19] have not provided a satisfactory conclusion to this line of inquiry.

When the cross sections for quarkonium production were measured at the Tevatron, the results were more than an order of magnitude different from the then current leading order predictions in the Color Singlet Model [1]. This spurred the development of further models and the calculations at higher order that have already been discussed. It is now clear that higher order corrections are extremely important, and while it is not always certain that a particular model will converge at higher order, the large theoretical uncertainties make it seem plausible that different models could each be capable of predicting the correct cross section.

The measurements of the polarization of $\Upsilon$ mesons and other quarkonia states from the Tevatron is a much thornier problem. In this case, not only is there a disagreement between theory and experiment, but the two experiments - CDF and D0 - disagree on the results [18, 20]. In Section 1.3, one can see the large discrepancy between the measured values of the parameter $\alpha$, which is identified with the parameter $\lambda_0$ defined in this thesis. It is important to make note of some drawbacks in these measurements.

Neither experiment measures the azimuthal decay anisotropy. Not only are the parameters $\lambda_0$ and $\lambda_{00}$ not measured, but it is assumed that any effect they have integrates to zero when projecting the decay distribution onto the $\cos\theta$ axis. This is potentially problematic because a limited detector coverage does not necessarily provide contributions from all values of $\phi$. Both experiments only perform the measurement in the helicity frame. The experiments also have different rapidity coverage.
- CDF measures only in the limited range $|y^\ell| < 0.6$ and D0 in the wider range $|y^\ell| < 1.8$. As discussed in Section 1.2.1, the values of the measured decay parameters can vary depending on the rapidity of the produced mesons. In the end there is no reason to expect the two necessarily to agree, and each measurement is incomplete.

![Graph of $\Upsilon(1S)$ polarization as a function of transverse momentum.](image)

Figure 1-12: Comparison of CDF and D0 results for $\Upsilon(1S)$ polarization as a function of transverse momentum, measuring only the polar anisotropy in the helicity frame. The CDF analysis uses a rapidity coverage of $|y| < 0.6$; D0 uses $|y| < 1.8$. This is compared to predictions from the color singlet model and NRQCD.

### 1.4 Summary

Production of quarkonia in hadron collisions is an area of QCD that is still not well understood. In the recent past many theoretical models have been developed, and many calculations extended to higher orders, in order to account for the cross sections measured at the Tevatron. All these efforts produce large theoretical uncertainties, and it is necessary to examine a different observable in order to discriminate between the different models.

The measurement of the spin alignments of the produced quarkonia is a key set of observables that can help in this regard. The predictions in different models diverge...
greatly, especially at high transverse momentum. For example, the Color Singlet Model predicts longitudinal polarization, while the NRQCD framework predicts a strong transverse polarization. However, disagreements in the two experimental results from the Tevatron leave this an open question.

This thesis will study the spin alignment of promptly produced $\Upsilon(1S)$ mesons by measuring the parameters that define the angular distribution of the decay to two muons using the CMS detector at the LHC. The analysis focuses on the fundamentals by removing as many possible assumptions about the underlying angular distribution as possible. The dependence of the polarization parameters on both the transverse momentum and rapidity will be examined. The analysis repeats the measurement in two different coordinate frames in parallel in order to maximize the sensitivity to variations. In the end, a frame invariant quantity will be calculated in each to provide an immediate self check and to allow for simpler comparisons with other experiments.
Chapter 2

The CMS detector at LHC

To carry out this measurement, the Compact Muon Solenoid (CMS) at the Large Hadron Collider (LHC) has been used. The LHC collides two proton beams of energy 3.5 TeV, for a total collision energy of 7 TeV, which is the highest in the world. It is also capable of high instantaneous luminosities - increasing so far to $L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$. This allows for the measurement of $\Upsilon$s out to large transverse momentum, where the predictions for polarization become stronger. CMS is one of two large, multipurpose detectors at the LHC. It is designed around a 4 T solenoidal magnet, with detector elements both inside and outside the magnet cylinder. This analysis depends on the ability of CMS to detect and trigger on relatively low momentum muons – the main focus of the experiment is on the detection of higher energy muons produced in electroweak and beyond the Standard Model processes. The fact that the analysis objects are near the physical limits of the detector will provide a number of experimental challenges that must be overcome in order to carry out the measurement. In the following sections, the accelerator and detector components will be described in detail.

2.1 The LHC

The Large Hadron Collider (LHC), located at the site of the European Organization for Nuclear Research (CERN) on the French-Swiss border, is the world’s largest and
most energetic particle accelerator. It is described in full detail in [21]. Two beams of protons are accelerated around a 27 km diameter underground tunnel. Originally planned to collide beams of 7 TeV protons for a total energy of 14 TeV, in the current configuration beams of 3.5 TeV are collided. In addition to proton collisions, the machine is also capable of producing collisions of lead ions with an energy of 2.76 TeV per nucleon for a total collision energy of 1.15 PeV.

There are four detectors located at the four beam interaction points around the ring. Two large, general purpose detectors are located on opposing sides of the circle - the Compact Muon Solenoid (CMS) and A Toroidal LHC ApparatuS (ATLAS). The CMS experiment is used to perform the analysis detailed here. The other two detectors are designed for more specialized purposes. The detector LHCb focuses on the physics of the bottom quark; A Large Ion Collider Experiment (ALICE) focuses on the physics of ion collisions.

The goal of the LHC is simple - to provide as high energy collisions as possible at the highest luminosity attainable. For CMS and ATLAS, the goal is to analyze collisions at a peak luminosity \( L = 10^{34} \text{ cm}^{-2} \text{s}^{-1} \), although this has not yet been reached in the data taking period so far. By the time of this writing, the luminosity has increased to \( L = 10^{33} \text{ cm}^{-2} \text{s}^{-1} \).

In order to produce the colliding beams, a multistep procedure involving different accelerator apparatuses is used. Protons from hydrogen are linearly accelerated to an energy of 50 MeV. These protons are injected into the Proton Synchrotron Booster (PSB), a ring that accelerates the protons to 1.4 GeV and then feeds them into the Proton Synchrotron (PS). There they are accelerated to 25 GeV and fed into the Super Proton Synchrotron (SPS) to be accelerated to 450 GeV. This is the injection energy for the LHC; further acceleration to 3.5 TeV takes place in the final ring itself. There are two injection points from the SPS to the LHC, one for each beam direction. An overview drawing of this procedure can be found in Fig. 2-1.

Because two proton beams travelling in opposite directions cannot be bent with the same magnetic field, two beam paths around the ring are required. Because of a lack of space in the tunnel and because it is more economical, the LHC uses twin-bore
magnets where both beams pass through the same overall mechanical and cryogenic systems through two beam paths with opposite magnetic fields. The magnets are designed to provide a peak bending field of 8.33T to allow eventual beams of 7 TeV. They utilize NbTi technology and are cooled with superfluid helium to a temperature of 2K.

The luminosity has been increased by increasing the intensity and number of bunches of protons in the LHC ring. Recent runs have had as many as 1380 bunches in each beam with $1.5 \times 10^{14}$ protons per bunch. The collision spacing has been reduced to 50ns. This has produced a total integrated luminosity by June 2011 of about 1.1 fb$^{-1}$ for use in this analysis.

2.2 CMS

The CMS detector is a large, general purpose particle detector located at one of the collision points in the LHC tunnel. It is described in full detail in [22]. It is as over 20 m long and almost 16 m high - it is only compact by virtue of the density of components inside its volume and by comparison to its rival experiment ATLAS. The cavern housing it is located approximately 100 m below ground beneath Cessy, France.

The detector is built around a solenoidal superconducting magnet with a radius of about 3 m. The magnet produces a 3.8 T field inside of its volume. Outside, circular
Iron return yokes are placed in order to channel the outer magnetic field and to provide approximately 2 T in the opposite direction. There are four major subdetector components to the CMS detector. The tracking system measures the momenta and directions of charged particles. The electromagnetic calorimeter measures the energy of showering photons and electrons. The hadronic calorimeter measures the energy of showering hadrons. The outer muon systems are used to identify and can also be used to measure the momenta of muons. Only the muon chambers are located outside the solenoidal magnet; they are placed in the return yoke. Schematic depictions of CMS in the transverse and longitudinal planes with respect to the beam axis are in Fig. 2-2.

![Image of CMS detector](image)

Figure 2-2: Schematic depictions of the CMS detector – (a) transverse to and (b) along the beamline. The solenoidal magnet is shown in gray, with the iron return yoke in red. Muon chambers are interspersed in the iron. The hadronic calorimeter is shown in yellow and the electromagnetic calorimeter in green. Inside the electromagnetic calorimeter is the inner tracker.

This analysis depends critically on the tracking and muon detection systems in CMS. It is necessary to identify and reconstruct muons with the lowest possible momentum transverse to the beam direction. The muon detectors are used to identify tracks found in the inner tracker as muons, and the inner track is used to reconstruct the path and momentum of the observed muon. The following sections will discuss in detail how this is done. The calorimeters are not used at all in this analysis, but will be included here for completeness.
2.2.1 The Silicon Tracker

The CMS inner tracker is entirely composed of silicon tracking sensors. The use of an all silicon tracker allows for precise measurement of each location where a charged particle passes through the sensor. These detectors are able to operate quickly and in a high radiation environment. Two types of these detectors are used. The innermost layers near the interaction point are made up of silicon pixels, and the outer layers are strip sensors.

The good position resolution allows for an excellent constraint of the direction of the charged particle track, including the measurement of the momentum in the non-bending direction by the combination of the transverse curvature and the angle with respect to the beamline. This results in very good reconstruction of primary and secondary vertices. And while the limited number of detector layers used due to space and cost constraints reduces the total number of hits assigned to each track, the precision of the transverse momentum measurement is still quite good. For muon tracks of a few GeV/c, this resolution has been measured and can be seen in Fig. 2-3. The resolution becomes worse at higher $\eta$ because of the decreased transverse path length and the effects of larger amounts of material passed through.

![Figure 2-3: Track momentum resolution for muons from $J/\psi$ decays in simulation and data as a function of pseudorapidity. These muons are similar to those from $\Upsilon$ decays in their $p_T$ range. The resolution is described by a parameterization derived from a fit to the line shape of the $J/\psi$.](image)

The tracker is made up of multiple layers of sensors with two main geometries. In the central part of the detector, there are cylindrically shaped "barrel" layers with
the long axis along the beam direction. At the ends of the cylinders are located disk shaped "endcap" layers perpendicular to the beam direction. A graphical representation of the tracker components sliced along the rz plane is found in Fig. 2-4. There are three layers of barrel pixels, and two disks of endcap pixels on each side of the detector. There are three layers of strips in the Tracker Inner Barrel (TIB) and six in the Tracker Outer Barrel (TOB). There are three disks of strip detectors at either end of the TIB called the Tracker Inner Disks (TID). The outer Tracker EndCap (TEC) is composed of nine disks on either side of the detector.

![Figure 2-4: Layout of CMS silicon tracker in the rz plane. The various subdetector elements are labelled. The small lines represent detector module. They are placed horizontally along the length of the cylindrically shaped barrel subdetectors; many modules together in a horizontal line is a tracker layer. They are placed vertically in the endcap; many modules together in a vertical line is an endcap disk. Each small line in one of the endcap disks belongs to one ring of the endcap disk.](image)

Each of the layers or disks of one of the strip subdetectors is subdivided into a number of modules that share electronic readout. In the endcap disks, these modules are arranged into a number of circular "rings." Depending on the z distance from the interaction point, rings are present or not present in the different endcap disks, so that complete coverage is provided out to a pseudorapidity of approximately 2.5. The barrel silicon strips are aligned parallel to the z-direction, giving a sensitive measurement of a hit in r$\phi$. The endcap strips are aligned radially, giving a sensitive hit measurement in z$\phi$. The innermost two layers of the TIB and TOB, and rings 1, 2, and 5 of TEC are double sided modules. The second side is mounted with a stereo
angle of 100 mrad with respect to the other. These modules then provide information about the third coordinate.

The first few layers of the inner tracker utilize silicon pixel sensors. This is necessary to reduce occupancy since these detectors lie at a radius to the interaction point less than 10 cm. This also provides excellent position resolution along both the φ and z directions. Each sensor is 100 × 150 μm². Charge interpolation between different pixels allows a spatial resolution of about 20 μm. The pixel detector enables excellent reconstruction of vertices in full three dimensional space, which will later be used to determine that both muons from the Υ decay come from a common point.

The strip detectors use a variety of different sensor thicknesses and pitches. The TIB uses a sensor thickness of 320 μm with pitches of 80 μm and 120 μm. This will lead to position resolutions of about 30 μm along the sensitive direction. The other subdetector elements use either the same 320 μm thickness or 500 μm in the case of the TOB and the last three rings of TEC. The sensors in TOB have pitches of 183 μm on the first four layers, and 122 μm on the outer two. The endcap sensors in TID and TEC use a variety of different pitches from about 100 μm up to 141 μm and 184 μm respectively.

Silicon tracking detectors, with the necessary electronics and cooling that come with them, are made up of an extremely large amount of material in comparison to gas detectors. In some regions of the tracker there is more than 1 radiation length of material before reaching the calorimeters. This leads to a large amount of ionization energy loss and multiple scattering that must be properly accounted during the reconstruction of tracks. This issue is compensated well by software modelling for muon tracks in the range of a few GeV such as are considered here; the final biases are known to be of the order of 0.1% [23].

2.2.2 The Muon Detectors

The detection of muons is of primary importance for CMS generally, and for this analysis in particular. To this end, CMS features a wide coverage of muon detection systems located outside the solenoid magnet, interspersed with the iron return yoke.
By placing these systems on the outside of so much material in the tracker and calorimeters, almost all non-muon particles will have completely showered; particles leaving signals in the muon detectors are then identified as muons simply by their presence there. The muon detectors cover a pseudorapidity range up to $|\eta| < 2.4$, and the full coverage region can be used to trigger collisions to be recorded.

The muon systems utilize three types of gas-based particle detectors in different regions. In the barrel region of $|\eta| < 1.2$, four layers of drift tube (DT) stations are used. In the endcap region from $|\eta| = 0.9$ to $|\eta| = 2.4$, cathode strip chambers (CSC) are used. In addition, resistive plate chambers (RPC) are used in the region $|\eta| < 1.6$. A schematic of the CMS muon system can be found in Fig. 2-5.

The DT system is used to provide information on the trajectories of passing muons in the barrel region. It is made up of four stations arranged cylindrically in between sections of iron, designed to each have a position resolution of approximately $100 \mu m$. The first three stations are made up of three “superlayers,” and the outermost is made of two such layers. Each superlayer has four layers of drift chambers. In the inner three stations, two of the superlayers have their wires arrayed along the $z$ direction to provide sensitive measurements in the $r\phi$ plane. The third superlayer is setup to provide a sensitive measurement in the $rz$ plane. This superlayer is not present in the outermost station.

The CSC system is used to provide information on the trajectories of passing
muons in the endcap. These are built into trapezoidal shaped panels arranged in groups along planes of constant $z$. The anode wires of the chambers are arranged along the azimuthal direction to provide the measurement along the radial direction, and the cathode is broken into radial strips to provide the measurement along the azimuthal direction. In each trapezoidal panel there are six gas gaps with anode wires. The CSCs are designed to provide a position resolution from about 75 $\mu$m to 150 $\mu$m.

The RPC systems are placed in both the barrel and endcap adjacent and parallel to DT or CSC chambers. These systems provide excellent timing resolution although their spatial resolution is worse than the other two muon systems. They are therefore most useful as an independent input to the trigger system and to help resolve ambiguities between different signals in the same chamber. The chambers used are double gap with a shared set of readout strips parallel to the $z$ direction in the barrel and radial in the endcap, and are operated in avalanche mode.

### 2.2.3 Electromagnetic and Hadronic Calorimeters

CMS features two main types of calorimeter. The innermost is the electromagnetic calorimeter (ECAL) and the outer is the hadronic calorimeter (HCAL). Both of these detectors are used to measure the energies of particle showers, the former resulting from the interaction of electrons and photons, and the latter resulting from hadrons. This analysis uses only muons to analyze the produced $\Upsilon$s, and does not depend in any way on the calorimeters. They will be briefly described for completeness sake.

The ECAL is made of over sixty thousand lead tungstate crystals. These crystals act both as absorber and active material in the calorimeter. This material has a high density (8.8 $g/cm^3$). The combination of a short radiation length (0.89 cm) and a small Molière radius of 2.2 cm allow a compact detector with a high granularity. These crystals are arranged to provide excellent granularity in the transverse shower direction. There are two main sections of the ECAL - the barrel (cylindrically shaped portion) and the endcap (covering either end of the cylinder). The barrel crystals have a depth of 23 cm or 25.8$X_0$, and a physical face of $22 \times 22 \text{mm}^2$ which corresponds
to approximately $0.0174 \times 0.0174$ in $\phi \times \eta$. The endcap crystals are slightly shorter with a slightly larger face.

The scintillation light from the crystals is detected by avalanche photo diodes in the barrel, and vacuum phototriodes in the endcap. These were designed especially for CMS, and chosen to be fast, radiation hard, and able to operate in the magnetic field.

The HCAL is made of four separate detector structures using different technologies in order to measure the energies of hadrons. As in the ECAL there are separate barrel and endcap structures inside the solenoid magnet. Here the calorimeter is made of brass as absorbing material and scintillator as active material. Brass was chosen in order to operate in the high magnetic field. The barrel as made up of a number of wedge shaped structures with tiles of scintillator in each wedge providing readout granularity of $0.087 \times 0.087$ in $\phi \times \eta$. The minimum depth of the barrel HCAL corresponds to 5.82 interaction lengths ($\lambda_f$). The endcap HCAL has similar granularity up to pseudorapidity $|\eta| < 1.6$, and $0.17 \times 0.17$ past that. The scintillator light is passed through wavelength shifting fibers before begin channeled to a hybrid photodiode for readout.

Because of space contraints between the tracker and ECAL inside and the magnet outside, the bulk of the HCAL inside the solenoid is limited in the number of interaction lengths it contains. There is therefore also an outer calorimeter placed outside the magnet in the barrel section in order to catch the tails of hadronic showers. This calorimeter uses iron as an absorber, and including the material of the magnet systems, the minimum interaction length is $11.8 \lambda_f$. There is also a forward calorimeter placed to cover the pseudorapidity range $3 < |\eta| < 5.2$. This forward calorimeter is based on a different technology - Cerenkov signals in quartz are used with iron as absorber.

### 2.2.4 Trigger System

The high collision rate at the LHC requires powerful, dedicated systems to decide which collision events are to be kept for later analysis. With the eventual 25ns
spacing between colliding bunches, the interaction rate will be 40 MHz. The rate of events that will eventually be kept must be of the order of hundreds of events per second. To accomplish this remarkable sifting, two levels of trigger systems are used to make the necessary decisions.

The first system, called Level 1 (L1), is made up of dedicated hardware systems. While the full detector readout signals are buffered in the readout electronics, a system made of different types of programmable logic circuits is used to select events based on the information contained in the muon systems and calorimeters. This system is made of multiple steps, and in the end builds objects corresponding to muons, electrons or photons, and jets. It also calculates global quantities, including all energy deposited in the calorimeters and the missing transverse energy of the event. In the end, these calculations are used to determine whether an event is accepted at L1, which causes the detector signals to be read out from the electronics. The L1 accept rate is limited to a maximum of 100 kHz, but has not yet been run at full capacity.

These detector signals are then brought to the High Level Trigger (HLT) computing farm. The HLT system is designed completely in software and run on commerical processors, with many events running in parallel. This allows as complicated a processing as is possible in the limited time allotted to make a final decision on an event. It is also possible to change the algorithms run in the HLT fairly easily to accommodate different running conditions. The HLT must cut down the rate of events to the final value of a few hundred per second. All events accepted by the HLT will be kept permanently for processing using the full software. A full description of the trigger systems used to identify the muons in this analysis is in the following chapter.

2.2.5 Simulation of the CMS Detector

In order to supplement the understanding of the details of the detector and various physics processes, CMS employs a complicated simulation of the entire detector. All physical elements of the detector are placed in as accurate a spatial map as possible. The GEANT4 program [24] is utilized to simulate the detector response to the passage of particles. The program provides simulation of random interactions in the material,
and creates simulated detector output signals that will be used to reconstruct events. The simulation contains also emulation of the trigger logic at all levels including the hardware L1 step. This part of the simulation is the least accurate however, and in general it is desirable to rely on trigger measurements using real data - as in Chapter 4. After simulation, the simulated detector response is processed in the same way as real data. The only exception is that of course one still has access to the information about the particle content of the central interaction. This simulation interfaces in a general way to many different types of “event generators” which provide this initial particle information.

This analysis attempts to have only minimal reliance on simulated events. It is necessary to understand the detector acceptance as will be discussed in Chapter 5. Beyond this it is only used to help quantify some of the systematic uncertainties as will be discussed in ???. The important parts of the efficiency determination will be measured with data.
Chapter 3

Reconstruction, Identification, and Selection of Muons

In order to identify events containing the muons from $\Upsilon$ decays, a series of procedures are needed. First, the detector signals left by passing muons are reconstructed to find trajectories belonging to muons. There are two major steps in this procedure. The muons must first be found when the detector is taking data in the CMS trigger systems, in order to flag a collision to be stored for later analysis. This step is referred to as “online” processing. This stored data is processed again “offline”, where there are less restrictions on computing time. It is during the offline step that the final muon objects are reconstructed and identified to be used in the analysis. Once these muon objects have been identified, pairs of muons likely to come from $\Upsilon$ decays are selected in order to carry out the measurement. These procedures will be discussed in detail in the following sections, beginning with the offline selection.

3.1 Offline Muon Reconstruction

While all events that are analyzed offline have passed trigger requirements, offline reconstruction of muons proceeds entirely independently from the steps taken in the online trigger processing, starting from the digitized detector signals stored for the selected events. This procedure is discussed first, because the the online reconstruc-
tion steps are similar but streamlined for faster processing time. Muon reconstruction depends critically on the operation of the silicon tracker and muon systems. The first step is to perform "local reconstruction," in which position measurements of charged particles are collected into hits and segments in the tracker and muon systems. Then software algorithms are used to identify the trajectories, giving the momentum and direction for likely muon candidates.

This analysis relies on the lowest momentum muons measurable with CMS, and always requires silicon tracks. These tracks are reconstructed separately without reference to any muon system information. Tracks and muon segments are then combined into muon trajectories with two major algorithms. One starts with the muon systems, reconstructing a "Stand-Alone Muon" with muon segments only, and then searching for a matching track and combining all the information into a "Global Muon." The second algorithm starts using tracks, and attempts to identify them as "Tracker Muons" by matching them to signals in the muon stations. These algorithms will now be discussed in more detail.

3.1.1 Track Reconstruction

To start the procedure of track reconstruction, charge deposits in the silicon pixels and strips must be converted into position measurements with uncertainties. Positions in the pixels are determined using a template method that uses the charge distribution to provide an estimate for the point, and provide similar resolution along both directions. In the strip tracker, the measurement along the sensitive direction is determined using the distribution of charge along adjacent strips. In the nonsensitive direction, it is only possible to place a hit as somewhere along a centimeters long module.

In the end the collection of hits must be turned into a collection of tracks. This proceeds in a number of steps, first to identify hits from the same track, and then performing a final fit to determine the track momentum and direction. The overarching algorithm used by CMS is an iterative procedure designed to identify as many tracks as possible subject to the time constraints imposed by the available computing power. High quality tracks and high momentum tracks are reconstructed first, and
the hits attached to them are removed before attempting to build other tracks. In the end there are six track building steps used in CMS reconstruction, using the same mechanics but different selections.

The first stage of each step in this procedure is seeding. The layers of the pixel tracker and some inner layers of the strip detector are used to form seeds. Triplets or pairs of hits in these layers that appear to come from a common particle are identified. Which layers are used and whether triplets or pairs are used depends on the step of the iterative procedure being carried out. These seeds also provide a rough momentum prediction based either on the helix of the three hits, or of the two hits with the defined beam spot for the event.

The next stage is pattern recognition. Layer by layer, the trajectories given by seeds are extrapolated outward. Hits in each layer compatible with the projected path are attached to the track. For seeds displaced from the innermost detector layer, the extrapolation is also carried out inwards to attach hits inside the seed. The end result is a set of hits considered likely to make up the trajectory of a single particle. It is this collection of hits that will be used to perform the final track fit.

The final fit employs two steps - the inside-out track fitting step and the outside-in track smoothing step. Starting from the seed trajectory, for each hit in the collection found during pattern recognition, an extrapolation is done from the last layer already considered to the one with the hit under consideration. This extrapolation must calculate the the particle’s bend in the magnetic field as well as the inflation of the uncertainties of the track parameters.

Additionally, in order to produce an accurate measurement of the track direction and momentum it is important to correctly account for the material that the particle passes through. The change in momentum from energy loss and the increase of the trajectory uncertainty from multiple scattering are accounted for in the track fitting procedure. In simulation a full description of the tracker in terms of the detector components and their constituent materials is implemented in GEANT4. While it is theoretically possible to use this model to assess material effects during trajectory propagation, it is practically impossible because of the long computing times.
necessarily involved.

In the track fit, a parameterized model of the tracker is used instead. The CMS simulation is used to determine the average amount of material passed through by particles entering a given set of modules of the tracker since leaving the previous tracker layer. The modules averaged over are chosen to have a similar material profile and are always part of the same layers or disks. Energy loss corrections for this amount of material and an increase in the uncertainty due to multiple scattering are calculated for each hit during the propagation.

After the propagation, the predicted trajectory must be combined with the measurement of the particle position from the hit. This is done with the Kalman Filter algorithm [25]. This is a standard mathematical procedure to produce an updated trajectory with uncertainties. This update is done using two dimensional information for each hit, except in the case of the barrel strips, in which it is done only along the sensitive direction.

Once all hits in the trajectory have been processed in this manner, the smoothing step begins. The final trajectory has its uncertainty increased by a large factor. Then each hit is considered starting from the outermost. In this case the propagation between hits reduces the uncertainty of the predicted trajectory because the assumption is still that the particle emerged from the primary interaction point. For each hit the update procedure is applied. After the innermost hit is processed, the final set of track parameters with uncertainties are obtained.

3.1.2 Offline Muon Reconstruction

Similar to how tracking begins, muon reconstruction starts by defining hits in the elements of the various muon stations. These hits are one dimensional in the DTs and RPCs, and two dimensional in the CSCs. They correspond to signals deposited in one part of the whole chamber’s readout. These hits undergo a second step of processing that is different than tracking - they are grouped together into “segments” containing hits in a single one of the muon stations.

These segments form the starting point for muon reconstruction in the muon sta-
tions. Particle trajectories are first reconstructed using these segments. By extrapolating towards the collision point, each segment has an estimate for the transverse momentum of the passing particle. These estimates are used as the seeds to do a trajectory fit on the segment level. If the segments are successfully fit, the procedure is repeated instead fitting the individual muon hits contained in the segments. The resulting trajectory is called a “stand-alone muon” track. These tracks alone are not used in this analysis; their momentum and direction resolution are not as good as tracks reconstructed using the silicon tracker.

In order to produce “global” muons that contain tracker information, the muon station tracks are matched with the collection of tracks in the silicon tracker. This is done first with a rough matching in $\eta \times \phi$, selecting a set of possible matching silicon tracks for each stand-alone muon track. Then all of these tracks are propagated to a common plane for detailed matching. This plane is chosen as the plane of the innermost muon station hit. A number of different matching criteria are calculated using the momentum and local positions with their uncertainties.

For each track candidate that passes stricter selection cuts on the matching variables, a global fit of the tracker and muon station hits is attempted. Afterwards, only the combined track with the lowest $\chi^2$ is chosen as a global muon. In this way there is only one global muon made for each stand-alone muon. While this global fit is useful for identification, it is not used in this analysis for the muon momentum measurement. It does not improve upon the performance of the silicon track alone due to the effects of multiple scattering and alignment between the tracker and muon stations. Only the measurement from the silicon track that was matched to the stand-alone muon for the global fit will be used.

In order to improve the reconstruction efficiency for low momentum muons, a full reconstruction in the muon stations and a global fit is not required. To broaden the pool of muon candidates, a separate algorithm is used to produce “tracker” muons. For each track in the silicon tracker collection, compatible muon information is searched for. This is done by propagating the track trajectory into the muon stations, then searching for nearby segments. If there is at least one segment close
enough in distance, each selected segment is associated to the track and a tracker
muon object is created. In these cases the silicon track provides all momentum and
direction information.

3.2 Online Muon Trigger

In many ways the algorithms used to identify muons in the trigger systems are similar
to those used offline. They are designed to run in the very limited time frame allowed
for making decisions during data taking. In the end, trigger decisions are used only
to decide which events should be kept. While the objects created in the trigger are
stored for use during the analysis (see in particular Chapter 4), these objects provide
no input to the offline reconstruction previously described. All of the muon detector
types in CMS participate in the trigger, giving pseudorapidity coverage equal to that
of the offline.

The muon trigger starts in the L1 hardware trigger system. This is implemented in
specially designed logical circuits at each stage of the trigger. Each of the DTs, CSCs,
and RPCs builds local segments corresponding to potential muon trajectories. Then
each detector type separately builds candidate muons from local segments. They
must then be combined in the muon global trigger. RPC candidates are merged with
matching DT candidates in the barrel or CSC candidates in the endcap. Thus at
L1, all muon candidates only involve muon system information. The logic allows up
to four candidates to be identified in an event. For the purposes of this analysis,
all events are required to have two L1 muons present without any selection on their
momentum.

In the HLT, the muons from L1 are used to begin the selection of the fully triggered
muons. These are used as seeds to reconstruct so-called Level 2 (L2) muons which
are similar to the stand-alone muons in the offline processing. The muons that will
be used in the final triggers used in this analysis are called Level 3 (L3) muons and
incorporate tracking information. In the HLT, tracking is done on a regional basis in
order to save processing time by ignoring other parts of the tracker that would not
contain information on the muons. Based on each L2 muon, hits located in parts of the tracker that could contain a corresponding muon track are used and silicon tracks made. These tracks are similar to offline silicon tracks, but the seeding requirements are stricter to save on computation time. After constructing potential tracks, and if one of the tracks is matched to the L2 muon, an L3 muon is created.

Because of the high rates of low momentum muons at CMS, it has been necessary to use strict trigger requirements in this analysis. It is completely unfeasible to trigger on the presence of only one of the two muons from an $\Upsilon$ decay without completely overwhelming the data taking abilities of CMS due to the large number of single muons from background sources like non-quarkonia heavy flavor and decays in flight. In the very early running in 2010 it was possible to only require the presence of two L1 muons for the HLT to accept the event, but quickly it became necessary to use L3 muons and finally to place stricter cuts on the pair properties.

During the 2011 running used here, it is necessary to use a set of triggers that use the same type of L3 muon objects with different requirements on the pair depending on which run period in which the data was taken. This began by requiring the muons to be opposite sign and to have a combined mass in the wide region from below the mass of the $J/\psi$ (3.1 GeV/$c^2$) to above the $\Upsilon(3S)$ (10.35 GeV/$c^2$), while each muon is also above 3 GeV/$c$ $p_T$. Then this region was forced to be further broken up with only a narrow mass window around the $\Upsilon$ states used for this analysis. Next, a change of strategy was used - there would no longer be cuts on the individual muon $p_T$, but instead only dimuon pairs in the $\Upsilon$ mass range that also had low rapidity ($|y^{\mu\mu}| < 1.3$) would be selected. And finally, the minimum pair $p_T^{\mu\mu}$ was set to 5 GeV/$c$.

Since theoretically the most interesting kinematic region is that of high $p_T$, and because in general the central part of the detector is best understood and has smaller background, this limited kinematic range provides the best possible measurement given the total rate constraints. This allows the total accepted rate to be kept roughly constant while the instantaneous luminosity grows, without sacrificing the ability to study these events in detail by placing strict requirements on the muons individually. All of these events from all of these triggers are combined into the selected dataset
used in this analysis.

### 3.3 Selection of $\Upsilon$ candidates

The selection used to identify dimuon pairs potentially from $\Upsilon$ decays is chosen to allow for the largest possible selected signal. The capabilities of the detector muon systems and the trigger selection as described in the previous sections are the most important elements driving the event selection. To begin, a set of quality muons are selected in each event of the trigger sample described in the previous section. The physical detector geometry implicitly defines a lower cut on the muon transverse momentum in order to be reconstructed. We apply an explicit kinematic selection inside of this allowed region that depends on $p_T^\mu$ and $\eta^\mu$. The determination of the kinematic selection is based on the efficiency to reconstruct the muons and will be discussed further in Chapter 4. Additional selection is applied to the silicon track part of the muon in order to select only high quality muons. The complete muon selection is as follows:

- Tracker muon or global muon
- Passes one of the kinematic selections:
  - $p_T^\mu > 3.75$ GeV/$c$ AND $|\eta^\mu| < 0.8$
  - $p_T^\mu > 3.5$ GeV/$c$ AND $0.8 \leq |\eta^\mu| < 1.6$
  - $p_T^\mu > 3.0$ GeV/$c$ AND $1.6 \leq |\eta^\mu| < 2.4$
- Silicon track has at least 10 tracker hits and 1 hit in the pixel tracker
- Silicon track normalized (by number of degrees of freedom) $\chi^2 < 5$
- Transverse impact parameter relative to the beam spot $< 0.2$ mm

After selecting the potential muons, pairs of them must be identified as $\Upsilon$ candidates. These muons must of course be of opposite charge, and their combined mass
must lie within a window around the mass of the $\Upsilon(1S)$, namely 8.5 to 11.5 GeV/$c^2$. This window includes also the $\Upsilon(2S)$ and $\Upsilon(3S)$. The $\Upsilon(2S)$ and $\Upsilon(3S)$ higher mass $b\bar{b}$ states also decay into muons, and are therefore also found when applying this selection. True muon pairs from $\Upsilon$ decays will emerge from a common location at the collision point, so in order to cut down on background pairs a vertex fit is performed. For each oppositely charged pair of muons within 4cm of each other at the beamline, the common vertex for the pair as well as the vertex probability is calculated. Pairs that do not correspond well to a common vertex will have probabilities near 0; the final cut is placed to keep only pairs with probability greater than 0.01. Figure 3-1 shows the inclusive mass spectrum of the selected $\Upsilon$ candidates, which includes more than $9 \times 10^5 \Upsilon(1S)$. In addition to the three prominent peaks, there is also a substantial background left.

![Inclusive mass distribution of selected $\Upsilon$ candidates.](image)

The measurement of the momentum of the muons used in this analysis comes solely from the silicon tracker. The muon detectors are used only to identify certain tracks as belonging to muons as described in the previous sections. The track momentum measurement is generally very good, but discrepancies from the true momentum value on the order of 0.1% are known to exist. This has been studied for tracks of a few GeV/$c$ using the position of the $J/\psi$ mass peak and its lineshape. A likelihood fit to this shape is used to measure the momentum resolution and bias, and to provide a
correction factor for each muon that depends on its kinematic variables [23]. In all steps of this analysis this correction is applied to the muons selected before applying any cuts.

For each pair of muons selected as a candidate $\Upsilon$, the decay angle variables in the dimuon rest frame ($\cos \theta_{HX}, \phi_{HX}, \cos \theta_{CS}, \phi_{CS}$) are calculated using the reconstructed momenta and directions of the two. Not all pairs of muons from $\Upsilon$ decays occurring at the interaction point will be present in this final sample. It is essential to understand how often the muons enter the volume occupied by CMS, and how often when they do will they actually be found and selected by the trigger and offline selection.
Chapter 4

Trigger and Reconstruction
Efficiencies for Single Muons

Not all muons that enter the CMS detector will be successfully identified and reconstructed by the trigger and offline systems or selected by the analysis cuts. It is essential to understand the efficiency with which they are properly found in order to correct the measured number of signal events to the true number entering the detector. While the measurement of the angular decay distribution is not reliant on the overall scale of the efficiency because only relative rates as a function of decay angle are needed, the muons studied are near the lower threshold of CMS capabilities where the efficiency to find them goes from 0% to the maximum efficiency at high transverse momentum. This necessitates a detailed understanding of the relative efficiency as a function of the muon kinematics. It is additionally very difficult to emulate the behavior of the trigger systems with such precision. To these ends, the efficiency to identify muons is measured using collision data with the “Tag and Probe” method. In the following sections, the details of this method and the results obtained will be discussed.
4.1 Tag and Probe Efficiency Measurement

The “Tag and Probe” method is a common technique used to study the efficiency to identify and reconstruct objects in collider detectors. It exploits the decay of well known unstable particles into two stable objects. One of these stable objects is identified by applying a high purity selection to it; this object is referred to as the tag. Then a candidate for the second leg, the probe, can be identified without applying strict requirements - except that, together with the tag, the pair mass corresponds to that of the well known decay. The number of probes that pass a particular selection compared with the total number identified will then correspond to the efficiency of that selection.

In general it is desirable that the probes used have the same kinematic distribution as the analysis objects for which the efficiency is needed. In the case of the $\Upsilon$, this would be possible using the dimuon pair from its own decay - a well identified muon in the detector provides an excellent tag, and the large $\Upsilon$ peaks can be used to identify the probe as coming from a muon. However, it is still necessary for the collision events used to perform the tag and probe procedure to be selected by some trigger, and whenever possible this trigger should only depend on the tag. As explained in Chapter 3, it is not feasible to take single muon triggers at the low momentum typical of dimuon quarkonia decays. Therefore, special trigger paths have been developed that identify one good muon as a tag, and require a probe object at the trigger.

Unfortunately, due to high rate these triggers are only run using the $J/\psi$ decay to two muons. In particular, these triggers identify one L3 muon and one silicon track in the HLT that together lie close to the $J/\psi$ mass of 3.096 GeV/$c^2$, and are known as “Mu+Track” triggers. In current running, it is required that the tag muon have a $p_T$ of at least 5 GeV/$c$ or 7 GeV/$c$, and the probe track have a $p_T$ at least 2 GeV/$c$ or 7 GeV/$c$ respectively. Both of these triggers are prescaled so that only a fraction of actual collision events can possibly fire it. The lower threshold pair has a higher prescale, so that fewer events that would pass it are actually kept. This is again necessary to lower the absolute rate of the trigger firing.
The dataset collected with these triggers is then used to measure the efficiency of the muon reconstruction and trigger identification of muons. The starting point for the efficiency measurement actually assumes that the muon being searched for left a silicon track in the tracker that was reconstructed without quality cuts during the offline processing. This starting point is used because the tracking efficiency has been measured separately and shown to be very high and uniform in the momentum region being used in this analysis [26]. Because this measurement is only sensitive to the variation of the efficiency as a function of the muon kinematics, and because the variation in tracking efficiency is very small compared to the total efficiency uncertainty, this variation is measured in simulation and discrepancies with the true efficiency are neglected.

The tag muon is identified as an offline global muon with the quality cuts listed in Section 3.3, with the additional requirement that the $\chi^2$ of the global tracker and muon system fit be smaller than 20. This muon must match the L3 muon object that fired that trigger by passing the requirements that $\Delta R = \sqrt{\Delta \phi^2 + \Delta \eta^2} < 0.1$ and $\Delta p_T/p_T < 0.2\text{GeV/c}$. The probe is identified as an offline track that matches the HLT track object that fired the trigger using the same matching requirements. It is possible that both muons from the $J/\psi$ decay in one event pass the tag requirement because both are selected as L3 muons and both pair up with tracks in the Mu+Track trigger (with the other muon corresponding to the track part). In this case, both combinations are considered tag-probe pairs, and both objects enter into the consideration of the probe efficiency.

Once the tag-probe pair has been identified, different levels of quality cuts can be applied to the probe. It is required that the track also be identified either as a tracker muon or as a global muon, that the muon pass the quality cuts of Section 3.3, and that the muon be selected as an L3 trigger muon online. Probes which meet all of these requirements are considered passing probes, and those which do not are failing probes. The efficiency is calculated as the number of passing probes that come from a $J/\psi$ decay, divided by the total number of probes from $J/\psi$ decays. This fit and calculation can be performed in arbitrary bins of the muon kinematics by applying
additional kinematic selections to the passing and failing probes.

It is possible to consider a series of separate classes of probe which pass one level of the selection but fail the next. When multiple classes are defined, a factorized sequence of efficiencies are obtained by simultaneously fitting each of these distinct subsets to extract all the efficiencies together. In the end this analysis does not do this; only the total efficiency for single muons is required. As will be discussed in the next section, a fit will be performed to the binned measurements to extract a more detailed dependence. Applying this fit only to the total efficiency instead of the separate factorized efficiencies simplifies the process of calculating the correct uncertainties on the efficiency.

4.2 Single Muon Efficiency Results

After the tag-probe pairs are separated into passing and failing categories, the efficiency is the number of passing probes divided by the total number. Because of the presence of background events it is necessary to fit the mass distribution of the tag-probe pairs to identify the pairs actually stemming from a \( J/\psi \) decay. This fit is performed simultaneously in the passing and failing categories with the total number of \( J/\psi \) decays and the efficiency as parameters. This fit is done separately in bins of \( p_T^\mu \) and \( |\eta^\mu| \) to trace out the kinematic dependence of the efficiency. The background is modelled with a linear dependence, and the signal peak with a “Crystal Ball” function [27] where a Gaussian core changes over to a power law dependence on the low mass side of the peak to account for final state radiation. The binned efficiency with its uncertainty is extracted directly from this fit. An example of these fits in one bin is found in Fig. 4-1, and a two-dimensional map of this efficiency is found in Fig. 4-2.

In each slice of \( \eta^\mu \) the efficiency as a function of \( p_T^\mu \) is fit using a turn on curve. Doing so offers the advantages of producing the best estimate of the efficiency possible for each particular muon and of lessening the dependence of the efficiency measurement on the spectrum contained inside of each bin. This is important for this analysis because the efficiency measurement is done using \( J/\psi \) decays which are expected to
Figure 4-1: Example fits to the mass distributions of $J/\psi$ tag and probe pairs for the bin $p_T^\mu$ (4.0,4.5) GeV/c, $|\eta^\mu|$ (0.8,1.2) – (a) probes passing muon and trigger selection and (b) probes failing.

have a different $p_T^\mu$ dependence than $\Upsilon$ decays. The curve is specified by an error function with three parameters of the form:

$$
\epsilon(p_T^\mu) = \frac{c_0}{2} \left(1 + erf \left(\frac{(p_T^\mu - c_1)}{\sqrt{2c_2}}\right)\right)
$$

(4.1)

There is one exception - in the furthest forward slice ($2 < |\eta^\mu| < 2.4$) the binned points do not well constrain the turn on since the measurement is only made above $p_T = 3$ GeV/c, and the turn on occurs below this value. In this slice, a flat efficiency function is instead used. The measured points together with the fit functions are shown in Fig. 4-3, and the fit coefficients used to determine the final measured efficiency are contained in Table 4.1. The levelling out to the plateau of maximum efficiency and the initial turn on from zero efficiency have obviously different curvatures. It would be difficult to account for this variation with a smooth efficiency curve for the entire range. However, the fit results will only be used in the selection region. The selection region is therefore defined so that the upper part of the turn on can be described well by the efficiency function. The fit function range in the figure corresponds to this range. Outside of the central region, it is possible to fit down to zero efficiency, so
the selection region is defined instead by the matching cutoff at 3 GeV/c.

The fit uncertainties are used to produce six fluctuation curves in each slice of $|\eta\mu|$. These curves are obtained from fluctuations up and down along the eigenvectors of the 3x3 covariance matrix of the function parameters. The size of the fluctuations correspond to the square root of the eigenvalues. In the case of the flat efficiency fit in the forward $|\eta\mu|$ slice, there are two fluctuations corresponding to the uncertainty on the efficiency value. These will be used in the determination of the efficiency systematic uncertainties as described in Chapter 6.

| $|\eta| < 0.8$ | $0.8 < |\eta| < 1.2$ | $1.2 < |\eta| < 1.6$ | $1.6 < |\eta| < 2.0$ | $2.0 < |\eta| < 2.4$ |
|----------|-----------------|-----------------|-----------------|-----------------|
| $c_0$ | 0.878 | 0.839 | 0.882 | 0.839 | 0.713 |
| $c_1$ | 3.894 | 3.860 | 2.984 | 2.280 | 0.000 |
| $c_2$ | 0.957 | 0.512 | 0.405 | 1.398 | 0.000 |

Table 4.1: Fitted efficiency parameters
Figure 4-3: Fits to the total efficiency for single muons as a function of $p_T^\mu$ in slices of $|\eta^\mu|$ for data taken in 2011. The blue curve is the central value of the fit, the six colored curves are the uncertainty curves corresponding to fluctuations up and down along the three covariance eigenvectors.
4.3 Efficiency Corrected Signal Yields

Once the efficiency for the single muons is measured, corrections can be applied to the selected signal candidate events in order to obtain the “true” signal yield. All dimuon candidates are weighted on a candidate by candidate basis as follows:

\[
 w = \frac{1}{\epsilon(p_{T1}^{\mu1}, \eta^{\mu1}) \cdot \epsilon(p_{T2}^{\mu2}, \eta^{\mu2})}
\]  

(4.2)

Weighting each dimuon pair by the efficiency measured for the particular kinematics of each muon separately allows the exploitation of the detailed efficiency dependence obtained through the turn-on curve fits. The mass distribution of this set of candidates with weights is then fit to extract the \( \Upsilon \) signal yield. This is necessary because of the lack of a robust prediction for the angular distribution of background events near the \( \Upsilon \) mass. This makes it impossible to fit simultaneously for the signal yield and polarization distribution as a function of mass and decay angle. Instead, the set of candidates is divided up into a number of kinematic bins of not only \( p_{T1}^{\Upsilon} \) and \( y^{\Upsilon} \) (for which the dependence of the polarization parameters will be measured), but also in the decay angles \( \cos \theta \) and \( \phi \) (all candidates are divided in parallel into helicity and CS decay angle bins). The signal yield in each bin is extracted using a fit to the efficiency weighted mass distribution.

In order to increase the number of candidates in each bin and therefore the robustness of the fit, it is advantageous to exploit symmetries in order to fold together bins that have the same polarization distribution and acceptance. Because the detector is close to symmetric for positive and negative rapidity, we combine both sides to bin in \( |y^{\Upsilon}| \). However, a simple combination will lead to the acceptance, the area of phase space covered by the detector, depending on the underlying polarization. This happens because the area of the detector covered at \( \phi = 0 \) (in the decay frame) is different for positive and negative rapidity events. It is necessary to reverse the direction of \( \hat{y} \) for negative rapidity events, essentially rotating them in the decay angle \( \phi \) by \( \pi \). This does not change the value of the \( \lambda_\theta \) and \( \lambda_\phi \) terms from Eq. (1.1) as these terms are unchanged by this rotation. However, it is important to note that
this changes the sign of the $\lambda_{\theta\phi}$ cross term for negative rapidity candidates. The symmetry of the proton-proton collision requires $\lambda_{\theta\phi}$ to take equal and opposite values for positive and negative rapidity candidates when using the same definition of $\hat{y}$, but the change of axis used to make the acceptance agree instead makes the parameters simply equal. Therefore, this term is still present in the final distribution using this angle redefinition when combining rapidity.

After this combination, the underlying polarization distribution features four fold symmetry in the angular space. With a proper rotation of the definition of $\phi$ for negative $\cos \theta$ candidates, one can use the ranges $[0 - \pi]$ and $[0 - 1]$ for $|\phi|$ and $|\cos \theta|$ respectively. This is done by taking the absolute value of $\phi$, and then changing this to $\pi - |\phi|$ for negative $\cos \theta$ candidates, while also taking the absolute value of $\cos \theta$. After these changes of variables, the number of candidates in equal size bins is maximized, making it easier to extract the signal yield. The final, four-dimensional binning used for the yield extraction and later for the polarization fits is then:

- $p_T^\Upsilon = \{0, 5, 10, 15, 20, 25, 30\} \text{ GeV/c}$
- $|y^\Upsilon| = \{0, 0.5, 1\}$
- $|\cos \theta| = \{0, 0.2, 0.4, 0.6, 0.8, 1\}$
- $|\phi| = \{0, \pi/5, 2\pi/5, 3\pi/5, 4\pi/5, \pi\}$

Separately in each bin, the efficiency weighted mass distribution is fit with an unbinned likelihood fit. The fit must include the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ peaks as well as a functional form for the background. Each signal peak is modelled with a Crystal Ball functional form with fixed power law tail parameters taken from a fit to all candidates in a $p_T^\Upsilon$ and $|y^\Upsilon|$ bin. The three peaks share a common resolution parameter, and the mass differences between the three are fixed based on the PDG values. This was found to be the simplest form that could still reproduce well the distributions in small amounts of data (as some bins have by construction). There are large variations in the shape of the background based on decay angle. To accommodate this, the overall mass window used is tightened to $8.5 \text{ GeV/c}^2$ to $11.5 \text{ GeV/c}^2$, and
a quadratic form is used for the background. In regions with accepted signal events, this provides a reasonable functional description of the background. Some examples of yield fits in different bins are found in Fig. 4-4. The resulting efficiency corrected $\Upsilon(1S)$ yields are used as the data input to the polarization fit, as will be discussed in Chapter 5.

Figure 4-4: Example fits to the efficiency corrected dimuon mass distributions in the kinematic bin $10 < p_T^X < 15$ GeV/c and $|y|^X < 0.5$ in two bins of decay angle in each analysis frame.
Chapter 5

Detector Acceptance and
Polarization Fit

The goal of the measurement is to use the reconstructed and efficiency corrected \( \Upsilon(1S) \) signal yields to access the underlying angular distribution of the muons caused by the spin alignment of the \( \Upsilon(1S) \). Values for the polarization parameters will be measured separately in each bin of \( p_T^\mu \) and \( |y^\mu| \), so each of these bins provides a different angular map of signal events. But not all parts of the angular distribution are accessible because the detector does not cover the full phase space. The region in kinematic and decay angle space in which the decay muons do enter the detector to possibly be reconstructed is called the acceptance region. In order to extract the polarization parameters it will be necessary to understand in detail how the acceptance sculpts an underlying angular distribution of muons to produce a measurable angular dependence. Then a fit must be performed using this information in order to find the best estimate of the true parameters.

5.1 Dimuon Detector Acceptance

There are two ways in which muons can fail to enter the data sample besides inefficiencies in the reconstruction and trigger. A muon may travel too close to the beamline; only muons inside the detector coverage with \( |\eta^\mu| < 2.4 \) are selected. A
muon may also be too soft; low $p_T$ muons will fail to enter the muon stations or will be unable to be reconstructed as muons because of scattering. The acceptance region is chosen so that inside of it, the efficiency may be well described by the measurement procedure of Chapter 4. The full kinematic selection in $p_T^\mu$ was detailed in Section 3.3. If either of the two muons from an $\Upsilon$ decay do not meet these requirements, the event is considered outside the acceptance region.

In each $p_T^\Upsilon$, $y^\Upsilon$ bin, the acceptance is determined with fine-grained detail. As discussed in Chapter 4, the track reconstruction single muon efficiency is included in the determination of the acceptance. Thus, we define acceptance in a bin as:

$$A_p^\Upsilon(\cos(\theta), \phi) = \frac{N_{\text{reco}}(\cos(\theta), \phi; (p_T^\Upsilon, y^\Upsilon) \in \text{bin}, \text{SiTrk pair passes cuts})}{N_{\text{gen}}(\cos(\theta)', \phi'; (p_T'^\Upsilon, y'^\Upsilon) \in \text{bin})}$$  \hspace{1cm} (5.1)$$

The unprimed variables are calculated using the simulated detector response with the same reconstruction of tracks as for real data, and the primed variables are calculated using “true” quantities from before detector simulation. The tracks have no extra quality cuts applied at this stage; the effects of the track quality are taken into account in the efficiency measurement. This acceptance is calculated using an angular binning much finer than that used during the signal extraction. This definition has the advantage of including a mapping between generator level quantities used in simulation and reconstructed quantities that are equivalent to real data in a well defined way.

Reconstructed tracks are needed in the definition of acceptance to be able to use the reconstructed quantities in the numerator. For this analysis, however, there are advantages to be found here. The track reconstruction efficiency in the $p_T$ range we are considering for the muons is known to be very uniform [26]. This can be measured using data for low $p_T$ muons, but it requires triggering on L2 muons and applying the tag and probe procedure. This makes it very difficult to measure the subtle variations in the tracking efficiency this way since the uncertainty in this method will be high. But the polarization measurement only depends on the relative differences in efficiency between different parts of the accepted region. As such, it is reasonable
to calculate the tracking efficiency from simulation since any difference with respect to data will be small compared to the other efficiency variations considered (muon identification and trigger efficiency). The tag and probe method used to measure single muon efficiencies in data also cannot be used to understand the efficiency for dimuon pair cuts like the vertex probability cut. However, the ones used in this analysis have a very high signal efficiency in simulation without strong dependence on the decay angles. These efficiencies are therefore determined in the simulation and included in the definition of the acceptance (which is always a dimuon quantity) for ease of computation.

To this purpose approximately 40 million single $\Upsilon(1S)$ events have been simulated to produce the acceptance maps. In this simulation, only $\Upsilon \rightarrow \mu\mu$ decays are produced, but otherwise the full CMS detector simulation is used as implemented in GEANT4. The initial two muons, as well as any possible final state radiation photons, are produced by the EVTGEN program [28] which implements the PHOTOS program [29] for the radiation. These $\Upsilon$ mesons are produced unpolarized (isotropic decay distribution), with flat $p_T$ and rapidity distributions. The $p_T$ distribution is weighted to correspond to that obtained with a fit to the differential cross section measurement on the 2010 data [30].

As described in Section 4.3, both the acceptance and yield are calculated in bins using the four-fold symmetry of the angular distribution. The unfolded acceptance distribution is found in Fig. 5-1 for one bin of $p_T$, $y$ with a comparison to the folded distribution in the same bin. The same acceptance region shape is found in each of the four quadrants of the $\cos \theta$, $\phi$ plane, up to a rotation or flip between them. The transformation can be used because the underlying angular distribution has the same symmetries.

Figure 5-2 contains examples of the acceptance maps for each analysis frame for two bins of $p_T$, $y$. The rest of the maps used in this analysis can be found in Appendix C. In general these acceptance maps are characterized by regions of very high acceptance (it would be equal to 1 except for the included efficiencies from simulation), and regions of zero acceptance, with very sharp turn-ons between them.
The extent of the turn-on is governed by the size of the bins in $p_T^F$, $y^T$; different parts of the bins have slightly different edges in acceptance. An infinitesimally small bin would have a step function turn-on. The helicity and Collins-Soper frames have very different shapes for the acceptances. In the CS frame, the whole range of $|\cos \theta|$ is accessible for some values of $|\phi|$, but in the helicity frame the acceptance drops out entirely near $|\cos \theta| = 1$. At higher $p_T^F$ the region of high acceptance grows larger. It is necessary to understand the acceptance very finely in order to properly apply it over the coarse binning necessary to perform yield extraction. Inside of the large bins, both the acceptance and underlying angular distribution can change by large amounts.

5.2 Fitting the Angular Distribution

The fine grained acceptance maps are combined with an angular distribution (of the form of Eq. (1.1)) to produce a probability density function (PDF) that can be used to fit the efficiency corrected signal yield maps already produced. This is done with a binned maximum likelihood fit. In each bin of $p_T^F$ and $|y^T|$ this fit is done using
Figure 5-2: Acceptance maps for the bin $p_T^\ell\epsilon (5,10)$ GeV/c, $|y^\ell|\epsilon (0,0.5)$ (a) in the helicity frame and (b) Collins-Soper frame, and for the bin $p_T^\ell\epsilon (20,25)$ GeV/c, $|y^\ell|\epsilon (0,0.5)$ (c) in the helicity frame and (d) Collins-Soper frame.
the acceptance map unique to that bin. The PDF prediction in each bin used for
signal extraction then corresponds to the integral inside the bin boundaries of the
underlying polarization distribution multiplied by the acceptance. Because of the
different acceptance maps, the function is essentially unique in each measurement
bin of $p_T^f$ and $|y^T|$. A representation of the construction of the fit function is found
in Fig. 5-3. By including the finely binned acceptance in the fit function, holes in
the acceptance are always properly and automatically accounted even when they
cover only part of a signal extraction bin. In this case only the parts of the angular
distribution with non-zero acceptance will be included in the integral. The set of
parameters $\{\lambda\}$ that produce the best fit are then the output from this procedure.
To visualize the result of the fit, examples of projecting the result of the fit onto
the $|\cos \theta|$ and $\phi$ axes can be found in Fig. 5-4; the plots include the projection of
maximal polarizations along $|\cos \theta|$ in each analysis frame as well as the isotropic
distribution.

![Acceptance Test $\lambda'$s]

Figure 5-3: The polarization PDF for a single $p_T^f, |y^T|$ bin is obtained by multiplying
the finely binned acceptance map for that bin by the angular distribution for a given
set of test parameters $\{\lambda\}$. The result is integrated inside each of the coarser bins
used for signal extraction, and a binned likelihood fit to the efficiency corrected signal
yields is used to determine the measured polarization parameters.

The fit parameters are not restricted to lie inside the physical region. The input
signal yields have both statistical and some systematic uncertainties (for instance
related to background subtraction) included from the signal extraction fit to the
weighted mass distributions. To determine the polarization parameters’ uncertainties,
(a) Helicity $15 < p_T^< 20 \text{ GeV}/c, |y^T| < 0.5$

(b) Helicity $15 < p_T^< 20 \text{ GeV}/c, |y^T| < 0.5$

(c) Collins-Soper $15 < p_T^< 20 \text{ GeV}/c, |y^T| < 0.5$

(d) Collins-Soper $15 < p_T^< 20 \text{ GeV}/c, |y^T| < 0.5$

Figure 5-4: Projections of the fit result in one bin of $p_T^$, $|y^T|$ in both analysis frames. In the $|\cos\theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
pseudoexperiments are conducted by refitting fluctuations of the input signal yields. The yield in each bin is fluctuated in a Gaussian distribution around the mean number of signal in that bin based on the uncertainty from the yield fit. This produces 10000 fluctuated signal yield maps for each bin of \( \pT \) and \( |y^\tau| \). These maps are then refit separately with the polarization PDF for that bin, and the distributions of the final parameters are used to determine the uncertainty on each parameter. This accounts for uncorrelated statistical and fit uncertainties in the different bins. Example histograms for the results from a set of these polarization fits in the helicity frame are found in Fig. 5-5.

![Histograms](Image)

**Figure 5-5:** Example results of polarization fits to 10000 signal yield maps obtained through random fluctuation of the fitted signal yields from data based on their uncertainties. Performed in the helicity frame for the bin \( 10 < \pT < 15 \text{ GeV/c} \) and \( |y^\tau| < 0.5 \).

This set of fits in each \( \pT, |y^\tau| \) bin then produces the final measured polarization
parameters' uncertainties. The angular distribution and therefore the fit PDF depend on three parameters. The frame invariant quantity $\tilde{\lambda}$ is calculated separately for each pseudoexperiment, and the distribution of results is used to determine the final uncertainty like for the other parameters. This then accounts for the correlations between the two parameters $\lambda_\theta$ and $\lambda_\phi$ used to calculate it.
Chapter 6

Sources of Systematic Uncertainty

Before considering the final results of these fits, it is necessary to explore the other contributing sources of uncertainty on the final parameters. In general this will be done by considering possible sources of error, and quantifying the degree to which fluctuations of each type change the polarization parameters. These uncertainties come in from all parts of the analysis - the measurement of the efficiency, the determination of the acceptance, the fits used at various stages, and more. In the following sections, the sources of uncertainty will be discussed in detail, and finally their contributions will be tabulated and combined.

6.1 Uncertainties in the Efficiency

Uncertainty in the determination of the single muon efficiencies directly affects the determination of the yields from the efficiency weighted signal yield fits. The polarization measurement is based on the relative numbers of signal events in different decay angle regions, and as such does not depend on the absolute efficiency of selection. Because the selection requires the use of low $p_T$ muons at the turn-on point of the muon and trigger efficiencies, variations in the shape of the turn-on curve can have effects on the measured polarization. As described in Section 4.2, the turn-on curves are fit with a functional form. There is one curve for each slice of $\eta$ in which the muon efficiency is measured. Uncertainties in the shape of the curve or on the
relative difference between curves in different slices directly affects the determination of the final parameters.

Estimating the uncertainty on the efficiency curves is one of the most difficult things to pin down properly. While the statistical elements of the efficiency uncertainties are roughly straightforward, it is difficult to pin down a complete set of systematic fluctuations that should cover everything. The strategy is developed thusly - make some changes in the derivation of the efficiency, combine these with the statistical fluctuations, and finally use simulation to difference of the methods used with respect to the true efficiency. While the particular value of the true efficiency in simulation need not equal the value using the real detector, the differences between the measured efficiency and true efficiency in simulation should be similar to the differences in the real detector. The sources of these errors will be the same, especially innate biases in the tag and probe method used, and the correlations between the two muons resulting from being close together in space.

The uncertainties on the efficiency curves along the eigenvectors of the covariance matrix are obtained from the fit, and were shown together with the nominal fits in Fig. 4-3. Because of computational restrictions, the eigenvector fluctuation with the smallest uncertainty (corresponding to the yellow curves) is discarded, and a limited set of 1σ fluctuations are used across the different slices of η. For each of these sets of fluctuations, the entire analysis chain is repeated using the fluctuated set of efficiency weights. This is necessary to ensure proper evaluation of the fitted signal yields for the different weights, as the relationship between the two is a complicated function of the kinematics. In each polarization measurement bin of $p_T^T$ and $y^T$, the largest change with respect to the central polarization parameter value as a result of these different run throughs is taken as a systematic uncertainty.

The tag and probe method is used to determine the single muon efficiency in bins of $p_T$. In order to fit these bins with a curve, the points must be placed correctly inside the bin. This is done by repeating the fit multiple times, and placing the points for successive iterations according to the position on the previous fit where the value in a particular bin is equal to the average across that bin [31]. To conservatively
estimate the uncertainty on this position, the initial (where the points are placed at bin centers) and final efficiency curve fits are both used to generate corrected data and perform the yield and polarization fits. The difference between the two results is taken as a systematic uncertainty. The effect of the bin boundaries on the efficiency curve fit are also investigated by performing the efficiency measurement with an alternate binning scheme. The first few bins are dictated by the selection cutoffs employed, but the remaining bins can be apportioned differently. This uncertainty is likewise quantified by comparing the final results using the two binning schemes.

In addition to uncertainties on the single muon efficiencies themselves, it is also important to estimate the effect of using the product of single muon efficiencies to measure the dimuon efficiency. This is done in simulation by comparing the calculation of the product of single muon efficiencies obtained through tag and probe on Monte Carlo $J/\psi$ to the true efficiency for the dimuon $\Upsilon$s. In some sense this becomes a catch-all for any remaining uncertainties. For instance, correlations between the two muons might impact the measured single muon efficiencies differently than the true dimuon efficiency. This is especially important because the spectra and separations between the decay muons will be different for the two mother particles. The strength of correlations likely depends on the separation between the two. The measured single muon efficiencies in the bins would vary slightly based on the underlying muon $p_T$ spectrum. Fitting the points with a curve attempts to alleviate this issue by allowing the center point to float according to the functional form of the efficiency as described in the previous paragraph. Any remaining discrepancy will show up when comparing the true efficiencies in simulation.

These and any other discrepancies related to the method in which the efficiencies are measured and applied will cause the product of the single muon efficiencies used to create the candidate weight to differ from the "true" dimuon efficiency for those kinematics. The single muon efficiency using the tag and probe method is obtained from $J/\psi$ simulation. Then a "$p$" factor is calculated as the correction factor that must be applied to the fitted yield in some bin of candidates weighted using the single
muon efficiencies to obtain the true track pair $\Upsilon$ dimuon yield in simulation:

$$
\rho_{\text{bin}}^{\text{tk pairs}} = \frac{\sum_{\text{tk pairs} \in \text{bin}} w_T^{\text{pair}}}{\sum_{\mu\text{ pairs} \in \text{bin}} \frac{w_T^{\text{pair}}}{\varepsilon_{(\mu_1)} \varepsilon_{(\mu_2)}}}
$$  \hspace{1cm} (6.1)

Both the numerator and denominator also have a weight $w(p_T)$, which is the same $p_T$ spectrum weighting used to calculate the acceptance and based on the differential cross section measurement from 2010.

The correction factor can be seen as a function of $p_T^\Upsilon$ and $y^\Upsilon$ in Fig. 6-1. When $\rho$ is greater than 1, it means the measured single muon efficiencies are overestimates of the true diuon efficiency because the corrected yield is then lower than the true yield (since the event weights go as $1/(\varepsilon_1 \varepsilon_2)$). This factor is calculated using a specially created simulation sample implementing a set of triggers emulating the 2011 trigger menus used in the real data selection. It is calculated in the bins of decay angle used in the yield fit, and the difference between the $\rho$-scaled polarization result and the nominal result without such scaling is taken as a systematic uncertainty.

![Figure 6-1: True $\Upsilon(1S)$ yield divided by the single muon total efficiency weighted yield as calculated using simulation as a function of $p_T^\Upsilon$ and $y^\Upsilon$.](image)

The size of the effect due to this mismeasurement of the efficiencies also depends on the underlying polarization of the $\Upsilon$ sample (whether it is simulation or data).
Angular decay distributions which have more events near the edges of the acceptance where the efficiency corrections are largest, will see a larger effect. This is demonstrated in Fig. 6-2. To test the whole procedure, particular polarizations are applied to a simulated sample through event weighting. When transverse polarizations are used, the resulting measured polarization is too low, and the effect becomes stronger for more polarized samples as in Fig. 6-2(a). But since this was done in simulation, one can repeat the procedure without using the efficiency correction. The data sample is then made up of untriggered track pairs as in Fig. 6-2(b). In this case very good results are obtained, showing that the effect seen is related to the efficiency. Therefore, it is absolutely necessary that the efficiency systematic uncertainties be evaluated in the neighborhood of the actual measured polarization. This is done automatically for uncertainties related to the efficiency turn-on curve, and in the end the $\rho$ factor is also calculated using a similar underlying angular distribution for the simulation as is measured in data.

(a) Simulated test of the polarization (b) Comparison of one of the simulated procedure including efficiency corrections tests of the polarization procedure including efficiency corrections to the result for samples weighted to have different polarizations with same sample using the true accepted pairs

Figure 6-2: Simulated tests of the polarization procedure, with and without efficiency corrections. The entire analysis is performed on simulation weighted to different underlying polarizations. "T" refers to transverse polarization ($\lambda_\theta = 1$), "L" refers to longitudinal polarization ($\lambda_\theta = -1$). In (a), the efficiency correction obtained from tag and probe on simulation is used; in (b), there is no efficiency correction applied because the selection is changed to be untriggered track pairs.
6.2 Uncertainties in the Acceptance

The systematic uncertainties of the acceptance in general come from differences between simulation and data. The acceptance must be calculated using simulated events, and requires two reconstructed tracks that together fit together to form an $\Upsilon$ candidate. Thus, the chief sources of systematic uncertainty for the acceptance are related to the modelling in simulation of $\Upsilon$ decays and the effects of including track reconstruction into the definition of acceptance. The modelling is affected chiefly by the description of final state radiation included in the simulation and the difference between the transverse momentum and rapidity spectra used in simulation with respect to the true, fundamental distribution. As defined in Section 5.1, accepted candidates require reconstructed tracks without quality requirements. The analysis is therefore dependent on the simulation description of the relative tracking efficiency in different parts of the kinematic acceptance region, and the efficiency to vertex two of them together, although these effects are expected to be smaller than the modelling ones. In general these differences can be explored by variations of the simulated input to produce alternate acceptance maps which are then used to refit the polarization distributions and extract new parameters.

The shape of the final state radiation (FSR) tail has a direct effect on the acceptance of $\Upsilon$ particles. Dimuon pairs of different mass due to the emission of a photon will have different muon kinematic distributions. If the simulated FSR tail used to determine acceptance differs from the true shape in data, then the result will not be precisely applicable. The simulation used includes FSR modelling through the PHOTOS program. The simplest comparison to make is the difference in the acceptance calculated with and without FSR; this has been done to produce a new acceptance map for each bin of $p_T^\Upsilon$, $|y^\Upsilon|$. This is obviously a very conservative estimate of the potential differences, but the difference is taken as a systematic uncertainty.

The other major source of modelling error comes from the difference in kinematic distribution of the simulated $\Upsilon$s used to measure acceptance. The sample used to determine acceptance was produced with a flat $p_T$ spectrum and is reweighted based
on the differential cross section measurement using CMS data in 2010 assuming unpolarized production. Changing the polarization hypothesis causes larger changes in the cross section measurement than any of the other systematic uncertainties. Therefore, this cross section was also measured using the hypotheses of maximal polarization along the helicity and Collins-Soper frames to calculate the acceptance (in that case acceptance depending only on $p_T^X$, $|y^T|$ and not the decay angles). These modified differential cross sections are fit separately, and all are used to weight the events used to determine acceptance. The difference between the weighting functions can be seen in Fig. 6-3. For each of the sets of acceptance maps produced using the different weights, the change in the polarization parameters is computed. The largest difference in each bin of $p_T^X$, $|y^T|$ is taken as a systematic uncertainty.

![Figure 6-3: Weight functions used to determine acceptance, as fit from the cross section measurement with different polarization hypotheses.](image)

### 6.3 Uncertainties in the Fitting Procedures

This analysis utilizes a number of different fits to extract information from the data. Besides the $J/\psi$ mass peak fits in the tag and probe method and the curve fits for the efficiency, there are the efficiency corrected signal yield fits, and the final polarization fits. The two steps involved in determining the efficiency contribute systematic uncertainties that are accounted by changing the efficiency as described in Section 6.1. The uncertainty in the $\Upsilon$ mass fits increase the uncertainty on the signal
yield. Parts of this uncertainty, like the background subtraction, show up directly in
the yield uncertainties returned by the fit and are thus automatically included in the
pseudoexperiments used to produce an uncertainty for the final parameters.

The polarization fit PDF is tested by generating pseudoexperiments with different
input values of the polarization parameters $\lambda$. In each bin of $p_T^\gamma$ and $y^\gamma$, the PDF
incorporating the acceptance for that bin is used to generate many simulated data
samples. The size of these data samples correspond to the total number of efficiency
corrected signal events in that bin. Values for the three parameters are chosen from
the set $\{-0.8, -0.3, 0.0, 0.3, 0.8\}$ such that three $\lambda$ parameters form a possible physical
polarization set. For each set of parameters, 100 simulated data samples are produced.
The residual distribution for each parameter over the many pseudoexperiments are
combined together. The bias in the distribution of the residuals for each parameter
and for $\lambda$ are taken as a systematic uncertainty. An example of these residuals in one
bin can be found in Fig. 6-4. This uncertainty is calculated separately in each bin of
$p_T^\gamma$ and $y^\gamma$ and for the fit performed in each analysis frame. The overall effect is very
small.

6.4 Other Sources of Systematic Uncertainty

There are a number of sources of uncertainty in the determination of the track mo-
momentum scale. They include material budget determination, magnetic field measure-
ment, and the track reconstruction algorithm. These sources of uncertainty affect
the determination of the track $p_T$ by introducing biases or changes in resolution. In
this analysis, the muon $p_T$ is corrected using the result of the MuscleFit procedure
(separate corrections for simulation and data). To conservatively estimate the effect
of these scale corrections, the entire procedure is redone using uncorrected muons,
where there are known discrepancies between the momentum scale as measured in
simulation and data on the order of 0.1%. The resulting change is usually very small,
but it is included as a systematic uncertainty.
Figure 6-4: Residuals of the fit parameters taken from pseudoexperiments performed using the polarization PDF for the helicity frame bin $10 < p_T < 15$ GeV/c and $|y^T| < 0.5$. The bias in the mean is taken as a systematic uncertainty.
6.5 Systematics Summary

The quantitative effect of these systematic uncertainties on the frame invariant parameter $\hat{\lambda}$ can be found in Tables 6.1 and 6.2. There are large changes in the sizes of the different uncertainties in different transverse momentum and rapidity bins. Usually the uncertainties associated with the determination of the efficiency dominate, but this is not always the case. The lowest $p_T$ bin has very large systematic uncertainties; the efficiency uncertainty is still sizeable and the effects of the acceptance also grow. As $p_T$ grows, in general the systematic uncertainties decrease, but in the central rapidity bin $|y^\gamma| < 0.5$, $25 < p_T^\gamma < 30$ GeV/c, the large effects of the $\rho$ factor correction dominate the uncertainty.
Table 6.1: Systematic uncertainties on the frame invariant parameter $\tilde{\lambda}$ in the helicity frame as a function of $p_T^T$ and $|y^T|$. Included are statistical fluctuations of the efficiency curves, the effect of point placement and bin boundaries on efficiency, the dimuon efficiency correction factor $\rho$, the $p_T$ spectrum used for acceptance calculation, the effect of FSR on acceptance, the momentum scale, and the polarization fit bias.
Table 6.2: Systematic uncertainties on the frame invariant parameter $\tilde{\lambda}$ in the Collins-Soper frame as a function of $p_T^X$ and $|y^X|$. Included are statistical fluctuations of the efficiency curves, the effect of point placement and bin boundaries on efficiency, the dimuon efficiency correction factor $\rho$, the $p_T$ spectrum used for acceptance calculation, the effect of FSR on acceptance, the momentum scale, and the polarization fit bias.
Chapter 7

Results

The results of applying the described polarization analysis method to approximately 1.1 fb$^{-1}$ of data taken in 2011 are presented here. The statistical uncertainties on the parameters are obtained from pseudoexperiments varying the signal yield input to the polarization fit as described in Chapter 5. In addition the systematic uncertainties described in Chapter 6 are included. The measurement is performed in six equal bins of $p_T$ from 0 to 30 GeV/c, and two slices of $|y|$, [0, 0.5], [0.5, 1].

The results for the three fit parameters $\lambda_\parallel$, $\lambda_{\parallel\perp}$, $\lambda_\perp$ in the helicity and Collins-Soper frame are found in Figs. 7-1 and 7-2. The comparison of the frame invariant $\tilde{\lambda} = (\lambda_\parallel + 3\lambda_\perp)/(1 - \lambda_\perp)$ calculated in each frame is found in Fig. 7-3. The results from the two frames generally agree, with the exception of one bin ($p_T < 5$ GeV/c, $|y| < 0.5$). Systematic uncertainties dominate at low $p_T$, while statistical uncertainties also play an important role at higher $p_T$. In the central rapidity slice, the highest point in $p_T$ shows odd behavior - jumping up to much higher values of $\tilde{\lambda}$ compared to previous points or the result for the rapidity slice $0.5 < |y| < 1$. There is no obvious cause for this, but the point does have a considerably larger uncertainty. The values of all the points with their statistical and systematic errors can be found in Tables 7.1 and 7.2.
Figure 7-1: Results for all three $\lambda$ parameters in the helicity frame as a function of $p_T$. The inner error bars on each point represent the systematic uncertainties, and the outer bars the total uncertainty.
Figure 7-2: Results for all three $\lambda$ parameters in the Collins-Soper frame as a function of $p_T^\tau$. The inner error bars on each point represent the systematic uncertainties, and the outer bars the total uncertainty.
Table 7.1: Values for the measured $\lambda$ parameters measured in the helicity frame with uncertainty in bins of $p_T$ for the two slices of rapidity. The first uncertainty is the statistical uncertainty, and the second is the total systematic uncertainty.
Table 7.2: Values for the measured $\lambda$ parameters measure in the Collins-Soper frame with uncertainty in bins of $p_T$ for the two slices of rapidity. The first uncertainty is the statistical uncertainty, and the second is the total systematic uncertainty.
Figure 7.3: Comparison of results for the frame invariant $\bar{\lambda}$ as a function of $p_T^\gamma$ in each analysis frame for each slice of $|y^\gamma|$. The inner error bars on each point represent the systematic uncertainties, and the outer bars the total uncertainty.
Chapter 8

Conclusions

The production of quarkonia in hadron collisions is still a poorly understood facet of QCD. The existence of many competing phenomenological models to describe it provides an opportunity for experimental measurements to be a deciding factor. While understanding the production cross sections of these particles is an important first step, most of the discriminating power lies with supplementary observables. To this end, this thesis examines the spin alignment of promptly produced (including feed-down) $\Upsilon(1S)$ mesons produced in proton-proton collisions at the LHC. The CMS detector has been used to identify and select these mesons using the decay to two muons. By examining the angular decay distribution of the muons in the rest frame of the parent quarkonium, its spin alignment is measured. Approximately $1 \text{fb}^{-1}$ of collision data taken during 2011 has been used in order to make this measurement.

There is no previous agreed upon result for this measurement. Similar analyses undertaken at the Tevatron proton-antiproton collider were limited by model dependent assumptions. Furthermore, the two experiments at that collider, CDF and D0, produce incompatible results. This thesis therefore builds a new strategy from the ground up to achieve a model independent result. This is done by always considering the full two dimensional space of muon decay angle rather than a single projection, by performing the analysis in parallel using two different coordinate frames, and by measuring a frame invariant observable in each to enhance the power of comparison.

The final result measures the three frame dependent polarization parameters $\lambda$
defining the angular distribution as a function of the transverse momentum and rapidity of the produced $\Upsilon(1S)$ meson. The range in $p_T$ extends from 0 to 30 GeV/c and in rapidity in the central region of the detector $|y^\Upsilon| < 1$. From these parameters is calculated the frame invariant quantity $\lambda$, allowing comparison between the results from the two frames, as well as between the two slices of rapidity used, and to other potential measurements. Overall, a very small, negative value is obtained for $\lambda$ as a function of $p_T^\Upsilon$. The azimuthal dependence (via $\lambda_\phi$ or $\lambda_{\theta\phi}$) is consistent with zero for the measurement in the helicity frame, and the measurement of the polar anisotropy $\lambda_\theta$ roughly agrees in the two rapidity slices. In the Collins-Soper frame, the value of $\lambda_\theta$ is seen to change as a function of rapidity, but the values of $\lambda_\phi$ or $\lambda_{\theta\phi}$ are still very small.

Overall, this result disfavors theoretical predictions of large polarization for prompt $\Upsilon(1S)$, especially if that polarization is transverse in the helicity frame. The experimental precision is not high enough to distinguish between an isotropic decay distribution and slightly longitudinal polarization. Relating back to the production mechanisms discussed in Section 1.1, the predictions of NRQCD of strong transverse polarization in the helicity frame do not agree with this measurement. Depending on the size of the contributions of transversely polarized quarkonia resulting from feed-down decays, the Color Singlet Model could produce predictions compatible with this result.

The possibility exists to make this experimental result more precise, at least in the higher $p_T$ region. With a much larger data set it would be possible to increase the selected muon $p_T$ threshold, lowering the uncertainty related to the determination of the efficiency turn on curve. This essentially would make the measurement impossible below $p_T^\Upsilon = 15$ GeV/c, but this might be difficult anyway because of the trigger thresholds that would be used at higher instantaneous luminosity. At the same time, the lower continuum background at these high dimuon $p_T$ regions may make it possible to achieve an adequate description of the angular distribution of the background. This would improve the measurement by allowing on to simultaneously fit for the mass and decay angles of the $\Upsilon$ signal. At the same time, the polarization
of the $\Upsilon(2S)$ and $\Upsilon(3S)$ could be measured, and these particles have very different contributions from feeddown. Combined with measurement of the spin alignments of the $J/\psi$ and $\psi'$, a more complete experimental description of the production of quarkonia in hadron collisions is feasible in the near future.
Appendix A

Measured Efficiency

Table A.1

| $p_T$ (GeV/$c$) | $0.0 < |\eta| < 0.8$ | $0.8 < |\eta| < 1.2$ |
|----------------|----------------------|----------------------|
|                | $\epsilon$          | $\delta \epsilon_+$ | $\delta \epsilon_-$ | $\epsilon$          | $\delta \epsilon_+$ | $\delta \epsilon_-$ |
| 3 - 3.25       | 0.0005               | +0.0004              | -0.0003              | 0.0103               | +0.0025              | -0.0021              |
| 3.25 - 3.5     | 0.0373               | +0.0037              | -0.0035              | 0.0901               | +0.0097              | -0.0085              |
| 3.5 - 3.75     | 0.1586               | +0.0092              | -0.0089              | 0.2683               | +0.0166              | -0.0178              |
| 3.75 - 4       | 0.3861               | +0.0184              | -0.0193              | 0.4735               | +0.0349              | +0.0000              |
| 4 - 4.5        | 0.5908               | +0.0155              | -0.0164              | 0.6462               | +0.0223              | -0.0219              |
| 4.5 - 5        | 0.7107               | +0.0175              | -0.0167              | 0.8080               | +0.0321              | -0.0531              |
| 5 - 7          | 0.8347               | +0.0096              | -0.0087              | 0.8207               | +0.0133              | -0.0155              |
| 7 - 10         | 0.8742               | +0.0064              | -0.0062              | 0.8523               | +0.0071              | -0.0073              |
| 10 - 100       | 0.8908               | +0.0071              | -0.0081              | 0.8362               | +0.0146              | -0.0157              |
### Table A.2

| \( p_T \ \text{GeV/c} \) | \( 1.2 < |\eta| < 1.6 \) | \( 1.6 < |\eta| < 2.0 \) |
|----------------|----------------|----------------|
| \( 3 - 3.25 \) | 0.5791, +0.0445, -0.0441 | 0.6174, +0.0308, -0.0298 |
| \( 3.25 - 3.5 \) | 0.7118, +0.0370, -0.0439 | 0.6286, +0.0417, -0.0419 |
| \( 3.5 - 3.75 \) | 0.8233, +0.0383, -0.0397 | 0.7335, +0.0427, -0.0650 |
| \( 3.75 - 4 \) | 0.8704, +0.0423, -0.0483 | 0.6208, +0.1315, -0.1034 |
| \( 4 - 4.5 \) | 0.8538, +0.0325, -0.0428 | 0.7927, +0.0596, -0.0639 |
| \( 4.5 - 5 \) | 0.8490, +0.0290, -0.0679 | 0.8416, +0.0403, -0.0485 |
| \( 5 - 7 \) | 0.8812, +0.0125, -0.0142 | 0.8252, +0.0135, -0.0142 |
| \( 7 - 10 \) | 0.8899, +0.0077, -0.0080 | 0.8580, +0.0111, -0.0173 |
| \( 10 - 100 \) | 0.8692, +0.0172, -0.0175 | 0.8035, +0.0202, -0.0218 |

### Table A.3

| \( p_T \ \text{GeV/c} \) | \( 2.0 < |\eta| < 2.4 \) |
|----------------|----------------|
| \( 3 - 3.25 \) | 0.8066, +0.0412, -0.0417 |
| \( 3.25 - 3.5 \) | 0.6178, +0.0963, -0.1494 |
| \( 3.5 - 3.75 \) | 0.7414, +0.0789, -0.1363 |
| \( 3.75 - 4 \) | 0.8418, +0.0509, -0.0645 |
| \( 4 - 4.5 \) | 0.7312, +0.0556, -0.1906 |
| \( 4.5 - 5 \) | 0.6886, +0.1243, -0.0707 |
| \( 5 - 7 \) | 0.8071, +0.0232, -0.1110 |
| \( 7 - 10 \) | 0.7208, +0.0233, -0.0254 |
| \( 10 - 100 \) | 0.6034, +0.0486, -0.0390 |

### Table A.4: Fitted efficiency parameters and fluctuations

| \( 0.0 \leq |\eta| < 0.8 \) | \( 0.8 \leq |\eta| < 1.2 \) | \( 1.2 \leq |\eta| < 1.6 \) | \( 1.6 \leq |\eta| < 2.0 \) | \( 2.0 \leq |\eta| < 2.4 \) |
|----------------|----------------|----------------|----------------|----------------|
| \( c_0 \) | 0.878 | 0.844 | 0.883 | 0.833 | 0.728 |
| \( c_1 \) | 3.921 | 3.868 | 2.930 | 2.352 | 0.000 |
| \( c_2 \) | 0.954 | 0.511 | 0.489 | 1.258 | 0.000 |
| \( c_0 \) | 0.879 | 0.845 | 0.884 | 0.828 | 0.745 |
| \( c_1 \) | 3.927 | 3.866 | 2.924 | 2.378 | 0.000 |
| \( c_2 \) | 0.867 | 0.467 | 0.319 | 0.639 | 0.000 |
| \( c_0 \) | 0.876 | 0.843 | 0.882 | 0.837 | 0.711 |
| \( c_1 \) | 3.916 | 3.871 | 2.937 | 2.326 | 0.000 |
| \( c_2 \) | 1.040 | 0.556 | 0.660 | 1.878 | 0.000 |
Table A.5: Fitted efficiency parameters fluctuations

| $|\eta| < 0.8$ | $|\eta| < 1.2$ | $|\eta| < 1.6$ | $|\eta| < 2.0$ | $|\eta| < 2.4$ |
|---------------|---------------|---------------|---------------|---------------|
| $c_0$ | $c_1$ | $c_2$ | $c_0$ | $c_1$ | $c_2$ |
| 0.868 | 3.951 | 0.955 | 0.887 | 3.892 | 0.952 |
| 0.848 | 3.845 | 0.513 | 0.840 | 3.892 | 0.510 |
| 0.859 | 2.899 | 0.490 | 0.906 | 2.962 | 0.488 |
| 0.889 | 2.429 | 1.261 | 0.777 | 2.274 | 1.255 |
| 0.745 | 0.000 | 0.000 | 0.711 | 0.000 | 0.000 |
## Appendix B

### Efficiency Corrected Yields

Table B.1: Efficiency corrected \( \Upsilon(1S) \) yields, \( 0 < p_T < 5, \ 0.0 < |y| < 0.5 \)

(a)

| \( 0 < p_T < 5 \) | \( 0.0 < |y| < 0.5 \) | \( \phi_{HX} \) | \( \phi_{CS} \) |
|------------------|------------------|------------------|------------------|
| \( \cos \phi_{HX} \) | \( \cos \phi_{CS} \) | \( \cos \phi_{HX} \) | \( \cos \phi_{CS} \) |
| 0.0-0.2 | 2050.6 \pm 65.9 | 11601.3 \pm 142.9 | 13541.2 \pm 142.1 | 11522.6 \pm 151.7 | 1447.1 \pm 59.0 |
| 0.2-0.4 | 2510.0 \pm 67.7 | 9811.2 \pm 127.4 | 13818.4 \pm 146.6 | 9738.6 \pm 151.2 | 608.0 \pm 39.4 |
| 0.4-0.6 | 3034.1 \pm 74.3 | 7801.0 \pm 113.7 | 13253.9 \pm 150.1 | 4386.6 \pm 121.2 | 231.8 \pm 24.1 |
| 0.6-0.8 | 1964.1 \pm 59.0 | 3069.1 \pm 75.7 | 2852.2 \pm 64.5 | 58.8 \pm 11.8 | 0.0 \pm 0.5 |
| 0.8-1.0 | 753.4 \pm 34.7 | 866.9 \pm 39.7 | 540.1 \pm 30.1 | 95.2 \pm 12.8 | 13.0 \pm 7.9 |

(b)

| \( 0 < p_T < 5 \) | \( 0.0 < |y| < 0.5 \) | \( \phi_{HX} \) | \( \phi_{CS} \) |
|------------------|------------------|------------------|------------------|
| \( \cos \phi_{HX} \) | \( \cos \phi_{CS} \) | \( \cos \phi_{HX} \) | \( \cos \phi_{CS} \) |
| 0.0-0.2 | 3780.1 \pm 79.8 | 10984.4 \pm 129.8 | 13506.3 \pm 141.2 | 11498.4 \pm 132.3 | 4066.3 \pm 83.0 |
| 0.2-0.4 | 2463.8 \pm 68.2 | 9248.3 \pm 125.0 | 14068.5 \pm 150.9 | 10651.1 \pm 134.4 | 3083.8 \pm 79.0 |
| 0.4-0.6 | 797.9 \pm 35.5 | 3969.6 \pm 78.7 | 15537.3 \pm 178.5 | 6350.0 \pm 121.4 | 1451.8 \pm 62.6 |
| 0.6-0.8 | 0.0 \pm 0.0 | 3.4 \pm 1.8 | 4734.1 \pm 133.1 | 0.0 \pm 2.7 | 11.1 \pm 3.7 |
| 0.8-1.0 | 0.0 \pm 0.0 | 0.0 \pm 0.0 | 17.0 \pm 4.7 | 0.0 \pm 0.0 | 0.0 \pm 0.0 |
Table B.2: Efficiency corrected $\Upsilon(1S)$ yields, $5 < p_T < 10$, $0.0 < |y| < 0.5$

(a)

| $5 < p_T < 10$ | $|\phi_{HX}|$ |
|-----------------|------------------|
| 0.0-0.2         | 3595.6 ± 102.5   |
| 0.2-0.4         | 23644.4 ± 186.6  |
| 0.4-0.6         | 6748.4 ± 98.9    |
| 0.6-0.8         | 688.8 ± 33.5     |
| 0.8-1.0         | 5.0 ± 0.0        |

(b)

| $5 < p_T < 10$ | $|\phi_{CS}|$ |
|-----------------|------------------|
| 0.0-0.2         | 18569.8 ± 158.0  |
| 0.2-0.4         | 15158.1 ± 145.4  |
| 0.4-0.6         | 11919.1 ± 88.7   |
| 0.6-0.8         | 3969.3 ± 115.7   |
| 0.8-1.0         | 4297.0 ± 81.1    |

Table B.3: Efficiency corrected $\Upsilon(1S)$ yields, $10 < p_T < 15$, $0.0 < |y| < 0.5$

(a)

| $10 < p_T < 15$ | $|\phi_{HX}|$ |
|-----------------|------------------|
| 0.0-0.2         | 9546.3 ± 114.1   |
| 0.2-0.4         | 9291.5 ± 114.0   |
| 0.4-0.6         | 4838.2 ± 82.7    |
| 0.6-0.8         | 30.4 ± 7.6       |
| 0.8-1.0         | 2897.0 ± 81.1    |

(b)

| $10 < p_T < 15$ | $|\phi_{CS}|$ |
|-----------------|------------------|
| 0.0-0.2         | 5538.6 ± 84.3    |
| 0.2-0.4         | 5276.7 ± 82.9    |
| 0.4-0.6         | 5067.0 ± 83.9    |
| 0.6-0.8         | 5301.5 ± 90.5    |
| 0.8-1.0         | 8501.2 ± 127.8   |
Table B.4: Efficiency corrected $\Upsilon(1S)$ yields, $15 < p_T < 20$, $0.0 < |y| < 0.5$

(a)

| $15 < p_T < 20$ | $0.0 < |y| < 0.5$ | $|\phi_{HX}|$ | $|\phi_{CS}|$ |
|------------------|------------------|-----------------|-----------------|
| 0.0-0.2          | 0.000-0.628      | 0.628-1.257     | 1.257-1.885     |
| 0.2-0.4          | 2403.2 ± 66.5    | 2647.8 ± 60.7   | 2388.1 ± 56.2   |
| 0.4-0.6          | 664.9 ± 36.2     | 2323.6 ± 58.7   | 2539.8 ± 58.4   |
| 0.6-0.8          | 0.0 ± 0.0        | 293.0 ± 20.5    | 1147.5 ± 40.2   |
| 0.8-1.0          | 0.0 ± 0.0        | 0.0 ± 0.0       | 0.0 ± 0.0       |

(b)

| $20 < p_T < 25$ | $0.0 < |y| < 0.5$ | $|\phi_{HX}|$ | $|\phi_{CS}|$ |
|------------------|------------------|-----------------|-----------------|
| 0.0-0.2          | 1806.0 ± 50.1    | 2479.8 ± 56.6   | 1872.1 ± 50.7   |
| 0.2-0.4          | 1737.9 ± 48.3    | 2515.1 ± 57.2   | 1859.3 ± 51.2   |
| 0.4-0.6          | 1923.2 ± 52.7    | 2588.5 ± 58.4   | 2250.9 ± 57.5   |
| 0.6-0.8          | 2382.4 ± 60.7    | 2606.4 ± 59.5   | 2438.6 ± 62.5   |
| 0.8-1.0          | 2676.6 ± 65.2    | 2648.7 ± 62.6   | 2803.1 ± 66.4   |

Table B.5: Efficiency corrected $\Upsilon(1S)$ yields, $20 < p_T < 25$, $0.0 < |y| < 0.5$

(a)

| $20 < p_T < 25$ | $0.0 < |y| < 0.5$ | $|\phi_{HX}|$ | $|\phi_{CS}|$ |
|------------------|------------------|-----------------|-----------------|
| 0.0-0.2          | 676.9 ± 29.2     | 641.8 ± 27.9    | 652.6 ± 28.1    |
| 0.2-0.4          | 603.5 ± 29.4     | 654.9 ± 28.1    | 678.8 ± 28.9    |
| 0.4-0.6          | 511.6 ± 28.3     | 645.8 ± 28.2    | 608.3 ± 28.3    |
| 0.6-0.8          | 42.9 ± 7.5       | 440.8 ± 24.2    | 414.3 ± 22.8    |
| 0.8-1.0          | 0.0 ± 0.0        | 0.0 ± 0.0       | 0.0 ± 0.0       |

(b)

| $20 < p_T < 25$ | $0.0 < |y| < 0.5$ | $|\phi_{CS}|$ |
|------------------|------------------|-----------------|
| 0.0-0.2          | 0.0 ± 0.0        | 0.0 ± 0.0       |
| 0.2-0.4          | 0.0 ± 0.0        | 0.0 ± 0.0       |
| 0.4-0.6          | 12.6 ± 3.6        | 657.5 ± 28.2    |
| 0.6-0.8          | 245.0 ± 19.5      | 698.7 ± 29.0    |
| 0.8-1.0          | 754.7 ± 31.7      | 638.3 ± 28.3    |
Table B.6: Efficiency corrected $\Upsilon(1S)$ yields, $25 < p_T < 30$, $0.0 < |y| < 0.5$

<table>
<thead>
<tr>
<th>$25 &lt; p_T &lt; 30$</th>
<th>$0.000-0.628$</th>
<th>$0.628-1.257$</th>
<th>$1.257-1.885$</th>
<th>$1.885-2.513$</th>
<th>$2.513-3.142$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.0-0.2$</td>
<td>209.7 ± 16.5</td>
<td>209.7 ± 16.2</td>
<td>208.5 ± 15.6</td>
<td>202.9 ± 15.7</td>
<td>238.0 ± 16.9</td>
</tr>
<tr>
<td>$0.2-0.4$</td>
<td>215.5 ± 17.4</td>
<td>215.3 ± 17.0</td>
<td>186.5 ± 15.0</td>
<td>209.9 ± 15.7</td>
<td>245.2 ± 18.1</td>
</tr>
<tr>
<td>$0.4-0.6$</td>
<td>252.2 ± 18.4</td>
<td>208.9 ± 16.9</td>
<td>224.2 ± 16.8</td>
<td>255.1 ± 17.6</td>
<td>216.6 ± 17.2</td>
</tr>
<tr>
<td>$0.6-0.8$</td>
<td>76.6 ± 10.4</td>
<td>180.4 ± 16.8</td>
<td>152.6 ± 14.5</td>
<td>228.3 ± 17.0</td>
<td>210.9 ± 18.3</td>
</tr>
<tr>
<td>$0.8-1.0$</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
<td>2.7 ± 1.7</td>
<td>0.0 ± 0.0</td>
</tr>
</tbody>
</table>

Table B.7: Efficiency corrected $\Upsilon(1S)$ yields, $0 < p_T < 5$, $0.5 < |y| < 1.0$

<table>
<thead>
<tr>
<th>$0 &lt; p_T &lt; 5$</th>
<th>$0.000-0.628$</th>
<th>$0.628-1.257$</th>
<th>$1.257-1.885$</th>
<th>$1.885-2.513$</th>
<th>$2.513-3.142$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.0-0.2$</td>
<td>5771.4 ± 114.5</td>
<td>13794.9 ± 160.3</td>
<td>14658.4 ± 159.0</td>
<td>12528.3 ± 164.5</td>
<td>3277.4 ± 100.9</td>
</tr>
<tr>
<td>$0.2-0.4$</td>
<td>13890.8 ± 152.7</td>
<td>14059.0 ± 159.5</td>
<td>14106.9 ± 164.7</td>
<td>11712.7 ± 185.0</td>
<td>2612.5 ± 199.7</td>
</tr>
<tr>
<td>$0.4-0.6$</td>
<td>2997.5 ± 72.7</td>
<td>8194.5 ± 123.1</td>
<td>15005.8 ± 193.1</td>
<td>10645.5 ± 205.5</td>
<td>1064.1 ± 88.8</td>
</tr>
<tr>
<td>$0.6-0.8$</td>
<td>5.3 ± 2.6</td>
<td>3.2 ± 5.7</td>
<td>4.2 ± 5.6</td>
<td>444.2 ± 60.5</td>
<td>6.6 ± 2.2</td>
</tr>
<tr>
<td>$0.8-1.0$</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
</tr>
</tbody>
</table>

| $0.5 < |y| < 1.0$ | $0.000-0.628$ | $0.628-1.257$ | $1.257-1.885$ | $1.885-2.513$ | $2.513-3.142$ |
|----------------|----------------|----------------|----------------|----------------|----------------|
| $0.0-0.2$ | 5055.8 ± 104.3 | 13246.8 ± 152.1 | 14685.6 ± 158.2 | 14009.1 ± 157.4 | 6277.4 ± 114.1 |
| $0.2-0.4$ | 2559.8 ± 83.3 | 9499.8 ± 142.1 | 13835.1 ± 167.7 | 13245.3 ± 167.8 | 4961.1 ± 117.3 |
| $0.4-0.6$ | 640.5 ± 37.0 | 3382.7 ± 80.9 | 15004.0 ± 200.9 | 9563.4 ± 182.6 | 2261.2 ± 107.3 |
| $0.6-0.8$ | 0.0 ± 0.7 | 3.2 ± 0.0 | 5176.8 ± 150.7 | 2414.1 ± 123.5 | 0.0 ± 19.0 |
| $0.8-1.0$ | 0.0 ± 0.0 | 0.0 ± 0.0 | 86.1 ± 17.7 | 2.8 ± 1.8 | 0.0 ± 0.0 |
Table B.8: Efficiency corrected $\Upsilon(1S)$ yields, $5 < p_T < 10$, $0.5 < |y| < 1.0$

(a)

<table>
<thead>
<tr>
<th>$5 &lt; p_T &lt; 10$</th>
<th>0.000-0.628</th>
<th>0.628-1.257</th>
<th>1.257-1.885</th>
<th>1.885-2.513</th>
<th>2.513-3.142</th>
</tr>
</thead>
<tbody>
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<td>y</td>
<td>&lt; 1.0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\cos\theta_{HX}$</td>
<td>0.0-0.2</td>
<td>25924.6 ± 211.5</td>
<td>36885.9 ± 226.8</td>
<td>29201.7 ± 236.6</td>
<td>1435.6 ± 68.7</td>
</tr>
<tr>
<td></td>
<td>0.2-0.4</td>
<td>22135.1 ± 188.4</td>
<td>37841.0 ± 233.4</td>
<td>32074.1 ± 266.1</td>
<td>7254.2 ± 163.4</td>
</tr>
<tr>
<td></td>
<td>0.4-0.6</td>
<td>8804.9 ± 118.4</td>
<td>35585.4 ± 233.5</td>
<td>32974.6 ± 292.4</td>
<td>14232.5 ± 275.4</td>
</tr>
<tr>
<td></td>
<td>0.6-0.8</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.5</td>
<td>3858.5 ± 70.2</td>
<td>6324.3 ± 134.7</td>
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<tr>
<td></td>
<td>0.8-1.0</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
</tr>
</tbody>
</table>

(b)

<table>
<thead>
<tr>
<th>$5 &lt; p_T &lt; 10$</th>
<th>0.000-0.628</th>
<th>0.628-1.257</th>
<th>1.257-1.885</th>
<th>1.885-2.513</th>
<th>2.513-3.142</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.5 &lt;</td>
<td>y</td>
<td>&lt; 1.0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\cos\theta_{CS}$</td>
<td>0.0-0.2</td>
<td>21651.6 ± 179.4</td>
<td>37488.7 ± 227.3</td>
<td>23657.1 ± 191.0</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>0.2-0.4</td>
<td>15997.3 ± 157.8</td>
<td>35160.9 ± 247.9</td>
<td>16110.3 ± 186.6</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>0.4-0.6</td>
<td>9693.6 ± 132.1</td>
<td>35160.9 ± 247.9</td>
<td>16110.3 ± 186.6</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>0.6-0.8</td>
<td>3271.5 ± 83.9</td>
<td>31217.1 ± 279.3</td>
<td>11078.6 ± 184.6</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>0.8-1.0</td>
<td>353.5 ± 36.7</td>
<td>2267.3 ± 88.5</td>
<td>18834.5 ± 287.0</td>
<td>9628.2 ± 223.0</td>
</tr>
</tbody>
</table>

Table B.9: Efficiency corrected $\Upsilon(1S)$ yields, $10 < p_T < 15$, $0.5 < |y| < 1.0$

(a)

<table>
<thead>
<tr>
<th>$10 &lt; p_T &lt; 15$</th>
<th>0.000-0.628</th>
<th>0.628-1.257</th>
<th>1.257-1.885</th>
<th>1.885-2.513</th>
<th>2.513-3.142</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.5 &lt;</td>
<td>y</td>
<td>&lt; 1.0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\cos\theta_{HX}$</td>
<td>0.0-0.2</td>
<td>8523.5 ± 114.4</td>
<td>9253.1 ± 109.7</td>
<td>8876.2 ± 116.3</td>
<td>6648.2 ± 124.4</td>
</tr>
<tr>
<td></td>
<td>0.2-0.4</td>
<td>7474.8 ± 106.5</td>
<td>9560.4 ± 111.9</td>
<td>8566.4 ± 112.9</td>
<td>8466.0 ± 139.1</td>
</tr>
<tr>
<td></td>
<td>0.4-0.6</td>
<td>84.7 ± 10.1</td>
<td>2972.3 ± 65.4</td>
<td>4177.9 ± 84.8</td>
<td>3675.7 ± 92.0</td>
</tr>
<tr>
<td></td>
<td>0.8-1.0</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
</tr>
</tbody>
</table>

(b)

<table>
<thead>
<tr>
<th>$10 &lt; p_T &lt; 15$</th>
<th>0.000-0.628</th>
<th>0.628-1.257</th>
<th>1.257-1.885</th>
<th>1.885-2.513</th>
<th>2.513-3.142</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.5 &lt;</td>
<td>y</td>
<td>&lt; 1.0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\cos\theta_{CS}$</td>
<td>0.0-0.2</td>
<td>6191.9 ± 91.7</td>
<td>9488.8 ± 110.8</td>
<td>6522.8 ± 95.2</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>0.2-0.4</td>
<td>5547.9 ± 87.5</td>
<td>9057.1 ± 109.7</td>
<td>6195.0 ± 95.9</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>0.4-0.6</td>
<td>4605.2 ± 82.6</td>
<td>8740.0 ± 111.4</td>
<td>5827.9 ± 98.8</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>0.6-0.8</td>
<td>4750.1 ± 87.9</td>
<td>8753.8 ± 116.6</td>
<td>6322.7 ± 112.9</td>
<td>91.7 ± 23.9</td>
</tr>
<tr>
<td></td>
<td>0.8-1.0</td>
<td>2027.7 ± 68.5</td>
<td>6951.6 ± 120.2</td>
<td>8660.1 ± 130.7</td>
<td>8389.2 ± 141.5</td>
</tr>
</tbody>
</table>
Table B.10: Efficiency corrected $\Upsilon(1S)$ yields, $15 < p_T < 20$, $0.5 < |y| < 1.0$

(a)

| $15 < p_T < 20$ | $0.5 < |y| < 1.0$ | $\phi_{HX}$ | $\phi_{HX}$ |
|-----------------|-----------------|-------------|-------------|
|                 |                 | 0.000-0.628 | 0.628-1.257 |
| $|\cos\theta_{HX}|$ |                 | 1.257-1.885 | 1.885-2.513 |
| 0.0-0.2         | 1976.4 ± 62.4   | 2291.3 ± 56.1 | 2227.9 ± 53.6 |
| 0.2-0.4         | 1197.1 ± 49.8   | 2073.1 ± 55.3 | 2227.9 ± 53.4 |
| 0.4-0.6         | 83.7 ± 15.9     | 1688.4 ± 50.9 | 2409.3 ± 55.7 |
| 0.6-0.8         | 0.0 ± 0.0       | 264.1 ± 19.9  | 1272.9 ± 42.3 |
| 0.8-1.0         | 0.0 ± 0.0       | 0.0 ± 0.0     | 0.0 ± 0.0    |

(b)

| $15 < p_T < 20$ | $0.5 < |y| < 1.0$ | $\phi_{CS}$ |
|-----------------|-----------------|-------------|
|                 |                 | 0.000-0.628 | 0.628-1.257 |
| $|\cos\theta_{CS}|$ |                 | 1.257-1.885 | 1.885-2.513 |
| 0.0-0.2         | 0.0 ± 0.0       | 1812.7 ± 48.9 | 2261.7 ± 53.3 |
| 0.2-0.4         | 0.0 ± 0.0       | 1596.7 ± 46.6 | 2301.9 ± 54.8 |
| 0.4-0.6         | 0.0 ± 0.0       | 1750.7 ± 48.2 | 2241.3 ± 54.3 |
| 0.6-0.8         | 39.2 ± 8.1      | 1962.5 ± 52.3 | 2123.1 ± 54.1 |
| 0.8-1.0         | 1546.2 ± 51.0   | 2157.3 ± 56.6 | 2140.7 ± 57.3 |

Table B.11: Efficiency corrected $\Upsilon(1S)$ yields, $20 < p_T < 25$, $0.5 < |y| < 1.0$

(a)

| $20 < p_T < 25$ | $0.5 < |y| < 1.0$ | $\phi_{HX}$ |
|-----------------|-----------------|-------------|
|                 |                 | 0.000-0.628 | 0.628-1.257 |
| $|\cos\theta_{HX}|$ |                 | 1.257-1.885 | 1.885-2.513 |
| 0.0-0.2         | 594.4 ± 30.7    | 638.5 ± 30.1 | 650.8 ± 28.8 |
| 0.2-0.4         | 580.3 ± 34.1    | 652.1 ± 31.1 | 698.0 ± 29.5 |
| 0.4-0.6         | 225.1 ± 22.5    | 652.1 ± 32.5 | 611.7 ± 28.2 |
| 0.6-0.8         | 0.0 ± 0.0       | 161.2 ± 16.6 | 482.7 ± 25.9 |
| 0.8-1.0         | 0.0 ± 0.0       | 0.0 ± 0.0    | 0.0 ± 0.0    |

(b)

| $20 < p_T < 25$ | $0.5 < |y| < 1.0$ | $\phi_{CS}$ |
|-----------------|-----------------|-------------|
|                 |                 | 0.000-0.628 | 0.628-1.257 |
| $|\cos\theta_{CS}|$ |                 | 1.257-1.885 | 1.885-2.513 |
| 0.0-0.2         | 0.0 ± 0.0       | 490.0 ± 26.0 | 667.1 ± 28.7 |
| 0.2-0.4         | 0.0 ± 0.0       | 524.1 ± 26.4 | 678.2 ± 29.0 |
| 0.4-0.6         | 6.2 ± 27        | 623.5 ± 29.6 | 710.5 ± 30.6 |
| 0.6-0.8         | 221.3 ± 17.9    | 692.6 ± 31.0 | 627.6 ± 29.2 |
| 0.8-1.0         | 608.2 ± 29.4    | 665.2 ± 30.6 | 610.8 ± 29.2 |
Table B.12: Efficiency corrected Υ(1S) yields, $25 < p_T < 30$, $0.5 < |y| < 1.0$

(a)

| $25 < p_T < 30$ | $|\phi_{HX}|$ |
|-----------------|-----------------|
| $0.5 < |y| < 1.0$ | 0.000-0.628 | 0.628-1.257 | 1.257-1.885 | 1.885-2.513 | 2.513-3.142 |
| 0.0-0.2 | 210.9 ± 18.4 | 259.9 ± 19.9 | 233.2 ± 18.3 | 236.9 ± 18.1 | 252.8 ± 18.5 |
| 0.2-0.4 | 241.3 ± 21.7 | 212.2 ± 17.3 | 262.6 ± 19.8 | 201.1 ± 17.7 | 259.6 ± 21.9 |
| 0.4-0.6 | 185.3 ± 20.4 | 256.1 ± 19.8 | 237.2 ± 18.1 | 243.6 ± 19.0 | 235.4 ± 18.9 |
| 0.6-0.8 | 10.5 ± 4.4 | 196.7 ± 18.9 | 199.1 ± 18.8 | 233.5 ± 18.4 | 241.3 ± 19.3 |
| 0.8-1.0 | 0.0 ± 0.0 | 0.0 ± 0.0 | 2.2 ± 2.4 | 22.6 ± 5.6 | 25.8 ± 6.0 |

(b)

| $25 < p_T < 30$ | $|\phi_{CS}|$ |
|-----------------|-----------------|
| $0.5 < |y| < 1.0$ | 0.000-0.628 | 0.628-1.257 | 1.257-1.885 | 1.885-2.513 | 2.513-3.142 |
| 0.0-0.2 | 0.0 ± 0.0 | 215.1 ± 19.2 | 259.3 ± 19.2 | 226.7 ± 18.9 | 0.0 ± 0.0 |
| 0.2-0.4 | 2.1 ± 0.0 | 221.7 ± 18.3 | 232.3 ± 18.1 | 311.8 ± 21.0 | 7.4 ± 2.9 |
| 0.4-0.6 | 25.4 ± 6.2 | 236.8 ± 18.8 | 237.0 ± 19.1 | 205.9 ± 18.4 | 60.9 ± 13.0 |
| 0.6-0.8 | 169.2 ± 15.6 | 230.1 ± 18.1 | 231.1 ± 18.1 | 234.3 ± 20.2 | 164.1 ± 20.8 |
| 0.8-1.0 | 204.1 ± 18.1 | 238.1 ± 20.1 | 279.3 ± 19.7 | 234.7 ± 18.3 | 221.4 ± 19.6 |
Appendix C

Acceptances

Figure C-1: Acceptance for $p_T^x < 5 \text{ GeV}/c$, $|y^x| < 0.5$

Figure C-2: Acceptance for $p_T^x < 5 \text{ GeV}/c$, $0.5 < |y^x| < 1.0$
Figure C-3: Acceptance for $5 < p_T^X < 10 \text{ GeV/c}, |y^X| < 0.5$

Figure C-4: Acceptance for $5 < p_T^X < 10 \text{ GeV/c}, 0.5 < |y^X| < 1.0$

Figure C-5: Acceptance for $10 < p_T^X < 15 \text{ GeV/c}, |y^X| < 0.5$
Figure C-6: Acceptance for $10 < p_{T}^{X} < 15 \text{ GeV}/c$, $0.5 < |y^{X}| < 1.0$

Figure C-7: Acceptance for $15 < p_{T}^{X} < 20 \text{ GeV}/c$, $|y^{X}| < 0.5$

Figure C-8: Acceptance for $15 < p_{T}^{X} < 20 \text{ GeV}/c$, $0.5 < |y^{X}| < 1.0$
Figure C-9: Acceptance for $20 < p_T < 25 \text{ GeV/c}, \mid y \mid < 0.5$

Figure C-10: Acceptance for $20 < p_T < 25 \text{ GeV/c}, 0.5 < \mid y \mid < 1.0$

Figure C-11: Acceptance for $25 < p_T < 30 \text{ GeV/c}, \mid y \mid < 0.5$
Figure C-12: Acceptance for $25 < p_T < 30 \text{GeV}/c$, $0.5 < |y^T| < 1.0$
Appendix D

Fit Projections
Figure D-1: Projections of the fit result in one bin of $p_T^\gamma$, $|y^\gamma|$ in both analysis frames. In the $|\cos\theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
Figure D-2: Projections of the fit result in one bin of $p_T$, $|y^\tau|$ in both analysis frames. In the $|\cos \theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
Figure D-3: Projections of the fit result in one bin of $p_T^X$, $|y^X|$ in both analysis frames. In the $|\cos \theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
Figure D-4: Projections of the fit result in one bin of $p_T$, $|y|$ in both analysis frames. In the $|\cos \theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
Figure D-5: Projections of the fit result in one bin of $p_T^\tau$, $|y^\tau|$ in both analysis frames. In the $|\cos \theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
(a) Helicity $|\cos \theta| < 10 < p_T^X < 15$ GeV/$c$, $0.5 < |y^Y| < 1$

(b) Helicity $|\phi| < 10 < p_T^X < 15$ GeV/$c$, $0.5 < |y^Y| < 1$

(c) Collins-Soper $|\cos \theta| < 10 < p_T^X < 15$ GeV/$c$, $0.5 < |y^Y| < 1$

(d) Collins-Soper $|\phi| < 10 < p_T^X < 15$ GeV/$c$, $0.5 < |y^Y| < 1$

Figure D-6: Projections of the fit result in one bin of $p_T^X$, $|y^Y|$ in both analysis frames. In the $|\cos \theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
(a) Helicity $|\cos \theta| < 0.5$

(b) Helicity $|\phi| < 0.5$

(c) Collins-Soper $|\cos \theta| < 0.5$

(d) Collins-Soper $|\phi| < 0.5$

Figure D-7: Projections of the fit result in one bin of $p_T^X$, $|y^X|$ in both analysis frames. In the $|\cos \theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
Figure D-8: Projections of the fit result in one bin of $p_T^X$, $|y^T|$ in both analysis frames. In the $|\cos \theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
Figure D-9: Projections of the fit result in one bin of $p_T^\gamma$, $|y^\gamma|$ in both analysis frames. In the $|\cos \theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
Figure D-10: Projections of the fit result in one bin of $p_T$, $|y^T|$ in both analysis frames. In the $|\cos \theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
Figure D-11: Projections of the fit result in one bin of $p_T^\gamma$, $|y^\gamma|$ in both analysis frames. In the $|\cos \theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
Figure D-12: Projections of the fit result in one bin of $p_T^\gamma$, $|y^\gamma|$ in both analysis frames. In the $|\cos \theta|$ axis, the fit result is compared to fully longitudinal and transverse polarizations in that frame as well as an isotropic decay distribution.
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