RENT PAYMENT AND HOUSING CONSUMPTION PATTERNS
OF RENTER HOUSEHOLDS

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ABSTRACT

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This is a study on how much of their income renter households are paying for their rent, and on housing expenditure patterns of renter households.

The result of this study will serve three purposes: In the first place, it will serve as a basic knowledge of renter households' behavior on rent expenditure. Secondly, the result will be helpful to see the effects of housing subsidy programs. Thirdly, given a set of reasonable contribution rates, it will serve in measuring the rent burden of household groups.

Two methods are adopted and used in this study. The first is the review and re-examination of existing housing consumption theories and of existing empirical works on housing consumption. The second is the cross-tabulation analyses on housing expenditure patterns of a one-out-of-a-hundred sample of renters in the Boston SMSA, and the corresponding regression analyses based on the same data of the Boston SMSA.

Rental expenditure patterns of renters by their income and characteristics such as household size, sex and race of head, and age of head are studied. In addition, regression equations, which are stratified by household size to estimate rent-income ratios of household groups, are specified. Also, basic information needed for running multiple regression are specified for future analyses. Results are summarized in the Conclusion section.

Several suggestions on further extension of this study follow.

Thesis Supervisor: Joseph Ferreira
Title: Assistant Professor of Urban Studies and Planning
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May 30, 1975    Hiroshi Ueno
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I. INTRODUCTION

This is a study on how much of their incomes renter households are paying for their rent, and on renter households' behavior concerning housing consumption; that is to say, the emphasis is on the effects of household characteristics on housing expenditures. There are two pressing reasons for pursuing this study.

In the first place, a direct housing assistance program, sometimes called a "housing allowance" program, requires us to set up a fair contribution rate for participants in the program. One of the first steps to develop reasonable contribution rates will be a study of housing consumption patterns of households.

The second issue is the consideration of equity in housing subsidy allocation. Equitable allocation of subsidy funds is a recent concern among housing researchers.\(^1\) Equitable allocation should be based on housing deprivations. One of the most important housing deprivations is the rent burden. In order to measure rent burden, one must know how much tenants are actually paying for their rent.

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I-A. Products Resulting from this Study

The direct objectives of this study will be well defined by stating the products of this study. There are two products, which are:

1) An empirical model of renter households' behavior with regard to housing consumption, and

2) A table of how much rent tenants pay by household size and income, or regression equations to estimate how much rent tenants pay.

I-B. Purposes

There are several reasons why the development of these products is necessary. First, to design an equitable housing program, it is necessary to set up a reasonable contribution rate for each type of household, as mentioned before. The conventional 25 percent (of income) contribution rate is too rough to be equitable.\(^2\) Reasonable contribution rates should be based on how much renters can afford to pay for rent. This is a question that is difficult to answer, but it is possible to answer the question of how much rents tenants of various types are paying at present. What they can afford to pay could be higher or lower than the present rent, however this study will show how renter households

\(^2\)I. Lowry, Rental Housing in New York City, the New York City Rand Institute, New York City, 1971, p. 140.
will behave on rent expenditure under the present conditions. Although the knowledge of renter's behavior is not directly related to what they should pay, without the basic knowledge of renter's behavior on rent expenditure, it would be difficult to establish criteria on what they should pay.

Secondly, a direct cash assistance housing program requires a model of renter behavior in order to predict the effects of the program. If the characteristics of a household and amount of its direct cash assistance are known, one can estimate how much the household will pay for rent, using the result of this study of households' behavior.

Thirdly, as stated above, quantitative measurements of rent burden are necessary to establish an equitable housing subsidy allocation. If we have a reasonable contribution rate, as given; for instance, standard budgets of low-and moderate-income households estimated by Bureau of Labor Statistics, results of this study (that is, how much rent tenants are paying) will show the degree of rent burdens borne by the various types of households. For example, such questions as, "Do aged households have a rent burden?" and, "How much of a burden do they have?" could be answered by the combination of this study and a rent burden criterion. Estimates of rent burden could be used as one of the indicators by which households could be judged to be assisted by a housing program.
I-C. Method

To develop the two products described in section I-A, the following two methods have been adopted:

1) Review and re-examination of existing housing consumption theories and existing empirical works on housing consumption, and

2) Development of graphs based on various tables derived from controlled and uncontrolled data, and regressions of some of those tabulated data.

Chapter II examines existing theories and empirical works. Section II-A reviews and discusses both the existing qualitative theories on household behavior in household consumption and some important concepts in studying housing expenditure. Section II-B reviews and analyzes existing quantitative theories on housing consumption. Section II-C reviews and criticizes existing empirical works on household behavior with regard to housing consumption and draws conclusions about household behavior, based on the re-examination of the literature.

Chapter III studies household behavior with regard to housing consumption using new data of the Boston SMSA, extracted from the Public Use Sample of the 1970 U.S. Housing

3"Controlled" and "uncontrolled" tables will be explained in Chapter III.
Census. Section III-A discusses specific objectives of the Boston study, the data and concepts used, and the methods. Section III-B analyzes the results of the cross-tabulations based on the controlled data. Section III-C analyzes the results of cross-tabulations and regressions based on the uncontrolled data.

Chapter IV contains the conclusions of this thesis based on Chapter II and Chapter III. Section IV-A summarizes the findings and postulates housing consumption models. Section IV-B summarizes the results of the research on how much rent tenants pay. Section IV-C tries to see an implication of this study on rent burden. In the final section, IV-D, what was accomplished by this study and what remains for further research are discussed as a conclusion.
II. HOW MUCH RENT TENANTS ARE PAYING: THEORIES AND EMPIRICAL WORKS

II-A. Factors Influencing Housing Expenditure: Qualitative Theory

II-A-1. Rent, income and price

a. Gross rent versus contract rent

Housing consumption is generally thought to include a package of goods, including the physical structure and various services and other amenities. In other words, housing is more than simply a physical shelter or space, it is a comprehensive concept. Therefore, in addition to costs included in contract rent, utility expenditures for water, electricity, gas, fuel, etc. and possibly transportation costs should be included in a measure of housing expenditure. Utility expenditures vary by region because of various factors such as winter temperature or average income of a region.¹

Gross rent is a more appropriate indicator of housing expenditure than contract rent. However, one must remember that even if gross rents are the same among a group of families, what each household gets may be considerably dif-

¹For example, see M. Reid, Housing and Income, University of Chicago Press, 1962, pp.46-47.
different. Some households may have less spacious housing of
good quality, while the others may have low quality housing
with relatively large sizes. In addition, some households
may have furnished housing with such conveniences as dish-
washers, air conditioning, refrigerators or other furniture,
while others may be paying for unfurnished housing.

b. Permanent income versus current income

Several studies have indicated that a family is likely
to determine the level of its housing expenditures based on
its "anticipated average income over time" rather than cur-
rent income. This "anticipated average income over time"
or "long-run expected income"\(^2\) is also called permanent in-
come in general. The concept that is needed in determining
housing expenditure is purchasing power. Permanent income
represents earning power but neglects wealth considerations;
therefore, it does not accurately represent purchasing
power. However, permanent income is generally regarded as
a better measurement than current income.

The concept of permanent income was proposed by Milton
Friedman. Friedman states that the current income of a
consumer unit \((Y_c)\) consists of two components; a permanent
component \((Y_p)\), which is the amount a consumer unit believes
that it could consume while maintaining its wealth intact,

and a transitory component \( (Y_t) \). Friedman postulates that
\[
Y_c = Y_p + Y_t. \tag{3}
\]

In explanation, Friedman states that:

The permanent component is to be interpreted as reflecting the effect of those factors that the unit regards as determining its capital value or wealth: the non-human wealth it owns; the personal attributes of the earners in the unit, such as their training, ability, personality; the attributes of the economic activity of the earners, such as the occupation followed, the location of the economic activity, and so on. It is analogous to the "expected" value of a probability distribution.

The transitory component is to be interpreted as reflecting all "other" factors, factors that are likely to be treated by the unit affected as "accidental" or "chance" occurrences...

In statistical data, the transitory component includes also chance errors of measurement... \( \text{4} \)

If we use this definition, however, we are still not sure how to measure permanent income empirically. One assumption underlying this definition is important to make the permanent income concept clear. He stated, just before this definition, that

...suppose Mr. A's measured income in any period is decidedly lower than the average measured income of a group of individuals who are similar to him in characteristics that we have reason to believe affect potential earnings significantly—for example, age, occupation, race, and location. It then seems reasonable to suppose that Mr. A's measured income understates his permanent income.

---


4Ibid.
For any considerable group of consumer units, the resulting transitory components tend to average out, so that if they, alone, accounted for the discrepancies between permanent and measured income, the mean measured income of the group would equal the mean permanent component, and the mean transitory component would be zero.5

The following picture (Figure II-1) may be helpful to understand this statement.

![Figure II-1](image)

**Figure II-1.** Probability Distributions of Current Incomes and Permanent Incomes In One Group of Households.

Note that the permanent income distribution is more concentrated, but both permanent and current income distribution have the same mean.

If a demographic group of households is defined small enough and precisely enough to suppose those households are really homogeneous in every characteristic with respect to incomes, then the permanent income of a household in the group is the same as the average current incomes of the group. Namely, the deviation of current income of a unit from the average current income of the group is totally attributable to the transitory income of the unit. This is a common assumption among researchers who used a concept of permanent income.

Friedman's second postulate is that permanent income and transitory income are uncorrelated with one another. The assumption implies that if we take a random sample of households from a particular demographic group, the households with higher current incomes are more likely to have positive transitory incomes. This bias toward average positive transitory income for high-income households drives estimates of income elasticity based on current incomes downward, as compared to those estimates that are based on permanent income. Because of this bias created by using current income, it is generally accepted among researchers that households' behavior in housing consumption should be based on their permanent income. There are,

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however, several problems associated with the permanent income concept. First, the procedure for estimating the permanent income of a household is not well defined by Friedman. Definition of groups or definition of computation forms are left to each researcher. The lack of consensus causes problems, because each researcher defines permanent income by his own method, thus making it hard to compare the results of the various researchers. There are two major methods used to define permanent income: to take an average of time series data of individuals or to take the average current income of a group at one point in time. The results of these methods differ from each other considerably.

There is also the problem of data gathering. To gather time series data of individuals requires a great deal of work. Therefore, most researchers use the group average method of estimating permanent income. For the purpose of making such estimates, however, they have to have groups sufficiently small that they can be assumed to be homogeneous in terms of earning powers. Since it is almost impossible to disaggregate data into such small groups, most researchers use fairly large groups for their data clusters. Those large groups, in fact, average out substantial variations in earning powers of households in the group. Consequently, estimated permanent incomes are too concentrated to the middle-income level.
In summary, past theoretical writings and past studies suggest that permanent income estimates based on time series individual data are the most reliable for a study on housing consumption behavior but difficult to measure.

c. Disposable income versus gross income

Among families whose gross incomes are the same, actual disposable incomes which can be used for consumption vary widely, depending on the source of incomes, because of the federal and local tax structures and differences in work related expenses such as commuting costs. If gross income were used to study rent-income ratios, then non-taxable income such as welfare payments would be treated in the same way as taxable income such as wage. This would cause rent-income ratios of households with non-taxable incomes to appear to be higher than the ratios of those with taxable incomes. Therefore, a better measure of purchasing power is disposable income. I. Lowry also recommends the use of disposable income in order to define a household's ability to pay rent. In his report, submitted to the City of New York, Lowry's definition of disposable income is gross income minus taxes, work-related expenses, and involuntary insurance. The definition of disposable income is mean-

ingful and workable.

One problem associated with estimating disposable income is how to measure total taxes paid, work-related expenses and involuntary insurance.

d. Price of housing

In addition to household income, price is one of the most influential factors in housing expenditures. Price varies by region and by sector in an SMSA. To measure the price of housing, one needs to be able to measure the quantity of housing. In most cases, housing quantity is measured by either total floor space or number of rooms in a unit. Neither measure is a rich enough measure of housing quantity to be satisfactory by itself.

In general, to determine how price varies by location, rents of units with the same structural characteristics such as same size, same type, same age, etc., but which are located in different sectors are compared to obtain relative prices of various units of housing. One problem in determining these relative prices is that, since housing in different locations differ from each other, it is hard to find comparable housing units in various sectors.

The effects of price on housing demand are generally described by the following:

\[ H_1 = Y_1 (P)^{B_1} \]  

\[ (2.1) \]
where

\( H_i \) = quantity of housing demanded by a household \( i \)
\( Y_i \) = income of a household \( i \)
\( P \) = price of housing in the area
\( \alpha \) = coefficient
\( B_1 \) = income elasticity
\( B_2 \) = price elasticity

If the price elasticity is unity, (2.1) can be transformed into

\[ R_i = H_i P = \alpha_1 (Y_i)^{B_1} \]

(2.2)

where

\( R_i \) = rental expenditure or rental cost of household \( i \).

This assumption of unity price elasticity implies that, even though housing consumptions vary as to the price of housing changes, rental expenditures are kept constant. Such an assumption solves the problem of price measurement.

For studies such as this one, which focus on the effects of household characteristics, including income, the above assumption greatly facilitates the process of the research and is not inconsistent with empirical findings.
II-A-2. Household Characteristics that Influence Housing Consumption Pattern

The variables that are briefly reviewed in this section are those which have already been revealed in existing studies to have effects on housing expenditures. There could be other minor variables which may have effects; however, the major ones are the following four variables, in addition to income and price, which have already been discussed.

a. Household size

Household size, i.e., number of persons in a household, is expected to have effects on housing consumption. One theory says that household size has negative effects on housing consumption because any additional person in a household tends to increase food consumption and other costs more than housing. Explanations in support of this theory are that two or three persons can share one room, and also that families can find larger units without increasing rents by choosing units in low quality structures or in low quality communities.

b. Age of head

It is commonly said that elderly households have excessively high housing expenditures. One explanation is that

---

elderly families prefer to live in good quality housing. Another example of driving the ratio of aged households high is that aged households tend to have less current income but relatively more wealth than younger households. At the same time, households with young heads are revealed in some researches\(^9\) to be paying high rents, although this finding is not commonly accepted among researchers.

c. **Sex of head**

It is also generally accepted that female-headed households are paying relatively high rent. An explanation in support of this theory is that females tend to put a high value on security; therefore, they tend to occupy high rent units. Another explanation for this is that discrimination against female-headed families may force them to live in high rent units instead of standard units.

d. **Race of head**

The race of the head of a household is also supposed to have effects on housing expenditures. Contrary to the common expectation, researchers have found that white households pay higher rents than non-white households. One explanation of this is that there is discrimination against non-white households, which forces non-white households to

live in deteriorated neighborhoods where relatively inexpensive units exist. As a consequence, such discrimination decreases rents by lowering housing quality. Another explanation is that white households prefer to live in high quality housing.
II-B. Price and Income: Quantitative Theory

In this section, a basic economic theory of housing expenditures of every household is presented and discussed. Before considering empirical research, a discussion about the theoretical model of household behavior is presented, and the implications of this model on housing expenditures is discussed, in order to clarify the hypotheses that empirical researchers should test. The discussion of the model includes consideration of the utility function of households, their implied housing demand function and the income elasticity of demand for housing.

II-B-1 Utility function and demand function

The theory of household rent expenditures presented in this thesis is based on the household's utility maximization theory, which, in turn, has been derived from neoclassic economics description of how individuals decide what goods and services to purchase, in terms of utility function of the purchased items. Utility is defined as the satisfaction individuals obtain through purchasing various goods and services. The same theory may be applied to households. Each household, assumed to be "rational", will try to purchase goods and services in order to maximize its total satisfaction subject to its income constraint. The simplest form of utility or satis-
faction is described by a Cobb-Douglas type utility function:

\[ U_i = H^{\alpha_i} X (1-\alpha_i), \quad (2.3) \]

where

- \( U_i \) = total utility of the \( i \)th type of household
- \( H \) = quantity of housing services
- \( X \) = quantity of non-housing goods and services
- \( \alpha_i \) = coefficient which represents the degree of preference and which has a value between 0 and 1.0.

The income constraint is described by the following function:

\[ Y_i = P_h H_i + P_x X_i, \quad (2.4) \]

where

- \( Y_i \) = income of the household
- \( P_h \) = price of housing \( H \)
- \( P_x \) = price of other goods and services \( X \).

From these two equations, the quantity of housing \( (H_i) \) purchased by the household can be derived. In addition, by multiplying \( H_i \) by \( P_h / Y_i \), the housing expenditure to income ratio can also be derived from (2.4) as follows:

\[ X_i = \frac{Y_i - P_h H_i}{P_x} \quad (2.5) \]

Substituting (2.5) for \( X \) in equation (2.3) we get:
The total utility of the household in terms of housing quantity can be maximized by setting
\[ \frac{\partial U_1}{\partial H_1} = 0, \]
yielding:
\[ \alpha_1 H_1 (\alpha_1 - 1) \left( \frac{Y_1 - P_h H_1}{P_x} \right)^{(1 - \alpha_1)} + H_1 \alpha_1 (1 - \alpha_1) \left( \frac{Y_1 - P_h H_1}{P_x} \right)^{-\alpha_1} \left( \frac{P_h}{P_x} \right) = 0 \]  
(2.7)

Multiplying (2.7) by
\[ H_1 (\alpha_1 - 1) \left( \frac{Y_1 - P_h H_1}{P_x} \right)^{\alpha_1} P_x, \]
we get
\[ \alpha_1 (Y_1 - P_h H_1) - H_1 (1 - \alpha_1) P_h = 0, \]
or
\[ H_1 (\alpha_1 - 1) P_h + P_h \alpha_1 Y_1 = 0. \]

Thus,
\[ H_1 = \frac{\alpha_1 Y_1}{P_h}, \]
\[ H_1 = \alpha_1 (Y_1)^{1}(P_h)^{-1}, \]  
(2.8)

\[ R = H_1 P_h = \alpha_1 Y_1, \]
and
(2.9)

\[ 1^{\text{Nkanta F. Ekanem, The Demand for Housing: An Analysis by Family Type, The Urban Institute, Washington, D.C., 1972, p. 2.}} \]
\[
\frac{R}{Y} = \frac{H_1 p_h}{Y_1} = \alpha_{11},
\]

where

\[R = \text{rental expenditure}.\]

As shown above, a housing demand function (2.8), a housing expenditure equation (2.9), and the rent-income ratio equation (2.10) can be derived from the special Cobb-Douglas type utility function. Equations (2.9) and (2.10) imply that housing expenditure of the \(i\)th type of household is always a constant proportion of that household's income. In other words, the housing expenditure-income ratio is constant for all households of \(i\)th type, regardless of their incomes. Equation (2.8) also implies that the income elasticity of housing demand is \(+1\) and the price elasticity is \(-1\).

The general Cobb-Douglas type utility function is given by

\[
U_i = \alpha_{11} H_1 \alpha_{i1} x_1 \alpha_{i2}.
\]  

If an \(i\)th type household maximizes its utility under the constraint of its income, then

\[
\frac{\partial U_i}{\partial H_1} = \alpha_i \left( \frac{Y_i - p_h H_1}{P_x} \right) \alpha_{i1} H_1 \alpha_{i1} - 1
\]

\[
+ \alpha_{i2} \left( \frac{Y_i - p_h H_1}{P_x} \right) \alpha_{i2} \left( \frac{p_h}{P_x} \right) = 0.
\]

Using the same procedure described in (2.7) through (2.10), we can obtain a housing demand function and a housing expen-
diture-income ratio equation as follows:

\[ H_i = \alpha_{11}(Y_i)^{+1}(P_{hi})^{-1}(\alpha_{11}+\alpha_{12})^{-1}. \]  

(2.13)

Since rent \( R = P_i H_i \), (2.13) can be rewritten in terms of household rent-income ratio so that

\[ \left( \frac{R}{Y} \right)_i = \alpha_{11}(\alpha_{11}+\alpha_{12})^{-1} \]

(2.14)

where \( \left( \frac{R_i}{Y_1} \right) = \left( \frac{R}{Y} \right)_i \).

As illustrated in equations (2.13) and (2.14), the general Cobb-Douglas type utility function suggests the same housing expenditure pattern as the special Cobb-Douglas type utility function does: namely, that the housing expenditure-income ratio is constant, and that the income elasticity and price elasticity are unity; i.e., +1 and -1, respectively. That is:

\[ \left( \frac{R}{Y} \right)_i = \alpha_1, \]  

(2.15)

\[ E^h_y = +1, \text{ and} \]

(2.16)

\[ E^h_p = -1, \]  

(2.17)

where

\[ E^h_y = \text{income elasticity of housing demand} \]

\[ E^h_p = \text{price elasticity of housing demand} \]

\[ \alpha_1 = \alpha_{11} \cdot (\alpha_{11} + \alpha_{12}). \]

A summary of equations (2.1) through (2.17) is contained in Figure II-2.
Figure II-2. Graphic Explanation of Utility Maximization Theory.
II-B-2. Two formulae of demand function

We have just shown that Cobb-Douglas utility functions imply unitary income and price elasticities and constant rent-income ratios. Although these rent-income ratios are constant for each household, the theory does in general allow $\alpha_1$, a housing preference coefficient, that is, the rent-income ratio, to vary by household types.

The coefficient $\alpha_1$, which represents the degree of preference, is dependent on household type. Thus, $\alpha_1$ can be a function of household characteristics. $\alpha_1$ can be described in the following two ways, depending on whether one assumes an additive or multiplicative effect of the various characteristics,

$$\alpha_1 = \alpha_0 + \alpha_1 c_1 + \alpha_2 c_2 + \ldots \quad (2.18)$$

or

$$\alpha_1 = \alpha_0 \cdot c_1 \alpha_1 \cdot c_2 \alpha_2 \cdot c_3 \alpha_3 \ldots \quad (2.19)$$

where

$\alpha_1, \alpha_2, \alpha_3 \ldots = \text{coefficients which represent the degree of preference}$

$C_1, C_2, C_3 \ldots = \text{household characteristics such as number of persons in a household or race of head}$

Rearranging equation (2.15) leads to

$$R_1 = \alpha_1 Y_1. \quad (2.20)$$

Substituting (2.18) and (2.19) for $\alpha_1$ contained in (2.20), we write
housing expenditure as:

\[ R_1 = Y_1(C_0 + \alpha_1 C_1 + \alpha_2 C_2 + \ldots), \]  

(2.21)

or

\[ R_1 = \alpha_0 Y_1 C_1 \alpha_2 C_2 \alpha_3 C_3 \ldots \]  

(2.22)

These formulae represent two theoretical expressions of housing expenditure, used in conjunction with Cobb-Douglas utility functions. Note that both forms assume that the effect of household characteristics will be to adjust the percentage of income devoted to rent. The difference lies in how the various characteristics (e.g., family size) interact. If all the \( C_i \)'s were dummy variables, their definition could be adjusted so that (2.20) and (2.22) are equivalent.

One basic problem with the Cobb-Douglas utility function approach stems from the unity elasticity assumption: Empirical research\(^2\) suggests that income and price elasticities are not unity but are somewhat less than 1.0 and slightly greater than -1.0, respectively. An alternative housing demand function that permits such assumption is

\[ H_1 = \alpha_1 (Y_1)^{B_1} (P_1)^{-B_2}, \]  

(2.23)

which implies that

\[ R = \alpha_1 (Y_1)^{B_1} (P_1)^{1-B_2}, \]  

(2.24)

where

\[ B_1 \text{ and } B_2 = \text{the income and price elasticities} \]

which are values between +1 and 0.

Note that (2.23) is a Cobb-Douglas type demand function. If
\[ B_1 = B_2 = 1, \] we have the same demand function as before.

Substituting (2.18) and (2.19) for \( \lambda_1 \) of (2.24) gives
us the new set of housing expenditure formulae, as expressed
in (2.25) and (2.26):

\[
R = Y^{B_1}(\lambda_0 + \lambda_1 c_1 + \lambda_2 c_2 + \ldots)(P)^{1-B_2} \tag{2.25}
\]

\[
R = \lambda_0 Y^{B_1} c_1 \lambda_1 c_2 \lambda_2 c_3 \lambda_3 \ldots(P)^{1-B_2} \tag{2.26}
\]

Equation (2.26) is a purely theoretical form of the
housing demand function of a household. Although economists
use various formulae of the demand function, the form from
which the other equations are derived is described by (2.26)\(^3\).
In addition, because (2.26) has a log-linear form, it is a
convenient formula for estimating income and price elastici-
ties through use of a regression analysis.

Though (2.26) is the most utilized form of the housing de-

\(^3\)For example, N. Ekanem, Op. cit., F. DeLeeuw, "The De-
mand for Housing: A Review of Cross-Section Evidence," The
Carliner, "Income Elasticity of Housing Demand," The Review
mand function, some researchers prefer the following special type demand function (2.27) for use in situations in which the price is constant:

\[ R = \alpha_0 + \alpha_1 Y + Y(\alpha_2 C_1 + \alpha_3 C_2 + \alpha_4 C_3 \ldots) \quad (2.27) \]

This formula is useful for four reasons. First, the results it predicts seem to fit the empirical data. Second, it is easier to interpret results of this formula than those of the log-linear form. Third, the formula allows us to do regression analysis. Fourth, this formula still maintains the important characteristic of \( Y \alpha_1 \): that is, that the effects of every household characteristic is always multiplied by the household's income. However, the theoretical justification of this formula has not yet been satisfactorily shown.

The major differences between the two formulae as expressed in (2.26) and (2.27) are: whether it has a constant term, and whether income elasticity is constant or variable. The first difference is clearly seen in equations (2.26) and (2.27). In order to understand the second difference, the income elasticity of (2.27) must be derived.

---


6 An income elasticity is defined as percentage difference in housing expenditure divided by percentage difference in income. Mathematically, it is defined as:
\[ E_Y^R = \frac{\partial R}{\partial Y} \left( \frac{Y}{R} \right) \]
\[ = \frac{(\alpha_1 + \alpha_2 \cdot c_1 + \alpha_3 \cdot c_2 + \ldots) \cdot Y}{\alpha_0 + (\alpha_1 + \alpha_2 \cdot c_1 + \alpha_3 \cdot c_2 + \ldots) \cdot Y} \]  
(2.28)

where
\[ E_Y^R = \text{income elasticity of housing demand} \]

This derivation (2.28) illustrates that the income elasticity is variable instead of constant, namely, the income elasticity of (2.27) is small at the low-income stage and then gradually increases close to 1.0 as income goes up, while the income elasticity of (2.26) has a constant value of \( B_1 \).

II-C. Empirical Work on Housing Consumption Patterns of Renter Households and Income Elasticity

There exist several empirical studies that focus on housing expenditure models. However, the results of those studies on housing consumption vary widely. In this section, the major empirical efforts on this subject are reviewed and compared, and several hypotheses are developed from those studies. In the next section, those hypotheses are examined using data from the 1970 U.S. Census.

\[ E_Y^R = \frac{\Delta R}{R} \cdot \frac{\Delta Y}{Y} \]

or
\[ E_Y^R = \frac{\partial R}{\partial Y} \cdot \frac{Y}{R} \]

where
\[ R = \text{incremental increase in housing expenditure} \]
\[ Y = \text{incremental increase in income} \]
Most works in the field of housing expenditures look at both the rent-income relation of renters and the housing expenditure-income relation of homeowners. In this paper, however, the rent-income relation of renters will be the main focus, because the housing expenditure of homeowners is hard to measure accurately and varies widely; therefore, it requires us lots of "courageous" assumptions to estimate housing expenditure of homeowners.

On the other hand, the gross rent of renters is easy to measure and is a relatively fair indicator of the housing expenditures of a household. In addition, because this thesis focuses on low-income families, the majority of whom are renters (e.g., in 1969, in the Boston SMSA, about 64 percent of all households whose income was under $10,000 were renters), the housing expenditures of renters are more relevant than those of homeowners.

Regarding past studies on this subject, the most extensive and intensive ones were done by G. Carliner, M. Reid, 7N. Ekanem, 8 and I. Lowry. 9 These studies, as well as research by others, have focused mainly on income elasticity. They are discussed

in the following sections.

II-C-1. Summary and analysis of studies by M. Reid

a. Database, concepts, and method

The various data, concepts, and methods used by Margaret Reid to study rent expenditure effects of income, price, family size, age of head, sex of head, and race of head are summarized and analyzed below.

b. Effects of income on rent expenditure

In a study done in 1962, Reid estimated income elasticities of average housing expenditures with respect to average housing income of tenants in several metropolitan areas. The data base and concepts used, and the formula applied by Reid are described in tables II-1 and II-2. Although the estimates of income elasticity she had obtained ranged from .860 to 1.226. Because of "heterogeneity of housing variables related to rent control",\(^\text{10}\) Reid judged that the result of renters were far from normal, and, paying less attention to the result of renters, stated that "the elasticity of housing with respect to normal income appears to be between 1.5 and 2.0".\(^\text{11}\) This is a misleading conclusion in regard to renters, because she paid minor attention to the result of renters and relied heavily on results of homeowners to derive this conclusion. Since this thesis is focusing on renters, the result of renters will be discussed in this section. The Reid's result suggests


\(^\text{11}\)Ibid., p.6, p.376 .
TABLE II-1

COEFFICIENTS OF REGRESSION BY M. REID

<table>
<thead>
<tr>
<th>Metropolitan Area</th>
<th>1.16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban Area</td>
<td>0.95</td>
</tr>
</tbody>
</table>

**Dependent Variable**: average contract rent of a group of households within areas. The groups are defined by housing quality.

**Y**: average income of a group as an indicator of permanent income.

**Formula**: not specified, but supposed to be $R = a \cdot Y^b$.

**Data**: 1950 Housing Census.

**Source**: based on M. Reid, *op. cit.*, *Housing and Income*, p. 162, p. 170.
TABLE II-2
RESULTS OF MULTIPLE REGRESSION BY M. REID

One of the results of M. Reid's study is as follows:

<table>
<thead>
<tr>
<th>Result</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>.995</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.470</td>
</tr>
<tr>
<td>B</td>
<td>1.002</td>
<td>.450</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.478</td>
</tr>
<tr>
<td>C</td>
<td>.860</td>
<td>.586</td>
<td>-.323</td>
<td></td>
<td></td>
<td></td>
<td>.478</td>
</tr>
<tr>
<td>D</td>
<td>.877</td>
<td>.558</td>
<td>-.306</td>
<td>.558</td>
<td></td>
<td></td>
<td>.538</td>
</tr>
<tr>
<td>E</td>
<td>1.226</td>
<td></td>
<td></td>
<td>1.385</td>
<td></td>
<td></td>
<td>.578</td>
</tr>
<tr>
<td>F</td>
<td>1.024</td>
<td>.446</td>
<td>-.236</td>
<td>.229</td>
<td></td>
<td>1.274</td>
<td>.612</td>
</tr>
</tbody>
</table>

Based on M. Reid, Op. cit., Housing and Income, pp. 149-151.

Data: 1950 Housing Census, United States, 30 Metropolitan Areas

Formula: $H = \alpha Y^b (A_f)^c (L_e)^d (B_1)^e (B_2)^f (P_r)^g$,

where

$H =$ median contract rent as an indicator of median rent of each metropolitan area

$Y =$ median money income before taxes as an indicator of permanent income

$A_f = 100 \cdot \left( \frac{\text{All households, excluding those headed by a male over 65 or all headed by a female}}{\text{households}} \right)$

$P_r =$ variable which indicates rent increase from 1941 to 1950

$B_1, B_2 =$ variables which indicate ages of the housing stock

$L_e =$ variable which indicates employment opportunity

$A, B, C, D, E, F =$ terms in stepwise regression.

Note: No indication of significance level of each coefficient.
that income elasticity is almost unity if we use "permanent income" as a dependent variable; that is to say, that the rent-income ratio is constant in terms of permanent income (see Table II-1).

However, several problems exist in Reid's estimation procedure. First, she used data from several metropolitan areas but did not attempt to insert any price factor into her estimating formula. Second, she grouped her data by housing quality, using the census categories under "dilapidated."\(^{12}\) Several researchers already suggested that this grouping introduces a bias in estimates.\(^{13}\)

Grouping by housing quality tends to classify households by the level of housing services they receive. The result of this method is that households with stronger preference for housing than other goods are concentrated in the high quality categories and these households, on the average, have high incomes. Likewise, households with a weaker preference for housing than for other goods are concentrated in low quality categories. As a consequence, the estimated elasticities have a bias toward high. Third, Reid grouped the data by metropolitan area, and assumed median income to be an indicator of permanent income. However, the median income of a

\(^{12}\) Housing categories were not specified beyond the use of the category titles. This is another problem of Reid's study.

whole metropolitan area can in no way be considered the permanent income of any group of households. Due to these problems, Reid's estimates of elasticity must be used with caution.

c. Effects of household characteristics on rent expenditure

Age of head. Reid studied the effects of the age of the head of households on the rent-income ratio, using cross-tabulations. One of her results is illustrated in the Figure II-3.\(^{14}\) Her conclusion was, in regard both to renters and owners, that "housing-income ratios tend to be high for households or consumer units with an aged head, moderately high for those with a young head, and relatively low for those with a middle-aged head,"\(^{15}\) and that "income varies markedly with age of head, while housing differs only slightly."\(^{16}\)

One problem associated with this method is that it is not clear whether the movement of the rent-income ratio is a result of head's age difference or a result of the difference in the median incomes of groups. Because Reid used aggregated data, it is difficult to separate reliably the effects of

\(^{14}\)M. Reid, Op. cit., Housing and Income, p. 60. The graphs of the Figure II-3 are based on Table 3 on p. 60. The data base is the 1950 United States Housing Census, using the median rent and income of each age strata of all husband-wife renters in all standard metropolitan areas.

\(^{15}\)Ibid., p. 61.

\(^{16}\)Ibid., p. 87.
Figure II-3. Rent-Income Ratio by Age of Head and Annual Income by Age of Head.

Source: Based on Table 3 on p.60 of M.Reid, op.cit.
head's age from the effects of income. Income and other factors which are supposed to have effects on rent-income ratio should be controlled for, in order to isolate the effects of the age of head.

**Size of household.** Reid's conclusion is that net effect of increase in number of persons per household is a decline in housing-income ratio for both renters and owners.\(^{17}\) In addition to the same type of cross-tabulation analysis as that described in the section on the effect of head's age, Reid used another source of data, which is illustrated in Table II-3.\(^{18}\) In this case, income is relatively controlled for, and therefore, Reid's conclusions are partly supported by the data. However, there remains the problem that the renters are not separated from the owners.

**Sex and race of head.** Female headed households showed a high rent-income ratio as compared with the ratio of male-headed households:\(^{19}\) This ratio was especially high for households with an aged female head. Reid's explanation of this result is that 1) single mothers have low incomes because of child-care time; and 2) the average income of single women is depressed by the probable incidence of negative transitory income. One problem of this result is that, again, income was

\[^{17}\text{M. Reid, Op. cit., Housing and Income, p. 88.}\]
\[^{18}\text{Ibid., p. 72.}\]
\[^{19}\text{Ibid., p. 67. The data base used here is the Housing Census, 1950.}\]
TABLE II-3  
MEAN HOUSING EXPENDITURE OF THE EIGHT INCOME CATEGORIES UNDER $10,000  
(Each category has a weight of one)  
(Irrespective of tenure)  

<table>
<thead>
<tr>
<th>Number of persons in a household</th>
<th>Housing expenditure of owners and renters</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>$484</td>
</tr>
<tr>
<td>Two</td>
<td>$482</td>
</tr>
<tr>
<td>Three</td>
<td>$462</td>
</tr>
<tr>
<td>Four</td>
<td>$438</td>
</tr>
<tr>
<td>Five</td>
<td>$413</td>
</tr>
<tr>
<td>Six or more</td>
<td>$366</td>
</tr>
</tbody>
</table>

not controlled for and therefore, it is not clear whether the high rent-income ratio is a function of income or of being female.

Reid also found that the housing expenditures of whites was much higher than that of non-whites when the current income of households was held constant, and when housing expenditure and current income of owners were compared. However, Reid disregarded this data. Basing her reasoning on average income and average housing expenditure of three household groups classified by housing quality, Reid concluded that little difference among two racial groups was likely to exist in housing expenditure if the permanent income was held constant.\textsuperscript{20} This conclusion is ill-reasoned because the average income and the average housing expenditure of household groups classified by housing quality are poor measures of permanent income and biased estimates of housing expenditure, as explained in section II-C-1-b.

In short, Reid gathered good data; however, she derived less reliable conclusions in most cases, since she heavily relied on the group average type permanent income concept. It is difficult to cluster small enough groups to obtain good estimates of permanent income.

Nkanta Ekanem studied variations in the proportion of income spent on housing by household type and by metropolitan areas, using data from the United States Census of 1960. He concluded that there was little effect of any metropolitan differences on housing expenditure-income proportions. However, Ekanem did find significant effects of household types within a metropolitan area on those proportions. He also found that income and price elasticities of demand for most types of households tended to be unity, except for non-elderly husband-wife homeowners. These conclusions are critically reviewed below.

a. Data base, method, and concepts of Ekanem's study

Ekanem studied both owners and renters; however, only renters will be discussed here. Ekanem conducted two research studies, one about the effects of household characteristics, and another about elasticities with respect to income and price, each of which was based on different assumptions from the other. The results of the two studies will be discussed in sections b and c, respectively.

In the first study, the rent-income ratio of each household's type was estimated by regression analysis separately for each of 16 SMSA's in the United States. The assumption underlying this first analysis was that income and price elasticities are unity. The following regression formula
was used:

\[
\log \left( \frac{R}{Y} \right) = \log \alpha_0 + \alpha_1 D_1 + \alpha_2 D_2 + \alpha_3 D_3 + \alpha_4 D_4 + \alpha_5 D_5, \tag{2.29}
\]

where

- \( R \) = gross rent per year
- \( Y \) = measured income or current income
- \( D_1 \) = elderly household: if a head is 65 years or more, \( D_1 = 1 \)
- \( D_2 \) = single-person household: if single person, \( D_2 = 1 \); in all other cases, \( D_2 = 0 \)
- \( D_3 \) = Non-husband and wife household: if a household is non-husband and wife, \( D_3 = 1 \); in all other cases, \( D_3 = 0 \)
- \( D_4 \) = female-headed household: if a head is female, \( D_4 = 1 \); in all other cases, \( D_4 = 0 \)
- \( D_5 \) = husband-wife household with head younger than 45 years old, 1 or 0: if head is younger than 45 years old, \( D_5 = 1 \); in all other cases, \( D_5 = 0 \)

\[\text{In order to avoid unnecessary confusion, I have skipped Ekanem's weighting procedure. The dependent variables that Ekanem used in his estimations was the logarithm of the rent-income ratio weighted by the square root of the number of renter households. Because this is a transitory process of regression computation, the principal formula is still equation (2.29). For a detailed but confusing discussion about this, see Ekanem, Op. cit., The Demand for Housing: An Analysis by Family Type, pp. 8-10.}\]
\[ \alpha_0, \alpha_2, \ldots, \alpha_5 = \text{regression coefficients} \]

This equation is a log-linear formula. The dummy variables can be transformed into the following form:

\[ \log\left( \frac{R}{Y} \right) = \log \alpha_0 + \alpha_1 \log D_1' + \alpha_2 \log D_2' + \ldots + \alpha_5 \log D_5', \]

(2.30)

where

\[ D_1', D_2', D_3', D_4', D_5' = e, \text{ if the household falls into one of the categories} \]

\[ D_1 \text{ through } D_5, \text{ as specified in (2.29)} \]

\[ 1, \text{ in all other cases.} \]

Therefore,

\[ \log\left( \frac{R}{Y} \right) = \log \left\{ \alpha_0 \cdot (D_1')^{\alpha_1} \cdot (D_2')^{\alpha_2} \ldots \cdot (D_5')^{\alpha_5} \right\}, \]

and consequently,

\[ R = (Y)^{+1} \cdot \alpha_0 \cdot (D_1')^{\alpha_1} \cdot (D_2')^{\alpha_2} \ldots \cdot (D_5')^{\alpha_5}. \]  

(2.31)

Equation (2.31) is exactly the same formula as (2.26) except for the price factor contained in (2.26). Because Ekanem analyzed each SMSA separately, the price factor could be assumed to be constant. Thus, (2.31) is a pure log-linear formula.

Two main problems associated with this regression model are evident. First, he assumed income elasticity to be +1.0. However, as Ekanem was aware,²² it is not certain whether

²²N. Ekanem, Ibid., p.5.
income elasticity is, in fact, +1.0. For example, Reid's conclusion was that the elasticity was between 1.5 and 2.0 and Lee's conclusion was that it was .65 for renters.\(^2\)\(^3\)

Thus, it is reasonable to assume that the income elasticity is not +1.0, and that therefore, the rent-income ratio that Ekanem estimated could have been already biased by income effects, because he used group data instead of individual data. Even if he had used individual data, disregarding the income factor in the regression would have changed the significance levels of the coefficients of the dummy variables.

Second, the factor of household size, except for single persons, was not included in Ekanem's calculations. Since household size is supposed to be a significant factor, Ekanem should have dealt with it in his study.

The data base Ekanem used for this analysis consisted of classification tables from the 1960 Census, which showed the estimated number of households in each SMSA. Since Ekanem used group data, his dependent variable, i.e., rent-income ratio, was an average which may introduce aggregation problems causing bias.\(^2\)\(^4\)

---


Ekanem also assumes that the choice of current income rather than permanent income would not have an effect on rent-income ratio, based on the argument that transitory incomes will be dissolved into error term of regression equations. However, this assumption is not valid, as you see in the argument in the II-A-1 section.

b. Results of the first study of Ekanem: Effects of Household Characteristics on Rent-Income Ratio

Ekanem found that variables $D_3$ (non-husband and wife households), and $D_5$ (husband-wife household with head younger than 45 years old) were not significant. The other factors; i.e., elderly-headed households ($D_1$), single person households ($D_2$), and female-headed households ($D_4$) had significantly positive effects on the rent-income ratios of all 16 SMSA's. Specifically, the variable of female-headed households had the highest positive effects on the rent-income ratio. Ekanem's results on the Boston SMSA revealed that standard households which had non-elderly male heads, and which were comprised of two or more persons were paying 18 percent of their income for rent. On the other hand, households headed by an elderly person paid 23 percent of their income, as did female-headed households. Single person households paid 25.5 percent of their income.

Ekanem also examined the effects of non-white character-
istics using the comparison between a weighted average rent-income ratio and an actual rent-income ratio of non-white households. Ekanem's results suggested that non-white households had higher rent income ratio than average households. This result differs from Reid's finding of no differences between white households and non-white households. Ekanem tried to explain his results but was unable to. This failure, again, was caused by his disregard of income effects and by group average data.

c. Ekanem's second study: income and price elasticities

In this study, Ekanem estimates inter-regional income and price elasticities for renters. The estimated elasticities are listed in Table II-4. The estimates were based on the following demand function:

\[
H_i = \alpha_i Y_i^{B_1} P^{-B_2},
\]

(2.32)

where

- \( H_i \) = housing demand of the ith type household
- \( \alpha_i \) = preference coefficient or constant term
- \( Y_i \) = permanent income of the ith type household
- \( P \) = price of one unit of housing services in a SMSA
- \( B_1 \) = income elasticity
- \( B_2 \) = price elasticity.

TABLE II-4
INTER-METROPOLITAN AREAS INCOME AND PRICE ELASTICITIES OF HOUSING DEMAND OF RENTERS

<table>
<thead>
<tr>
<th>Type of household</th>
<th>Income elasticity</th>
<th>Price elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>.93</td>
<td>-.93</td>
</tr>
<tr>
<td>Elderly</td>
<td>.92</td>
<td>-1.35</td>
</tr>
<tr>
<td>Single</td>
<td>.97</td>
<td>-.51</td>
</tr>
<tr>
<td>Female</td>
<td>.87</td>
<td>-.91</td>
</tr>
</tbody>
</table>

Based on N. Ekanem, *The Demand for Housing: An Analysis by Family Type*, p. 29.

Standard household = a nonelderly male-headed household of two or more persons.
In order to use current income, Ekanem transformed (2.32) into:

\[ H_1 = \alpha_i P^{-B_2} Y_{1c}^{1-B_1 (1+U)}^{1-B_1} \]  

(2.33)

where \( U \) = a stochastic variable with zero mean

\( Y_{1c} \) = current income of the ith type household.

Thus, we can see that

\[ \frac{R}{Y_{1c}} = \frac{H_1 P}{Y_{1c}} = \alpha_i (P)^{1-B_2} (Y_{1c})^{1-B_1 (1+U)}^{1-B_1} \]  

(2.34)

Ekanem regressed the log-linear form of the equation (2.34) using group data classified by four household types which were shown in Table II-4. For this regression, rent income ratios expressed as \( R/Y_{1c} \) were estimated by household types, as in the first study, for the 16 metropolitan areas.

The median incomes of the four types of households were expressed as \( Y_{1c} \), and the relative price of housing of each SMSA was expressed as \( P \).

The results of this regression are listed in Table II-4. Ekanem concluded, based on these results, that income and price elasticities were not terribly different from unity among renter households.\(^{27}\)

\(^{26}\) Rationale and problems to do with this transformation has been discussed in II-A-1.

This conclusion does not follow from the procedure of his computation. What Ekanem analyzed was income and price elasticities among geographical areas, not these elasticities among households. Because he relied on the group data classified by household type regardless of income, and because he regressed each household type separately using the 16 SMSA data, the outcomes were necessarily inter-regional income elasticities, instead of elasticities among households. This confusion about inter-regional and "among-households" income elasticities stemmed from using group data and raises questions about his conclusions.

II-C-3. Summary and analysis of studies by I. Lowry, J. DeSalvo, and B. Woodfill

Ira S. Lowry and his colleagues tried to develop logical criteria for distinguishing those who need help with their housing expenses from those who do not. As one part of this effort, Lowry et al analyzed the rent-income ratios of tenants in New York City, as well as the rental expenditure patterns of various types of households. The effects of submarkets on rental expenditures were also studied by dividing New York City into four submarkets and by running regression analyses separately for those submarkets.

a. Data, method, and concepts used by Lowry et al

The distinguishing characteristics of Lowry's study on rent-income ratios were: 1) that the individual data, rather
than group data were regressed; 2) that the income effects among households within a city, rather than cross-sectional income effects were analyzed; and 3) a linear formula, rather than a log-linear formula, was used as the regression equation.

However, Lowry and his colleagues used current income, which tended to raise the rent-income ratio for households with low current income and to lower this ratio for households with high current income.

The regression formula used was developed by trial and error and has the following linear form:28

\[
\frac{R}{Y} = \alpha_1 + \alpha_0 \left(\frac{1}{Y}\right) + (\alpha_2 S + \alpha_3 S^2 + \alpha_4 A + \alpha_5 A^2 + \alpha_6 N + \alpha_7 F),
\]

(2.35)

where

- \( R \) = gross rent per year of each household
- \( Y \) = current income per year in $1,000
- \( S \) = size of a household; i.e., the number of persons in a household
- \( A \) = age of a household head
- \( N \) = dummy variable: race of head; if non-white or Puerto Rican, then \( N = 1 \)
- \( F \) = dummy variable: sex of head; if female, then \( F = 1 \)

The underlying demand function, income elasticity function, and price elasticity function are as follows:

\[
H = \left\{ \alpha_0 + \alpha_1Y + Y(\alpha_2 + \alpha_3 + \ldots + \alpha_7) \right\}(P)^{-B},
\]

(2.36)

\[
E^H_P = B, \text{ and }
E^H_Y = \frac{(\alpha_1 + \alpha_2 + \alpha_3 + \ldots + \alpha_7)Y}{\alpha_0 + (\alpha_1 + \alpha_2 + \alpha_3 + \ldots + \alpha_7)Y},
\]

(2.37)

where

- \(P\) = price of one unit of housing services
- \(E^H_P\) = price elasticity of housing demand
- \(E^H_Y\) = income elasticity of housing demand
- \(H\) = annual gross rent
- \(B\) = price elasticity.

A distinctive characteristic of this demand function (2.36) is the existence of an intercept \(\alpha_0\). If \(\alpha_0\) were deleted from the function, the function would simply be a demand function with unity income elasticity. By inserting \(\alpha_0\) into the function, Lowry allowed the income elasticity to be smaller than 1.0. As discussed in section II-B, a linear type demand function like Lowry's formula has no supporting theo-
retical argument. Therefore, one must be careful about adopting a linear form such as this one.

b. Effects of income

Lowry's regression results on the effects of income are summarized in Table II-5. Lowry did not examine income elasticities of housing demand but, instead, examined the effects of income on rent expenditure. Contrary to Reid's and Ekanem's conclusions, which were based on inter-regional analysis of income elasticities, Lowry's conclusions were that, within any of four sub-markets, income had the greatest effects on changes in rent-income ratios. For example, except for Manhattan, income of households explained 81 percent of the observed variance in the rent-income ratios. This means that, outside of Manhattan, almost all of the explained variance (which is 81 percent, based on the data of R² in the Table II-5) are attributed to income for tenants in controlled housing, and that 81 percent out of 82 percent of the explained variance are attributed to income for tenants in uncontrolled housing.

Income had negative effects on the rent-income ratio in all four sub-markets; i.e., the rent-income ratio declined significantly as income rose, although gross rent expenditures increased as income rose. Thus, this finding contradicts Reid's and Ekanem's conclusions that rent-income ratio is constant over changes in household income. Using current instead of permanent income might account for the decline.

29I. Lowry, Ibid., p. 74.
### TABLE II-5

RENT/INCOME RATIOS AS FUNCTIONS OF INCOME AND HOUSEHOLD CHARACTERISTICS: FOUR HOUSING SUBMARKETS IN NEW YORK CITY, 1968

<table>
<thead>
<tr>
<th>Item</th>
<th>Controlled Housing</th>
<th>Uncontrolled Housing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Manhattan</td>
<td>Non-Manhattan</td>
</tr>
<tr>
<td>Regression Parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regression constant</td>
<td>.17784</td>
<td>.00917</td>
</tr>
<tr>
<td>1/(Income in $1,000)</td>
<td>.76638</td>
<td>.90718</td>
</tr>
<tr>
<td>Number of persons</td>
<td>.01788</td>
<td>.01534</td>
</tr>
<tr>
<td>(Number of persons)²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age of household head</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Age of household head)²</td>
<td>.00536</td>
<td>.00000</td>
</tr>
<tr>
<td>Sex of household head</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>.00000</td>
<td>.00000</td>
</tr>
<tr>
<td>Female</td>
<td>.02923</td>
<td>.01004</td>
</tr>
<tr>
<td>Ethnic group of household:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>White non-Puerto Rican</td>
<td>.00000</td>
<td>.00000</td>
</tr>
<tr>
<td>Non-white or Puerto Rican</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.03195</td>
<td>.00067</td>
</tr>
<tr>
<td>Regression Statistics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient of determination (R²)</td>
<td>.66</td>
<td>.81</td>
</tr>
<tr>
<td>Standard error of estimate (Sₑ)</td>
<td>.10</td>
<td>.08</td>
</tr>
<tr>
<td>Number of observations</td>
<td>2,009</td>
<td>4,470</td>
</tr>
</tbody>
</table>

Source: Calculations by NYCRI from unit records of the New York City Housing and Vacancy Survey of 1968.

*a Not significantly different from zero at the .01 level of significance. 23

23 Extracted from the Appendix C, Table C-7, I. Lowry, Rental Housing in New York City, p. 237.
c. Effects of other household characteristics

Race

In Lowry's study, the secondarily influential variable was race, although it was far less influential than the income variable. Lowry's study showed that, in all four sub-markets, and at every level of income, housing expenditures of non-white and Puerto Ricans were less than their white counterparts. This finding is counter to what Ekanem found; that is, that non-whites were paying more for housing than whites were.

Female

Lowry concluded that, in all four sub-markets, female-headed households spent more than male-headed households, regardless of race. This is completely consistent with Reid's and Ekanem's conclusions. However, Lowry's conclusion is an overstatement of his results. He could have concluded that females spent more than males, but he should not have concluded that this was true for both ethnic groups. But the latter part of this conclusion cannot be adequately addressed using his regression model. Lowry's model does not allow one to separate the effects of white or non-Puerto Rican females from the effects of non-white or Puerto Rican females. Lowry was assuming implicitly the same female effects for both ethnic groups. In order to separate the effects of these two, the model should have had an interactive term of sex and ethnicity. But, since it did not, Lowry could not have based his conclusions about

30I. Lowry, Ibid., p. 67.
the ethnic difference of female effects on any empirical data. Lowry's results on ethnicity and sex are illustrated in the Figure II-4.31

**Household size**

The results of Lowry's analysis on household size are illustrated in the Figure II-5.32 Lowry concluded that if other factors were controlled, the housing expenses for each added person to a household were modest; that peak expenses were paid by households with four, five, or six persons; and that large households spent less. These conclusions suggest increasing housing expenses as household size increases from one to four or five, as opposed to Reid's conclusion which suggests decreasing housing expenses.

**Age of head**

Like the effects of household size, the effects of age were not strong in Lowry's results, as is illustrated in the Figure II-6.33 In a submarket which had no rent control, the young and the old paid more than the rest of the population if other characteristics were held constant. This is consistent with Reid's and Ekanem's conclusions and is more accurate than those. Therefore, this conclusion seems to be quite plausible.

---

Figure II-4. Gross Rent Expenditures as a Function of Income.

For four person households with head 42 years of age, located in NYC, outside of Manhattan, and not regulated by rent control.

Source: Based on Lowry, op.cit.
Figure II-5. Gross Rent Expenditure as a Function of Household Size.

For households headed by white males 42 years of age with annual incomes of $5,000 and located in an uncontrolled submarket.

Figure II-6. Gross Rent Expenditure as a Function of Age of Head.

For two person households headed by white male with annual incomes of $5,000 and located in an uncontrolled submarket.
II-C-4 Summary and conclusion

The studies by Reid, Ekanem, and Lowry et al are summarized in Table II-6.

a. Conclusions

Income effects

Because Reid's and Ekanem's studies were inter-regional analyses that were based on group data, they do not adequately derive the income elasticity of households. However, other studies conclude that income elasticity of housing for renters is less than 1.0.34

Therefore, a review of the literature suggests that income elasticity of housing for renters is less than 1.0, using current incomes. This means a rent-income analysis should include an income variable among its independent variables. However, two questions are still left for further study. These questions are: How much is the income elasticity? And, is the elasticity constant or variable?

Size

Reid's conclusion is consistent with our conventional understanding that food, clothing, and some other goods are

### TABLE II-6
RESEARCH CHARACTERISTICS AND CONCLUSIONS OF THE THREE STUDIES

<table>
<thead>
<tr>
<th>Research Characteristics:</th>
<th>Reid</th>
<th>Ekanem</th>
<th>Lowry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data base</td>
<td>Group average or group median.</td>
<td>Midpoint of group (a kind of group average)</td>
<td>Individual.</td>
</tr>
<tr>
<td>Control of income</td>
<td>No, in most cases.</td>
<td>No, in the first study.</td>
<td>Yes.</td>
</tr>
<tr>
<td>Control of other factors</td>
<td>Some yes, some not, Yes.</td>
<td>Yes, in the second.</td>
<td>Yes.</td>
</tr>
<tr>
<td>Income assumptions</td>
<td>No assumptions.</td>
<td>Income elasticity of housing is 1.0.</td>
<td>Income elasticity of housing is less than 1.0 and more than zero.</td>
</tr>
<tr>
<td>Income effect analysis</td>
<td>Inter-regional.</td>
<td>Inter-regional.</td>
<td>Intra-regional (among households)</td>
</tr>
</tbody>
</table>
| Conclusion:               | $E_X^H=1.5 \sim 2.0$ (cross-sectional) but data suggest: $E_X^H=.860 \sim 1.226$ based on grouped data. | $E_X^H=1.0$ (cross-sectional) | $E_X^H=0 \sim 1.0$ (cross-sectional,)
| Income elasticity ($E_X^H$) | | | |


TABLE II-6 (continued)

<table>
<thead>
<tr>
<th>Conclusion:</th>
<th>Reid</th>
<th>Ekanem</th>
<th>Lowry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size vs. rent income ratio (R/Y).</td>
<td><img src="image1" alt="Graph" /></td>
<td>Not analyzed</td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>Age vs. R/Y</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
<td><img src="image5" alt="Graph" /></td>
</tr>
<tr>
<td>Sex</td>
<td>Female $\rightarrow$ high</td>
<td>Female $\rightarrow$ high</td>
<td>Female $\rightarrow$ high</td>
</tr>
<tr>
<td>Race</td>
<td>No difference between two.</td>
<td>Non-white $\rightarrow$ high</td>
<td>White $\rightarrow$ high</td>
</tr>
</tbody>
</table>
more necessary than housing. On the other hand, because Lowry had a fairly good data base, his conclusion, shown in Table II-6, may be more reliable than Reid's. However, there is not enough evidence to decide if either of these conclusions is plausible. Extra data pertinent to these questions from the Boston analysis are presented in the next chapter.

**Age**

No significant conflict regarding the effects of age exists among the three studies. This review suggests that aged households tend to pay high rents; young households, moderately high rents; and middle households, relatively low rent. It also suggests that the conventional notion that aged-headed households pay a severely high proportion of their income for rent is somewhat exaggerated.

**Sex**

Regarding the effects of sex, there exists no conflict, either. Reid, Ekanem, and Lowry et al agree that female-headed households tend to pay higher rents.

**Race**

Each of the three studies came to different conclusions regarding the effects of race. However, because of the reasons

---

35 At the same time, there is another question. Food and clothing are necessary goods. However, if a household's income is very high, then food and clothing expenditure will no longer be a budget constraint on housing expenditure. In this case, it would be reasonable to expect that rent increases as size of household increases for high-income households.
explained in the respective sections, the conclusions of Reid and Ekanem are less reliable than those of Lowry et al. Therefore, it is reasonable to conclude that households headed by whites tend to pay higher rents. However, further studies are necessary.

Model specification

One major question remains. That is, does the demand function have a linear form or a log-linear form?

The hypotheses and questions stated in this section will be tested in the following section, using new data obtained from the 1970 U.S. Census.
III. EMPIRICAL ANALYSIS OF BOSTON SMSA: QUALITATIVE ANALYSIS BY CROSS-TABULATION

III-A. Introduction

III-A-1. Objectives and questions

a. Objectives

In this section, rent-income ratios of each household type are re-examined, using new group average data derived from individual data of Boston SMSA in 1970. This additional analysis of rent-income ratios is necessary because there were wide variations among the conclusions of Reid, Ekanme and Lowry et al., as described in the section II-B-4 of this thesis. Therefore, in those cases in which the most likely conclusions have been identified, it will be helpful to test them using new data. Similarly, for cases where the conclusions are uncertain, we need further analyses to establish a concrete housing consumption pattern. Second, since there is a debate over the model specification, we shall begin an analysis by studying cross-tabulation of rent-income ratios (or rent) and other household characteristics. Doing this will help in identifying the nature of the association among the variables and the type of specification to be tested. After this qualitative analysis of relations between independent variables and a dependent variable, we can then proceed to quantitative analyses,
using regressions. Thirdly, further exploration of the relationship between variables is necessary in order to specify a better model. Finally, because it has a big influence, it is necessary to see whether there is an intercept term—or a constant term—in the relation between the independent variable of income and the dependent variable of gross rent. In summary, the objectives of this further analysis are: to test existing empirical theories based on new individual data from the 1970 U.S. Census, and to specify a more accurate model by cross-tabulations.

b. Specific questions

Specific questions to be addressed in this analysis are:

1) In regard to income, whether the linear demand function used by Lowry is consistent with the Boston data. In particular, is there an intercept term or not?

2) Holding other factors constant, is the relation between rent and income linear, exponential, or logarithmic?

3) Holding other factors constant, what is the likely relation between size of household and rent-income ratio?

4) Are there interactive effects among the race and
sex variables?

5) For continuous independent variables, are there any unique changes in their movement such as a peak or an inflection point?

6) In regard to age and sex, are the conclusions in II-C-4 consistent with the Boston data?

III-A-2. Data, concept of income, and methods of the analysis

a. Data

The data used in this study are from the Public Use Sample, one out of a hundred, County Group and SMSA tapes of the U.S. Housing Census in 1970.¹ All renters in the Boston SMSA were extracted from the Public Use Sample. Those renters with no incomes or minus incomes and those with no cash rent were deleted from the sample.² Because this study is focusing on how much tenants are paying or how much they are willing to pay, such renters as the above will cause biases in estimates if they are included in the sample. Number of cases; that is, all renters in the Boston


²Number of cases of no-income renters was about 70. Number of cases deleted by the other reasons could not be identified because Public Use Sample does not specify those.
SMSA, excluding those who are deleted by the procedure above, classified by income and household size are listed in Table III-1. We have 4,834 cases, in total. Multiplying 4,834 by 100 leads to the actual number of total renter households in Boston SMSA, excluding those who are deleted by the procedure above. From 4,834, households with rent-income ratios equal to or more than 100 percent were deleted in the analysis. Number of households with rent-income ratio equal to or more than 100 percent and less than 1,000 percent were 286. Among 286, 146 were households with income ranging from $1 to $999, 125 were those with income from $1,000 to $1,999, 14 were those with income from $2,000 to $2,999, and 1 was a household with income from $3,000 to $3,999. Therefore, the rent-income ratio over 100 percent is a phenomenon among low-income households especially those whose income were below $2,000. Possible causes of this high incidence of the extremely high rent-income ratio among low-income households are: 1) a possible tendency of low-income households to understate their income, and 2) having used current income instead of permanent income. This high incidence suggests
<table>
<thead>
<tr>
<th>Household Size</th>
<th>Income ($)</th>
<th>1.0-999</th>
<th>1k-1999</th>
<th>2k-2999</th>
<th>3k-3999</th>
<th>4k-4999</th>
<th>5k-5999</th>
<th>6k-6999</th>
<th>7k-7999</th>
<th>8k-8999</th>
<th>9k-9999</th>
<th>10k-10999</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 person</td>
<td>112.</td>
<td>253.</td>
<td>199.</td>
<td>155.</td>
<td>124.</td>
<td>139.</td>
<td>109.</td>
<td>86.</td>
<td>78.</td>
<td>42.</td>
<td>33.</td>
<td></td>
</tr>
<tr>
<td>2 persons</td>
<td>32.</td>
<td>72.</td>
<td>107.</td>
<td>104.</td>
<td>122.</td>
<td>115.</td>
<td>98.</td>
<td>125.</td>
<td>111.</td>
<td>106.</td>
<td>86.</td>
<td></td>
</tr>
<tr>
<td>3 persons</td>
<td>17.</td>
<td>18.</td>
<td>47.</td>
<td>36.</td>
<td>42.</td>
<td>49.</td>
<td>59.</td>
<td>81.</td>
<td>56.</td>
<td>66.</td>
<td>56.</td>
<td></td>
</tr>
<tr>
<td>4 persons</td>
<td>14.</td>
<td>19.</td>
<td>18.</td>
<td>24.</td>
<td>24.</td>
<td>28.</td>
<td>43.</td>
<td>40.</td>
<td>49.</td>
<td>36.</td>
<td>43.</td>
<td></td>
</tr>
<tr>
<td>5 persons</td>
<td>5.</td>
<td>5.</td>
<td>4.</td>
<td>17.</td>
<td>18.</td>
<td>17.</td>
<td>14.</td>
<td>26.</td>
<td>25.</td>
<td>27.</td>
<td>24.</td>
<td></td>
</tr>
<tr>
<td>7 persons</td>
<td>3.</td>
<td>1.</td>
<td>3.</td>
<td>2.</td>
<td>6.</td>
<td>5.</td>
<td>6.</td>
<td>9.</td>
<td>9.</td>
<td>7.</td>
<td>5.</td>
<td></td>
</tr>
<tr>
<td>8 or more</td>
<td>2.</td>
<td>1.</td>
<td>1.</td>
<td>3.</td>
<td>3.</td>
<td>3.</td>
<td>0.</td>
<td>2.</td>
<td>7.</td>
<td>10.</td>
<td>2.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>11k-11999</th>
<th>12k-12999</th>
<th>13k-13999</th>
<th>14k-14999</th>
<th>15k-15999</th>
<th>16k-16999</th>
<th>17k-17999</th>
<th>18k-18999</th>
<th>19k-19999</th>
<th>20k</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.</td>
<td>29.</td>
<td>11.</td>
<td>9.</td>
<td>11.</td>
<td>4.</td>
<td>2.</td>
<td>2.</td>
<td>2.</td>
<td>21.</td>
<td>1442.</td>
</tr>
<tr>
<td>2</td>
<td>78.</td>
<td>75.</td>
<td>54.</td>
<td>42.</td>
<td>33.</td>
<td>30.</td>
<td>19.</td>
<td>11.</td>
<td>15.</td>
<td>24.</td>
<td>50.</td>
</tr>
<tr>
<td>3</td>
<td>43.</td>
<td>45.</td>
<td>29.</td>
<td>34.</td>
<td>17.</td>
<td>18.</td>
<td>11.</td>
<td>15.</td>
<td>7.</td>
<td>44.</td>
<td>790.</td>
</tr>
<tr>
<td>4</td>
<td>39.</td>
<td>32.</td>
<td>29.</td>
<td>14.</td>
<td>21.</td>
<td>11.</td>
<td>8.</td>
<td>7.</td>
<td>5.</td>
<td>25.</td>
<td>529.</td>
</tr>
<tr>
<td>5</td>
<td>10.</td>
<td>15.</td>
<td>15.</td>
<td>5.</td>
<td>10.</td>
<td>4.</td>
<td>9.</td>
<td>2.</td>
<td>4.</td>
<td>24.</td>
<td>280.</td>
</tr>
<tr>
<td>6</td>
<td>12.</td>
<td>3.</td>
<td>4.</td>
<td>8.</td>
<td>1.</td>
<td>7.</td>
<td>1.</td>
<td>4.</td>
<td>1.</td>
<td>9.</td>
<td>160.</td>
</tr>
<tr>
<td>7</td>
<td>10.</td>
<td>4.</td>
<td>3.</td>
<td>3.</td>
<td>1.</td>
<td>0.</td>
<td>2.</td>
<td>0.</td>
<td>7.</td>
<td>89.</td>
<td></td>
</tr>
<tr>
<td>8+</td>
<td>2.</td>
<td>4.</td>
<td>3.</td>
<td>1.</td>
<td>1.</td>
<td>0.</td>
<td>1.</td>
<td>0.</td>
<td>1.</td>
<td>4.</td>
<td>50.</td>
</tr>
<tr>
<td>T.</td>
<td>215.</td>
<td>207.</td>
<td>147.</td>
<td>116.</td>
<td>97.</td>
<td>75.</td>
<td>51.</td>
<td>43.</td>
<td>44.</td>
<td>184.</td>
<td>4834.</td>
</tr>
</tbody>
</table>

Source: Table developed from 1/100 Public Use Sample of 1970 U.S. Housing Census using EFFECT program.
that rent and income data of households with income below $2,000 are less reliable to analyze.

In the following analysis, the dependent variable will be either rent-income ratio or annual rent itself. To avoid heteroscedasticity, rent-income ratios rather than annual rents were analyzed as a major dependent variable, while annual rents were analyzed as a supplemental variable when it was necessary to see rents as well as rent-income ratios. The explaining variables will be annual income, household size, age of head (so far, variables are continuous variables), race of head, and sex of head (latter two are dummy variables). A summary of definitions of all the variables is contained in Table III-2. "Income" is annual, gross and current income including all kinds of transfer payments. Rent is annual gross rent.

b. Concept of income

In this study, current income, rather than permanent income, is used for the following reasons:

1) Data problem: It is generally accepted among researchers that housing expenditures are spent by house-
TABLE III-2
DEFINITION OF VARIABLES

<table>
<thead>
<tr>
<th>Sample</th>
<th>Renters or tenants or households</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All renters in the Boston SMSA, excluding &quot;No cash rent renters&quot;, inhabitants of group quarters, and those renters with no income or with minus income.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variables</td>
<td>k</td>
<td>Rent or annual rent</td>
<td>-Annual gross rent = monthly gross rent ( \times 12 ), which is equivalent to (contract rent + average monthly cost of utilities)( \times 12 ). No cash rent renters have been excluded.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rent income ratio</td>
<td>( R/Y ) or ( \frac{1}{N} \sum \left( \frac{R_i}{Y_i} \right) ), (not ( \frac{\sum R}{\sum Y} )) ( (N = \text{total number of entries}) ) Households with rent-income ratio over 100% have been excluded.</td>
</tr>
<tr>
<td>Explaining variables</td>
<td>Y</td>
<td>Annual income</td>
<td>-Annual, gross, and current income, excluding no income or minus income, &quot;in kind&quot; income and sale of property income. However, net self-employment income, and income from social security, welfare, investments, pensions, and unemployment insurance are included.</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>Household size</td>
<td>-Number of persons in housing unit. Excluding group quarter.</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>Age of head</td>
<td>-Age of head Excluding under 14 years of age.</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>Non-white (or race)</td>
<td>-Race of head All races except white.</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>Female (or sex)</td>
<td>-Sex of head Female</td>
</tr>
</tbody>
</table>
holds based on their long-run income expectation rather than on current income which may include transitory plus or minus components. However, one problem associated with permanent income is the difficulty of measurement, since permanent income is defined as a long-run expected income. Thus, to measure permanent income, we must have time series income data for several years. Such data is really difficult to gather. The best available data at this moment is the Public Use Sample, one out of a hundred, of the 1970 U.S. Census, which does not show time series data. Therefore, this study has to rely on the current income. However, at the end of this section a possibility of modifying estimates of income elasticity that is based on current income into estimates based on permanent income will be discussed.

2) Problem of group data

To get estimates of permanent income without studying time series data, group average or group median methods have been used by many researchers. For example, Ekanem, Reid, and others attempted to reduce the bias of current income by using group medians. They argue that the transitory components of current income are dissolved into an error term, leaving permanent income. Nevertheless, biases can still

---

arise and another problem also develops. Generally, groups are so large that much of the variation in permanent income among households is also lost. Therefore, it is more accurate to use individual data of current income than to use group average data of current income. This is especially true in cases such as this study in which income is expected to explain a large portion of the deviations in housing expenditures among various household groups. In this study, average of individual rent-income ratios in a group stratified by income was used to construct cross-tabulations.

c. Modification of income elasticity

In this study, current income is used. In order to estimate income elasticity with respect to permanent income, the income elasticity with respect to current income should be adjusted in some way, because using current income instead of permanent income results in underestimating the income elasticities of housing demand. G. Carliner\(^5\) studied the differences among income elasticities derived from different income definitions. Based on four-year individual data, Carliner concluded that income elasticity with respect to permanent income for renters is 0.5. A summary of Carliner's results are listed in the Table III-3. This table shows how much estimates of income elasticity with respect

### TABLE III-3

**INCOME ELASTICITY OF HOUSING DEMAND**

<table>
<thead>
<tr>
<th>Income Definition</th>
<th>Regressions with Demographic Terms</th>
<th>Regressions Without Demographic Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Owners</td>
<td>Renters</td>
</tr>
<tr>
<td>$Y_e$</td>
<td>0.631 (.023)</td>
<td>0.520 (.023)</td>
</tr>
<tr>
<td>$Y_d$</td>
<td>0.619 (.023)</td>
<td>0.521 (.023)</td>
</tr>
<tr>
<td>$Y_m$</td>
<td>0.499 (.021)</td>
<td>0.439 (.020)</td>
</tr>
<tr>
<td>$Y_r$</td>
<td>0.746 (.023)</td>
<td>-</td>
</tr>
</tbody>
</table>


$Y_e$ = permanent income which is the simple four-year average of measured household income.

$Y_d$ = permanent income which is the weighted four-year average. Income for the current year has a weight of 0.4; for the previous year, 0.3; for the year before that, 0.2; and for the first year, 0.1.

$Y_m$ = current year's measured income.

$Y_r$ = permanent income including imputed rent (income concept for homeowners).

Numbers in parentheses are standard errors.
to current income differ from the one with respect to permanent income. Specifically, an estimate of income elasticity based on permanent income is about 117 to 118 percent of an estimate of income elasticity based on current income, according to Carliner. This gives us some sense about how much estimates of elasticity based on current income should be adjusted in order to give estimates of elasticity based on permanent income. As a consequence, it suggests rent-income ratios based on permanent income also should be increased by the same rate as the rate used to adjust the elasticity; that is, by 117 or 118 percent.

d. **Target geographical area**

In this study, the Boston SMSA is analyzed as a whole instead of being subdivided into several submarkets. The reasons for this are the following:

1) **Focus on the effects of household characteristics**

Although most other researchers have assumed that housing price is constant within an SMSA, Lowry et al. suggested that housing prices differ between a central business district (CBD) (Manhattan, in his research) and the suburbs within a city. Lowry's argument is reasonable with respect to the price factor. However, the main intention of this study is to see the effects of household characteristics, including income, on rental expenditures (namely quantities multiplied by prices). In this study, I assumed the price
elasticity of housing demand in Boston SMSA is 1.0. Then the
rents paid by households with the same incomes but located at
different parts of the SMSA are the same. As a consequence,
the same rent-income ratio is maintained for all households in
the SMSA. This will delete the effects of price changes in
the SMSA, and allow us to look at effects of household charac-
teristics on rents for the whole SMSA.

2) **Data problem**

Even if this study were intended to analyze the effects
of prices in submarkets, one should not underestimate the
difficulties of measuring the prices, gathering data, and de-
fining the CBD. And also, the Public Use Sample does not
allow us to divide the data according to location in the CBD
or in the suburbs.

3) **CBD of Boston is not so strong**

The central business district of Boston is not as dis-
tinctive as Manhattan. It is reasonable to assume that there
are less differences between housing prices in the Boston CBD
and in the Boston suburbs than are found in New York City.

e. **Gross income**

In this study, gross income is used instead of dispos-
able income. Because there is no good way to estimate dis-
posable income, and because estimating disposable income in-
volves value judgment on what are the inevitable expendi-
tures, estimates of disposable income become contingent to
policy factors. Once rent, income, and size association are examined, it may be possible to develop a notion of disposable income.

f. Method

In this study, controlled cross-tabulation was used. Specifically, taking the rent-income ratio and rent itself as dependent variables, various cross tables by either of two dependent variables and by one of explaining variables (such as income or household size) are constructed. For each cross-tabulation, the other variables were controlled to be constant to the extent possible.

The EFFECT program at MIT was used for computation and tabulation. Developing graphs of rent-income ratios versus, for example, income from these tables facilitates development of model specification for later use for regression analysis. For example, they help in determining by which line among linear and non-linear lines a good fit can be expected, and also in what range a linear line (or non-linear line) fits to data and in what ranges it does not fit. One problem of these graphs is that the curves will have wide fluctuations due to error terms or "noises", and consistent judgments about errors arising from sample fluctuations are difficult to obtain.

III-B. Results of Cross-Tabulation

III-B-1. Effects of Income

The results of the cross-tabulations of rent-income
ratios by incomes and of rent by income are illustrated in Figures III-1-A, B and III-2-A, B. Plots in the graphs are average rent-income ratios (that is, \( \frac{1}{N} \sum_{i=1}^{N} \frac{R_i}{Y_i} \)) for income groups with \( N \geq 0 \), where \( N \) is a member of cases in each group. The graphs show that, when household characteristics are held constant, rent-income ratios dramatically increase as income decreases.

a. Role of income in consumption behavior

The first thing suggested by the results is not surpising; that is, an income variable plays a significant role in rent-income ratios and should be included in any model for estimating rent-income ratios. In other words, rent-income estimates that disregard the income factor are less reliable. For instance, Ekanem's rent-income estimates of elderly families or female families are overestimations of the effects of those characteristics because it ignores the effects of high incidence of low-income among those families.

b. Income elasticity of housing demand assuming constant prices

Both rapid decrease in rent-income ratios along with increase in incomes and stable rent-income ratios around 10 percent for high-income households suggest that income elasticities of housing demand is somewhat high for high-income households and close to zero for the lowest income households, and that income elasticity is below 1.0 for all
Figure III-1-A. Average Rent-Income Ratio by Annual Income Stratified by Household Size.

For households headed by white male 18-64 years of age excluding households with R/Y over 100%.

(E(Entry of each point)): 10 ≤ E ≤ 87.
Figure III-1-B. Average Rent-Income Ratio by Annual Income Stratified by Household Size.
Figure III-2-A. Average Annual Gross Rent by Income Stratified by Household Size.

For households headed by white males 18-64 years of age, excluding households with R/Y over 100%.

E(Entry of each point): $13 \leq E \leq 167$. 
Figure III-2-B. Average Annual Gross Rent by Income Stratified by Household Size.
income households. This implies that income elasticities could be variable rather than constant. It also suggests, in opposition to Reid's conclusions, that for low-incomes housing is really one of the necessity goods.

c. Intercept and regression formula

The question discussed in this section is "What is the best regression formula in regard to income, or more specifically, which is more plausible a simple linear formula or a log-linear one?" The curves shown in the Figure III-i-A and -B suggest that an inverse of income (that is, 1/Y) is appropriate as an explaining variable of regressions. Namely,

\[ R/Y = a + b \cdot \left( \frac{1}{Y} \right) \]  

or

\[ R/Y = a' \left( \frac{1}{Y^{b'}} \right) \]  

where \( b > 0, b' > 0 \), are the possible formulae.

Multiplying them by \( Y \) leads to the following housing expenditure formulae:

\[ R = b + a.Y \]  

or

\[ R = a' \cdot (Y)^{1-b'} \]  

The differences between these two formulae are the intercept term \( b \) and the exponent of \( Y \): that is, the choice of an appropriate formula depends on whether there is intercept
b or not, and whether or not b' = 1. The figure III-2-A and -B suggest intercept b exists. Those curves do not seem to extend down to zero rent. On the contrary, they suggest that households tend to maintain a certain level of rent expenditure, even at the lowest income. As a consequence, it can be said that the formula (3.3) is supported more strongly by the data as a rent regression formula than is the (3.4).

The figure III-2-A and -B show graphs of rents versus incomes stratified by household sizes. Those graphs show a wide range of fluctuations and it is hard to tell whether the graphs are linear or non-linear. Therefore, we are not sure whether the formula (3.3) is appropriate or not. In this case, it is better to use a more general formula than (3.3); that is,

\[ R = b + a.Y^c \]  (3.5)

As a conclusion, the best formula for regression is (3.5) and regressions should be run separately for different household sizes.

One problem here is that the formula (3.5) cannot be regressed using linear regression. Formulae we can run are, again, (3.1) or (3.2). Therefore, it should be re-examined whether the constant term b is important or the exponential form \( Y^c \) excluding b is important in regression. There are two evidences which
are not definitive but suggestive to this re-examination. One is the Figure III-8. One graph in the figure shows the relation of rents to incomes based on uncontrolled data; that is, based on all renters in Boston SMSA instead of the white, male-headed non-aged renters. It shows a clear straight line and suggests the index c of the equation (3.5) is 1.0.

Another is trial runs of single regressions using both formulae (3.1) and (3.2) Taking the data of two-person households illustrated in the Figure III-2-A, the two formulae were regressed. Results showed that coefficients of the determinant (R$^2$) of (3.1) and (3.2) were .99 and .97, respectively. The high R$^2$ were obtained partly because the regression were run using the average of rents of households grouped by income class. This result suggests that, again, both R$^2$ are close to each other and difficult to separate, but also it suggests that (3.1) could be a better formula than (3.2), if one uses current income instead of permanent income.

The overall conclusion about regression formulae is that the best formula is (3.5), and that (3.1) could be a better formula than (3.2). To confirm this conclusion, however, further regression analyses are necessary.

d. **Income and household size**

Look at Figure III-1-A and B. The figures suggest that, as a whole, household size has very little effect on rent-income ratios. However, upon closer inspection, it is evident that household size becomes fairly influential as
incomes decrease. For example, one-person households with incomes of $3,000 to $4,000 paid about 35 percent of their income for rent, while 3-to 4-person households with the same income paid as much as 50 percent. This conclusion will be supported by the later section which will discuss the effects of household size.

The effects of household size are not simple. For households of up to 3-4 persons, household size has a positive effect on rent income ratios, but for households larger than 3-4 persons, there seem to be negative effects.

e. **Elasticity and household size**

Look at the Figure III-2-A and -B again. It is clear that rent expenditures are inelastic in terms of income for large size households, while they are comparatively elastic for small-size households.

f. **Rent-income ratio of low-income households**

It is surprising to see how much of their incomes low-income households are paying for housing. They are paying for rents by really high ratios. For example, households with annual incomes of $3,000 are paying 50 percent of their income for rent, regardless of household size. Does such a figure represent actual payment? There are two possible kinds of noises in these data: that is, welfare payments and mis-reporting of income. If there were any kind of
housing earmarking in welfare payments, it would increase rent expenditures of welfare recipients. Massachusetts has basically two welfare programs for low-income households: Aid to Families with Dependent Children (AFDC) and the General Relief. Both programs have no housing earmarking.\(^6\) As a consequence, welfare programs do not necessarily inflate rental expenditure. How about mis-reporting of income among low-income families? It is quite possible that low-income families tend to understate their incomes. One piece of evidence for this is that, among renter households headed by white male 18-64 years of age and with incomes under $2,000, 69 out of 89 cases reported rental payments higher than their incomes, based on the 1970 U.S. Census Public Use Sample. Low-income families could have a tendency to not report incomes from part-time jobs.\(^7\) Although income understatement is quite possible among low-incomes, it is also confirmed by several housing officials\(^8\) based on their empirical knowledge that rent payments could easily be as high as 50 to 60 percent of their incomes for low-income families.

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\(^6\) Based on an interview with M. Ringer, Housing Unit Section, State Welfare Department.
\(^7\) Based on an interview with M. Hobbs, Massachusetts Housing Finance Agency.
\(^8\) Based on an interview with M. Ringer, Mr. Crowley of Boston Housing Authority, etc.
III-B-2. Effects of race and sex

a. Results

The results of the cross-tabulations with respect to effects of race and sex of household heads on rent-income ratios is illustrated in Figures III-3 and -4, from which households with rent-income ratio over 100% were already excluded. In this study, race and sex are considered dichotomous variables: race of a head of a household is defined as either white or non-white, and its sex is defined as either male or female. In order to see interactive effects of race and sex, as well as effects of race or sex, four cross-tables are constructed for households headed by white females, white males, non-white females and non-white males.

When household size and age of head are held roughly constant, as a whole, race and sex have a little effect on rent-income ratios. For middle- and high-income households whose incomes are over $8,000, both race and sex seem to have no effect on the rent-income ratio; however, for households whose incomes are under $8,000, these variables do have effects.

Among low- and moderate-income households, those who are headed by whites tend to pay higher rents than those headed by non-whites. However, one new finding of this study is that the difference in rent-income ratios occurs only between white female-headed households and non-white female-
Figure III-3. Rent-Income Ratio by Income Stratified by Race and Sex.

For 3-6 person household with head 18-64 years of age.

\( E(\text{Entry of each point}): 10 \leq E \leq 232. \)

Excluding households with \( R/Y \) over 100%.
Figure III-4. Annual Gross Rent by Income Stratified by Race and Sex.

For 3-6 person household with head 18-64 years of age.

E(Entry of each point): 10 ≤ E ≤ 232.

Excluding households with R/Y over 100%.
headed households. For male-headed households, the race of the head of household has no effect, although a slight difference between white males and non-white males can be seen in Figure IV-4. If this finding is correct, it suggests that racial differences in rent-income ratio are evident only for females.

Among low-and moderate-income households, white female heads have higher rent-income ratios. This finding agrees with the conclusion cited in Section II-C-4. On the contrary, non-white female heads have lower rent-income ratios. This is a new finding and unexpected, according to the results reported in the literature reviewed in the previous chapter. Thus, this finding requires further examination.

b. Effects of public housing

Possible effects of public housing were examined in reference to the lower rent-income ratios of non-white women. We know that public housing accommodates many households headed by non-white females. The tenants are supposed to pay 25 percent of their incomes on rent. Therefore, public housing has a negative effect on the rent-income ratio of non-white female-headed households who were previously

9To state the situation more exactly, tenants in public housing are required to pay 25 percent of their adjusted incomes. In this study, however, computation was done assuming that 25 percent of gross income was required, mainly for convenience.
paying more than 25 percent of their income for rent on the average. It could be a cause of low rent-income ratios of non-white females. To see whether the low rent-income ratio of non-white females was caused by public housing, two data sets (set A and set B; to know what they are, look at Tables III-4-A and -B) which showed significantly low rent-income ratios of non-white females in the Figure III-3 were adjusted in order to exclude the effects of public housing.

Adjustment was based on two assumptions. First, the City of Boston was assumed to be able to fairly represent the whole Boston SMSA in terms of the households' numbers in the public housing against the total renter households, so as to allow this study to use Boston data in order to adjust rent-income ratios. Secondly, the number of households in public housing (both federal and state) in 1969 was the same as those in 1970, so as to allow this study to use the data in 1970. Based on the data of the Boston Housing Authority, a probability of a renter to be living in the public housing instead of a private rental unit was calculated for each of four groups: white female, white male, non-white female, and non-white male-headed households. Then rent-income ratios of Boston city (ratios which are identical to

10 The Table, Head of Household by Race (in 1970 in the City of Boston), constructed by Mr. Saffadini, Boston Housing Authority.
those of Boston SMSA) were adjusted by the formula of

$$R' = \frac{R - R(P) \cdot p(P)}{P(P)}$$  \hspace{1cm} (3.6)$$

where

- $p(P)$ = probability of a household in a group to be living in public housing
- $R$ = measured rent-income ratio of a group (not adjusted)
- $R(P)$ = rent-income ratio of households in public housing; that is, .25
- $P(P)$ = probability of a household in a group to be not living in public housing, that is, $1.0 - p(P)$
- $R'$ = rent-income ratio of households in a group and not in public housing; that is, an adjusted rent-income ratio.

Results are illustrated in Table III-4-A and -B. The precise procedure of computation is explained in Appendix A. The adjusted rent-income ratios of data set B show that the rent-income ratio of non-white female-headed households is close to that of the white females; that is, the non-white females no longer show lower rent-income ratios. However, the non-white females in data set A still show lower rent-income ratios. Therefore, although it is hard to conclude, based on only one set of data, that having a non-white female head has negative effects on the rent-income ratio,
### TABLE III-4-A
**RENT-INCOME RATIOS OF HOUSEHOLDS WITH INCOME $4,000–$6,000 (DATA SET A)**
(3–6 person households with a head 18–64 years of age)

<table>
<thead>
<tr>
<th></th>
<th>Original Rent-Income Ratios *1</th>
<th>Adjusted Rent-Income Ratios *2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>white-headed</td>
<td>non-white headed</td>
</tr>
<tr>
<td>Male headed</td>
<td>.320</td>
<td>.310</td>
</tr>
<tr>
<td>Female headed</td>
<td>.350</td>
<td>.260</td>
</tr>
</tbody>
</table>

*1 Extracted from the Figure III-3.
*2 See Appendix A.

### TABLE III-4-B
**RENT-INCOME RATIOS OF HOUSEHOLDS WITH INCOME $2,000–$4,000 (DATA SET B)**
(3–6 person households with a head 18–64 years of age)

<table>
<thead>
<tr>
<th></th>
<th>Original Rent-Income Ratios *1</th>
<th>Adjusted Rent-Income Ratios *2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>white-headed</td>
<td>non-white headed</td>
</tr>
<tr>
<td>Male headed</td>
<td>.530</td>
<td>N.A.</td>
</tr>
<tr>
<td>Female headed</td>
<td>.500</td>
<td>.420</td>
</tr>
</tbody>
</table>

N.A. = invalid data because of a too small entry.
it is plausible to conclude that having a non-white female head has no positive effects on rent-income ratios.

III-B-3. Effects of household size

The results of this study are illustrated in Figure III-5. Among households headed by white males 18-64 years of age, household size has no effect on the rent-income ratios for households whose incomes are over $6,000. This means that households with the same income pay their rent by the same ratio regardless of their size, provided their income is over $6,000. As income decreases below $6,000, household size becomes influential in a special way: namely, households of 2-4 persons have high rent-income ratios, while households with one person or 5-or-more persons pay comparatively less rent than those with 2-4 persons. This result is consistent with the result of Lowry's study in regard to low-income households, but in regard to middle- and high-incomes, the results of this study are not consistent with his results. The curve of rent income ratios for middle- and high-income households can easily be explained. First, as the size of a household increases, space needs increase; however, at the same time, needs for food, clothing, etc. increase.\(^{11}\) As a consequence, expenditure prefer-

Figure III-5. Rent-Income Ratio by Size of Household Stratified by Income.

For households headed by white male 18-64 years of age.

Entry: $12 \leq E \leq 236$.

Excluding households with R/Y over 100%.
ences for housing or other goods remain the same. Second, more space could be obtained for a larger household without extra expenditure by choosing housing in a different neighborhood. The curve of rent-income ratios for low-incomes could reflect two things. At the beginning, an additional person to a household requires more housing than other goods, but after household sizes have reached to 3-4 persons, need for other goods by an additional person to a household becomes overwhelming.

III-B-4. Effects of age of head

The results of this study on the effects of the age of head are illustrated in Figure III-6. For households with incomes higher than $10,000, the age of heads has no effect on rent-income ratios for two-person households, and have slightly negative effects on the ratios for 1 and 3-4 person households. An exception to this is the curve for the 3-4 person household with incomes from $15,000 to $20,000, the curve which shows strong negative effects of age on rent-income ratios. This is a different finding from that of Lowry. For households with incomes below $10,000, however, the age of heads has clear effects: age has a negative effect on rent-income ratios until the age of 60 years. Thereafter, the age of head has a positive effect on the ratio. As a consequence, for households are
Figure III-6. Rent-Income Ratio by Age of Head Stratified by Income and Household Size.

For households headed by white male.
E(Entry): $10 \leq E \leq 177$.
Excluding households with R/Y over 100%.
controlled to be headed by white males and to have incomes below $10,000, rents are paid by the lowest ratios by households with heads 60 years old; by the highest ratios by young households, and by the intermediate ratios by aged households. This finding is basically consistent with the findings cited in II-C-4, although it deviates in one aspect: i.e., this study suggests that aged households are paying rent by the moderate ratios instead of by the highest ratios, as compared with young and middle-aged households.

One of the possible reasons that young households were found to be paying rent by the highest ratios may be caused by the high permanent income of young households. It is reasonable to assume that young households have higher expected future incomes; that is, permanent incomes, than the older households. Thus, this high permanent income may cause high housing expenditures.

III-C. Cross-Table Analysis of Uncontrolled Data

Thus far, in this study on the Boston SMSA, the data analyzed were renter households headed by white males of 18 to 64 years of age, for all the studies except the study for the race-sex analysis and the age analysis. In this section, analyses were done based on uncontrolled data\textsuperscript{12} stratified

\textsuperscript{12}"Uncontrolled data" means the data which consists of all renter households in Boston SMSA instead of those headed by white males of 18 to 64 years of age, that were referred to as "controlled".
by income and size, in order to examine whether or not the conclusions based on the controlled data are generalizable to other types of households. And also, this section intends to see general effects of income and household size. Since these two are major policy factors in housing assistance programs, and since the results of the uncontrolled data will be a convenient material for quick references to see how much rent tenants with certain income and household size are paying on the average in the Boston SMSA.

Results of this section suggest that the earlier findings are generalizable in terms of effects of incomes and household sizes. Because incomes and household sizes of uncontrolled households showed the same effects on rent-income ratios as those of controlled households did. It means that, even after aggregating the deviations of rent-income ratios from averages by races, sexes, and ages of heads, household sizes and incomes maintained the same effects on rent-income ratios. As we have seen in previous sections, there are deviations by races, sexes, and ages of heads, but those deviations are small component to the income effect.

III-C-1. Effects of income

In Section III-B-1, households headed by white males of 18-64 years of age were studied. (see Figures III-1-A and -B). In this section, all renter households in the
SMSA are examined. The results are illustrated in figures III-7-A and -B, which show exactly the same pattern as in figures III-1-A and -B. The graphs show clear hyperbolic shapes rather than straight declining lines. The deviations by household sizes are reduced from the deviations in figures III-1-A and -B.

Integrating household size categories leads to the overall average rent-income ratios by income, which are illustrated in Figure III-8.\(^{13}\) This figure also shows average rent expenditures by income. Those two curves in Figure III-8 are surprisingly simple and strongly support that a linear demand equation such as (3.3) will fit the data better than the log-linear form given in (3.4). These results are consistent with the conclusions expressed in Section III-B-1 based on the controlled data. The relationship between annual rent and income suggested by Figure III-8 is:\(^{14}\)

\[
R = 1.25 + (.047)Y, \quad (3.5)
\]

---

\(^{13}\)Two curves of rent-income ratios and rents in Figure III-8 were constructed from the identical data but by different computation procedures. Therefore, two comparable data points on each curve do not exactly fit each other.

\(^{14}\)Based on the results of regressions summarized in Table III-5. The last row of the table for "all households" shows the regression coefficients of Figure III-8.
Figure III-7-A. Rent-Income Ratio by Annual Income Stratified by Household Size.

For all Boston renters, excluding households with a rent-income ratio over 100%.

E(Entry): $15 \leq E \leq 354$. 

R/Y (Rent-Income Ratio)
Figure III-7-B. Rent-Income Ratio by Annual Income Stratified by Household Size.
For all Boston households, excluding households with income below $2,000.
E(Entry); $15 \leq E \leq 354$
Figure III-8. Rent-Income Ratio and Annual Rent by Annual Income.
For all Boston renters excluding households with income below $2,000.
E(Entry): $73 \leq B \leq 731$.

Note: Two vertical axes of R/Y and R have no relation each other.
where

\[ R = \text{annual rent in units of } $1,000 \text{ and} \]
\[ Y = \text{annual income in units of } $1,000. \]

a. Rent-income ratio and elasticity

For purposes of convenience, the six rent-income curves in Figure III-7-A, -B, and Figure III-8 were regressed based on the following bivariate regression formula using group average data shown in Table IV-2,

\[ \frac{R}{Y} = \alpha + B\left(\frac{1}{Y}\right). \] (3.6)

The results of this regression are summarized in Table III-5. A coefficient of determinant (that is \( R^2 \)) in Table III-5 indicates how well an equation fits to data. For example, \( R^2 = .9684 \) indicates that 96.84 percent of the variation of rent-income ratios are explained by the inverse of income. The \( R^2 \) in the table are very high, because aggregated data (which is shown in Table IV-2) was used, where all individual households within the same income class are grouped together. If individual data were used, \( R^2 \) would be much lower than those in the Table.

If the formula (3.6) is transformed into a rent regression formula like (3.5), \( B \) in Table III-5 indicates the intercept and \( \alpha \) indicates the slope. As one can see in this table, 1-person households and 6-7-person households have low intercepts (\( B \)) and high slopes (\( \alpha \)), while 2-5-person households have high intercepts and low slopes. Al-
TABLE III-5
RENT-INCOME RATIO REGRESSION RESULTS (COEFFICIENTS OF REGRESSION)

<table>
<thead>
<tr>
<th>Household Type</th>
<th>Regression Coefficients</th>
<th>Coefficient of Determinant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha )</td>
<td>B</td>
</tr>
<tr>
<td>1 person household</td>
<td>.0587</td>
<td>1.1407</td>
</tr>
<tr>
<td>2 person household</td>
<td>.0470</td>
<td>1.3136</td>
</tr>
<tr>
<td>3 person household</td>
<td>.0331</td>
<td>1.3955</td>
</tr>
<tr>
<td>4-5 person household</td>
<td>.0343</td>
<td>1.3227</td>
</tr>
<tr>
<td>6-7 person household</td>
<td>.0478</td>
<td>1.2135</td>
</tr>
<tr>
<td>All households</td>
<td>.0473</td>
<td>1.2485</td>
</tr>
</tbody>
</table>

Formula: \( R/Y = \alpha + B \left( \frac{1}{Y} \right) \)

Source: All Boston SMSA renters from the 1970 Census

Data: Based on group data stratified by $2,000 increments. The data base is shown in Table IV-2.

All coefficients are significant at .01 level.
though it is hard to explain these findings, they suggest that 2 to 5 person households are relatively inelastic to their incomes with regard to their housing expenditures.

To confirm the above argument and also to see variations of income elasticities by income changes, income elasticities of each type of household by three levels of income of $5,000, $10,000 and $15,000 have been computed, and are shown in Table III-6. This table confirms the existence of low elasticities among 2 to 5 person households. The table also suggests considerably low income elasticities for all households, since none of the elasticities exceeds the .5 level. Because this study is based on current income, the elasticity estimates will be below those based on permanent income (see Section II-A-1-b and Section III-A-2-b).

To get a rough idea of permanent income elasticities, we could increase those elasticities in the Table III-6 by 20 percent based on the hypothesis stated in the Section III-A-2-c and estimates by Carliner.\textsuperscript{15}

Even when increased by 20 percent, the income elasticities of this study are still low. Only one elasticity in Table III-6 exceeds the 0.5 level.

TABLE III-6

INCOME ELASTICITY BY HOUSEHOLD SIZE AND BY ANNUAL INCOME

<table>
<thead>
<tr>
<th>Household Type</th>
<th>Income Elasticity by Annual Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$5,000</td>
</tr>
<tr>
<td>1 Person household</td>
<td>.2046</td>
</tr>
<tr>
<td>2 Person household</td>
<td>.1517</td>
</tr>
<tr>
<td>3 Person household</td>
<td>.1060</td>
</tr>
<tr>
<td>4-5 Person household</td>
<td>.1148</td>
</tr>
<tr>
<td>6-7 Person household</td>
<td>.1645</td>
</tr>
<tr>
<td>All household</td>
<td>.1593</td>
</tr>
</tbody>
</table>

Data Base: Table III-5

Computation: The regression equation in Table III-5 is

\[ \frac{R}{Y} = \alpha + B \left( \frac{1}{Y} \right). \]

Multiplying it by \( Y \) leads to \( R = B + \alpha Y \).

Therefore, an income elasticity \( \frac{dR}{dY} \frac{Y}{Y} \) is

\[ \frac{dR}{dY} = \frac{Y}{B + \alpha Y}. \]
III-C-2. Effects of household size

The average effects of household size on rent-income ratios, using all renters in the Boston SMSA, are illustrated in Figure III-9. Adding households headed by non-white, female, or aged persons to those used in Figure III-5 did not change the pattern of rent-income ratios and households size relationships: namely, for high-income households, size has no effect on the average rent-income ratios, and for low-income households, size does have positive effects until the sizes reach three persons per household, after which household size has a negative effect.
Figure III-9. Rent-Income Ratio by Household Size Stratified by Annual Income.
For all Boston renters excluding households with income below $2,000.
E(Entry): $11 \leq E \leq 199.$
IV. CONCLUSION

IV-A. Conclusions on Housing Consumption Patterns of Renter Household

Based on the study of renters in Boston SMSA and the review of past studies, the following conclusions about the relation between housing consumption and household characteristics are obtained. These final conclusions are illustrated in Table IV-1.

Rent-income ratios and household income are inversely related, and the relation between income and rent-income ratio is hyperbolic. The formula that best fits the data is most likely of the form:

\[ \frac{R}{Y} = a + b \cdot \left( \frac{1}{Y^c} \right) \]  

where

- \( \frac{R}{Y} \) = rent income ratio of a type of household whose income is \( Y \)
- \( Y \) = income
- \( a, b, c \) = coefficients.

This formula, however, cannot be specified by linear regression analyses. An alternative form that works well is

\[ \frac{R}{Y} = a + b \cdot \left( \frac{1}{Y} \right) \]  

This formula is slightly better than the formula of

\[ \frac{R}{Y} = a' \cdot \left( \frac{1}{Y^{b'}} \right) \]
Table IV-1. Conclusions about Housing Consumption and Household Characteristics.

<table>
<thead>
<tr>
<th>Rent-Income Ratio vs. Income</th>
<th>Relation is hyperbolic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{R}{Y}$</td>
<td>$\frac{1}{Y}$</td>
</tr>
<tr>
<td>$4.7%$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\frac{R}{Y}$</td>
<td>$\frac{1}{Y}$</td>
</tr>
<tr>
<td>$1,250$</td>
<td>$Y$</td>
</tr>
</tbody>
</table>

- There is an intercept.

- The best fit equation is most likely to be

$$\frac{R}{Y} = a + b\left(\frac{1}{Y}\right).$$

- Better regression formula is

$$\frac{R}{Y} = a + b\left(\frac{1}{Y}\right)$$

rather than

$$\frac{R}{Y} = a \cdot \frac{1}{Y}$$

for households with income over $2,000 and less than $2,000.

Although the difference between two equations is slight and both fit well to the data, for multiple regression analysis, a linear formula rather than a log-linear formula will serve well.
Table IV-1 (continued)

- The regression formula for all renters in Boston SMSA is

\[ \frac{R}{Y} = 0.0473 + 1.248 \left( \frac{1}{Y} \right) \]

where \( R \) and \( Y \) are measured in $1,000.

| Income Elasticity vs. Current Income and vs. Household Size. | - Elasticities may well vary as a function of income.  
- Elasticities are considerably low ranging from .1 to .5 as incomes change from $5,000 to $15,000.  
- Elasticities are high for 1 person or 6 or more person households, and are relatively low for 2 to 5 person households. |
|-----------------------------------------------------------|------------------------------------------------|
| Rent-Income Ratio vs. Household Size.                     | - As a whole sizes have little effects on rent-income ratios compared to the effect of income.  
- Household size is influencial for households with incomes under $6,000.  
- Relation is polinomial. |

\[ \frac{R}{Y} \]

- low income ≤ $6,000
- middle
- high

Size
Table IV-1 (continued)

-For low income households, 2 to 5 person households have higher ratios.

-For multiple regressions, the results suggest distinguishing the data by sizes of 1, 2 to five and 6 or more person households, or inserting an interactive term of income and size (a * $S^2_Y$).

Race and Sex vs. Rent-Income Ratio.

-As a whole, race and sex have little effect on rent-income ratios.

-For low income households with income below $8,000, race and sex have effects on rent-income ratios.

-White female headed household pay rents higher than average, while non-white female headed household pay below average rents.

-Effects of nonwhite female heads may well not be significant, because much of this difference is explained by the high incidence of nonwhite female headed households in public
Table IV-1 (continued)

- Public housing has significant contribution to lowering the rent-income ratio of nonwhite female headed households.

- Male headed households of both sexes have similar rent-income ratios.

For multiple regressions, distinguishing by sex and race, especially by white females, nonwhite female and all males, is suggested.

- Alternatively two interactive terms of race and sex are required. However all such terms may well not be significant.

Rent-Income Ratio vs. Age of Head

- For highincome households over $10,000, the age of the head of household has no effect or only slight negative effect. For lowincome households under $10,000, the age of the head has negative effect until age 60, and
Table IV-1 (continued)

thereafter it has a positive effect.

-For multiple regressions, distinguishing by either age groups or by incomes (devided into "under $10,000" and "over $10,000") is suggested.
for households with an income of from $2,000 to $20,000. For all renters in Boston SMSA, regardless of household characteristics, the equation (4.2) is specified as

\[ \frac{R}{Y} = 0.0473 + 1.248\left(\frac{1}{Y}\right), \]  

(4.4)

where

rents (R) and incomes (Y) are measured in $1,000.\textsuperscript{1}

Note that (4.4) implies a rent of $1,248 plus 4.7 percent of the household's income. The fit for equation (4.3) had an \( R^2 \) of .97 instead of .99 (again, using aggregated data from Table IV-1). The fitted equation (4.3) implied a constant income elasticity, but fitted value was again low (about 0.20) and included in the range implied by (4.4).\textsuperscript{2}

The income elasticities found in this study are very low as compared to the results of studies by Reid and Ekanem, but are comparable to implied elasticities by the study of Lowry. For all households with incomes of from $5,000 to $15,000, income elasticities range from 0.11 to 0.44. For households with income of $10,000, income elasticities are about 0.20, which is very low. Because this study was based on current and not permanent income.

Household size also has an effect on income elasticity.\textsuperscript{3}

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\textsuperscript{1}See Table III-5.
\textsuperscript{2}See Table III-6.
\textsuperscript{3}See Table III-6.
Income elasticity decreases as household size increases up to three-person households by .10 to .18; however, income elasticity increases again as household size increases more than three persons.

Deviating from the analyses by Lowry, household sizes, sex and race of head, and age of head have effects on rent-income ratios only for households with low-and moderate-incomes less than $10,000 or $6,000. These characteristics have almost no effect on the rent-income ratio for those with high incomes.

In regard to households headed by a female, white female heads have a positive effect on rent-income ratios, while non-white female heads could have a negative effect or at least have no positive effect. In the Boston SMSA data, public housing had the significant effect of lowering the rent-income ratio of non-white female-headed households; as a consequence, the results of this study (under the effects of public housing) show clear low rent-income ratios for non-white female-headed households.

IV-B. Facts: How Much Rent Tenants Are Paying

In addition to housing consumption patterns, actual facts on how much rent tenants are paying is a major interest of this study. The rent-income ratios by which renters are paying can be estimated by two methods.
One is by cross-tables themselves constructed from the Public Use Sample of the 1970 U.S. Census. The table which shows rent-income ratios by household size and by income class is shown in Table IV-2. This table shows an average rent-income ratio by which each type of household paid their rent in 1969, regardless of household characteristics such as age, sex, or race of head. These data were used in the graphs in Figure III-7-A and-B and in Figure III-8.

Another method of estimating rent-income ratios is the regression equations shown in Table III-5. These equations give us more internally consistent estimates of rent-income ratios than those derived from Table IV-2.

The procedure to obtain a rent-income ratio is as follows:

To estimate the rent-income ratio of a one-person household with an income of $6,000, for instance, the equation which should be used is

\[ \frac{R}{Y} = 0.0587 + 1.1407 \left( \frac{1}{Y} \right), \]

where

\[ R, Y = \text{annual rent and income, respectively, measured in } \$1,000. \]

Substituting 6.0 (i.e., $6,000) for \( Y \) leads to \( \frac{R}{Y} = 0.2488 \).

As a result, the average rent-income ratio of one-person households with an annual income of $6,000 is estimated to
### TABLE IV-2.

RENT INCOME RATIO (%) BY INCOME AND BY HOUSEHOLD SIZE

- All Boston SMSA renters
- Excluding households with \( R/Y \geq 100\% \)
- Blanks in the table mean invalid data because of \( R/Y \geq 100\% \) or too small entries in the cells

<table>
<thead>
<tr>
<th>Annual Income ($)</th>
<th>1P</th>
<th>2P</th>
<th>3P</th>
<th>4-5P</th>
<th>6-7P</th>
<th>8 or more</th>
<th>All Renters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0-1999</td>
<td>60.24</td>
<td>62.79</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2k-3999</td>
<td>43.78</td>
<td>48.49</td>
<td>49.06</td>
<td>46.76</td>
<td>45.63</td>
<td>46.06</td>
<td></td>
</tr>
<tr>
<td>4k-5999</td>
<td>29.10</td>
<td>31.62</td>
<td>33.02</td>
<td>30.76</td>
<td>30.34</td>
<td>30.63</td>
<td></td>
</tr>
<tr>
<td>6k-7999</td>
<td>21.74</td>
<td>23.43</td>
<td>23.47</td>
<td>23.24</td>
<td>22.27</td>
<td>22.87</td>
<td></td>
</tr>
<tr>
<td>8k-9999</td>
<td>19.04</td>
<td>18.12</td>
<td>17.43</td>
<td>18.13</td>
<td>19.68</td>
<td>18.25</td>
<td></td>
</tr>
<tr>
<td>10k-11999</td>
<td>15.54</td>
<td>16.26</td>
<td>16.33</td>
<td>15.60</td>
<td>17.70</td>
<td>16.11</td>
<td></td>
</tr>
<tr>
<td>14k-15999</td>
<td>13.55</td>
<td>13.94</td>
<td>12.59</td>
<td>11.09</td>
<td></td>
<td>12.80</td>
<td></td>
</tr>
<tr>
<td>16k-17999</td>
<td>13.00</td>
<td>11.01</td>
<td>11.36</td>
<td></td>
<td></td>
<td>12.12</td>
<td></td>
</tr>
<tr>
<td>18k-19999</td>
<td>12.24</td>
<td>11.28</td>
<td>9.96</td>
<td></td>
<td></td>
<td>11.25</td>
<td></td>
</tr>
</tbody>
</table>

**E (Entry):**

| 15 \leq E \leq 354 | 87 \leq E \leq 731 |

**Comparable Graph**

- Figure III-7-A
- Figure III-7-B
- Figure III-8
be about 25 percent.

As one can see in Table IV-1 and in various other graphs in this study such as III-1-A and -B or III-7-A and -B, households with incomes of less than $6,000 (about 41.8 percent of all renters in the Boston SMSA) are predicted to pay more than 25 percent of their gross income on rent, and those with income less than $4,000 (about 26.7 percent of all renters in the Boston SMSA) are predicted to pay more than 40 percent.

IV-C. A Comparison of What Renters Pay and What Renters Should Pay

To see approximately how many families of each household size experience rent burden, a comparison of what households are paying and what they should pay was done. Its results are shown in Table IV-3. The data of what households are paying were obtained from my study (that is, Table IV-1) and expressed by percentage rent-income ratios. The data of what they should pay were based on Income Available for Rent (1971) proposed by Matt Hobbs in Massachusetts Housing Finance Agency.⁴ and were expressed by percentage rent-income ratios using gross annual income. For convenience, it is assumed that what they should pay in 1971 can

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⁴ One who has interests in this should refer to Matt Hobbs, Memorandum: Idealized Housing Subsidy Program, 1972.
### TABLE IV-3.
COMPARISON OF WHAT TENANTS ARE PAYING AND WHAT TENANTS SHOULD PAY

<table>
<thead>
<tr>
<th>Persons per Household</th>
<th>Gross Income by $1,000</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3-4</td>
<td>4-5</td>
<td>5-6</td>
<td>6-7</td>
<td>7-8</td>
<td>8-9</td>
<td>9-10</td>
<td>10-11</td>
<td>11-12</td>
<td>12-13</td>
<td>13-14</td>
</tr>
<tr>
<td>1 P. Hh. Are Paying</td>
<td>.44</td>
<td>.29</td>
<td>.22</td>
<td>.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Should Pay</td>
<td>.00</td>
<td>.03</td>
<td>.15</td>
<td>.23</td>
<td>.25</td>
<td>.25</td>
<td>.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 P. Hh. Are Paying</td>
<td>.48</td>
<td>.32</td>
<td>.23</td>
<td>.18</td>
<td>.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Should Pay</td>
<td>.00</td>
<td>.03</td>
<td>.12</td>
<td>.19</td>
<td>.24</td>
<td>.24</td>
<td>.24</td>
<td>.24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 P. Hh. Are Paying</td>
<td>.49</td>
<td>.33</td>
<td>.23</td>
<td>.17</td>
<td>.16</td>
<td>.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Should Pay</td>
<td>.00</td>
<td>.08</td>
<td>.15</td>
<td>.20</td>
<td>.22</td>
<td>.23</td>
<td>.23</td>
<td>.23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 P. Hh. Are Paying</td>
<td>.47</td>
<td>.31</td>
<td>.23</td>
<td>.18</td>
<td>.06</td>
<td>.12</td>
<td>.17</td>
<td>.21</td>
<td>.21</td>
<td>.22</td>
<td></td>
</tr>
<tr>
<td>Should Pay</td>
<td>.00</td>
<td>.05</td>
<td>.10</td>
<td>.15</td>
<td>.19</td>
<td>.21</td>
<td>.21</td>
<td>.21</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 P. Hh. Are Paying</td>
<td>.47</td>
<td>.31</td>
<td>.23</td>
<td>.18</td>
<td>.00</td>
<td>.14</td>
<td>.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Should Pay</td>
<td>.00</td>
<td>.06</td>
<td>.12</td>
<td>.17</td>
<td>.21</td>
<td>.22</td>
<td>.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 P. Hh. Are Paying</td>
<td>.46</td>
<td>.30</td>
<td>.22</td>
<td>.20</td>
<td>.18</td>
<td>.13</td>
<td>.13</td>
<td>.13</td>
<td>.13</td>
<td>.13</td>
<td>.13</td>
</tr>
<tr>
<td>Should Pay</td>
<td>.00</td>
<td>.05</td>
<td>.07</td>
<td>.14</td>
<td>.17</td>
<td>.20</td>
<td>.20</td>
<td>.20</td>
<td>.20</td>
<td>.20</td>
<td>.20</td>
</tr>
<tr>
<td>7 P. Hh. Are Paying</td>
<td>.46</td>
<td>.30</td>
<td>.22</td>
<td>.20</td>
<td>.18</td>
<td>.13</td>
<td>.13</td>
<td>.13</td>
<td>.13</td>
<td>.13</td>
<td>.13</td>
</tr>
<tr>
<td>Should Pay</td>
<td>.00</td>
<td>.03</td>
<td>.08</td>
<td>.12</td>
<td>.15</td>
<td>.18</td>
<td>.19</td>
<td>.20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Entries are rent-income ratios

Sources: What tenants are paying is based on Table IV-1.
What tenants should pay is based on Income Available for Rent (1971) proposed by Matt Hobbs (MIHA)
be applicable to 1969.

One can say that a household paying higher rent than he should pay is experiencing rent burden. The solid dark line in the table shows separation of rent burden from non-rent burden. Using this line and the Table III-1, which shows numbers of households in every cell, one can compute how many households out of all households with a certain household size are experiencing rent burden. The results are shown in the last column of Table IV-3. For example, 68.1 percent of one-person households are experiencing rent-burden. Similarly, 85.4 percent for seven-person households. The results suggest that most of renter households are experiencing rent-burden.

This short analysis has shown one of many ways of how the results of this study can be used.

IV-D What Was Done in This Study and What Remains to Be Researched

IV-D-1 What was done in this study

In this study, patterns of housing consumption among rental households and the rent-income ratios by which renters pay their rent have been examined. A set of housing consumption patterns and the rent-income ratios of Boston renters have been specified.

It should be noted, once more, that the above conclu-
sions are based on specific conditions. First, all analyses on Boston renters were based on current income as an income concept. As suggested in several sections, time series permanent income is a more appropriate income concept than current income. Current income tends to overestimate rent-income ratios of low-income households and to underestimate those of high-income households. As a consequence, use of current income tends to underestimate overall elasticities of housing expenditures.

Secondly, it also should be remembered that the analyses of Boston renters were done for the whole SMSA, and data disaggregated by geographical areas were not studied. In other words, the price elasticity of housing expenditure in Boston SMSA was assumed to be unity. However, price elasticity has not yet been proved to be 1.0, estimates of price elasticity vary widely among different studies from 0.7 to 1.5. If the price elasticity of Boston SMSA is significantly different from 1.0, then analyses based on the entire SMSA, as a whole, would be distorted by the geographical concentration of any type of household. For instance, aged households could be concentrated in the central business district where housing price is supposed to be high. These

high prices would then cause high rental expenditures of aged households, and consequently, would cause high income ratios. Thus, geographical aggregation could cause biases in estimating rent-income ratios. Thirdly, the Boston analysis was done based on averages of individual rent-income ratios, which are grouped by income.

IV-D-2. What remains to be researched

Although several simple regressions were run, a direct consequent step which should be pursued in the next study is a multiple regression analysis based on the various suggestions in Section IV-A and based on individual data. This step is necessary to confirm the significance levels of the qualitative conclusions contained in Section IV-A and to quantify the qualitatively expressed relations in the conclusion. This study has generated sufficient information and bases for specification of regression models.

A further step after the quantification by multiple regression is an exploration of implications of the results on housing policies. Possible implications are: 1) suggestions on maximum contribution rates by which tenants should pay for their rents by themselves; 2) assessment of participation possibility of renters in any proposed housing policy with specific contribution rate such as the one shown in Section IV-C; and 3) measurement of rent burden given criteria from other researches.

Each of these topics could be an independent study.
theme. It is sincerely hoped that this study will help in creating more equitable and more rational housing assistance policies.
APPENDIX A: ADJUSTMENT OF OBSERVED RENT-INCOME RATIOS IN ORDER TO EXCLUDE THE EFFECT OF PUBLIC HOUSING

As mentioned in Section III-B-2-b, public housing has an effect to decrease the rent-income ratios for low-income households who paid more than 25 percent of their income on rent. Especially the public housing in the Boston SMSA could have strong negative effect on the rent-income ratios of non-white female-headed households whose income is below $6,000 (As one sees in Figure III-3, non-white female-headed households with income below $6,000 paid over 25 percent).

In order to find the true effects of sex and race of heads on rental expenditure, effects of the public housing have to be eliminated from the observed rent-income ratios. The exclusion procedure of the public housing effects are illustrated in the tables from A-1 to A-7. A key table is A-3, which shows the probabilities of living in the public housing for renters with income below $6,000 by race and sex of head. Tables A-1 and 2 compute necessary data for calculating the probabilities in Table A-3, the probabilities which were represented by p(P) in the equation (3.6). For convenience, (3.6) is rewritten here with small modification as

\[ \hat{R} = \frac{R - R(P) \cdot p(P)}{1 - p(P)} \]

(A.1)
### TABLE A-1

**COMPUTATION OF NUMBER OF RENTER HOUSEHOLDS WITH INCOME BELOW $6,000 AND WITH HEAD OF MALE AND FEMALE, IN THE CITY OF BOSTON**

<table>
<thead>
<tr>
<th>Renter Households</th>
<th>At all income levels (1)</th>
<th>With income under $6,000 = (1)×R_1 = (2)</th>
<th>With Y ≤ $6,000 headed by male = (2)×(1−R_2) = (3)</th>
<th>With Y ≤ $6,000 headed by female = (2)×R_2 = (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All renters</td>
<td>158,400</td>
<td>66,528</td>
<td>33,929</td>
<td>32,599</td>
</tr>
<tr>
<td>White headed renter</td>
<td>129,300</td>
<td>54,306</td>
<td>27,696</td>
<td>26,610</td>
</tr>
<tr>
<td>Non-white headed renter</td>
<td>29,100</td>
<td>12,222</td>
<td>6,233</td>
<td>5,989</td>
</tr>
</tbody>
</table>

where

\[
R_1 = \left( \frac{\text{all renters with income under $6,000}}{\text{all renters}} \right) \text{ in 1969, in Boston SMSA} = \frac{170,600}{407,600} = 0.42
\]

\[
R_2 = \left( \frac{\text{all renters with income under $5,000 headed by female}}{\text{all renters with income under $5,000}} \right) \text{ in 1969, in Boston SMSA} = \frac{68,322}{140,129} = 0.49
\]

Data sources:

(1): Table H-1, Census Tract, Boston SMSA, PHC (1)-29, U.S. Census, 1970

R_1: Table A-3, Metropolitan Housing Characteristics, Boston SMSA, HC(2)30, U.S. Census, 1970

R_2: Table A-7, Metropolitan Housing Characteristics, Boston SMSA, HC(2)30, U.S. Census, 1970

It is assumed that a half of one-person households were female.
TABLE A-2

COMPUTATION OF NUMBER OF RENTER HOUSEHOLDS IN PUBLIC HOUSING WITH INCOME BELOW $6,000 AND WITH HEAD OF MALE AND FEMALE, IN THE CITY OF BOSTON

<table>
<thead>
<tr>
<th>Renter Households in Public Housing</th>
<th>Headed by male at all income levels (11)</th>
<th>Headed by Female at all income levels (12)</th>
<th>Headed by male with income below $6,000 (11)xR₃=(13)</th>
<th>Headed by female with income below $6,000 (12)xR₃=(14)</th>
<th>With income below $6,000 (13)+(14)=(15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All renters</td>
<td>3394</td>
<td>5354</td>
<td>2681</td>
<td>4230</td>
<td>6911</td>
</tr>
<tr>
<td>White headed renters</td>
<td>2239</td>
<td>2600</td>
<td>1769</td>
<td>2054</td>
<td>3823</td>
</tr>
<tr>
<td>Non-white headed renters</td>
<td>1155</td>
<td>2754</td>
<td>912</td>
<td>2176</td>
<td>3088</td>
</tr>
</tbody>
</table>

where

\[ R₃ = \left( \frac{\text{all households in public housing with income below $6,000}}{\text{all households in public housing}} \right)_{1970} \]

\[ = \frac{6838}{8708} = 0.79 \]

Data sources: (11), (12): Table "Head of Household by Race as of 1970," constructed by Mr. Saffadini, Boston Housing Authority

R₃: Table "Anticipated Income by Number of Workers as of 1970," constructed by Mr. Saffadini, Boston Housing Authority.
### TABLE A-3

PROBABILITY OF LIVING IN PUBLIC HOUSING FOR RENTERS WITH INCOME BELOW $6,000 BY RACE AND SEX OF HEAD, IN THE CITY OF BOSTON

<table>
<thead>
<tr>
<th>Renter household</th>
<th>With income below $6,000 = ( \frac{15}{22} )</th>
<th>With ( Y \leq 6,000 ) headed by male = ( \frac{13}{23} )</th>
<th>With ( Y \leq 6,000 ) headed by female = ( \frac{14}{24} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>All renters</td>
<td>.10</td>
<td>.08</td>
<td>.13</td>
</tr>
<tr>
<td>White headed renters</td>
<td>.07</td>
<td>.06</td>
<td>.08</td>
</tr>
<tr>
<td>Non-white headed renters</td>
<td>.25</td>
<td>.15</td>
<td>.36</td>
</tr>
</tbody>
</table>

Data sources: (15), (13), (14): Table A-2.
(2), (3), (4): Table A-1.
For explanation of the variables, look at the explanation of (3.6). Observed or measured rent-income ratios (that are \(R\) in the equation A.1) in tables A-4 and A-6 are adjusted by the probabilities in Table A-3 (that are \(p(P)\) in (A.1)) based on the equation (A.1), generating adjusted rent-income ratios (that are \(R'\)) in tables A-5 and A-7. Two tables A-4 and A-5 are the original data for Table III-4-A, and two tables A-6 and A-7 for Table III-4-B.

Because of imperfections of available data, this adjustment procedure stands on numerous assumptions, major ones of which are discussed in the following section: 1) It was assumed that participation rates of renters in the public housing in the City of Boston could be applicable to the entire Boston SMSA. In other words, the probabilities of living in the public housing for renters with income below $6,000 in Boston city (shown in Table A-3) were assumed to be the same as those in the Boston SMSA. 2) In the computation in Table A-1, it was assumed that the ratio of renters with income below $6,000 to all renters in the SMSA could be applicable to the ratio in the City of Boston. 3) At the same time, it was also assumed that the ratio of renters with income below $5,000 headed by females to those headed by persons of any sex in the SMSA can be applicable to the ratio of renters with income below $6,000.
### TABLE A-4

**AVERAGE RENT-INCOME RATIOS OF ALL RENTERS WITH INCOME FROM $4,000 TO $6,000 BY RACE AND SEX OF HEAD, IN THE BOSTON SMSA**

<table>
<thead>
<tr>
<th></th>
<th>Headed by male (33)</th>
<th>Headed by female (34)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White headed renters</td>
<td>.32</td>
<td>.35</td>
</tr>
<tr>
<td>Non-white headed renters</td>
<td>.31</td>
<td>.26</td>
</tr>
</tbody>
</table>

*Data sources: (33), (34): A cross-table based on uncontrolled data of all renters in the Boston SMSA derived from Public Use Sample, 1970 U.S. Census of Housing, using the EFFECT program of MIT.*

### TABLE A-5

**ADJUSTED RENT-INCOME RATIO OF ALL RENTERS WITH INCOME FROM $4,000 TO $6,000 BY RACE AND SEX OF HEAD, IN THE BOSTON SMSA**

<table>
<thead>
<tr>
<th></th>
<th>Headed by male (\frac{(33)-(23)x(.25)}{(1.0-(23))}) (43)</th>
<th>Headed by female (\frac{(34)-(24)x(.25)}{(1.0-(24))}) (44)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White headed renters</td>
<td>.325</td>
<td>.359</td>
</tr>
<tr>
<td>Non-white headed renters</td>
<td>.321</td>
<td>.266</td>
</tr>
</tbody>
</table>

*Data sources: (33), (34): Table A-4.*

(23), (24): Table A-3.
### TABLE A-6

AVERAGE RENT-INCOME RATIOS OF ALL RENTERS WITH INCOME FROM $2,000 TO $4,000 BY RACE AND SEX OF HEAD, IN THE BOSTON SMSA

<table>
<thead>
<tr>
<th></th>
<th>Headed by male (53)</th>
<th>Headed by female (54)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White headed renters</td>
<td>.53</td>
<td>.50</td>
</tr>
<tr>
<td>Non-white headed renters</td>
<td>N.A.</td>
<td>.42</td>
</tr>
</tbody>
</table>

N.A.: Invalid data because of a too-small entry.

Data sources: (53), (54): Ibid., look at the data source of Table A-4.

---

### TABLE A-7

ADJUSTED RENT-INCOME RATIO OF ALL RENTERS WITH INCOME FROM $2,000 TO $4,000 BY RACE AND SEX OF HEAD, IN THE BOSTON SMSA

\[
\text{Headed by male} = \frac{(53)-(23)x(.25)}{(1-(23))} = (63)
\]

\[
\text{Headed by female} = \frac{(54)-(24)x(.25)}{(1-(24))} = (64)
\]

<table>
<thead>
<tr>
<th></th>
<th>Headed by male</th>
<th>Headed by female</th>
</tr>
</thead>
<tbody>
<tr>
<td>White headed renters</td>
<td>.548</td>
<td>.522</td>
</tr>
<tr>
<td>Non-white headed renters</td>
<td>N.A.</td>
<td>.516</td>
</tr>
</tbody>
</table>

Data sources: (53), (54): Table A-6.

(23), (24): Table A-3.
headed by females in the City of Boston. 4) And was also assumed that the two ratios described in 2) and 3) have no differences between white-headed and non-white headed households. 5) In the computation in Table A-2, it was also assumed that the number of renters in the public housing in 1970 were good estimates of those in 1969 for the City of Boston. 6) It was assumed that the ratio of renters in the public housing with income below $6,000 to all renters in the public housing had no difference between white-headed and non-white-headed renters. Finally, 7) in the computation in tables A-5 and A-7, it was assumed that all renters in the public housing in the City of Boston paid 25 percent of their gross annual income on rent, and that 8) probabilities of living in public housing for renters with income below $6,000 could be applicable to those for renters with income from $2,000 to $4,000 and also for renters with income from $4,000 to $6,000.

Although there were lots of assumptions described above, the results in tables A-5 and A-7 are the best estimates of rent-income ratios excluding the effect of public housing, because of imperfect data. As discussed in III-B-2, at the income level of $4,000 to $6,000, non-white female-headed households show fairly low rent-income ratios, while, at the income level of $2,000 to $4,000, those house-
holds do not show a significant difference from white female-headed households.
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