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Citation: Wilczek, Frank. "Quantum Physics: Majorana Modes Materialize." Nature 486.7402 (2012): 195–197.

As Published: <http://dx.doi.org/10.1038/486195a>

Publisher: Nature Publishing Group

Persistent URL: <http://hdl.handle.net/1721.1/78563>

Version: Original manuscript: author's manuscript prior to formal peer review

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Majorana Modes Materialized

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The condensed matter physics community has been galvanized by the apparent experimental discovery of Majorana modes [1]. These entities, whose existence had been anticipated theoretically [2], dramatically expand the known possibilities of quantum behavior. They could become important components in quantum engineering; specifically, they might provide the basic elements – the qubits – for a quantum computer [3] [4]. Mourik *et al.* [1], realizing theoretical designs [5] [6], studied the electrical properties of nanowires with strong spin-orbit coupling (InSb) lying atop a (conventional) superconductor. Interaction with the substrate electrons then induces superconductivity in the wire’s electrons as well, through the well-known proximity effect. Mourik *et al.* observe that electrons can tunnel into and out of the wire at zero energy (that is, in the middle of the superconducting gap). This shows, quite directly, that there are unusual new states associated with the ends of the wire, as predicted. Details of the behavior, including its sensitivity to external magnetic fields, make the identification of those states as degrees of freedom associated with Majorana modes compelling. Two other experiments, not yet published, also report evidence for Majorana modes, in different but related systems [7] [8].

Over the last few decades physicists have discovered that particle-like excitations (i.e., quasiparticles) in condensed matter systems can have strange, fascinating, and possibly useful properties, including fractional electric charge and unconventional quantum statistics (anyons). Majorana modes are a major new addition to this universe of exotica.

To set the context and avoid misunderstanding, a brief reflection on the fundamental nature of quasiparticles is in order. The elementary building blocks of material systems are not negotiable: Electrons, photons, and atomic nuclei are what we get to work with. Furthermore, the basic interactions among those ingredients are known to high accuracy: Maxwell and Schrödinger rule. So how, within that familiar and reliable framework, can new and exotic “particles” possibly arise?

Phonons are the original examples of quasiparticles. Phonons were introduced, conceptually, by Albert Einstein in 1907 [9]. Two years before, he had introduced the idea that light has particle-like properties, being created and transmitted in discrete units, namely photons. Generalizing that intuition, he proposed that the vibrations of solids come in discrete packets, which we now call phonons. Einstein used this idea to explain an otherwise mysterious deficit of vibrational motion in diamond at low temperatures. In the same way that quantization of radiant energy removes the “ultraviolet catastrophe” of black body radiation, quantization of vibrational energy suppresses the low-temperature specific heat of diamond. Einsteins work predated the modern understanding of solids (for example, the atomic nucleus was only discovered in 1911!), but its central concepts endure. Holes are another very important kind of quasiparticle [10]. As their name suggests, holes represent the absence of an electron where one would normally be expected. Holes come in discrete localized units and move as if they were particles.

Those two classic examples well illustrate noteworthy peculiarities of quasiparticles. Phonons, though they arise from the correlated motion of atomic nuclei, are particle-like objects that in no way resemble atomic nuclei. Holes, though they arise from the correlated motion of electrons, have dramatically different properties from electrons - indeed, they have the opposite electric charge. Quasiparticles transubstantiate the elementary units from which they are built. They are emergent objects, embodying recognizable – that is, reproducible and long-lived – organizations of energy within structured materials.

Different materials can support different kinds of quasiparticles. Each material’s quasiparticles, of course, depend on the structure of that material, and they exist only within that material. Modern physicists can exploit their deep understanding of matter to design materials whose quasiparticles will have interesting or useful properties. Transistors, which orchestrate holes, are an outstanding but by no means singular success story.

Majorana modes add a striking new variation to the quasiparticle theme. Appearing as solutions of equations of a type invented by Ettore Majorana in 1938 [11] [12], Majorana modes represent forms of excitation predicted - and now observed - to be available to a few very special and specific kinds of quasiparticles. Majorana modes were discovered as mathematical possibilities within quantum field theory [13], then in theoretical models of possible exotic

$(p_x + ip_y)$ superconductors and of the $\nu = \frac{5}{2}$ quantum Hall effect [14]. In a brilliant paper Alexei Kitaev [2] showed how Majorana modes could arise at the ends of superconducting wires. His reasoning was (relatively) simple and transparent, and is at the heart of most subsequent proposals for realization, including the ones that underly [1] [7] [8].

In getting acquainted with Majorana modes, it may be helpful to draw upon their analogy to a more familiar property possessed by many particles and quasiparticles, namely spin. Thus for example an electron at a given position can be in either of two states, with spin up or down. Spin opens up new dynamic possibilities to particles that have it, i.e. the possibility of spin-dependent interactions (including interactions that change the spin's direction). Majorana modes generate, for quasiparticles that support them, a kind of emergent spin. It is convenient to have a word for the concept “quasiparticle that supports Majorana modes”, and I have suggested “mode-icule” [15]. To describe the quantum state of a mode-icule, we must specify a two-component wave function, just as we do for a (spin-ful) electron.

The analogy to spin proves inadequate, however, when we come to describe systems of several identical mode-icules. Crucially, the emergent “Majorana” degrees of freedom attached to separate mode-icules, unlike ordinary spins, are not entirely independent. On the contrary: Those emergent degrees of freedom are, unavoidably, highly entangled. Here a different analogy may be helpful. When we have a system containing several electrons, their states are not entirely independent. Specifically, multi-electron states are constrained by the Pauli exclusion principle, which forbids two electrons from occupying the quantum state, and more generally by Fermi statistics. Majorana degrees of freedom display a considerably more intricate and interesting form of entanglement. The quantum state of $2n$ mode-icules is described by a 2^n -dimensional wave function, whereas $2n$ independent spins would require 2^{2n} dimensions [16]. This constraint arises because mode-icules are nonabelian anyons [17] (as opposed to fermions or bosons). Although entanglement drastically reduces the wave function's dimensionality, it remains exponentially large. Interchange of mode-icules is accompanied by complicated but perfectly predictable evolution of their wave functions in Hilbert space, described by the mathematics of Clifford algebra [18].

Theorists have developed ingenious, ambitious proposals to exploit mode-icules, with their controllable entanglement, to enable quantum computation [3] [4]. Until recently their beautiful mathematical ideas have been loosely tethered, at best, to laboratory reality. Much difficult terrain separates proof of principle from the promised land of complex, scalable mode-icule circuits, but at last a beachhead has been established.

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