FREQUENCY ANALYSIS OF AN UNDERWATER EXPLOSION

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TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>I. INTRODUCTION</th>
<th>Pg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>II. PRELIMINARY STATEMENT OF RESULTS</td>
<td>3</td>
</tr>
<tr>
<td>III. DISCUSSION OF DATA</td>
<td>4</td>
</tr>
<tr>
<td>IV. DETERMINATION OF ENERGY DISTRIBUTION</td>
<td>5</td>
</tr>
<tr>
<td>V. APPLICATION OF METHOD TO DATA</td>
<td>9</td>
</tr>
<tr>
<td>VI. CONCLUSIONS</td>
<td>11</td>
</tr>
<tr>
<td>TABLES AND GRAPHS</td>
<td>13</td>
</tr>
<tr>
<td>DATA AND ANALYTIC APPROXIMATIONS</td>
<td>18</td>
</tr>
<tr>
<td>APPENDIX A</td>
<td>26</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>27</td>
</tr>
</tbody>
</table>
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I. INTRODUCTION

The object of this thesis is to determine the distribution of energy in the frequency spectrum of an underwater explosion from a pressure vs time curve, and to present a preliminary study of this distribution as a function of the size of the charge.

In order to completely cover the question of energy distribution in the frequency spectrum it is necessary to consider the energy expressed as a percentage of the total, and also the distribution of absolute energy expressed in milliwatt-seconds. It is the absolute energy which is of greatest interest but the percentage distribution is useful in correlating the results.

The need for this frequency analysis arose as a result of recent investigations in the transmission of sound in sea water conducted by the Woods Hole Oceanographic Institution. A knowledge of the energy distribution will facilitate the study of long range signalling and will also aid in the study of absorption and reflection phenomena.

Former work on this problem is non-existing since heretofore underwater explosions were not used as a method of signalling and thus the frequency spectrum was of no significance. However, as soon as the transmission of sound
becomes the objective, a knowledge of the frequency spectrum of the source is of importance.

The ultimate objective of this work is to know the energy-frequency characteristics of an explosion, given the size, type and depth of the charge. Future work to obtain this objective may be divided into two parts:

1. The taking of a sufficient amount of data to make a systematic study of frequency distribution as a function of type, depth and size of charge.

2. Apply the method presented in this paper to determine the energy distributions.

Sufficient data should be taken to obtain reliable results.
II. PRELIMINARY STATEMENT OF RESULTS

1. The conditions giving the maximum absolute energy of 12.9 milliwatt-seconds occurred for the largest charge, 460 lbs. of TNT, in the 400-800 cps. octave.

2. Increasing the size of the charge tends to increase the percentage energy in the low frequency end of the spectrum and diminish the percentage energy in the higher frequency end (above 500 cps.).

3. The maximum percentage energy for the three charges, (60 lbs, 225 lbs and 296 lbs of TNT), are very nearly equal and occur in the vicinity of 400 cps.

4. 40% of the total energy occurs between 100 and 800 cps for the three charges mentioned above.
III. DISCUSSION OF DATA

The data for this Thesis was supplied by the U.E.R.L. Laboratories at the Woods Hole Oceanographic Institution, Woods Hole, Massachusetts. This data consisted of pressure vs time photographs of explosions of 2 lbs, 60 lbs, 225 lbs, 296 lbs, and 460 lbs of TNT. All of these records, except the 2 lb charge were taken at a distance of 20 feet from the explosion and at a depth of 41 feet below the surface. The 2 lb charge record was taken 4 ft from the explosion and also at a depth of 41 ft.

As is usually the case in underwater measurements, a large amount of data should be observed and averaged to eliminate the possibility of using erroneous data if only a few records are taken for a given set of conditions. Unfortunately sufficient data to obtain averages was not available so that it is felt that the actual numerical results obtained in this thesis may be considerably in error. Quantitative results will be stated but the emphasis will be placed on general trends.

In one case where two pressure-time curves were obtained for identical conditions, the results calculated from the two curves differed by 100%. This emphasises the need for taking sufficient data to obtain a reliable average.
IV. DETERMINATION OF ENERGY DISTRIBUTION

From a pressure-time curve of an underwater explosion it is desired to obtain the distribution of energy in the frequency spectrum. The method presented in this paper consists of obtaining the magnitude of the Fourier Integral, $F_f$, as a function of frequency, using this function to obtain the percentages of the total energy in octave bandwidths and then returning to the pressure-time curve to obtain the total energy.

Undoubtedly the most desirable method would be one which uses the pressure-time curve directly. A machine, the Cinema Integraph, is capable of performing the desired Fourier Integral, but unfortunately this machine is not in operation at the present time. A graphical method employing the original curve is quite unsatisfactory since it would consist of graphically integrating under curves of the type, $\int f(t) \cos \omega t \, dt$ and $\int f(t) \sin \omega t \, dt$ which would be a cumbersome task. Thus it is highly desirable to obtain an analytic expression which closely approximates the actual curve.

From a glance at a pressure-time curve it seems logical to center the attack on the exponential curve.

---

1 See Reference 1.
A single exponential can be found which closely approximates
the initial slope of the curve and another exponential can
be found which approximates the final slope. Thus it
seems logical that the sum of two exponentials might
closely match the curve at all points. This is actually
the case. The method used to match the sum of two exponentials
to the actual curve is illustrated in Appendix A.

Thus, choosing \( F(t) = a_1 e^{-b_1 t} + a_2 e^{-b_2 t} \) it is possible
to obtain the Fourier Integral of this function by
standard methods. The majority of the straight-forward
mathematics will be omitted, but it seems advisable to
include some of the intervening equations which might
be helpful in interpreting the energy equations.

\[
F(t) = a_1 e^{-b_1 t} + a_2 e^{-b_2 t}
\]

where \( a_1, a_2, b_1, \text{ and } b_2 \)
are determined by the
method presented in Appendix A.

\[
F_f = \int_{-\infty}^{\infty} F(t) e^{-j2\pi ft} dt
\]

\[
= \frac{a_1}{b_1 + j2\pi f} + \frac{a_2}{b_2 + j2\pi f}
\]

\[
|F_f| = \sqrt{\frac{A + B}{(b_1^2 + 4\pi^2 f^2)(b_2^2 + 4\pi^2 f^2)}}
\]

where \( A = (a_1 b_2 + a_2 b_1) \)
\( B = (a_1 + a_2)^2 \)

\( \text{1 Only the initial pulse of the explosion is considered.} \)
The percentage of the total energy which occurs in an octave bandwidth is given by the expression:
\[
\frac{\int_{f_0}^{2f_1} |F_f|^2 \, df}{\int_0^{\infty} |F_f|^2 \, df}
\]

Expanding the expression for $|F_f|^2$, obtained on the preceding page, as a sum of partial fractions and then performing the integration indicated above, the expression for percentage energy becomes:

\[
\left[ \frac{a_1^2}{360b_1} + \frac{a_1a_2}{180(b_1+b_2)} \right] \left[ \tan^{-1}\frac{2\pi f}{b_1} - \tan^{-1}\frac{2\pi f}{b_1} \right] + \left[ \frac{a_2^2}{360b_2} - \frac{a_1a_2}{180(b_1+b_2)} \right] \left[ \tan^{-1}\frac{2\pi f}{b_2} - \tan^{-1}\frac{2\pi f}{b_2} \right]
\]

\[
\frac{a_1^2}{4b_1} + \frac{a_1a_2}{b_1+b_2} + \frac{a_2^2}{4b_2}
\]

where the inverse tangents are expressed in degrees.

It now remains to obtain the expressions for total energy. The analytic approximation will also be used in this derivation which consists of evaluating the expression:

\[
\text{Total Energy} = \frac{4\pi R^2}{\rho c \times 10^7} \int_0^\infty F^2(t) \, dt
\]

\[
= \frac{4\pi R^2}{\rho c \times 10^7} \left[ \int_0^\infty a_1^2 e^{-2\pi f t} \, dt + \int_0^\infty a_2^2 e^{-2\pi f t} \, dt + \int_0^\infty a_1 a_2 e^{-2\pi f t} \, dt \right]
\]

\[
= \frac{8\pi R^2}{\rho c \times 10^7} \left[ \frac{a_1^2}{4b_1} + \frac{a_2^2}{4b_2} + \frac{a_1a_2}{b_1+b_2} \right]
\]

$F_t$ is in cgs units.
The expression in parenthesis in the last equation is recognized as the same expression which appears in the denominator of the percentage energy equation. Thus, the absolute energy in the octave $f_1$ to $2f_1$ can be expressed as:

$$\frac{8 \pi R^2}{c \times 10^2} \left[ \left( \frac{a_1^2}{360 b_1} + \frac{a_2}{180(b+b_1)} \right) \left( \tan^{-1} \frac{4\pi f_1}{b_1} - \tan^{-1} \frac{2\pi f_1}{b_1} \right) + \left( \frac{a_2^2}{360 b_2} + \frac{a_1 a_2}{180(b+b_2)} \right) \left( \tan^{-1} \frac{4\pi f_1}{b_2} - \tan^{-1} \frac{2\pi f_1}{b_2} \right) \right]$$

where $R$ is the distance of the pressure element from the explosion in centimeters.

$c = 1.5 \times 10^5$ cgs units.

It is convenient to evaluate the energies for defined octaves. For this purpose the octaves 25 cps to 50 cps, 50 to 100, 100 to 200, 200 to 400, 400 to 800, 800 to 1600, 1600 to 3200 and 3200 to 6400 were chosen.
V. APPLICATION OF METHOD TO DATA

Having developed a method of attack, the remainder of this paper is concerned with applying it to the data and then correlating the results as far as is consistent, as previously discussed.

Photographs of the actual data and the comparisons of the actual and the approximate curves are found on Pages 18 to 25 inclusive.\(^1\) In every case the analytic curve is observed to compare favorably (with) the actual curve except in the vicinity of \(t=0\). To attempt to match in this region would be unwarranted since it is very likely that the initial peak reading is in error due to the poor response of the hydrophone at very high frequencies.

The resulting percentage energy distributions and absolute energy distributions are tabulated in Tables I and II respectively, on Page 13.

Once again it should be mentioned that the results hardly warrant plotting and the connecting lines on Graphs C and D merely serve to indicate the general trend.

The plot of percentage energy distribution for the 2 lb charge is observed to possess two maxima. Physically there is no explanation for this, but mathematically, assuming the actual curve can be represented by the sum of two exponentials, the reason is obvious. As the difference

\(^1\) Photographs of the 2 lb and the 225 lb charge were not available.
between the values of the slopes of the two component
exponentials becomes greater, the effect is to increase
the spacing between the maxima of the two inverse tangent
curves in the resulting expression for percentage energy
distribution. In the 2 lb case this difference was
sufficient to cause the sum of the two inverse tangents
to have two maxima. Thus it appears as though this method
fails under such conditions.
VI. CONCLUSIONS

The percentage energy distributions for the 60 lb, 225 lb, and the 296 lb charges can be correlated very effectively, but the 2 lb and the 460 lb charges do not seem to follow the general trend. Since this lack of consistency also appears in the data it is probably a combination of two reasons. First, the possibility of erroneous data and secondly, the effects of the non-linear properties of seawater.

For the three percentage energy distributions plotted on Graph A, the following observations should be noted: In each case the maximum percentage energies are very nearly the same and occur in the vicinity of 400 cps. At higher frequencies the percentage energy for the largest charge decreases most rapidly, while in the low frequency range the percentage energy for the smallest charge decreases most rapidly with decreasing frequency.

It is also interesting to note that approximately 40% of the total energy is in the range between 100 and 800 cps. Another point of interest is the rapidity with which the percentage energy decreases after the maximum has been reached. The percentage energy in the last three octaves during which the energy is increasing is approximately 1.5 times the percentage energy in the first three octaves during which the energy is decreasing. This indicates that the energy is more concentrated in the
lower portion of the spectrum.

Although the percentage energy correlation is interesting it is really the absolute energy which is of interest. Graphs C and D show that the maximum energy is obtained in the 400 to 800 cps octave for the largest charge. The real value of this plot would only be realized if the calculated points were sufficiently reliable to produce a smooth curve. Thus it would be possible to give the effectiveness of any size charge between 2 lbs and 460 lbs at a depth of 41 ft below the surface, in any frequency range. However, as the graphs stand now, they are not sufficiently accurate to indicate more than the order of magnitude and the general trend.
### Size of Charge

<table>
<thead>
<tr>
<th>Octaves (cps)</th>
<th>2 lbs</th>
<th>60 lbs</th>
<th>225 lbs</th>
<th>296 lbs</th>
<th>460 lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 - 50</td>
<td>4.5 %</td>
<td>7.0 %</td>
<td>8.4 %</td>
<td>10.5 %</td>
<td>9.2 %</td>
</tr>
<tr>
<td>50 - 100</td>
<td>7.7</td>
<td>10.3</td>
<td>10.6</td>
<td>13.5</td>
<td>11.5</td>
</tr>
<tr>
<td>100 - 200</td>
<td>10.8</td>
<td>12.6</td>
<td>12.6</td>
<td>15.5</td>
<td>12.1</td>
</tr>
<tr>
<td>200 - 400</td>
<td>11.0</td>
<td>13.6</td>
<td>15.8</td>
<td>15.7</td>
<td>13.8</td>
</tr>
<tr>
<td>400 - 800</td>
<td>9.7</td>
<td>15.1</td>
<td>16.2</td>
<td>13.6</td>
<td>14.9</td>
</tr>
<tr>
<td>800 - 1600</td>
<td>10.7</td>
<td>14.1</td>
<td>11.7</td>
<td>8.6</td>
<td>11.9</td>
</tr>
<tr>
<td>1600 - 3200</td>
<td>12.5</td>
<td>9.6</td>
<td>6.6</td>
<td>4.6</td>
<td>7.2</td>
</tr>
<tr>
<td>3200 - 6400</td>
<td>11.9</td>
<td>5.3</td>
<td>3.5</td>
<td>2.3</td>
<td>3.8</td>
</tr>
</tbody>
</table>

### PERCENTAGE ENERGY DISTRIBUTION

**TABLE I**

<table>
<thead>
<tr>
<th>Octaves (cps)</th>
<th>2 lbs</th>
<th>60 lbs</th>
<th>225 lbs</th>
<th>296 lbs</th>
<th>460 lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 - 50</td>
<td>2.46</td>
<td>104</td>
<td>395</td>
<td>765</td>
<td>1542</td>
</tr>
<tr>
<td>50 - 100</td>
<td>4.22</td>
<td>153</td>
<td>495</td>
<td>986</td>
<td>1925</td>
</tr>
<tr>
<td>100 - 200</td>
<td>7.45</td>
<td>187</td>
<td>593</td>
<td>1128</td>
<td>2030</td>
</tr>
<tr>
<td>200 - 400</td>
<td>6.06</td>
<td>198</td>
<td>742</td>
<td>1143</td>
<td>2320</td>
</tr>
<tr>
<td>400 - 800</td>
<td>5.37</td>
<td>224</td>
<td>764</td>
<td>994</td>
<td>2500</td>
</tr>
<tr>
<td>800 - 1600</td>
<td>5.87</td>
<td>209</td>
<td>564</td>
<td>625</td>
<td>1990</td>
</tr>
<tr>
<td>1600 - 3200</td>
<td>6.92</td>
<td>142</td>
<td>312</td>
<td>337</td>
<td>1198</td>
</tr>
<tr>
<td>3200 - 6400</td>
<td>6.56</td>
<td>78.6</td>
<td>163</td>
<td>165</td>
<td>632</td>
</tr>
</tbody>
</table>

**ABSOLUTE ENERGY DISTRIBUTION (milliwatt-sec)**

After used logarithm on log
and lost 10² elementary

1542 milliwatt-sec \( \rightarrow \) 1.83 kWh
Energy vs. Size of Charge for Various Octaves

Graph C

Energy (milliwatt-seconds x 10^2)

Size of Charge (lbs of TNT)
Energy vs Size of Charge
for Various Octaves

Graph D

Energy (m.w.e. x 10^-2)

Size of Charge (lbs of TNT)
Shot No 370   Film 232
Charge TNT
Wt 2*   R 4'
Actual Points 0
Analytic Curve
Shot No. RE194    Film No. R632
Charge TNT
Wt 60*    R 20'

Actual Points o
Analytic Curve

Pressure in psi

Time in msec
Shot No RE136    Film No R101
Charge TNT-A MK41
Wt 225*    R 20'

Actual Points  
Analytic Curve
Shot No.: RE131   Film No. R383

Charge: TNT-A MK 6

wt 296#   R 20'

Actual Points 0
Analytic Curve
Shot No RE181  Film R580

Charge TNT-A  MK29DB

wt 460kg  R 20'

Actual Points
Analytic Curve

Time in msec
METHOD FOR MATCHING THE SUM OF TWO EXPONENTIAL CURVES TO THE ACTUAL CURVE

The method of matching used in this thesis consisted of the following procedure:

1. Two points were plotted on semi-log paper which define the tail of the actual curve, i.e. "a" and "b" in Fig. 1.

2. Two other points were chosen at positions approximately those of "c" and "d" Fig. 1. From these values the ordinates of Fig. 2, at the corresponding times, were subtracted.

3. Plot these differences on the semi-log paper thus defining the second exponential.

Thus the actual curve is matched at the points a, b, c, and d. The slope of the second exponential is usually so steep that it does not effect points "a" or "b". A proper selection of points is essential to give a good approximation, but after a few attempts the proper choice is usually obvious.
BIBLIOGRAPHY