A DESIGN OF STRUCTURAL PRECAST REINFORCED CONCRETE

FACADE FOR A HIGH RISE URBAN HOUSING TYPE

by

Yusing Yiu-Sing Jung

B. Arch., University of Toronto

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Signature of Author

Yusing Y. Jung
Department of Architecture, July 16, 1962

Certified by

Thesis Supervisor

Accepted by

Chairman, Departmental Committee on Graduate Students
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ABSTRACT

The problem of utilizing a system of precast reinforced concrete window units to form the structural facade of a high rise building is considered mainly from the standpoint of its capacity to satisfy the structural requirements. Its architectural performance, its adaptability to the integration with mechanical services, and the feasibility of such system as a method of construction dictate the form of each part as well as the expression of the whole.

A design of a structural facade system for a high rise urban housing type is presented with a structural analysis.

Thesis Supervisor: Eduardo Fernando Catalano
Professor of Architecture
July 16, 1962

Pietro Belluschi, Dean
School of Architecture and Planning
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Dean Belluschi

In partial fulfillment of the requirements for the degree of Master in Architecture, I hereby submit this thesis entitled, "A Design of Structural Precast Reinforced Concrete Facade for a High Rise Urban Housing Type."

Respectfully

(Yusung Y. Jung)
ACKNOWLEDGEMENT

The encouragement of Professor Eduardo F. Catalano is gratefully acknowledged; his advice and guidance played an important part in determining the scope and general nature of the design.

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I. INTRODUCTION

The idea of precasting reinforced concrete parts of a structure is considered nearly as old in fact as reinforced concrete itself. Precast units erected as complete structures had been used before the turn of this century. A recent example, the Medical Research Laboratory at the University of Pennsylvania designed by Louis I. Kahn, Architect, and Dr. August E. Komendant, Structural Consultant, had demonstrated the feasibility of precasting all the structural units for the entire eight-story rigid frame. With the advance of prestressing technology, it became evident that there is no doubt that the advantages of precast prestressed concrete structural elements will eventually be more and more utilized in buildings of much greater height. The Norton building in Seattle, a 20-story structure by Skidmore, Owings & Merrill, Architects, uses precast prestressed concrete girders to span 70 feet, allowing great flexibility in floor layout. The proposed Metropolitan Tower in Honolulu, a 30-story structure by I. M. Pei and Associates, will likely use precast prestressed floor panels and beams to save weight and construction time. But, so far, a completely precast structural system has not
been built for a building over 120 feet high in the United States, in spite of the main claims for prefabrication are said to be its speed, the saving in formwork and scaffolding, and economies gained by the production on an industrial basis and the use of machinery for construction.

Undoubtedly, local building codes have prevented many designs from being realized. Costs of construction in many cases are too high, being influenced by the costs of handling and transportation of the units. But, the core of the problem is quite often the uncertainty about the feasibility of using an entirely precast system for such high rise structure in view of its lack of stiffness and inherent difficulties at the junctions of the members.

On the other hand, precast concrete cladding and curtain wall for high rise building has long been accepted due to the advantages of weather resistant exposed aggregate finishes and dimensional control. It is an inevitable outcome that once precasting is accepted, a system of combining cladding and structural framing in a single precast structural facade will be demanded by the architects. Needless to say, structural facade has been one of the oldest expressions of architectural form in history. The characteristic new turn, which gave such great importance to this concept, was the possibility and the development of exposed precast concrete structural members.
Eero Saarinen and Associates' U. S. Embassy buildings in both London, England and Oslo, Norway are two recent examples of many low buildings employing the structural facade principle. No exterior cladding was used or needed on the precast window units. In the field of high rise building, again only structural facade in monolithic concrete cast-in-place has been constructed. Examples are the Kips Bay Plaza Apartments, New York City and the Earth Science Building at M.I.T., Cambridge (under construction), both designed by I. M. Pei and Associates.

The question is therefore unavoidable. "What are the problems involved in designing a precast structural facade for a high rise building?" Hoping that the experience gained from answering this question would pave the path toward future studies of precast system for a complete high rise structure, it is felt that the aim of this study will be better limited to the design of this particular aspect.
II. GENERAL DATA

Kurt Billig, in his paper entitled "Structural Precast Reinforced Concrete", which was issued about 1947, outlined the main advantages in using precast parts for the erection of reinforced concrete structures as follows:

"1. Economy in moulding and scaffolding--this point being of special importance for structures of considerable heights and spans

2. Economy in labour obtained by a far-reaching standardization of the precast units and by the extensive employment of machinery for their manufacture and erection

3. The production of precast parts in the workshops is independent of weather conditions, especially frost; so the seasonal nature of construction work in countries with severe climates may be changed into a permanent industry.

4. Skilled workers are permanently occupied in the workshops. By using modern erection plants, only a small number of workers need be employed on site.

5. Speed of erection is greatly increased. The charges for capital, overhead, and supervision on the site are correspondingly reduced.

6. Technical control in the workshop is better than on the site, resulting in higher quality concrete and in a more accurate placing of the reinforcement. The improved quality of precast articles is recognized in the building regulations of various countries by an increase in the permissible stresses."
7. The influence of shrinkage is practically eliminated, and the effect of temperature is no longer of importance because of the numerous joints. Expansion joints may, therefore, be omitted.

8. The quantity of concrete and mortar used on the site is greatly reduced. The building site is therefore drier and cleaner.

9. Only finished units are transported to the site. That means a reduction in transport to, and storage on, the building site for by comparison a good percentage of the building materials used for structures cast in-situ, which is required only temporarily or is waste, and is therefore transported twice.¹

At the same time he also brought up the two disadvantages:

1. "Repeated handling of the precast units, their additional transport from the workshop to the building site, and breakage of units in transit.

2. The difficulty of producing satisfactory connections between the precast units which will provide perfect continuity and frame effects in the finished structure equivalent to those in a monolithic structure cast in-situ."²

On the other hand, architects motivated by a search for a new architectural expression of boldness and visual strength in cast-in-place concrete structural facade often are dissatisfied with the many drawbacks and deficiencies of con-

¹,²Structural Precast Concrete. Kurt Billig. Chapter 1. pgs. 1 and 2.
struction. The inadequate colour control, surface spalling in climates of frequent freeze-thaw cycles, uncontrollable shrinkage and thermal cracks, rusting caused by insufficient concrete cover, cold joints caused by partial set of the first layer of concrete before the succeeding layer is placed, leakage caused by improperly designed formwork, staining by water run-off, honeycombing, sand streaking, entrapped air, and colour match of patching compound are the common faults.\(^3\)

It is true that all of these may be eliminated with intelligent field supervision, proper use of admixtures and surface water repellents, and by closer co-ordination of contractor's and architect's design activities, particularly in formwork detailing. Nevertheless, all these factors tend to favour a precasting method for an exposed concrete finish.

Kurt Billig in his discussion of joints of precast members emphasized the fact that precast multistory buildings, without heavy junctions cast in place, work as hinged systems, no member taking bending moments from its neighbour. The inner play of forces is simple, clear and definite, more so than in a framed building, although in the latter case the forces may be analyzed by accurate calculations. The exact loads and reactions in the precast members are known and

these units can be designed quite economically with a constant margin of safety throughout. Whereas, in a monolithic frame cast in place, it is almost impossible to allocate the materials in such a manner, as the coefficient of safety changes between rather large limits in the various parts. The unquestionable advantage of the monolithic frame is its stiffness. But, sufficient stiffness may also be obtained in precast structures by various means, for example, by using the floor structures and the cross walls for this purpose.  

In pursuit of this approach, it seems that a system of combination of precast structural facade with cast-in-place concrete flat slab is an attractive method for this study. A flat slab construction will conveniently eliminate the problem of beams and girders, limiting the connection to a condition of edge support. The concept of this approach is to utilize the erected structural precast concrete units to make up a continuous self-supporting facade and at the same time to form the exterior support for the floor structure to be cast in place. While retaining the advantages of the "hinged concept" in the design of the units, as mentioned by Kurt Billig, the system as a whole appears to have at the same time sufficient rigidity without resorting to a heavy

---

cast-in-place junction where a column, beam, and floor meet.

This method would not be applicable without the recent development of high-strength materials and methods of connection. Improved production techniques now make available precast concrete with a compressive strength of 7,500 p.s.i. or more and reinforcing steel with a yield point strength of 75,000 p.s.i. It means that by careful control of the variation of both concrete strength and the type of steel, it is feasible to maintain a smaller uniform size of precast facade unit in contrast to the increasing load requirement. It also means that economy would be gained from the standardization of moulds. Furthermore, in the research of composite connections of precast and cast-in-place concrete, Arthur R. Anderson in his paper entitled "Composite Designs in Precast and Cast-in-Place Concrete"\textsuperscript{5} which he presented in 1960 before the Structural Division Session on Composite Design in Building Construction at the A. S. C. E. Annual Convention in Washington, D. C., confirmed the workability of various methods of connection with laboratory tests results.

Unfortunately, the designer, as yet unguided by codes and textbooks, must resort to imagination and ingenuity in order to capitalize on the potential of a new method of connection.

\textsuperscript{5}Progressive Architecture, September 1960.
construction. Without the benefit of any existing example, the remaining doubt whether a system of structural precast reinforced concrete facade is workable in combination with cast-in-place flat slab, can only be cleared by a trial design and structural analysis.
III. CONCEPT AND CRITERIA

3.1 Building Type

In view of the use of a flat slab construction, a prototype high rise urban housing tower is considered appropriate from the standpoint of its universal application. It is felt that such a building should have maximum flexibility to accommodate changes and combinations of different ratio of efficiency unit, one-bedroom suites, two-bedroom and three-bedroom apartments. Its height should be determined by structural efficiency and the selection of elevators in view of the lack of site restriction. Although the main purpose of the building is to set a certain limitation for the application of a structural precast facade system, the size of the building and the proportion of rental area to the total should be designed to be comparable to the marketable requirement of a minimum of 85% ratio.

3.2 Structural System

A system of cast-in-place reinforced concrete flat slab without drop panels is assumed in conjunction with the structural precast reinforced concrete facade.
Slab thickness is determined by the bay size and the consideration of the economy of reinforcing steel. ACI code\textsuperscript{6} requires that slab shall not be less than L/36 nor 5 in. in thickness.

Structural precast reinforced concrete facade units must be designed to comply with all structural requirements upon which the quality of the whole structure depends. These requirements are the axial load, wind stresses, gravity bending and shear, and thermal stresses. Upper columns must transfer their loads as direct as possible to the lower ones without local overstress. Floor slab should have sufficient area of support with good anchorage. And lastly, the junction should be simple and convenient for erection.

3.3 Partitioning

A module of 5'-10" is selected from the survey of existing examples of I. M. Pei and Associates' projects. The basic module of 5'-10" will allow partitioning in multiplies of two and three, 11'-8" and 17'-6" center to center of partitions. These are considered suitable for full bedroom and living room widths. The module of

\textsuperscript{6} Proposed Revision of (ACI 318-56)
5'-10" is also selected from the standpoint of feasible weight and size limit of precast facade units. Architecturally, the acceptance of this module will undoubtedly determine the exterior rhythm and character of the building.

3.4 Mechanical Integration

Most building codes require an opening equal to 5% of the room area for natural ventilation where mechanical ventilation is not provided. In this case, the building is designed for complete air-conditioning in summer and winter. It is assumed that a Fan-coil unit system is selected and perimeter pipe space will be incorporated into the design. A central duct system will also supplement the supply and exhaust, but it is felt that outside air intake at the Fan-coil unit should be incorporated. A 0-100% proportioning damper for rear inlet is desirable from the standpoint of its adaptability to different regional and climatic operations. It is also felt that with minimum modification of the structural precast facade unit, an Induction unit system of air-conditioning could be accommodated. Since the purpose of this study is to evaluate the adaptability of the design, no comparative selection between the two air-
conditioning systems is attempted on the performance basis. Thermal insulation will be provided to maintain a $U$ value of 0.14 or better for the exterior wall.

3.5 Fabrication and Erection

The critical stage of the following operations must be carefully considered:

1. Initial crane lifting from mould--units must be designed to withstand stresses due to bending and shear. Modulus of rupture of concrete at the critical section must provide the resisting strength necessary to lift the panel without cracking the concrete.

2. The quality of construction is greatly dependent upon the transport of units to site, erecting by crane, bracing and joining of units. Pick-up loops and bracing anchors must be placed to minimize local stress and allow for most straight forward placing into final position by crane.

3. Profile of precast units must take into account of convenient mould removal and the constant exposure to damage before and after installation. It is assumed that steel moulds will be used.

4. Welding of high strength reinforcing steel must
conform to "Schedule of Preferred Joint Details for Arc-welded Splices in Reinforcing Bars". 7

Space allowance for field operation must be considered for the joints of precast units. Grout spline should be concealed to provide more effective weather seal.

Furthermore, a maximum weight of about 3 tons is considered the limit from the standpoint of easy transport and erection by a center crane method.

7Appendix D High Strength Steel Concrete Reinforcing Bars. p. 13.
IV. DESIGN PROPOSAL

4.1 Drawings

1. Plans - Typical floors
2. Elevation
3. Section
4. Facade

Photographs

5. Assembly of facade units
6. Model
7. Model
8. Model
4.2 General Description

A 23-story tower accommodating an average of 140 residential units:

Dimension--90.5' x 90.5'
Height--230'
Floor to floor height--9'

Typical two-bedroom unit:

<table>
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<tr>
<th>Room</th>
<th>Dimensions</th>
<th>Area</th>
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<tr>
<td>LR</td>
<td>16 x 23.5</td>
<td>376</td>
</tr>
<tr>
<td>DR</td>
<td>16 x 13</td>
<td>208</td>
</tr>
<tr>
<td>K</td>
<td>10 x 7.5</td>
<td>75</td>
</tr>
<tr>
<td>BR1</td>
<td>16 x 11.6</td>
<td>186</td>
</tr>
<tr>
<td>BR2</td>
<td>16 x 11.6</td>
<td>186</td>
</tr>
</tbody>
</table>

Unit living area 1031 sq. ft.
Gross unit area 1294 sq. ft.
Ratio of 80%
Gross area / Frontage 17 ft.

Height of tower is determined mainly by the maximum capacity of a minimum allowable column area (Appendix B) at 5'-10" center to center. Three 2000 lb. 500 FPM elevators are selected on the basis of total travel of 250 feet.

4.3 Design Explanation

1. Square tower

The advantages of a square plan are the concentration of services, maximum perimeter for unit living area, and the economy of proportion of floor areas. Structurally, especially for structural facade application,
its box girder shape is extremely efficient in wind resistance, taking the maximum advantage of the moment of inertia of all exterior columns in both axes.

2. Bay size

A 17' - 6" square bay is determined by the following reasons:

a. The limit of economical bay size for concrete flat slab construction is around 18' x 18'.

b. Maximum depth of bedroom and width of living room in no case needs to be larger than 16' - 0". The placing of column along the separation of living area and service section does not interfere with the maximum flexibility of conversion of units.

c. 6" flat slab is required for a 17' - 6" panel. (L/36) however, 7" flat slab is selected to allow for coverage of the trap of the bath. Additional saving of steel is also achieved by the increase of depth.

3. Flat slab connection

An effect of direct connection from floor slab to the exterior column (effective column area of the jambs of two adjoining precast units) is avoided for the
following reasons:

a. To allow mechanical pipe space to be incorporated behind two adjoining precast unit jambs.

b. To maintain the same strength of concrete, \( f_y = 3000 \) p. s. i., for all floor slab. Since higher strength concrete is only needed for the facade unit below the seventeenth floor, crushing of floor slab must be avoided.

c. A hinged connection with shear key at the head of the precast unit will transfer the load from the floor slab indirectly to the effective column area. Consequently no gravity moment due to floor slab is carried to the structural facade.

d. Deflection due to thermal contraction of facade will be absorbed by the rotation at the joint between the floor slab and the head of the precast unit.

4.4 Window Unit

As a result of the structural analysis, (Table III, Appendix B) effective column area of 100 sq. in. is
maintained for all units. A system of precast reinforced concrete window units is preferred over the 'Column and Beam' or 'Tee Column' systems, because of the advantages of fewer joints and greater rigidity. The design of the jamb section of the precast unit is based on the concept that by connecting the two jambs at the two floor lines and at mid-section with ties and grout pads, a single column effect can be achieved. Since the jambs are laterally restrained by the head and sill, and the ends of the combined jamb column are definitely restrained by the welded splice of reinforcing steel and grout pads, h'/t considering each jamb (half area of column) alone has a value of 12, far below the allowable limit of 18. It is also believed that uneven load arriving at each jamb will be balanced and transferred evenly through the two grout pads before such load reaches the effective column below.

A separate precast concrete panel under the window sill is designed for two purposes, namely (1) to provide a drip and cover for the head of the precast facade unit, and (2) to replace the conventional inlet air grille for the Fan-coil unit. A control damper can effectively alter the path of outside air according to
the need of the season. In the summer when outside air should be chilled before it is delivered to the room, air is directed to the rear inlet of the Fan-coil unit. In the winter when there is a danger of freezing of the coil, a controlled amount of outside air can be brought in bypassing the coil.

4.5 **Load Transfer on Ground Floor**

One of the main problems of a structural system employing a precast structural facade is the final transfer of the facade load. On the ground floor, it is often desirable from the point of view of visual quality to have fewer columns, and it is almost customarily demanded on the ground of spatial freedom to introduce a grander scale. The critical wind moment here can only be resisted by the fewer number of base columns with the aid of the central core.

An attempt has been made in this design to collect both the exterior facade load and the interior column load by 12 inverted pyramids. The facade load is uniformly distributed along one edge and the interior column load is placed at the opposite vertex of each triangle. Eccentric loading is balanced with respect to both axes. A continuous hinge connection is provided under
the precast facade and a hinge under each interior column to ensure no secondary bending is introduced above the pyramid.

4.6 **Construction Sequence**

A slip-forming method is proposed for the construction of the core. A crane will be centrally supported by the core which goes up in advance of the rest of the structure. Precast window units will be erected by the central crane at a rate of 20 windows per day, and 3 days per floor. It is estimated that 3 days will give sufficient time for the casting of interior columns and forming for the flat slab. One day is required for casting of the 8000 sq. ft. floor slab. In the meantime the core rises at the same rate to keep the crane a constant elevation above each floor. The entire process can be repeated at a rate of one floor per week.
V. STRUCTURAL ANALYSIS

5.1 Assumptions

For the purpose of this analysis, the following assumptions are made:

a. The building is located in a region where snow load is computed at 40 p.s.f., maximum velocity of wind is 100 m.p.h.

b. The National Building Code of Canada is used for the computing of wind load and the reduction of live load.

c. Proposed Revision of Building Code Requirements for Reinforced Concrete (ACI 318-56) is accepted.

d. Wind moment is resisted by the members in proportion to the ratio of their moment of inertia.

e. The amount of reaction of the exterior columns to the live and dead load of the slab has been calculated on the assumption of a simply-supported span, thus providing a constant margin of safety throughout.

f. The analysis is limited to the checking of stresses in critical section of the exterior columns only. Working stress design method is used.

5.2 Axial Load

Axial load varies from 6.95 kips to 200.95 kips for exterior columns (Appendix B, Table I). It is obvious, that in order to maintain a constant column area, a certain percentage of columns will be oversized. Table II shows
the range of allowable axial unit stress, $F_a$, obtained by
the combination of various compressive strengths of
concrete $f'_c$, ratio of steel area $p_q$, and yield point of
steel $f_y$. Table III presents the minimum column area $A_g$
possible for each floor and the minimum steel area $A_s$.
Since the minimum steel area required by the code is 1.24
sq. in., the selection of higher strength steel will prove
to be advantageous between the tenth and the seventeenth
floors from the standpoint of saving steel.

A minimum of 96 sq. in. is required by the code.
In this case 100 sq. in. is considered the best choice
to cover the entire range with minimum oversizing from
the second to the eighteenth floors.

Approximate $A_g$ for the interior columns and
the center core area are computed in order to establish
their moment of inertia.

5.3 Approximate Wind Stresses Analysis

The advantage of the structural facade is
particularly apparent when the ratio of moment of inertia
is compared (Appendix B). Because of the large number
of columns on each side of the core, it is assumed that
no bending will take place for each individual column,
but tension and compression will act as a couple on the
windward and leeward sides of the building. Shear caused by wind load is not considered critical as it is in effect resisted by two exterior frames (and core) behaving like Vierendeel trusses in two directions; and shear key at head of each precast unit ensures rigid bracing by all floor slabs.

Maximum compression force computed from wind moment is 23.4 kips for each second floor column. ACI code allows for \( L. L. + D. L. + \text{Wind}, P=1.333 \alpha_g F_a \). Since the column load at second floor level is 200.95 kips, the additional compression is within the allowable additional stress limit.

5.4 Gravity Bending and Shear in Window Unit

It is assumed that when the precast window unit is erected and before additional column load is superimposed to give each column sufficient restrain, the head of the window which carries the immediate floor load on it, should be designed for positive bending moment as a simple support beam. Again it is checked for negative bending moment for end restrain (Appendix B). Shear is found to be negligible. Since there is a slight torsion developed by the rotation of the head of the window unit to absorb thermal movement of the structural facade, it
is advisable to provide stirrups of minimum size at 9"c.c. in order to prevent cracks from occurring on the surface.

5.5 Erection Stresses in Window Unit

Since the unit is designed to support L. L. and D. L. of 1.53 kips per foot when in place, there is no doubt that it will have sufficient resistance to bending when it is lifted in an upright position. However, the pick-up points are located to ensure minimum bending moment, developed due to the weight of the unit.

5.6 Combined-loading at Base Column

Since each of the base columns is braced and connected to others and the core, the exact distribution of moment is difficult to estimate. An approximate calculation by the ratio of moment of inertia indicates a critical moment of 5,580'k. However, this amount will be considerably reduced by far more complex bending effects with the pyramids. Therefore, it is estimated safe enough to design for a resisting moment of 3000'k along X-X axis, and 443'k along Y-Y axis for the purpose of tentative sizing of base columns.

It is also expected that bending moment will be carried up the pyramid walls for a certain distance. Since superimposed load on the pyramids may not be uniformly
balanced at all times, there is danger of buckling of the pyramid walls. No attempt is made to calculate this structural element in view of the fact that it is a very complicated process, so the checking of the pyramid walls has therefore been limited to the computed axial load only to ensure that the compressive stress is within the allowable limit. Cross diaphragms are provided inside the pyramids to ensure walls against buckling.

5.7 Thermal Movement

Thermal expansion or contraction in this case is not prevented by the floor structure, consequently no stress is produced. A total movement of 1 5/8" for a 100°F temperature differential should be prepared for the structural facade.
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APPENDIX A

List of Symbols

\( A_g \) = gross area of tied column  
\( A_s \) = area of reinforcement  
\( B \) = trial factor between 3 and 3.5  
\( b \) = width of rectangular flexural member  
\( C_h \) = velocity height variation coefficient  
\( C_s \) = shape or form factor  
\( D. L. \) = dead load  
\( d \) = distance from extreme compression fiber to centroid of tension reinforcement  
\( e_{x,y} \) = eccentricity of the resultant load on a column, measure from the gravity axis along the X or Y axis  
\( f_a \) = axial load divided by area of member, \( A_g \)  
\( f_a \) = allowable axial unit stress  
\( f_{b_x}, f_{b_y} \) = bending moment about the X or Y axis divided by the section modulus of the transformed section  
\( f_b \) = allowable bending stresses that would be permitted for bending alone  
\( f_c \) = compressive stress in concrete  
\( f_c' \) = compressive strength of concrete at 28 days  
\( f_s \) = nominal allowable stress in vertical column reinforcement  
\( f_y \) = tensile stress in web reinforcement  
\( f_y \) = specified yield point of reinforcement  
\( I \) = moment of inertia of beam or column  
\( j \) = ratio of distance between centroid of compression and centroid of tension to the depth, \( d \)  
\( k \) = ratio of distance from extreme compressive fiber to neutral axis to the depth, \( d \).  
\( k_{si} \) = kips per square inch  
\( L. L. \) = live load  
\( l \) = span length of slab or beam  
\( M \) = Bending moment  
\( N \) = load normal to the cross section of a column  
\( n \) = ratio of modulus of elasticity of steel to that of concrete
APPENDIX A (cont.)

\[ P = \text{allowable load on a reinforced concrete column without reduction for length or eccentricity} \]

\[ P_g = \text{ratio of cross-sectional area of vertical reinforcement to the gross area, } A_g \]

\[ \text{psf} = \text{pound per square foot} \]

\[ \text{psi} = \text{pound per square inch} \]

\[ Q = \text{static moment} \]

\[ t = \text{over-all depth of rectangular column} \]

\[ V = \text{total shear} \]

\[ V_{30} = \text{velocity of wind at 30 ft. above ground} \]

\[ v = \text{nominal shear stress as a measure of diagonal tension} \]

\[ v_c = \text{shear stress assumed carried by concrete} \]

\[ W = \text{total dead and live load} \]

\[ w = \text{uniformly distributed unit dead and live load per unit of length} \]

\[ \Sigma = \text{summation} \]
APPENDIX B - SAMPLE CALCULATIONS

I. Axial Load on Column

Area of load per col.
= 5.83' x 8.75'
= 51 sq. ft.

Assume 7" flat slab.

Fig. B-1

Roof load:
Snow 40 psf
Roofing 6
Insulation 3
7" concrete 87.5
= 136.5

136.5 x 51 = 6.95 k

Dead load:
Flooring 5 psf
Plaster 5
7" concrete 87.5
= 97.5

97.5 x 51 = 4.97 k

Live load:
Live load 40 psf
Partition 18

58

58 x 51 = 2.96 k

Add:
Window Unit,
Glass .217
Concrete 1.4
= 6.59 k

Weight of column:

Assume 100 sq. in.
= .93 k

Weight of 23rd floor ext. panel:
Assume 6" panel = 1 k

Additional D.L. + L.L. for Mechanical Floor: 4 k
<table>
<thead>
<tr>
<th>FLR</th>
<th>D. L. TOTAL</th>
<th>L. L. TOTAL</th>
<th>% OF REDUCTION</th>
<th>NET L. L. TOTAL</th>
<th>D. L. + L. L.</th>
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<td>6.95</td>
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<td>24.86</td>
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<td>57.14</td>
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<td>176.42</td>
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</table>
TABLE II ALLOWABLE AXIAL UNIT STRESS

\[ P = A_g \left( 0.18 f_c^1 + 0.32 p_g f_y \right) \] For Tied Column

\[ f_a = 0.32 f_y \] not to exceed 24,000 psi

\[ p_g = \frac{A_g}{A_g} \] between 0.01 and 0.04

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<th>TYPE</th>
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<th>( f_y = 40,000 )</th>
<th>( f_y = 50,000 )</th>
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<th>( f_y = 75,000 )</th>
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<td>( F_a )</td>
<td>( p_gf_a )</td>
<td>( F_a )</td>
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<td>960</td>
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<td>758</td>
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<td>790</td>
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<td>( F_a(3) )</td>
<td>( F_a(4) )</td>
<td>( A_g )</td>
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<td>22</td>
<td>21.99</td>
<td>( P_g = 0.01 )</td>
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<td>*1.24, 4 -5/8&quot;</td>
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<td>19</td>
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<td>76.0</td>
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<td></td>
</tr>
<tr>
<td>18</td>
<td>59.17</td>
<td>( P_g = 0.01 )</td>
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<tr>
<td>17</td>
<td>67.72</td>
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<td>*1.24, 4 -5/8&quot;</td>
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</tr>
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<td>16</td>
<td>75.99</td>
<td>( P_g = 0.02 )</td>
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<td>*1.24, 4 -5/8&quot;</td>
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<td></td>
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<td>83.95</td>
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<td>91.0</td>
<td>*1.24, 4 -5/8&quot;</td>
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<tr>
<td>14</td>
<td>92.95</td>
<td>( P_g = 0.02 )</td>
<td>89.0</td>
<td>*1.24, 4 -5/8&quot;</td>
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<td>13</td>
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<td>( P_g = 0.02 )</td>
<td>93.5</td>
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<td>12</td>
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<td>92.5</td>
<td>*1.24, 4 -5/8&quot;</td>
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<tr>
<td>11</td>
<td>119.95</td>
<td>( P_g = 0.02 )</td>
<td>93.5</td>
<td>*1.24, 4 -5/8&quot;</td>
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<tr>
<td>10</td>
<td>128.95</td>
<td>( P_g = 0.02 )</td>
<td>98.0</td>
<td>*1.24, 4 -5/8&quot;</td>
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<td></td>
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<tr>
<td>9</td>
<td>137.95</td>
<td>( P_g = 0.03 )</td>
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<td>1.77, 4 -3/4&quot;</td>
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<tr>
<td>8</td>
<td>146.95</td>
<td>( P_g = 0.03 )</td>
<td>99.5</td>
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<td>7</td>
<td>155.95</td>
<td>( P_g = 0.04 )</td>
<td>93.5</td>
<td>2.00, 4 -3/4&quot;</td>
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<td>6</td>
<td>164.95</td>
<td>( P_g = 0.04 )</td>
<td>98.7</td>
<td>2.76, 4 -3/4&quot;</td>
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<td>5</td>
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<tr>
<td>4</td>
<td>182.95</td>
<td>( P_g = 0.04 )</td>
<td>91.0</td>
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<td>2</td>
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<td>98.5</td>
<td>3.94, 4 -7/8&quot;</td>
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</tbody>
</table>

* Minimum steel area 4- 5/8" bars = 1.24 sq. in.
II Axial Load on Interior Column

Assume area of load per column = 17.5 x 17.5 = 306 sq.ft.
Assume column forms are reused for 7 floors, critical column area
is checked at 2nd, 9th, and 16th floors.

<table>
<thead>
<tr>
<th>FLR</th>
<th>TOTAL D. L.</th>
<th>TOTAL L. L.</th>
<th>D. L. + L. L.</th>
<th>COLUMN WEIGHT</th>
<th>N</th>
<th>F_a(5)</th>
<th>A_g</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>230 k</td>
<td>65.7 k</td>
<td>295.7 k</td>
<td>24.8 k</td>
<td>320.5 k</td>
<td>1142</td>
<td>280.0 (18&quot;x15.5&quot;)</td>
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<tr>
<td>9</td>
<td>431 k</td>
<td>119.5 k</td>
<td>550.5 k</td>
<td>50.6 k</td>
<td>601.1 k</td>
<td>1398</td>
<td>430.0 (18&quot;x24&quot;)</td>
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<tr>
<td>2</td>
<td>632 k</td>
<td>179.2 k</td>
<td>811.2 k</td>
<td>84.0 k</td>
<td>895.2 k</td>
<td>1590</td>
<td>563.0 (18&quot;x31.5&quot;)</td>
</tr>
</tbody>
</table>

III Interior Core

Assume L.L. + D.L. = 150 psf

Total load per 1'-0" strip of wall = \( \frac{w_1 + w_3}{2} \) \( \frac{2}{3} \)
= \( \frac{150 \times 6 + 150 \times 24}{3} \) = 2.1 k

Assume wall thickness is reduced at 2nd, 9th, and 16th floors.

<table>
<thead>
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<th>FLR</th>
<th>TOTAL D.L. + L.L.</th>
<th>WEIGHT OF WALL</th>
<th>N</th>
<th>F_a(5)</th>
<th>A_g*</th>
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<td>16</td>
<td>16.8 k</td>
<td>6 k</td>
<td>24.8 k</td>
<td>.758</td>
<td>32.6 (12&quot;x6&quot;)</td>
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<tr>
<td>9</td>
<td>31.5 k</td>
<td>10.7 k</td>
<td>42.2 k</td>
<td>790</td>
<td>53.5 (12&quot;x8&quot;)</td>
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<tr>
<td>2</td>
<td>46.2 k</td>
<td>15.5 k</td>
<td>61.7 k</td>
<td>870</td>
<td>71.0 (12&quot;x10&quot;)</td>
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</table>

*Approximately 40% allowed for elevators, doors and ducts opening.
Area in ( ) indicates gross A_g per 1'-0" strip of wall.
IV Approximate Wind Stresses Analysis

Wind load = \(0.00256 \times c_s (c_h v_{30})^2\) or \(c_s p\)

where \(p = 0.00256 (c_h v_{30})^2\) from Table 41Al, National Building Code of Canada, 1953. \(c_s = (+.5) + (+.2) = .7\)

Assume 100 mph wind velocity.

Wind Load

\[p = 36.1\] \(\times .00256 (1.312 \times 100)^2 = 30 \text{ psf}\)

\(p = 33.9\) \(\times .00256 (1.312 \times 100)^2 = 25.3\)

\(p = 29.6\) \(\times .00256 (1.312 \times 100)^2 = 22.8\)

\(\Sigma M about A = 20 \times 90 \times 30 \times 15 + 25 \times 90 \times 50 \times 55 + 30 \times 90 \times 125 \times 142.5 = 55,091,250 \text{ ft.lb.}\)
Moment of Inertia $I = Ad^2 + \frac{bt^3}{12}$ at 2nd floor level A.

Since $\frac{bt^3}{12}$ is comparatively very small, it can be neglected.

$I_{\text{ext.col.}} = 2 \times 16 \times 100 \times \frac{1}{4} \times (540)^2 = 935,000,000 \text{ in}^4$

$I_{\text{int.col.}} = 2 \times 4 \times 565 \times (330)^2 = 491,000,000 \text{ in}^4$

$I_{\text{core}} = \frac{288^4 - 272^4}{12} = 103,000,000 \text{ in}^4$

$I_{\text{ext.col.}} + I_{\text{int.col.}} + I_{\text{core}} = 1,529,000,000 \text{ in}^4$

Moment about A resisted by exterior columns will be:

$\frac{55,091,250 \times 935,000,000}{1,529,000,000} = 33,605,000 \text{ ft.lb.}$

In a plane A, this moment is resisted by the moment of a couple.

This couple, tension or compression, is the additional load on 16 exterior columns on each side.

$\frac{374 \times 1000}{90} = 374,000 \text{ lb.}$

$\frac{374k}{16} = 23.4k < .33 \text{ of } 200.95k \text{ (N at 2nd floor)}$
Shear stress due to Wind Load at section X - X:

Since all longitudinal reinforcement acts in compression, this allows

\[ v_c = 1.75 \sqrt{\frac{f'_c (1 + 0.004 N/A_g)}{3,500 (1 + 0.004 \times 2,040)}} \]

Therefore,

\[ v_c = 1.75 \sqrt{3,500 (1 + 0.004 \times 2,040)} \]

= 312 psi

Transverse shear V due to Wind Load is,

\[
V = (20 \times 30 \times 90) + (25 \times 50 \times 90) \\
+ (30 \times 125 \times 90)
\]

= 504,000 lb.

At the neutral axis,

\[
Q = Q_{flange} + Q_{web}
\]

= \[
(2 \times 8 \times 100 \times 45 \times 12) \\
+ (2 \times 8 \times 100 \times 22.5 \times 12) \\
+ (4 \times 563 \times 27.5 \times 12) \\
+ (2 \times 563 \times 10 \times 12) + (4,680 \times 108.3)
\]

= 2,675,000 in\(^3\).

The shear stress is,

\[
v = \frac{VQ}{bhI}
\]

\[
= \frac{504,000 \times 2,675,000}{2 \times 10 \times 1,529,000,000}
\]

= 44 psi, smaller than \(v_c\). O.K.
GRAVITY BENDING AND SHEAR IN WINDOW UNIT

Typical unit, 18th to 22nd floors.

D.L. + L.L. = 4.97 + 2.96 = 7.93 k per unit

Uniform load = \( \frac{7.93}{5.16} = 1.53 \) kips per foot.

Condition (a) - simply-supported beam,

\[ M = \frac{w l^2}{8} = \frac{1.53 \times (5.16)^2 \times 12}{8} = 61,000 \text{ #} \]

Condition (b) - when ends are restrained,

\[ M = \frac{w l^2}{12} = 40,600\text{ #} \]

Width of beam \( b = 13" \),

\( f' = 3,000, \quad f_c = 45 f' = 1,350 \)

\( f_s = 20,000, \quad n = 30 \times 10^6 = 10, \quad \frac{60,000}{f_s} \)

\( k = 0.403 \), and \( j = 0.866 \)

For \( +M \), \( d = \frac{61,000}{5 \times 1350 \times 405 \times 866 \times 15} = 4.5" \)

\[ A_s = \frac{61,000}{20000 \times 866 \times 4.5} = 0.785 \text{ sq.in.} \]

For \( d = 6" \), \( A_s = 0.59 \text{ sq.in.} \)

For \( d = 7" \), \( A_s = 0.305 \)

For \( d = 8" \), \( A_s = 0.442 \)

In order to provide a shear key for the floor slab, \( d = 7" \) is needed.

For \( -M \), when \( d = 7" \), \( A_s = 0.336 \text{ sq.in.} \)

Shear \( v = \frac{V}{bd} = \frac{3950}{13 \times 7} = 43.5 \text{ psi} \), smaller than \( v_c \) allowed.

Shear Key, \( V = 1.53 \text{ kips per 12" strip}, \)

\[ d^k = \frac{V}{v_c b} = \frac{1530}{55 \times 12} = 2.3" \]
VI ERECTION STRESSES IN WINDOW UNIT

Weight of unit-

Jamb, \(80 \text{ sq. in.} \times 150 \times 9' = 750 \text{ lb.}\)

1/2 Sill, \(1 \times 50 \times 150 \times 2.375 = 62 \text{ lb.}\)

D.L. each side = 812 lb.

Head, \(w = 164 \text{ sq.in.} \times 150 = 171 \text{ plf}\)

\(R_1 = R_2 = 812 + \frac{171 \times 4.75}{2} = 1219 \text{ lb.}\)

Since the unit is designed for \(+M\) and \(-M\) due to \(w = 1.55 \text{ kips}\), it is obvious that \(A_g\) provided is ample to handle the erection stresses. Assume two pick-up points as shown,

\(-M\) due to weight of unit = \(\frac{171 \times 4.75}{2} = 321 \text{ ft.lb}\)

Extreme fiber stress in tension in concrete without reinforcement,

\(f = \frac{Mc}{I} = \frac{321 \times 12 \times 4.7}{\frac{1}{12} \times 17.5 \times 9.45} = 15.4 \text{ psi. O.K.}\)

Pick-up loops should be located so that \(x\) is less than 3" to ensure shear is resisted by the jamb.

INITIAL PICK-UP

\(M_o = 1219 \times 2.425 - 814 \times 2.05 - 166.6 \times 2.425^2\)

\(= 798 \text{ ft.lb. (for two jambs)}\)

Extreme fiber stress in concrete in tension without reinforcement,

\(f = \frac{798 \times 12}{6 \times 10^2} \times \frac{1}{2} = 47.8 \text{ psi / jamb}\)
VII COMBINED AXIAL LOAD AND BENDING OF BASE COLUMN

\[ M_y = 3,000 \text{ ft.k.} \quad \text{and} \quad M_x = 443 \text{ ft.k.} \quad N = 1700 \text{ k.} \]
\[ e_y = \frac{M_y}{P} = \frac{3000}{1700} = 1.77' \text{ or } 20" \]
Assume \( t = 70" \) is smaller than \( \frac{2t}{3} \), \( e = 0.28 \)
\[ e_x = \frac{M_x}{P} = \frac{443}{1700} = 0.26' \text{ or } 3.13" \]
Assume \( t = 36" \) is smaller than \( \frac{2t}{3} \), \( e = 0.086 \)

For combined loading,
\[ \frac{f_a}{F_a} + \frac{f_{bx}}{F_b} + \frac{f_{by}}{F_b} = 1 \]

(1) Use Trial Factor method \( P = N \left( 1 + \frac{B_e}{t} \right) \), where \( B_e \) is the numerical sum of \( B_e \) in both directions. \( B \) is between 3 and 3.5
Assume \( b = 3.5 \)
\[ B_e = 3.5 \times 0.28 + 3.5 \times 0.086 = 1.26 \]
\[ P = 1,700,000 \left( 1 + 1.26 \right) = 3,840,000 \]
\[ p = 0.4, f_c = 5,000, \quad n = 7 \]
\[ P = 0.8 A_c \left( 0.225 f_c + f_s p_g \right) = 3,840,000 \]
\[ A_c = 2,500 \text{ sq.in.} \]

Value of concrete,
\[ 0.8 \times 225 \times 5000 \times 2500 = 2,260,000 \]
Value of steel,
\[ 0.8 \times 20,000 A_s = 1,580,000 \]
\[ A_s = 99 \text{ sq.in.} \]
(2) \[ f_a = \frac{1,700,000}{2,500} = 680 \text{ psi} \]

\[ F_a = \frac{2,260,000 + .8 \times 99 \times 20,000}{2,500} = 1,535 \text{ psi} \]

\[ I_x = \frac{1}{12} \times 36 \times 72^3 + 99 \times (6 - 1) \times 33.5^2 \]
\[ = 1,868,000 \text{ in}^4 \]

\[ S = \frac{I}{c} = \frac{1,868,000}{36} = 52,000 \text{ in}^3 \]

\[ f_{bx} = \frac{3,000,000 \times 12}{52,000} = 692 \text{ psi} \]

\[ F_{b'} = 0.45 \frac{f_c}{c} = 2,250 \text{ psi} \]

\[ I_y = \frac{1}{12} \times 72 \times 36^3 + 99 \times (6 - 1) \times 16.5^2 \]
\[ = 416,450 \text{ in}^4 \]

\[ S = \frac{I}{c} = \frac{416,450}{18} = 23,100 \text{ in}^3 \]

\[ f_{by} = \frac{443,000 \times 12}{23,100} = 230 \text{ psi} \]

\[ F_b = 2,250 \]

\[ \frac{f_a}{F_a} + \frac{f_{bx}}{F_b} + \frac{f_{by}}{F_b} \]

\[ = \frac{680}{1,535} + \frac{692}{2,250} + \frac{230}{2,250} = .852 \text{ smaller than 1} \]
Axial Load on Pyramid walls

Assume symmetrical loading as shown on Fig. 8-8.

To balance superimposed loads with respect to $X'-X'$ axis,

$M = 0$

$820 \times x = 900 \times y$

$x : y = 900 : 820$

$x = \frac{900 \times 18}{1720} = 9.42'$

$y = \frac{820 \times 18}{1720} = 8.58'$

$\tan \theta_1 = \frac{9.42}{15}$, $\theta_1 = 32.1^\circ$

$\tan \theta_2 = \frac{8.58}{15}$, $\theta_2 = 29.7^\circ$

Since $R_1 \cos 32.1^\circ = 820$

$R_1 = 968$ k.

$R_2 \cos 29.7^\circ = 900$

$R_2 = 1,035$ k.

Tension on top of pyramid,

$= 968 \sin 32.1^\circ$

$= 1035 \sin 29.7^\circ$

$= 514$ k.

This represents the components of the sum of all forces along the $Y'-Y'$ axis only acting on the base column.

The pyramid wall must take into account the change of angle along section A-A. True angle at A-A = $45^\circ$. For each facade column,

$R_a = \frac{205}{\cos 45^\circ} = 290$ k  compared to $R_1 = \frac{205}{\cos 32.1^\circ} = 242$ k.

For a 12" strip of wall, $f_c = 3,500$ psi, $f_y = 75,000$ psi, $p_e = 0.04$

$A_{e} = \frac{P}{f_a} = \frac{290,000}{1,590} = 183$ sq.in. (12" x 15.3" thick)
VIII THERMAL STRAIN

\[ \varepsilon_T = \alpha(\Delta T) \]

where \( \varepsilon_T \) = Thermal strain
\( \alpha \) = Coefficient of linear expansion
\( \Delta T \) = Change of temperature

Assume in a critical winter condition,
\[ t_{\text{int.}} = 70^\circ \text{F} \]
\[ t_{\text{ext.}} = -30^\circ \text{F} \]
\( \alpha \) of concrete = \( 6 \times 10^{-6} / \text{F} \)

For a total height of building = 230' or 2,760'',
total thermal contraction of the exterior facade will be,

\[ \frac{6 \times 100 \times 2,760}{1,000,000} = 1.6'' \]

In a condition of complete constraint, \( \varepsilon \) must be zero, and,

\[ \varepsilon = \frac{\sigma}{E} + \alpha(\Delta T) = 0 \]

a thermal stress, \( \sigma \), will be present.

\[ \sigma = - E \alpha(\Delta T) = - (4.6 \times 10^6)(6 \times 10^{-6} \times 100) \]
\[ = -2,760 \text{ psi tension} \]

However, since a hinged condition exists on each floor, there is no stress developed.
APPENDIX C

Examples of Structural Facade in Housing Projects...

(a) Kips Bay Plaza, New York, N. Y.

(b) Society Hill, Philadelphia, Pa.
**BUILDING**  Kips Bay Plaza Apartments, New York, N. Y.

**ARCHITECT**  I. M. Pei & Associates, New York

<table>
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<tr>
<th>NUMBER OF FLOOR</th>
<th>FLR TO FLR HEIGHT</th>
<th>BAY SIZE</th>
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<tbody>
<tr>
<td>21</td>
<td>8'- 9&quot;</td>
<td>17' x 17'</td>
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**STRUCTURAL FRAMING**
Reinforced Concrete

**STRUCTURAL FACADE**
Cast-in-place reinforced concrete

**SPAN TO INTERIOR SUPPORT**
17'- 0"

**STRUCTURAL MULLION SPACING**
5'- 8" o.c.

**WINDOW**
Fixed glass with hopper unit

**MECHANICAL SYSTEM**
Room air-conditioner
Heating--continuous convector type

**REMARK**
Exterior base column at 17'- 0" o.c.
Fig. 1
Kips Bay Plaza.

Fig. 2  Half-plan of typical floor of Kips Bay Plaza apartments.
      Each floor contains 28 apartments.
**BUILDING**  Society Hill Towers, Philadelphia, Pa.

**ARCHITECT**  I. M. Pei & Associates, New York

<table>
<thead>
<tr>
<th>NUMBER OF FLOOR</th>
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<tbody>
<tr>
<td>32</td>
<td>8'- 9&quot;</td>
<td>17&quot;- 6&quot; x 17'- 6&quot;</td>
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</table>

**STRUCTURAL FRAMING.**
- Reinforced concrete

**STRUCTURAL FACADE**
- Cast-in-place reinforced concrete

**SPAN TO INTERIOR SUPPORT**
- 17'- 6"

**STRUCTURAL MULLION SPACING**
- 5'- 10" o.c.

**WINDOW**
- Fixed glass with hopper unit

**MECHANICAL SYSTEM**
- Continuous heat and A. C. unit

**REMARK**
- Exterior base columns at 11'- 8" o.c.
Fig. 3 Society Hill towers.

Fig. 4 Typical floor plan of Society Hill towers. Each tower contains 240 apartments (8 per floor).
Fig. 5 Kips Bay Plaza facade at the time when windows were being installed.
Fig. 6 Drawing from a study of facade for the Society Hill project.
1. Kips Bay Plaza apartments project (Fig. 1 and 2) is a completed example of several housing projects by I. M. Pei and Associates, Architects, where an exposed cast-in-place concrete structural facade system has been employed. It demonstrates a search for a fair-face concrete surface out of the form, free of surface defects, and requiring no further surface treatment other than a wash down with a detergent, oxalic acid, or muriatic acid as performed on brick masonry.

2. The same system is employed for the University Apartments, Chicago, and the proposed Society Hill project in Philadelphia (Fig. 3 and 4).

3. The facade for Kips Bay Plaza project (Fig. 5) was originally designed to be built in precast concrete, but this was prevented by the costs and codes. The exterior columns, flush at the building face, are reduced in depth at the fifth and tenth floors to express their diminishing loads. Glass line was set back 14½" to provide shielding of sky glare. 3500 p. s. i. stone concrete was used throughout the whole structure.

4. Although the basic facade system is the same as that of Kips Bay Plaza project, the module and details of Society
Hill project vary to conform with forming refinement and aesthetic experimentation. The exterior columns, also flush at the building face, are reduced from 1'- 0" x 3'- 0" to 1'- 0" x 2'- 0" at the thirteenth floor. This difference in the positioning of glass line will result in a richer variation of shadow lines.

5. It is claimed that the economic prognosis of Kips Bay Plaza project is promising and indicates a possible saving in ultimate cost to the owner due to the speed of construction over the conventional concrete frame, brick cladded type.

6. No doubt significant is the architectural concept and the new character of its structural form.