INVESTIGATION OF A HYDRAULIC-BOOST
AILERON CONTROL SYSTEM

by

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CONFIDENTIAL
Prof. Joseph S. Newell
Secretary of the Faculty
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Dear Professor Newell:

In accordance with the regulations of the faculty, we hereby submit a thesis entitled "Investigation of a Hydraulic-Boost Aileron Control System", in partial fulfillment of the requirements for the degree of Master of Science.
ABSTRACT

An investigation of a power-boosted aileron control system was conducted to determine its static and dynamic characteristics. The control system is that installed in the U.S. Air Force medium bomber, the B-45A. The actual system used for the study was a laboratory replica of that used in the airplane. Actual components were assembled together with a representative inertia and damping consisting of a steel beam loaded with springs and containing a viscous damper.

The laboratory set-up included only the control system for one aileron.

A mathematical derivation of the system performance function is given for two cases:

1. when the affects of fluid compressibility are considered

2. when compressibility is neglected.

Experimental transient responses of the control system were obtained by applying to the system a step input function of torque. By using these experimental transient responses, the frequency response of the system is derived for various loading conditions. It is found that the performance function of the system is not unique but varies as the operating conditions of the system vary.

Computation tables are provided for evaluating numerical coefficients used in the theoretical derivations.

An investigation to determine the cause of reported oscillations in the actual system installed in the airplane was made. The results of the investigation revealed that the oscillation could be caused by
two factors:

(1) hydraulic oscillations could occur within the control valve due to fluid flow around sharp edges
(2) oscillations could be induced in the system through the coupling action between the mass and the elasticity of the input system, and through the coupling action between the valve and output inertia and the elasticity of the aileron beam.

Suggestions are given for the redesign of the valve and the linkage system to overcome these oscillations.

It is recommended that further study be performed on the actual installation in the airplane. Then, the equipment as it actually exists with its normal parameters of mass and cable flexibility would be more accurately tested under actual conditions. This would do away with approximating the aerodynamic loads, and would also permit the effects of structural deformation of the airplane while in flight to be taken into account.
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OBJECT

The object of this investigation was to perform an analytical and an experimental study on the hydraulic power-boosted aileron control system of the U.S. Air Force medium bomber B-45A, in order to obtain its static and dynamic characteristics.

It was also desired to investigate reported oscillations of the system in order to determine their cause and means of eliminating them.

An auxiliary aim was to evaluate an analog method of investigation.
CHAPTER I

INTRODUCTION

Aircraft have consistently increased in size since the day the first one took to the air. As the trend toward building larger planes continued, it was possible for the pilot to actuate the control surfaces directly through the cockpit controls and connecting cables and linkages. Auxiliary aids such as Flettner tabs on the control surfaces increased the size of aircraft which could be flown manually without requiring excessive pilot forces. However, finally aircraft became so large that it was necessary to supplement the pilot's effort by power boost systems.

The requirements of a satisfactory boost system include stability, smooth operation, and fast response of the control surface to the pilot's demand. These characteristics must be acceptable in operation over a wide temperature range, at reduced atmospheric pressures, large loading conditions, and the elastic deflections of the aircraft structure.

Closed-loop systems are often used in boost controls in order that their inherent advantages may be obtained. In these systems a device is present which compares the system response with the command input. This device, called the comparator, measures the difference between the response and the input. This difference, called the error, is utilized to control the output from a power source which in turn drives the output member.

The dynamic factors involved in the boost system sometimes augment the dynamics of the loading on the control surface and the aileron inertia,
and sometimes result in a system which is not stable.

The subject of this study is the aileron hydraulic boost system of a heavy high speed aircraft. Instability conditions exist occasionally and do not appear to be correlated to any one particular flight attitude, altitude or airspeed. This thesis is a part of a continuing study of the system to investigate the dynamic behavior of the system and determine those parameters which contribute to instability.

The control system is of the reversible type. That is, for any loading condition imposed on the aileron, a portion of this load is fed back and is "felt" by the pilot. Briefly, the system consists of the pilot's wheel and cables leading to a mechanical linkage system which applies a force on a hydraulic valve and also imparts a small percentage of the total aileron hinge moment directly to the aileron. The force on the valve establishes a pressure difference across a cylinder piston combination which contributes the major portion of the total hinge moment.

Since the airplane was not available for test, experimental tests were made using the mechanical linkage system, and the hydraulic system which is incorporated in the aircraft. A balanced beam, with a damper and restrained by springs, was used to simulate the aileron inertia and the aerodynamic loading and damping.

A mathematical analysis of the system was conducted, and from this analysis it was proposed to set up an electronic analog. With the analog it would have been possible to investigate the effect of variation of various system parameters with relative ease. Time did not permit the analog phase of the study; however, the various system constants for
setting on the analog have been determined and a description of the analog components are included in this report.

With the type of linkage system used, several non-linear parameters are involved. Non-linearities are also introduced in the variable orifice valve and the cylinder piston combination. The overall system has been tested and analysed on the assumption of a linear system. This assumption has been justified to a degree by limiting the input disturbances to small values from various initial conditions of deflection. As long as small disturbances occur, the variation of the non-linear factors is not great and average values can be assumed.
CHAPTER II

DESCRIPTION AND OPERATION OF THE AILERON CONTROL SYSTEM

A. General Description

The aileron control system consists of two similar mechanical linkage systems and hydraulic boost units. One set being attached to each aileron and the two units being interconnected through cables. These cables are attached to the pilot's control wheel and the automatic pilot drive motor so that either manual or automatic control may be obtained.

A schematic diagram of the system for one aileron is shown in Fig. II-1. The principal components and their functions are:

(1) The pilot's control wheel or the automatic pilot drive motor and the connecting cables to the aileron sector provide the means for transmitting the control forces to the sector.

(2) The sector linkage system provides a twofold purpose; (a) establishes a hinge moment directly on the aileron, and (b) establishes a force on the hydraulic system valve. Since the linkage system provides a method by which the pilot applies a moment directly to the aileron hinge, (although only a fraction of the moment applied through the boost cylinder) that important characteristic of pilot's "feel" is maintained. Also in the event of hydraulic failure of one or both hydraulic systems, some control is maintained.
FIG. II-1. SCHEMATIC DIAGRAM OF AILERON CONTROL SYSTEM.
(3) The hydraulic system is controlled by the force established in a connecting rod from the linkage system to the valve mechanism. The pressures established within the valve are transmitted to a boost cylinder which is also attached to the aileron.

As the boost cylinder deflects the aileron, the movement is transferred back through the linkage system and varies the valve position until an equilibrium position is reached where the aileron hinge moment due to the applied pilot force plus the moment established by the boost cylinder balance the moments due to the aerodynamic and aileron inertia forces.

B. Sector Linkage System

1. Description

The sector and related linkage is shown in Fig. II-2. The assembly is supported by a bracket attached to the rear wing spar. Cables from the control wheel are secured to a sector which is free to rotate about a shaft supported by the bracket assembly. Rigidly attached to the under side of the sector P is a short arm S. Pivoted about the other end of this shaft is a floating Link DO₁. To one end of this link is attached a rod which actuates the valve. The other end of the floating link is attached to the link DF. Link DF in turn is attached to the rigid link FED. This rigid link is pivoted about the same shaft which supports the sector. The pivot point has been designated as point O. The link EG connects point F to a crank attached to the aileron.
FIG. II-2. SECTOR LINKAGE SYSTEM.
Figure II-3a is a schematic diagram in which corresponding parts bear the same notation as in Fig. II-2. Treating the sector linkage as a free body, and summing the moments about the supporting shaft at point 0, it is seen that the moment applied to the sector is transferred to the point P (also fixed to the sector) and is opposed by the moment established by the force in the rod EG acting on the link FEO. The moment about point 0 due to the force in the valve rod may be neglected since the movement of point 0₁ from 0 is very small. The force in the rod EG is transferred to the floating link, 0₁D at D through links FEO and FD. The floating link pivoted at P then assumes an equilibrium position when it is balanced by the force in the rod leading to the valve assembly.

2. Operation

Operation of the sector linkage system may best be described by assuming that a step input function of torque is suddenly applied to the sector when the ailerons are in the neutral position. After the torque has been applied in the short time interval before the hydraulic boost is effective, the aileron deflection is small because of the aerodynamic forces and the existing pressures in the boost cylinder. Conditions at this time may be represented by Fig. II-3a, where the valve is slightly open due to the small displacement of 0₁ from 0, and the sector is in the neutral position. In this position the valve rod is approximately normal to the floating link D0₁. A short time later, the boost cylinder is effective and causes the aileron to deflect. This displacement is fed back through the rod EG, and then links FEO and FD
a) NO SECTOR ROTATION - VALVE OPEN.

b) SECTOR ROTATED - VALVE OPEN

FIG. II-3. SCHEMATIC DIAGRAM OF SECTOR LINKAGE SYSTEM.
to the point D on the floating link. The orientation of the system can now be represented by Fig. II-3b. In this position the force applied on the floating link at D remains the same because FD is approximately normal to the floating link at all times. However, the force in the valve rod is no longer normal to the link and in order to hold the link DO\textsubscript{1} in equilibrium, the force in the rod must increase thus actuating the valve still further. The actual operation of the valve does not involve sudden changes in this manner but is a continuous action until the final aileron displacement is obtained.

C. Hydraulic System

1. Description

The principal components of the hydraulic system are shown in Fig. II-4. This diagram does not include additional components actually in the system that allow an interflow between the two sides of the cylinder in the event the flow to the valve falls below a set point. This precludes the aileron being locked in a fixed position in the event of a hydraulic failure.

The pump manufactured by Vickers Inc. is of the fixed stroke type having an output flow of 3.72 gallons per minute when operated at 3,420 revolutions per minute. Pump pressures are limited by means of a pressure relief valve having a cracking pressure of 3,000 pounds per square inch. The reservoir from which the pump is supplied has a capacity of 1.3 gallons above the outlet pipe to the pump. The boost cylinder has inlet ports so that pressures are applied from the valve to either end. The piston face has an area of 3.56 square inches. Forces from the piston are transmitted through a rod integral with one side of the piston face. The area of the
FIG. II-4. SCHEMATIC DIAGRAM OF HYDRAULIC SYSTEM.
rod is 0.995 square inches and leaves an effective area on the rod side of the piston of 2.565 square inches. Pressure applied to the side of the piston with the largest effective area tends to displace the aileron in the up position. Piston movement from the neutral position to the full up position of the aileron is 2.557 inches; movement to the full down position is 1.568 inches giving a total allowable movement of the piston of 4.125 inches.

The valve shown schematically in Fig. II-5 is of the variable orifice type. The orifice openings are dependent upon the position of the plungers. The plungers in turn are positioned by a fitting attached to the top of the body at the pivot point 0. This fitting is hereafter designated as the valve lever arm. The angular position of the lever arm is determined by the force applied through the control rod from the sector linkage by the forces exerted by the rubber bumpers, and by the internal valve pressures applied on the plungers.

As shown in the schematic sketch, the valve is supplied with fluid from the pump at the point \( P_H \), and the return oil to the reservoir flows by the point \( P_R \). Lines to the "up" and "down" side of the cylinder are indicated at \( P_U \) and \( P_D \). The four variable orifices have been numbered 1, 2, 3, and 4. The cross hatched pistons are free to move in the valve body. Their upward movement being restricted by a fitting in which the plungers move at the point a, downward motion is restricted when either orifice 1 or 3 is completely closed. When the plungers are in the neutral position, the inlet pressure \( P_H \), being higher than other pressures within the valve, force the pistons up against the plunger fitting. Passages within the valve body interconnect the lower orifices with the \( P_U \) and \( P_D \).
FIG. II-5  AILERON CONTROL VALVE
outlets. The pistons have been machined so that the lower and upper orifices are connected as shown. If the valve lever arm is rotated in the counter-clockwise direction, the plunger motion is such that the opening of orifice #4 is decreased and that of #2 is increased, and #1 and #3 remain unchanged. Continued movement will completely close orifice #4 and will decrease the opening of #1. In the limiting position both #4 and #1 will be completely closed, #2 completely opened and #3 unchanged. A similar sequence of orifice opening variations is obtained for clockwise rotation of the valve lever arm.

2. Operation

The interconnection of the valve orifices, supply and return lines, and the connecting lines to the cylinder may best be represented by a bridge circuit as shown in Fig. II-6. Since the valve is being supplied by a pump which delivers a constant flow, the inlet pressure \( P_H \) is a function of the overall impedance to the flow offered by the bridge circuit, which is altered as the orifice openings are changed.

Assume that the valve lever arm is rotated in the counter-clockwise direction so that the opening of orifice #4 is decreased and that of #2 is increased. The effect of the unbalance of the bridge between points \( P_U \) and \( P_D \) is such that a pressure rise exists at \( P_H \), a pressure drop at \( P_D \) and an increase in the effective pressure in the cylinder which is measured by \( P_U \) and \( P_D \). If the valve lever arm motion is continued until orifice #4 is completely closed and the piston is not in motion, the pressure at \( P_U \) is the same as the inlet pressure while the pressure at \( P_D \) has continued to drop because orifice #2 continues to open.
FIG. II-6. EQUIVALENT CIRCUIT OF VALVE AND BOOST CYLINDER
This effect continues as orifice #1 closes but becomes more drastic since the overall impedance to flow of the valve is rising and the inlet pressure $P_U$ becomes greater. In the limiting position both orifices #4 and #1 are closed in which case $P_U = P_H$ and $P_D = P_R$, or the differential pressure in the cylinder is nearly equal to the cracking pressure of the relief valve of 3,000 pounds per square inch.
A. Apparatus

The aileron control system was simulated in the laboratory by a test rig. An overall view of the system is shown in Fig. III-1. Components of the test rig, which are used in the aircraft, are the linkage system and the following hydraulic units, the pump, the relief valve, the reservoir, the control valve, and the boost cylinder.

The hydraulic pump was driven by a 5-horsepower, 3-phase, 230-volt, 60-cycle motor. A view of the motor with its supporting structure, pump, relief valve, and reservoir is shown in Fig. III-2. Figure III-3 gives a more detailed view of the relief valve, the pump, and the reservoir.

The remainder of the test rig is supported on a welded structural steel framework. The aileron is simulated by a balanced beam which is keyed to a supporting shaft. This shaft is supported by bearings attached to the framework. The beam assembly is shown in Fig. III-4. Weights are attached to each end of the beam by means of a bolt that passes through a slot machined in the beam. By varying the position of these weights relative to the center of rotation of the shaft, the moment of inertia of the beam could be adjusted to that of the aileron. Airloads on the aileron are simulated by
FIG. III-1. OVERALL VIEW OF SYSTEM
FIG. III-2. VIEW OF PUMP, RELIEF VALVE, RESERVOIR AND MOTOR
FIG. III-3. VIEW OF PUMP; RELIEF VALVE AND RESERVOIR
FIG. III-4. BEAM AND AIRLOAD SPRINGS
springs that are attached to the beam and to the base of the supporting structure. Since the airloads on the aileron are not linear functions of the aileron deflection as indicated by Fig. III-11, two sets of springs are used to represent the loading conditions which exist for each airspeed. Springs are available to represent the loading for three different airspeeds. Each spring of the first set, (inboard pair shown in Fig. III-4) is under tension throughout the full range of aileron deflection. These springs simulate the loading conditions for small deflections from the neutral position. The second set of springs, (outboard pair) offer no restraint on the beam in the neutral position and for small deflections from it. However for larger deflections in either the aileron up or down position, one of the springs restrains the beam while the other does not.

Figure III-5 shows the manner in which the boost cylinder and the piston is attached to the supporting structure and how the piston rod is connected to the beam crank. This crank is keyed to the shaft that supports the beam. Also keyed on the opposite side of the beam is another crank, (see Fig. III-6) connecting the beam with the sector linkage system.

The sector linkage system shown in Fig. III-7, with its supporting bracket is secured to a plate fixed to the supporting framework. Also shown are the rods connecting to the beam crank and the hydraulic valve.

The control valve, Fig. III-8 is secured to the same plate to which the sector linkage is fixed.
FIG. III-5. BOOST CYLINDER
FIG. III-6. VIEW SHOWING CONNECTION FROM BEAM CRANK TO SECTOR LINKAGE
FIG. III-7. SECTOR LINKAGE SYSTEM
ANES
FROM PUMP TO RESERVOIR
LINES
TO UP AND DOWN SIDES OF CYLINDER
ROD CONNECTING SECTOR LINKAGE
FIG. III-8. CONTROL VALVE
B. Test Procedure

When the tests were conducted, the input to the control system was a moment applied to sector and the output was the angular deflection of the beam. Instrumentation was set up to measure these two quantities as illustrated in the schematic sketch of Fig. III-9. The deflection of the beam was measured by the use of a microsyn signal generator or pickoff. This electromagnetic device produces a voltage at the secondary winding which is proportional to the angular deflection from a given null point, when the primary coils are excited by an alternating current. The excitation for the pickoff was obtained from a variac connected to the 110-volt, 60-cycle supply with the excitation current limited to approximately 75 milliampere. The output signal from the pickoff was fed through a Model T31 W03 Thordarson amplifier to a Heiland Research Corporation, Type A500-R12 oscillograph from which permanent photographic records could be obtained.

Moments were applied to the sector by means of stranded wire cables attached to the sector arms. These cables were led over pulleys (See Fig. III-1) that were secured to the framework. Since the moment on the sector is the product of the tensile force in the cable and the constant sector radius, the sector moment is proportional to the tension in the cable. The cable tension was measured by the amount of unbalanced voltage of a bridge circuit which consisted of two strain gages (Balwin Locomotive Works, Type A-7 SR-4) and a bridge balancing circuit. (Consolidated Engineering Corporation, Type 8-103C).
FIG. III-9. SCHEMATIC DIAGRAM OF INSTRUMENTATION.
One arm of the bridge consisted of a strain gage glued to a thin bakelite strip. This strip was 1/8 of an inch thick, 6 inches long, 1\(\frac{1}{2}\) inches wide at the ends, and tapered at the center so that the cross sectional area, where the gage was attached, was 0.050 square inches. One end of the strip was attached to the cable leading to the sector and the other end was attached to the cable passing over the pulleys. By pulling on this cable, a tensile stress is applied to the bakelite strip and the resultant strain in the gage changes the resistance of the gage. Connected in series with this gage was another gage used as a dummy, which was similar to that described. Connections from the common junction of the gages and from the two remaining leads were made to the bridge balancing unit. The bridge circuit was excited from a 110:6.3-volt center-tapped step down transformer. The excitation voltage was again 60 cycles. The output from the bridge circuit was amplified by a Miller Corporation Type A, Model AA6 amplifier before being connected to the oscillograph.

At the time the tests were conducted suitable equipment was not available; frequency response characteristics could not be determined directly.

Transient response records were obtained by two methods. The first method consisted of attaching a weight to the sector cable containing the strain gage, manually raising the weight until the cable tension was relieved, and then releasing the weight. Weights on both cables are shown in Fig. III-10. This method did not give a satisfactory step input function because the weight is accelerated during the time the beam is moving, which tends to decrease
FIG. III-10. WEIGHTS AND MOTOR USED TO APPLY MOMENTS TO SECTOR
the moment from its steady-state value during this acceleration period. Oscillations in the cable tension also occur due to the exchange of kinetic energy, from the falling weights, to potential energy in the stranded wire cables acting as springs. This method was also faulty because the weights could not be instantaneously released or raised manually in a shorter time interval then the time interval required for the system to respond.

The second method of obtaining an input moment was by using a 1/4 horsepower direct-current motor. The sector cable was secured to a pulley fastened to the motor shaft as shown in Fig. III-10. By opening or closing a switch in the motor circuit, the stall torque of the motor was transferred to the sector. This method was better than the first method described because a more sudden input was obtained. However, because of the motor armature inertia, the disadvantages of the first method were still present. Pulse inputs to the system could be obtained by rapidly closing and opening the switch.

Static characteristics of the system were obtained by placing calibrated weights on the sector cables and noting the beam deflection. These runs were conducted on the system in its normal closed-loop arrangement and also with the feedback path opened. The open-loop test was made by disconnecting the rod that couples the sector linkage system to the beam at the beam crank and clamping it to the supporting framework so that the lower portion of the sector linkage system was fixed.
FIG. III-11. AILERON HINGE MOMENT VS SURFACE DEFLECTION AT VARIOUS AIRSPEEDS

AILERON UP → AILERON DOWN

AILERON HINGE MOMENT IN POUNDS

SURFACE DEFLECTION IN DEGREES
CHAPTER IV
CHARACTERISTICS OF THE CONTROL VALVE

The control valve places at the disposal of the system the energy made available by the engine-driven hydraulic pump. This energy is required by the pilot to overcome the loads on the aileron. In fact, at high air loads, the hydraulic system is providing almost all the energy to the system. The force exerted directly by the pilot contributes little, and is being used almost entirely as a means of governing the output of the hydraulic portion of the mechanism.

The manner in which the control valve functions is an important part of the operation of the system. Essentially, the action of the plungers is to change the openings of the orifices in the valve. The pressure drops existing across the orifices are related to the dynamic loads on the system. Since flow through an orifice is a function of its opening and the pressure drop across it, the operation of the valve becomes a function of the plunger geometry and the forces called for to overcome the loads placed on the system.

To determine the hydraulic characteristics of the valve, the valve was removed from the system and tests were conducted on it. The valve was connected to a hydraulic table in such a manner that various rates of flow and pressure differences could be obtained. The rocker arm and rubber bumpers were disconnected and each orifice was investigated separately. Plunger position was controlled and measured by means of a calibrated gear. Rate of fluid flow was measured with a standard flow
meter and pressures were measured with suitable gauges.

In general, the two lower orifices, namely, numbers one and three had identical characteristics; and likewise for the upper orifices, namely, numbers two and four. (See Fig. IV-1) Therefore, the individual data for the similar orifices has been combined.

Rates of flow through the orifices were measured as a function of the plunger position or orifice opening with the pressure drops across the orifice maintained constant. A number of measurements of this type were performed for various values of pressure drop across the orifice within the range of the test equipment available. The results are shown in Fig. IV-1 and Fig. IV-2. The test equipment was capable of producing a maximum pressure drop of about two thousand pounds per square inch. The actual system could develop three thousand pounds per square inch. However, the test data forms a family of curves which should be capable of extrapolation if values in the higher region are desired.

Another series of tests were made keeping the orifice opening constant and varying the pressure drop. This series of test were repeated for various size of orifice openings. The results are shown in Fig. IV-3, and Fig. IV-4.

For computing the numerical coefficients found in the theoretical performance function, the partial derivatives of the rates of flow with respect to orifice opening and to pressure drop are desired. This information is represented by the slopes of the respective curves given in Fig. IV-1 through Fig. IV-4. Each curve has been approximated by a straight line and the slopes of these lines plotted as the partial derivatives. They are shown in Figs. IV-5 through IV-8.
Other static characteristics of the valve include those of Fig. IV-9 which were obtained by applying various forces to the valve at the point where the rod from the sector linkage system is connected. Pressures from the valve outlets to the up and down pressure sides of the cylinder were measured with pressure gages. Movement of the plungers was determined by means of a dial micrometer. These two variables, the plunger movement and the differential pressure existing across the valve were plotted as a function of the applied valve force. It will be noticed that if the zero point of the valve force were moved approximately 5 pounds in the aileron down direction then each curve would be of approximately the same form either side of this displaced zero. The rubber bumpers were purposely adjusted to give this unbalance so that the beam would remain in the horizontal position in the no-load condition. If the unbalance were not made, then the pressures on either side of the boost cylinder would be the same, but because of the unequal areas on opposing sides of the piston a torque would be applied to the beam deflecting it from the horizontal position. The flat portion of the differential pressure curve shows no change in differential pressure and the portion of the plunger movement curve that shows a small change in plunger movement are caused by the closing of the upper orifices. For the higher valve forces the differential pressure curve is quite linear. Figure IV-10 shows the variation of valve differential pressure plotted as a function of the plunger deflection.
FIG. IV-I. RATE OF FLUID FLOW THRU LOWER ORIFICES AS A FUNCTION OF PLUNGER POSITION.
FIG. IV-2. RATE OF FLUID FLOW THRU UPPER ORIFICES AS A FUNCTION OF PLUNGER POSITION.
FIG. IV-3. RATE OF FLUID FLOW THRU LOWER ORIFICES AS A FUNCTION OF PRESSURE DIFFERENCE ACROSS ORIFICE
FIG. IV-4. RATE OF FLUID FLOW THRU UPPER ORIFICES AS A FUNCTION OF PRESSURE DIFFERENCE ACROSS ORIFICE
FIG. IV-5  PARTIAL DERIVATIVE OF RATE OF FLUID FLOW
WITH RESPECT TO ORIFICE OPENING, \( \frac{\partial Q}{\partial y} \), AS A FUNCTION
OF PRESSURE DIFFERENCE—FOR LOWER ORIFICES

PRESSURE DIFFERENCE ACROSS ORIFICE IN psi
FIG. IV-6. PARTIAL DERIVATIVE OF RATE OF FLUID FLOW WITH RESPECT TO ORIFICE OPENING, $\frac{\partial Q}{\partial y}$, AS A FUNCTION OF PRESSURE DIFFERENCE—FOR UPPER ORIFICES

\[ \frac{\partial Q}{\partial y(\text{UPPER})} \]

IN GALS PER MIN./INCH

PRESSURE DIFFERENCE ACROSS ORIFICE IN psi
FIG. IV-7. PARTIAL DERIVATIVE OF RATE OF FLUID FLOW WITH RESPECT TO PRESSURE DIFFERENCE, $\frac{\delta Q}{\delta (\Delta P)_{(\text{LOWER})}}$, AS A FUNCTION OF PLUNGER POSITION - FOR LOWER ORIFICES
FIG. IV-8. PARTIAL DERIVATIVE OF RATE OF FLUID FLOW WITH RESPECT TO PRESSURE DIFFERENCE, $\frac{\delta Q}{\delta (\Delta P)_{\text{UPPER}}}$, AS A FUNCTION OF PLUNGER POSITION – FOR UPPER ORIFICES.
FIG. IV-9.
VALVE STATIC CHARACTERISTICS
FORCE IN VALVE CONTROL ROD VS
VALVE PRESSURE DIFFERENTIAL AND VALVE PLUNGER MOVEMENT

PLUNGER DEFLECTION IN INCHES

PRESSURE DIFFERENTIAL

VALVE PLUNGER DEFLECTION

FORCE IN VALVE CONTROL ROD IN POUNDS
FIG. IV-10.
VALVE STATIC CHARACTERISTICS
VALVE PLUNGER DEFLECTION VS
VALVE PRESSURE DIFFERENTIAL

AILERON UP ——> AILERON DOWN

VALVE PLUNGER DEFLECTION IN INCHES
CHAPTER V
MATHEMATICAL DERIVATION OF MECHANISM PERFORMANCE FUNCTION

A. Introduction

The moment applied to the sector by the pilot, \((SM)_{app}\), is considered as the basic input to the boost system and the angular deflection of the aileron, \(A_{ail}\), is considered as the output of the system. Then, by definition,

\[
[PF]_{[(SM)]_{app}, A_{ail}} = \frac{A_{ail}}{(SM)_{(app)}},
\]

It is the purpose of this chapter to establish the form of this performance function.

Many of the coefficients appearing in the derivation arise from non-linear relationships among the system variables. To carry through this non-linearity completely would complicate the analysis considerably and would lead to variable coefficient differential equations. These differential equations would be awkward to use and their solutions would be beyond the scope of most engineering mathematics. To overcome this difficulty, linear approximations have been made wherever feasible. In those cases where it was not justifiable to linearize coefficients, the range of variation was restricted so that specific values might be used. This thereby places restrictions on the validity of the resulting equations. They will only be true in that range of operation of the
system where the necessary conditions are fulfilled. The bases for making these linear approximations are contained in separate figures at the end of this chapter. Sample values of the variables have been evaluated. If operation outside of these ranges is of interest, different numerical values may have to be used.

The complete mathematical treatment is lengthy and involves numerous simple algebraic manipulations which, for brevity, are condensed and only indicated here. When these abridged steps result in new groupings of constants, a new subscript is generally used in the resulting equations. Figure V-1 presents an equivalent circuit of the control valve and boost cylinder upon which the analysis is based.

B. Derivation

Summing moments about the aileron hinge

\[(HM)_{\text{pilot}} + (HM)_{\text{boost}} = (HM)_{\text{aero}} + (HM)_{\text{inertia}} \quad (V-1)\]

From Fig. V-2,

\[(HM)_p = K_1 (SM)\]

From Fig. V-3,

\[(HM)_b = K_2 F_b\]

The aerodynamic and inertia moments are

\[(HM)_{\text{aero}} + (HM)_{\text{inertia}} = (I_p^2 + B_p + K) A_{(a)}\]
\( P_H \) = pressure at inlet to valve.
\( P_{H_0} = P_H \) at \( t=0 \)
\( P_U \) = pressure at entrance to "up" supply line
\( P_{U_0} = P_U \) at \( t=0 \)
\( P'_U \) = pressure in "up" side of cylinder
\( P_D \) = pressure at exit of "down" supply line
\( P_{D_0} = P_D \) at \( t=0 \)
\( P'_D \) = pressure in "down" side of cylinder
\( P_R \) = pressure at exit of valve
\( Q_H \) = rate of fluid flow at inlet to valve
\( Q_U \) = rate of fluid flow into "up" supply line
\( Q_{CU} \) = rate of fluid flow compressed in "up" supply line
\( Q'_U \) = rate of fluid flow into "up" side of cylinder
\( Q_D \) = rate of fluid flow at exit of "down" supply line
\( Q_{CD} \) = rate of fluid flow compressed in "down" supply line
\( Q'_D \) = rate of fluid flow out of "down" side of cylinder
\( Q_I \) = rate of fluid flow through orifice No. 1, etc.
\( A_U \) = area of "up" side of piston
\( A_D \) = area of "down" side of piston
\( F_b \) = force in piston arm
\( \Delta X \) = displacement of piston from neutral
\( y \) = displacement of plunger from neutral
\( \Delta \) = incremental change
\( p \) = derivative with respect to time
\( V \) = volume of supply line; up and down lines have approximately equal volumes
\( B \) = bulk modulus of hydraulic fluid

**FIG. V-1. EQUIVALENT CIRCUIT OF CONTROL VALVE AND BOOST CYLINDERS**
SUMMING MOMENTS ABOUT POINT O:

\[(SM)_{app} = F_p \times e\]

\[F_p = \frac{(SM)}{e}\]

\[(F_p)_d = \frac{(SM)}{e} \times \cos A_{ail}\]

\[(HM)_p = (F_p)_d \times d\]

\[= \frac{(SM)}{e} \times \cos A_{ail} \times d\]

**For \(-5^\circ < A_{ail} < +5^\circ\)**

\[d = 6.4 \text{ INS}\]
\[e = 3.4 \text{ INS}\]
\[\cos A_{ail} = 0.98 \text{ INS}\]

\[(HM)_p = \frac{(6.4)(0.98)}{3.4} = 1.84(SM)\]

\[(HM)_p = K_1(SM)\]
\[K_1 = 1.84 \text{ INS}\]

**FIG. 3-2. DERIVATION OF AILERON HINGE MOMENT DUE TO PILOT**
(FORCE APPLIED TO AILERON THRU BOOST CYLINDER ARM)

\[ F_b \]

\[ (HM)_b = F_b \times n \]

\[ = K_2 \times F_b \]

AILERON

\[ \text{FOR} \ -5^\circ < A_{(ail)} < +5^\circ \]

\[ n = 5.3 \text{ INCHES} \]

\[ (HM)_b = K_2 F_b \]

\[ K_2 = 5.3 \text{ INS.} \]

FIG. V-3. DERIVATION OF AILERON HINGE MOMENT DUE TO HYDRAULIC BOOST CYLINDER, \( (HM)_b \).
where \( I \) is equal to the effective moment of inertia of the aileron, and \( B \) and \( K \) are aerodynamic loading coefficients. Substituting these relations in eq (V-1)

\[
K_1(SM) + K_2 F_b = (I_p^2 + B_p + K) A_{(a)i1}
\] (V-2)

The evaluation of \( F_b \) in this equation is obtained as follows:

Summing the forces on the boost piston and neglecting the mass of the piston,

\[
F_b = A_u P_u' - A_D P_D'
\] (V-3)

Since the flow is laminar, and if the forces involved in accelerating the fluid are neglected, use of the Hagen-Poiseuille law gives,

\[
P_U' = P_U - G Q_U
\] (V-4a)

\[
P_D' = G Q_D + P_D
\] (V-4b)

where \( G \) is the resistance coefficient. Substituting eqs (V-4a) and (V-4b) in eq (V-3) gives

\[
F_b = A_u P_u - A_D P_D - A_u G Q_U - A_D G Q_D
\] (V-5)

Substituting the value of \( Q_U \) and \( Q_D \) from Fig. V-4 and rewriting,

\[
F_b = [A_u - A_u \frac{V_G}{B} p]\left[\frac{P_u}{V_G - p + 1}\right] + \left[\frac{A_D V_G}{B} p - A_D\right] \left[\frac{P_D}{V_G - p + 1}\right] \left[\frac{K_5 G (A_u^2 + A_d^2)}{p}\right] A_{(a)i1}
\] (V-6)
\[ Q_U = Q_{U}' + Q_{CU} \]
\[ Q_{U}' = K_S A_{u(p)} \]
\[ Q_{CU} = \frac{V}{B} p (P_u + P_{u}') \]

From Fig. V-8, \( x = K_S A_{(ail)} \)

From eq (V-4a), \( P_{u}' = P_u - GQ_u \)

\[ Q_{CU} = \frac{V}{B} p (P_u + P_{u} - GQ_u) = \frac{V}{B} p (2P_u - GQ_u) \]
\[ Q_U = K_S A_{u(p)} A_{(ail)} + \frac{V}{B} p (2P_u - GQ_u) \]

Solving for \( Q_U \)

\[ Q_U = \frac{K_S A_{u(p)} A_{(ail)} + \frac{2V}{B} p P_u}{\frac{VG}{B} p + 1} \]

In a similar manner,

\[ Q_{U}' = \frac{K_S A_{D} p A_{(ail)} - \frac{2V}{B} p P_D}{\frac{V}{B} p + 1} \]

FIGURE V - 4

DERIVATION OF \( Q_U \) AND \( Q_D \) BASED ON BOOST CYLINDER DYNAMICS AND FLUID COMPRRESSIBILITY
The factors \( \frac{P_U}{V G_{B}^{p+1}} \) and \( \frac{P_D}{V G_{B}^{p+1}} \) are evaluated as follows:

From Fig. V-5,

\[
Q_{D} = c_1 y + c_2 P_D + c_3 P_U + c_4 \tag{V-7}
\]

from Fig. V-4,

\[
Q_{D} = \frac{K_5 A_{D P} A_{(a i l)} - 2V_{B} P_D}{V G_{B}^{p+1}} \tag{V-8}
\]

Equating eqs (V-7) and (V-8) and solving for \( P_U \),

\[
P_U = \frac{C_5 p}{V G_{B}^{p+1}} A_{(a i l)} + \frac{C_6 P + C_7}{V G_{B}^{p+1}} P_D + C_8 y + \text{CONSTANT} \tag{V-9}
\]

In a similar manner, using the values for \( Q_U \), the following can be obtained

\[
P_D = \frac{C'_5 p}{V G_{B}^{p+1}} A_{(a i l)} + \frac{C'_6 P + C'_7}{V G_{B}^{p+1}} P_U + C'_8 y + \text{CONSTANT} \tag{V-10}
\]

Solving eqs (V-9) and (V-10) simultaneously for \( \frac{P_U}{V G_{B}^{p+1}} \) and \( \frac{P_D}{V G_{B}^{p+1}} \)

\[
\frac{P_U}{V G_{B}^{p+1}} = \frac{C_9 P^2 + C_{10} P}{V G_{B}^{p+1}} A_{(a i l)} + [C_{11} P + 12] y + \text{CONSTANT} \tag{V-11}
\]
The flows through the orifice are functions of the plunger displacement $y$, and the pressure drop across the orifice $\Delta P$. Then since

$$Q_1 = f(y, P_H - P_D)$$
$$Q_2 = f(y, P_D - P_R)$$

$$\Delta Q_1 = \frac{\partial Q_1}{\partial y} \Delta y + \frac{\partial Q_1}{\partial (\Delta P)} \Delta (P_H - P_D)$$

If the change is measured from $t = 0$, at which time the plunger is in neutral, then $\Delta y = y$ and $\Delta (P_H - P_D) = (P_H - P_D) - (P_H - P_D_0)$

Thus

$$\Delta Q_1 = \frac{\partial Q_1}{\partial y} y + \frac{\partial Q_1}{\partial (\Delta P)} (P_H - P_D - (P_H - P_D_0))$$

Similarly,

$$\Delta Q_2 = \frac{\partial Q_2}{\partial y} y + \frac{\partial Q_2}{\partial (\Delta P)} (P_D - P_D_0)$$

if $P_R$ is assumed approximately equal to zero. Substituting these in the original equation gives

$$Q_D = \left[ \frac{\partial Q_2}{\partial y} - \frac{\partial Q_1}{\partial y} \right] y - \frac{\partial Q_1}{\partial (\Delta P)} P_H + \left[ \frac{\partial Q_2}{\partial (\Delta P)} + \frac{\partial Q_1}{\partial (\Delta P)} \right] P_D + \text{constant}$$

From Fig. V-6

$$P_H = K_1 + K_2 y + K_3 P_D + K_4 P_U$$

Substituting this value, rewriting, and redefining constants gives

$$Q_D = C_1 y + C_2 P_D + C_3 P_U + C_4$$

In a similar manner

$$Q_U = C_1' y + C_2' P_D + C_3' P_U + C_4'$$

FIGURE V - 5

DERIVATION OF $Q_D$ AND $Q_U$ BASED ON FLOW THROUGH THE VALVE ORIFICES
\[ Q_H = Q_1 + Q_3 \]
\[ Q_1 = Q_{1o} + \Delta Q_1 \]

From Fig. V-5

\[ \Delta Q = \frac{\partial Q_1}{\partial y} y + \frac{\partial Q_1}{\partial (\Delta P)} P_H - \frac{\partial Q_1}{\partial (\Delta P)} P_D - \frac{\partial Q_1}{\partial (\Delta P)} (P_H - P_D) \]

Then

\[ Q_1 = Q_{bo} + \frac{\partial Q_1}{\partial y} y + \frac{\partial Q_1}{\partial (\Delta P)} P_H - \frac{\partial Q_1}{\partial (\Delta P)} P_u - \frac{\partial Q_1}{\partial (\Delta P)} (P_{H_0} - P_{U_0}) \]

Similarly

\[ Q_3 = Q_{3o} + \frac{\partial Q_3}{\partial y} y + \frac{\partial Q_3}{\partial (\Delta P)} P_H - \frac{\partial Q_3}{\partial (\Delta P)} P_u - \frac{\partial Q_3}{\partial (\Delta P)} (P_{H_0} - P_{U_0}) \]

Thus

\[
Q_H = \left[ \frac{\partial Q_1}{\partial (\Delta P)} + \frac{\partial Q_3}{\partial (\Delta P)} \right] P_H - \left[ \frac{\partial Q_1}{\partial y} + \frac{\partial Q_3}{\partial y} \right] y - \frac{\partial Q_1}{\partial (\Delta P)} P_o - \frac{\partial Q_3}{\partial (\Delta P)} P_u + \text{constant}
\]

Solving for \( P_H \)

\[
P_H = \frac{1}{\left[ \frac{\partial Q_1}{\partial (\Delta P)} + \frac{\partial Q_3}{\partial (\Delta P)} \right]} \left[ Q_H - \left( \frac{\partial Q_1}{\partial y} + \frac{\partial Q_3}{\partial y} \right) y \frac{\partial Q_1}{\partial (\Delta P)} P_o + \frac{\partial Q_3}{\partial (\Delta P)} P_u + \text{constant} \right]
\]

Redefining coefficients gives

\[
P_H = K_1 + K_2 y + K_3 P_D + K_4 P_u
\]

FIGURE V - 6

DETERMINATION OF \( P_H \)

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\[
\left[ \frac{p_D}{v_{GB} p+1} \right] = \frac{\left[ c_q p^2 + c_{10} p \right] A_{(ai1)} + \left[ c_1 p + c_{12} \right] y + \text{CONSTANT}}{D} \tag{V-12}
\]

where,
\[
D = \left[ \frac{v_G}{B} p+1 \right]^2 - \left[ c_6 p + c_7 \right] \left[ c_6' p + c_7' \right]
\]

Substituting eqs (V-11) and (V-12) into eq (V-6) and rewriting,
\[
F_b = \frac{c_{13}}{D} + \left[ \frac{c_{14} p^2 + c_{15} p + c_{16}}{D} \right] y + \left[ \frac{c_{17} p^3 + c_{18} p^2 + c_{19} p}{D (\frac{v_G}{B} p+1)} \right] A_{(ai1)} \tag{V-13}
\]

Substitution of the value of \( y \) given in Fig. V-7 and in eq (V-13) and then expanding, rewriting, multiplying by \( K_2 \), and redefining coefficients gives
\[
K_2 F_b = \frac{c_20 p^2 + c_{21} p + c_{22}}{(\frac{v_G}{B} p+1) (K_5 p + K_6)} [\text{SM}] + \frac{c_{23} p^4 + c_{24} p^3 + c_{25} p^2 + c_{26} p}{D (\frac{v_G}{B} p+1) (K_5 p + K_6)} A_{(ai1)} + K_6 \tag{V-14}
\]

Substituting eq (V-14) in eq (V-2)
\[
K_i [\text{SM}] + \frac{c_20 p^2 + c_{21} p + c_{22}}{(\frac{v_G}{B} p+1) (K_5 p + K_6)} [\text{SM}]
\]
\[
+ \frac{c_{23} p^4 + c_{24} p^3 + c_{25} p^2 + c_{26} p}{D (\frac{v_G}{B} p+1) (K_5 p + K_6)} A_{(ai1)} + K_6 = (I p^2 + B p + K) A_{(ai1)} \tag{V-15}
\]
Equating (A) and (B)

0.034 \( (P_D - P_U) = 0.138 \text{(SM)} \)

or \( \text{(SM)} = K_7 (P_D - P_U) \)

Substituting the values of \( P \) and \( P \) obtained from equations II and I2, p., and solving for \( y \) gives

\[
y = \frac{1}{\left(\frac{\nu c}{B} + 1\right)(k_5 P + k_6)} K_7 D \text{(SM)} + (k_8 P^2 + k_9 P) A_{\text{ail} + k_{10}}
\]

**FIG. V-7. DETERMINATION OF PLUNGER DISPLACEMENT, \( y \).**

\[ K_7 = 0.246 \]
FIG. V-8. PISTON DISPLACEMENT, $x$, VERSUS AILERON DEFLECTION, $A_{(ail)}$.

$$x = K_5 A_{(ail)}$$

$K_5 = .0912$
From the initial adjustment of the system, \( A(ail) \) is zero when \( (SM) \) is zero, therefore \( K_6 \) must be zero. Setting \( K_6 \) equal to zero, rewriting, and solving for \( \frac{A(ail)}{(SM)} \) gives

\[
[PF \ (SM) \ A(ail)] = \frac{n_4 p^4 + n_3 p^3 + n_2 p^2 + n_1 p + n_0}{d_6 p^6 + d_5 p^5 + d_4 p^4 + d_3 p^3 + d_2 p^2 + d_1 p + d_0}
\]

(V-16)

where the expressions for the coefficients are given in Tables V-2 and V-3.
\( A_U = 3.55 \text{ sq. in.} \); area of up side of piston.

\( A_D = 2.55 \text{ sq. in.} \); area of down side of piston.

\( G = 2.23 \frac{\text{lb-sec}}{\text{in}^5} \); fluid resistance coefficient for AN-0-366 fluid.

\( V = 9.95 \text{ in}^3 \); average volume under compression.

\( B = .24 \times 10^6 \frac{\text{lb}}{\text{in}^2} \); bulk modulus of fluid, AN-0-366.

\[ \frac{\partial Q_1}{\partial y} = \text{partial derivatives of rate of fluid flow} \]
\[ \frac{\partial Q_2}{\partial y} \]
\[ \frac{\partial Q_3}{\partial y} \]
\[ \frac{\partial Q_4}{\partial y} \]
with respect to plunger position; \( \frac{\text{in}^2}{\text{sec}} \).

Values are determined from previous figures for conditions of valve operation being studied.

\[ \frac{\partial Q}{\partial (\Delta P)} \]
\[ \frac{\partial Q_5}{\partial (\Delta P)} = \]
\[ \frac{\partial Q_6}{\partial (\Delta P)} \]
\[ \frac{\partial Q_7}{\partial (\Delta P)} \]
partial derivatives of rate of fluid flow with respect to pressure drop; \( \frac{\text{in}^5}{\text{lb-sec}} \).

See previous figures for values.

\( K_5 = \) see Fig. V-8

\( K_2 = \) see Fig. V-3

\( K_7 = \) see Fig. V-7

\( I = \text{lb-in-sec}^2 \); total effective moment of inertia.

\( B = \text{lb-in-sec} \); total effective damping.

\( K = \text{lb-in} \); total effective spring constant. See Fig. III-22

**TABLE V - 1**

**FACTORS NEEDED FOR COMPUTATION OF COEFFICIENTS**

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TABLE V-2. COMPUTATION SCHEDULE FOR ANALYSIS INCLUDING EFFECTS OF FLUID COMPRESSIBILITY.
TABLE V-3. COMPUTATION SCHEDULE FOR ANALYSIS NEGLECTING EFFECTS OF FLUID COMPRESSIBILITY.
DERIVATION OF PERFORMANCE FUNCTION WITHOUT COMPRESSIBILITY EFFECTS

Consideration of the compressibility of the fluid yields a considerably complicated performance function to evaluate. There are ranges of operation of the valve where fluid compressibility is quite small and its effects can be neglected. This leads to a simpler performance function which is derived below.

Since \( Q_{CU} \) and \( Q_{CD} \) are now negligible, Fig. V-4, becomes simply,

\[
Q_u = Q_u' = K_5 A_u p A_{(ail)}
\]

\[
Q_D = Q_D' = K_5 A_D p A_{(ail)}
\]

(V-17)

Substituting these in eq (V-5) of the previous analysis gives,

\[
F_b = A_u P_u - A_D P_D - A_u G [K_5 A_u p A_{(ail)}] - A_D G [K_5 A_D p A_{(ail)}],
\]

or

\[
F_b = A_u P_u - A_D P_D - K_5 G [A_u^2 + A_D^2] p A_{(ail)}
\]

(V-18)

\( P_u \) and \( P_D \) are evaluated as follows:

from Fig. V-5,

\[
Q_p = C_1 y + C_2 P_D + C_3 P_u + \text{constant}
\]

(V-19)

Equating (V-17) and (V-19) and solving for \( P_u \),

\[
P_u = C_5 p A_{(ail)} + C_6 y + C_7 P_D + \text{constant}
\]

(V-20)
In a similar manner the following can be obtained,

\[ P_0 = c_5 p A_{(a\|l)} + c_6 y + c_7 P_U + \text{CONSTANT} \]  \hspace{1cm} (V-21)

Solving eqs (V-20) and (V-21) simultaneously, for \( P_U \) and \( P_D \):

\[ P_U = c_8 p A_{(a\|l)} + c_9 y + \text{CONSTANT} \]  \hspace{1cm} (V-22)

\[ P_D = c_8' p A_{(a\|l)} + c_9' y + \text{CONSTANT} \]  \hspace{1cm} (V-23)

Substituting these into eq (V-18) and rewriting;

\[ F_b = c_{10} p A_{(a\|l)} + c_{11} y + \text{CONSTANT} \]  \hspace{1cm} (V-24)

The quantity, \( y \), is obtained from Fig. \( V-7 \), by using the new values of \( P_U \) and \( P_D \) given in eqs (V-22) and (V-23), this results in

\[ y = c_{12} (\text{SM}) + c_{13} p A_{(a\|l)} + \text{CONSTANT} \]  \hspace{1cm} (V-25)

Substituting eq (V-25) in eq (V-24) gives,

\[ F_b = c_{14} p A_{(a\|l)} + c_{15} (\text{SM}) + \text{CONSTANT} \]  \hspace{1cm} (V-26)

Returning to the original derivation and using eq (V-26) in eq (V-2) yields,

\[ K_1 (\text{SM}) + K_2 \left[ c_{14} p A_{(a\|l)} + c_{15} (\text{SM}) \right] + \text{CONSTANT} = \left[ I_p^2 + B_p^2 + K \right] A_{(a\|l)} \]  \hspace{1cm} (V-27)
This, when rewritten, has the form,

\[
[pF][^{(SM)}_{A_{(ai)}}] = \frac{A_{(aih)}(r)}{(SM)} = \frac{a_0}{Ip^2 - B' p + K} \quad \text{(V-28)}
\]

The coefficients can be evaluated by using the accompanying computation Table V-3.
CHAPTER VI
EVALUATION OF SYSTEM PARAMETERS

Before a suitable laboratory model can be constructed, the physical constants and variable parameters of the system must be known. This information is also necessary when the theoretical performance function is to be evaluated and an analog is to be constructed. In general, the information was obtained from two sources:

(1) from experimental measurements on the components of the system

(2) from engineering data supplied by the manufacturer.

From the experimental measurements, the characteristics of the control valve were obtained; from the engineering data, such items as lengths of lever arms, the moment of inertia of the aileron, etc. were obtained. The coefficient of the aerodynamic damping on the aileron is an example of some information that could not be obtained readily. If the wind tunnel data cannot supply this information, a value for the coefficient may have to be assumed. This assumption would be based upon theoretical considerations or upon past experiences. This is one of the drawbacks of using a laboratory model. Direct measurements on the airplane in flight would produce the exact conditions and no assumptions or approximations would be necessary. In accordance with general aerodynamic practice, the authors have considered the aerodynamic damping of the aileron to be small. When this damping was compared to the other forms of damping in the system, it was considered to be negligible. In the laboratory, the viscous damper was not used on the beam shaft.

Much of the engineering data was furnished by the North American
Aviation Company in their Report No. NA-48-761.

The computation tables and numerical values given in the previous chapter have been designed along a consistent set of units. The usual care must be observed that all values used in the tables are in these units, namely, pounds, feet, and seconds.

The following procedure is suggested when the computation tables are used:

1. In order that a linear system may be considered, a small operating range of the system should be selected and investigated.

2. This selection of range should then be examined to determine if fluid compressibility is of sufficient importance to be considered. This information then determines the choice of the computation table to be used.

3. Based upon the range of operation, flow coefficients must then be selected from the graphs given; for example, if in the range under study orifice #4 is closing and orifice #2 is opening, this will govern the selection of the values of the partial derivatives.

4. Various geometrical parameters vary as the aileron assumes different values of deflection; the values given in the previous chapter are mostly for \(-5^\circ < A(ail) < 5^\circ\). However, if other values of deflection are of interest, proper selection from the appropriate curve can be made.

The hydraulic fluid used was AN-0-366 and from its properties the constants would necessitate the adjustment of these values.

Numerical values can be found in Table V-1.
A. Introduction

The operation of the system depends upon two factors, the variation of moment applied to the sector by the pilot, and the loading on the aileron. Essentially the system provides a variable mechanical advantage to the pilot. For small loads on the system the pilot is capable of supplying sufficient moment directly to the aileron to produce the desired result. As the airspeed or desired deflection of the aileron increases, increasing the load on the system, a point is reached where the pilot is not able to provide sufficient moment directly to the aileron to produce the desired deflection. Through the opening of the control valve, additional moment is supplied to the aileron to assist in overcoming the loading. As the load continues to increase, the hydraulic system provides a greater and greater proportion of the moment necessary to deflect the aileron. At the highest speeds and largest deflections, the ratio of the moment applied by the boost system to the moment applied directly by the pilot is in the order of seventy to one.

The output of the valve depends upon the relationship existing between the position of the plunger and the force on the valve arm. Since the characteristics of the valve change as this relationship changes, the overall performance of the boost system also changes. For example, a large moment applied to the sector by the pilot might cause the valve to open fully before the aileron had deflected appreciably, thereby applying the maximum hydraulic pressure to the boost cylinder. If then,
The applied moment were increased, no change in the valve output would occur. Similarly, if only a small moment is applied to the sector, little or no change would occur in the output of the valve and the effect of the valve dynamics would not be present in the overall response of the system. Thus, the overall system dynamics depend in part on the size of the input.

All this means that the system is non-linear in its action. Therefore, an analysis must proceed with due regard to these effects. The methods adopted by the writers were to examine the performance of the system in small regions of operation in which the system could be considered approximately linear within the region. By so doing, a complete coverage of the entire range of operation could be made. Then, by correlating all these results the overall performance of the system could be obtained. Limitations of time and available test equipment did not permit this complete coverage to be made. However, sufficient information was obtained to establish some prominent characteristics of the system.

B. Static Characteristics

The static characteristics were obtained by measuring the deflection of the beam for various static values of moment applied to the sector. The tests were performed under two operating conditions:

(a) The system in its normal condition.
(b) The arm from the sector to the beam was disconnected so that the only moment applied to the beam was that due to the boost cylinder. In effect, this corresponded to open-loop operation of the system.

The results are given in Fig. VII-1. It is to be observed that the characteristics are the same under both conditions.
FIG. VII-1. STATIC CHARACTERISTICS OF TEST RIG CONTROL SYSTEM.
BEAM DEFLECTION VS. SECTOR MOMENT FOR OPEN AND CLOSED LOOP OPERATION.
C. **Transient Response**

Dynamic inputs were applied to the system in two ways. First, weights were attached to cables connected to the sector and then released suddenly. Second, a sector cable was attached to the shaft of a direct-current electric motor whose field was then pulsed producing a pulse or step function of torque on the sector. Simultaneous records of this applied moment and the resulting aileron deflection were made. Fig. VII-2 is a record taken by releasing a 50 pound weight equivalent to an applied moment of 225 pound-inches. The loading springs on the beam were equivalent to an airspeed of 150 miles per hour. The initial deflection of the aileron was 330 milliradians (mils). The input was completed in about 1/8th of the time necessary for the aileron to come to rest. Because of the limitation imposed by handling the weights manually, the step is not very sharp. In an endeavour to produce a better step function the weight input system was replaced with the electric motor system. Fig. VII-3 is a record made with this system. It can be seen from this figure that the step is considerably steeper. (Note: The two pips appearing in the applied moment curve after the step is completed are believed due to irregularities present in the strain gage circuit when light or zero loads are applied to the gage.) This record was taken for a loading equivalent to an airspeed of 300 miles per hour. The applied moment was 212 pound-inches and the initial deflection was 64 mils.

D. **Frequency Response**

Although transient responses provide much useful information about the system, a frequency response is desirable when the system is to be used in conjunction with other components such as an automatic pilot, or for compensating the existing system when synthesizing a new system.
FIG. VII-2 TRANSIENT RESPONSE DUE TO INPUT MOMENT OF TWO HUNDRED TWENTY-FIVE POUND-INCHES. INITIAL DEFLECTION-330 MILS. EQUIVALENT AIRSPEED- 150 MPH.
FIG. VII-3 TRANSIENT RESPONSE DUE TO INPUT MOMENT OF TWO HUNDRED AND TWELVE POUND-INCHES. INITIAL DEFLECTION-64 MILS. EQUIVALENT AIRSPEED-300 MPH.
from the old. A particularly useful concept is that of the Performance Function developed by Dr. C. S. Draper and his associates at the Instrumentation Laboratory, Massachusetts Institute of Technology, Cambridge, Mass. To obtain the phase angle and amplitude ratio of the performance function, recourse was made to a method developed by Prof. R. C. Seamans, Jr., B. P. Blasingame, and G. C. Clementson. The method is given in detail in reference (4)*. This method permits determination of the performance function directly from the results of transient responses of the system to pulse or step inputs. The method is particularly useful when the system under investigation presents an awkward or a difficult evaluation through means of sinusoidal inputs, but to which pulse or step inputs can easily be applied. This is the case here.

Figure VII-4 is the performance function obtained from the transient response of Fig. VII-2. The amplitude ratio and the phase angle drop off fairly steeply in the frequency range between 0 and 7 radians per second. Then, from 7 radians per second the amplitude ratio approaches zero and the dynamic phase angle approaches $-180^\circ$. The performance function is behaving like a second order system.

Figure VII-5 is the performance function obtained from the transient response of Fig. VII-3. In general, it is similar in shape to frequency response of Fig. VII-4. However, it is spread out more over the frequency range.

* See Bibliography
FIG. VII-4. AMPLITUDE RATIO AND PHASE ANGLE OF PERFORMANCE FUNCTION OBTAINED FROM TRANSIENT RESPONSE OF FIG. VII-2
FIG. VII - 5. AMPLITUDE AND PHASE ANGLE OF PERFORMANCE FUNCTION OBTAINED FROM TRANSIENT RESPONSE OF FIG. VII - 3.
CHAPTER VIII

OSCILLATIONS IN THE SYSTEM

It had been brought to the attention of the writers that oscillations of the boost control mechanism had been found to be present in the operation of the system under actual conditions of the installation in the airplane. No specific reports were available and the only information given was that the oscillations occurred in a random fashion. Sometimes they would occur on the ground and sometimes in the air. These oscillations would be most undesirable in the system and during this investigation various attempts were made to locate and determine the causes of any oscillations.

Oscillations were encountered during the tests. In general, they were believed to be due to two causes, hydraulic phenomena within the valve, and an interchange of energy between the input system and the aileron beam.

When the valve was disconnected from the system and tested separately on the hydraulic table, oscillations in the valve occurred. Although no direct evidence was obtainable, the oscillation was believed to be caused by an irregular flow of the hydraulic fluid around the various sharp edges of the orifice. Under normal conditions the position of a piston in the valve is determined by the position of the plunger pressing against its upper surface, there being sufficient hydraulic forces on the bottom surfaces of the piston to keep it against the plunger. However, if a lower orifice should be very nearly closed, it is possible for the flow of hydraulic fluid

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through this orifice to take an irregular pattern. That is, the presence of the sharp edges in the orifice would permit critical changes in the flow pattern such that there would be local fluctuations in pressure. Since the piston has four different areas upon which various different hydraulic pressures are acting, the position of the piston, when not in contact with the plunger, is determined by the net hydraulic force acting on it. Normally, the net hydraulic force keeps the piston against its upper seat, if the upper orifice is still open, or against the plunger if the upper orifice is closed. However, if the net hydraulic force on the piston is not upward, because of the local pressure variations occurring at the sharp edges of an orifice, the piston would float and its position would not be a function of the plunger position. This readjustment of the piston position would tend to increase the net upward force on the piston and would tend to return it to its normal position, where if the necessary flow conditions can again exist across the orifice, the cycle would repeat itself. This is apparently what occurred in the laboratory. The frequency of the oscillation and the measurements of the pressure variations during the oscillation were not obtainable with the equipment immediately available.

Another type of oscillation was encountered when the complete system was being tested. It was found that sustained oscillations could be induced in the system when weights were suspended from the input cables and an additional force was applied to one of the cables. The magnitude of the additional force necessary to cause oscillations depended upon the magnitude of the weights suspended from the cables.
The frequency of oscillation was determined by using a stroboscope and it was found to be 30 cycles per second. This frequency of oscillation was found to be independent of the magnitude of the suspended weights and the size of the load springs used. The magnitude of the oscillations increased with the magnitude of the suspended weights, and was also dependent upon the amount of damping present in the system. For example, if the rubber bumpers were removed from the control valve, the magnitude of oscillation increased; if the viscous damper was placed on the beam shaft the magnitude was reduced. A Frahm-type vibration absorber was attached to the valve lever arm but did not reduce the oscillations. It was found that the system would oscillate at the frequency for which the absorber was tuned. The magnitude of oscillation increased as the frequency decreased. When the connecting link between the sector and the beam was disconnected, the system could not be made to oscillate. Thus this oscillation appears due to an interchange of energy between the input system and the aileron beam.
A. Discussion

The previously derived theoretical performance function, neglecting fluid compressibility, indicated a second-order system. The experimental responses showed that for low ranges of frequency the performance function has second-order characteristics. When compressibility was taken into account, the derived performance function has a fourth-order numerator that can be capable of producing the higher frequency effects exhibited in the experimental performance function.

Care must be exercised in attempting a closer correlation between the theoretical and the experimental determinations. The assumptions under which the derivation holds true must exist when the experimental tests are made. One of these assumptions restricts the plunger travel to a certain range, which may be difficult to maintain for higher loadings on the beam.

Likewise, restrictions were placed upon the interpretation given to the experimental response. The method of analysis used in preparing the frequency response is based upon linear differential equations with constant coefficients. If the transient response is made under conditions of small departures from linearity, it will yield reliable information; otherwise it can be misleading.

An important result of the investigation was the observation that the two performance functions found were not the same, but instead had different frequency characteristics. This stems from the non-linear operation of the system throughout the range of loadings. This variation plays an
important part when a mating of the boost system with an automatic pilot or control system is attempted because the two systems must be compatible for optimum results. This performance function variation also complicates the design of other equipment to be used in the control system. To permit a thorough evaluation of the system, the performance function should be established throughout the entire range of operation of the system.

In general, there are four methods available for determining the performance function:

1. by using theoretically derived equations
2. by the construction of a laboratory model of the system
3. by the construction of an analog
4. by conducting flight tests on the airplane using the actual system.

Each of these methods has its individual advantages and disadvantages.

Theoretical equations rest upon assumptions and linear approximations. Although this limits their accuracy the equations can still yield useful engineering results. However, the computations necessary for the evaluation of the equations are tedious and time-consuming. In addition, the evaluation procedure must be repeated in entirety in order to study the effect of any change in a parameter. For the boost system under study the non-linearity and the variation of the coefficients makes the evaluation of the equations particularly awkward and limits the accuracy of the results. The advantage of the theoretical method is that it provides a means of estimating the performance of the system before it is built.

Design of a laboratory model of this system requires similar assumptions and approximations as in the theoretical approach. However, many actual components of the system can be used thereby reducing the number of approximations that are necessary. Problems of instrumentation
and measurement now arise, but in general are capable of being satisfactorily solved. The accuracy of the results is still limited. It would be either impractical or difficult to change the parameters of a system in order to improve its operation. For example, changing the flow characteristics of the control valve would entail a redesign of the valve which would require costly machine work. The advantages of using the laboratory installation are the ease and convenience with which the investigation can be performed, and the increased accuracy that can be obtained by using actual components of the system.

The analog method still necessitates carrying along assumptions and approximations and its results are thereby limited in accuracy. The practical problems involved may be considerably different from the laboratory installation. Modern and high-speed electric analog equipment presently available provides a flexible and fast method of covering large ranges of parametric variations. It can be used to synthesize changes in the dynamic relationships between the variables of the system. By this method, for example, a change in the valve characteristic could be provided by a twist of a knob. Should the affect of providing some means of compensation to the system such as integral or lead networks be desired, the analog, by insertion of standard components, can easily handle the change.

Use of the actual system itself does not require any approximations or assumptions. It, thereby, provides the most accurate method of investigation. Depending upon the particular physical system, it may be either difficult or impossible to perform the necessary tests and to carry out the instrumentation with ease and convenience. However, in contrast with all the previous methods, the physical parameters of the system now are present in their actual environment. Variation of loadings to cover the range of operation of the system are readily controlled. The increased difficulties of instrumentation and measurement are offset by the greater accuracy of the
results. One drawback to the method is that the airplane to be tested must exist and must be capable of being flown safely. With experimental models of new or radical design, preliminary work on laboratory models may be the only means available.

In the system under investigation, preliminary design has already been carried out and the system constructed. The limitations of the mathematical method now outweigh its advantages. It has been included here primarily because it provides a necessary basis for evaluating factors that must go into the design of an analog and because its previous existence is not known. The analog method has been briefly discussed by the authors and a discussion of a suitable analog is given in Appendix A. It is recommended that the analog be studied further particularly for the affect of changing the characteristics of the control valve.

The authors feel that the flight test method now presents the greatest advantages. The laboratory investigation has indicated that the characteristics of the input components to the sector play an important role in inducing oscillations. In addition, the flight tests take into account many of the factors that were approximated in the laboratory and some that were omitted. Among these are:

1. The actual installation in the airplane consists of a complete sector, a valve, and a linkage for both the right and left aileron, and these are coupled together; the laboratory model consisted of only one of these systems.

2. Flight tests provide the actual aerodynamic loads on the system; in the laboratory these are difficult to approximate closely.

3. The structural flexibility and deflections of a large airplane of the B-45 class should be taken into account; the laboratory model does not do this.

4. Damping provided by the cables and other linkages should also be taken into account.
Some of these factors can be included in further laboratory work, but advantages to be gained by flight testing are considered to predominate. Therefore, the authors recommend that further determination of the performance function be done in the actual airplane.

Returning to the problem of oscillations present in this system, two suggestions are made for providing some improvement along these lines. First, the source of the hydraulic oscillations is believed to be due to the presence of sharp edges on the piston and the plunger that go to make up the orifices. It is felt that redesign of these parts in such a manner that the fluid flow would be more streamlined would lessen or even eliminate some of these oscillations. This would entail rounding the end of the plunger and its seat in the piston and also sounding the bottom of the piston and its seat in the body of the valve. Second, it is felt that the oscillations due to the coupling between the input system, namely, the pilot's yoke, the cables, etc., and the aileron system can be eliminated by opening up the link from the sector to the aileron. This would do away with the feedback path to the valve and also the pilot's feel. The pilot's feel could be restored by an equivalent spring system. A disadvantage would be the loss of control should the hydraulic system fail. This is a serious disadvantage and involves the safety of the flight. Means must be provided to actuate the aileron system in the event of power failure. Opening of the link could be done in such a manner that it could be closed again manually by the pilot in the event of power failure and thereby return the system to its present design.

B. Conclusions

From this investigation the authors arrived at the following conclusions:

1. The non-linear operation of the boost system produces a variable performance function. This variation in the performance function
complicates the design of suitable automatic control equipment to be used with this boost system.

2. Oscillations can be produced in the system through two means:

   (a) local variations in fluid flow through the orifices that set the pistons in the valve into oscillation

   (b) coupling between the input system to the sector and the aileron mass that produce oscillations in the system
CHAPTER X
RECOMMENDATIONS FOR FURTHER STUDY

The following tests are proposed for further study on the system:

1. Instrument the laboratory test system so that sinusoidal inputs can be used for obtaining the following frequency response characteristics:

   (a) Apply sinusoidal displacement inputs to a device to simulate the pilot's wheel or the automatic pilot drive motor. Connecting cables to the sector should duplicate the elasticity of the control cables used in the aircraft. Record the beam and input displacements, and from these obtain the system frequency response characteristics. Input displacements should be limited to small values so that the effect of the system non-linearities may be minimized. Test runs could also be performed with the initial beam deflection set at various initial positions. The resultant family of curves will indicate the variation of the dynamic characteristics throughout the range of operation due to non-linearities.

   (b) Repeat (a) above, except that the system is to be operated as an open-loop as previously described. Comparison of the frequency response characteristics obtained from this set of runs compared with those of
(a) will determine if the system response is materially changed due to the feedback loop in the system.

(c) Obtain the frequency characteristics of the valve by applying sinusoidal inputs of either displacement or force to the valve and recording the output valve pressures $P_U$ and $P_D$.

(d) Investigate the variation in the valve characteristics due to the variation in the length and size of tubing between the pump and the valve and between the reservoir and the valve.

(e) Determine the effect of variation in the length and size of lines between the valve and the cylinder by recording the pressures at the cylinder.

(f) Determine the effect on the valve characteristics due to various flow rates of fluid supplied to the valve. This can be accomplished conveniently by installing a variable displacement pump in place of the present fixed displacement pump.

2. If analog components are available it is recommended that an analog system be constructed and used for the investigation of the system using the basic equations and the system constants derived in this report.

3. Minor modifications in the valve design can decrease the oscillatory nature of the valve. The change proposed at the present time is to bevel the sharp-edged valve orifices and mating plungers so that a more stream-line flow is obtained.
4. In the analysis of the system and in the laboratory tests, no attempt was made to include or duplicate the elastic deformations of the aircraft, the inertia of the connecting cables and sprockets, and the friction in the pulleys and moving parts. Further study may indicate that these items should be given more consideration.

5. A study should be conducted for the design of a suitable vibration absorber to be mounted on the sector linkage system or on the valve to reduce the magnitude of the inherent oscillations occasionally occurring in the valve.

6. It is also recommended that more detailed information be obtained from the aircraft manufacturer regarding the nature of the oscillations of the control system.

7. If an aircraft is available it is suggested that a similar investigation be made using the actual control system installed therein.

8. It is recommended that hydraulic tests be performed on the valve with the lower orifices almost closed to determine if cavitation occurs.
APPENDIX A

ANALOG COMPONENTS

The analog (1, 2) that was to be used in conjunction with the mathematical analysis utilizes electrical inputs to an interconnected network of operational units. The output from this system is a voltage which can be applied to an oscilloscope giving an image similar to the output of the physical system when subjected to the same form of inputs.

Individual parameters of the physical system can be represented by individual units of the analog. Thus the system response with variation of different parameters can be quickly obtained by the variation of the individual analog components.

A sketch of a basic component is shown in Fig. A-1.

![Diagram of a basic component](image)

**Fig. A-1.**

The box marked A is a high gain direct-current amplifier having an odd number of stages. The input voltage at 0 is connected to the grid of
the first tube of the amplifier. The boxes marked $Z_1$ and $Z_2$ represent impedances. If the grid of the first tube is negatively biased, grid current flow is negligible and

$$i_1 = i_2$$

Or in terms of the impedance functions and the voltage drops across them

$$\frac{e_1 - e_2}{Z_1} = \frac{e'_1 + e_2}{Z_2} \quad (A-1)$$

Also the output voltage of the amplifier is given by

$$e_2 = A e'_1 \quad (A-2)$$

So substituting the value of $e'_1$ from eq (A-2) into eq (A-1) gives

$$e_2 = \frac{Z_2}{Z_1} \left[ \frac{1}{1 + \frac{1}{A} \left(1 + \frac{Z_2}{Z_1} \right)} \right] e_1$$

If the gain of the amplifier is sufficiently large, the bracketed term becomes negligible giving;

$$e_2 = \frac{Z_2}{Z_1} e_1 \quad (A-3)$$
If the impedances of eq (A-3) are unequal resistances, then

\[ e_2 = \frac{R_2}{R_1} e_1 \]  \hspace{1cm} (A-4)

Thus, a means has been derived by which a quantity can be multiplied by a constant. This constant multiplying factor can easily be changed to other values as desired by making either \( R_1 \) or \( R_2 \) variable resistors or by using a dividing potentiometer connected across the output \( e_2 \).

The accuracy of the multiplying factor is determined only by the accuracy of the resistors and not by the amplifier, as long as the gain of the amplifier remains large.

By suitable choices for \( Z_1 \) and \( Z_2 \), the operation of integration and differentiation can be accomplished. Thus if \( Z_2 \) is a capacitor whose impedance, expressed by the use of the Heaviside operator, is \( 1/C_p \) and \( Z_1 \) is a resistance \( R \), then eq (A-3) becomes

\[ e_2 = \frac{1}{(RC)p} e_1 \]  \hspace{1cm} (A-5)

If the product \( RC \) is chosen as unity, the unit will perform as an integrator. If the elements of \( Z_1 \) and \( Z_2 \) are interchanged from the previous case, the output voltage can be expressed as

\[ e_2 = (RC)p e_1 \]  \hspace{1cm} (A-6)

thus comprising a unit that performs the basic operation of differentiation.
The operation of summation of a number of voltages from various sources can be accomplished by the arrangement shown in Fig. A-2.

The input voltages are represented by $e_a$, $e_b$, and $e_c$. The current equation can be written at the junction point $0$.

$$i_a + i_b + i_c = i_2 \quad (A-7)$$

Writing these currents as a function of the applied voltages and their respective impedances

$$\frac{e_a}{Z_a} + \frac{e_b}{Z_b} + \frac{e_c}{Z_c} = \frac{e_2}{Z_2} \quad (A-8)$$
If the impedances in the above equation are all equal resistances, then eq (A-8) becomes
\[ e_2 = e_a + e_b + e_c \quad (A-9) \]

Thus, a method is available for addition. If voltages of the proper polarity are applied, this method will also perform the operation of subtraction.

Multiplication of two variables can be accomplished by varying the resistance \( R_2 \) of eq (A-4) by means of a servo drive system. When the input to the servo system is varied, the variable resistance \( R_2 \) at the output of the servo system varies a proportional amount. Thus an output voltage is obtained which is proportional to the variable input voltage \( e_1 \) and to the variable resistor \( R_2 \).

Non-linear circuit elements that vary with variable circuit parameters can be approximated by a method described by Welti\(^{(3)}\). By this method, which utilizes cathode follower circuits, a curve can be approximated by a number of straight lines. The greater the complexity of the curve the larger is the number of approximating lines necessary for its analysis.

The analog is well suited when a system is analysed with a step input function. The input to the analog is a series of rectangular step pulses. The response of the analog network to a series of these pulses is illustrated in Fig. A-3 where the design of the analog components is such that the transient variation has effectively decreased to zero after each quarter cycle.
If the output of the analog is applied to an oscilloscope which has been so adjusted that only the first quarter cycle of each period is viewed, then a persistent image of the output response can be obtained.
APPENDIX B

BIBLIOGRAPHY


