6.003 Homework 11

*Please do the following problems by **Wednesday, April 28, 2010**. You need not submit your answers: they will NOT be graded. Solutions will be posted.*

Problems

1. **Impulsive Input**

   Let the following periodic signal

   $$x(t) = \sum_{m=-\infty}^{\infty} \delta(t-3m) + \delta(t-1-3m) - \delta(t-2-3m)$$

   be the input to an LTI system with system function

   $$H(s) = e^{s/4} - e^{-s/4}\,.$$

   Let b_k represent the Fourier series coefficients of the resulting output signal $y(t)$. Determine b_3.

2. **Fourier transform**

   Part a. Find the Fourier transform of

   $$x_1(t) = e^{-|t|}\,.$$

   Part b. Find the Fourier transform of

   $$x_2(t) = \frac{1}{1+t^2}\,.$$

   Hint: Try duality.

3. **Fourier representations of the pulse**

   In this problem you connect the four Fourier representations, as we did for you in Lecture 20, where we used a triangle as the canonical function. Here, you use a pulse:

   $$f(t) = \begin{cases} 1 & \text{for } -1 < t < 1; \\ 0 & \text{otherwise.} \end{cases}$$

   Part a. Find $F(j\omega)$, the continuous-time Fourier transform of $f(t)$. State Parseval's theorem for the Fourier transform, and check that it works by applying it to $f(t)$ and $F(j\omega)$.

   Part b. Sketch $f_p(t)$, a periodic version of $f(t)$ with period $T = 4$. Find the Fourier transform of $f_p(t)$ and compare it to the Fourier-series coefficients f_k for $f_p(t)$.

   State Parseval's theorem for Fourier series.

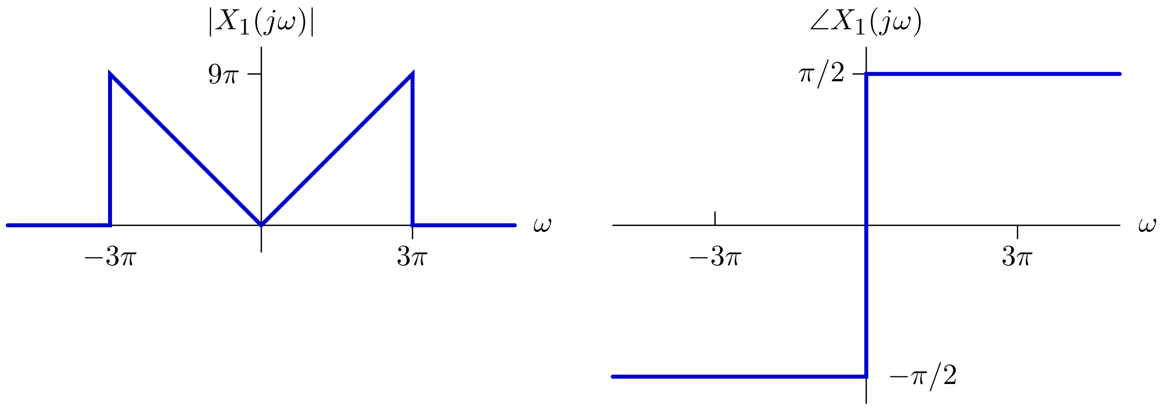
   Applying Parseval's theorem to $f_p(t)$ and f_k and thereby do an interesting sum.

   Part c. Make $f_s(t)$ by multiplying $f(t)$ by a train of impulses, each with unit area and period $T = 2/3$. [So $f_s(t)$ should be composed of delta functions.] Give the corresponding discrete-time signal $f[n]$.

   Find the Fourier transform of $f_s(t)$ and compare it to the discrete-time Fourier transform of $f[n]$.

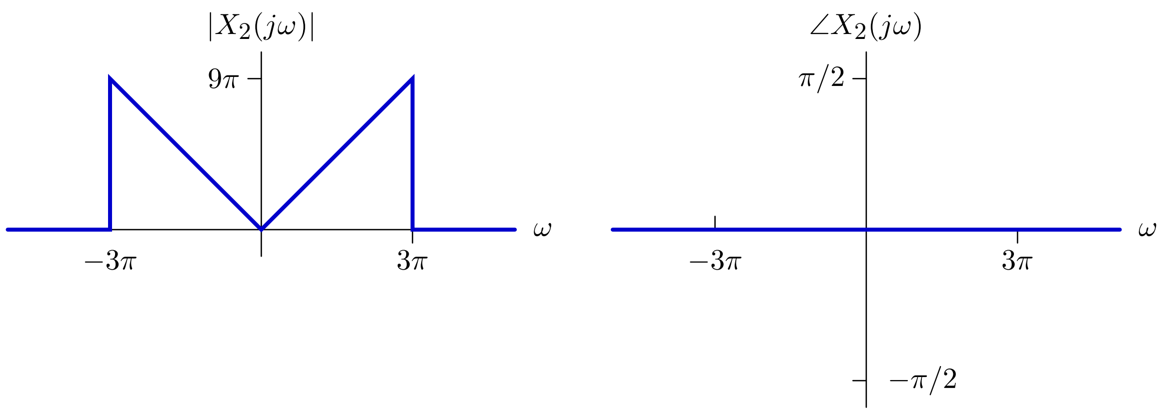
4. Fourier transform

Part a. Determine $x_1(t)$, whose Fourier transform $X_1(j\omega)$ has the following magnitude and angle.



Express $x_1(t)$ as a closed-form and sketch this function of time.

Part b. Determine $x_2(t)$, whose Fourier transform $X_2(j\omega)$ has the following magnitude and angle.

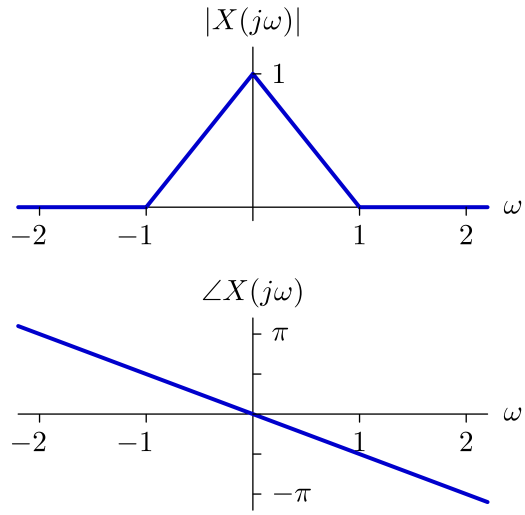


Express $x_2(t)$ as a closed-form and sketch this function of time.

Part c. What are important similarities and differences between $x_1(t)$ and $x_2(t)$? How do those similarities and differences manifest in their Fourier transforms?

5. Fourier Transforms

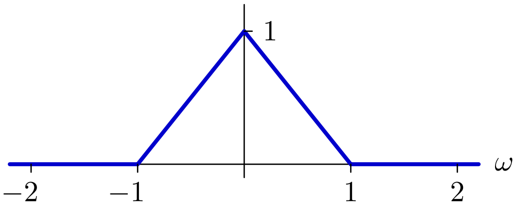
The magnitude and angle of the Fourier transform of a signal $x(t)$ are given in the following plots.



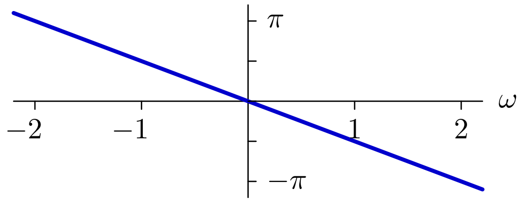
Five signals are derived from $x(t)$ as shown in the left column of the following table. Six magnitude plots (M1-M6) and six angle plots (A1-A6) are shown on the next page. Determine which of these plots is associated with each of the derived signals and place the appropriate label (e.g., M1 or A3) in the following table. Note that more than one derived signal could have the same magnitude or angle.

signal	magnitude	angle
$\frac{dx(t)}{dt}$		
$(x * x)(t)$		
$x\left(t - \frac{\pi}{2}\right)$		
$x(2t)$		
$x^2(t)$		

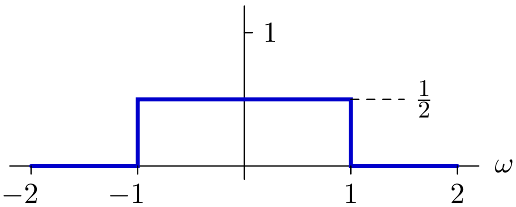
M1



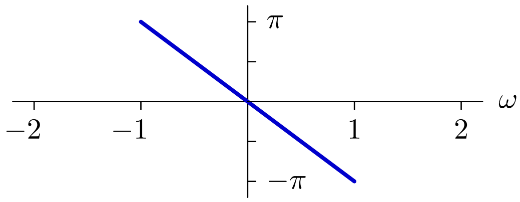
A1



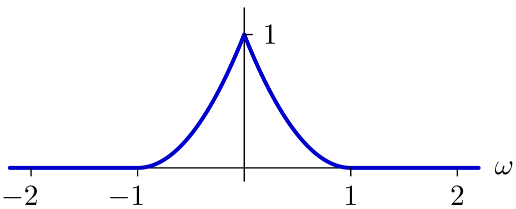
M2



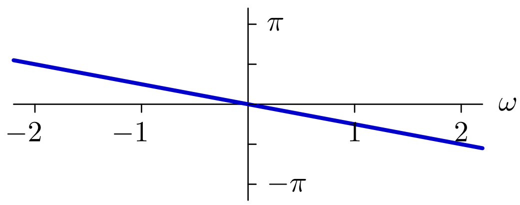
A2



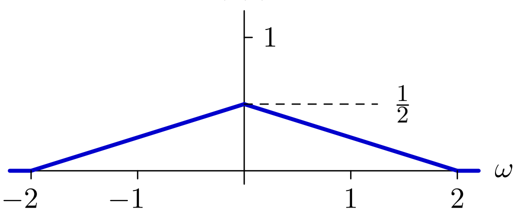
M3



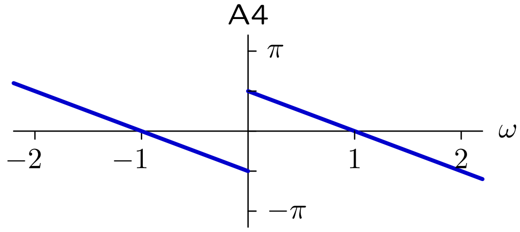
A3



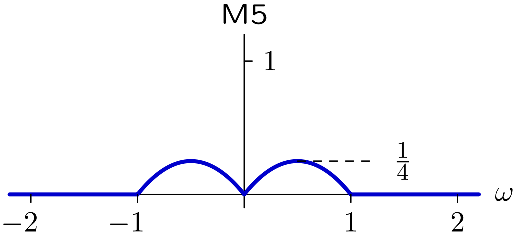
M4



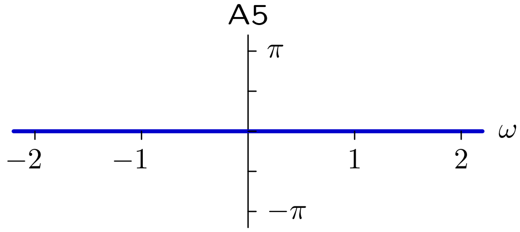
A4



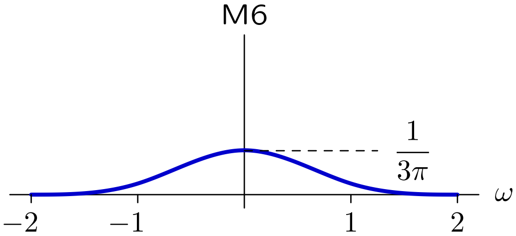
M5



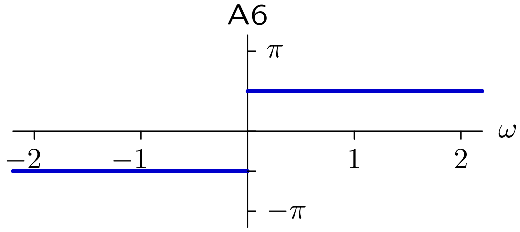
A5



M6



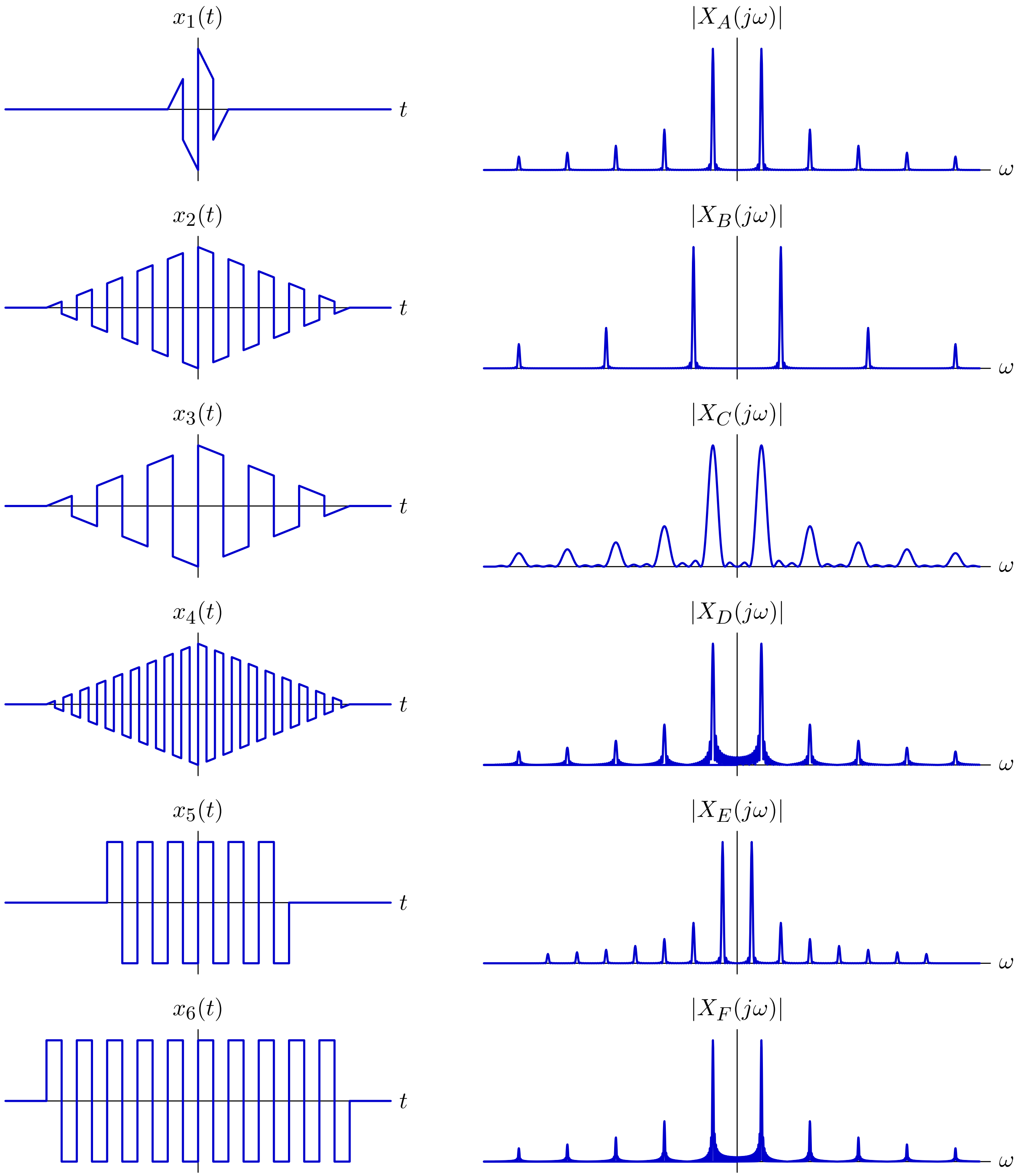
A6



6. Patterns

The time waveforms for six signals are shown in the left panels below. The right panels show the magnitudes of the Fourier transforms of $x_1(t)$ to $x_6(t)$, however, the order has been shuffled. For each panel on the left, find the corresponding panel on the right.

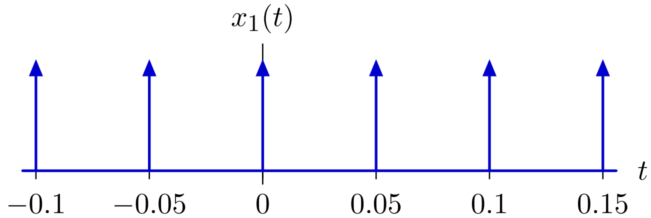
All of the time functions are plotted on the same time scale. Similarly, all of the frequency functions are plotted on the same frequency scale.



7. Inputs and Outputs

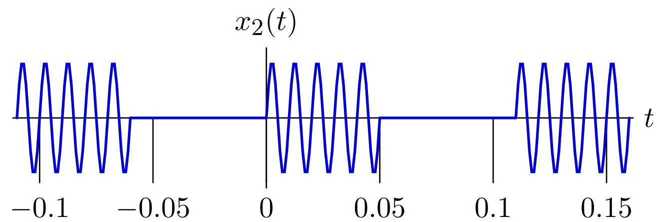
A **causal, stable** LTI system with frequency response $H(j\omega)$ has input $x(t)$ and output $y(t)$. The problem is to determine which of the following inputs can or cannot give rise to the output $y(t) = \sin(2\pi \cdot 100 \cdot t)$. For each part of the problem, determine if the statement is True (T) or False (F) and give an explanation.

Part a. $x_1(t)$ is a periodic impulse train of period 0.05 s.



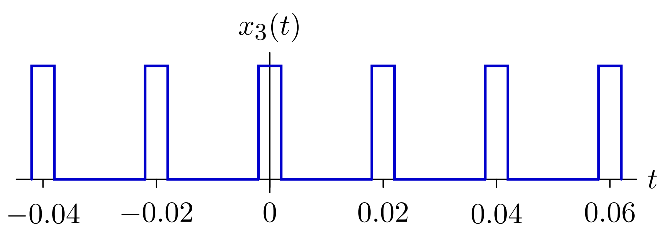
(T or F) $x_1(t)$ **can** generate the response $y(t) = \sin(2\pi \cdot 100 \cdot t)$.

Part b. $x_2(t)$ is a periodic function of period 0.11 s. Each period consists of five cycles of a sinewave of the form $\sin(2\pi \cdot 100 \cdot t)$.



(T or F) $x_2(t)$ **can** generate the response $y(t) = \sin(2\pi \cdot 100 \cdot t)$.

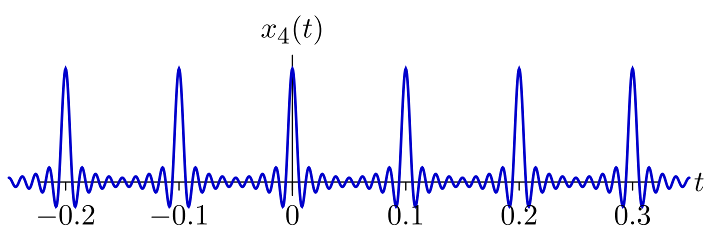
Part c. $x_3(t)$ is a periodic pulse train of period 0.02 s. Each pulse has duration 0.004 s.



(T or F) $x_3(t)$ **can** generate the response $y(t) = \sin(2\pi \cdot 100 \cdot t)$.

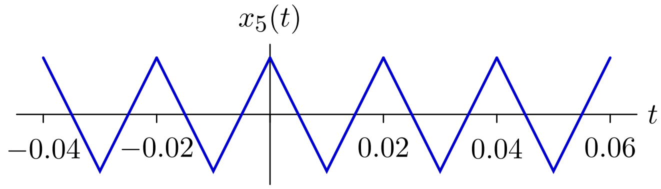
Part d. $x_4(t)$ is a periodic sinc pulse train of period 0.1 s. Each sinc pulse has the formula

$$\frac{\sin(\pi \cdot \frac{t}{0.006})}{\pi \cdot \frac{t}{0.006}}$$



(T or F) $x_4(t)$ **can** generate the response $y(t) = \sin(2\pi \cdot 100 \cdot t)$.

Part e. $x_5(t)$ is a periodic triangular wave of period 0.02 s.



(T or F) $x_5(t)$ **can** generate the response $y(t) = \sin(2\pi \cdot 100 \cdot t)$.

6.003 Signals and Systems
Spring 2010