TESTS OF AN EMPIRICAL INFLUENCE FUNCTION

FOR REGIONAL PROJECTIONS

by

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May 20, 1961

Professor John T. Howard
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Dear Professor Howard:

I hereby submit my thesis entitled, Tests of an Empirical

Influence Function for Regional Projections, in partial fulfillment

of the requirements for the degree of Master in City Planning.

Respectfully yours,

Jean C. MacCarthy

<u>ABSTRACT</u>

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Submitted to the Department of City and Regional
Planning on May 20, 1961, in partial fulfillment
of the requirements for the degree of Master in
City Planning.

Projections of regional productivity have not attempted
to consider the interactions between regions. The value and
validity of a method for predicting interregional productivity by use
of an income influence function has been examined within this study.
Within the limitations of the data used, it has been found to be of
no value.

THESIS SUPERVISOR:.......................

Dr. Aaron Fleisher
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CHAPTER I

INTRODUCTION

INTRODUCTION

Projections of regional productivity are usually made by extrapolating the latest national trend on the assumption that as the nation goeth so doth the region. Sometimes, as an alternative method, the trend of the region itself is determined. Missing from both of these methods is any attempt to consider the interactions between the regions.

The influence one region has on another would, on intuitive grounds, be expected to decline as distance increases. This distance effect has been verified for a variety of socio-economic factors, such as: population migration[1]; residential propinquity of applicants for marriage licenses[2]; change of family residence[3]; bus, railway, and airline passenger movements[4]; and attendance by state at universities[5].

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[1] E. G. Ravenstein, "The Laws of Migration", Journal of the Royal Statistical Society, 1885.

[2] J. H. S. Bossard, "Residential Propinquity as a Factor in Marriage Selection," American Journal of Sociology, XXXVIII, 1932; M. R. Davie and R. J. Reeves, "Propinquity of Residence Before Marriage," American Journal of Sociology, XLIV, 1939; and R. H. Abrams, "Residential Propinquity as a Factor in Marriage Selection," American Sociological Review, VIII, 1943.

[3] S. A. Stouffer, "Intervening Opportunities: A Theory Relating Mobility and Distance," American Sociological Review, 1940.

[4] G. K. Zipf, Human Behavior and the Principle of Least Effort, (Cambridge: Addison-Wesley Press), 1949.

[5] John Q. Stewart, "The Development of Social Physics," American Journal of Physics, XVIII, No. 5, 1950.

Isard and Freutal[1] have made this distance effect the basis

of a model for regional productivity projections. It is the purpose

of this study to verify the accuracy of this model against a

particular set of data - a large portion of which was obtained from

an investigation which used this same measure of influence for regional

population projections.

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[1] Walter Isard and Guy Freutal, "Regional and National Projections and Their Interrelations," Long-Range Economic Projection, National Bureau of Economic Research, (Princeton: Princeton University Press), 1954.

CHAPTER II

FORMULATION OF THE PRODUCTIVITY

PROJECTION MODEL

FORMULATION OF THE PRODUCTIVITY PROJECTION MODEL

Isard and Freutal defined productivity of a region using the technique of the Council of Economic Advisors (CEA)[1]: productivity is the product of the region's total labor force and average productivity per worker.[2] If the productivity of a region should increase or decrease at the same rate as national productivity during a certain period, then:

$$_i GRP_{t+\theta} = \lambda_\theta \; _i GRP_t \qquad \text{where} \qquad \lambda_\theta = \frac{GNP_{t+\theta}}{GNP_t}$$

where $\quad _i GRP = {}_i L \; _i T;$

$\quad _i L$ = number of workers in region i;

$\quad _i T$ = average productivity per worker in region i;

$\quad GNP = L \; T;$

$\quad L$ = number of workers in the nation;

$\quad T$ = average productivity per worker in the nation;

$\quad t$ = base year;

$\quad \theta$ = forecast period; and

$\quad t+\theta$ = forecast date.

If, however, the growth or decline of a region's productivity depended also on the effect of productivity changes in surrounding regions, then the authors suggested that a productivity projection for the region should measure the effect of external, as well as internal, changes.

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[1] *Annual Economic Review*, Council of Economic Advisors, (Washington, D.C.: United States Government Printing Office), January 1950, p. 76.

[2] Isard and Freutal, op. cit., p. 47.

The projection model would then be written as:

$$_i GRP_{t+\theta} = \lambda_\theta \,_i GRP_t + \,_i X$$

where $_i X$ = a measure of the effect of external changes
of productivity on region i.

Isard and Freutal termed the measure proposed to account for

interregional economic relations the "potential of income" - a measure

analogous to Stewart's population potential.

On the basis of field studies, John Q. Stewart (in 1939)

suggested that a group of people exert influence on another group in

direct proportion to their size and in inverse proportion to their

separation.[1] He later referred to this population/distance ratio as

the "coefficient of influence"[2] Drawing an analogy to physical

concepts, Stewart and others referred to this influence ratio as

"population potential". Isard's and Freutal's concept of "income

potential" - obtained by substituting some form of income for

population in Stewart's ratio - will be called an "income influence"

in this thesis[3] The sum of the influences on a particular region i

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[1]
John Q. Stewart, "Empirical Mathematical Rules Concerning the
Distribution and Equilibrium of Population", Geographical Review, XXXVII,
1947, American Geographical Society, p. 471.

[2]
John Q. Stewart, "A Measure of the Influence of a Population at a
Distance," Sociometry, V, 1942, p. 66.

[3] While an analogy between a concept in one field with a concept in the
physical sciences is desirable to enlarge the knowledge and
applications of the former field, strict adherence to the analogy may
defeat this intent. The analogy tends to obliterate the fact that the
concept's validity is not attached to the concept by the analogy, but
must be determined by testing. In this instance, the physical analogy
has also tended to obliterate the fact that this influence function was
a measure fitted to demographic statistics and did not proceed from
a priori assumptions.

of the income of region i and the surrounding regions will be considered as "total income influence" on region i; that is,

$$_iV_T = \frac{_1Y_T}{_{i1}D} + \ldots + \frac{_iY_T}{_{ii}D} + \ldots + \frac{_nY_T}{_{in}D} = \sum_{j=1}^{n} \frac{_jY_T}{_{ij}D},$$

where $\quad _iV_T$ = total income influence on region i at time T;

$\quad _jY_T$ = income of region j at time T; and

$\quad _{ij}D$ = distance between region i and region j.

Isard's and Freutal's definition of $_iX$ dictated that this measure be zero when the change in total income influence on region i was proportionate to national productivity growth, positive when it was greater, and negative when it was less; that is,

$$_iX = 0 \qquad \text{when} \qquad _iGRP_{t+\theta} = \lambda_\theta \, _iGRP_t$$

$$_iX > 0 \qquad \text{when} \qquad _iGRP_{t+\theta} > \lambda_\theta \, _iGRP_t$$

$$_iX < 0 \qquad \text{when} \qquad _iGRP_{t+\theta} < \lambda_\theta \, _iGRP_t \, .$$

The authors defined the following factor as their representation of $_iX$:

$$_i\beta \left(\frac{_iV_{t+\theta}}{\rho_\theta \, _iV_t} - 1 \right) _iGRP_t = \, _iZ,$$

where $\quad _i\beta$ = a positive measure of each region's resources;

$\dfrac{_iV_{t+\theta}}{_iV_t}$ = relative income influence; and

$\rho_\theta = \dfrac{_{US}Y_{t+\theta}}{_{US}Y_t} \qquad$ where Y = income of the United States.

This choice for $_iX$ exactly satisfied the required constraints; for if a change in total income influence upon region i were the same as national income growth, then

$$\frac{_iV_{t+\theta}}{\rho_\theta \,_iV_t} = 1 \qquad \text{and} \qquad _iZ = 0.$$

If the change were greater than national income growth, then

$$\frac{_iV_{t+\theta}}{\rho_\theta \,_iV_t} > 1 \qquad \text{and} \qquad _iZ > 0.$$

When the nation's growth was greater than the change in total income influence on region i, then

$$0 < \frac{_iV_{t+\theta}}{\rho_\theta \,_iV_t} < 1 \qquad \text{and} \qquad _iZ < 0 .$$

Isard and Freutal offered no other justification for this choice of their representation of $_iX$. Obviously, their weighting of the "modified relative income influence" $\left(\text{i.e.,} \quad \frac{_iV_{t+\theta}}{\rho_\theta \,_iV_t} - 1 \right)$ by regional productivity satisfied this factor's need for a dimension of productivity. The authors' use of $_iV_{t+\theta}$ — total income influence upon region i at the <u>projected</u> date — is a serious weakness. A factor within a projective model which itself needs projection is feasible only if this factor is easily predicted; that is, if it increases or decreases at an unvarying rate, is not subject to sudden changes — is "stable" over time. Similarly, the use of the factor

$$\left(\frac{_iV_{t+\theta}}{\rho_\theta \, _iV_t} - 1 \right)$$ in the model is feasible only if the determination of $_i\beta$

from past behavior applies as well to its future behavior. Since the modified relative income influence measure must account for the positive or negative values of $_iZ$, $_i\beta$ must also always be positive. But, negative values are not entirely impossible. If the viewpoint were generalized slightly and $_iZ$ were considered as a term in a linear regression model, then $_i\beta$ could assume negative values; but $_iZ$ would then lose its economic interpretation. Thus, the workability of Isard's and Freutal's theoretical proposal for $_iX$ depends upon a determination of the stability over time of $_iV_t$ and $_i\beta$, and the validity of the model requires that $_i\beta$ always be positive.

The theoretical projection model proposed by Isard and Freutal would have the following form:[1]

$$_iGRP_{t+\theta} = \lambda_\theta \, _iGRP_t + _i\beta \left(\frac{_iV_{t+\theta}}{\rho_\theta \, _iV_t} - 1 \right) _iGRP_t \; .$$

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[1]
Isard and Freutal did not present their projection model in the above form, but instead in the following manner:

$$_iGRP_{t+\theta} = _iGRP_t(1+p)^\theta (1+z)^\theta + _i\beta \left(\frac{_iV_{t+\theta}}{(1+\rho'_\theta)_iV_t} - 1 \right) _iGRP_t$$

where   p = rate of national population change;
z = rate of national productivity per worker change; and
ρ'_θ = rate of national income change.

Isard in his book Methods of Regional Analysis, the Boston City Planning Board in their Staff Report "Income Potential and Regional Productivity Projections", and Isard and Freutal in the above model defined the variables p, z, and ρ'_θ as the rate of national population change, productivity change, and income change, respectively. This definition

In an attempt to verify the accuracy of this model, the modified relative income influence factor $\left(\dfrac{_i V_{t+\theta}}{\rho_\theta \; _i V_t} - 1 \right)$ will be evaluated. If

$_i V_{t+\theta}$ proves to be a stable quantity and if the modified relative income influence factor has meaning, then $_i \beta$ should be examined. If $_i V_{t+\theta}$ is not predictable, or if the modified relative income influence factor has no meaning, then serious doubt would arise as to the appropriateness of this quantity for projecting regional productivity.

1(continued)
is dimensionally incorrect, however; for (1+p) must be dimensionless because the number one has no dimension. The z and ρ_θ' are subject to the same criticism. It would seem that these authors intended p, z, and ρ_θ' to be defined as the <u>fractional rate</u> of national population, productivity, and income change; and the p in the expression (1+p) to be tacitly multiplied by one year. Now (1+P) is dimensionless. A similar multiplication is necessary for z and ρ_θ' to be dimensionless.

The factor $(1+p)^\theta (1+z)^\theta$ was utilized by the CEA in their projection of the Gross National Product; that is,

$$GNP = L \cdot T$$

where L = number of workers in the nation;
T = average productivity per worker in the nation;

and
$$GNP_{t+\theta} = L(1+p)^\theta \, T(1+z)^\theta.$$

The use of p was under the assumption that the labor force participation would remain constant. Another assumption by the CEA was that the number of hours worked would remain constant. Therefore, from the above:

$$\frac{GNP_{t+\theta}}{GNP_t} = (1+p)^\theta (1+z)^\theta = \lambda_\theta.$$

Since ρ_θ can also be shown equal to $(1+\rho_\theta')$, the two models are equivalent:

1. $\rho_\theta' = \dfrac{\Delta Y}{Y \, \Delta T} \cdot \Delta T$ By DEFINITION

2. $1 + \rho_\theta' = 1 + \dfrac{\Delta Y}{Y} = \dfrac{Y + \Delta Y}{Y}$

3. But, $\dfrac{Y + \Delta Y}{Y} = \dfrac{Y_{t+\theta}}{Y_t} = \rho_\theta$ By DEFINITION

CHAPTER III

DATA AND COMPUTATIONS

APPROACH TO THE DETERMINATION OF THE STABILITY OF $_iV_t$

An investigation of the stability of $_iV_t$ requires the
determination of $_iV_t$ at several time periods. As mentioned earlier, an
empirical test of the use of this income influence measure within
population projections was conducted by Gerald A. P. Carrothers.[1]
He considered personal income as the measure embracing all determinants
of population change and rail distances as the measure of friction
against human interaction; that is, personal income/distance. He
gathered income and distance data for the United States for ten year
periods by which he computed the total income influence. This paper
shall utilize Carrothers' income influence data for the years 1920,
1930, 1940, and 1950; extend it through another ten year period; and
determine the stability of the measures over these time periods. The
use of this influence function within the projection model in the forms
of relative income influence and modified relative income influence
also shall be investigated for stability.

DEFINITION OF INPUTS

An investigation of the total income influence on a
particular region requires defining the regions, type of income, and
distance to be utilized. Although regions ought to be chosen to suit
the analysis proposed, income data availability limits the choice of
regions to states or functions of states. Income data collection also

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[1]
Gerald A. P. Carrothers, Forecasting the Population of Open Areas,
doctoral dissertation, Massachusetts Institute of Technology, Cambridge,
Massachusetts, 1959.

dictates that a particular kind of income be taken as the measure of state income — personal income of the residents. While the measurement of distance does not present a problem, the definition of it does. Carrothers chose rail distances. The computations shall be extended to 1959 using rail distance and airline distance.

The eastern half of the United States was chosen as the area for investigation (Map A). This area contained twenty-three of the thirty-one regions for which income influence data were available in Carrothers' study (compare Maps A and D).

The central points of each of these twenty-three regions coincided as nearly as possible with the centers of gravity of population. Since the gravity center of population would be expected to be the gravity center of personal income of the residents, these "centers" will be assumed applicable within this study (Table I and Map A).

A problem immediately arose from this decision to consider only two-thirds of the thirty-one regions, because the total income influence, relative income influence, and modified relative income influence measures had been computed upon the basis of the thirty-one regions. Could the same total income influence values of thirty-one regions be used as the total income influence values of twenty-three regions? A computation was made of the total income influence values of each of the twenty-three regions for four different time periods by subtracting the influence values of the eight regions not included in this study (Table XVI). In order to determine if some relation existed between the total income influence values of the thirty-one regions and of the twenty-three regions upon each of these

twenty-three regions, the two sets of values for the four time periods were mapped. Since Carrothers had chosen to approximate the average distance between all points in one region and all points in another by the measure of the shortest rail distance between the "centers", the subsequent values of the total income influence measure would be concentrated at these "centers." Lines connecting the "centers" of equal value would yield a graphic representation of the income influence upon each region by the region itself and the surrounding regions. From these "iso-influence" lines, an estimation could be made of the total income influence of either the thirty-one regions or the twenty-three regions on any point in this eastern section of the United States. A comparison of these two sets of maps (Maps E through H and J through N) disclosed that the values of total income influence for twenty-three regions produced a pattern which was quite similar to the pattern for the values of total income influence for thirty-one regions; but, more important, the pattern for the twenty-three regions varied over the time periods in the same manner as the pattern for the thirty-one regions. This similarity in patterns suggested a determination of the ration of the total income influence of the thirty-one regions and of the twenty-three regions. This computation (Table XXIII) showed that the ratios were quite similar both for all regions within each time period and for each region over the four time periods. The ratios for each region over the thirty year period differed by such a small increment, that each region's ratio of total income influence of thirty-one regions to total influence of twenty-three regions was assumed constant over this period (Constant column in Table XXIII). And, since these constant values ranged between 1.01 and 1.20, the

assumption was made to consider the arithmatic average of these values

(1.13) as the figure representing the error within each region's total

income influence values by the utilization of total income influence of

thirty-one regions as total income influence of twenty-three regions.

Since the exact values of the income influence are not as important

within this investigation as the comparison between the values and

because the ratio of the total income influence of the thirty-one

regions and the twenty-three regions was the same for each of the

twenty-three regions over the thirty year period, total income

influence of the thirty-one regions will be assumed equivalent to total

income influence of the twenty-three regions.

DETERMINATION OF $_iV_{1959}$

An appropriate extension of the income influence data would

have been from 1950 - 1960. However, the inability to obtain 1960

state personal income data necessitated the use of 1959 data. The

difference between this 1959 average and the 1960 Census figure was

not anticipated to be too great, since the latter would have been

collected only three months after the former had been determined. A

more significant problem involved the decision by the Census Bureau in

1955 to revise the type of income collected for states. Previous to

this time, state income data was tabulated according to income payments

to individuals. In 1956 a complete restudy of state income from 1929 -

1955 was published in the form of "personal income"[1] — a form which

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[1] Personal Income By States Since 1929, A Supplement to the Survey of Current Business, United States Department of Commerce, Office of Business Economics, 1956, p. 10.

contained nonmonetary income and other forms of personal income not
counted within income payments to individuals. Since the latter was
not collected after 1955, a choice had to be made between a
recomputation of Carrothers' data utilizing 1930, 1940, and 1950 values
of personal income or an acceptance of the discrepancy anticipated by
a comparison of dissimilar measures.

An investigation was conducted of the ratio of personal
income to income payments to individuals within five regions and the
United States for 1930, 1940, and 1950. These results (Table IX)
showed such a small increment of difference between the values for each
region and the United States over the time periods and within each ten
year period, and such a small variation from unity, that the decision
was made to consider personal income for 1959 as a measure of state
income equivalent to income payments to individuals. Moreover, the
Census Bureau stated that since the small percentage of difference
between these two values would itself vary from state to state, it could
be considered negligible.[1]

The total income influence function includes not only a
measure of the effect of the surrounding regions upon a particular
region, but also a measure of the influence of the income within the
region itself; that is, the "self-influence." Carrothers chose to
determine this by forming concentric rings fifty miles in width
emanating from the "center" of each region (Map A) and computing a
separate total income influence for each region; that is

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[1] Personal Income By States Since 1929, op. cit., p. 59.

$$i^{v}t = \sum_{s=1}^{m} \frac{s^{y}t}{is^{d}}$$

where $_{i}v$ = self-influence of region i;

s^{y} = income of each ring s in region i;

$_{is}d$ = distance from the "center" of region i
to the center of each ring s; and

m = number of fifty mile wide rings emanating
from the "center" of the region.

Carrothers assumed a uniform distribution of income within the whole region and computed the total income of each ring as the product of the population of the counties in the ring and the per capita personal income for the region. The distance from the center of each ring to the "center" of the region was taken to represent the average distance from the "center" to all points in the ring.

Define $_{ii}D$ such that:

$$_{ii}D = \frac{i^{Y}t}{i^{V}t}$$

where $_{i}Y$ = total income of region i and

$_{i}V$ = self-influence of region i.

A computation of this measure $_{ii}D$ for each region over the four time periods disclosed that the four values were nearly constant (Table XIV). Thus, $_{ii}D$ for each region could be considered as an equivalent stable value over the thirty year period. This then defines an equivalent distance for the purpose of computing the self-influence measure. The assumption was made to utilize each region's 1950 value of $_{ii}D$ as the equivalent distance factor for the 1959 determination of the region's

influence upon itself, since the 1950 measure would be expected to be

more closely analogous to 1959 than the 1920, 1930, or 1940. figures,

or an average of these values. The self-influence of each region for

1959 was then computed as the quotient of the region's total 1959

personal income and its computed value of $_{ii}D$ (Table XV).

The sum of each region's self-influence together with the

income influence of the other twenty-two regions upon the region yields

the total income influence upon each region. The value of each income

influence measure of the surrounding regions is determined as the

quotient of the region's income and the distance between the "center"

of the particular region and each surrounding region. The 1959 income

data for the regions composed of entire states were easily obtained,

for the data were available in this form. Regions 1 - 5, however,

were composed of parts of states and necessitated a separate

determination of the income for each region. Carrothers' method for

obtaining the income for these regions will be followed within this

investigation for comparative purposes. Region 1 is composed of

several counties of Connecticut, New York, Pennsylvania, and Virginia;

the District of Columbia; and the entire states of New Jersey, Delaware,

and Maryland (Table I). Carrothers assumed a uniform distribution of

income and determined the income of the counties as the product of the

counties' population (Table IV) and the per capita personal income of

the respective states (Table VI). Since the counties in Pennsylvania,

were more closely associated with the New Jersey area than with the

rest of Pennsylvania, New Jersey per capita personal income was used

for the five Pennsylvania counties. Similarly, District of Columbia's

per capita income was used as the measure of personal income within the Vir-

ginia areas. For the counties in New York, per capita personal income of the

state was used. The sum of the resulting incomes of the several parts

of states within this region together with the total personal income of

the District of Columbia, New Jersey, Delaware, and Maryland yielded

total personal income of Region 1 (Table VI). The total income of

Regions 2 - 5 were determined as residues of the total states' incomes.

Region 2's income was determined as the sum of the incomes of the

entire states of Rhode Island, Massachusetts, Vermont, New Hampshire,

and Maine together with the difference between the Connecticut total

personal income and that part contained in Region 1 (Table VI). The

incomes of Regions 3, 4, and 5 were composed of the residues of

New York's, Pennsylvania's, and Virginia's total personal income,

respectively (Table VI).

Distances used in this investigation are the same as those

used by Carrothers and are listed in Table XI. Utilizing these values,

the 1959 total income influence could now be determined for each of

the twenty-three regions as the sum of the self-influence measure of

the region itself and the individual values of the income influence

of the surrounding regions (Table XVIII). The individual influence

values between regions together with the self-influence measures are

available in Table XXVIII. The values of the 1959 total income

influence upon each region were mapped in the same manner as the total

income influence values for 1920, 1930, 1940, and 1950 were mapped

(Map I).

The ratio of the total income influence upon each region at

the end and the beginning of each ten year period for the years

1920 - 1930, 1930 - 1940, 1940 - 1950, and 1950 - 1959 was tabulated
in Table XXIV. The ratio of United States' personal income at the
end and the beginning of these same ten year periods was obtained from
Carrothers' material for 1920 - 1950 and computed for 1959 (Table X).
Unity subtracted from the values of the quotient of this ratio of
national income and the relative income influence measures for each
region (Table XXVI).yielded values for the factor $_iZ$ (Table XXVII) —
Isard's and Freutal's measure of the effect of the external changes of
productivity upon region i.

CHAPTER IV

FINDINGS AND CONCLUSIONS

INVESTIGATION OF $_iV_t$

As stated in Chapter I, the workability of Isard's and
Freutal's productivity projection model depends upon a determination
of the predictability of $_iV_{t+\theta}$, for a factor within a projective model
which itself needs projecting is feasible only if this factor is
easily predicted. Before this investigation is attempted, it seems
relevant to examine $_iV_t$ itself to determine what it means, what its
errors are, and how these errors are determined.

The accuracy of the total income influence upon a region i,

$$_iV_t = \sum_{j=1}^{n} \frac{_jY_t}{_{ij}D} ,$$ depends upon the errors involved in the estimate

of income data and distance data. The error in the compilation of
personal income is unknown. The Department of Commerce stated that
"the many source materials and procedures utilized in the State income
estimate are not of such a nature as to permit calculations of error"[1]
and that "the user must study the estimates and then decide for himself
whether they are sufficiently reliable for the purpose he has in mind."[2]
The errors involved in the distance estimate are threefold: the
measurement of distance, differences in definition of distance, and
approximations in computing distance. While the error in measurement
can be assumed to be small, the errors involved in definition and
computation will be investigated to determine their size and influence

[1] Personal Income By States Since 1929, op. cit., p. 67.
[2] Ibid.

upon the value of the total income influence measure.

The distance measure within the total income influence factor was defined as the shortest rail distance between the "centers" of the regions. Straight line distance between the "center" of the region itself and its emanating rings was used for the distance values of the self-influence. In order to determine the effect a different type of distance measure would have upon the total income influence values, the 1950 and 1959 total income influence measures were recomputed using domestic airline distances (Table XVIII). The air mileage between the "centers" of the regions was used as the measure of distance between a particular region and its surrounding areas (Table XII). Since air transportion is unlikely to be used within a region itself, the self-influence values of the previous computation utilizing straight line distances between the "center" of the region and its rings were assumed applicable within this determination of total income influence (Table XXIX). These total income influence values for 1950 and 1959 on the twenty-three regions were mapped (Maps O and P). A comparison of this mapping with the mapping of the values obtained for 1950 and 1959 with distance defined as rail measurements disclosed that the definition of distance had very little effect upon the pattern of total income influence (compare Maps H and I with Maps O and P).

The method of measuring distance both within the total income influence and the self-influence measures involved computational approximations. The assumption that the distance between the "centers" of the surrounding regions and a particular region would approximate the distance between all points in the regions increases in validity as the distance between the regions increases, for the two regional masses

could then be considered as point masses. But, for regions immediately surrounding the region under investigation, this assumption of regional masses concentrated at the "centers" may imply a substantial error in distance measurement. A similar statement could be made concerning the measurement of distance in the self-influence determination; that is, the assumption that the distance between the "center" of the region and the rings approximated the distance between the "center" and all points in the region may be much less valid for the rings immediately surrounding the "center" than for the rings at the outer edge of the region.

The fractional analysis of the errors within the total income influence measure is limited because all reasonable definitions of distance cannot be investigated and the error in income compilation is unknown. The error determination within the total income influence measure itself, $_iV_t$, would seem more feasible. Another study using this same income influence measure was available. William Warntz had estimated the total income influence for 1950. A comparison of his values with the total income influence values computed within this study was chosen as the method of determining the error in the influence measure.

William Warntz determined the 1950 total income influence measures of the United States using 115 control points. He utilized the Standard Metropolitan Area (SMA) classification to define these regions. Warntz used 1950 personal income of the residents as his income measure (Table VIII). This measure has been shown to be approximately equal to the income payment to individuals measure of income on the scale of the state.(Section 3 in Chapter III). Warntz

defined distance between the "centers" of his regions as the mileage along the great circle curves; i.e., air mileage. The previous section has shown that this measure of distance had little effect upon the values of total income influence determined by rail distance. Warntz simplified the method of self-influence determination: the quotient of the region's personal income and one-half the radius of the region's area. Warntz's definition of $_{ii}D$ as the average distance from the "center" to any point within the region — one-half the distance from the "center" to the edge of the region — can be assumed to be subject to the same magnitude of error as the equivalence measure of $_{ii}D$, since Warntz's regions are approximately the size of the "center" and several of its immediately surrounding rings. Thus, the data in the two measures are comparable.

Within the eastern section of the United States used in this study, Warntz determined the total income influence for seventy-seven regions as compared with twenty-three (Tables II and XIX and Map B). A grouping of these seventy-seven regions within the twenty-three regions was done (Table III and Map C). Since eighteen of these twenty-three regions were composed of entire states, the total income influence of each of the SMA's in the state weighted by their areas and the total income influence of the residual of the state weighted by its area were combined. This central total income influence value for each region compared with the single value of each of the eighteen regions (Table XX). Within the areas not composed of whole states, Regions 1 - 5, the following procedure was utilized.

Since the parts of the states in Region 1 were not the same as those investigated by Warntz, the SMA's most closely analogous to the areas were chosen (compare Table I with Table III). The sum of these influence values weighted by their areas and the weighted influence values of the SMA's and the residual areas of each of the entire states. in the region was assumed to approximate the total income influence of an area analogous to Region 1 (Table XX). Similar computations were made for the total income influence value of Region 2. The values for Regions 3 - 5 were determined as the weighted sum of the SMA's and the residual of the state not included in Region 1 (Table XX). Although the results were not completely comparable in Regions 1 - 5, the assumption was made to consider these two studies as an estimation of the same quantity within the same area. A comparison of a mapping of Warntz's combined data within this eastern United States area with a mapping of the data for the twenty-three regions disclosed that the patterns were not essentially different (compare Map H with Map N). These two data maps are essentially two versions of the same thing.

The difference in the corresponding values of the income influence of these two maps divided by the values derived from Carrothers' data is the measure of the error within the income influence values (Table XXX). The arithmatic average of these values (24.2%) was taken to be the percentage of error in the determination of $_iV_t$.

Another determination was made of the error of $_iV_t$, using the total income influence values determined by rail and air distances. A computation of the difference between the 1950 values determined by air distance and rail distance divided by the total income influence

values determined by air mileage produced an arithmatic average error of 16% (Table XXX). The difference between the two values of error of $_iV_t$ could be speculated as the result of the measures of income, since the 25% represented two different measures of income and the 16% represented the same. Further comparison of the 1959 total income influence values utilizing air distances with the 1959 values of total income influence using rail distance disclosed that the error involved was only 10% (Table XXX). The reason for this diminishing error could be speculated as the result of the use of the 1959 values of total income influence which were computed on the basis of twenty-three regions whereas the 1950 values represented the influence of thirty-one regions. Thus, the error becomes so small that the variation of distance can be assumed to yield an error of a negligible magnitude.

STABILITY OF $_iV_t$

The contours of total income influence for each region of the years 1920, 1930, 1940, 1950, and 1959 are shown in Maps E - I. The resemblance between them is striking. Values of the contours change but the pattern remains essentially the same.

A closer look at the variation of total income influence in time is obtained by computing the increments and percentage of change of these values at different times. These are shown in Tables XXI and XXII. The increments themselves show very little; however, the percentage of change of the values of the influence from one tim e to the next is approximately the same for all the regions for the same time interval.

The relative income influence $\dfrac{_iV_{t+\theta}}{_iV_t}$ must display

the same stability as the percent change of the total income

influence[1] (Table XXIV). The values displayed in Figure 1 show that

the relative income influence has much the same value for all regions

over the same interval. Therefore, if the value of the total income

influence function for any one region were predictable, the values

for all the regions would be predictable; because it would be

justified to say that the ratio of $\dfrac{_iV_{t+\theta}}{_iV_t}$ for this region is

close to the ratio for all the other regions. Figure 2 shows that at

least one value of $_iV_{t+\theta}$ must be known, for the factor follows no

pattern susceptible to prediction.

The relative income influence values were also computed

for the 1950 and 1959 total income influence measures determined

by air distance. The values of this ratio (Table XXV) were close

to constant and essentially equal to the same constant that was

obtained for rail distances (compare Table XXIV and XXV).

THE MODIFIED RELATIVE INCOME INFLUENCE FUNCTION

The computed values of the modified relative income influence

function for rail distances are listed in Table XXVII. (The values

for air distances cannot be significantly different because the

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[1] $\dfrac{_iV_{t+\theta} - _iV_t}{_iV_t} = \dfrac{_iV_{t+\theta}}{_iV_t} - 1$.

Figure 1. The Relative Income Influence[a] of the Twenty-three
Regions[b] for the Census Years 1920 - 1950, and 1959.
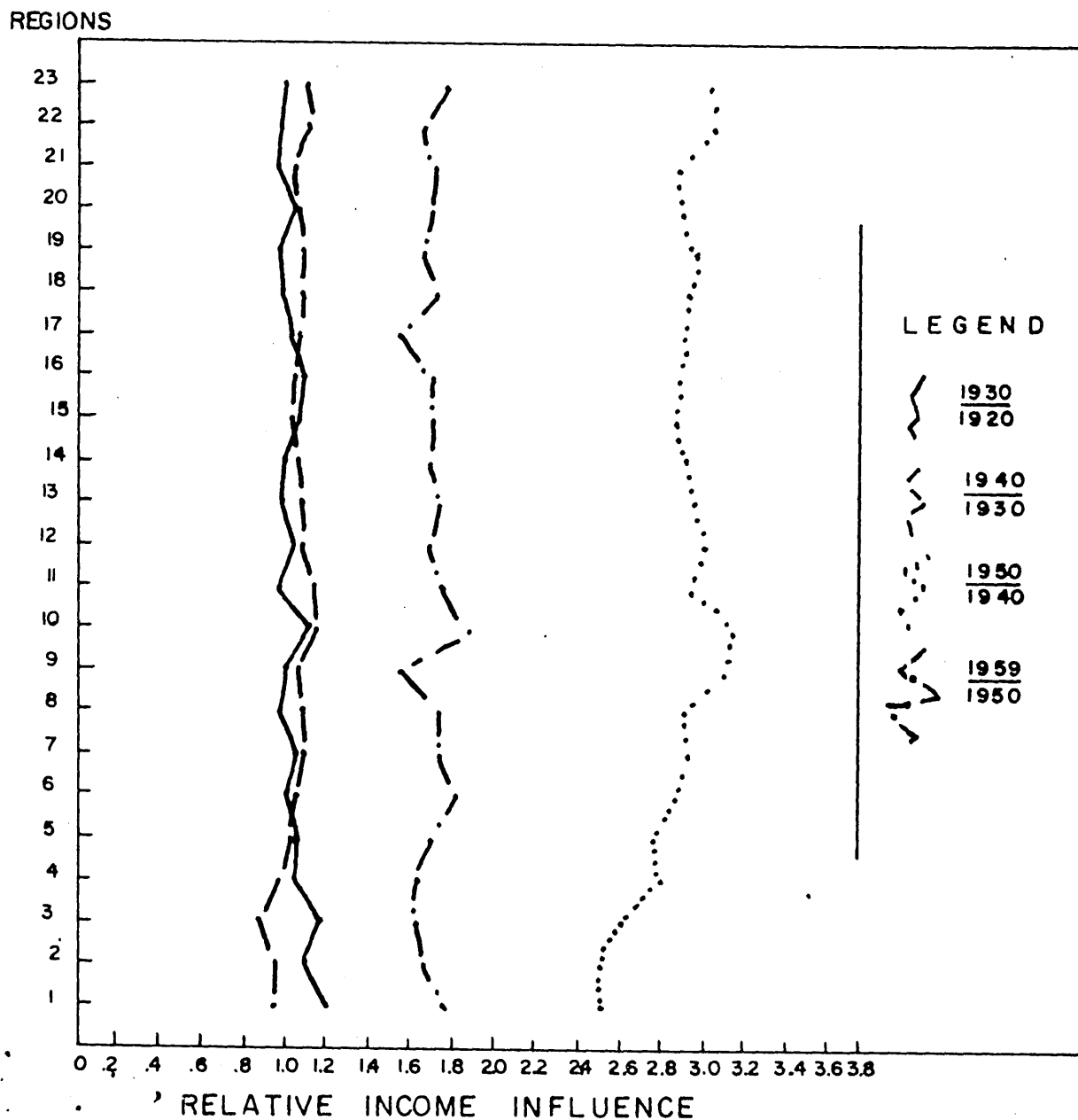
REGIONS

LEGEND

1930 / 1920

1940 / 1930

1950 / 1940

1959 / 1950

0 .2 .4 .6 .8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8 3.0 3.2 3.4 3.6 3.8

RELATIVE INCOME INFLUENCE

[a]
    Relative income influence data in Table XXIV.
[b]
    See Table I and Map A for definition of the regions.

Figure 2. The Relative Income Influence[a] Average Over Twenty-three
Regions[b] for Four Ten Year Periods as a Function of
Time Intervals.

SPACE
AVERAGE
OF
RELATIVE
INCOME
INFLUENCE

3.0

2.5

2.0

1.5

1.0

.5

| 1930 | 1940 | 1950 | 1959 |
| 1920 | 1930 | 1940 | 1950 |

TIME INTERVALS

_____
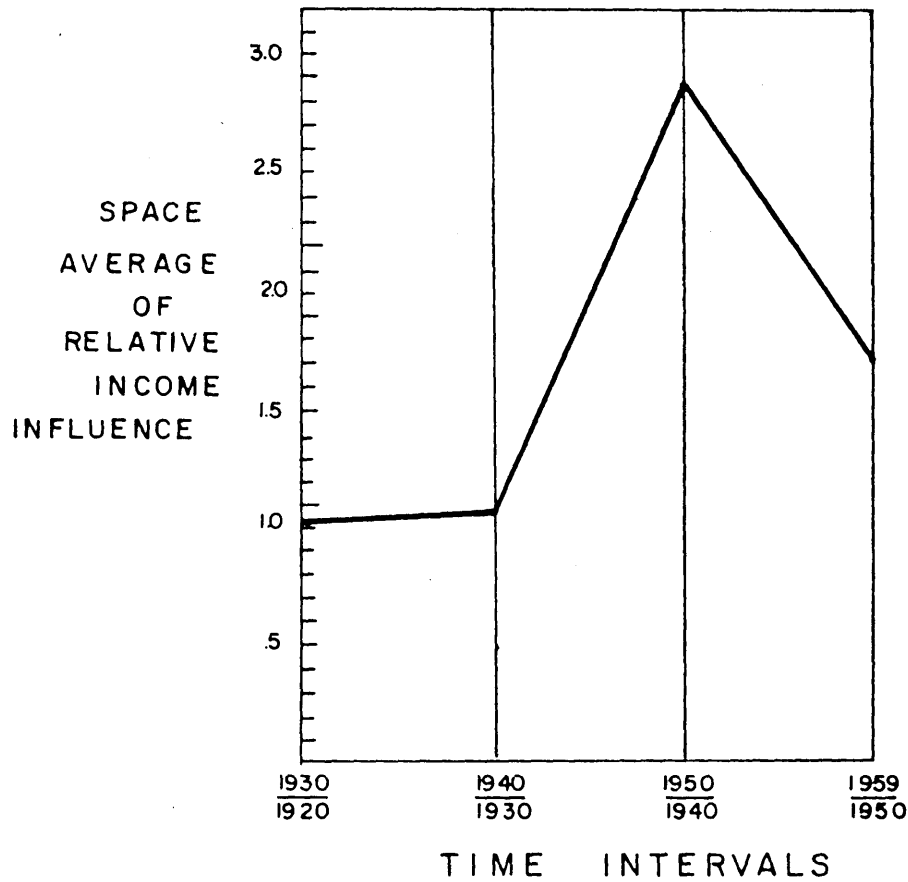
[a]
Average relative income influence data in Table XXIV.
[b]
See Table I and Map A for definition of the regions.

relative income influence functions for rail and air distances are essentially the same.) The values seem to cluster rather close to zero. It is extremely important to determine whether these values are significantly different from zero, for it is only in this case that the productivity projection model can have any predictive value beyond an extrapolation based on the changes in national productivity.

The modified relative income influence function

$$\frac{i^V{}_{t+\theta}}{\rho_\theta \; i^V{}_t} = 1$$

depends upon the income influence values. Any error in the total income influence therefore will appear as an error in this function. The variable part of it is

$$\frac{i^V{}_{t+\theta}}{\rho_\theta \; i^V{}_t} = G.$$

Therefore the error in G shall be discussed. Application of the propogation of error analysis (Table XXXI) showed that:

$$\% \text{ error in } G = \% \text{ error in } {}_iV_t \left(\sqrt{\frac{1}{\rho_\theta^2} + 1} \right).$$

The percent error in G is therefore at least as great as the percent error in ${}_iV_t$ (since ρ_θ is at least greater than one) which was found to be about 25%. Therefore, since $G \approx 1$ (Table XXVI), G can vary between .75 and 1.25. The corresponding variability of the modified relative income influence function runs between -.25 and +.25, from which it can be concluded that the values of it listed in Table XXVII are not significantly different from

zero.[1] Thus, the Isard and Freutal model, at least with respect to these data, has no projecting value.

CONCLUSION

The conclusion of the preceeding section suggests questioning of whether the failure of the modified relative income influence factor is a radical or a relative failure. It is possible that if the grain of income data were finer, income data more accurate, and a proper measure of distance chosen, the model would be useful for projections. A means of testing the hypothesis that the scale of data is too gross is available in Warntz's investigation.

A study of Warntz's map of seventy-seven control points should be conducted to determine if this more intense collection of data yields income influence values which have meaning in the modified relative income measure. This could be accomplished through a computation of Warnta's income influence measures for 1960 and an analysis of the modified relative income data for the base year 1950 and the projected year 1960. If this measure has meaning within $_jX$, then future users of this productivity model should be aware of the limitations of this model and utilize small regions within their investigations.

The lack of income data availability for any regions but states or parts of states (and just personal income of the residents for these regions) limits any practical investigation of the influencing

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[1] The inconclusiveness of the results of Carrothers' study of a population projection model which used this same value of the modified relative income influence factor appears to be a direct result of his acceptance of the validity of this influence factor without testing.

effect of other types of income as an aid to productivity projections.
A state or other regional collection of productivity data would provide
the measure suggested by Isard and Freutal within their theoretical
proposal. Other types of income (e.g., corporate) while difficult to
compute on a regional scale might make some account of the external
changes of productivity until such time as regional productivity data
were available.

Another possibility is that the income influence function is
simply an inadequate measure of the effect of distance. It is interest-
ing to notice that if the income data alone were plotted, the resulting
pattern resembles fairly closely the pattern of the income influence
function. A mapping of the income values for 1950 and 1959 (Maps Q and R)
disclosed that while the pattern is not exactly the same within the areas
of lower income as the pattern of total income influence, the effect of
distance does little to disrupt the pattern of the higher income value
gradients in the northern half of the study area. These gradients of
income not only behaved similarly to total income influence gradients in
space, but also over time — as a comparison of the 1950 income map with
the 1950 total income influence map and the 1959 income map with the 1959
total income influence map verified. Thus, the pattern of the total in-
come influence measure seems to be determined by the pattern of the type
of income utilized and not by the definition of distance.

If neither of these corrections improve the ability of the
modified relative income influence measure to account for
interregional relations, then the ability of this concept to measure
these relations could be questioned. The measure might be too gross
to detect these changes and a more penetrating measure might be needed

within the interregional factor $_iX$.

This paper has not attempted to investigate the stability of the coefficient $_i\beta$ which is specified as a positive constant — but which Carrothers found had many negative values in his investigation —, since the factor $_iX$ reduced to zero as a result of the values of the modified relative income influence and its components required no further investigation. If, however, the results of an extension of Warntz's study proved more acceptable, then the appropriate form of this constant, together with its stability over time, would need to be established for its feasible use in Isard's and Freutal's productivity projections.

APPENDIX I

DEFINITION OF REGIONS

TABLE I. Twenty-three Regions in the Eastern Section of the United States.[a]

Region Number	"Center" of Region[b]	States Included in Region[b]
1	New York City	Connecticut – Fairfield County New Haven County Delaware District of Columbia Maryland New Jersey New York – Bronx County Kings County Nassau County New York County Orange County Putnam County Queens County Richmond County Rockland County Suffolk County Westchester County Pennsylvania – Bucks County Chester County Delaware County Montgomery County Philadelphia County Virginia – Alexandria City Arlington County Fairfax County Falls Church Town
2	Boston	Connecticut – remainder[c] Maine Massachusetts New Hampshire Rhode Island Vermont
3	Syracuse	New York – remainder[c]
4	Pittsburgh	Pennsylvania – remainder[c]
5	Richmond	Virginia – remainder[c]

(a)
   See Map A of regions and "centers".
(b)
   Regions and "centers" defined in Carrothers' Thesis, pp. 146 and 147.
(c)
   Includes the portion of the state not in Region 1.

TABLE I. (continued)

Region Number	"Center" of Region[a]	States Included in Region[a]
6	Charleston	West Virginia
7	Salisbury	North Carolina
8	Columbia	South Carolina
9	Atlanta	Georgia
10	Tampa	Florida
11	Birmingham	Alabama
12	Jackson	Mississippi
13	Nashville	Tennessee
14	Louisville	Kentucky
15	Columbus	Ohio
16	Detroit	Michigan
17	Indianapolis	Indiana
18	Chicago	Illinois
19	Milwaukee	Wisconsin
20	Des Moines	Iowa
21	Jefferson City	Missouri
22	Little Rock	Arkansas
23	New Orleans	Louisiana

[a] Regions and "centers" defined in Carrothers' Thesis, pp. 146 and 147.

TABLE II. Seventy-seven Regions in the Eastern Section of the United States.[a]

Region Number	Area Included in Region [b]	Region Number	Area Included in Region [b]
1	Birmingham SMA	39	Kansas City SMA
2	Rest of Alabama	40	St. Louis SMA
3	Arkansas	41	Rest of Missouri
4	Hartford SMA	42	New Hampshire
5	Rest of Connecticut	43	Rest of New Jersey
6	Delaware	44	Albany SMA
7	Washington D.C. SMA	45	Buffalo SMA
8	Jacksonville SMA	46	New York SMA
9	Miami SMA	47	Rochester SMA
10	Tampa SMA	48	Syracuse SMA
11	Rest of Florida	49	Rest of New York
12	Atlanta SMA	50	Charlotte SMA
13	Rest of Georgia	51	Rest of North Carolina
14	Chicago SMA	52	Cincinnati SMA
15	Peoria SMA	53	Cleveland SMA
16	Davenport, Moline and Rock Island SMA	54	Columbus, Ohio SMA
		55	Dayton SMA
17	Rest of Illinois	56	Toledo SMA
18	Evansville SMA	57	Rest of Ohio
19	Ft. Wayne SMA	58	Erie SMA
20	Indianapolis SMA	59	Pittsburgh SMA
21	South Bend SMA	60	Scranton SMA
22	Rest of Indiana	61	Philadelphia SMA
23	Des Moines SMA	62	Rest of Pennsylvania
24	Rest of Iowa	63	Providence SMA
25	Louisville SMA	64	Rest of Rhode Island
26	Rest of Kentucky	65	South Carolina
27	New Orleans SMA	66	Chatanooga SMA
28	Rest of Louisiana	67	Knoxville SMA
29	Baltimore SMA	68	Memphis SMA
30	Rest of Maryland	69	Nashville SMA
31	Maine	70	Rest of Tennessee
32	Boston SMA	71	Vermont
33	Springfield SMA	72	Richmond SMA
34	Rest of Massachusetts	73	Norfolk SMA
35	Grand Rapids SMA	74	Rest of Virginia
36	Detroit SMA	75	West Virginia
37	Rest of Michigan	76	Milwaukee SMA
38	Mississippi	77	Rest of Wisconsin

(a)
    See Map B of regions.
(b)
    Data obtained from personal communication with William Warntz.

TABLE III. Twenty-three Regions Consisting of Combinations of the Seventy-seven Regions in the Eastern Section of the United States.[a]

Region Number	Area Included in Region	Region Number	Area Included in Region
1	Hartford SMA Delaware District of Columbia SMA Baltimore SMA Rest of Maryland Rest of New Jersey New York City SMA Philadelphia SMA Richmond SMA	10	Jacksonville SMA Miami SMA Tampa SMA Rest of Florida
		11	Birmingham SMA Rest of Alabama
2	Rest of Connecticut Maine Boston SMA Springfield SMA Rest of Massachusetts New Hampshire Providence SMA Rest of Rhode Island Vermont	12	Mississippi
		13	Chatanooga SMA Knoxville SMA Memphis SMA Nashville SMA Rest of Tennessee
		14	Louisville SMA Rest of Kentucky
3	Albany SMA Buffalo SMA Rochester SMA Syracuse SMA Rest of New York	15	Cincinnati SMA Cleveland SMA Columbus SMA Dayton SMA Toledo SMA Rest of Ohio
4	Erie SMA Pittsburgh SMA Scranton SMA Rest of Pennsylvania	16	Grand Rapids SMA Detroit SMA Rest of Michigan
		17	Evansville SMA Fort Wayne SMA Indianapolis SMA South Bend SMA Rest of Indiana
5	Norfolk SMA Rest of Virginia		
6	West Virginia		
7	Charlotte SMA Rest of North Carolina	18	Chicago SMA Peoria SMA Davenport, Moline, and Rockport SMA Rest of Illinois
8	South Carolina		
9	Atlanta SMA Rest of Georgia		

[a]
    See Map C of regions.

TABLE III. (continued)

Region Number	Area Included in Region
19	Milwaukee SMA Rest of Wisconsin
20	Des Moines SMA Rest of Iowa
21	Kansas City SMA St. Louis SMA Rest of Missouri
22	Arkansas
23	New Orleans SMA Rest of Louisiana

APPENDIX II

POPULATION

TABLE IV. Population[a] of the Counties in Region 1[b] for 1960

Counties	Population
Connecticut - Fairfield County	653.589
New Haven	660,315
New York - Bronx County	1,424,815
Kings County	2,627,319
Nassau County	1,300,171
New York County	1,698,281
Orange County	183,734
Putnam County	31,722
Queens County	1,809,578
Richmond County	221,991
Rockland County	136,803
Suffolk County	666,784
Westchester County	808,891
Pennsylvania - Bucks County	308,567
Chester County	210,608
Delaware	553,154
Montgomery County	516,682
Philadelphia County	2,002,512
Virginia - Alexandria City	91,023
Arlington County	163,401
Fairfax County	275,002
Falls Church Town	10,192

[a] Population-Preliminary Reports, Bureau of Census, U.S. Dept. of Commerce, 1960.
[b] See Table I and Map A for definition of Region 1.

APPENDIX III

INCOME

TABLE V. Income Payments to Individuals,[a] by States and Regions,[b]
for Census Years 1920 to 1950. (In $ millions)

Region Number	States Included in Region	1920	1930	1940	1950
1	Connecticut - part	581	705	747	1,879
	Delaware	158	182	239	628
	District of Columbia	495	644	905	2,093
	Maryland	1,053	1,036	1,222	3,420
	New Jersey	2,564	3,081	3,138	7,777
	New York - part	6,543	8,306	7,649	18,212
	Pennsylvania - part	1,944	2,364	2,228	5,368
	Virginia - part	63	90	143	602
	Total	13,401	16,408	16,271	39,979
2	Connecticut - remainder[c]	509	632	670	1,719
	Maine	469	432	431	1,067
	Massachusetts	3,494	3,512	3,309	7,535
	New Hampshire	291	279	269	682
	Rhode Island	513	527	511	1,217
	Vermont	204	195	187	438
	Total	5,480	5,577	5,377	12,658
3	New York - remainder[c]	4,114	5,040	4,181	10,169
4	Pennsylvania - remainder[c]	4,546	4,274	3,997	10,816
5	Virginia - remainder[c]	907	770	984	2,949
6	West Virginia	750	682	760	2,115
7	North Carolina	907	812	1,131	3,859
8	South Carolina	565	365	545	1,763
9	Georgia	1,007	798	986	3,336
10	Florida	423	635	900	3,387
11	Alabama	736	617	763	2,581
12	Mississippi	504	385	444	1,527
13	Tennessee	844	743	927	3,203
14	Kentucky	968	794	880	2,688
15	Ohio	4,070	4,251	4,448	12,620
16	Michigan	2,637	2,940	3,425	10,242
17	Indiana	1,706	1,595	1,858	5,780
18	Illinois	5,377	5,903	5,740	15,400
19	Wisconsin	1,599	1,587	1,622	4,962
20	Iowa	1,356	1,248	1,233	3,725
21	Missouri	1,987	1,984	1,914	5,570
22	Arkansas	576	393	493	1,578
23	Louisiana	767	725	847	2,848
	Total United States	69,772	73,325	75,852	217,828

(a)
    Income data derived from Carrothers' Thesis, pp. 592 and 593.
(b)
    See Table I and Map A for definition of Regions.
(c)
    Includes the portion of the state not in Region 1.

TABLE VI. Personal Income,[a] by States and Regions,[b] for 1959.
(In $ millions)

Region Number	State	Personal Income	Income per capita
1	Connecticut - part	3,701	2,817
	Delaware	1,314	2,946
	District of Columbia	2,210	2,943
	Maryland	7,108	2,343
	New Jersey	15,429	2,608
	New York - part	29,850	2,736
	Pennsylvania - part	9,365	2,222
	Virginia - part	1,588	1,816
	Total	70,565	
2	Connecticut - remainder[c]	3,203	2,817
	Maine	1,713	1,768
	Massachusetts	12,380	2,444
	New Hampshire	1,200	2,010
	Rhode Island	1,837	2,156
	Vermont	694	1,789
	Total	21,027	
3	New York - remainder[c]	15,253	2,736
4	Pennsylvania - remainder[c]	16,752	2,222
5	Virginia - remainder[c]	6,078	1,816
6	West Virginia	3,053	1,635
7	North Carolina	6,771	1,485
8	South Carolina	3,148	1,332
9	Georgia	6,081	1,553
10	Florida	9,273	1,980
11	Alabama	4,607	1,409
12	Mississippi	2,528	1,162
13	Tennessee	5,362	1,521
14	Kentucky	4,548	1,514
15	Ohio	21,979	2,328
16	Michigan	17,493	2,253
17	Indiana	9,714	2,102
18	Illinois	25,734	2,610
19	Wisconsin	8,258	2,116
20	Iowa	5,398	1,953
21	Missouri	9,248	2,145
22	Arkansas	2,370	1,322
23	Louisiana	5,169	1,575
	Total United States	383,287	

(a)
Survey of Current Business, U.S. Dept. of Commerce, August 1960, p. 17.
(b)
See Table I and Map A for definitions of Regions.
(c)
Includes the portion of the state not in Region 1.

TABLE VII. Personal Income[a] by States Since 1929. (In $ millions)

States	1930	1940	1950
Maine	460	444	1,087
New Hampshire	302	285	699
Vermont	205	184	448
Massachusetts	3,588	3,385	7,799
Rhode Island	540	534	1,287
Connecticut	1,493	1,566	3,860
New York	13,186	11,713	28,054
New Jersey	3,495	3,433	8,699
Pennsylvania	6,904	6,417	16,477
Delaware	203	270	689
Maryland	1,176	1,309	3,755
District of Columbia	616	807	1,774
Michigan	3,186	3,610	10,803
Ohio	4,472	4,606	12,891
Indiana	1,681	1,898	6,006
Illinois	6,235	5,964	15,984
Wisconsin	1,754	1,740	5,060
Minneapolis	1,423	1,467	4,184
Iowa	1,255	1,272	3,799
Missouri	2,073	1,982	5,705
North Dakota	208	224	781
South Dakota	248	230	793
Nebraska	713	578	1,949
Kansas	882	762	2,643
Virginia	933	1,267	4,024
West Virginia	712	777	2,203
Kentucky	853	914	2,834
Tennessee	850	995	3,288
North Carolina	929	1,171	4,108
South Carolina	421	584	1,869
Georgia	897	1,060	3,510
Florida	683	982	3,632
Alabama	705	801	2,659
Mississippi	407	474	1,590
Louisiana	753	861	2,937
Arkansas	415	501	1,539
Oklahoma	884	867	2,514
Texas	2,399	2,776	10,375
New Mexico	142	199	798
Arizona	223	248	979
Montana	271	318	957
Idaho	222	242	757
Wyoming	132	152	474
Colorado	603	617	1,930
Utah	257	269	892
Washington	1,043	1,152	3,986
Oregon	593	677	2,451
Nevada	76	99	314
California	5,079	5,839	19,627
Total United States	76,780	78,522	225,473

(a)
    Personal Income By States Since 1929, U.S. Dept. of Commerce, pp. 140 and 141.

TABLE VIII. Personal Income[a] by Regions[b] for 1950. (In $ millions)

Region Number	Area Included in Region	1950
1	Birmingham SMA	680.4
2	Rest of Alabama	1979.6
3	Arkansas	1532.0
4	Hartford SMA	1114.4
5	Rest of Connecticut	2733.6
6	Delaware	688.0
7	Washington D.C. SMA	2833.7
8	Jacksonville SMA	444.2
9	Miami SMA	796.3
10	Tampa SMA	613.1
11	Rest of Florida	1787.8
12	Atlanta SMA	1019.0
13	Rest of Georgia	2461.6
14	Chicago SMA	11142.4
15	Peoria SMA	477.9
16	Davenport, Moline and Rock Island SMA	456.2
17	Rest of Illinois	4107.4
18	Evansville SMA	296.1
19	Ft. Wayne SMA	388.1
20	Indianapolis SMA	1186.4
21	South Bend SMA	357.4
22	Rest of Indiana	3080.4
23	Des Moines SMA	419.0
24	Rest of Iowa	3067.5
25	Louisville SMA	928.6
26	Rest of Kentucky	1738.3
27	New Orleans SMA	1028.0
28	Rest of Louisiana	1909.1
29	Baltimore SMA	2406.1
30	Rest of Maryland	794.8
31	Maine	1088.0
32	Boston SMA	4981.9
33	Springfield SMA	767.5
34	Rest of Massachusetts	2049.8
35	Grand Rapids SMA	454.1
36	Detroit	5557.4
37	Rest of Michigan	4805.5
38	Mississippi	1588.0
39	Kansas City SMA	1464.5
40	St. Louis SMA	3005.8

[a] Data obtained from personal communication with William Warntz.

[b] See Table II and Map B for definition of Regions.

TABLE VIII. (continued)

Region Number	Area Included in Region	1950
41	Rest of Missouri	2161.2
42	New Hampshire	704.0
43	Rest of New Jersey	1599.9
44	Albany SMA	834.5
45	Buffalo SMA	1699.7
46	New York SMA	26741.1
47	Rochester SMA	859.7
48	Syracuse SMA	476.0
49	Rest of New York	3833.5
50	Charlotte SMA	322.9
51	Rest of North Carolina	3791.1
52	Cincinnati SMA	1684.3
53	Cleveland SMA	2873.0
54	Columbus SMA	903.9
55	Dayton SMA	745.3
56	Toledo SMA	729.9
57	Rest of Ohio	6227.0
58	Erie SMA	350.5
59	Pittsburgh SMA	3745.6
60	Scranton SMA	404.8
61	Philadelphia SMA	6538.6
62	Rest of Pennsylvania	6112.1
63	Providence SMA	1120.1
64	Rest of Rhode Island	166.9
65	South Carolina	1859.0
66	Chatanooga SMA	319.8
67	Knoxville SMA	431.1
68	Memphis SMA	727.3
69	Nashville SMA	461.7
70	Rest of Tennessee	1380.2
71	Vermont	445.0
72	Richmond SMA	567.4
73	Norfolk SMA	553.3
74	Rest of Virginia	2398.3
75	West Virginia	2207.0
76	Milwaukee SMA	1607.5
77	Rest of Wisconsin	3362.5

TABLE IX. Ratio of Personal Income[a] to Income Payments to Individuals.[b]

Ratio of Income, $\dfrac{_iY_t}{_iY'_t}$, where Y = personal income

Y' = income payments to individuals

Region Number	Area Included in Region	1930	1940	1950	Arithmatic Average
11	Alabama	1.14	1.05	1.03	1.07
15	Ohio	1.05	1.04	1.02	1.04
18	Illinois	1.06	1.04	1.14	1.08
21	Missouri	1.05	1.04	1.02	1.04
22	Arkansas	1.06	1.02	.98	1.02
	United States	1.05	1.04	1.04	1.04

[a] Personal Income data in Table VII.

[b] Income Payments to Individuals data in Table V.

TABLE X. Ratio of the Income of the Residents in the United States
for 1920 - 1930, 1930 - 1940, 1940 - 1950, and 1950 - 1959.

Ratio of National Income, $\rho_\theta = \dfrac{US^Y_{t+\theta}}{US^Y_t}$

Period θ	Ratio of Increase ρ_θ
1920[a] - 1930[a]	1.05
1930 - 1940[a]	1.03
1940 - 1950[a]	2.87
1950 - 1959[b]	1.76

[a]
Income payments to individuals data for 1920, 1930, 1940, and 1950 in Table V.
[b]
Personal income data for 1959 in Table VI.

APPENDIX IV

DISTANCE

TABLE XI. Shortest Rail Distances[a] Between "Centers" of Regions.[b]
(Figures rounded to nearest ten miles).

Region Number / Region Number

Region Number		23	22	21	20	19	18	17	16	15	14	13		Region Number
1	New York	-	-	530	810	1040	1010	920	860	1090	950	750	560	23
2	Boston	230	-	-	460	690	720	630	590	840	740	510	370	22
3	Syracuse	290	350	-	-	400	480	400	350	600	570	390	440	21
4	Pittsburgh	440	670	430	-	-	440	360	540	630	680	610	660	20
5	Richmond	340	570	630	410	-	-	90	270	340	410	380	530	19
6	Charleston	610	840	770	330	370	-	-	180	270	320	300	450	18
7	Salisbury	560	790	850	640	300	500	-	-	300	140	110	300	17
8	Columbia	700	930	990	770	360	730	150	-	-	190	370	560	16
9	Atlanta	860	1090	1080	810	580	690	300	250	-	-	230	420	15
10	Tampa	1190	1420	1480	1260	850	1220	640	490	560	-	-	190	14
11	Birmingham	990	1220	1070	800	740	690	470	420	170	650	-	-	13
12	Jackson	1230	1460	1460	1020	980	910	720	670	420	900	250	-	12
13	Nashville	950	1180	890	620	700	500	510	510	290	850	210	450	13
14	Louisville	870	1050	710	430	700	320	570	580	470	1040	390	590	14
15	Columbus	630	820	470	190	570	200	640	700	610	1170	600	820	15
16	Detroit	650	750	400	300	710	400	830	840	750	1310	740	960	16
17	Indianapolis	820	960	620	370	690	310	680	680	590	1150	500	700	17
18	Chicago	910	1020	670	470	860	490	860	870	730	1290	650	740	18
19	Milwaukee	990	1100	760	550	950	570	950	950	820	1380	740	820	19
20	Des Moines	1270	1380	1030	830	1220	840	1170	1170	950	1510	820	860	20
21	JeffersonCity	1160	1320	970	720	1030	660	940	950	730	1240	590	630	21
22	Little Rock	1290	1510	1220	940	1040	830	840	810	550	1030	390	340	22
23	New Orleans	1360	1570	1430	1150	1070	1040	790	750	490	820	360	180	23
		1	2	3	4	5	6	7	8	9	10	11	12	

Region Number

(a)
    Data obtained from Carrothers' Thesis, p. 603.
(b)
    See Table I and Map A for definition of Regions and "Centers".

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TABLE XII. Airline Route[a] Distances[b] Between "Centers" of Regions.[c]

Region Number		Region Number											Region Number
		23	22	21	20	19	18	17	16	15	14	13	
1	New York	-	-	350	592	1158	940	858	734	959	846	651	499	23
2	Boston	184	-	-	240	460	618	552	483	723	624	435	325	22
3	Syracuse	194	264	-	-	220	360	330	330	549	519	320	320	21
4	Pittsburgh	334	495	169	-	-	300	300	462	519	584	571	697	20
5	Richmond	289	474	386	243	-	-	82	244	228	336	353	479	19
6	Charleston	440	622	433	168	236	-	-	162	219	284	271	397	18
7	Salisbury	270	771	645	412	298	190	-	-	225	181	111	251	17
8	Columbia	302	791	678	448	317	220	135	-	-	160	313	465	16
9	Atlanta	762	945	781	527	470	302	277	194	-	-	199	347	15
10	Tampa	1014	1198	1099	877	722	353	560	425	410	-	-	152	14
11	Birmingham	869	1053	875	600	596	442	411	334	134	461	-	-	13
12	Jackson	1069	1250	1075	800	796	642	561	534	334	661	200	-	12
13	Nashville	765	949	739	464	526	322	163	274	213	623	177	380	13
14	Louisville	669	835	603	336	458	225	393	395	321	731	329	532	14
15	Columbus	479	642	874	145	317	133	397	425	489	899	524	727	15
16	Detroit	511	641	354	212	445	284	554	585	603	1013	638	787	16
17	Indianapolis	659	821	567	325	496	262	478	489	432	842	428	562	17
18	Chicago	724	868	592	403	620	400	637	652	592	1001	574	688	18
19	Milwaukee	738	869	594	433	670	461	770	732	674	1083	656	770	19
20	Des Moines	1024	1168	1092	703	920	700	987	952	892	1301	874	620	20
21	Jefferson City	1054	1198	1122	733	778	545	483	594	533	943	497	420	21
22	Little Rock	1081	1259	1038	779	851	642	688	688	457	798	337	200	22
23	New Orleans	1185	1369	1187	922	895	754	699	599	427	489	322	150	23
		1	2	3	4	5	6	7	8	9	10	11	12	

Region Number

(a)
See p. A-16 for method of determining air mileage for regions not on direct airline routes.
(b)
    Data obtained from United States Domestic Airline Mileage Tables in the Official Airline Guide and Manuals at offices of Eastern, United and American Airlines at Logan Airport in Boston.
(c)
    See Table I and Map A for definition of regions and "centers".

TABLE XII. (continued) Method of Determining Air Mileage for Regions
not on Direct Airline Routes.

==

Regions 1, 2, 3, 4, and 6.

        to Region 7: `Charlotte + 50 miles
        to Region 12: Birmingham + 200 miles
        to Region 20: Chicago + 300 miles
        to Region 21: Chicago + 330 miles

Region 5.

        to Region 7: Charlotte + 50 miles
        to Region 12: Birmingham + 200 miles
        to Region 20: Chicago + 300 miles
        to Region 21: Louisville + 320 miles

Region 7.

        to Region 12: Birmingham + 200 miles

Region 8.

        to Region 7: Charlotte + 50 miles
        to Region 12: Birmingham + 200 miles
        to Region 19: Chicago + 80 miles
        to Region 20: Chicago + 300 miles
        to Region 21: Nashville + 320 miles
        to Region 22: Charlotte + 50 miles to Memphis + 125 miles

Regions 9, 10, and 11.

        to Region 7: Charlotte + 50 miles
        to Region 12: Birmingham + 200 miles
        to Region 20: Chicago + 300 miles
        to Region 21: Nashville + 320 miles
        to Region 22: Memphis + 125 miles

Regions 13 and 14.

        to Region 7: Charlotte + 50 miles
        to Region 12: Memphis + 180 miles
        to Region 20: Chicago + 300 miles
        to Region 21: 320 miles

Region 15.

        to Region 7: Charlotte + 50 miles
        to Region 12: Memphis + 180 miles
        to Region 20: Chicago + 300 miles
        to Region 21: Louisville + 320 miles

TABLE XII. (continued)

Region 16.

        to Region 7: Charlotte + 50 miles
        to Region 12: Memphis + 180 miles
        to Region 20: Chicago + 300 miles
        to Region 21: Chicago + 330 miles

Region 17.

        to Region 7: Charlotte + 50 miles
        to Region 12: Memphis + 180 miles
        to Region 20: Chicago + 300 miles
        to Region 21: 330 miles

Region 18.

        to Region 7: Charlotte + 50 miles
        to Region 12: Memphis + 180 miles
        to Region 20: 300 miles
        to Region 21: 330 miles

Region 19.

        to Region 7: Charlotte + 50 miles
        to Region 8: Chicago + 80 miles
        to Region 12: Memphis + 180 miles
        to Region 20: 300 miles
        to Region 21: 360 miles

Region 20.

        to Region 7: Chicago + 300 miles to Charlotte + 50 miles
        to Region 12: 200 miles + 140 miles + 180 miles
        to Region 21: 220 miles
        to Region 22: 240 miles + 220 miles

Region 21.

        to Region 7: Charlotte + 50 miles to Memphis + 125 miles
        to Region 22: 240 miles

Region 22.

        to Region 7: Charlotte + 50 miles to Nashville + 320 miles

TABLE XIII. Area[a] of Regions.[b]   (In square miles)

Region Number	Area	1/2 radius in miles
1	1118	9
2	51078	64
3	52675	65
4	346	5
5	4899	20
6	1978	13
7	1488	11
8	777	8
9	2054	13
10	1304	10
11	54262	66
12	1138	10
13	58483	68
14	3617	17
15	1277	10
16	873	9
17	55935	67
18	241	4
19	671	7
20	402	5
21	467	6
22	36205	54
23	594	7
24	56045	67
25	908	9
26	39864	57
27	1118	9
28	45162	60
29	1106	9
30	9942	28
31	31040	50
32	770	8
33	333	5
34	7867	25
35	862	8
36	1965	13
37	57002	68
38	47248	62

(a)
     Data obtained from personal communication with William Warntz.
(b)
     See Table II and Map B for definition of regions.

TABLE XIII. (continued)

Region Number	Area	1/2 radius in miles
39	1643	11
40	2520	14
41	69226	74
42	9017	27
43	7522	24
44	1405	11
45	1587	11
46	3939	18
47	673	7
48	792	8
49	47944	62
50	542	7
51	49097	63
52	730	8
53	688	7
54	538	7
55	881	9
56	343	5
57	41000	57
58	812	8
59	3053	16
60	454	6
61	3550	17
62	45045	60
63	494	6
64	1058	9
65	30305	49
66	1024	9
67	1428	11
68	751	8
69	533	7
70	41797	58
71	9278	27
72	734	8
73	667	7
74	39893	57
75	24080	44
76	239	4
77	54705	66

TABLE XIV. Distance for Self-Influence of the Regions[a] in 1950.
(In miles)

$$\text{Distance, } _{ii}D = \frac{_iY_t^{(b)}}{_iV_t^{(c)}} \text{ , } \quad \text{where} \quad \begin{array}{l} _iY = \text{personal income of region i;} \\ _iV = \text{self-influence of region i; and} \\ t = 1920, \ 1930, \ 1940, \text{ and } 1950. \end{array}$$

Region Number	1920	1930	1940	1950
1	33.4	32.9	34.3	34.9
2	34.5	34.4	34.9	35.8
3	54.1	54.0	54.4	54.0
4	52.5	51.6	50.8	49.1
5	62.6	59.2	56.9	55.1
6	49.7	49.1	48.1	46.8
7	60.5	58.0	57.1	55.2
8	55.9	57.7	58.3	58.0
9	58.2	60.0	58.0	56.9
10	77.2	68.6	71.3	72.0
11	55.8	54.6	53.7	52.3
12	69.4	70.0	68.3	67.3
13	67.0	68.8	69.7	70.4
14	53.5	58.0	58.7	53.5
15	72.9	74.7	74.1	73.7
16	47.6	41.7	41.7	40.1
17	50.2	51.1	50.1	51.0
18	41.0	38.0	38.2	37.5
19	50.1	47.2	47.3	45.7
20	70.3	69.3	64.6	66.5
21	77.3	79.7	77.8	78.4
22	60.8	63.4	63.0	60.7
23	56.9	56.5	57.3	55.4

(a)
  See Table I and Map A for definition of regions.
(b)
  Income payments to individuals data for 1920, 1930, 1940, and 1950 in Table V; personal income data for 1959 in Table VI.
(c)
  Self-influence data in Table XV.

APPENDIX V

INCOME INFLUENCE, RELATIVE INCOME INFLUENCE,
AND MODIFIED RELATIVE INCOME INFLUENCE

TABLE XV.. Self-Influence at Each Region[a] for 1920, 1930, 1940, 1950, and 1959.　　　　　　　　　(In $ millions per mile)

Self-Influence, $_i v_t = \dfrac{_i Y_t^{[b]}}{_{ii}D^{[c]}}$, where $_i Y$ = personal income of region i ; $_{ii}D$ = distance constant of region i; and t = 1920, 1930, 1940, 1950, and 1959.

Region Number	1920[d]	1930[d]	1940[d]	1950[d]	1959
1	401.	498.	474.	1140.	2020.
2	159.	162.	154.	353.	587.
3	76.0	93.3	76.0	188.	282.
4	86.7	82.8	78.7	220.	341.
5	14.5	13.0	17.3	53.5	110.
6	15.1	13.9	15.8	45.2	65.3
7	15.0	14.0	19.8	69.9	123.
8	10.1	6.33	9.35	30.4	54.3
9	17.3	13.3	17.0	58.6	107.
10	5.48	9.26	12.6	47.1	129.
11	13.2	11.3	14.2	49.4	88.2
12	7.26	5.50	6.50	22.7	37.6
13	12.6	10.8	13.3	45.5	76.1
14	18.1	13.7	15.0	50.2	85.0
15	55.8	56.9	60.0	171.	298.
16	55.4	70.5	82.2	255.	436.
17	34.0	31.2	37.1	113.	190.
18	131.	155.	150.	410.	686.
19	31.9	33.6	34.3	109.	181.
20	19.3	18.0	19.1	56.0	81.2
21	25.7	24.9	24.6	71.0	118.
22	9.48	6.20	7.83	26.0	39.0
23	13.5	12.8	14.8	51.4	93.2

[a]
　　See Table I and Map A for definition of regions.

[b]
　　Income payments to individuals data for 1920, 1930, 1940, and 1950 in Table V; personal income for 1959 in Table VI.

[c]
　　Distance constant in Table XIV.

[d]
　　Carrothers' Thesis, p. 204.

TABLE XVI. Total Income Influence at Each Region[a] of Twenty-three Regions for Census Years 1920 - 1950. (In $ millions per mile)

Total Income Influence, $_iV_t = \sum_{j=1}^{23} \frac{_jY_t^{(b)}}{_{ij}D^{(c)}}$

Region Number	1920	1930	1940	1950
1	485	584	559	1372
2	264	282	275	665
3	189	219	204	526
4	203	209	208	571
5	120	128	133	367
6	132	132	137	384
7	101	103	113	323
8	90	91	98	280
9	97	94	98	297
10	54	59	64	193
11	94	92	103	299
12	72	73	79	230
13	107	107	114	331
14	145	145	149	432
15	189	193	197	563
16	189	201	213	615
17	180	183	190	547
18	239	266	266	733
19	168	178	179	510
20	90	92	94	268
21	110	104	105	301
22	73	73	78	233
23	68	69	72	217

[a]
    See Table I and Map A for definition of regions.
[b]
    Income data in Table V.
[c]
    Distance data in Table XI.

TABLE XVII. Total Income Influence at Each Region[a] of Thirty-one
Regions for Years 1920, 1930, 1940, 1950, and 1959.
(In $ millions per mile)

$$\text{Total Income Influence, } _iV_t = \sum_{j=1}^{31} \frac{_jY_t^{[b]}}{_{ij}D^{[c]}}$$

Region Number	1920[d]	1930[d]	1940[d]	1950[d]	1959[e]
1	492	591	567	1400	2450
2	271	289	282	689	1170
3	198	227	213	556	905
4	213	218	218	606	1010
5	128	136	141	390	692
6	142	142	148	419	705
7	110	112	122	355	622
8	99.0	97.8	106	309	544
9	107	104	109	337	533
10	60.5	65.6	71.2	221	420
11	107	105	116	342	597
12	86.3	86.7	92.4	277	472
13	120	120	127	375	645
14	156	156	162	475	810
15	201	205	209	602	1020
16	191	212	224	652	1110
17	194	196	203	592	925
18	257	283	283	785	1340
19	185	194	196	564	935
20	114	115	118	342	568
21	128	121	124	359	607
22	94.3	91.6	97.2	299	495
23	81.4	81.1	86.8	264	465

(a)
    See Table I and Map A for definition of regions.
(b)
    Income data in Table V and Table VI.
(c)
    Distance data in Table XI.
(d)
    Data obtained from Carrothers' Thesis, op. cit., p. 204.
(e)
    Individual income influence values between regions and self-influence
measures tabulated in Table XXVIII.

TABLE XVIII. Total Income Influence at Each Region[a] of Thirty-one
Regions for Years 1950 and 1959. (In $ millions per mile)

$$\text{Total Income Influence, } _iV_t = \sum_{j=1}^{31} \frac{_jY_t^{[b]}}{_{ij}D^{[c]}}$$

Region Number	1950[d]	1959[d]
1	1449.	2527.
2	741.8	1255.
3	648.2	1070.
4	707.9	1185.
5	469.8	818.1
6	559.9	949.2
7	478.7	827.9
8	429.4	746.0
9	364.1	633.5
10	231.8	442.5
11	344.6	597.3
12	267.5	514.3
13	411.7	666.0
14	501.3	852.5
15	649.1	1060.
16	706.5	1197.
17	586.5	992.2
18	799.4	1343.
19	587.6	994.5
20	316.6	523.3
21	336.0	564.5
22	277.1	466.3
23	246.5	417.5

[a]
   See Table I and Map A for definition of regions.
[b]
   Income payments to individuals data for 1950 in Table V; personal
income data for 1959 in Table VI.
[c]
   Distance data in Table XII.
[d]
   Individual income influence values between regions and self-influence
measures tabulated in Table XXIX.

TABLE XIX. Total Income Influence of Seventy-seven Regions[a] for Year 1950.
(In $ millions per mile)

$$\text{Total Income Influence,}^{(b)}\ _iV_t = \sum_{j=1}^{115} \frac{_jY_t^{(c)}}{_{ij}D^{(d)}}$$

Region Number	Self[e]	Base	Total
1	76	394	470
2	31	349	380
3	24	336	360
4	223	938	1161
5	137	938	1075
6	53	749	802
7	258	718	976
8	56	296	352
9	61	206	267
10	61	266	327
11	27	260	287
12	102	361	463
13	36	376	412
14	655	473	1128
15	48	482	530
16	51	491	542
17	61	482	543
18	74	493	567
19	55	631	686
20	237	626	863
21	60	661	721
22	57	626	683
23	60	465	525
24	46	380	426
25	103	542	645
26	30	506	536
27	114	275	389
28	32	284	316
29	267	753	1020
30	28	893	921
31	22	320	342

(a) See Table II and Map B for definition of regions.
(b) Data obtained from personal communication with William Warntz.
(c) Income data in Table VIII.
(d) Distance data not available.
(e) Distance data for self-influence in Table XIII.

TABLE XIX.   (continued)

Region Number	Self[a]	Base	Total
32	623	558	1181
33	154	818	972
34	82	726	808
35	57	579	636
36	427	544	971
37	71	444	515
38	26	330	356
39	133	364	497
40	215	445	660
41	29	381	410
42	26	490	516
43	67	1133	1200
44	76	657	733
45	155	551	706
46	1486	520	2006
47	123	570	693
48	60	660	720
49	62	587	649
50	46	452	498
51	60	420	480
52	211	584	795
53	410	615	1025
54	129	593	722
55	83	626	709
56	146	664	810
57	109	593	702
58	44	611	655
59	234	604	838
60	67	796	863
61	385	800	1185
62	102	613	715
63	187	732	919
64	19	774	793
65	38	376	414
66	36	436	472
67	39	553	592
68	91	373	464
69	66	461	527
70	24	437	461
71	16	487	503
72	71	562	633
73	79	484	563
74	42	530	572
75	50	562	612
76	402	555	957
77	51	399	450

[a] Distance data for self-influence in Table XIII.

TABLE XX. Total Income Influence Values of the Seventy-seven Regions[a] Combined for the Twenty-three Regions.[b]

$$\text{Total Income Influence,} \quad _iV_t = \sum_{j=1}^{n} \frac{_jY_t}{_{ij}D}$$

Region Number	Areas Included in Region	Income Influence[c] of Each Region	Weights[d] (Area of Region)	Weighted Income Influence
1	Hartford SMA	1161	578.6	
	Delaware	802	1978.0	
	District of Columbia	976	520.0	
	Baltimore SMA	1020	577.1	1082
	Rest of Maryland	921	9309.9	
	New Jersey	1200	7522.0	
	New York City SMA	2006	1364.6	
	Philadelphia SMA	1185	1021.3	
	Richmond SMA	633	303.9	
2	Connecticut - remainder[e]	1075	4320.4	
	Maine	342	31040.	
	Boston SMA	1181	1062.3	
	Springfield	972	529.2	
	Rest of Massachusetts	808	6315.5	515
	New Hampshire	516	9024.	
	Providence SMA	919	505.3	
	Rest of Rhode Island	793	552.7	
	Vermont	503	9278.	
3	Albany SMA	733	463.5	
	Buffalo SMA	706	473.4	
	Rochester SMA	693	305.9	651
	Syracuse SMA	649	226.5	
	New York - remainder[e]			

(a) See Table II and Map B for definition of the regions.
(b) See Table III and Map C for definition of the combined regions.
(c) Income Influence data of seventy-seven regions in Table XIX.
(d) Area data obtained from U.S. Census, 1940, Population - Volume I, Bureau of Census, United States Department of Commerce, p. 18.
(e) That portion of the remaining area of the region not included in Region 1.

TABLE XX. (continued)

Region Number	Areas Included in Region	Income Influence(a) of Each Region	Weights(b) (Area of Region)	Weighted Income Influence
4	Erie SMA	655	88.5	721
	Pittsburgh SMA	838	1624.5	
	Scranton SMA	863	385.5	
	Pennsylvania - remainder(c)	715	41925.2	
5	Norfolk SMA	563	441.9	572
	Virginia - remainder(c)	572	39153.2	
6	West Virginia	612	—	612
7	Charlotte SMA	498	47.	480
	Rest of North Carolina	480	49095.	
8	South Carolina	414	—	414
9	Atlanta SMA	463	257.5	412
	Rest of Georgia	412	58260.5	
10	Jacksonville SMA	352	242.2	285
	Miami SMA	267	164.	
	Tampa SMA	327	262.8	
	Rest of Florida	287	53073.	
11	Birmingham SMA	470	347.6	381
	Rest of Alabama	380	50730.4	
12	Mississippi	356	—	356
13	Chatanooga SMA	472	531.8	462
	Knoxville SMA	592	200.9	
	Memphis SMA	464	288.2	
	Nashville SMA	527	315.9	
	Rest of Tennessee	461	40624.2	
14	Louisville SMA	645	454.4	537
	Rest of Kentucky	536	39654.6	

(a)
   Income Influence data of seventy-seven regions in Table XIX.
(b)
   Area data obtained from U.S. Census, Population - Volume I,
Bureau of Census, United States Department of Commerce, p. 18.
(c)
   That portion of the remaining area of the region not included in Region 1.

TABLE XX. (continued)

Region Number	Areas Included in Region	Income Influence of Each Region (a)	Weights (b) (Area of Region)	Weighted Income Influence
15	Cincinnati SMA	702	39645.2	
	Cleveland SMA	795	521.9	
	Columbus SMA	1025	336.2	
	Dayton SMA	722	219.8	703
	Toledo SMA	709	194.8	
	Rest of Ohio	810	204.1	
16	Grand Rapids SMA	636	142.9	
	Detroit SMA	971	856.3	522
	Rest of Michigan	515	56022.8	
17	Evansville SMA	567	176.8	
	Fort Wayne SMA	686	141.1	
	Indianapolis SMA	863	315.8	684
	South Bend SMA	721	155.6	
	Rest of Indiana	683	35415.7	
18	Chicago SMA	1128	1184.2	
	Peoria SMA	530	109.	555
	Davenport, Moline, and Rockport SMA	542	192.2	
	Rest of Illinois	543	54461.6	
19	Milwaukee	957	250.3	
	Rest of Wisconsin	450	54464.7	452
20	Des Moines SMA	525	210.2	
	Rest of Iowa	426	55775.8	426
21	Kansas City SMA	497	500.4	
	St. Louis SMA	660	956.	414
	Rest of Missouri	410	67813.6	
22	Arkansas	360	—	360
23	New Orleans	389	333.8	
	Rest of Louisiana	316	44843.2	

(a)
Income influence data of seventy-seven regions in Table XIX.
(b)
Area data obtained from U.S. Census, Population - Volume I, Bureau of Census, United States Department of Commerce, p. 18.

TABLE XXI. Total Income Influence Change at Each Region[a] for Four
Ten Year Periods. (In $ millions per mile)

Income Influence Change, $_iV_{t+\theta} - {}_iV_t$ [b]

Region Number	1920-1930	1930-1940	1940-1950	1950-1959
1	99.1	23.9	832.	1050.
2	18.2	7.65	408	479
3	28.7	13.6	343	349
4	5.41	.408	388	400
5	8.03	5.50	249	301
6	.725	5.83	271	286
7	2.36	9.76	233	267
8	- 1.21	8.37	203	235
9	- 2.67	4.63	228	197
10	5.06	5.63	150	199
11	- 2.27	11.1	226	255
12	.357	5.72	185	195
13	- .768	7.50	248	269
14	- .583	6.80	313	335
15	4.54	3.77	393	422
16	21.0	12.5	427	457
17	1.86	7.53	389	333
18	25.9	.076	503	555
19	9.29	2.27	368	371
20	.639	3.07	224	225
21	6.06	2.60	235	248
22	2.71	5.65	202	196
23	.288	5.69	177	200

[a]
See Table I and Map A for definition of regions.
[b]
Total income influence values in Table XVII.

TABLE XXII. Total Income Influence Percentage Change at Each Region[a]
for Four Ten Year Periods.

$$\% \text{ of change,} \quad \frac{V_{i\,t+\theta} - V_{i\,t}}{V_{i\,t}} \quad \text{[b]}$$
[c]

Region Number	1920-1930	1930-1940	1940-1950	1950-1959
1	20.1	- 4.05	147.	75.0
2	6.73	- 2.65	145	69.4
3	14.5	- 5.99	161	62.9
4	2.54	- .19	179	66.0
5	6.28	4.05	176	77.3
6	.51	4.09	183	68.3
7	2.15	8.72	192	75.1
8	- 1.22	8.56	191	76.1
9	- 2.50	4.45	210	58.4
10	8.36	8.58	211	89.6
11	- 2.13	10.63	197	74.7
12	.41	6.60	200	70.2
13	- .64	6.26	195	71.9
14	- .37	4.37	193	70.4
15	2.26	1.84	188	70.1
16	11.0	5.89	190	70.0
17	.96	3.84	191	56.2
18	10.0	.03	178	70.7
19	5.03	1.17	187	65.7
20	.56	2.67	190	65.9
21	4.74	2.14	242	69.0
22	2.88	6.17	232	65.6
23	.35	7.01	204	75.8

[a]
    See Table I and Map A for definition of regions.
[b]
    Increments of change in Table XXI.
[c]
    Total income influence values in Table XVII.

TABLE XXIII. Ratio of the Income Influence of Thirty-one Regions[a] to
the Income Influence of Twenty-three Regions[b].   (c)

$$\text{Income Influence Ratio,} \quad \frac{_iV}{_iV'} = \frac{\displaystyle\sum_{j=1}^{31} \frac{_jY}{_{ij}D}}{\displaystyle\sum_{k=1}^{23} \frac{_kY}{_{ik}D}}$$

Region Number	1920	1930	1940	1950	Constant
1	1.01	1.01	1.01	1.02	1.01
2	1.03	1.02	1.02	1.04	1.03
3	1.05	1.04	1.04	1.06	1.05
4	1.04	1.04	1.05	1.06	1.05
5	1.07	1.06	1.06	1.06	1.06
6	1.08	1.08	1.08	1.09	1.08
7	1.09	1.09	1.08	1.10	1.09
8	1.10	1.08	1.08	1.10	1.09
9	1.10	1.10	1.11	1.13	1.11
10	1.13	1.12	1.11	1.14	1.12
11	1.13	1.14	1.13	1.14	1.13
12	1.19	1.19	1.16	1.20	1.18
13	1.12	1.12	1.11	1.13	1.12
14	1.08	1.08	1.09	1.10	1.09
15	1.06	1.06	1.06	1.07	1.06
16	1.01	1.05	1.06	1.06	1.04
17	1.08	1.07	1.07	1.08	1.07
18	1.08	1.06	1.06	1.07	1.07
19	1.07	1.09	1.09	1.11	1.09
20	1.27	1.25	1.26	1.28	1.27
21	1.16	1.16	1.18	1.19	1.17
22	1.29	1.26	1.24	1.28	1.27
23	1.19	1.18	1.21	1.22	1.20

Arithmatic Average = 1.13

(a)
Total income influence of thirty-one regions in Table XVII.
(b)
Total income influence of twenty-three regions in Table XVI.
(c)
See Table I and Map A for definition of region.

TABLE XXIV. Relative Income Influence[a] at Each Region[b] for Census Years 1920 - 1950, and 1959.

Relative Income Influence, $\dfrac{iV_{t+\theta}}{iV_t}$

Region Number	1920-1930	1930-1940	1940-1950	1950-1959
1	1.20	.960	2.47	1.75
2	1.07	.974	2.45	1.69
3	1.14	.940	2.61	1.63
4	1.03	.998	2.78	1.66
5	1.06	1.04	2.76	1.77
6	1.01	1.04	2.83	1.68
7	1.02	1.09	2.92	1.75
8	.988	1.09	2.91	1.76
9	.975	1.04	3.10	1.58
10	1.08	1.11	3.11	1.90
11	.979	1.11	2.95	1.75
12	1.00	1.07	3.00	1.70
13	.994	1.06	2.95	1.72
14	.996	1.04	2.93	1.70
15	1.02	1.02	2.88	1.70
16	1.11	1.06	2.90	1.70
17	1.01	1.04	2.91	1.56
18	.996	1.04	2.93	1.71
19	.994	1.06	2.95	1.66
20	1.01	1.03	2.90	1.66
21	.949	1.02	2.90	1.69
22	.971	1.06	3.08	1.66
23	.997	1.07	3.04	1.76
Arithmatic Average	1.03	1.04	2.88	1.70

[a]
Total income influence values in Table XVII.
[b]
See Table I and Map A for definition of regions.

TABLE XXV. Relative Income Influence[a] at Each Region[b] for Years 1950 and 1959.

Relative Income Influence, $\dfrac{V_{i\,t+\theta}}{V_{i\,t}}$

Region Number	1950-1959
1	1.74
2	1.69
3	1.65
4	1.67
5	1.74
6	1.70
7	1.73
8	1.74
9	1.74
10	1.91
11	1.73
12	1.92
13	1.62
14	1.70
15	1.63
16	1.69
17	1.69
18	1.68
19	1.69
20	1.65
21	1.68
22	1.68
23	1.69

[a] Total Income influence values in Table XVIII.
[b] See Table I and Map A for definition of regions.

TABLE XXVI. Quotient of the Relative Income Influence[a] and the Ratio of National Income Growth[b] at Each Region[c] for Census Years 1920 - 1950, and 1959.

$$\frac{{}_iV_{t+\theta}}{\rho_\theta \, {}_iV_t} \, , \quad \text{where} \quad \rho_\theta = \frac{{}_{US}Y_{t+\theta}}{{}_{US}Y_t}$$

Region Number	1920-1930	1930-1940	1940-1950	1950-1959
1	1.14	.928	.859	.972
2	1.02	.941	.852	.956
3	1.09	.909	.908	.918
4	.976	.965	.968	.939
5	1.01	1.01	.961	.999
6	.956	1.01	.985	.952
7	.972	1.05	1.02	.989
8	.940	1.05	1.01	.995
9	.928	1.01	1.08	.896
10	1.03	1.05	1.08	1.06
11	.931	1.07	1.03	.989
12	.956	1.03	1.04	.964
13	.946	1.03	1.03	.974
14	.948	1.01	1.02	.966
15	.973	.984	1.00	.964
16	1.06	1.02	1.01	.964
17	.961	1.00	1.01	.885
18	1.05	.967	.968	.969
19	.999	.978	1.00	.940
20	.957	.993	1.01	.940
21	.903	.987	1.01	.958
22	.924	1.03	1.07	.938
23	.948	1.03	1.06	.996

(a)
Relative income influence data in Table XXIV.
(b)
Ratio of the income of the residents in the United States in Table X.
(c)
See Table I and Map A for definition of region.

TABLE XXVII. Modified Relative Income Influence at Each Region[a] for Census Years 1920 - 1950, and 1959.

Modified Relative Income Influence,$\quad \dfrac{V_{i\,t+\theta}}{\rho_\theta\,V_{i\,t}} - 1$ [b]

Region Number	1920-1930	1930-1940	1940-1950	1950-1959
1	+ .143	− .073	− .141	− .029
2	+ .016	− .059	− .148	− .044
3	+ .089	− .091	− .093	− .082
4	− .024	− .035	− .032	− .061
5	+ .011	+ .006	− .039	− .001
6	− .044	+ .006	− .015	− .048
7	− .028	+ .051	+ .016	− .011
8	− .060	+ .049	+ .014	− .005
9	− .072	+ .010	+ .078	− .104
10	+ .031	+ .050	+ .083	+ .063
11	− .069	+ .069	+ .029	− .011
12	− .045	+ .030	+ .044	− .036
13	− .055	+ .027	+ .028	− .026
14	− .052	+ .009	+ .019	− .034
15	− .027	− .016	+ .003	− .036
16	+ .056	+ .024	+ .010	− .036
17	− .039	+ .004	+ .012	− .115
18	+ .048	− .033	− .032	− .031
19	− .001	− .022	+ .000	− .060
20	− .043	− .008	+ .010	− .060
21	− .097	− .013	+ .010	− .042
22	− .076	+ .026	+ .071	− .062
23	− .052	+ .034	+ .060	− .004

[a] See Table I and Map A for definition of regions.

[b] Values of this factor plus unity in Table XXVI.

TABLE XXVIII. Income Influence[a] at Each Region for 1959.
(In $ millions per mile)

Income Influence, $_iV_t = \sum_{j=1}^{n} \frac{_jY_t^{(b)}}{_{ij}D^{(c)}}$

Region Number	Income Influence Region 1[d]	Income Influence Region 2[d]	Income Influence Region 3[d]
1	3.2813	3.1692	3.5557
2	12.8591	12.2522	13.9848
3	2.6938	2.5766	2.9767
4	3.1735	3.0024	3.6029
5	8.7396	8.0626	10.6009
6	3.0271	2.7519	3.5613
7	2.6025	2.3781	2.9557
8	11.0006	9.6994	11.4910
9	3.8007	3.2923	3.6146
10	1.8372	1.5695	1.9426
11	7.9724	7.0060	9.5340
12	4.2503	3.9115	5.2407
13	8.3414	7.5072	10.8657
14	26.9123	23.3240	43.7325
15	28.2791	25.2294	38.4089
16	11.8439	10.1167	15.6645
17	34.8873	26.8036	46.7638
18	5.2275	4.3314	6.4056
19	5.6442	4.5440	6.0247
20	2.0552	1.7315	1.7395
21	4.6535	3.7762	4.3056
22	7.0709	5.5788	5.6305
23	7.7924	6.5302	6.2655
24	4.4971	3.3849	3.1797
25	12.0910	8.5708	7.9658
26	17.8764	10.6631	9.6476
27	5.0049	3.6345	3.9649
28	38.0727	25.0029	38.9581
29	52.5965	43.5800	282.4303
30	91.4217	587.0184	60.0771
31	2019.7669	306.8043	243.3276
	2449.2730	1167.8036	905.2108

[a] Self-influence data in Table XV

[b] Income data in Table VI.

[c] Distance data in Table XI.

[d] See Table I and Map A for definition of region.

TABLE XXVIII. (continued)

Region Number	Income Influence Region 4[a]	Income Influence Region 5[a]	Income Influence Region 6[a]
1	3.8220	3.2919	3.7936
2	14.8339	13.8450	15.2702
3	3.2924	2.7622	3.2924
4	4.1107	3.3469	4.1952
5	13.0102	9.0149	13.0102
6	4.4145	3.3634	4.8159
7	3.5982	2.9769	3.8314
8	13.9852	12.9791	15.2889
9	4.4947	4.8308	4.9701
10	2.5212	2.2788	2.8554
11	12.8444	8.9786	14.0121
12	6.5036	4.4245	6.4261
13	15.0145	8.6926	14.4877
14	58.3100	24.6380	43.7325
15	54.7531	29.9232	52.5183
16	26.2486	14.0753	31.3290
17	115.6789	38.5596	109.8950
18	10.5767	6.4971	14.2125
19	8.6483	7.6600	10.7240
20	2.4784	2.5795	2.7780
21	5.7587	6.2256	6.6768
22	7.5074	10.4844	8.8130
23	7.3595	10.9094	7.6008
24	4.0883	8.7444	4.3123
25	10.5796	22.5700	13.5420
26	14.8243	110.3111	16.4270
27	9.2515	8.2513	65.2908
28	340.5118	40.8585	50.7636
29	35.4720	24.2111	19.8090
30	31.8358	36.8894	25.0321
31	160.3750	207.5441	115.6803
	1006.7034	691.7176	705.3913

[a] See Table I and Map A for definition of regions.

TABLE XXVIII. (continued)

Region Number	Income Influence Region 7[a]	Income Influence Region 8[a]	Income Influence Region 9[a]
1	3.2603	3.3791	3.6446
2	14.8339	15.4405	17.0225
3	2.8467	2.8718	3.1800
4	3.5201	3.5000	4.0032
5	9.0149	9.0149	10.0429
6	3.6222	3.5024	4.4145
7	3.4483	3.5672	4.5472
8	15.9654	16.7046	21.7361
9	6.5430	6.8920	10.5489
10	2.8214	2.9259	4.3090
11	9.8382	9.7347	12.6684
12	4.6136	4.6136	5.6821
13	8.6926	8.6926	10.0707
14	21.0759	20.8250	23.3240
15	29.9232	29.5793	35.2520
16	14.2823	14.2823	16.4610
17	34.3421	31.3985	36.0311
18	7.9789	7.8413	9.6765
19	10.5137	10.5137	18.4896
20	3.5111	3.7731	6.0190
21	9.8021	10.9690	27.1000
22	20.2700	24.3240	35.7705
23	14.4890	18.9244	27.1000
24	20.9866	54.2572	12.5920
25	122.5964	45.1440	22.5700
26	20.2600	16.8833	10.4793
27	6.1060	4.1821	4.4246
28	26.1750	21.7558	20.6814
29	17.9447	15.4070	14.1231
30	26.6164	22.6096	19.2908
31	120.0089	100.8071	82.0523
	621.9031	544.3160	533.3073

[a] See Table I and Map A for definition of region.

TABLE XXVIII. (continued)

Region Number	Income Influence Region 10[a]	Income Influence Region 11[a]	Income Influence Region 12[a]
1	3.0830	3.8220	4.0177
2	14.1275	18.2973	19.5061
3	2.5464	3.4130	3.5237
4	3.0321	4.4708	4.6052
5	6.7347	10.8009	10.4081
6	2.9227	5.2975	5.5763
7	2.9769	5.5918	5.9114
8	13.7717	27.3348	35.3745
9	6.3036	14.3583	28.7166
10	2.3009	6.0769	6.9705
11	7.4580	15.6745	14.6793
12	3.5748	6.5829	6.2767
13	5.9840	11.1594	10.0707
14	13.3534	23.6391	18.2218
15	19.9488	39.5907	34.7756
16	8.4452	19.4240	14.8742
17	18.7854	36.6316	26.8036
18	4.3730	11.6615	7.7084
19	6.3082	25.5333	11.9155
20	2.8088	10.1120	37.5855
21	7.0876	88.1554	18.4280
22	10.8589	35.7705	14.4785
23	128.8192	14.2661	10.3033
24	6.4244	7.4952	4.6985
25	10.5796	15.4063	9.4041
26	7.1505	8.2135	6.2020
27	2.5024	4.4246	3.3549
28	13.2952	20.9400	14.9539
29	10.3060	14.2551	10.4472
30	14.8077	17.2352	14.4020
31	59.2983	71.2778	57.3699
	419.9689	596.9130	471.5637

[a]
     See Table I and Map A for definition of region.

TABLE XXVIII. (continued)

Region Number	Income Influence Region 13[a]	Income Influence Region 14[a]	Income Influence Region 15[a]
1	4.0657	4.1483	4.0019
2	17.8261	16.8841	16.2246
3	3.4492	3.8122	3.5818
4	4.9395	5.1470	4.7480
5	13.4694	16.3557	15.6835
6	6.3253	6.8354	5.8861
7	6.6684	4.7563	4.4978
8	25.0569	20.9779	16.5513
9	9.2303	6.8920	5.4410
10	6.4054	4.6470	3.2027
11	21.0181	23.7128	16.2245
12	8.1787	8.8491	7.9382
13	15.5811	21.7315	20.1414
14	31.2375	47.2783	92.0684
15	57.1866	85.7800	80.4187
16	32.3733	88.2909	69.3714
17	52.3309	95.5608	298.2225
18	23.9368	84.9617	19.7739
19	76.1431	28.2210	12.7666
20	5.6177	4.2847	3.0829
21	21.9380	11.8128	7.6783
22	20.9689	12.9382	9.9688
23	10.9094	8.9163	7.9256
24	6.1725	5.4275	4.4971
25	13.2764	11.8789	10.5796
26	8.6828	8.6828	10.6631
27	6.1060	9.5406	15.2650
28	27.0193	38.9581	88.1684
29	17.1382	21.4830	32.4531
30	17.8194	20.0257	25.6426
31	74.2789	81.1092	112.0079
	645.3498	809.8998	1024.6767

[a] See Table I and Map A for definition of region.

TABLE XXVIII. (continued)

Region Number	Income Influence Region 16[a]	Income Influence Region 17[a]	Income Influence Region 18[a]
1	4.0657	4.1995	5.6386
2	15.7927	17.1632	17.4516
3	3.6418	3.8803	4.3172
4	4.7115	5.3260	5.9466
5	16.8367	19.4050	27.9243
6	5.3645	7.3068	8.1500
7	4.0174	5.3051	6.2379
8	15.0341	18.9905	18.5989
9	4.7422	6.0104	5.6184
10	2.8214	4.0169	3.7619
11	15.4133	26.4228	23.1200
12	8.5682	9.9962	14.9944
13	24.2882	30.5851	91.7555
14	436.0169	58.3100	64.7888
15	95.3111	142.9666	685.5087
16	32.3733	190.3194	53.9555
17	115.6789	156.9928	68.6843
18	12.2918	41.3454	15.1600
19	9.5750	17.8733	11.9155
20	2.6333	3.6114	3.4162
21	6.2256	9.2140	7.0876
22	8.1080	10.3067	8.3301
23	7.0786	8.0634	7.1883
24	3.7476	4.6294	3.6183
25	8.1578	9.9573	7.8733
26	8.5605	8.8086	7.0674
27	7.6325	9.8483	6.2306
28	55.8400	45.2756	35.6425
29	38.1325	24.6016	22.7656
30	28.0360	21.9031	20.6147
31	108.5615	86.0549	77.5440
	1109.2586	925.0653	1340.9067

[a] See Table I and Map A for definition of region.

TABLE XXVIII. (continued)

Region Number	Income Influence Region 19[a]	Income Influence Region 20[a]	Income Influence Region 21[a]
1	4.7910	5.3151	4.7686
2	17.0225	20.0652	19.9687
3	4.0743	5.5717	4.7239
4	5.5180	9.0073	7.5617
5	33.6735	42.4037	17.0880
6	7.0633	14.6137	17.6583
7	4.7022	7.2596	4.9261
8	1.7181	24.3797	22.0012
9	5.1178	4.9701	6.3814
10	3.2916	3.4347	5.1521
11	19.2666	23.1200	117.8990
12	12.2681	81.1851	13.4950
13	180.8189	18.7681	17.2041
14	51.4500	27.7666	29.1550
15	285.9333	71.4833	64.3350
16	35.9703	17.9851	27.7485
17	53.6073	32.3220	38.5596
18	11.9684	7.4557	11.6615
19	10.1169	8.1242	12.1863
20	3.0829	2.9395	4.0126
21	6.2256	3.9395	7.8084
22	7.4158	6.4010	8.3301
23	6.7195	6.1417	7.4782
24	3.3136	2.6905	3.3136
25	7.1273	5.7871	7.2031
26	6.3978	4.9819	5.9009
27	5.3561	3.6345	4.6257
28	30.4581	20.1831	23.2666
29	20.0697	14.8087	15.7247
30	19.1154	15.2369	15.9295
31	71.2778	55.5630	60.8319
	934.9317	567.5383	606.8993

[a] See Table I and Map A for definition of region.

TABLE XXVIII. (continued)

Region Number	Income Influence Region 22[a]	Income Influence Region 23[a]	Income Influence Region 8[b]
1	4.3241	3.7380	4.5762
2	21.0837	19.4088	25.7981
3	4.1000	3.5237	4.8649
4	5.6192	4.5708	7.2916
5	12.3107	8.9445	11.3356
6	7.5678	4.5085	7.5678
7	11.8228	5.5918	17.2416
8	53.0617	35.3745	224.5581
9	9.7528	93.2191	10.1352
10	39.0380	4.4716	6.9705
11	20.1043	11.4172	11.2780
12	7.8231	5.1903	7.2945
13	11.4694	8.1762	7.8647
14	20.8250	16.0486	14.5775
15	40.8476	27.9717	26.5298
16	16.4610	11.2930	10.2231
17	29.7013	23.1357	20.1642
18	8.9176	6.0640	5.2883
19	14.4918	9.5750	7.4472
20	7.4352	14.0444	4.9568
21	11.8128	12.7972	6.9803
22	11.0563	12.4102	7.3265
23	9.0029	11.3085	7.0786
24	3.8864	4.1973	2.9148
25	8.0607	8.5708	5.9920
26	5.8442	5.6803	4.3726
27	3.6783	2.9355	2.5872
28	17.8212	14.5669	12.9860
29	12.5024	10.6664	9.7152
30	13.9251	13.3929	11.3048
31	54.7016	51.8860	43.0274
	495.1570	464.6794	551.4644

[a]
    See Table I and Map A for definition of region.
[b]
    See Map D for definition of region.

TABLE XXVIII. (continued)

Region Number	Income Influence Region 7[a]	Income Influence Region 6[a]	Income Influence Region 5[a]
1	4.7910	5.2066	5.7011
2	25.3262	22.3306	20.1626
3	5.2152	5.7184	4.7239
4	8.2770	10.7456	6.8055
5	13.7939	20.4446	154.8837
6	12.8424	79.8869	7.5678
7	73.0450	12.5393	4.9855
8	75.1708	32.2160	17.8623
9	6.9851	5.4989	4.0382
10	6.7714	4.2321	2.5483
11	19.2666	38.5333	13.8029
12	9.4701	18.6137	19.9925
13	9.3840	13.7633	24.2882
14	16.9834	22.1430	25.7250
15	32.5746	49.4884	62.7658
16	12.4512	16.7448	16.4610
17	23.8902	30.5263	30.1082
18	5.2275	7.3354	6.4971
19	7.3452	8.0029	6.3082
20	3.6114	3.3263	2.2981
21	6.2256	5.7587	4.3462
22	6.6824	6.3343	5.3342
23	6.6712	6.3951	5.4547
24	2.7137	2.6016	2.4787
25	5.6425	5.7871	5.3314
26	4.3726	4.8238	4.7858
27	2.8268	3.4693	3.4693
28	14.5669	17.4500	19.0363
29	10.8950	12.8176	14.1231
30	12.0844	13.6538	14.8077
31	44.3805	51.4036	53.8664
	485.1718	537.7913	270.5597

[a]
    See Map D for definition of region.

TABLE XXVIII. (continued)

Region Number	Income Influence Region 4[a]	Income Influence Region 3[a]	Income Influence Region 2[a]
1	7.4489	11.5965	12.0058
2	27.6900	43.7210	331.6962
3	11.4368	59.8018	6.8621
4	95.2843	10.7456	4.0833
5	12.7211	8.2963	5.5577
6	7.4350	3.7175	2.2784
7	5.5918	3.3104	2.5231
8	21.4761	1.3463	11.2055
9	3.8574	2.7940	2.4154
10	2.1743	1.4905	1.2030
11	11.4172	6.7014	4.4461
12	7.9382	4.6136	2.6077
13	7.4396	5.1612	3.3844
14	13.4561	9.7726	6.6513
15	24.9845	17.0423	10.8126
16	8.4452	5.7809	4.0132
17	17.0379	12.0763	8.5855
18	3.8218	2.6596	1.8487
19	4.3241	2.8370	2.3012
20	1.9007	1.3664	1.1924
21	3.3627	2.4120	2.0295
22	3.9745	2.9663	2.4922
23	4.5905	3.6222	3.1540
24	1.7988	1.3867	1.1702
25	3.8913	2.9567	2.4182
26	3.3213	2.5754	2.0260
27	2.0910	1.5419	1.1224
28	11.2429	8.4606	5.9828
29	8.9723	6.9648	5.1356
30	10.3073	8.3110	6.2026
31	36.5622	29.1591	21.8468
	385.9958	285.1861	478.9839

[a]
    See Map D for definition of region.

TABLE XXVIII. (continued)

Region Number	Income Influence Region 1[a]
1	159.0555
2	48.8647
3	7.4079
4	4.4708
5	6.3960
6	2.1622
7	1.9472
8	8.0901
9	1.8934
10	1.0042
11	4.3214
12	2.8114
13	3.8769
14	6.9693
15	11.6972
16	3.9967
17	8.6192
18	1.8487
19	2.1362
20	.9952
21	1.7254
22	2.1717
23	2.8015
24	1.0423
25	2.1632
26	1.9606
27	1.1349
28	6.2741
29	5.3146
30	6.5301
31	22.6897

342.3678

TABLE XXIX. Income Influence[a] at Each Region[b] for Years 1950 and 1959. (In $ millions per mile)

$$\text{Income Influence,} \quad {}_iV_t = \sum_{j=1}^{n} \frac{{}_jY_t{}^{(c)}}{{}_{ij}D^{(d)}}$$

Region Number	Income Influence at Region 1 1950	1959
1	1144.0	2020.0
2	68.79	114.3
3	52.42	78.62
4	32.38	50.16
5	10.20	21.03
6	4.807	6.939
7	14.29	25.08
8	5.838	10.42
9	4.378	7.980
10	3.340	9.145
11	2.970	5.301
12	1.428	2.365
13	4.187	7.009
14	4.018	6.798
15	26.35	45.89
16	20.04	34.23
17	8.770	14.74
18	21.27	35.54
19	6.724	11.19
20	3.638	5.271
21	5.285	8.774
22	1.460	2.192
23	2.403	4.362
	1448.986	2527.336

[a]
    Self-influence data in Table XV.
[b]
    See Table I and Map A for definition of region.
[c]
    Income data in Table VI.
[d]
    Distance data in Table XII.

TABLE XXIX. (continued)

Region Number	Income Influence at Region 2	
	1950	1959
1	217.3	383.5
2	353.4	587.0
3	38.52	57.78
4	21.85	33.84
5	6.222	12.82
6	3.400	4.908
7	5.005	8.782
8	2.229	3.980
9	3.530	6.435
10	2.827	7.740
11	2.451	4.375
12	1.222	2.022
13	3.375	5.650
14	3.219	5.447
15	19.66	34.24
16	15.98	27.29
17	7.040	11.83
18	17.74	29.65
19	5.710	9.50
20	3.189	4.622
21	4.649	7.720
22	1.253	1.882
23	2.080	3.776
	741.851	1254.789

TABLE XXIX. (continued)

Region Number	Income Influence at Region 3 1950	1959
1	206.1	363.7
2	47.95	79.65
3	188.3	282.4
4	64.00	99.12
5	7.640	15.75
6	4.885	7.051
7	5.983	10.50
8	2.600	4.643
9	4.271	7.786
10	3.082	8.438
11	2.950	5.265
12	1.420	2.352
13	4.334	7.256
14	4.458	7.542
15	14.44	25.15
16	28.93	49.42
17	10.19	17.13
18	26.01	43.47
19	8.354	13.90
20	3.411	4.943
21	4.964	8.242
22	1.520	2.283
23	2.399	4.355
	648.191	1070.346

TABLE XXIX. (continued)

Region Number	Income Influence at Region 4	
	1950	1959
1	119.7	211.3
2	25.57	42.48
3	60.17	90.25
4	219.9	340.5
5	12.14	25.01
6	12.59	18.17
7	9.367	16.43
8	3.935	7.027
9	6.330	11.39
10	3.862	10.57
11	4.302	7.678
12	1.909	3.160
13	6.903	11.56
14	8.000	13.54
15	87.03	151.6
16	48.31	82.51
17	17.78	29.89
18	38.21	63.86
19	11.46	19.07
20	5.299	7.679
21	7.599	12.62
22	2.026	3.042
23	3.089	5.606
	707.882	1184.942

TABLE XXIX. (continued)

Region Number	Income Influence at Region 5 1950	1959
1	138.3	244.2
2	26.70	44.36
3	26.34	39.52
4	44.51	68.94
5	53.52	110.3
6	8.962	12.94
7	12.95	22.72
8	5.562	9.931
9	7.098	12.94
10	4.691	12.84
11	4.338	7.730
12	1.918	3.176
13	6.089	10.19
14	5.869	9.930
15	39.81	69.33
16	23.02	39.31
17	11.65	19.58
18	24.84	41.51
19	7.41	12.33
20	4.049	5.867
21	7.159	11.89
22	1.854	2.785
23	3.182	5.775
	469.821	818.094

TABLE XXIX. (continued)

Region Number	Income Influence at Region 6 1950	1959
1	90.86	160.4
2	20.35	33.81
3	23.48	35.23
4	64.38	99.71
5	12.50	25.75
6	45.24	65.29
7	20.31	35.64
8	8.014	14.31
9	11.05	20.14
10	9.595	26.27
11	5.839	10.42
12	2.379	3.938
13	9.947	16.65
14	11.95	20.21
15	94.89	165.3
16	36.06	61.60
17	22.06	37.08
18	38.50	64.34
19	10.76	17.91
20	5.321	7.711
21	10.22	16.97
22	2.458	3.692
23	3.777	6.855
	559.920	949.226

TABLE XXIX. (continued)

Region Number	Income Influence at Region 7	
	1950	1959
1	148.1	261.4
2	16.42	27.27
3	15.77	23.65
4	26.25	40.66
5	9.896	20.40
6	11.13	16.07
7	69.87	122.6
8	13.06	23.32
9	12.04	21.95
10	6.048	16.56
11	6.280	11.21
12	2.722	4.506
13	19.65	32.90
14	6.840	11.57
15	31.79	55.36
16	18.49	31.58
17	12.09	20.32
18	24.18	40.40
19	6.44	10.72
20	3.774	5.469
21	11.53	19.15
22	2.294	3.445
23	4.074	7.395
	478.738	827.905

TABLE XXIX.   (continued)

Region Number	Income Influence at Region 8	
	1950	1959
1	132.4	233.7
2	16.00	26.58
3	15.00	22.50
4	24.14	37.39
5	9.303	19.17
6	9.614	13.88
7	28.59	50.16
8	30.38	54.26
9	17.20	31.35
10	7.969	21.82
11	7.728	13.79
12	2.860	4.734
13	11.69	19.57
14	6.805	11.51
15	29.69	51.72
16	17.51	29.90
17	11.82	19.87
18	23.62	39.47
19	6.78	11.28
20	3.913	5.670
21	9.377	15.57
22	2.294	3.445
23	4.755	8.629
	429.438	745.968

TABLE XXIX. (continued)

Region Number	Income Influence at Region 9	
	1950	1959
1	52.47	92.60
2	13.39	22.25
3	13.02	19.53
4	20.52	31.79
5	6.274	12.93
6	7.003	10.11
7	13.93	24.44
8	9.088	16.23
9	58.61	106.83
10	8.261	22.62
11	19.26	34.38
12	4.572	7.569
13	15.04	25.17
14	8.374	14.17
15	25.81	44.95
16	16.99	29.01
17	13.38	22.49
18	26.01	43.47
19	7.362	12.25
20	4.176	6.052
21	10.45	17.35
22	3.453	5.186
23	6.670	12.11
	364.113	633.457

TABLE XXIX.   (continued)

Region Number	Income Influence at Region 10	
	1950	1959
1	39.43	69.59
2	10.57	17.55
3	9.253	13.88
4	12.33	19.10
5	4.084	8.418
6	5.992	8.649
7	6.891	12.09
8	4.148	7.407
9	8.14	14.83
10	47.05	128.8
11	5.599	9.993
12	2.310	3.824
13	4.831	8.087
14	3.677	6.22
15	14.04	24.45
16	10.11	17.27
17	6.865	11.54
18	15.38	25.71
19	4.582	7.625
20	2.863	4.149
21	5.907	9.807
22	1.977	2.970
23	5.824	10.57
	231.844	442.529

TABLE XXIX. (continued)

Region Number	Income Influence at Region 11	
	1950	1959
1	46.01	81.20
2	12.02	19.97
3	11.62	17.43
4	18.03	27.92
5	4.948	10.20
6	4.785	6.907
7	9.389	16.47
8	5.278	9.425
9	24.90	45.38
10	7.347	20.11
11	49.39	88.16
12	7.635	12.64
13	18.10	30.29
14	8.170	13.82
15	24.08	41.94
16	16.05	27.42
17	13.50	22.70
18	26.83	44.83
19	7.564	12.59
20	4.262	6.176
21	11.21	18.61
22	4.683	7.033
23	8.845	16.05
	344.646	597.271

TABLE XXIX. (continued)

Region Number	Income Influence at Region 12	
	1950	1959
1	37.40	66.01
2	10.10	16.78
3	9.460	14.19
4	13.52	20.94
5	3.705	7.636
6	3.294	4.755
7	6.879	67.71
8	3.301	5.895
9	9.988	18.21
10	5.124	14.03
11	12.91	23.04
12	22.70	37.59
13	8.429	14.11
14	5.053	8.549
15	17.36	30.23
16	13.01	22.23
17	10.28	17.28
18	22.38	37.40
19	6.444	10.72
20	6.008	8.706
21	13.26	22.02
22	7.890	11.85
23	18.99	34.46
	267.485	514.341

TABLE XXIX. (continued)

| Region | Income Influence at Region 13 | |
Number	1950	1959
1	52.26	92.24
2	13.34	22.16
3	13.76	20.64
4	23.31	36.10
5	5.606	11.56
6	6.568	9.481
7	23.67	41.54
8	6.434	11.49
9	15.66	28.55
10	5.437	14.88
11	14.58	26.03
12	4.019	6.653
13	45.48	76.14
14	17.68	29.92
15	36.37	63.34
16	22.03	37.62
17	23.03	38.70
18	38.79	64.82
19	10.36	17.24
20	5.344	7.745
21	17.41	28.90
22	4.855	7.292
23	5.707	10.36
	411.700	666.015

TABLE XXIX. (continued)

Region Number	Income Influence at Region 14 1950	1959
1	59.76	105.5
2	15.16	25.18
3	16.86	25.30
4	32.19	49.96
5	6.439	13.27
6	9.400	13.57
7	9.819	17.23
8	4.463	7.970
9	10.39	18.94
10	4.633	12.69
11	7.845	14.00
12	2.870	4.752
13	21.07	35.28
14	50.21	84.96
15	63.42	110.4
16	32.72	55.89
17	52.07	87.51
18	56.83	94.96
19	14.06	23.39
20	6.524	9.454
21	17.41	28.90
22	3.628	5.448
23	4.375	7.940
	501.346	852.494

TABLE XXIX. (continued)

Region Number	Income Influence at Region 15 1950	1959
1	104.6	147.3
2	19.72	32.75
3	11.64	17.45
4	74.59	115.5
5	9.303	19.17
6	15.90	22.95
7	9.720	17.06
8	4.148	7.407
9	6.822	12.44
10	3.768	10.31
11	4.926	8.792
12	2.100	3.477
13	9.231	15.45
14	13.51	22.85
15	171.2	298.2
16	64.01	109.3
17	31.93	53.67
18	54.23	90.61
19	14.77	24.58
20	6.378	9.243
21	10.73	17.82
22	2.529	3.798
23	3.366	6.110
	649.121	1060.127

TABLE XXIX. (continued)

Region Number	Income Influence at Region 16	
	1950	1959
1	78.24	138.1
2	19.72	32.80
3	28.73	43.09
4	51.02	79.02
5	6.627	13.66
6	7.447	10.75
7	6.966	12.22
8	3.014	5.381
9	5.532	10.08
10	3.344	9.154
11	4.045	7.221
12	1.940	3.212
13	6.888	11.53
14	8.588	14.53
15	78.88	137.4
16	255.3	436.0
17	25.69	43.17
18	70.32	117.5
19	21.76	36.22
20	7.177	10.61
21	10.15	16.85
22	2.183	3.278
23	2.970	5.390
	706.531	1197.166

TABLE XXIX. (continued)

Region Number	Income Influence at Region 17	
	1950	1959
1	60.67	107.1
2	15.42	25.61
3	17.93	26.90
4	33.28	51.54
5	5.946	12.25
6	8.073	11.65
7	8.073	14.17
8	3.605	6.438
9	7.722	14.08
10	4.023	11.01
11	6.030	10.76
12	2.717	4.498
13	12.76	21.36
14	24.22	40.97
15	69.72	121.4
16	45.52	77.75
17	113.3	190.3
18	95.06	158.9
19	20.34	33.84
20	8.063	11.68
21	16.88	28.02
22	3.267	4.907
23	3.880	7.042
	586.499	992.175

TABLE XXIX. (continued)

Region Number	Income Influence at Region 18 1950	1959
1	55.22	97.47
2	14.57	24.22
3	17.12	25.77
4	26.84	41.57
5	4.756	9.803
6	5.288	7.633
7	6.058	10.63
8	2.704	4.828
9	5.635	10.27
10	3.384	9.264
11	4.497	8.026
12	2.219	3.674
13	8.07	13.51
14	9.919	16.78
15	44.44	77.39
16	46.77	79.88
17	35.68	59.96
18	410.2	685.5
19	60.51	100.7
20	12.42	17.99
21	16.88	28.02
22	2.859	4.293
23	3.319	6.024
	799.358	1343.205

TABLE XXIX. (continued)

Region Number	Income Influence at Region 19	
	1950	1959
1	54.17	95.62
2	14.57	24.20
3	17.12	25.67
4	25.00	38.69
5	4.401	9.072
6	4.588	6.623
7	5.011	8.794
8	2.408	4.301
9	4.950	9.02
10	3.127	8.562
11	3.934	7.023
12	1.983	3.283
13	6.687	11.19
14	7.615	12.88
15	37.56	65.41
16	44.92	76.72
17	23.69	39.81
18	187.8	313.8
19	108.6	180.8
20	12.42	17.99
21	15.47	25.69
22	2.553	3.835
23	3.030	5.499
	587.607	994.482

TABLE XXIX. (continued)

| Region | Income Influence at Region 20 | |
Number	1950	1959
1	39.04	68.91
2	10.84	18.00
3	9.312	13.97
4	15.39	23.83
5	3.205	6.607
6	3.021	4.361
7	3.910	6.860
8	1.852	3.307
9	3.740	6.817
10	2.603	7.128
11	2.953	5.271
12	2.463	4.077
13	4.594	7.693
14	4.708	7.96
15	21.61	37.64
16	19.73	33.71
17	12.51	21.03
18	51.33	85.78
19	16.54	27.53
20	56.02	81.19
21	25.32	42.04
22	3.430	5.152
23	2.459	4.464
	316.580	523.327

TABLE XXIX. (continued)

Region Number	Income Influence at Region 21 1950	1959
1	37.93	66.95
2	10.57	17.55
3	9.063	13.59
4	14.76	22.85
5	3.790	7.812
6	3.881	5.602
7	7.990	14.02
8	2.968	5.300
9	3.740	6.817
10	2.603	7.128
11	2.953	5.271
12	2.463	4.077
13	4.595	7.693
14	8.400	14.21
15	24.32	42.35
16	18.66	31.86
17	17.52	29.44
18	46.67	77.98
19	13.78	22.94
20	16.93	24.54
21	71.01	117.9
22	6.575	9.875
23	4.811	8.731
	335.982	564.486

TABLE XXIX. (continued)

Region Number	Income Influence at Region 22	
	1950	1959
1	36.98	65.28
2	10.92	16.70
3	9.797	14.69
4	13.88	21.50
5	3.465	7.142
6	3.295	4.755
7	5.609	9.842
8	2.563	4.576
9	7.300	13.31
10	4.244	11.62
11	7.659	13.67
12	7.635	12.64
13	9.855	16.50
14	6.179	10.46
15	20.22	35.22
16	14.17	24.20
17	11.97	20.11
18	27.90	46.62
19	8.029	13.36
20	8.098	11.73
21	23.21	38.53
22	25.99	39.04
23	8.137	14.77
	277.075	466.265

TABLE XXIX. (continued)

| Region | Income Influence at Region 23 | |
Number	1950	1959
1	33.74	59.55
2	9.246	15.36
3	8.567	12.85
4	11.73	18.17
5	3.295	6.791
6	2.805	4.049
7	5.521	9.687
8	2.943	5.255
9	7.813	14.24
10	6.926	18.96
11	8.016	14.31
12	10.18	16.85
13	6.419	10.75
14	4.129	6.986
15	14.92	25.98
16	10.68	18.24
17	7.88	13.23
18	17.95	29.99
19	5.279	8.79
20	3.217	4.661
21	9.409	15.62
22	4.509	6.771
23	51.36	93.22
	246.534	417.460

APPENDIX VI

ERROR DETERMINATION

TABLE XXX. Error in the Income Influence Factor,(In percentage)

% of error, $\dfrac{{}_iV_t - {}_iV'_t}{{}_iV_t}$ (a) (b)	% of error, $\dfrac{{}_iV_t - {}_iV''_t}{{}_iV''_t}$ (a) (c)	
where V = influence using Carrothers' data; V'= influence using Warntz's data; and t = 1950.	where V = influence using Carrothers' data; V''= influence using air mileage; and t = 1950 and 1959.	
Region Number 1950	1950	1959
1 22.7	5.3	3.1
2 25.3	10.4	6.9
3 17.1	18.8	15.4
4 19.0	19.4	15.0
5 46.7	21.9	15.4
6 46.1	31.4	25.7
7 35.2	32.6	24.9
8 34.0	24.7	27.1
9 22.3	18.4	15.9
10 29.0	16.8	5.2
11 12.0	13.3	0.0
12 28.5	14.2	8.2
13 23.2	19.7	3.3
14 13.1	13.8	5.0
15 16.8	13.3	3.4
16 19.9	13.0	7.4
17 15.5	6.8	6.8
18 29.3	8.3	.2
19 19.9	13.3	6.0
20 24.6	15.5	8.4
21 15.3	10.4	7.3
22 20.4	15.9	6.2
23 20.1	12.1	11.0
Arithmatic Average 24.2	16.0	9.5

(a)
   Total income influence data in Table XVII.
(b)
   Total income influence data in Table XX.
(c)
   Total income influence data in Table XVIII.

TABLE XXXI. Method of Determining Error in the Modified Relative Income Influence Factor.

Modified Relative Income Influence, $\dfrac{_iV_{t+\theta}}{\rho_\theta \; _iV_t} - 1$

Let $G = \dfrac{_iV_{t+\theta}}{\rho_\theta \; _iV_t}$. (1)

Then $\sigma_G^2 = \left[\dfrac{\partial G}{\partial \, _iV_t}\right]^2 \sigma_{_iV_t}^2 + \left[\dfrac{\partial G}{\partial \, _iV_{t+\theta}}\right]^2 \sigma_{_iV_{t+\theta}}^2$. (a) (2)

Assume $\sigma_{_iV_t}^2 = \sigma_{_iV_{t+\theta}}^2 = \sigma_{_iV_T}^2$.

This assumption is conservative because the error in $_iV_{t+\theta}$ being a predicted quantity will certainly be larger than the error in $_iV_t$. Therefore, this estimate of the error in the modified relative income influence will be an understatement.

(4)

Differentiating G: $\dfrac{\partial G}{\partial \, _iV_t} = -\dfrac{_iV_{t+\theta}}{\rho_\theta \; _iV_t}$; $\dfrac{\partial G}{\partial \, _iV_{t+\theta}} = \dfrac{1}{\rho_\theta \; _iV_t}$.

Substitute (4) in (2)

$\sigma_G^2 = \sigma_{_iV_T}^2 \left[\left(\dfrac{_iV_{t+\theta}}{\rho_\theta \; _iV_t^2}\right)^2 + \left(\dfrac{1}{\rho_\theta \; _iV_t}\right)^2\right]$ (5)

(a)
Yardley Beers, Introduction to the Theory of Errors, (Reading, Mass.: Addison-Wesley Publishing Company, Inc.), 1957, p. 28.

TABLE XXXI. (continued)

Squaring the two factors within the bracket in (5) and removing ρ_θ^2

$$\sigma_G^2 = \frac{\sigma_{iV_T}^2}{\rho_\theta^2} \left[\frac{{}_iV_{t+\theta}^2}{{}_iV_t^4} + \frac{1}{{}_iV_t^2} \right]. \tag{6}$$

Multiplying the second factor within the bracket by $\dfrac{{}_iV_{t+\theta}}{{}_iV_{t+\theta}}$ and removing $\dfrac{{}_iV_{t+\theta}^2}{{}_iV_t^2}$

$$\sigma_G^2 = \frac{\sigma_{iV_T}^2}{\rho_\theta^2} \cdot \frac{{}_iV_{t+\theta}^2}{{}_iV_t^2} \left[\frac{1}{{}_iV_t^2} + \frac{1}{{}_iV_{t+\theta}^2} \right]. \tag{7}$$

Since $\rho_\theta^2 \cong \left(\dfrac{{}_iV_{t+\theta}}{{}_iV_t} \right)^2$ [a], then ${}_iV_{t+\theta}^2 \cong \rho_\theta^2 \, {}_iV_t^2$. \hfill (8)

Substitute (8) in the second factor in the bracket in (7)

$$\sigma_G^2 = \frac{\sigma_{iV_T}^2}{\rho_\theta^2} \cdot \frac{{}_iV_{t+\theta}^2}{{}_iV_t^2} \left[\frac{1}{{}_iV_t^2} + \frac{1}{\rho_\theta^2 \, {}_iV_t^2} \right].$$

Remove ${}_iV_t^2$ from the bracket and combine with ρ_θ

$$\sigma_G^2 = \frac{\sigma_{iV_T}^2}{{}_iV_t^2} \cdot \frac{{}_iV_{t+\theta}^2}{\rho_\theta^2 \, {}_iV_t^2} \left[1 + \frac{1}{\rho_\theta^2} \right].$$

[a] Values of $\dfrac{{}_iV_{t+\theta}}{\rho_\theta \, {}_iV_t}$ in Table XXVI cluster around unity.

TABLE XXXI. (continued)

Since $\quad G^2 = \dfrac{{}_iV_{t+\theta}^2}{C_\theta^2 \; {}_iV_t^2}$ $\quad$ by (1) ,

$$\frac{\sigma_G^2}{G^2} = \left(\frac{\sigma_{{}_iV_T}^2}{{}_iV_t^2}\right)\left(1 + \frac{1}{C_\theta^2}\right) .$$

Taking the square root of both sides of the equation

$$\frac{\sigma_G}{G} = \frac{\sigma_{{}_iV_T}}{{}_iV_t}\sqrt{\frac{1}{C_\theta^2} + 1}$$

Therefore, the $\quad$ % of error of G $\;=\;$ % of error of ${}_iV_t \sqrt{\dfrac{1}{C_\theta^2} + 1}$.

APPENDIX VII

MAPS

MAP A. TWENTY-THREE REGIONS AND THEIR "CENTERS"

IN THE EASTERN HALF OF THE

UNITED STATES[a]

0 50 100 150 200 250
MILES

13

SYRACUSE
3

30   2   BOSTON

MILWAUKEE
19

14   DETROIT
16   29

12   28

DES MOINES
●20   CHICAGO
18   4
●NEW YORK
1 CITY
●PITTSBURGH
31

15   INDIANAPOLIS   COLUMBUS   27
17   15

16   LOUISVILLE   CHARLESTON   5
14   ●6   RICHMOND

18   26

21   13   19   7   25
JEFFERSON CITY   NASHVILLE   SALISBURY

11

LITTLE ROCK   8
22   COLUMBIA

20   9   ATLANTA   24
BIRMINGHAM   11   9   22
10   21

JACKSON
12

9

NEW ORLEANS   23
23

10
●TAMPA
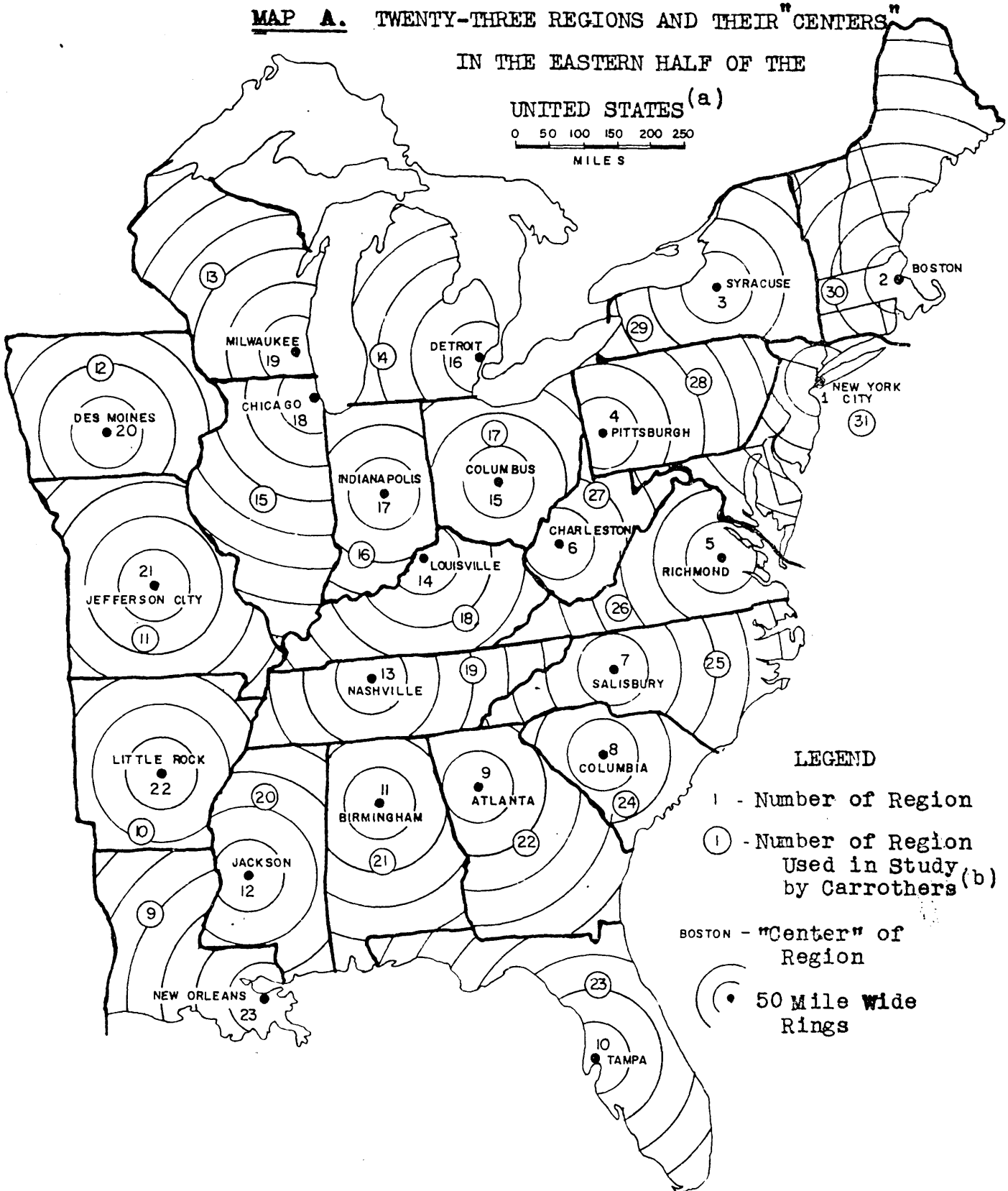
LEGEND

1 - Number of Region

① - Number of Region
Used in Study
by Carrothers[b]

BOSTON - "Center" of
Region

● 50 Mile Wide
Rings

―――――――――――――――――――――――――――――――

(a)
Region boundaries and "centers" in Carrothers' Thesis,
pp. 146 - 147; see Table I for definition of regions and "centers".
(b)
See Map D.

MAP B. SEVENTY-SEVEN REGIONS IN THE EASTERN HALF OF THE UNITED STATES(a)

LEGEND

1 - Number of Region

0 50 100 150 200 250

MILES
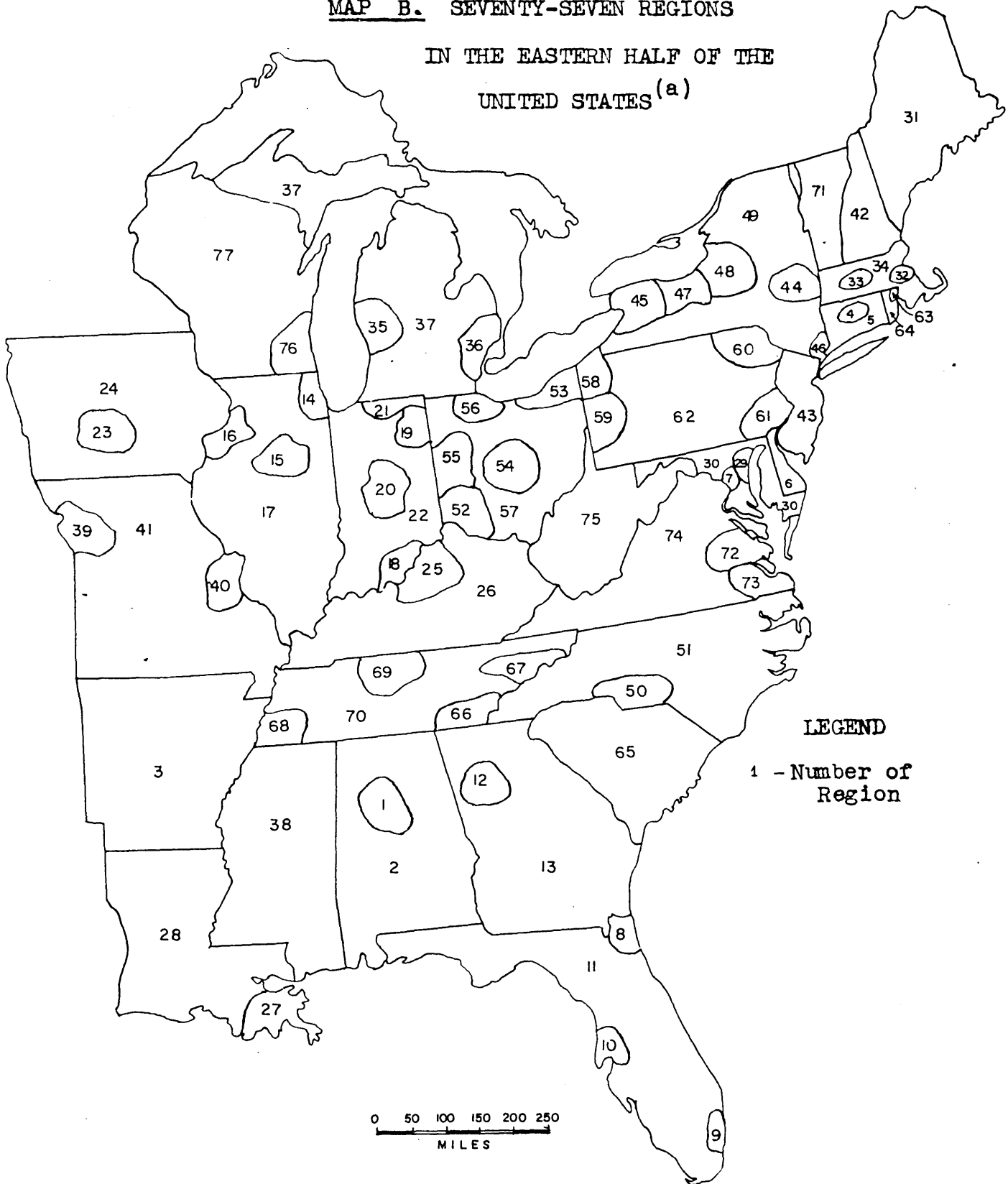
(a) Region boundaries obtained from personal communication with William Warntz; see Table II for definition of regions.

MAP C. TWENTY-THREE REGIONS[a] CONSISTING

OF COMBINATIONS OF THE

SEVENTY-SEVEN REGIONS[b]

0 50 100 150 200 250

MILES
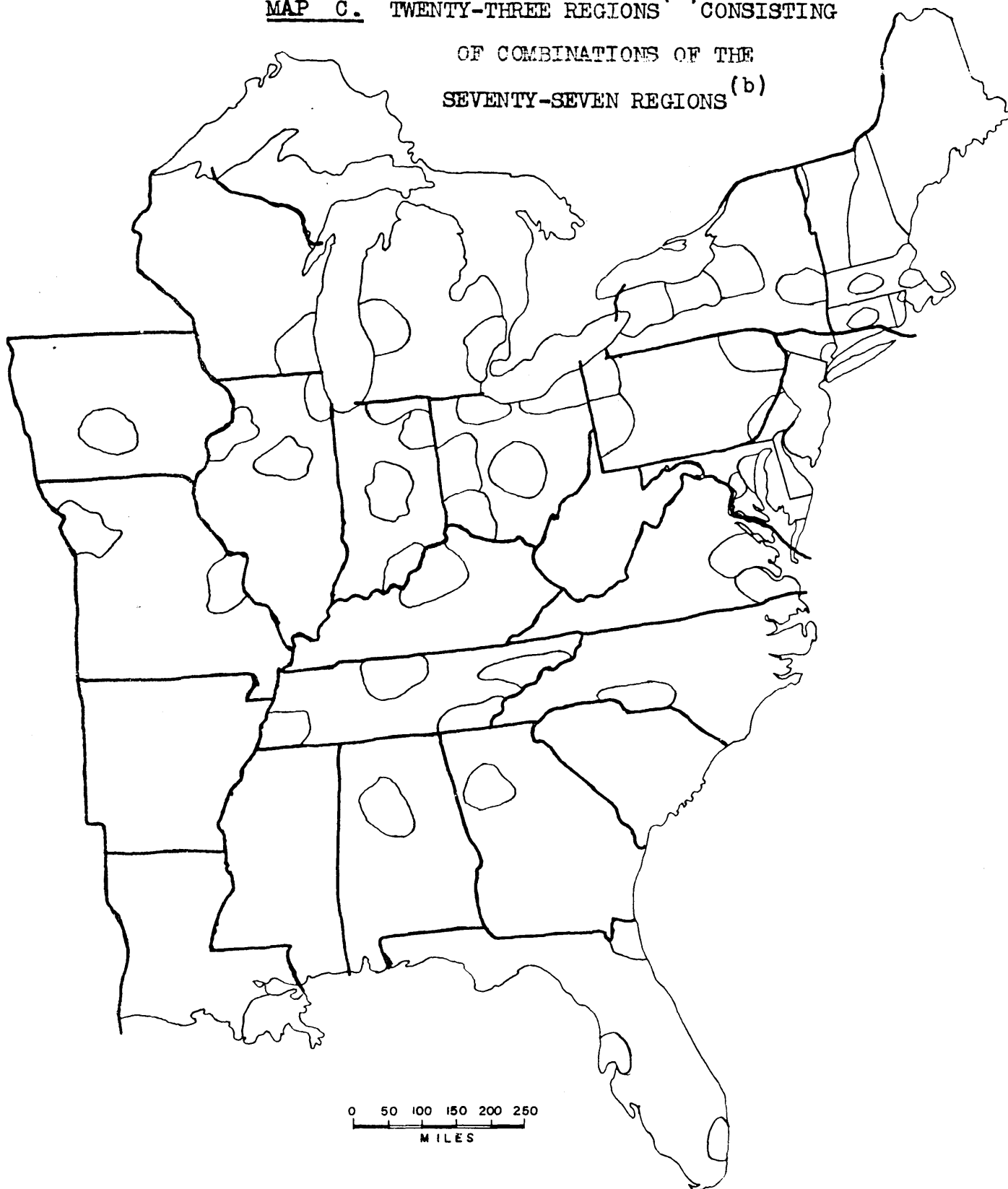
[a]
    See Map A for definition of regions.
[b]
    See Map B for definition of regions; see Table III for definition of combined regions.
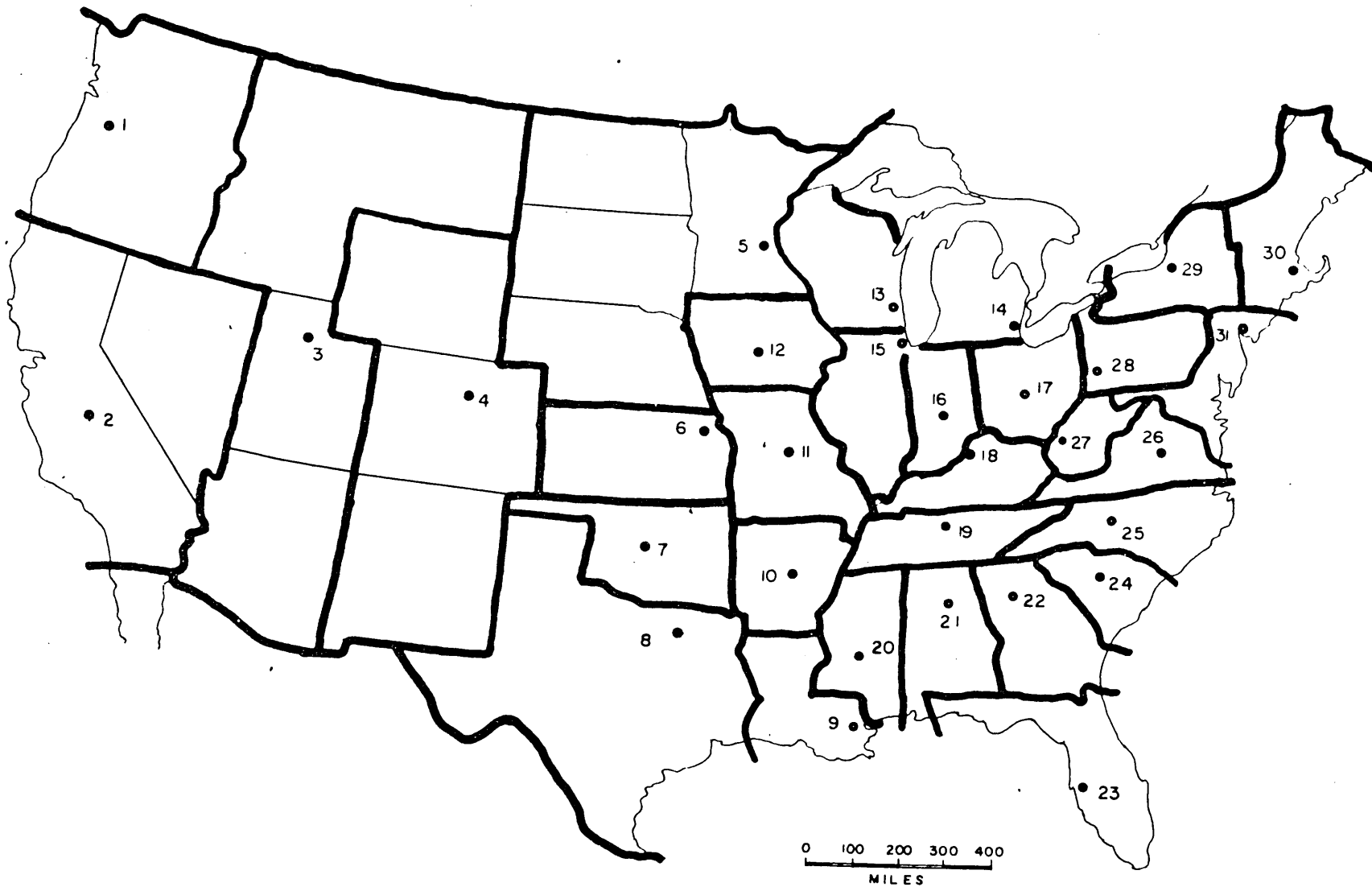
MAP D. THIRTY-ONE REGIONS IN THE UNITED STATES [a]

(a)
    Map prepared by Gerald A. P. Carrothers, Thesis, p. 147.

MAP E. EASTERN UNITED STATES[a]

1920 Total Income Influence[b]

Of United States

(In $ millions per mile)
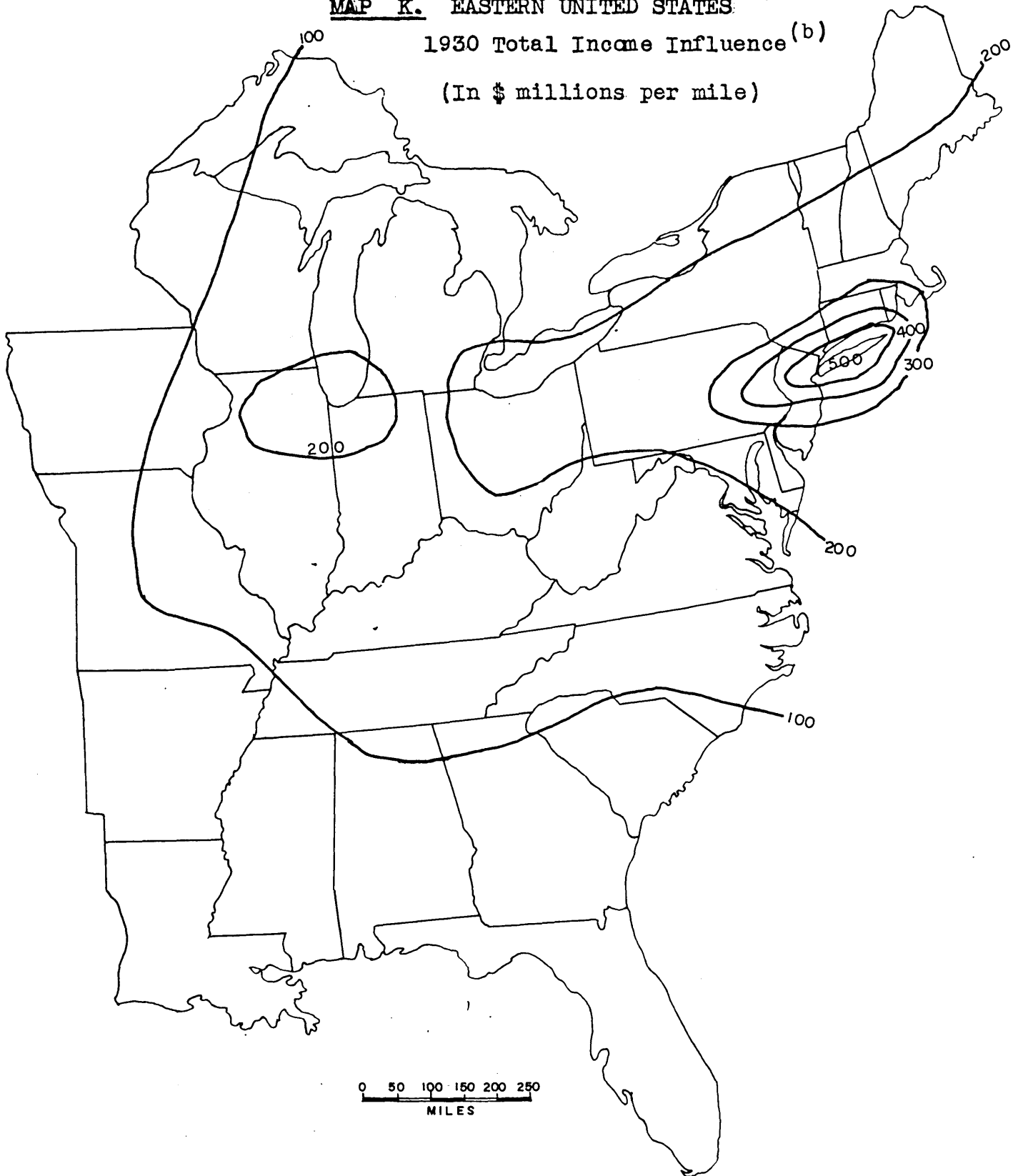

0 50 100 150 200 250

M I L E S
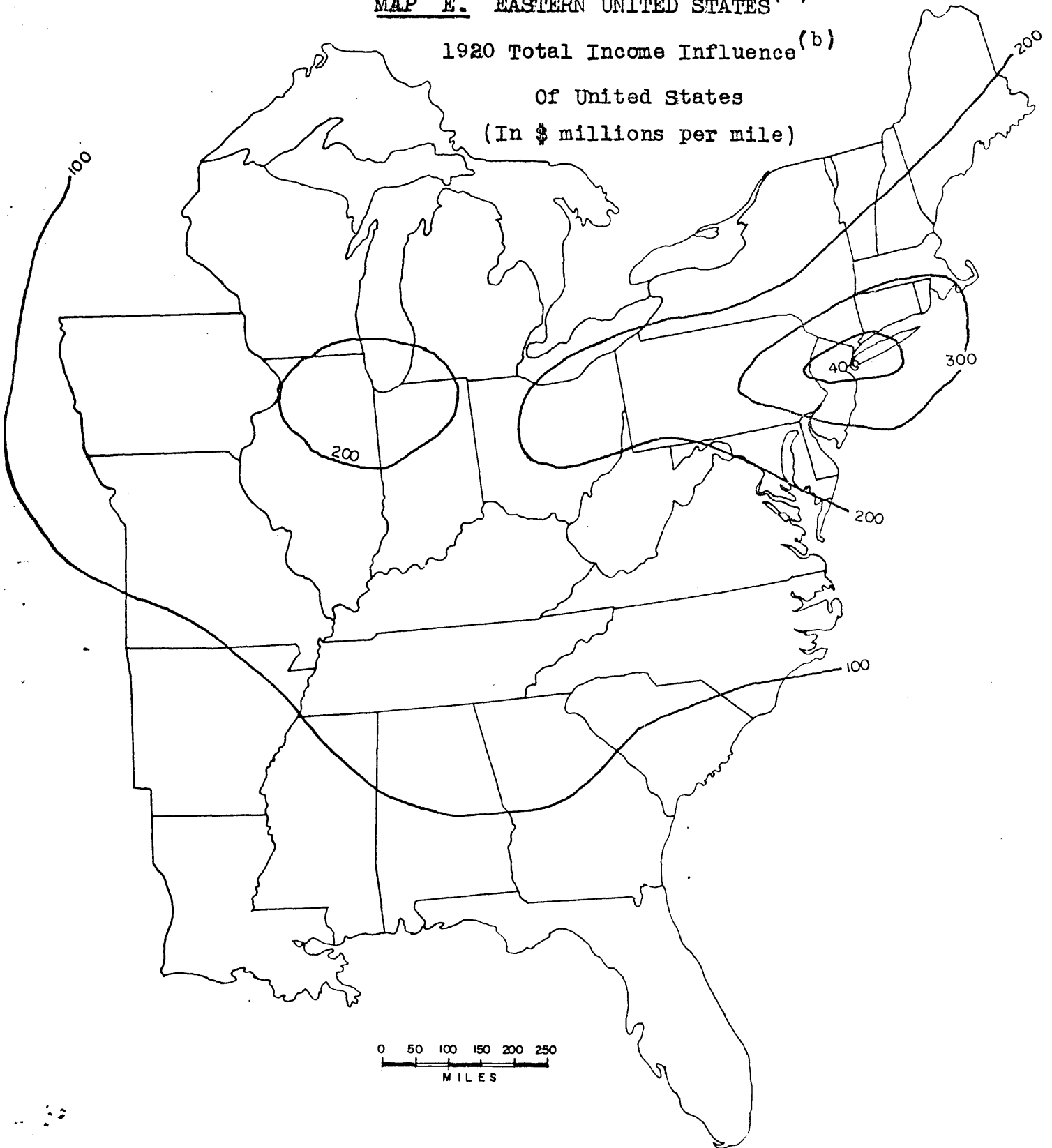
[a]
    See Table I and Map A for definition of regions.
[b]
    Total income influence data in Table XVII.

MAP F. EASTERN UNITED STATES[a]

1930 Total Income Influence[b]

Of United States

(In $ millions per mile)

0 50 100 150 200 250
MILES
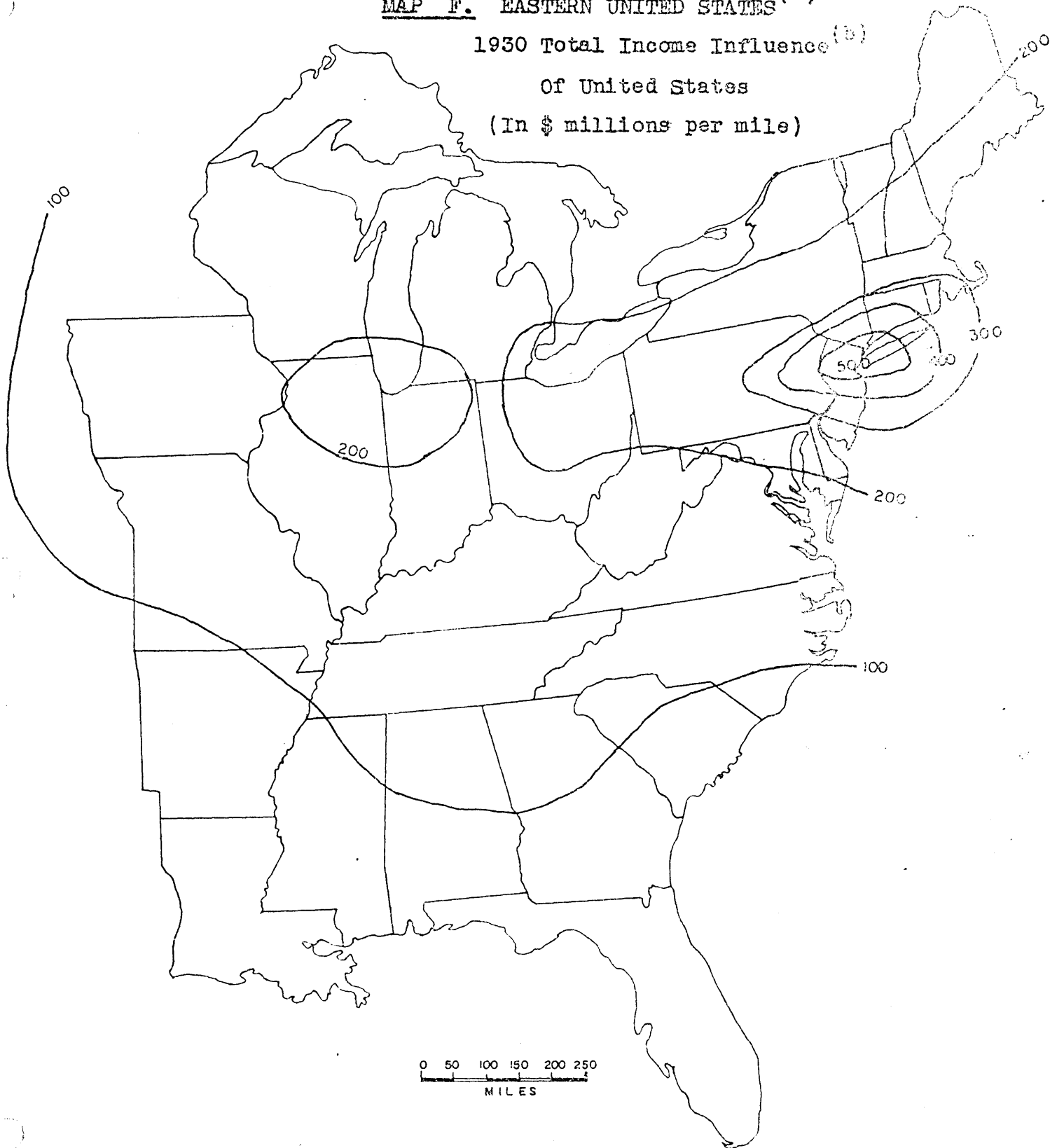
[a]
    See Table I and Map A for definition of regions.
[b]
    Total income influence data in Table XVII.

MAP G. EASTERN UNITED STATES [a]

1940 Total Income Influence [b]

Of United States

(In $ millions per mile)

————————————————————————

(a)
    See Table I and Map A for definition of regions.
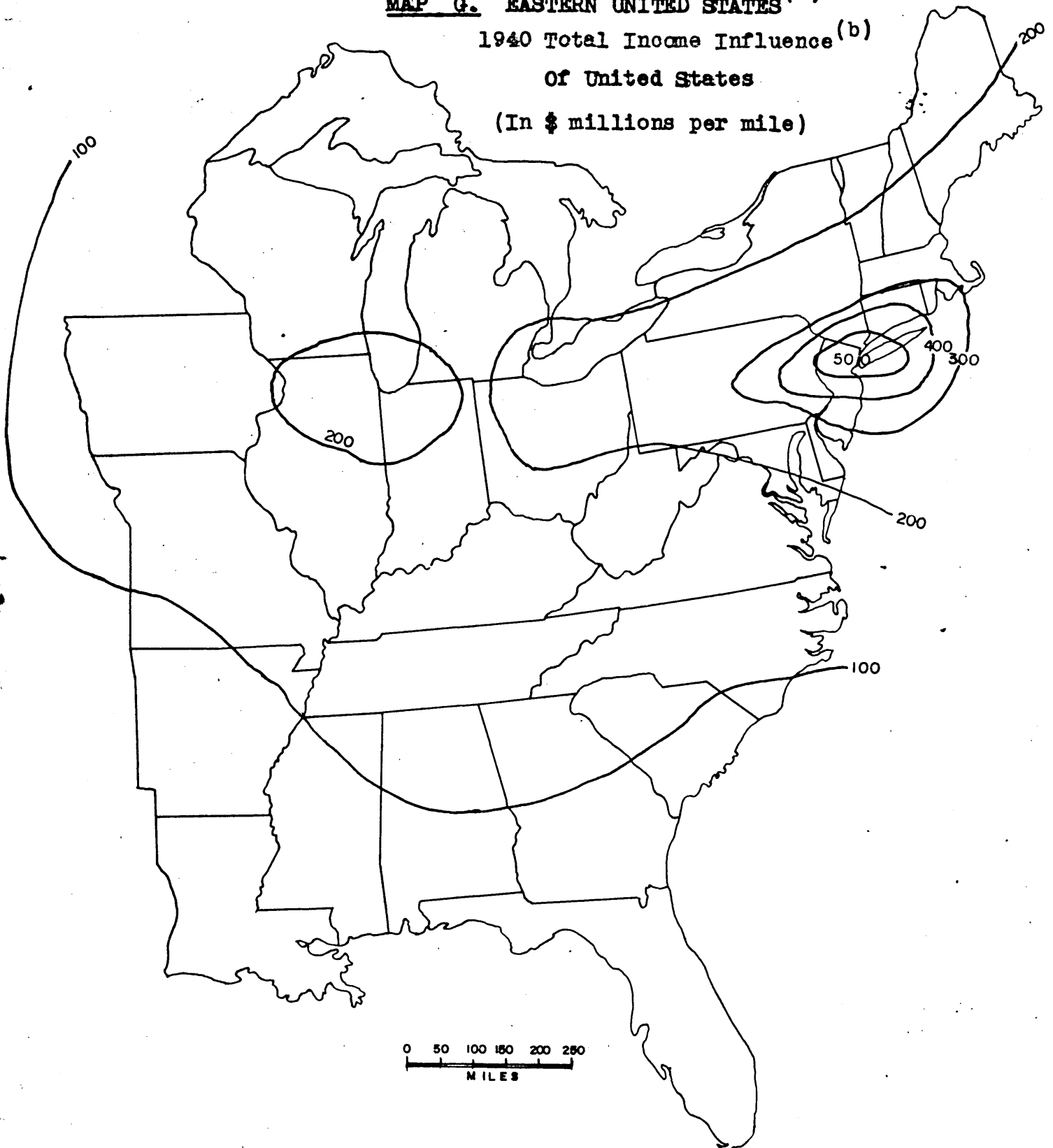
(b)
    Total income influence data in Table XVII.

MAP H. EASTERN UNITED STATES (a)

1950 Total Income Influence (b)

Of United States

(In $ millions per mile)
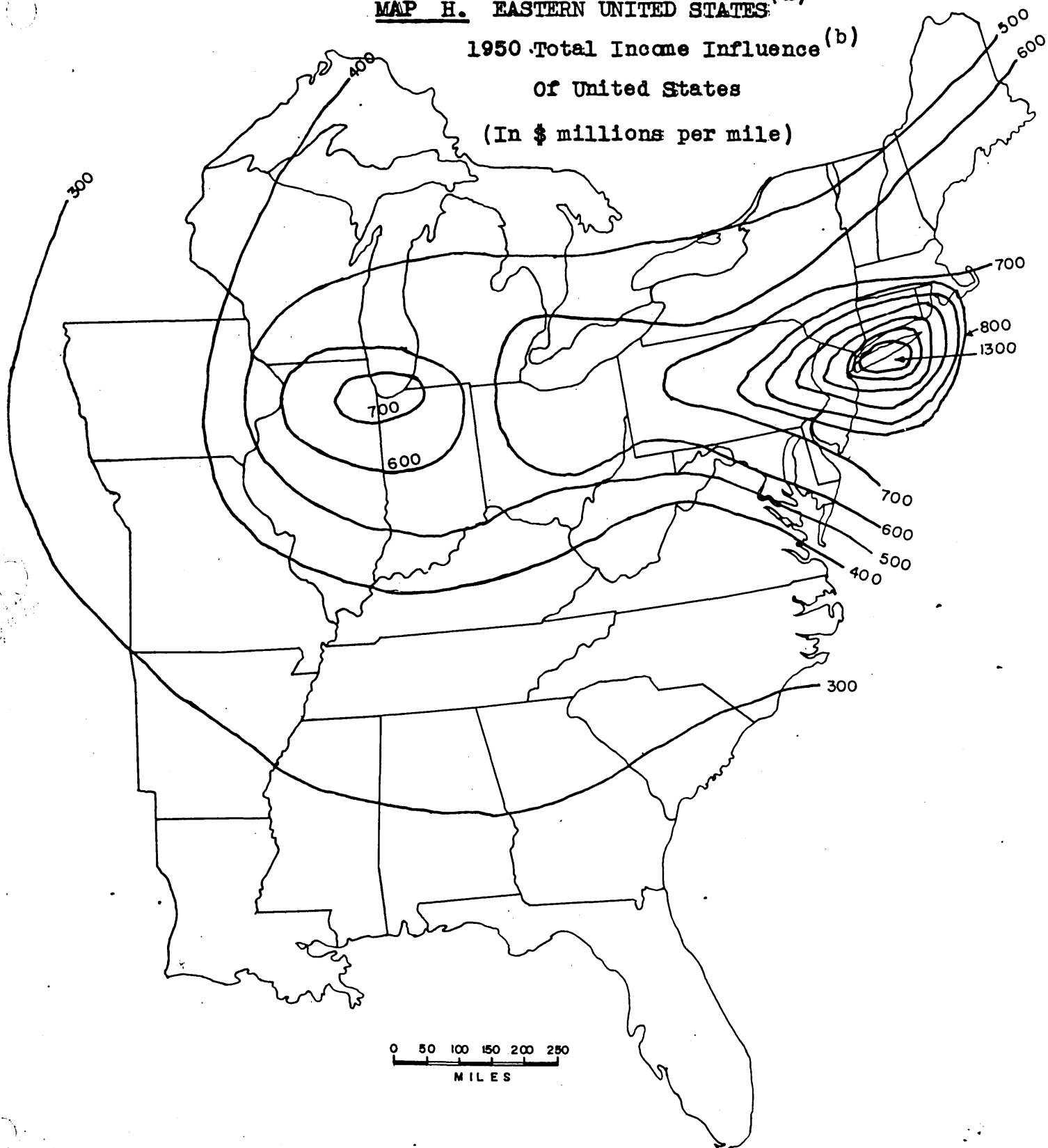
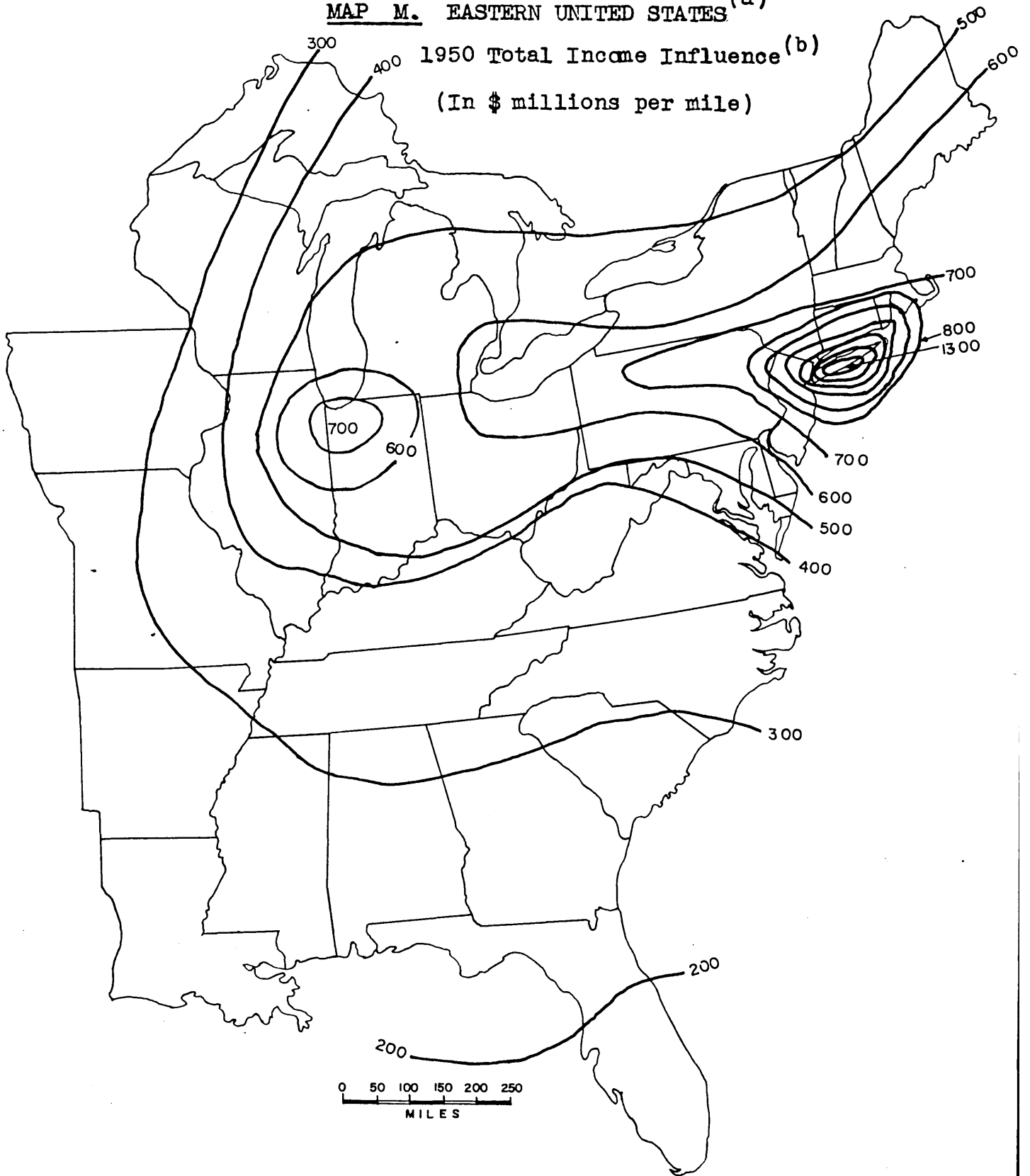

(a)
See Table I and Map A for definition of regions.
(b)
Total income influence data in Table XVII.

MAP I. EASTERN UNITED STATES(a)
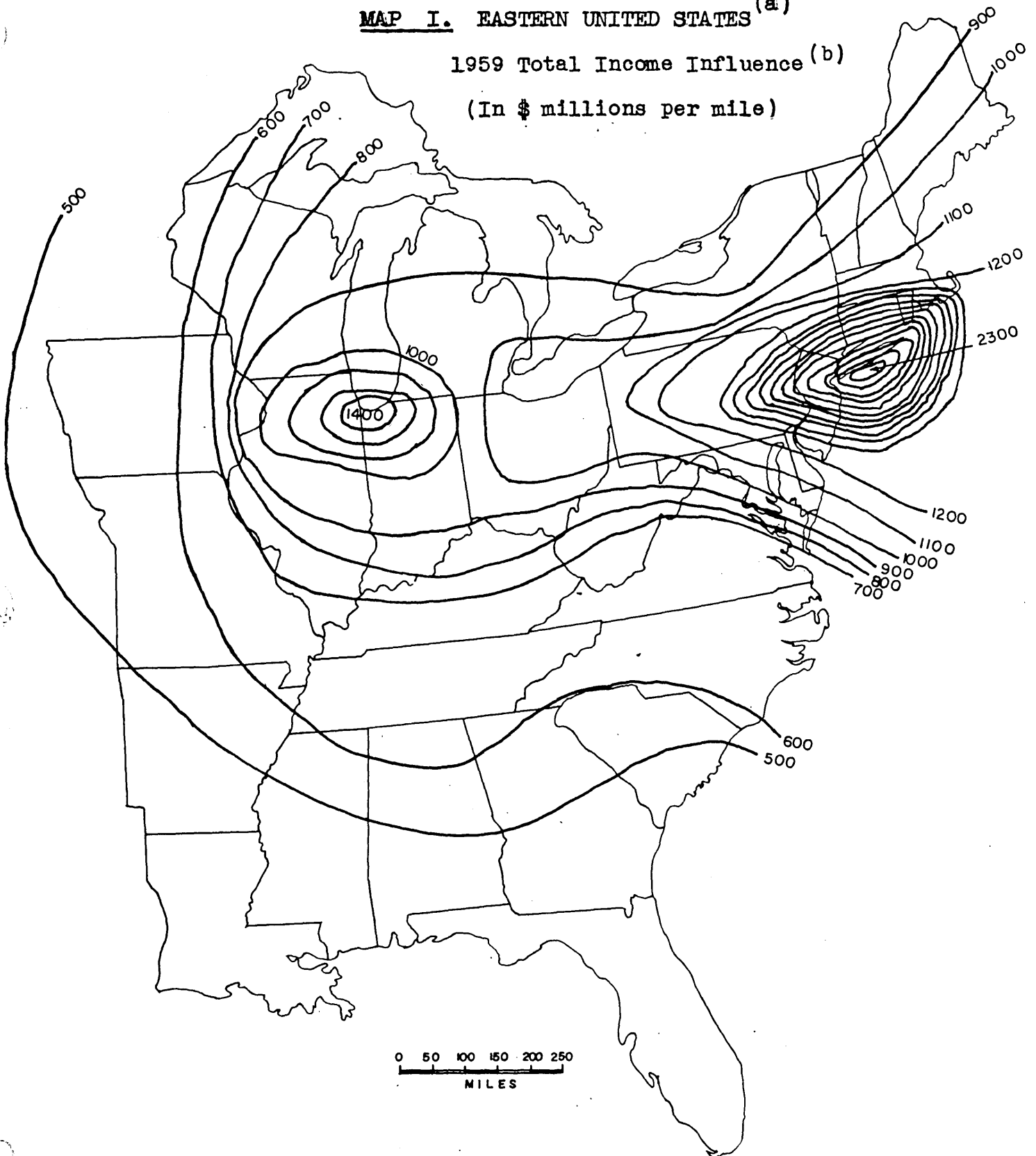
1959 Total Income Influence(b)

(In $ millions per mile)

0 50 100 150 200 250

MILES

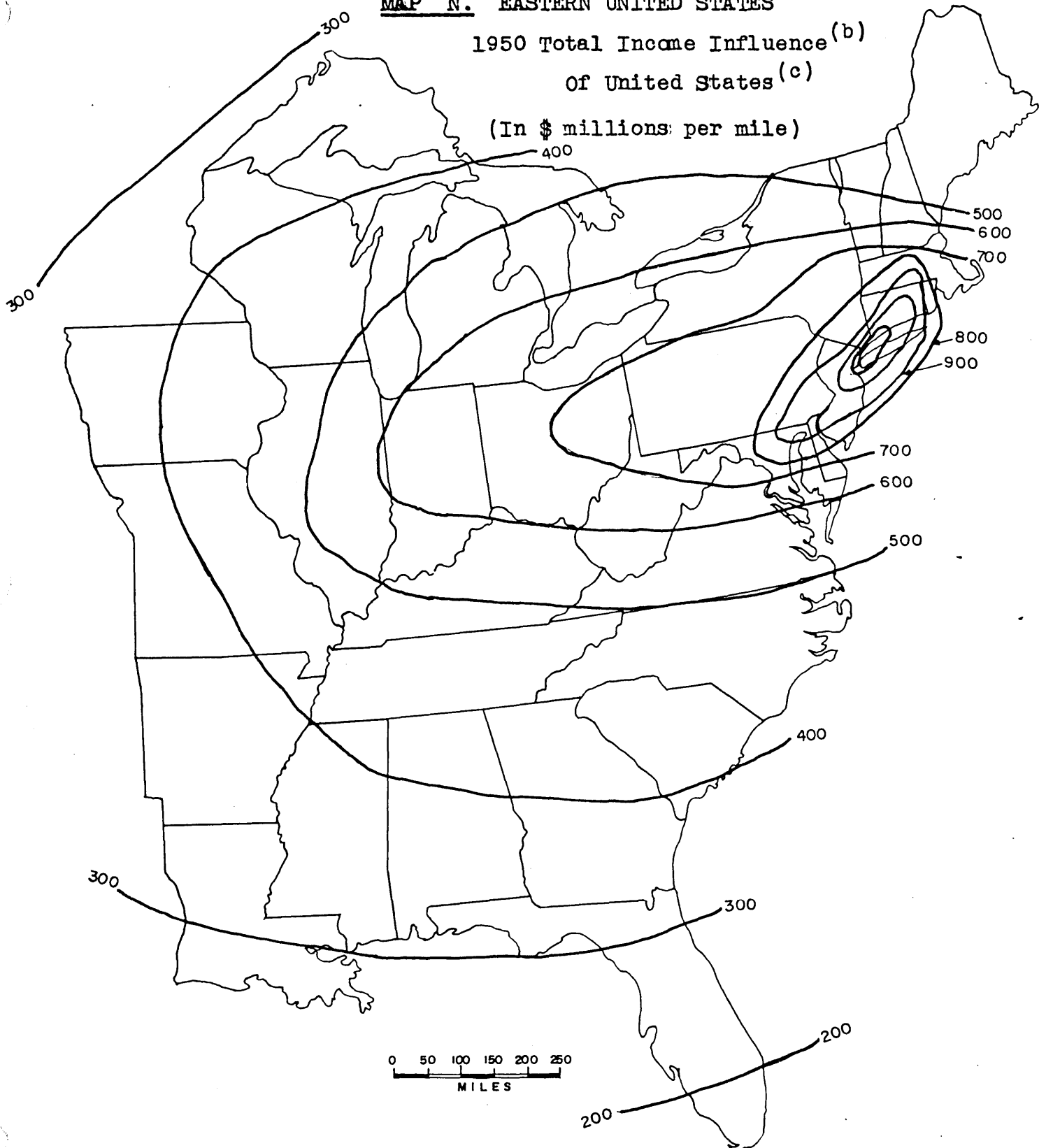
(a)
    See Table I and Map A for definition of regions.
(b)
    Total Income Influence data in Table XVII.

MAP   J.   EASTERN UNITED STATES[a]
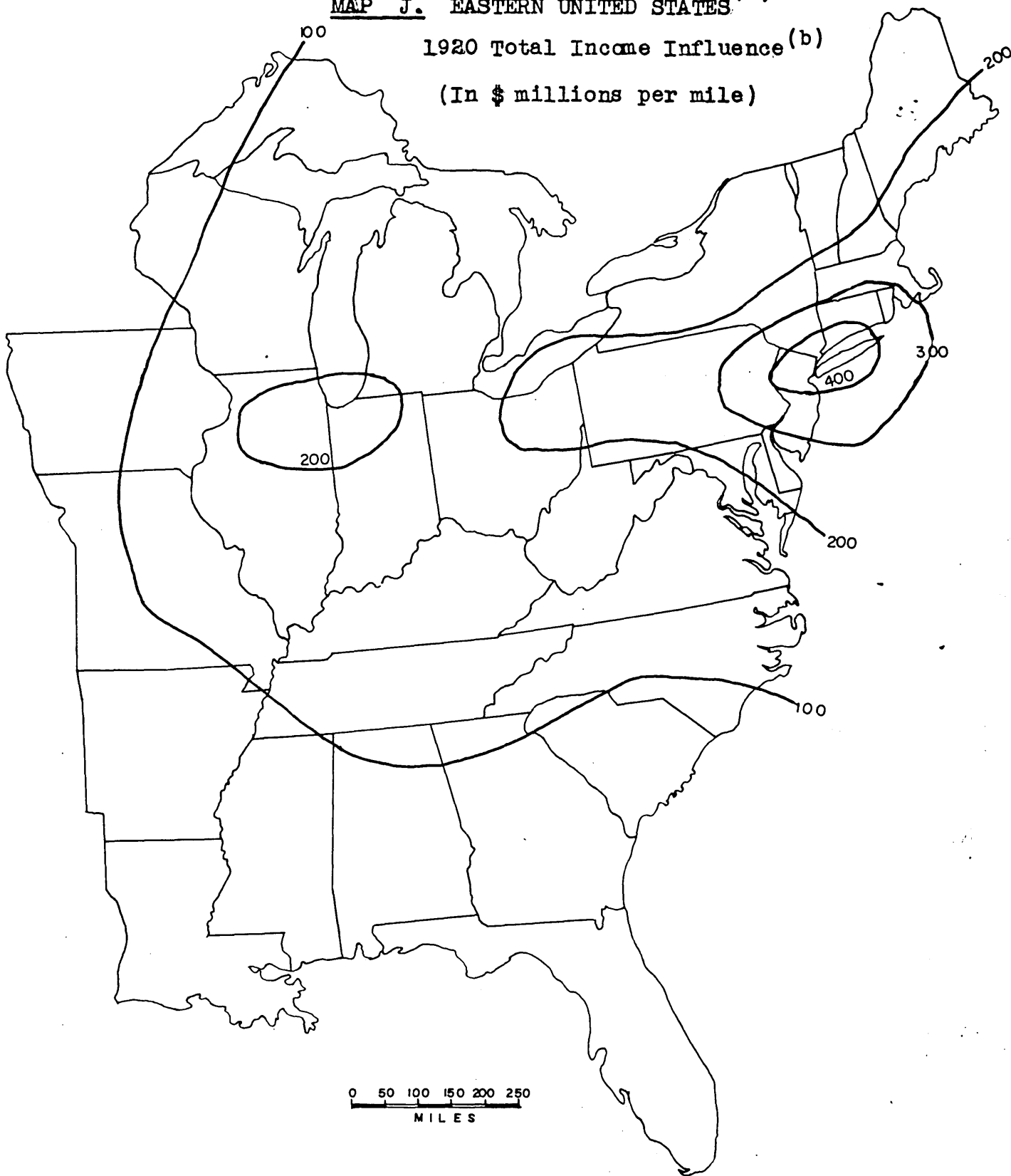
1920 Total Income Influence[b]

(In $ millions per mile)

0 50 100 150 200 250
MILES

[a]
See Table I and Map A for definition of regions.
[b]
Total income influence data in Table XVI.

MAP K.   EASTERN UNITED STATES(a)

1930 Total Income Influence(b)

(In $ millions per mile)

(a)
    See Table I and Map A for definition of regions.
(b)
    Total income influence data in Table XVI.

MAP L. EASTERN UNITED STATES [a]
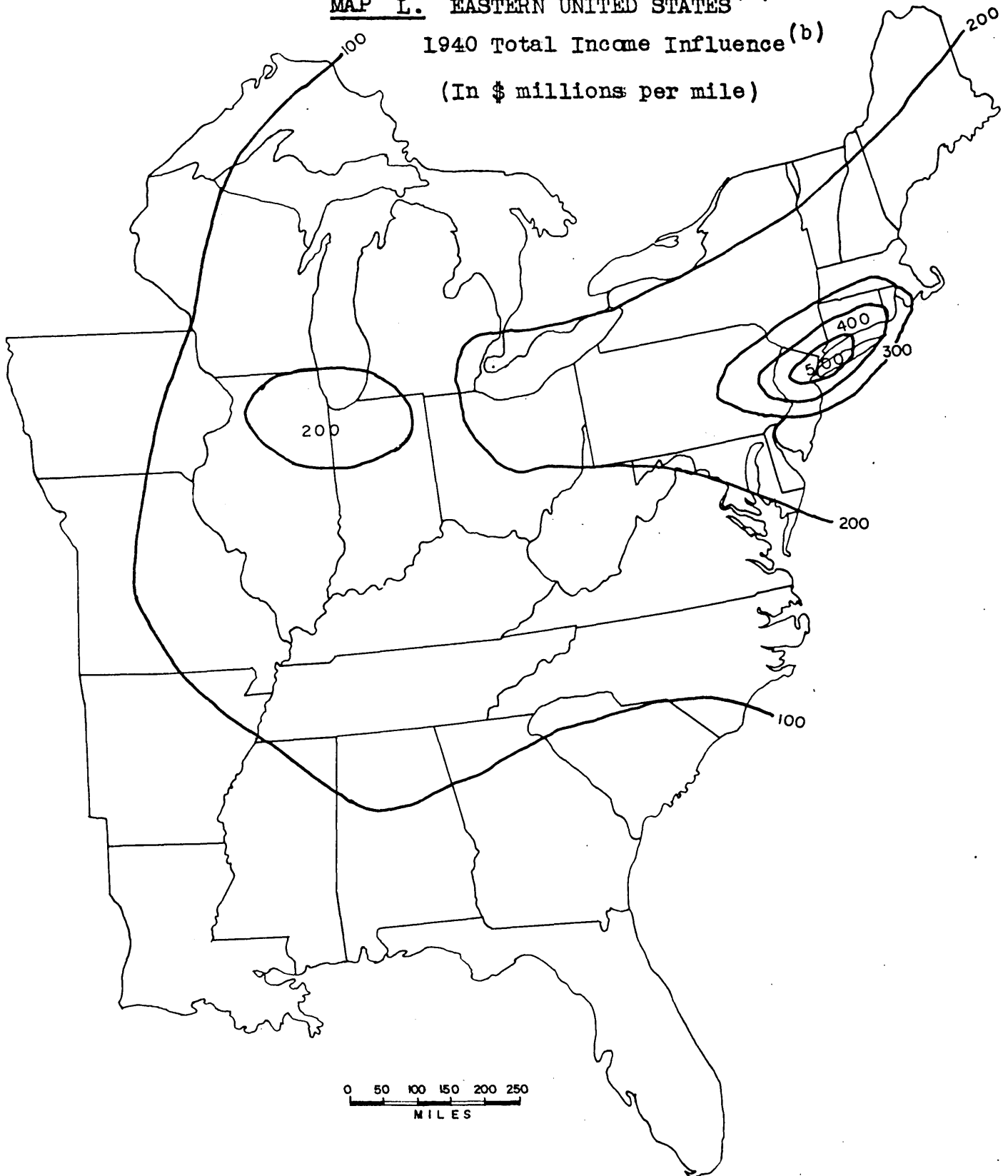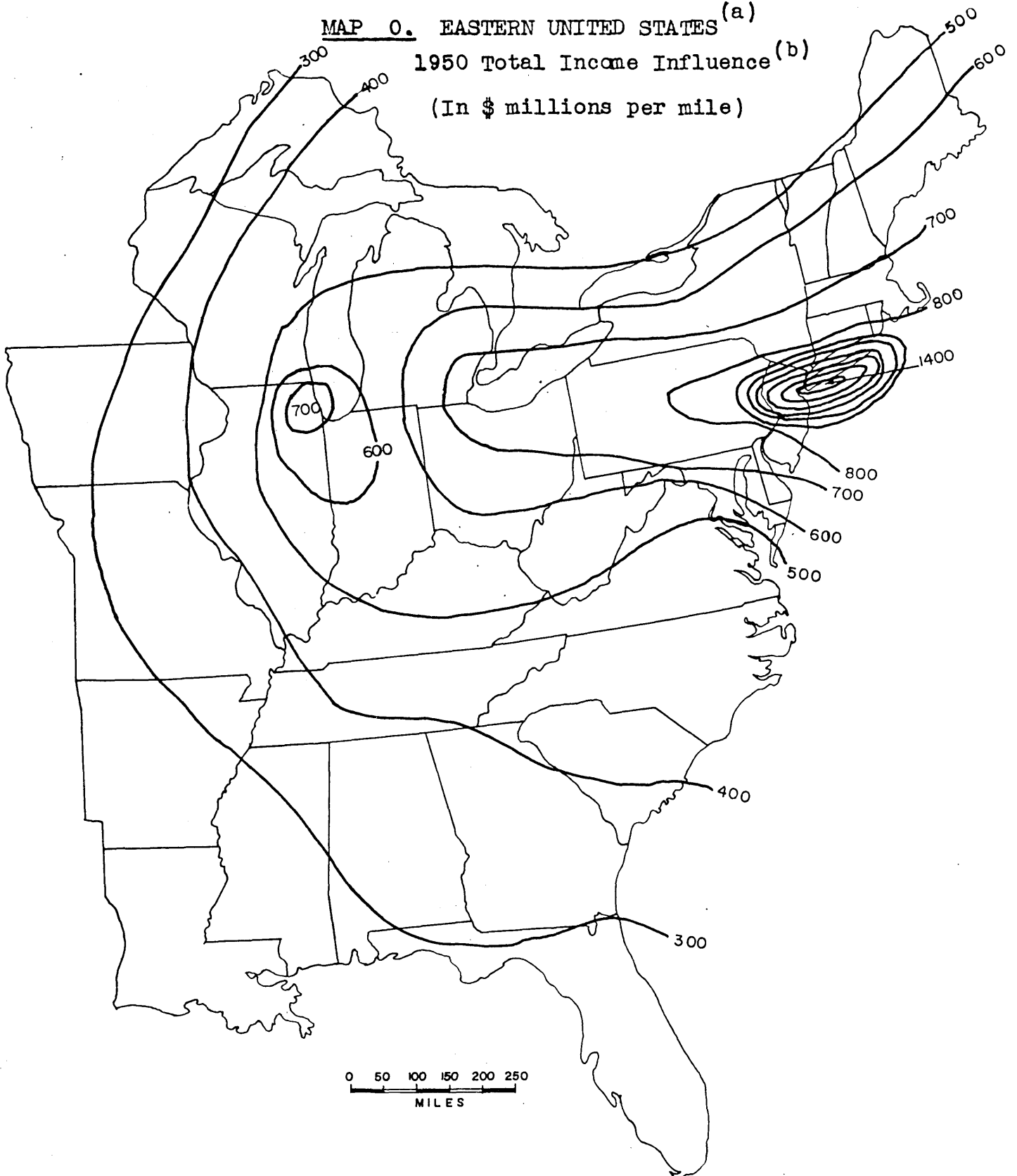
1940 Total Income Influence [b]

(In $ millions per mile)

0 50 100 150 200 250
MILES

---

[a]
    See Table I and Map A for definition of regions.
[b]
    Total income influence data in Table XVI.

MAP M. EASTERN UNITED STATES[a]

1950 Total Income Influence[b]

(In $ millions per mile)

300 400 500 600

700 800 1300

700 600 500 400

300

200

200

0 50 100 150 200 250
MILES

[a]
    See Table I and Map A for definition of regions.
[b]
    Total income influence data in Table XVI.

MAP N.   EASTERN UNITED STATES (a)

1950 Total Income Influence (b)

Of United States (c)

(In $ millions per mile)

0 50 100 150 200 250

MILES

(a) See Table II and Map B for definition of regions.

(b) Total income influence data in Table XIX.

(c) Map prepared by William Warntz, Geographical Review, 1958.

MAP O. EASTERN UNITED STATES (a)

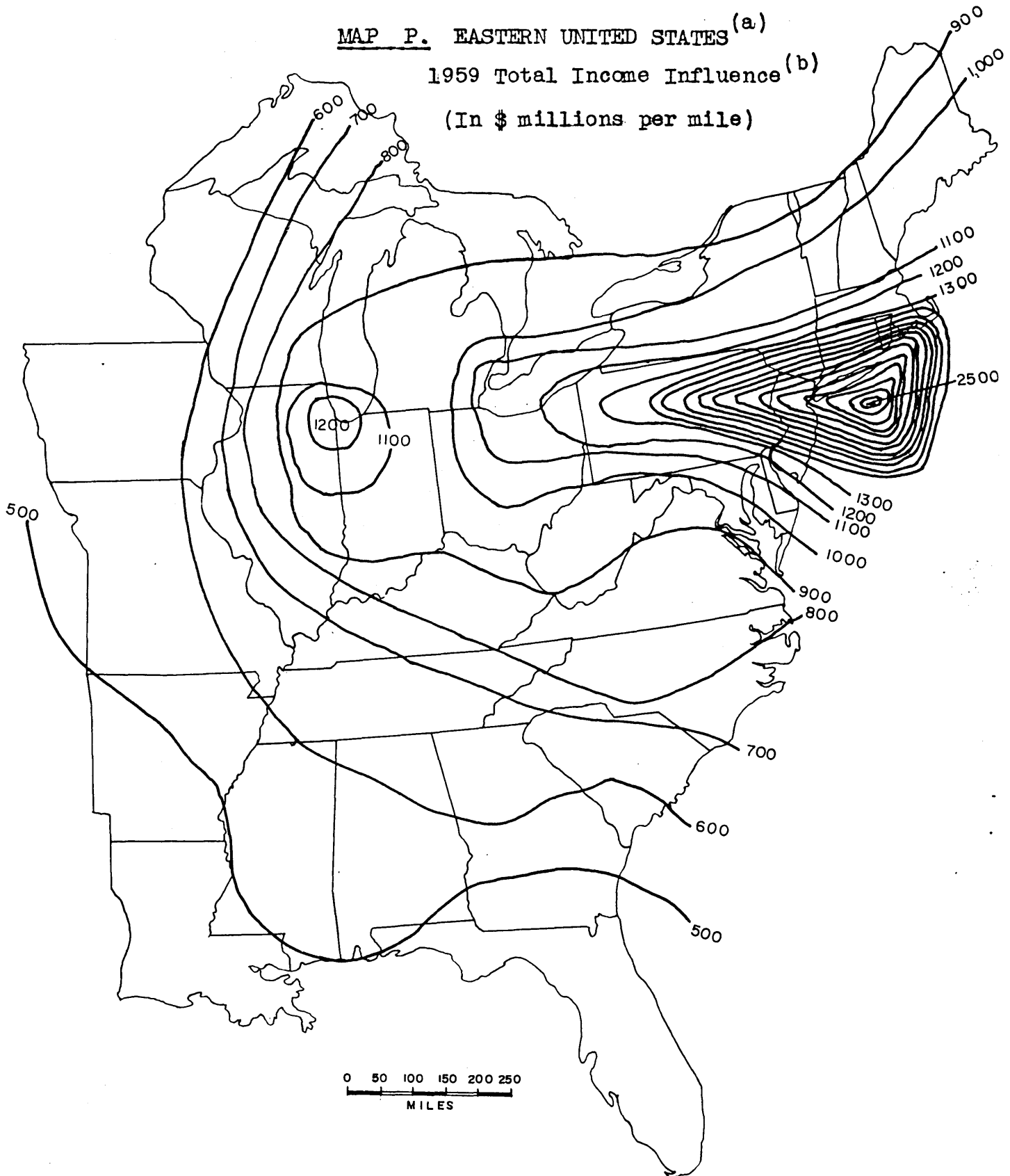1950 Total Income Influence (b)

(In $ millions per mile)

_____

(a)
    See Table I and Map A for definition of regions.
(b)
    Total income influence data in Table XVIII.

MAP P.   EASTERN UNITED STATES[a]

1959 Total Income Influence[b]

(In $ millions per mile)
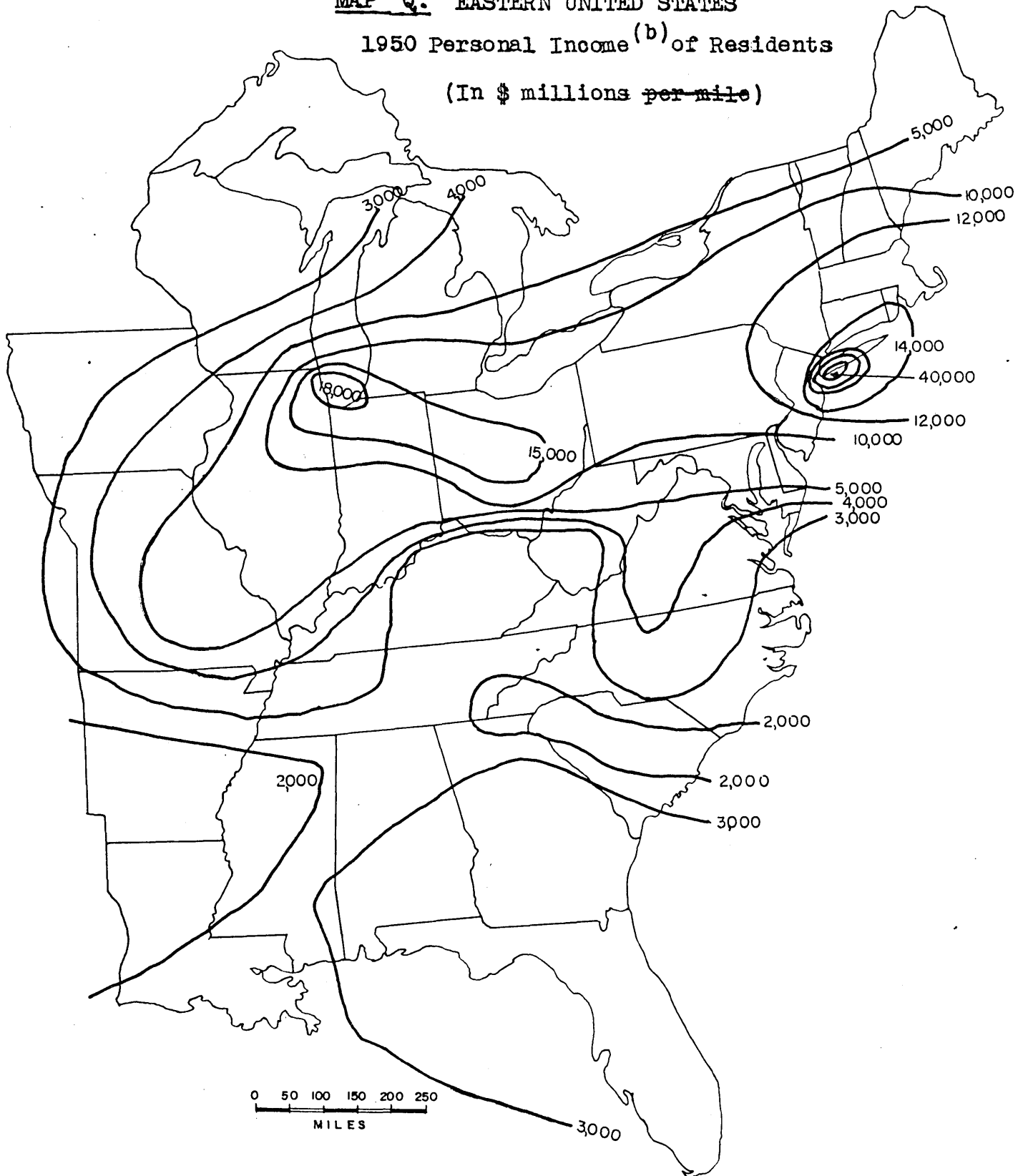


```
0   50  100  150  200 250
|___|___|___|___|___|
         MILES
```

(a)
    See Table I and Map A for definition of regions.
(b)
    Total income influence data in Table XVIII.

MAP Q. EASTERN UNITED STATES [a]

1950 Personal Income [b] of Residents

(In $ millions per mile)

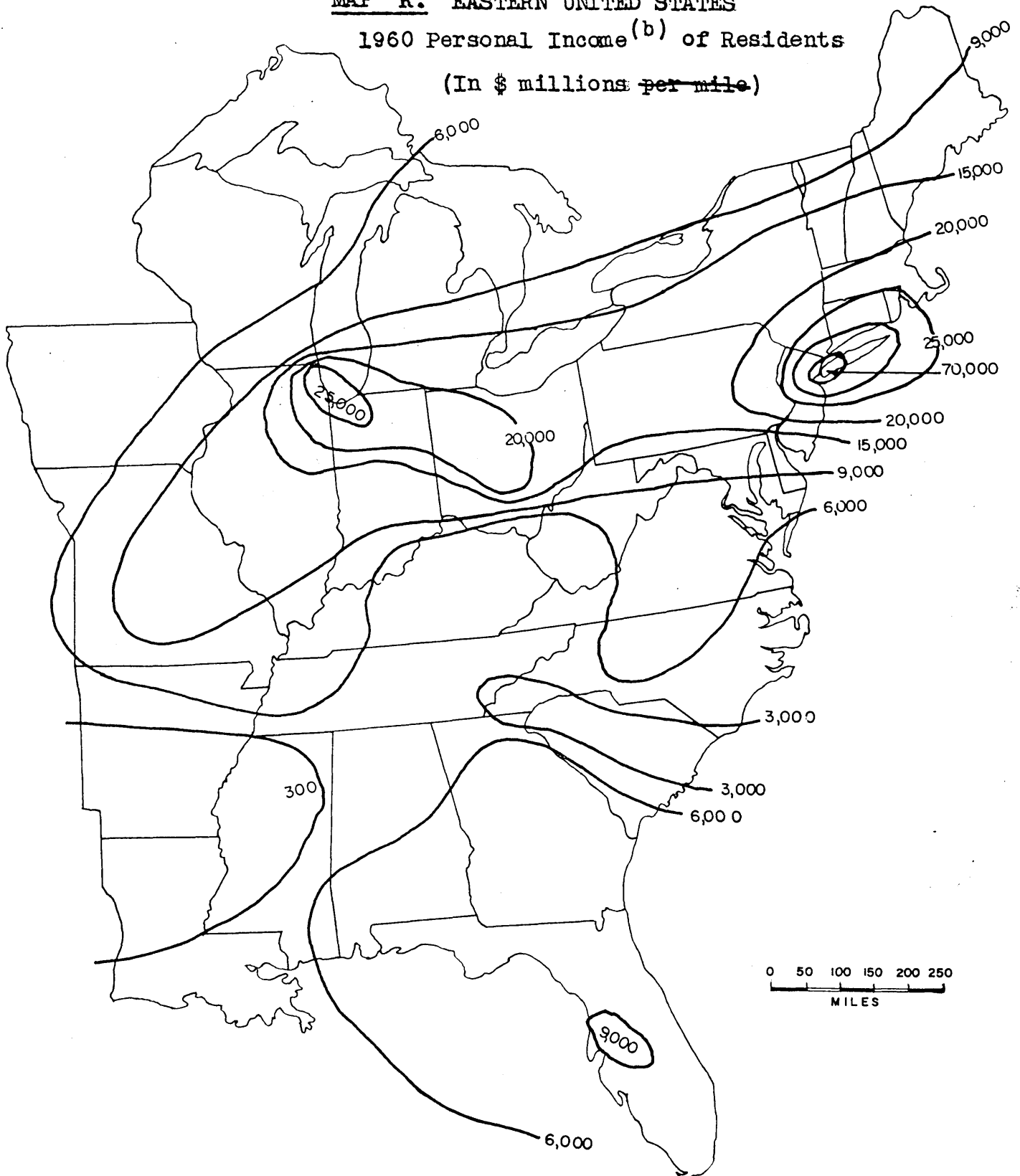0 50 100 150 200 250
MILES

(a)
    See Table I and Map A for definition of regions.
(h)
    Income data in Table V.

MAP R. EASTERN UNITED STATES (a)

1960 Personal Income (b) of Residents

(In $ millions ~~per mile~~)

9,000

6,000

15,000

20,000

25,000

70,000

20,000

15,000

9,000

6,000

25,000

20,000

3,000

300

3,000

6,000

9,000

6,000

0   50   100 150 200 250
MILES

(a)
 See Table I and **Map A** for definition of regions.
(b)
 Income Data in Table VI.

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