AUTOMORPHS

BY

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ABSTRACT

The purpose of this thesis is the delineation and investigation of a general pattern or mechanism which I have termed 'automorphic,' a word derived from the Latin roots, 'autos' meaning self, and 'morphe' referring to form. The thesis further characterizes the mechanism as 'recursive,' 'self-similar,' 'scaling' -- adjectives referring to form (and) phenomenon in or from which the same configuration is repeated many times at many different scales. The attempt is to maintain the generality of the term in order to establish it as a fundamental attribute of form or persistant structure, (or as a necessary component to a conception of form, as inherent to "order" as "modular coordination" or symmetry).

I will explore briefly several disciplinary fragments of contemporary physical theory where this mechanism can be said to be operational including an analogy to basic life processes -- the most elegant of the physical automorphisms. Its analytic and thus generative power in fields as diverse as astrophysics, geomorphology, biology and particle physics, carry important implications for the understanding our own human physical and cognitive processes and subsequently important bearings on the artifacts we generate.

The study will then focus on those topics specific to built form particularly that of this type of pattern's inherent structural and energetic stability. Central to this section is a note on spatial perception (and therefore 'space' itself) as a function of automorphism, or inherent recursive perceptual thresholds.

A graphic presentation of two built projects is meant as an attempt at unified synthesis and application.

Thesis Supervisor: John R. Myer
Title: Professor of Architecture, Department Head

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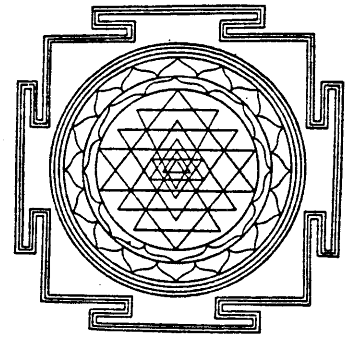
PRETEXT

"Reality Affirms Itself By Degrees But Without Ceasing to Be One."

Fritjof Schuon in "The Transcendental Unity of Religions"



That there exists a basic unity within the manifold diversities of this world is one of the oldest observations of mankind. Through this observation comes into being the sacred traditions which although outwardly different, act as bridges from the periphery to the centre--from relative to absolute, from multiplicity to unity. Unity, whether designated as a divinity or unified operational field, is the aggregate and the connection of all things. It envelops, contains, and is thus content of all things. While formless, devoid of differentiation, we find it manifest in the forms of nature. The order of multiplicity as so many reflections of the centre--its source and terminus.

It is out of inner necessity, I believe, that we as humans inherently posess image of wholeness or unity, and that the nature of our existence is the restoration of the fragments of our corporeal existence to wholeness.[1] We are ourselves, physical and metaphysically pattern or antientropic agents. We establish order, if not out of purpose, out of necessity.

One of the primary characteristics of conceptual (or spiritual) ordering systems and thus life itself, is repitition or self replication. This is in the field of geometry, known as symmetry and its most general implication according to Liebnitz was related to the indiscernibility of differences.[2] The principle indiscernibility is the fact that the world is modular, as

[1] An etymology of the world "whole"--it comes from the Indo-European root "kailo" signifying holy or healthy. The human body--a multiplicity of cells, organs, and simultaneous functions--is nevertheless clearly recognizeable as an individual, an integrity, and a whole, and the concept of health is based on such a fundamental wholeness and coordinated activity.

[2] Morrison, Philip. On Broken Symmetries, in On Resthetics in Science ed. Wechsler, Judith M.I.T. Press, Cambridge, 1979.

well as our knowledge of the world. Analysis implies atoms of knowing. The apparent prodigality of our world is a rodigality of combination of a few simple modules, if not one, depending on either one's theosophical inclinations or intuition concerning further resolution of "elementary" particles by the scientific ocmmunity.

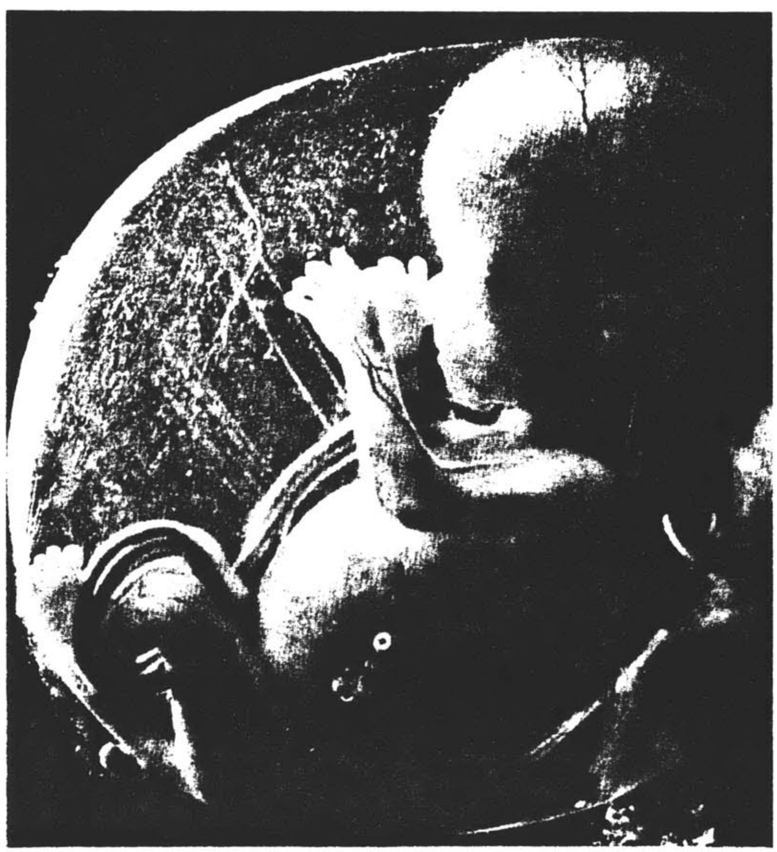
While the principle types of self-replication, rotation, or mirroring, and translation are well worth investigation they are not the principle topic of this essay. I am proposing that there is a third very profound type of symmetry which I will refer to as automorphism. This term in its most general sense refers to the replication of an initial motif and more specifically to replication of a number of different scales. It is by nature, mechanism and consequence of successive reorganizations and approximations, a generic cascade, commonly referred to on the science of pattern or mathematics as recursive. (Very generally, recursion can be defined as nesting and variations on nesting--stories inside stories, paintings inside paintings--even parenthetical comments inside parenthetical comments.)[3]

What this does is introduce the notion of hierarchy or differentiated levels. To the first principle of order or conceptual structure which is modularity. Automorphism is not an assumption of hierarchy, it is a basic symmetry operation which explains the formation of hierarchy. It is a principle component of the most archetypal notions of order--thus an archetype itself of our own existence.

[3] Or Footnotes[4]

[4] For Footnotes

Even so, it is only recently that this notion of structure has begun to assume any prominance in the natural sciences, and I believe this is so because of the inference which an automorphic or recursive mechanism has towards infinite regress or circularity. In fact, as I outline in the following section, it is only since the beginning of this century that scientific community has become comfortable with the mathematics of the infinite--comfortable with the impossibility of strict objectification. As it turns out, there is a crucial difference between recursive systems and circular (infinite) ones. That is, with recursive systems, there is either some part of the definition which avoids self reference, so the act of structuring out an object will eventually "bottom out", or there is a functional limit to the definition dependant on our own human perceptual resolvability with or without instrument extension. In the case of archetypal automorphism, or the fetus in the womb, the fetus may or may not have the potential for imprégnation or the capabiliity for carrying another being within itself but it is only potential and thus the system bottoms out due to the temporal limitations of gestation and maturation. This particular example while being the most accutely direct is also the most complicated, and I'll return to it later on, but it raises the question of what constitutes a "replication" or "copy". The baby, while similar to the mother; differs not only in size, but in the relaive proportion of its anatomy and in other basic physiological, not to mention



Automorphism as Archetype.
Self-regeneration is the principle attribute of "life" or living systems. In addition humans may singularly claim to be capable of generating a host of virtual presences, alternative identities, conflicting versions of who they are, dreams of what they may become.

emotional or intellectual parameters. The position taken here will be that a copy is constituted by a transformation which preserves all the functional information of the configuration from which it is generated. This type of transformation is called an isomorphism.

The function of this thesis will be a further delineation of this notion of automorphism --further, in the sense of proceeding towards a generalization. The method undertaken to accomplish this is the investigation of the phenomenon of form covering a very large range in scale. At this point our perception/conception due to instrument extension scales from a largest dimension of one billion light years (10^{25} meters) to a smallest of 10^{-16} meters. Our unaided perceptual grasp covers a mere 6 magnitudes of scale.[5] It is interesting to note that at the two presently knowable extremes (from quarks to golactic distribution), the nature of phenomenon appears to be that of a huge number of similar interacting particles. It is for this reason that the presently most effective simulations or conceptualizations may take place at all, at any scale. They are distributions of identical components given knowledge only of the properties of an individual component (and or its isomorphisms)[6] The purity of these essential relations is expressed through geometry or pattern.

The extremes represent the general case and the assumption is that we can establish its presence throughout the range of scales (or disciplines, which may be distinguished by their

[5] Eames, Ray and Charles, Morrison, Philip and Phylis. Powers of Ten, About the Relative Size of Things in the Universe, W. H. Freeman and Co. New York, 1982.

[6] We will take up this very important concept of isomorphism later on where it is defined as information preserving transformation.

The world over a cup of coffee. The notion of automorphism assumes that there are similar behaviors at different scales. For the most part the separation in scales is not as great as that between coffee aroma and global weather systems. In fact in the most useful cases there is very little scale separation.

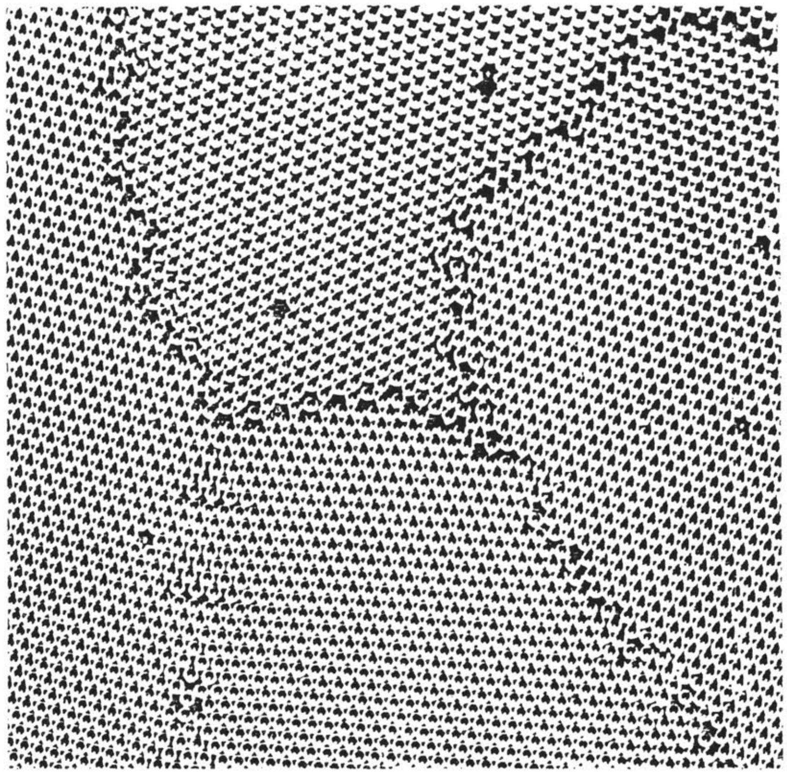
scale of investigation). The science of pattern or geometry is the lowest common denominator of all the scientific disciplines and this essay makes use of it to isolate the particular pattern of automorphism in a variety of diverse fields. The pattern itself of automorphism implies its operation at different scales and as we shall see it is a conceptual mechanism which enables us, as humans operating in a somewhat limited perceptual range, to come to terms with a number of new scales and dimensions which technological extension has brought us. It builds a conceptual continuity, a ladder by which we may traverse the complete range of physical phenonemnon and inhabit any one of them without any loss in our most precious possession of modular order.

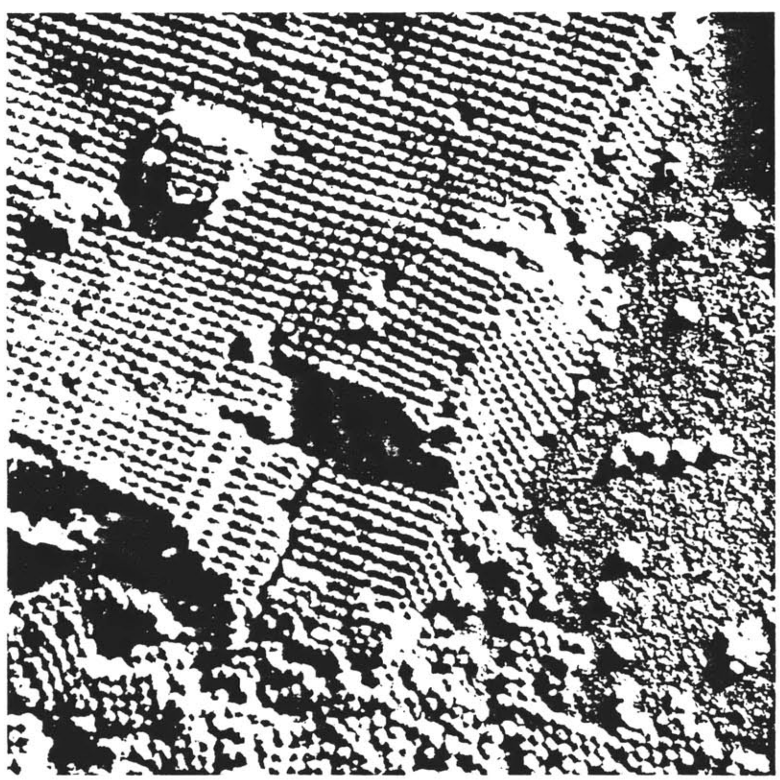
One may say, that there are many examples, especially those of our own visual domain where modular order if it is there at all in any scale besides its constituent elementary particles is certainly not apparent. Even in the soap bubble array shown in the facing illustration, order has broken down to produce territories whose edges are extremely irregular. This is where the real utility of automorphism becomes apparent, allowing description of the highly irregular patterns of nature while maintaining a fundamental simplicity of mathematical structure. It allows the return of visualization through geometry to stochastic and other critical processes.

It was the discovery of these structures which marks the paradigm shift separating the classical mathematics of the 19th century from the so-called modern mathematics of the 20th.

[7] Specifically Cantor's Set Theory--This domain of thinking stipulates that individual elements in something like a straight line, which posess continuity, are infintely more abundant than those in something like the realm of rational numbers, which does not; that the number of odd integers is the same as the number of odd plus even integers; that there are just as many points in lines of various lengths; that an infinte set stays the same despite much thinning--and Peano's plane-filling curve.

In both the structure of matter and in the human understanding of matter there exists an interplay between simple, predictable replication and the "accidents" that allow, and sometimes force the formation of new structure. But as we find out these new structures (in this case discontinuities) may be made to be described commensurably with a modular order.





Ironically, these 'pathological" structures remained without application until very recently, when Benoit Mendelbrot grouped the odds and ends of men such as Weirstrauss, Cantor, Peano, Lebnesque, Haussdorf, Koch, Sierpinski, Perrin, and Poibncare together under the term "fractal".[8] He applied it to descriptions of a variety of diverse irregular natural phenomenon from galaxy clusters, cloud boundaries, turbulent wakes, to our own vasular system, the convolutions of our brain and the substance of our flesh. It may also prove to be highly informative about the way we ourselves structure information.

[8] Mandelbrot, Benoit. Fractals: Form, Chance, and Dimension

My training is for the most part archtectural as I imagine to be the case for anyone who happens to pick up this thesis, if not architects then those in some manner interested in form for its own sake (philomorphs) or those interested in the production of form or artifacts. I think that no one would argue the importance or relevance of investigations outside the realm to which they have made themselves directly responsible for the relevence of their whole field of study is directly dependent on the external correspondences it can establish. And in the case of architecture and other systems of form production it is necessary that these systems be conceptually compatible with the observed physical phenomenon.

Be that as it may the next sections are somewhat large digressions into fie.d of mathematics, which the reader may not find as fascinating as I, and it is for this reason that I shall summarize the findings very briefly here.

The reader may proceed directly to the section concerning support/structure/stability although a perusal of the images may be quite informative. If one does feel inclined to read on, it will be found that althought many of the terms are mathematical one needs very little experience with numbers to get through. One of the biggest difficulties is that it rearticulates or tries to dispel some widely-held notions of "conventional" Euclidean wisdom, and they are presented here anyway.

The specific themes of this "automathematic" which differentiate from Euclid are:

1. It employs self-similarity. A very simple initiator can generate very complex artifacts as long as the initiator can be applied repeatedly. Both the visual (spatial) manifestation and the algebraic interaction display this quality of potential infinite repetition and invariance. This is in other words, a critical dependence on intial conditions.

2. There is an implied spatial homogeneity and homogenous distribution on a line plane, or space has two very desirable properties. It is invariant under displacement and is invariant under change-of scale (scaling).

3. The notion of dimension (meaning number of dimensions or dimensionality) turns out to have many mathematical facets which are not only conceptually distinct but may lead to different numerical values. The implied multiplicity, if not continuity of dimensions includes transitions between the well-defined discrete dimensions of topology. Underlying the mathematical notion is an intuitive "effective dimension" as a matter of

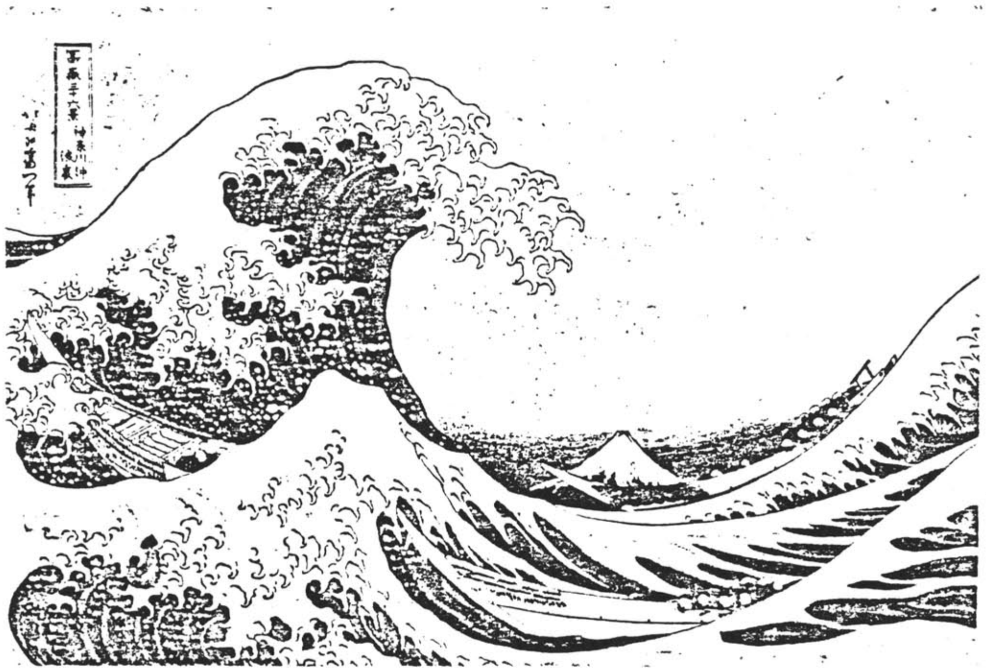
approximation and therefore of degree of resolution.[9] Similarity dimension is introduced as a function of a systems degree of articulation or frequency modulation.

4. In all cases of measurement of extent quantifies (length, area, volume, mass) there is a divergence syndrome. (That is some quantity that is commonly expected to be positive and finite turns out either to be infinite or to vanish.)

5. In cases concerning natural phenomenon, there is not a strict self similarity. Simulated chance or randomness is postulated and employed to model the unknown and unpredictable "will." Inversely, higher evolutions of a large number of curves which are strictly or geometrically self-similar look "chaotic" or "stochastic." This leads to postulations concerning chaos not as disorder, but as a higher form of order-emergent order or entelechy.

These themes may be further and more directly summarized in the print by Hokusai on the facing page. The turbulance of "The Great Wave" is represented by smaller self-similar waves. An even more elemental analysis is based on segments of the catenary type curve of Mount Fuji. The clue to this is Fuji's self-replication in the surge in the foreground. It is interesting to note that this visualization predated its adoption in the scientific community by about 200 years.

[9] Consider the several distince effective dimentions implicit in a ball of thread. To an observer placed far away, the ball appears as a zero--dimensional figure: a point as seen from a distance of 10 cm resolution, the ball of thread is a three-dimensional figure. At 10 mm, it is a pattern of one dimensional threads. At .1 mm. each thread becomes a column, and the whole becomes a three-dimensional figure again, and so on with the dimension crossing over repeatedly from one value to another. A larger analogous sequence scales and their particular dimension can be seen in Powers often. Eames, Morrison.



AUTOMATHEMATA

self similarity

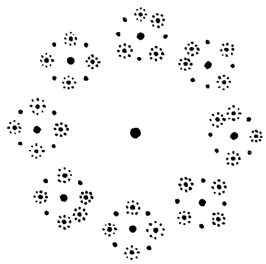
> "The endless embedding of the shape into itself gives us indeed an idea of what Tennyson describes as the inner infinity, which is after all the only one we could conceive in nature. Such similarty between the whole and its parts, even its infinitesmal parts, leads us to consider the triadic Koch curve as truly marvelous. Had it been given life, it would not be possible to do away with it, without destroying it altogether for it would rise again and again from the depts of is triangles as life does in the universe."[10]

Self similarity is by no means a new idea. It has undoubtedly been part of the collective unconscious, for quite some time although the earliest documentation of it seems to occur about 1700 . It occurred to Leibniz as he attempted to "tighten" the axioms of Euclid:

> "I have diverse definitions for the straight line. The straight line is a curve, any part of which is similar to the whole, and it alone has this property, not only among curves but among sets."

It's generalization beyond lines and planes is about one hundred years old. Leibniz also implies a self-similar scaling in his "monadology," where it is stated that minute portions of the world are precisely as complex and organized as large portions. Leibniz saw the world as a collection of beings called "monads" whose activity consisted in the perception of one another on the laws of preestablished harmony laid down by God. It is interesting to note that he thought of this action largely in optical

10 Reprinted in Mandelbrot, B. Form. Chance and Dimension, Freeman Press (San Francisco) 1975.



terms, because at the same time, Huygens was developing the primitive form of what is now known as "Huygen's principle," which states that light spreads from a source by forming around something like a small sphere consisting of secondary sources which in turn propagate light like the primary sources.[11]

A thought related to scaling also occurred to Laplace. I quote a remark found in his "System of the World," published in 1842.

> "One of (the) remarkable properties (of Newtonian attraction) is, that if the dimensions of all the bodies in the universe, their mutual distances, and their velocities were to increase or diminish proportionally, they would describe curves entirely similar to those which they are present describe; so that the universe reduced to the smallest imaginable space would always present the same appearance. To observers the laws of nature therefore only permit us to observe relative dimensions..."

Let me present one final remark by Kant in the same tradition.

> "That part of my theory which gives it its greatest charm...consists of the following ideas..It is natural to regard (the nebulus) stars as being...systems of many stars-...(they) are just universes and so to speak, milky ways...It might further be conjectured that these higher universes are not without relation to one another, and that by this mutual relationship they constitute again a still more immense system-...which perhaps like the the former, is yet again but one member n a new combination of numbers. We see the first members of a progressive relationship of world's and systems; and the first part of this infinite progression enables us already to recognize what must be conjectured of the whole. There is no end but an abyss..without bound."

[11] Weiner, Norbert. Human Use of Human Beings. Avon Books, New York, 1967.

It took until 1926 for the notion to appear solidly and quantitativedly within the realm of the physical sciences, when Lewis F. Richardson postulated that over a wide range of scales, turbulence is decomposable into self-similar eddies by a mechanism which Richardson referred to, not surprisingly as a "cascade."

In the same way the Koch curve may be used to model real coastlines,[13] it's major defects are in that its parts are identical to each other, and the self-similarity ratio r must be part of a strict scale of the form b^{-k}, where b is an integer, namely, 1 3, $(1\ 3)^2$, and so on. When we look at a sequence of various scaled maps, while they can be of the same generic detail, they can be very different in specific. Furthermore, the real coastline may have more than one D --one each associated wtih the concept of interface between land, water, and air.

It would seem at first glance then that the Koch curve while less complex than real coastlines, is far more complex than the standard curves of Euclid. However, when complexity is measured by the algorithms (set of instructions for the construction of any figure) length, it would seem that the Koch curve is less complex than a circle.[13] If we use a method to trace a regular polygon with a finite number of strokes, each defined by a finite number of lines of instruction, the construction of the Koch curve is seen to be a task of finite complexity.[14] By contrast, a circle involves, an infinite number of infinitely short strokes, hence is a curve of infinite complexity.

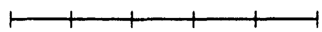
[12] For a more complete and rigorous study see; Mandelbrot, B. The Practical Geometry Nature. Freeman.

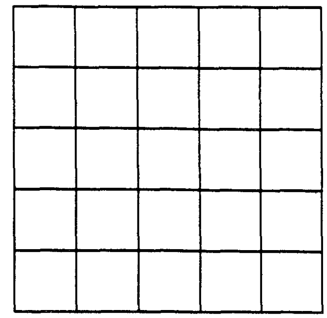
[13] Theory due to Kolmogorov and Chaitin, Mandlebrot. B. Practical Geometry of Nature, p. 41.

[14] This algorithm for the Koch curve (written in the LOGO programming language) demonstrates what is typical of recursive or looping programs. One can see that the subprogram "to side" repeatedly calls itself as an instruction.

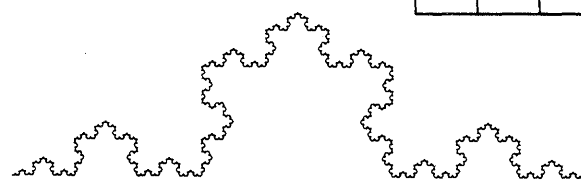
```
To Koch curve (size, level)
  Repeat
    side (size, level)
    right 120

To side (size, level)
  If level=0 then   forward size
return
  Side (size 3, level-1)
  Left 60
  Side (size 3, level-1)
  Right 120
  Side (size 3, level-1)
  Left 60
  Side (size 3, level-1)
```

These figures illustrate several types of self similarity and introduce us to the "similarity dimension." Because a straight line's euclidean dimension is 1, it follows for every integer b (here b=5) a straight interval of unit length may be divided into N=b subintervals of length r = ^{1} b In a similar manner, because a plane's euclidean dimension is 2, it follows for any b that unit square can be divided into $N=b^2$ squares of side r=1 b.

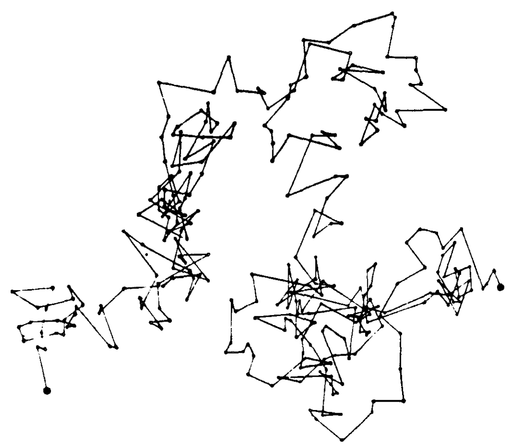




For a rectangular paralelepiped, the same argument gives us r(N)=1 N^{1} 3. So in general for D dimensions r(N) = 1 N^{1} P and the shapes similarity dimension is log N log (1 r), a notion which has been ignored since its value is the same as its euclidean dimension. The botton figure is a triadic Koch curve. It also can be decomposed into reduced size pieces, with N=4 and r=1 3. The similarity dimension D=6g N log 1 r is not an integer (D=1.2618) yet preserves the ordinary dimensions role as an exponent in defining measure.



The Brownian motion trail is topologically a curve, of dimension 1, however being practically plane filling, it is fractally of dimension 2. It is also important to note that it is self similar. If position were plotted 100 times more frequently, each interval would be replaced by a curve 100 times smaller than the original but just as complicated.



dimension

For discussion of some alternate facets of the idea "dimension", let me outline what the word has conventionally signified. The first writing on the subject is attributed to Plato (427-347 B.C.) when he comments to Socrates in "The Republic," that,

> "After plane surfaces...the right way is next in order after the second dimension to take the third..., the dimension of cubes and of everything that has depth."

Surfaces are defined as the boundaries of solids or pieces of space, curves as the boundaries of surfaces, points as the boundaries of curves or to quote Poincare in an elaboration of Euclid,

> "To divide space, cuts which are called surfaces are used; to divide surfaces, cuts that are called surfaces are required; to divide curves, points would be used; and a point cannot be divided not being a continuum. Since a curve can be divided by cuts which are not continua, they are continua of dimension one; since surfaces can be divided by continuous cuts of dimension one, they are continua of dimension two; and finally space can be divided by continuous cuts of two dimension, it is a continuum of dimension three."

In a more general sense a configuration was called N-dimensional if the least number of parameters needed to describe it's points was N. On the whole, this notion remained unchallenged until the last part of the nineteenth century, the previously mentioned discoveries of Cantor and Peano. The first destroyed the notion that a plane is richer in points than a line, and showed that dimension can be changed by a one-to-one transformation. The second contradicted the belief that dimension can be defined as the least number of continuous real parameters required to describe a space.

At the same time mathematicians were discovering that topological dimension fails to discriminate important differences between curves. The topological dimension of a coastline is the same as that of a circle: equal to 1.

If we attribute to matter the infinitely granular structure of the instrument extended world not only is there a problem of indecernability with the use of conventional dimension, but measures in these dimenisns become arbitary and uninformative. If one attempts to measure a natural coastline's length given an anthropocentric unit of measure, it becomes apparent that as the unit of measure is made smaller and smaller, the approximate length tends to increase without limit. The same misbehavior occurs concerning the second and the third power measures as they refer to "continuum" surfaces or solids; as magnification is increased they either vanish or grow without bound. Similar observations are applicable to properties such as velocity, pressure, and temperatures.[15]

This brief historical account I hope illustrates that not only is a multiplicity of dimensions possible but is necessary in a high resolution universe. In 1926 through the independant works of Lewis Richardson and Felix Hausdorff the Hausdorff dimension (synonymous with similarity dimension (see facing page)) was developed as a

[15] Fuller, R.B. Synergetics. Macmillan Publishing Company, New York, 1978.pp. 63-67. See also 432.00. 519.03 960 00 On Powers and Dimensions. And 527.70 Synergetics II. Macmillan Publishing Co. New York, 1979.

R.B. Fuller has expressed very similar assumptions concerning physical phenomenon which appear as "synergetics corallaries."

240.08 "There are no solids or particles--no things."

240.09 "A point is an as yet undifferentiated focal star embracing a complex of local events."

240.10 "There are no indivisible points."

240.11 "Unities may be treated as complex star points."

240.46 Physics has never made an experimentally demonstrable discovery of a straight line. Physics has found only waves and frequencies.

240.48 "There is no dimension without time."

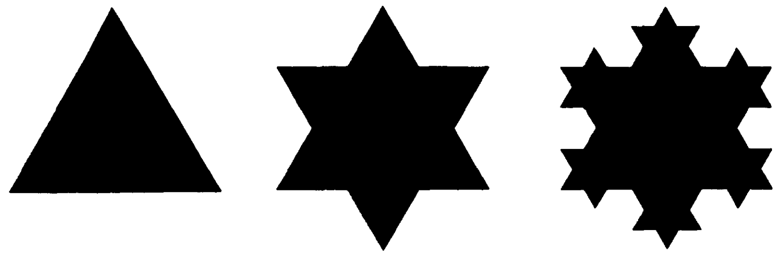
One should note that he comes to terms with the divergence syndrome of extent measurement with corallary 240.48. It is amiguous whether he is referring to "dimension" as exponent or measure, but I will assume he refers to measure. But the key point is that dimension(s) may be expressed only in magnitudes of time or energy, the parameters of which are frequency and angular modulation. Extent measurements are durations of experience.

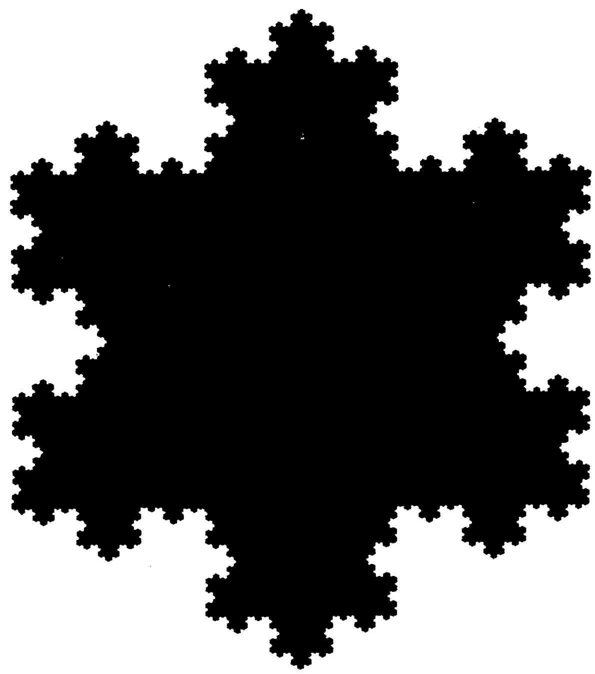
function which represents any physical property (such as length) of infinitely discontinuous matter, as a continuum.

On closer inspection (similariity) dimension (D = log N log 1 r) refers to frequency subdivision or modulation.

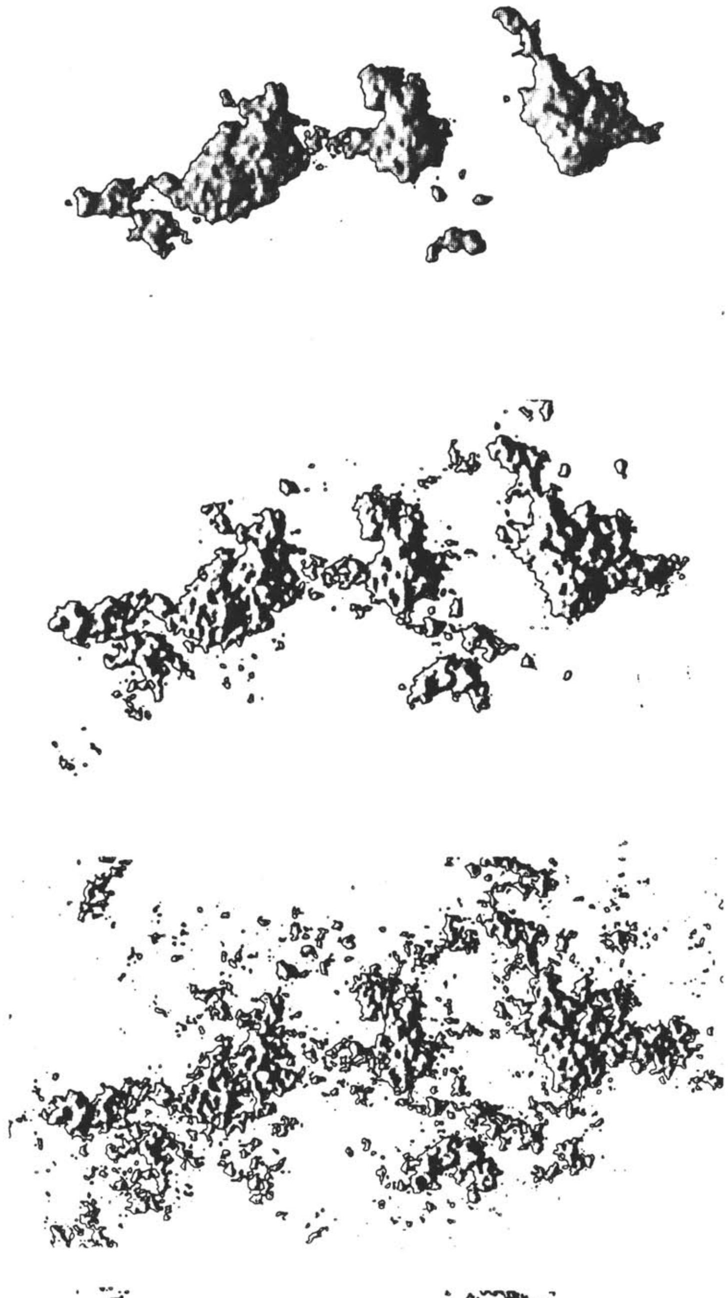
It appears, our discussion of dimension has curiously led us back to the notion of self-similarity, self-representation. The exponent of dimension refers to the frequency to which the "self" is represented. (e.g., second powering is self x self). The fractional dimension indicates the scale at which the self is rearticualted. Thus dimension in general may be said to be a time related parameter, and measures occuring within any of the dimenisons to remain unambiguous, are those of cyclic duration.

Triadic Koch coastline or snowflake. Original construction by Helge Von Koch. The construction begins with an "initiator" (equilateral triangle) with sides of unit length, and a generator. The latter is an oriented broken line made up of N equal sides of length r . (here N = 4 r = 1 3 and D = (log N log 1r) = 1.2618)





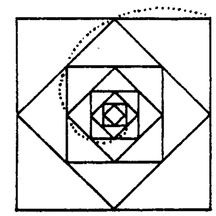
Random Koch Coastline. Construction by Benoit Mandelbrot D = 1.4800. This figure illustrates an attempt to improve the coastline model without making the algorithm significantly more complicated and preserving its deterministic character. Two generators (which fit roughly within the same outline) are chose randomly throughout the construction. The generators are drawn on two sides of an equilateral triangle. The overall form of the Koch Island is still very dependant on the initial shape.

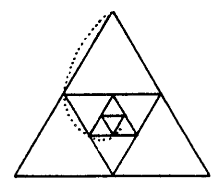


Brownian Landscapes construction by Mandlebrot. Progressive variation of D. While the p ·evious plate makes use of probability in a "control program" which acts on the Koch loop, this one demonstrates full-fledged recursive probability. Recursivebecause the algorithm is in the form on a loop, and unlike other physical theories where change predominates at the microscopic level, the importance of change remains constant on all levels, thus "scaling."

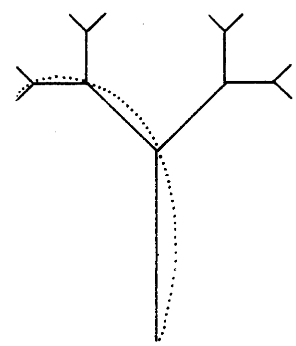
isomorphism

The notion of automorphism or self replication is most directly diagrammed and therefore evidenced in those instances which take the form of a spiral (or continued fraction) of a tree or branched structure. The continued fraction or spiral has engaged the attention of mathematicians since Phthagoras, if not the Egyptians before him (see Atlantis example). But it was in 1917 that D'Arcy Thompson in his treatise "on growth and form" articulated it's mathematical generality as intrinsic to a variety of not only biologically remote but physically different forms.[16] I quote,





> "In the growth of a snail, we can conceive no simpler law than this, namely, that it shall widen and lengthen in the same unvarying proportions: and this simplest of simple laws is that which nature tends to follow. The shell, like the creature within it, grows in size, but does not change its shape; and the existence of this constant relativity of growth, or constant similarity of form, is of the essence, and may be the basis of a definition, of the equiangular spiral."



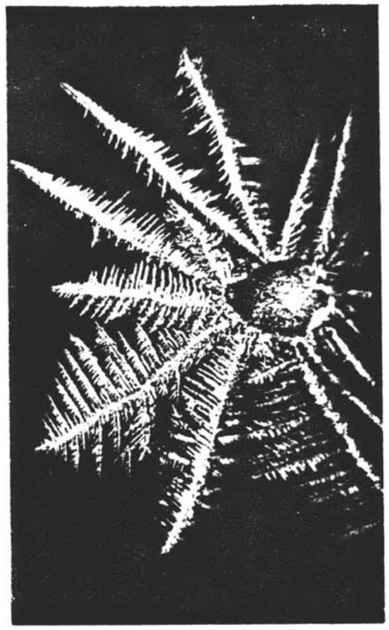
The tree or branced structure was similarly used by Darwin as an intrinsic image for his Theory of Evolution.[17] Its value for Darwin was in its depiction of the irregularity and or "complexity" of organic nature in conjunction with, if not the result of, a few simple deterministic laws in the spirit of Newtonian science. The difference this essay makes in the use of this "ideal type" is that for Darwin the tree was a diagram of the indeterminate movement of the

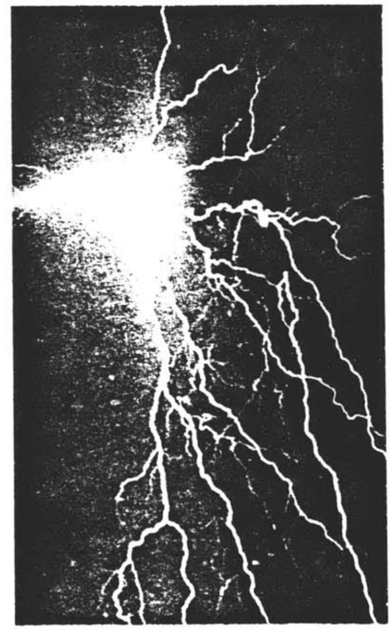
16 Thompson, Sir Darcy Wentworth, On Growth and Form (Chapter VI, The Equiangular Spiral p. 179), Cambridge University Press, 1961.

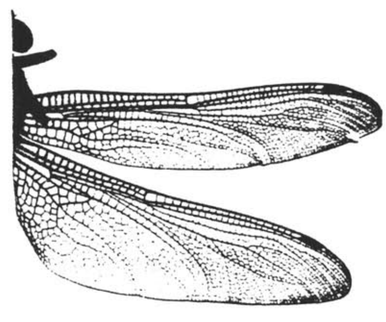
17 Gruber, Howard E. in the essay "Darwin's Tree of Nature and Other Images of Wide Scope" in Aesthetics in Science. ed, Wechsler, Judith, M.I.T. Press, Cambridge. 1978.

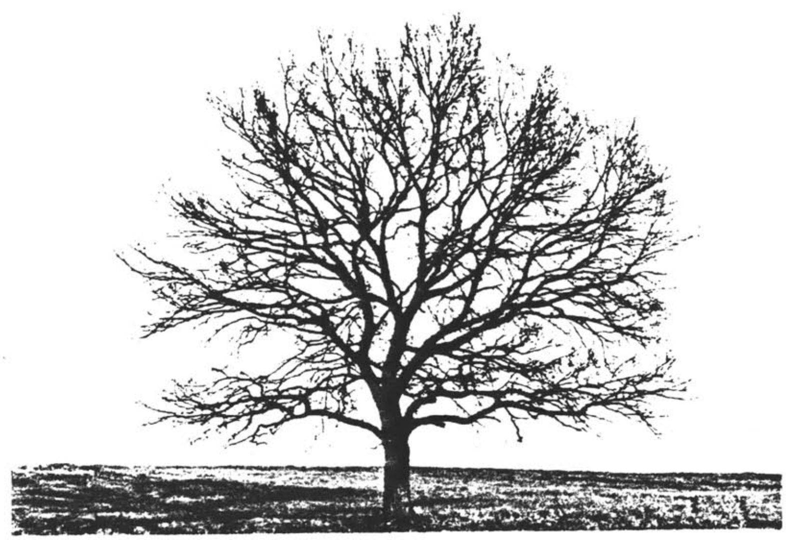
Tree Structures

It is interesting to note the etymology of the word "topology." It formerly referred to a mnemotechnical process, where the thread of reasoning involving several alternatives in the course of a mediative exercise was likened to the forks of a beaten path. Quite possibly the humble beginning of the electronic computer and its logic tree.









whole of organic evolution, here it is the diagram of the simple invariant mechanism which gives rise to the complex whole or heterarchy.

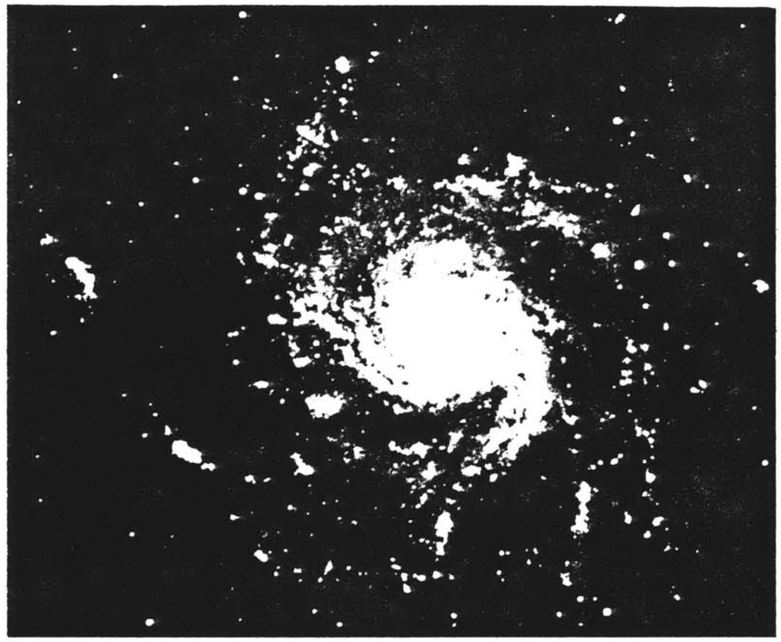
I would like to suggest the spiral and the branched structure are in fact the same ideal type and that type is the automorphic (recursive, self-similar, scaling) mechanism. The algorithm for each, like the Koch curve, display this prototypical mechanism, in that copies of the algorithm are contained within the algorithm itself and are called as procedures. I present them here for comparison.

```
To    Branch (size, level)
      if level = 0 then return
      forward size
      left 45
      branch (size 2, level-1)
      right 90
      branch (size 2, level-1)

To    Spiral (size, angle, scale)
      forward size
      left angle
      spiral (size*scale, angle, scale)
```

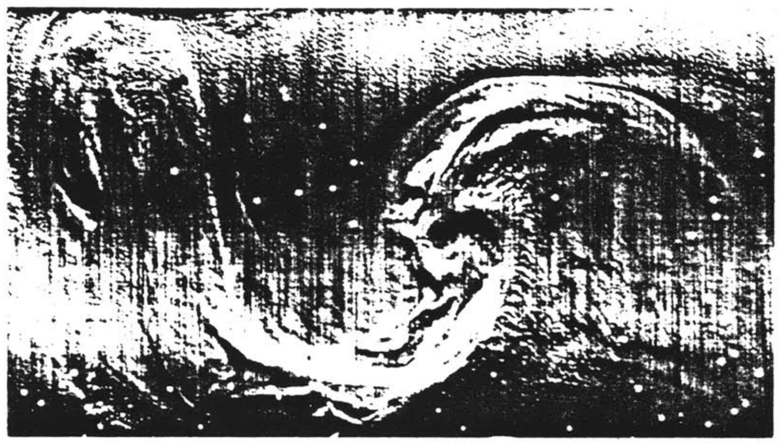
We can see that while these programs are different in their written and resulting physical or graphic manifestation, they have the same intrinsic structures. One can see their correspondence when their diagrams are overlayed.

Before proceeding to the next section concerned with automorphogenisis, I would like to point out another correspondence that is the correspondence between the written algorithm and its diagram. We can say that one is in fact a direct copy of the other. The diagram contains all the information which the algorithm has.

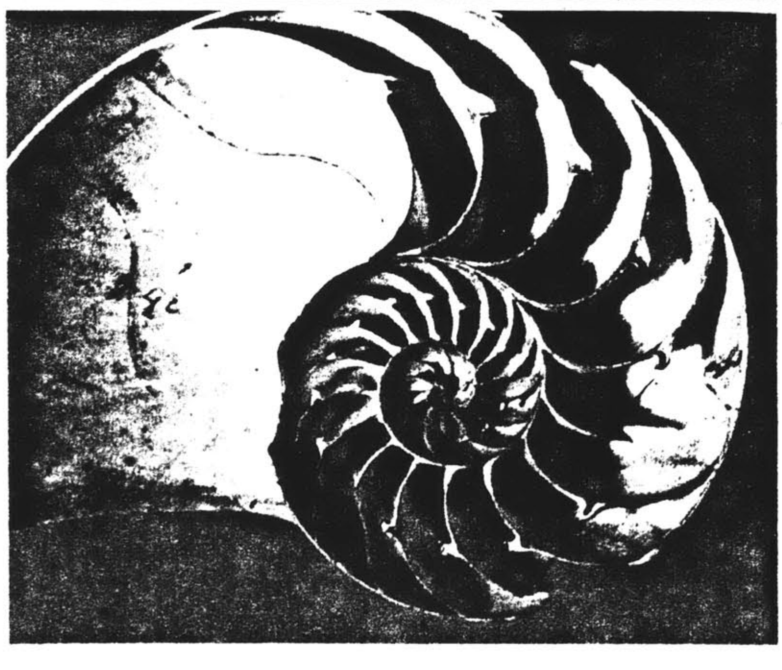
Spiral Nebula



Infrared photograph of an ocean eddy 200 miles east of Cape Cod.



Nautilus Shell

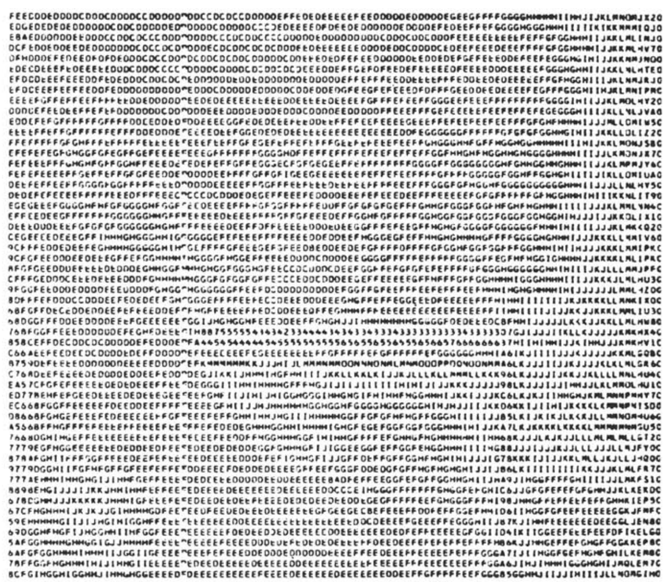


This one-to-one transformation is termed an isomorphism. Thus, isomorphism (or "copy", "self") admits of transformation, such as changes in size, relative proportion, and relative orientation; reflection, reversal, or inversion; and translation or relative position; and dimension. As long as each functional part of one maps directly onto a part of the other playing a similar functional role in its respective structure.

Perception of isomorphisms between two known structures represents the most important way by which science advances itself, and as Douglas Hofstadter has suggested, it is such perceptions which create meaning.[18]

The most familiar type of isomorphism is the storing of information in the modular form of script, or "aperiodic crystals." Such an example is illustrated on the facing page where a satellite has transmitted an image of the eastern Mediterranean by electronic impulse to an earth--based computer which transforms it into both a printed output and a pictorial reconstruction of apparent grey tones. To improve the images by adding more subtle visual detail, to represent the most delicate of palettes, to present not merely a flat image, but a sculpture "in the round" one needs merely to demand a numerically greater list of letters. Since every letter is transcribable into a choice of one among twenty--six options, and each of these options may be described with a handful of true-false, on-off, or yes-no choices, it is not hard to prove that

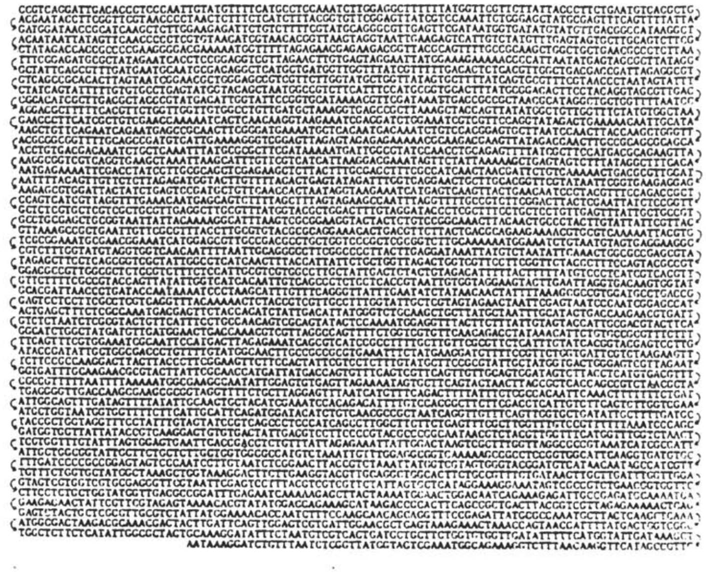
[18] Hofstadter, Douglas C. Godel, Escher, Bach - An External Golden Braid, Basic Books, New York, 1979.

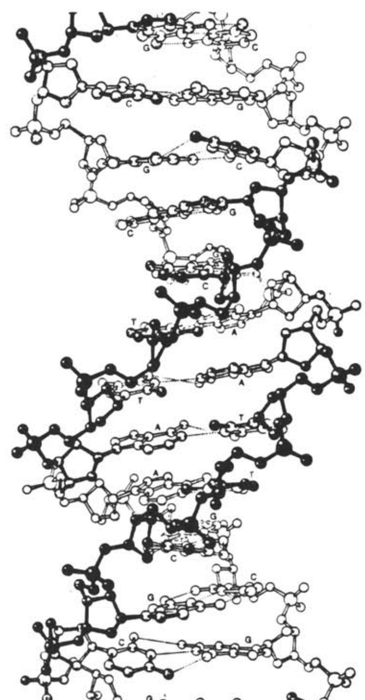


every image and every piece of knowledge is expressible through the intermediation of isomorphic relations into the same binary form. This conjunction of diversity with "invariance" or "unity" is essential if we are to have any hope of modelling or communicating any thoughts of "natural structure." This thesis itself may be said to be an aperiodic crystal.

The second aperiodic crytstal show here is the base sequence for the chromosome of bacteriophage 0x174. It is the first such genome ever mapped out for any organism. It would take about 500,000 of these pages to show the base sequence of the DNA of a single human cell. Given this one can see how enormously complicated the isomorphism from DNA to the human organism can be, nevertheless this process is timed, or given dimension, not only by the isomorphism[19] into binary units but by simple, orderly, invariant automorphic behavior, which is the topic we shall take up in the next section.

19 Even before Oswald Avery had established the connection between genese and DNA, the physicist Erwin Schrodinger predicted on purely theoretical grounds, that genetic information would have to be storted in "aperiodic crystals" in his book, What is Life?





AUTOMORPHOGENISIS

a reconstruction

I will freely quote here a very simplified account of the origin of life told by Richard Dawkins in his book The Selfish Gene,[20] before presenting a formal simulation obtained by simple recursive rules. Their correspondence is difficult to ignore.

> "...laboratory simulation of the chemical conditions of earth[20] before the coming of life have yielded organic substances called purines and pyramidines. These are building blocks of the genetic molecule, DNA itself."

> Processes analagous to these must have given rise to the "primeval soup" which biologists and chemists believe constituted the seas some three to four thousand million years ago. These organic substances became locally concentrated, perhaps in drying scum around the shores, or in tiny suspended droplets. Under the further influence of energy such as ultraviolet light from the sun, they combined into larger molecules..." "and drifted unmolested through the thickening broth."

> " At some point a particularly remarkable molecule was formed by accident. We will call it the replicator. It was not necessarily the biggest or most complex molecule around, but it had the extraordinary property of being able to make copies of itself."

> "...actually a molecule which makes copies of itself is not as difficult to imagine as it seems at first, and it had only to arrive once. Think of the replicator as a mold or template. Imagine it as a large molecule consisting of a complex chain of various sorts of building block molecules. The small building blocks were abundantly available in the soup surrounding the replicator. Now suppose each building block has an affinity for its own kind. Then whenever a building block from out in the soup...lands up next to the part of the replicator for

[20] Dawkins, Richard The Selfish-Gene, Oxford University Press, New York, 1976.

[21] "...A few simple gases in the atmosphere and some volcanoes, sunlight, or thundery weather."

which it has an affinity, it will tend to stick there. The building blocks which attach themselves in this way will automatically be arranged in a sequence which minics that of the replicator itself. It is easy to think of them joining up to form a stable chain just as on the formation of the original replicator.[22] This process could continue as a progressive stacking up, layer upon layer. This is how crystals are formed. On the other hand, the two chains might split apart, in which case we have two replicators, each of which can go on to make further copies.

"...suddenly a new type of 'stability' came into the world...as soon as the replicator was born it must have spread its copies rapidly over the seas, until the other larger molecules were formed more and more rarely."

"...erratic copying in biological replicators can in a real sense give rise to improvement, and it was essential for the progressive evolution of life that some errors were made."

"as miscopyings were made and propagated, the primeval soup became filled by a population not of identical replicas, but of several stable varieties or replicating molecules, all 'descended' from the same ancestor...Stable in that either the individual molecules lasted a long time, or they replicated rapidly, or they replicated accurately."

"...When the replicators became numerous, building blocks were used up at such a rate that they became a scarce and precious resource. Different varieties of strains of replicators must have competed for them," in a struggle for existence. Struggle, "...in in a sense that any miscopying which resulted in a new higher level of stability, or a new way of reducing the stability of rivals, was automatically preserved and multiplied." "Some of them may have even 'discovered' how to break up molecules of rival varieties chamically, and to use the building blocks so released for making their

[22] To remind us of the centrality of frequency modulation on the subject, the images of a templates is a static one. Norbert Wiener in Cybernetics (M.I.T. Press, Cambridge, 1948) makes the tentative suggestion that frequencies,..."let us say frequencies of molecular spectrum, may be the pattern which carry the identity of biological substances, and the self-organization of genes may be a manifestation of the self organization of frequencies.

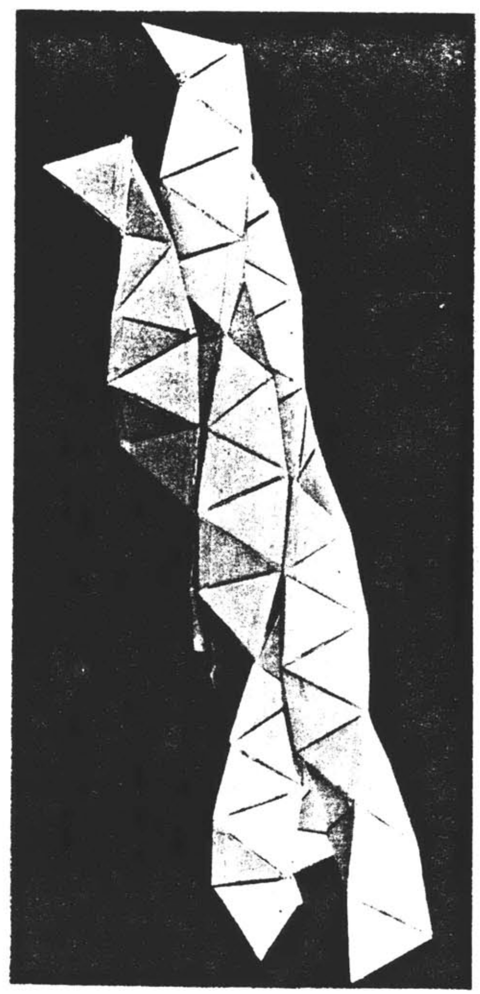
> own copies..." "Other replicators perhaps discovered how to protect protect themselves, either chemically or by building a physical wall of protein around themselves."
>
> Replicators began not merly to exist, but to construct for themselves containers, vehicles for their continued existence. Survival machines got bigger and more elaborate, and process was cumulative and progressive.
>
> "Four thousand million years on, what was to be the fate of the ancient replicators? They did not die out, for they are the past masters of the survival arts. But do not look for them floating loose in the sea; they gave up that cavalier freedom long ago. Now they swarm in huge colonies, safe inside gigantic lumbering robots, sealed off from the outside world, communicating with it by torturous indirect routes, manipulating it by remote control. They are in you and in me; they created us, body and mind; and their preservation is the ultimate rationale for our existence. They have come a long way, those replicators. Now they go by the name of genes, and we are their survival machines."

Dawkins goes on to propose that a new kind of replicator has appeared on our planet and this is due to the establishment of a new primeval soup. This new soup is the soup of human culture, and the replicator is a unit of cultural transmission, or unit of immitation which he calls a "meme". Examples of memes are tunes, ideas, catch phrases, fashions, or fads, or quotations. N.H. Humphreys neatly sums up this notion.

"...Memes should be regarded as living structures, not just metaphorically but technically. When you plant a fertile meme in my mind, you literally parasitize my brain, turning it into a vehicle for the meme's propogation in just the same way that a virus may parasitze the genetic mechanism of a host cell, and this isn't just a way of talking--the meme for example, say, 'belief in life after death' is actually realized physically, millions of times over, as a structure in the nervous systems of individual men the world over."

The Replicator

Buckminster Fuller has modelled the template process of DNA replication with helical columns of triple bonded tetrahedra. The tetrahedron corresponds to the four-dimensional pattern instruction contained within the chemical compounds G C T A. When grouped in nestled columns the dihedral angles of five tetrahedra around a transverse axis are 7° 20' short of 360°. This may explain the spring loaded separation of the helix after replication is complete.



cellular automata

"While the whole universe, and all closed systems in universe tend naturally to deteriorate and lose their distinctiveness--there are local enclaves whose direction seems opposed to that of the universe at large, and in which there is a limited and temporary tendency for organization to increase. Life finds its home in some of these enclaves."

Norbert Weiner

Every individual is an evolutionary pattern integrity."

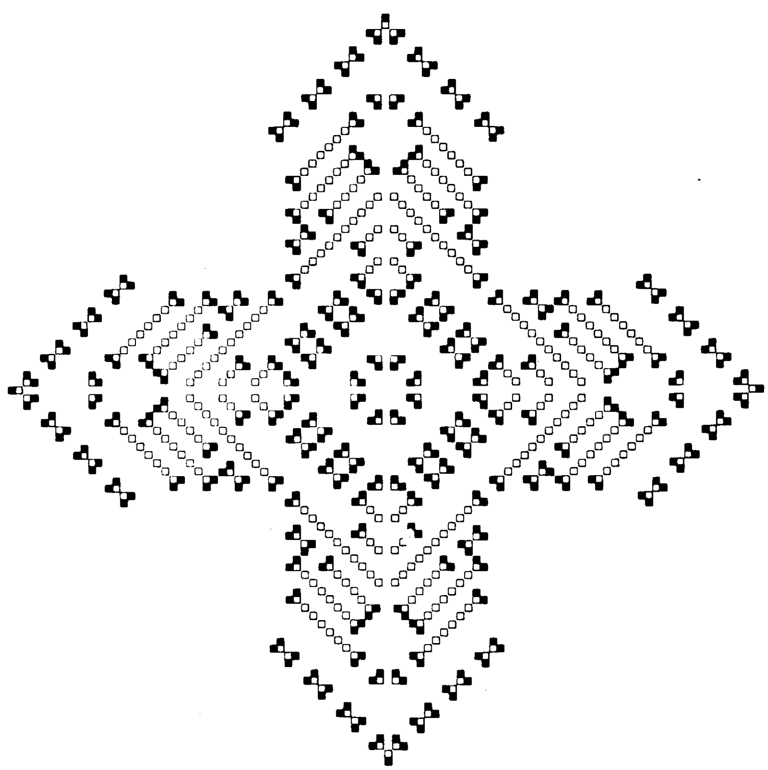
R.B. Fuller

The simulation I would like to present here is called very appropriately "The Game of Life" and was invented by John Horton Conway, a mathematician at the University of Cambridge. The game takes place on what is generally called a uniform cellular space which in this case is an infinite orthoganal grid. Each "cell" defined by the grid can assume one of two states: active (live) or quiescent (dead). The state of each cell is influenced by the state of its eight neighbors. The pattern of states changes in discrete time steps according to a set of transition rules that apply simultaneously.[23] These transition rules, or genetic laws for births, deaths, and survivals read as follows:

1. Births	each empty cell adjacent to exactly 3 neighbors is a birth cell and a counter (to signify the live state) is placed on it the next move.
2. Survivals	Every counter with 2 or 3 neighboring counters survives for the next generation.

[23] Games such as chess,, checkers and go can also be regarded as cellular automata games in which ther are more complicated neighborhoods and transition rules and which players choose among alternative next states in an attempt to reach a certain state which "wins". In contradistinction the transition rules for life are designed to produce a maximum of change and unpredictability.

Generation 45 is a cellular game devised by Stanislaw Ulam. That started with one counter on the central cell. As in the game of "life" the cells have two states but the neighborghood is one of the four orthogonal cells. Births occur on cells with only one neighbor and all live cells of generation N vanish when generation N+2 is born (that is only the two most recent generations survive). This pattern displays an extremely strong symmetry characteristic of most "stable" cellular automata.



3. Deaths — Each empty cell with 4 or more neighbors dies from overpopulation. Every counter with one neighbor or none dies from isolation.

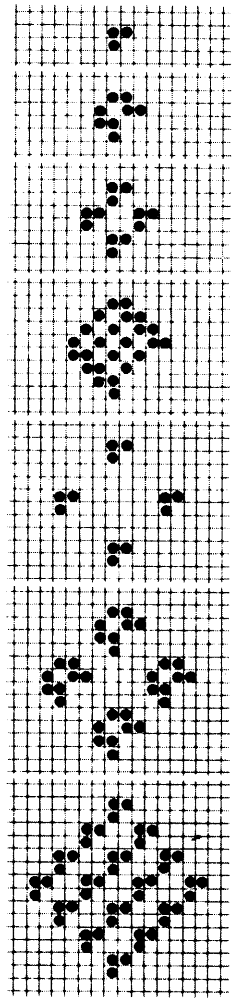
Starting with any simple initial configuration or population of counters each application of the genetic laws will define successive accretions or generations--a "tick" in the complete life history of the particular, initial, configuration.

Populations constantly undergo unusual, sometimes beautiful, and always unpredictable change. In a few cases the society eventually dies (a single organism vanishes after one generation), but the majority of starting patterns either reach stable figures ("still lifes") that cannot change or patterns that oscillate forever. In general, patterns with no initial symmetry tend to become symmetrical. Symmetry is never lost, although it may change or increase in richness. I have plotted out the successive generations of a very simple life configuration in the top corners of the pages of this section. After ten generations it evolves to a "stable" set of four "blinkers".

One of the most remarkable of Conway's discoveries is the 5 counter glider (see illustration). After two ticks it has shifted slightly and been reflecteed in a diagonal line (a glide reflection). After two more ticks the glider has righted itself and moved one cell diagonally down and to the right from its initial position. There are three other spaceships (in

The self replication of a tromino, by Edward Fredkin of M.I.T. uses two state cells and a neighborhood of four orthogonally adjacent cells and the following parity rule: each cell with an even number of live neighbors (0, 3, 4) at times t becomes or remains empty at time t + 1, and each cell wtih an odd number of neighbors (1, 3), at time t becomes or remains live at time t + 1. Fredkin proved that after 2^n ticks (n varying with different patterns) any initial pattern will reproduce itself four times--above, below, left, an right of any empty space it formerly occupied. The four replicas will be displaced 2^n cells from the vanished original. The new pattern will replicate after another 2^n steps, so that the duplicates keep quadrupling in the in the endless series 1, 4, 16, 64...

Fredkin's rule has beebn generalized to other neighborhoods, any number of dimensions and cells' with any prime number of states.



addition to the glider or "featherweight" spaceship); lightweight, featherweight, and heavyweight, all of which "apparently" move as a result of self-replication.[24]

In 1970, William Gosper of M.I.T. discovered a "glider gun"--an oscillator of period 30 that ejects a new glider every 30 ticks. Life enthusiasts have investigated thousands of ways that gliders and spaceships can collide to produce an incredible variety of stable patterns. (Including the null pattern, and patterns that produce new gliders and or spaceships.)

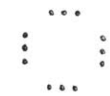
The existence of glider guns allows the simulation of a Turing machine (an idealized automation capable of performing any idealized calculation). The trick is to use gliders as unit pulses for storing and transmitting information and performing required logic functions (e.g., it is possible to arrange a formation of glider guns and other life forms so that a stream of gliders will calculate pi, e, 2 or any other real number to any number of decimal places). The embededing of a universal computer in the configuration allows the production of a universal constructor capable of producing any desired configuration through self-replication.[25]

Conway writes,

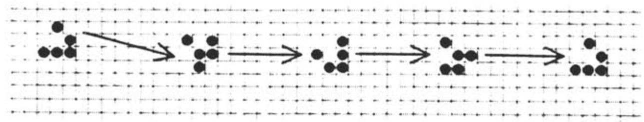
"It is possible, given a large enough 'life space,' initially in a random state, that after a time, intelligent self-reproducing animals will emerge and populate some parts of the space."

[24] Conway has speculated that in fact all "motion" is simulated--microlevel "spaceships" types may appear to move on the macrolevel whereas actually there is only an alternation of states of basic space-time cells in obediance to highly complex transition rules.

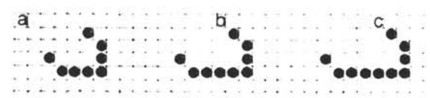
[25] This was first accomplished by John Von Neuman the pioneer of cellular automata theory, in the mid fifties, with a "kinematic" model. A machine would roam around a warehouse of parts, select needed components and put together an exact duplicate of itself. A more elegant proof for the existence of self-replicating machines was accomplished using the uniform cellular space of "life." Von Neuman, applying transition rules to a space in which each cell has 29 states and 4 orthogonglly adjacent neighbors, proved the existence of a configuration of about 200,000 cells that would self-reproduce.



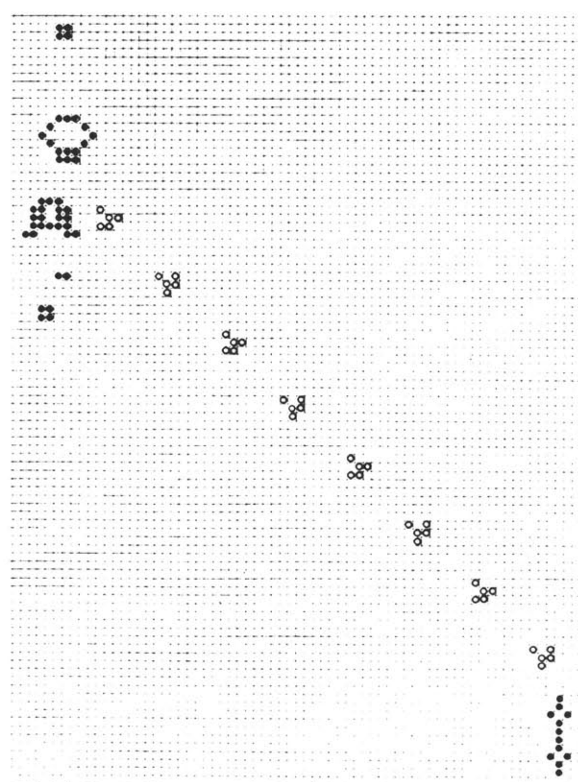
The glider of featherweight spaceship moves across the field by self replication.



Lightweight, middleweight, and heavyweight spaceships, move horizontally across the field at twice the speed of the glider. These are the only figures which may produce movement by self-replication, although longer spaceships may exist if they are accompanied by escorts to prevent blocking counters.



Pentadecathlon "eats" gliders fired by the gun. It may also be positioned such that it reflects a glider at 180 degrees, making it possible for two pentadecathlons to shuttle a glider back and forth between them. It is this notion and variations of it that allow the construction of a universal calculator with gliders serving as unit pulses of information.



It was with this in mind that Piers Anthony wrote "OX", a story of science fiction involving intelligent, sentient beings called "pattern entities" or "sparkle clouds," that have evolved in just this process in a higher dimension than our space-time. One of the sparkle clouds, in a moment of introspection, comments:

> ...stable or recurring form was an attribute of sentience, of pattern, blight was lack of pattern."

The notion of space time as being composed of discrete units, and universe as a vast cellular automation is in fact, as physicist Claudio Rebbi has explained in "The Lattice Theory of Quark Confinement," (Scientific American Feb. '83) one of most experimentally and theoretically potent ways of modelling the interaction in an electrodynamic field. The one he describes specifically consists of a discrete lattice of four dimensional cubes with the particles such as quarks situated at the vertex crossings or "lattice sites"--a space time life.

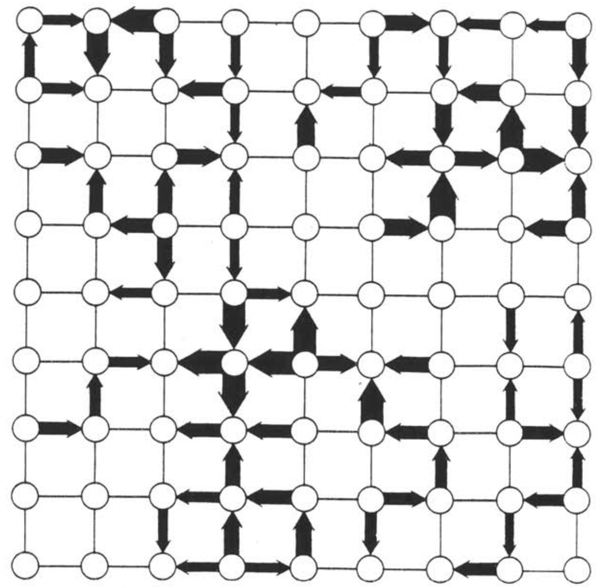
In the same way Fuller describes the game of "universe" as being like chess with 92 unique men.[26]

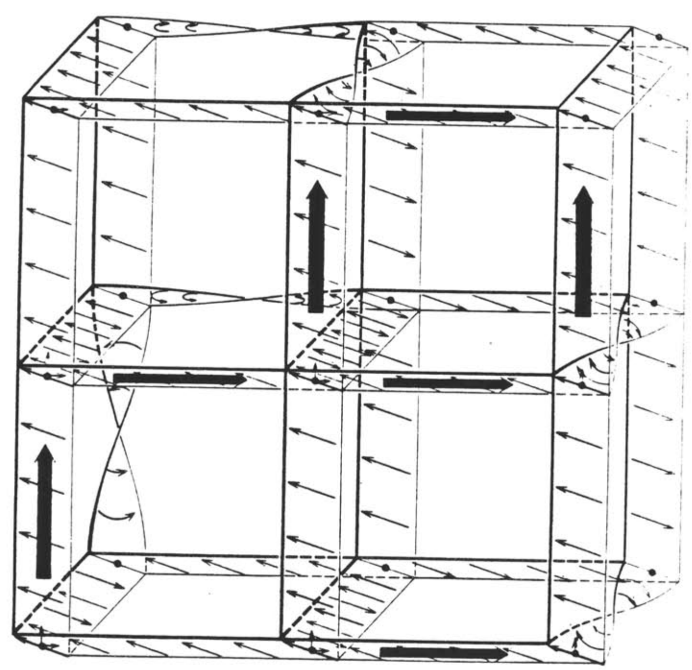
> "We start with universe as a closed system of complementary patterns--i.e., regenerative, i.e. adequate to itself--that has at any one moment for any one of its subpatterns 12 degrees of freedom. There is an enormous complexity of choice. We start playing the game, the most complicated game of chess that has ever been played. We start to play the game universe, which requires absolute integrity. You start with 12 alternate directions and multibillions of

[26] Referring to the 92 regenerative chemical elements as a pattern integrity. Fuller, R.B. Synergetics, Macmillan New York, 1975.

The lattice of points is a schematic way of representing space and time, and with such a lattice physicists seek to understand the field of force that gives rise to the permanent confinement of quarks, the constituents of the neutron and proton. The field responsible for the confinement is called the chromoelectric field and is classified mathematically as a gauge field -- gauge referring to the comparison of physical properties of different lattice sites which in turn gives rise to the lines of force represented by arrows in the diagram.

Shrinking the lattice on which particles and fields have been defined gives a closer approximation to the effect in a "continuous" space time.





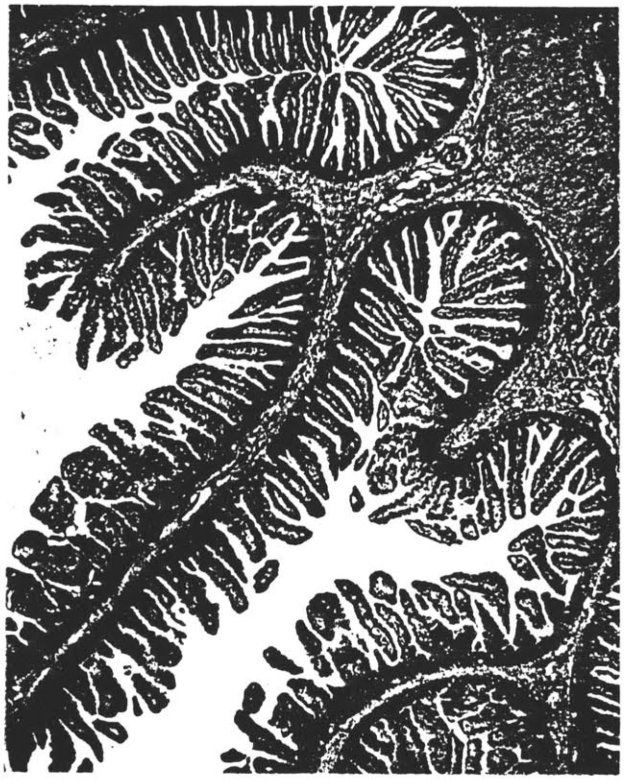
> frequency options for your first move, and you have again the same multioptions at each of your successive moves."
>
> "The vector equiilibrium becomes the omnidirectional checker frame and you can change the frequencies to suit conditions. But you must observe and obey the complexity of mass attraction and the critical proximity between precessing and falling in."

Herman Hesse in his book Magister Ludi--The Glass Bead Game has suggested the bead game in its conceptual extreme wherein all structures--musical, mathematical, social, political, visual, physical, biological, and chemical--can be presented in a single way, so that players can see all fields at once, from a single point of view, and make he contents of these many fields act on one another within the framework of the game. If such a game was in fact before us now, I suggest that pattern, whether physical metaphysical in all of these fields would exist due to these patterns intrinsic ability to self-reproduce. Pattern is stability and stability is self-regeneration.

SUPPORT

STRUCTURE

STABILITY

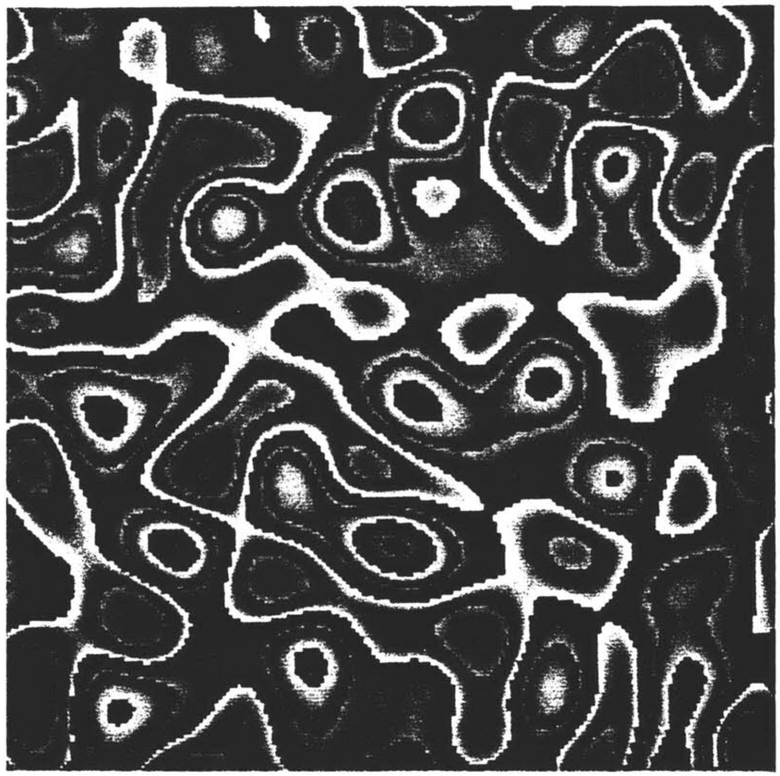


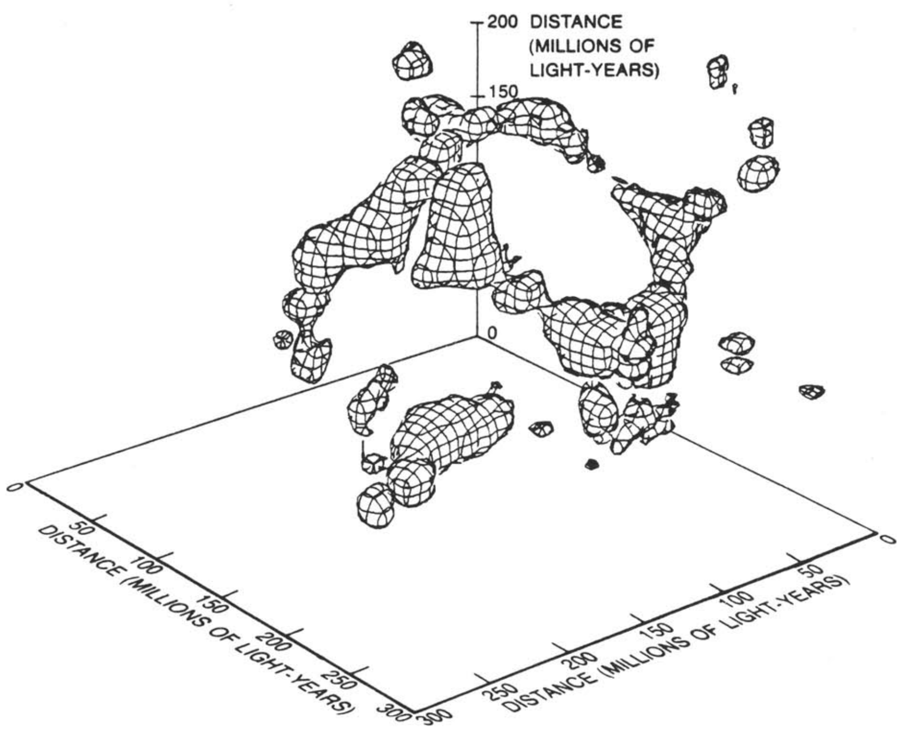
This sequence of illlustrations is intended to depict form falling into stability (therefore as stability) at various scales. It is only through self stability that forms exist at all and self stability is a result of automorphic behavior (see section on Automorphic Structure). This notion is most visually manifest in the structure of hemoglobin which is branched.

The tree structure of a hemoglobin molecule is not a haphazard, approximate pattern but a definite invariant structure, repeated over six thousand million, million, million times in the average human body. Hemoglobin demonstrates the principle that atoms fall into stable patterns. It does so at the rate of four hundred million million per second, and others are being destroyed at the same rate. The point to be made is that if a group of atoms in the presence of energy falls into a stable pattern it will tend to stay that way. Survival of the stable is the principle law of natural selection.

Cay and decay: Webster's Dictionary defines "cay" as "emergent reef of coral or sand." It's earlier etymologcal meaning is a "growth," a coming together of parts.

According to the big-bang theory the universe began as a singular point of infinite density some 10 to 20 million yrs. ago and pulsed into being in a vast explosion that continues on this day. Small perturbations in the density of matter and energy must be assumed to explain the coalescence of matter into stars, galaxies, clusters of galaxies, and superclusters. The perturbations can be understood at random wavelike fluctuations of the density around an average value (top illustration). The lower figure similates the clustering of galaxies into filamentary structures"vaguely" reminiscent to what we observed with hemoglobin.





built precedent

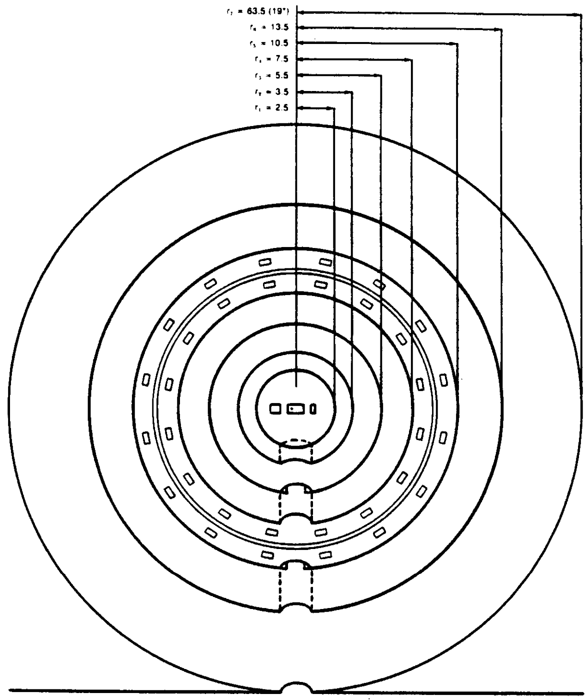
So far I have discussed the principle of automorphism though the most comprehensive and abstract science of mathematics or pattern. The science of pattern is the most general case, the fundamental communications system between all the sciences. By this means I have attempted to show how automorphic behavior is a lowest common denominator in the somewhat less general studies of physiochenmical, geomorphic, and information phenomena.

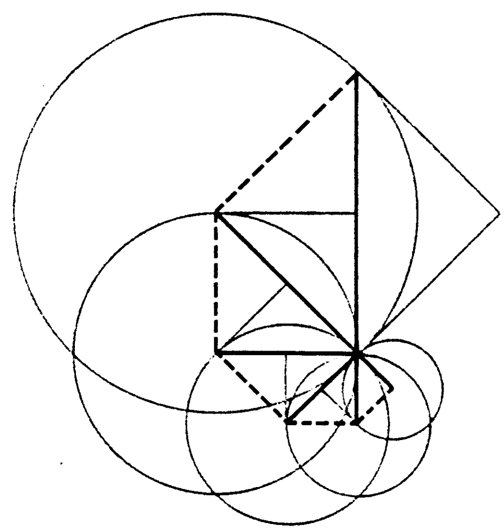
I would now like to introduce automorphic behavior to the analysis and subsequent synthesis of support structures for human inhabitation (survival machines for survival machines). This involves a much more local set of considerations or parameters, and I will discuss them under the broad categories of: individual (human) dimensional requirement and its perception, energetic requirements, structural requirements. Self--stability achieved through self-replication, remains as the "apparent continuity" for each of these locally distinct sciences.

Before entering into these more specific digressions, I'd like to present a brief archeology of previously built form, (largely through illustration) which exhibit characteristics of scaling and self similarity. The first example is that of Atlantis, and I have gone into it in greater detail not only because it occurs first chronologically but it reintroduces some of the form ideals I mentioned earlier.

Harold A.T. Reiche has delineated this relationship in his article, "Language of Archaic Astronomy" in Astronomy of the Ancients, ed. Brecher, K. and Feirtag M., M.I.T. Press, Cambridge, 1981. "Poseidon's construction comprises six (2 x 3) parts: a central island with a diameter of five (2 + 3) stades; and then five concentric bands, three of them to be filled with water and the remaining two, to be annular bands of land. The three bands of water have an aggregate width of six (2 x 3) stades; the two bands of land have an aggregate width of five (2 + 3) to top it off, the area of the outermost band of water is 216 square stages (2^3 x 3^3) in the Pythagorean-Platonic system two and all numbers divisible by two are termed 'female' and all odd numebrs (starting with three) are termed 'male;' their sum (five) and product (six) are consequently spoken of as being 'marriage' numbers. Thus Poseidon has surrounded his marriage bower with the numeric equivalents of marriage."

Construction by Harold A.T. Reiche in his article "The Language of Archaic Astronomy."





In the first place, the "ideals" originate from pure number.[1] Furthermore, this island within an island configuration very simply illustrates what is meant by the term recursion.

Lastly, Atlantis introduces us to the "recursively ennumerable set" in the progression of radii of the annular bands. It appears that each of the radii is greater than the preceeding radius by a factor of approximately 2 ; that is, the progression of radii can be expresed in the form $r_{n+1} = 2r_r$. As illustrated in the geometrical construction called "poseidon's conch,"[2] each radius turns out to be the side of a square whose diagonol is the next larger radius. The Pythagorean mathematicians believed that progressions of intervals recurred throughout the universe: in the successive realms of number, figure, solid, vibrating string, and revolving planet.

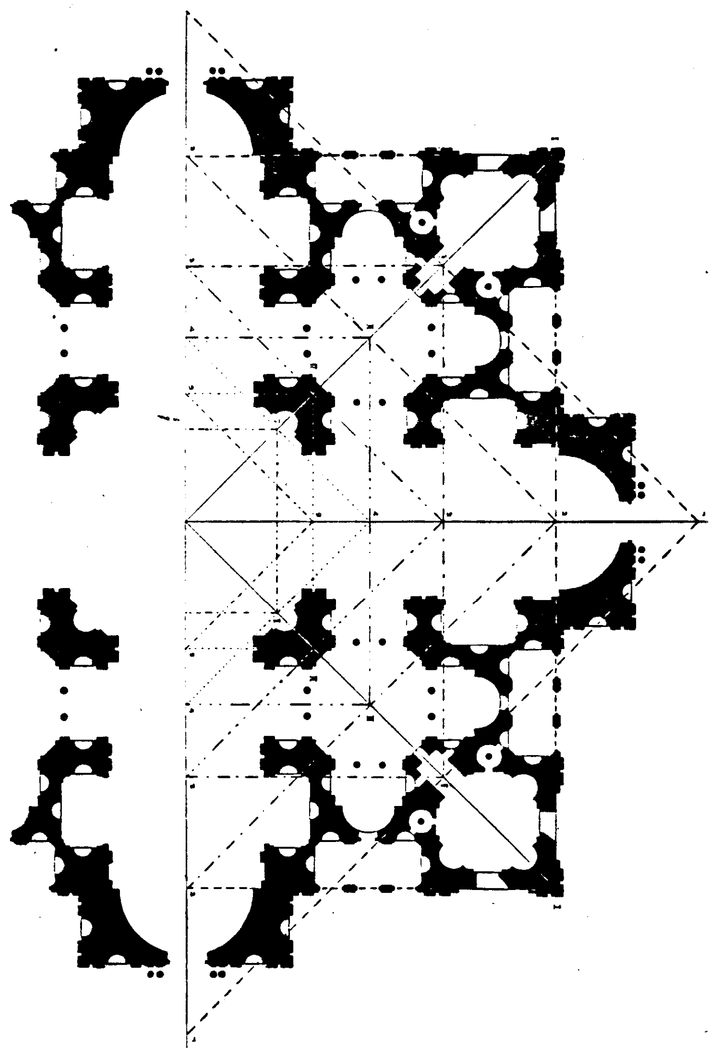
These progressions (recursively ennumerable sets) are generated from a set of starting points (axioms) by repeated application of rules of inference, each new element being compounded somehow out of previous elements in a "mathematical snowball." The most famous of these sequences is the Fibonacci series of 1, 2, 3, 5, 8, 13, 21, 34 which can be defined by the formula $n = (n - 1) + (n - 2)$ for n 2 , which defines a relationship between consecutive terms as $0 = 1.618...$. Like the relationship found in the radii at Atlantis it is "irrational and thus a horrifying "incommensurability" in the Pythagorean universe of "commensurability."

One last example of recursion in number theory may indicate the direction I want to move, from Atlantis to the construct I will propose at the end of this thesis. Consider the following definition of a function: $n = (n - (n - 1)) + (n - (n - 2))$ for $n \quad 2$. It is reminiscent of the Fibonacci definition in that each new value is a sum of two previous values--but not of the immediately previous two values. Instead, the two immediately previous values tell how far to count back to obtain the numbers to be added to make the new value. The sequence runs as follows: 1, 1, 2, 3, 3, 4, 5, 5, 6, 6, 6, 8, 8, 8, 10, 9, 10, ...

The apparent chaos and unpredictability of this sequence indicates that recursively defined sequences may possess some inherently increasing complexity, so that the further one goes, the less predictable they get. Is it possible that a sufficiently complicated system could break out of any predetermined pattern?

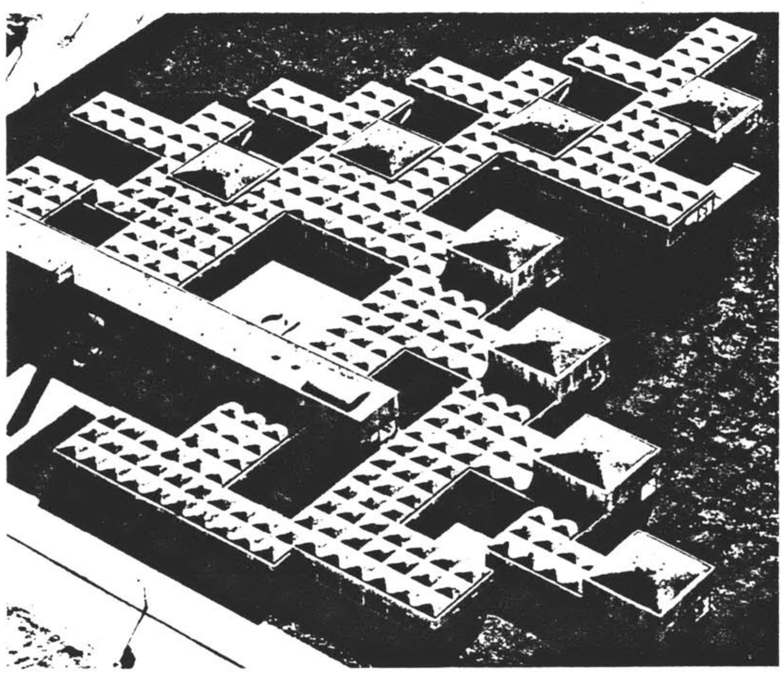
Palmanova, Italy 1953 by Scamozzi as pure territorial object. The fortified city was a tool by which 16th century despots either maintained or advanced their dominions. In a sense it was a vertex representation of this larger dominion. The self reference replication was continued not only in the concentric repitition of walls and ditches but in several forms of outerworks called "ravelins" and "tenailles." Within the bastion itself, its form was repeated in a raised tier caleld a cavalier. This was referred to as "defence in depth" during the renaissance and one can immediately see its tactical advantage--agressor's now had to overcome not one but several fortified cities. So we can speak of the edge as consisting of many versions of itself as well as the town (and vice versa) and the town as a version of the territory. Self replication is the language of territorial behavior. It is the means by which a form signifies or "speaks of itself."

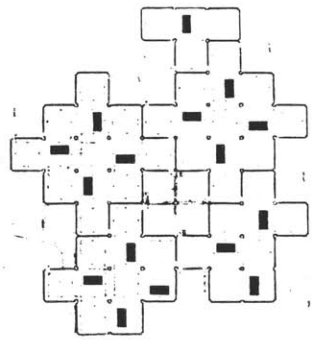
Bramante, in a proposal for St. Peters used a 2 progression indicated by the regulating lines. It is also apparent in the form itself by the repition of the central crossing at 1 1 scale in each of the four quadrants. The pythagorean concept of the temple as link to the cosmos is here articulated as a 2 stairway or ladder. Construction by Forster, Otto H. Bramante Wien, A. Schroll 1956.

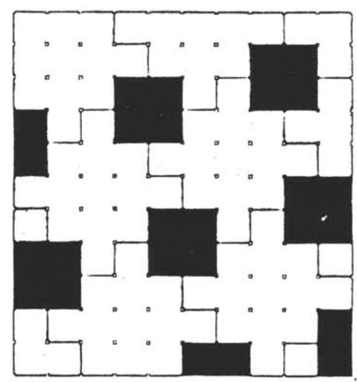


The return of scaling. Self similarity in the 20th centry is centered with Aldo Van Eyck and the "Dutch Structuralists." Anthropological concerns of dimension, place, and occasion are brought to form through geometric discipline. The large organizational form of Van Eycks orphanage clearly resemble a self-intersecting tree. A more rigorous aspect of scaling will be seen if one considers the individual roof room units as elemental in the spatial structural hierarchy. of these rooms build the larger collective territories of classroom (identified by the larger domed roof) and its reciprocal courtyard.

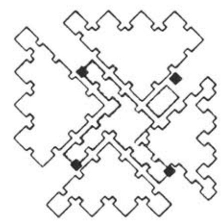
The large central court associated with the whole is a dimension of 4 of these classrooms or 36 of the smallest room units. And finally the territory which interfaces the city itself is partially built with 12 of the room units to a side or virtually 144. The progression of the collective territories is thus $(3)^2$ $(6)^2$ $(12)^2$.

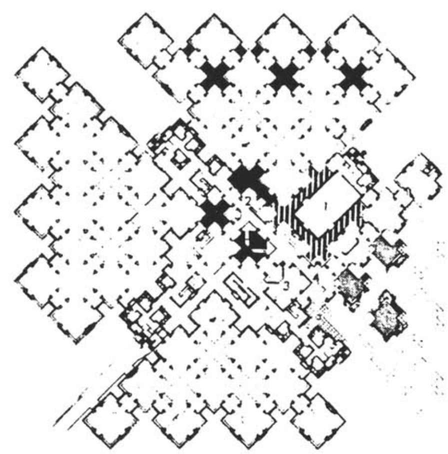


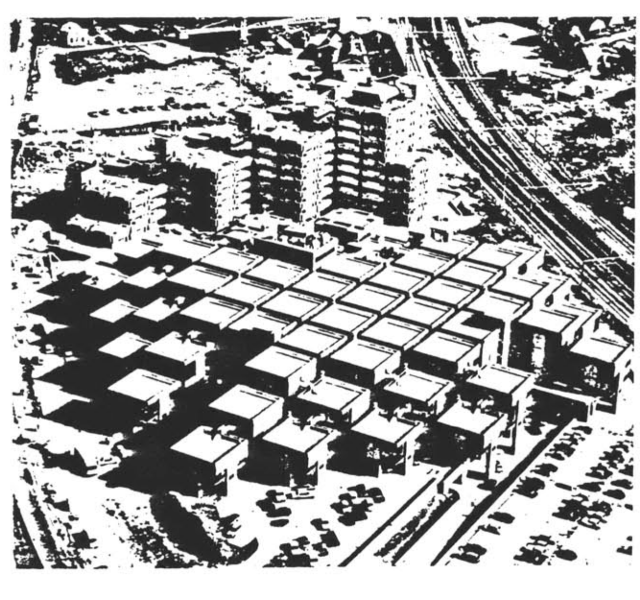




Fernando Domeyko in his library proposal employed the spatial lattice in a similar manner to Van Eyck. He established larger "tiles" by the aggregation of 19 room-sized spatial units in a mastaba (or stepped pyramid). The conjunction of four of these "tiles" builds the next larger territory leaving a outdoor court 4 times the size of the room-sized primitive. The growth process may continue upwards to establish larger hierarchical territories. One should also note the "nested" skylight system which re-establishes the mastaba configuration within the spatial primitive.

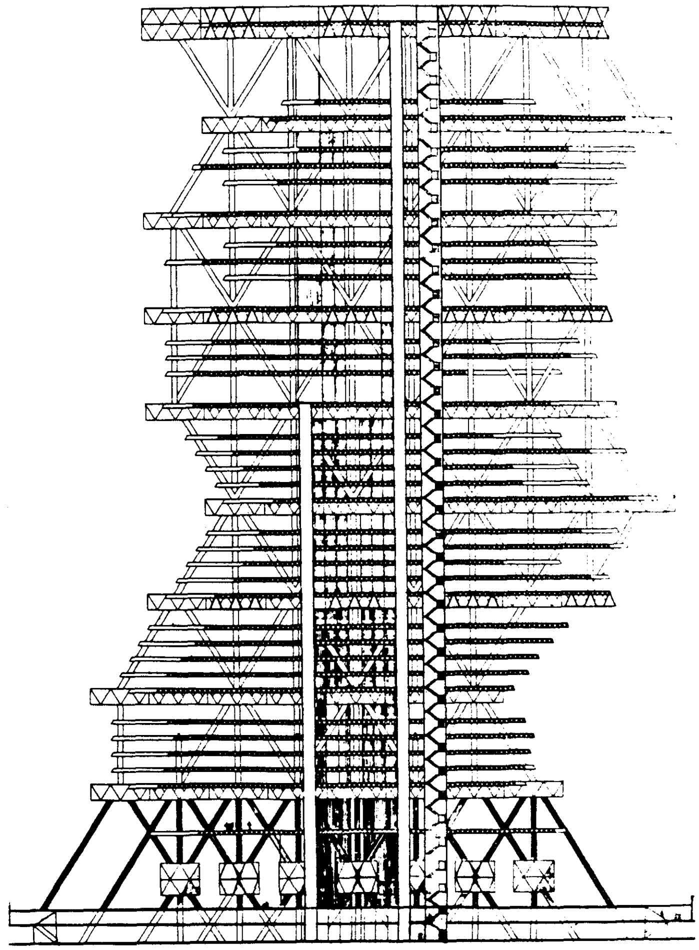


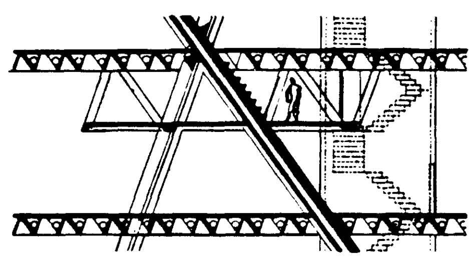




The Central Beheer office building by Herman Hertzberger is one of the most rigorous examples of spatial scaling. Four workstations for 1-4 people aggregate to form the first structural bay. Four structural bays aggregate to form the central space marking the crossing of the two major circulations. These circulations delinete 4 quadrants of 25 virtual structural bays of which only half (10 or 14) are actually articulate. The circulation corss occurs at all levels thus forming a tree-like lattice.

In the proposed office tower for Philadelphia by Louis I. Kahn, spatial scaling was established by hierarchical structural limitations. The three-fold hexagonal plan of the structure rotates every 66 ft. This results from the natural completion of the large scale triangulated space frame in its upward helical movement. The hierarchical sequence is from the main 66 ft. increments, to the hollow triangulated capitals (which are single floor height), to the 3 ft. deep space frame ceilings.





human dimensional requirement

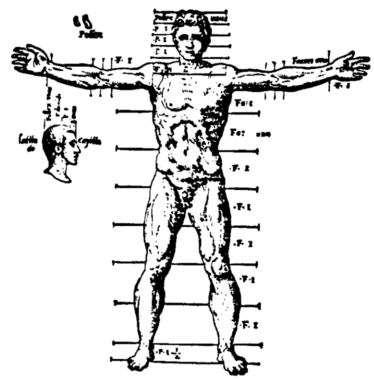
Throughout history, the human body has been a source of relative dimensions or increments for an archatype of space. Vitruvius wrote,

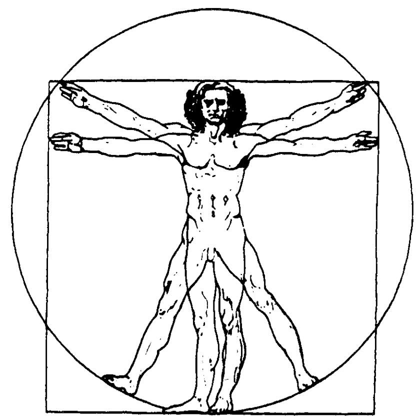
> "For if a man be placed flat on his back, with his hands and feet extended, and a pair of compasses centered at his navel, the fingers and toes of his two hands and feet will touch the circumference of a circle therefrom. And just as the human body yields a circular outline, so too a square may be found from it."

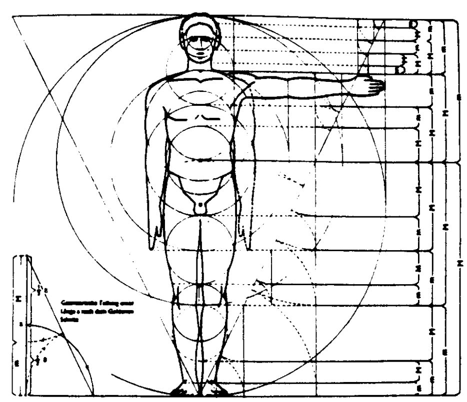
Leonardo's drawing depicts this relation. Although Barbaro's vitruvious shows the human body with thumb and face as static repetitive modules, Zeising's drawing from the 1880's subdivides the body into a linked hierarchy of extreme and mean divine proportion ratios. The case for a "dynamic module" of human proportion was furthered by Hambidge and others, the most recent of which was Le Corbusier. His "Modulor" was a double Fibonacci series of continued subdivision of a six foot man.

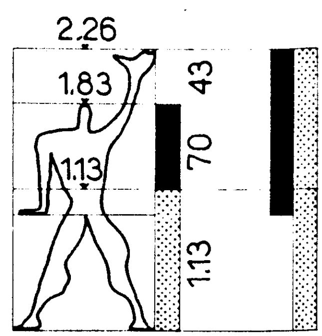
The importance of these studies I believe lies in the intuition that increments concerning the effective dimension or "territory" of the human species manifest as continuing fractions as a result of natural resonating human perceptual thresholds, also understood to occur in continuing fractions. The Weber-Fechner law, discovered in the 1840's, states that "there is a logarithmic relation of stimulus to sensation. For the senses of sight, hearing, taste, smell, and touch."[27] We perceive (or sense) increases in light or sound as equal increments when the

[27] Tyng, A.G. Resonance Between Eve and Archtype, in Via 6. Architecture- and Visual Perception, M.I.T. Press, Cambridge, 1983.








2.26
1.83
1.13
43
70
1.13

actual increase (stimuli) becomes progressively greater in geometric increments. The summation of natural perceptual thresholds for the senses, in which each new increment of perception is equal to the two previous increments may in fact proceed, at higher levels of information (frequency-) processing, to a specific proportional dominance. Typical of a stable condition in the brain the bio-engineer S. Deutsh has found an amplifier gain at the synapse of .618 (or the ratio of 1 to 1.618--the "divine proportion") in multisynaptic neuron chains with feedback. Deutch says that instability exists when this ratio is exceeded and that "several well-known pathological, states may be instances of nervous system instability."[28]

"Space"is primarily a visual phenomenon. Thus if there is no light (or sound, or touch or any other radiation phenomenon) there is no "space." Our eyesight is stroboschopic at 60 cycles sec. Thus, our awareness of "space" is frequency-modulated by our sensing equipment, as well as by our processing equipment. Thus any synthesis of support structure must at least be potentially multidimensionally resonant.[29]

Increments have in fact been established quantitively concerning effective dimension or "territory" of the human species. E.T. Hall, in the "Hidden Dimension" delineates a series of these effective dimensions which roughly correspond to the continuing fraction of the Fibonacci series. His "intimate close" of 0 ft. and his "intimate not close" of 1 ft. begin the

28 Deutsch, S. Models of the Nervous-System. Wiley, New York, 1967.

29 In analogy, the underlying physical basis for resonance in both music and color can be measured. In music, the ratios between lengths of string which produce harmonic resonance (1:2 for the octave, 2:3 for the fifth, and 3:5 for the sixth) were discovered by the Pythagoreans. The specific physical aspects of color were noted by Goethe and later by Itten at the Bauhaus. They found that the intensity ratio of green to red is 1:1, that of blue to orange is 1:2 and that of violet to yellow is 1:3. Itten wrote "Back in 1915, studies of depth effect in colors led me to the conclusion that the six fundamental rules on a black ground conform to the ratio of the golden section (divine proportion) in their gradation of depth." The Elements of Color. trans. E. Hagan Von Nostrand Reinhold. New York, 1970.

sequence. His "personal close" of 1.5 to 2.5 ft averages 2, and his "personal not close" of 2.5 to 4.0 ft. approximate 3. His "social consulting close" of 4 to 6.5 ft. approximates 5, and his "social connsultive not close" of 6.5 to 10 ft. approximates 8. His "public close" from 10 to 22 ft. includes 13 and 21 ft. Hall's public not close begins at 30 to 40 ft. approximating 34 ft.

A strong perceptual connection between the individual and the larger collective, social, conscience and its territory is established by the continued "spatial" ratio. Levels of identity resonate with the intuitive capacity to perceive them. The individual and his shelter acquire dimension by their relative position and the discrete perceptual steps between them.

"Space is its own Appreciation"
Aldo Van Eyck

energy

The generation of shelter or support structures requires the basic recognition that in general, the desirable internal state of this shelter is steady while the outside goes through variable frequencied perturbations. Variations in sun, water, and wind are recurrent or cyclic and it is this fact which enables adaptive response.[30] A single stress occurrence either lies within the response capacity of the sytstem, or it destroys the system.

Adaptive response for a support structures is through spatial transformation and differentiation and varies in evolutionary speed from the trial and error procedures over a span of years resulting in the peublos of Arizona and Colorado,[31] to the flipping open of an umbrella at the first drops of rain. Whatever the period of transformation is, if it results in a reduction of environmental stress, it is retained and reinforced as a stable configuration in the evolving form. The energy required to sustain a built configuration at a steady internal state seeks a minimum. Ralph Knowles has developed this into a general principle:

> "A minimum differentiation of form in relation to force action produces a maximum variation in the force effect; a maximum differentiation of form in relation to force action produces a minimum of variation in the effect of that force."

[30] Wiener, Norbert, Cybernetics, M.I.T. Press, Cambridge, 1948.

[31] Ralph Knowles in his book Energy and Form M.I.T. Press, Cambridge, 1974, has studied the development of Peublo Bonito, Acoma Peublo, and Longhouse Peublo. Differentiation of these settlements occurred in an unselfconscious way over a period of time, with each discrete corrective measure accompanied by an apparent decrease in susceptibility to environmental forces, yet without evident overall direction. He terms this "self-organized."

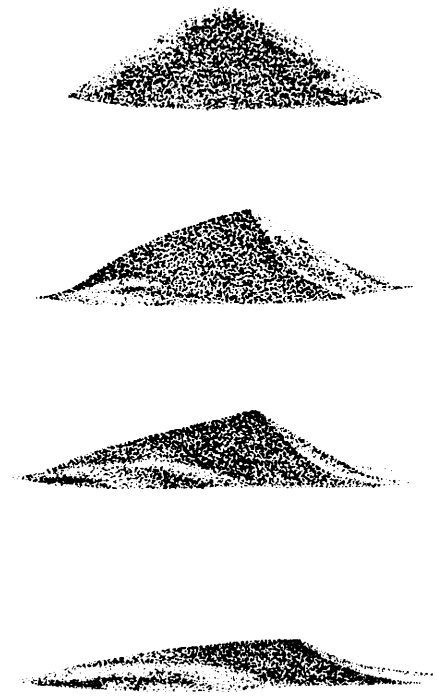
In the criterion for development of a support structure is the minimization of energy required to maintian equilibrium then. A composite of all recurrent forces must be considered. As insolation is the major recurrence the primary criterion is the minimization if not mitigation of its variation.

Upon further consideration we see that insolation recurs with two recognizable periods--daily and seasonaly. Interestingly enough, there are two functions concerning building form which control both of these recurrences. They are the most familiar frequency modulation of the surface and or volume, and orientation. Knowles develops the ratio of surfaces to volume as an expression of a forms susceptibility to environmental perturbation. The higher the ratio the more susceptible it is. Thus, high-frequency geodesics, which contain a very high volume per given surface unit, experience less perturbation than a simplex such as the tetrahedron.[32] The opposite is true if one thinks of susceptibility in a structural sense--the tetrahedron being the most structurally stable of all configurations.

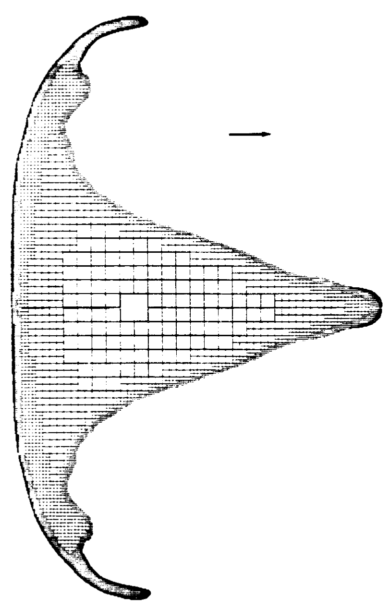
As I mentioned before, frequency modulation of a "boundary" is frequency modulation of a spatial "zone". As we move inward and away from the system's extremity, the descriptive increment need not be so refined. Spaces deeper within a building experience less variation in their state than do spaces near the surface. Consequently, the larger and deeper a building, the more of it's volume tends toward a steady state. Inside exceedingly large buildings, the thermal lag is so great that variations in conditions that recur

[32] Knowles reaches the oposite conclusion regarding faceting, but he is using facets to approximate a pure surface generated by the shadows from a point above the site whose altitude is determined such that it is a maximum without casting shadows outside its given domain or site. This is in fact the procedure concerning orientation which mitigates seasonal variation of insolation--the building as

The stable sand pile can be destabilized by the addition of another environmental force, wind. In the early stages of this transformation, surface area increases rapidly and volume is quickly decreased through loss of sand. As a more stable shape is attained, the rate of change in S V diminishes, approaching a steady state. After the force has stabalized, the range of wind pressure conditions is minimized by comparison with the original range.



The further away from the boundary of two diverse states, the larger the descriptive increment, corresponding to the diminishing information that defines the boundary -- in this example, of the above sand shape.

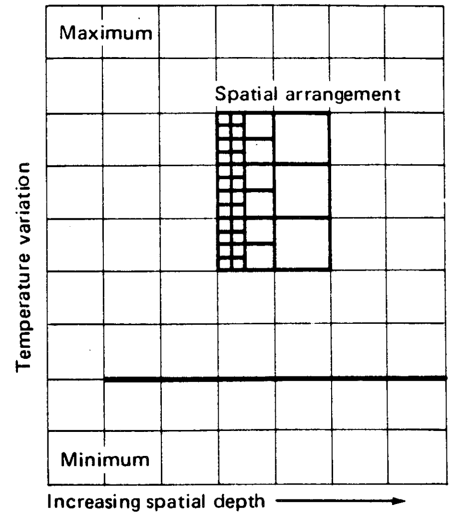


at an interval of one day may have very little effect on the internal volume. Theoretically, if the building gets deep enough, there may be portions inside which do not even respond to seasonal variations.

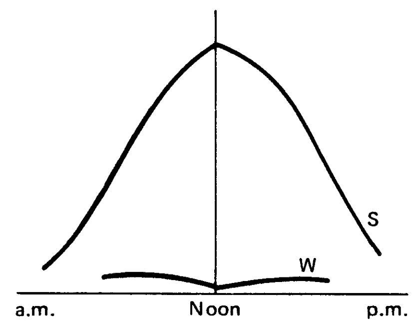
If the goal is to equalize the amount of variation within the system itself then the spatial encremaent should be differentiated from outside to inside. Near the outside wall where variation of temperature is inclined to be great, the spatial frequency should be great; toward the center where the temperature drop is in smaller increments, the spatial frequency is smaller (increments are larger), thus attaining the same amount of temperature drop per increment. Such a differentiation would clearly minimize daily variation.

Thus support systems can be thought of as frequency modulators controlling the effect of external fluctuation, and purposeful frequency or dimensional modulation is a result of true automorphic behavior--self-replication and feedback.

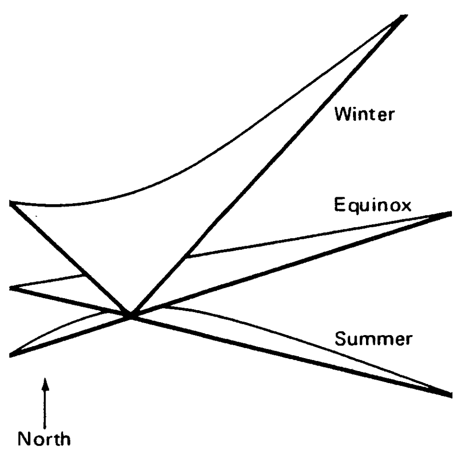
The magnitude of temperature variation between on space and another may be held constant by a differentiation of spatial increment from outside to inside.

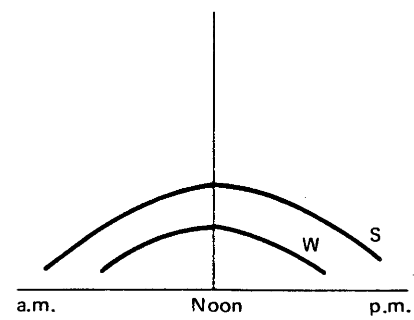


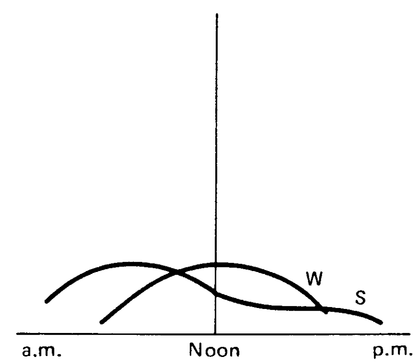
Three different seasons are chosen to cast shadows from vertex of an imaginary pyramid to the surface of the ground. A surface that is generated during the winter solstice has a much higher incident energy gain in the summer than it does in the winter, while a surface generated closer to the summer solstice tends to minimize this difference. Thus a self replication of the sun's path allows a means by which to mitigate the seasonal variation of insolation.



Diagrams are from work conducted by Ralph Knowles.







structure

The position of Robert Le Ricolais is central to the discussion of structural automorphism because it was he, through mathematical intuitionism mediated by analogical references to organic structure, reduced the concept of automorphism from a largely mathematical field to a specifically structural one. The premises for Le Ricolais work were laid by Hilbert, Maxwell and Cremona. Maxwell and Hilbert for their logic of continuous space and the elementory apparatus of topology,[33] and again Maxwell along with Cremona for their original studies on the "corrrespondant figure" (dual) or transformation groups.

> "Two systems of reference are equally admissable if all the physical and geometrical universal laws retain in both the same algebraic expression. The transformaitons, in passing from one to the other of these systems form the group of physical automorphisms of space: the laws of nature are invariants of the transformations of this group."

This proposition of H. Weyl's delineates Le Ricolais system of reference in which radiolarias and crystals, domes and bridges can described by a single apparatus of terms, concepts and relations.

Although, "space" remains undefinable, we nevertheless may speculate as to how space is populated. (As a matter of fact, it is not so much these entities themselves that count, but rather the rules of their manipulation.) If we are dealing in "flat space" the three sole entities are distinguished by whole numbers:

[33] Hilbert writes of topology: "It deals with geometric facts that do not even require concepts of uprights and planes to be studied, solely the existence of a continuous connection between the points of a figure."

1. Number of points determined by the intersections of lines indicated by C.

2. Number of incidental segments indicated by E.

3. Number of regions constituted by open or closed domains, these domains being marked by the previous segments of finished lengths, or of unlimited lengths in the case of open domains.

Between these three numbers, C, E, and P, a fundamental relation exists, called the Euler Relation, whereby, whatever the layout of straight or curved segments happens to be, one will always get:

$$P + C - E = 1$$

In the case of 3-dimensional space, Euler's law is usually written in the form:

$$F - E + V = 2$$

where F, by replacing P, has become the "number of faces" of a polyhedron. E is the "number of edges" and V the "number of summits."

These two relations play a fundamental role in the relatively new mathematical discipline designated by the name "Theory of Graphs" and subsequently to Le Ricolais "image method." This theory deals with the different connections possible between points and lines. In connections of this kind it is not a question of the notion of measuring lengths or surfaces but only of the conditions of existence of these configurations and of their qualitative properties.

For the sake of simplification, I will confine myself to a summarized aspect of his method. Consider the configuration or the "graph" represented by the letter Y. Such a configuration responds to $p = 3$, $E = 3$, $C = 1$

If we affect a transformation denoted by the operation

$$\begin{array}{lcl} P = 3 & & P^1 = 1 \\ E = 3 & \longrightarrow & E^1 = 3 \\ C = 1 & & C^1 = 3 \end{array}$$

whereby the three regions (which are metrically infinite), become a region, the sole point of Y is changed into three points. The resulting configuration is a triangle, and is the "dual" (or "image) of Y. So in general, the dual transformation is one in which the points become regions, conversely, where the regions become points, the only invarient element being the number of incidental segment. The question is what purpose can such a transformation serve?

The answer was supplied by the geometrician Cremona and the physicist Maxwell: The pattern Y represents a configuration of bars, and the pattern △ the force existing in these bars, the condition of equilibrium demanding the closure of the △ . So from this geometrical concept sprang the utilitarian branch: "Graphic Statics."

According to Hilbert's definition, an automorphic configuration is one where the initial motif, while increasing or decreasing, repeats itself and maintains the same incidence, that is, each knot C is characterized by an identical number of branchces, The autòmatism of the repetition in a triangulated network must be matched by an automatism in the distribution of stresses. Thus the representation of forces in an automorphic network forms an automorphism. If

"Poly Ten: or automorphic queen post truss by Robert Le Ricolais,

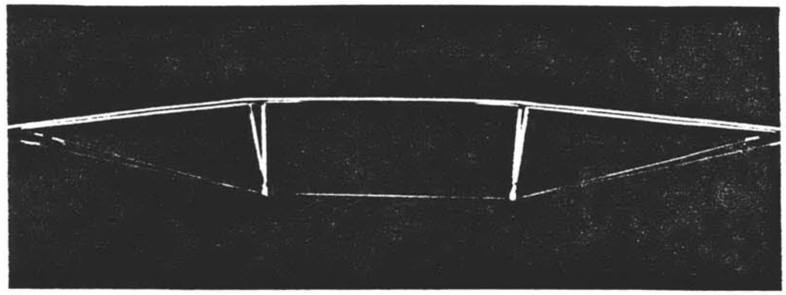
if L is the span

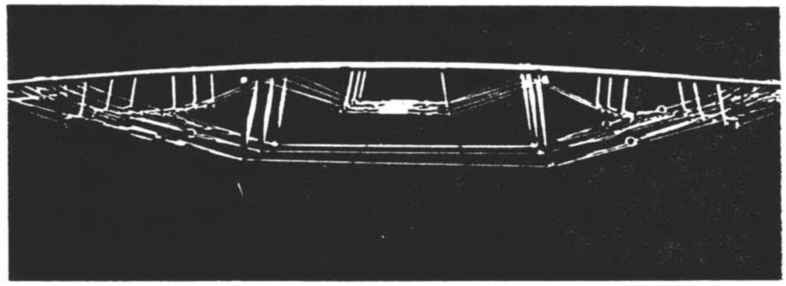
and L^1 is the reduced span

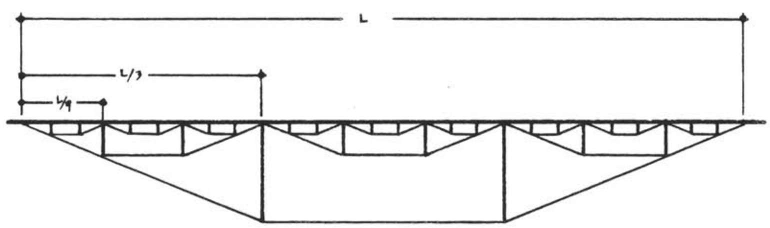
$$L^1 = L/3^n = L/27$$

The corresponding moment reduction is

$$M^1/M = 1/27^2 = 1/729$$







one will admit that each knot in a triangulated system represents an equation and that there are as many equations as there are knots, the number which often exceeds several hundred, one can grasp how much time is gained in the calculus operations.

Le Ricolais furthered the application of the "duals" using them as a means of transformation from planar to spatial configurations. If we take our example configuration and effect the addition:

$$P + P^1 = 4 = F$$
$$E + E^1 = 6 = E$$
$$C + C^1 = 4 = V$$

We get the characteristics of a spatial configuration--the tetrahedron. In general, by adding a unit to each of the elements P and V, we obtain a polyhedron of dimension 3.

The application of the foregoing principle holds for three-dimensional systems. That is, if the network is in itself an automorphism in the three dimensions, the image of the forces to which it is subjected will constitute an automorphism in the plan, due to symmetrically distributed loads. If the representation is congruous, each bar in the system must correspond to a segment which on the given scale will shown the amount of force acting on each bar. (See "Octen" illustration).

The aforegoing summary obviously suffers from many axiomatic deficiencies, but I believe it introduces the key themes developed in Le Ricolais' structural researchers over a period of several decades,[34] and from which I will try to elaborate.

34 For a more complete exposition see the essays, Towards a Technique - of Forms and Forms and Structures by Robert Le Ricolais in Zodiac 22:- Light Structures. Milan, 1973

Inside perspective view of the "octen" antenna by Robert Le Ricolais. Generated bay graphic inteerpretation by the image method. It is composed of octahedronal units, pretensioned by 6 high tensile steel cables at the 6 verticles of each octahedron.

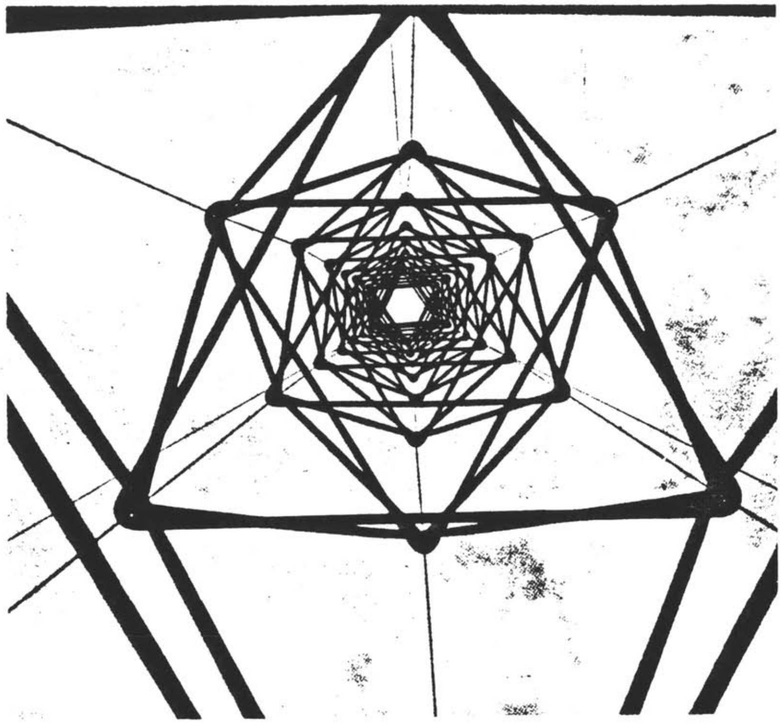
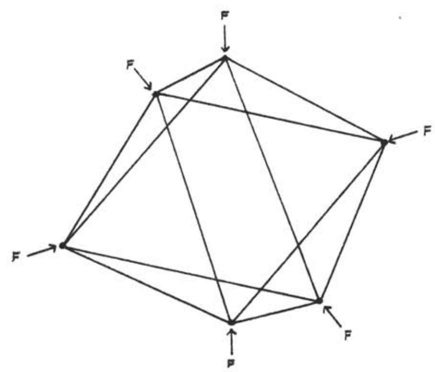


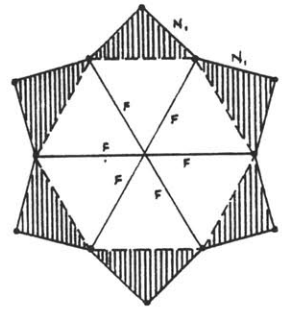
Image Method for Regular Octahedron:
--assuming a regular octahedronal frame constrained by 6 external forces F by symmetry: $-F = 2\sqrt{2}N$, $N_1 = -F\cdot\sqrt{2}/4$ where N_1 is the component of F_1 along the octahedron edges.
--considering figure(A), taking into account as segments, the 6 forces F acting at the apexes we have:

F = 8+6=14 P=6
E = 12+6=18 E=18
V = 6+0=6 C=13

(Because we are dealing with a planar image, one apex is missing wherefrom C=F-1).

--The construction is given in Figure (B) where we note the inversion of F. Metrically,
$N_1 = \sqrt{2}/4 \cdot -F$
--The shaded areas where W is the internal work, giving $W/2 = 3N_1^2$, are immediately readable, the angle of 90 degrees being kept in the transformation.





Before presenting a more graphic study of my own of the spatial automorphisms (a generality of the dual transformation of the regular Platonic "solids") I would like to discuss the notion of structural scaling in automorphic network (that is, networks which retain an equivalent incidence at each of it's joints or branch points).

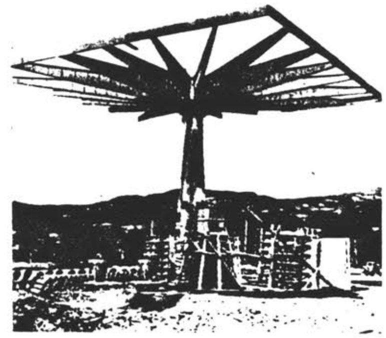
Consider the tree structure already known to the builders designers of the gothic cathedrals. It emerges as a result of having to perform the function of connecting a large number of points to one concentration point. In the tree structure by Frei Otto, the roof is conceived as a surface comprising a large number of points of application of forces in space, which have to be transmitted to fixed points. It is characterized by short buckling lengths of the compression members and small bending movements in the roof slab. The buckling lengths are shortest at the top and increase lower down where the members become thicker. (It is advantageous to keep the free buckling length of compressive members as short as possible, as a large proportion of the required cross-sectional area of the material serves merely to prevent buckling.) Only in "airborne" pneumatic structures is each individual elementary area of the roof surface supported directly by the air. Thus tree structures could theoretically be regarded as a transition from solid support at a small number of points into gaseous support at an infinitely large number of points.

Buckminster Fuller expresses the same notion with an unlimited frequency subdivision of geodesic tensegrities.[35]

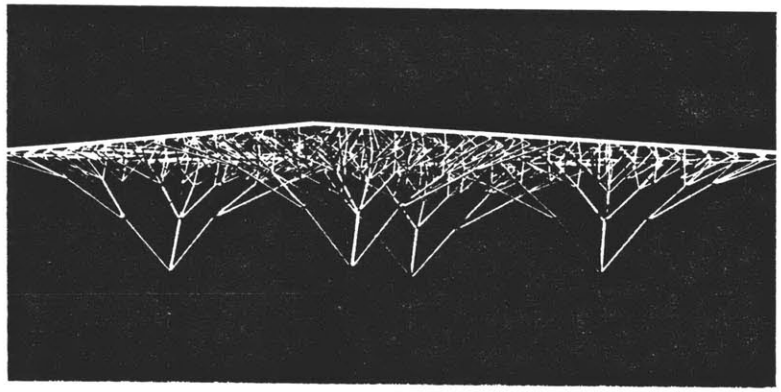
[35] Fuller. R.B Synergetics. sec. 740.00. MacMillan, New York, 1975.

Salisbury Cathedral

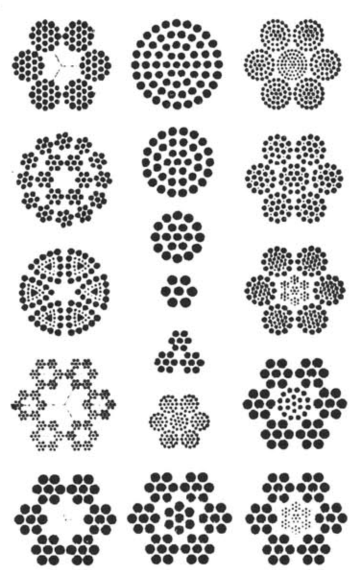
Tree Type Structural Bay in Turin by Nervi



Tree Structure. Demonstrating the "Continuity" between column and platform



Cross sections of steel cable--cables have the highest strengths of all commercially available structural materials and in general are more economical for tension members than for example solid round steel bars. This is due to the high surface to volume ratio. Compare the configurations with diagram of a photon (dissolution of a point) for Huygens.



It is obvious that each of the seemingly 'solid' compression struts in tensegrity island complexes could be replaced by miniature tensegrity masts. There is nothing to keep us from doing this but technological techniques for operating at microlevels. Every time we can see a separate strut and can devise a means for making a tensegrity strut of that overall size, we can substitute it for the previously 'solid' strut. By such a process of progressive substitutions in diminishing order of sizes, leading eventually via sub-sub-sub-miniaturize tensegrities to the discovery of the last remaining stage of the seemingly 'solid' struts, we find that there is a minimum 'solid-state' struts column diameter, which corresponds exactly with two diameters of the atoms of which it is constructed. And this is perfectly compatible, because discontinuity characterizes the structuring of the atoms. The atom is tensegrity, and there are no 'solids' left in the entire structural system."

So, as far as tensegrities go, the higher the degree of scaling the system accomodates, the greater the proportion of structure which is invested in tensional components. This is advantageous in that tensional components are unlimited in length in proportion to their cross-section diameter-to-length ratios. As we increase the frequency of each tension member, it is parted into a plurality of fibers, each of whose strength is multiplied up to hundreds and thousandfold that of the originally considered unit section (because of the increased surface to mass ratio). Most importantly, as the degree of scaling in a structure increases, the relative overall weight (density) rapidly diminishes in contrasts to all previous structural experience, increasing return is realized in the direction of the structures increasing dimension.

F.J. Dyson in his essay "The Search for Extra-Terestrial Technology"[36] establishes the upper limit for scaling structures. He suggests that given a very large and conspicuous technology, one of the things it might be interested (or find necessary) to do would be to build large, rigid structures (for reasons of colonization or industrilization) within "reasonable" proximity to its mother planet. The physical limit could be approximated by the available mass and energy.[37]

The formula for maximum size (D) of such structures is

$$(gD)^2 = (gR)\ (T/\rho) \quad g = GM/R^2$$

where

- g - local gravitational accelleration
- T - tensile or compressive stress in structural material
- ρ - density of the structural material

contrast this with the formula for structures built on earth,

$$gD_E = T/\rho$$

For good structural materials such as steel and fiberglass,

$$T/\rho \approx 10^9\ (cm^2\ \ sec^2)$$

so, $D_E = 10$ km which is correct for the highest mountains on earth.

For a structure in a low orbit about earth

$$D = \sqrt{D_E R} \quad 300\ \text{km}$$

and for a structure in an orbit around the sun at about the earth's distance

[36] Technology as opposed to intelligence, as technology would be the physical manifestation of other intelligence from Perspectives in Modern Physics: Essays in Honor of Hans A. Bethe. Ed., Marshak, R.E. Interscience, New York, 1966.

[37] In the case of the human species we have available a mass power ratio of 15 kg kw (mass of the earth sun power); the present industrial base uses an energy of about $3x10^{14}$ erg sec operating on a mass of 10^{10} gm. Thus in principle, there is available to us a growth by a factor of 10^{12} without going beyond our solar system.

$g = GM/R^2 = 1 \quad R = 1.5 \times 10^{13}$

we calculate $D \approx 10^6$ km

Given these as upper units, suppose we try to make a structure of size D as light as possible with building blocks of a given property. Dyson proposes an example using steel rods of 100 cm length, 1 cm diameter. Now take 12 of these rods to form an octahedron of size

$D = 100$ cm

$M = 10$ kg

Next, put 100 octahedra together to form the next size unit rod and stick 12 of these together to make a bigger octahedron, where

$D_2 = 10^4$ cm

$M_2 = 10^4$ kg

and so on.

Each generation multiplies the linear size by a factor $q = 100$ and the mass by a factor $q^r = 900$, say 1000, allowing for the joints.

So the Nth generation has:

A linear size	$D_n = q^n$ cm
A mass	$M_n = 10\ (q^r)^n$ gm
A volume	$V_n = q^{3n}$ cm^3
And a density	$n = y = 10(q^{-2r})^n$ g cm^3

We have $r = y = 10$. Hence at each generation

$$M = 10D^{3/2} \qquad = 10D^{-3/2}$$

so things get lighter as it get bigger!

For the 300 km structure near earth, the mass would be 10^{12} gm.

$M = 10D^{3\ 2}$ is the minimum for these structures. If one wants to intercept sunlight on the areea D^2, the structural voids must be filled with membrane. So the mass will be proportional to D^2 rather than $D^{3/2}$.

The example here uses a generating configuration based on the octahedron, but whatever generator is used, it remains true that minimum mass of a rigid framework, achieved by strict scaling, varies roughly as the three halves power of the linear size.

PERIODIC SPATIAL SYSTEMS

I discussed previously the principle of duality in regards to Le Ricolais' 'image method' which demonstrated two important attributes of the correspondent or dual configurations. The first is that for a given form, the dual is a graphic representation of it's forces. Secondly, the dual relationship provides a means of jumping dimensions -- that is, moving from planar to spatial configurations. Further, it appears to be the mechanism for transformation between all periodic spatial systems. That is, the 'dual' relationship is not one (as the name seems to imply) which exists between a discrete pair of configurations, but one which generates an intertransformable continuity of all configurations (this is known to be the case among the regular (platonic) and semi-regular (archimedian configurations).

The 'dual relationship' then merely designates two resultant states of an operation. There have been several different descriptive mechanisms ascribed to this relationship.

Arthur Loeb demonstrates duality by truncation-stellation. That is if two polyhedra are each others duals, then the stellation of one is dual to the truncation of the other.[38] Duality may also be achieved by screw displacement. In this mechanism, the edge length is maintained but is rotated by 90°. Haresh LaLvani postualtes a third mechanism which he designates as implosion-explosion.[39] This is a continuous dynamic transformation from one configuration to its dual moving through an 'equilibrium' configuration and a multiplicity of 'transpolyhedra.'

[38]Loeb, Arthur L. Space Structures Their Harmony and Counterpoint. Addison-Wesley Publishing Company, Reading, MA. 1976 Truncation replaces a vertex by a face by 'subtracting a vertex' and stellation replaces a face by a vertex by 'adding a face."

[39]Lalvani, Haresh Transpolyhedra Haresh Lalvan, New York, 1977

Given the various descriptions of the mechanism, there remains an invariant reciprocity of relationships between the two dual sets.

1. Number of faces of one is the same as the number of vertexes of the other, while the number of edges remain the same.
2. Sum of the dihedral angle of one and the central angle of the other is always 180^0.
3. The product of the insphere radius of one and the circumsphere radius of the other always equals the product of the two intersphere radii.

And in even more general terms, the mechanism is one which involves the replacement of all or part of the elements of a given dimensionality by elements of a different dimensionality (eg. vertices interchange with faces).

Given the dimensional jump outlined by Le Ricolais as a clue, if one initiates an automorjphism to each of the 'faces' (that is, connecting the midpoints of each edge) of a regular polygon we have immediately the description of the intersection of the two dual figures. Which is a new semiregular polyhedron. The compound figure (of two duals) inherently describes a vertex figure. This vertex figure is the dual of the figure of intersection. Thus, any regular or semiregular polyhedron will not only generate its own dual but implies also the next pair of duals in the hiearchy. For example, the tetrahedron generates itself as its own dual. The compound figure (of two tetrahedrons) generates the octahedron at the intersection and the cube as vertex figure. The compound of the outahedron and the cube generate the cuboctahedron at the intersection and the rhombic dodecahedron at the apices.

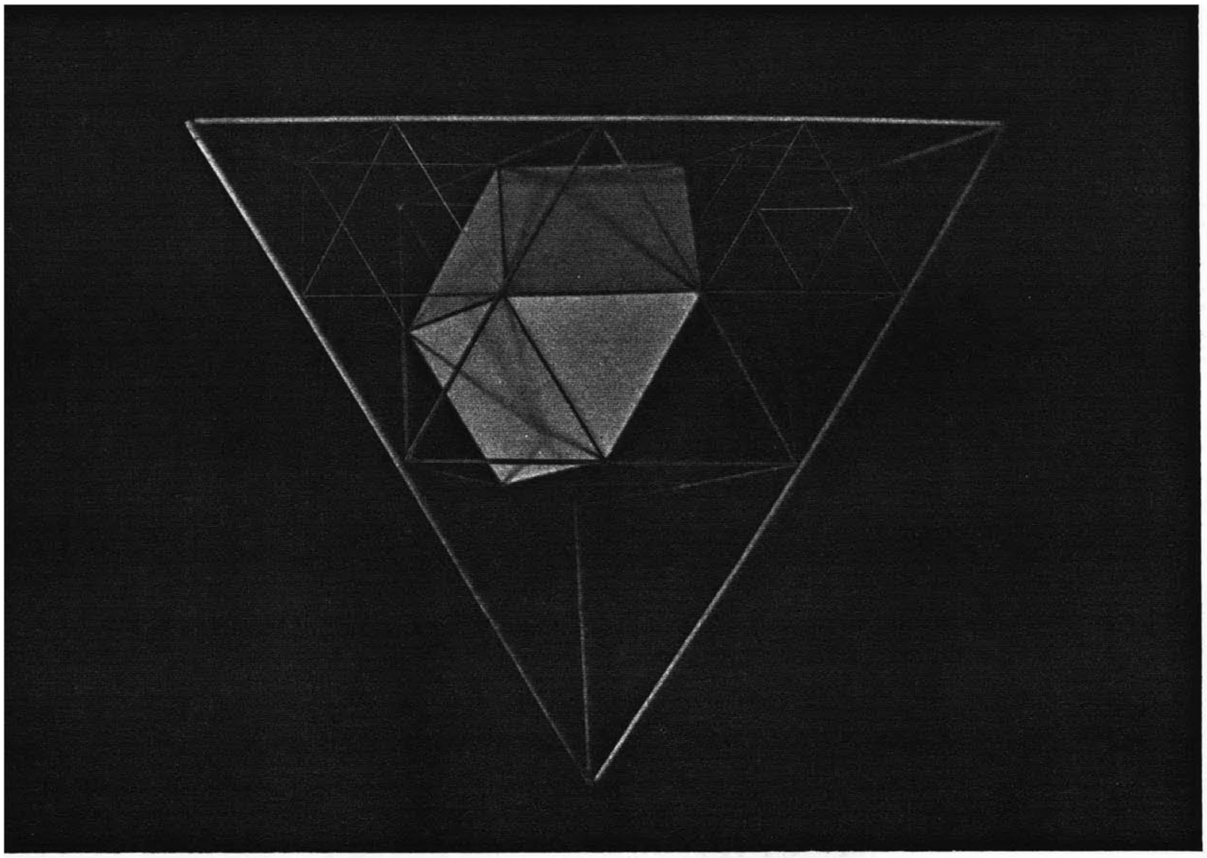
The automorphism of the faces not only depicts the dual configuration but its repeated application serves as a vehicle for movement among the continlia of periodic spatial systems.

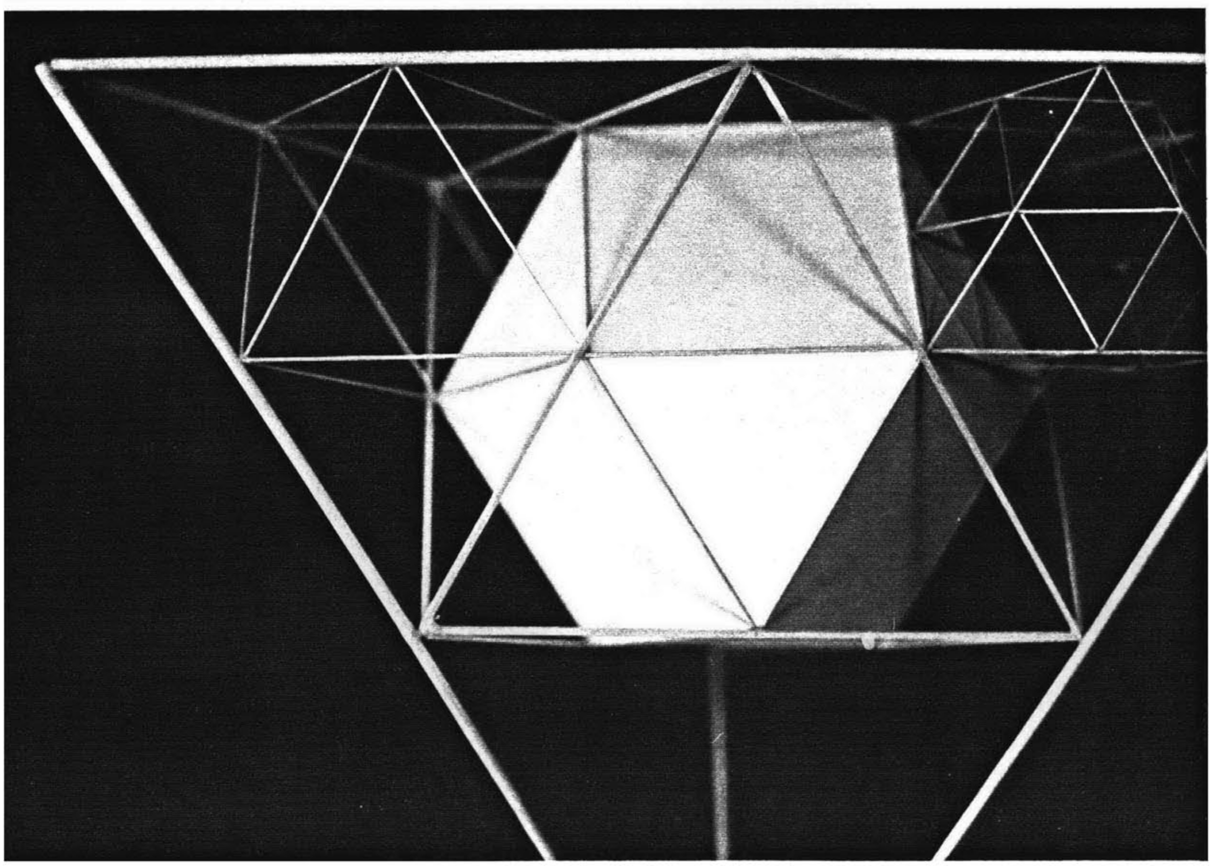
An automorphism of the faces of regular polyhedron may have different transformational implications than the quantized steps through the configurations. The initial configuration may be thought of as 'stable,' and as I mentioned before, stability involves self replication. The configuration may stabilize by the growth of smaller versions of itself along some or all of its inherent axis.

In the simplest case of the tetrahedron, first level growth is depicted by placing half-size (referring to edge length) tetrahedra on each of the four faces. Second level growth by placing quarter-size tetrahedra on each 'open' face of each of the first level tetrahedra. This may be continued ad infinitum but notice that the growth proceeds to certain limits, which in this case is inscribed by a cube. Similar growth sequences for the octahedron and the cuboctahedron are illustrated in the following photographs, and are indicative of the generality of the process as it applies to all regular and semi-regular configurations. It should be noted that there are three sequences shown for the cuboctahedron each growing to a different limit figure depending on which axis are 'open' to growth.

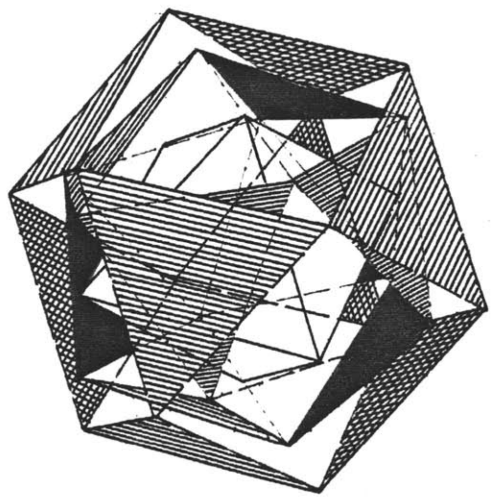
The objects thus defined seem to be, so to say, intermediate in complexity between inorganic patterns like those of crystals and the more varied intricacies of organic molecules and structures. Analogous to the game of life, these structures may shed some light on how much information is necessary given the simple automorphic mechanism, to describe the seemingly enormously elaborate structures of living objects.

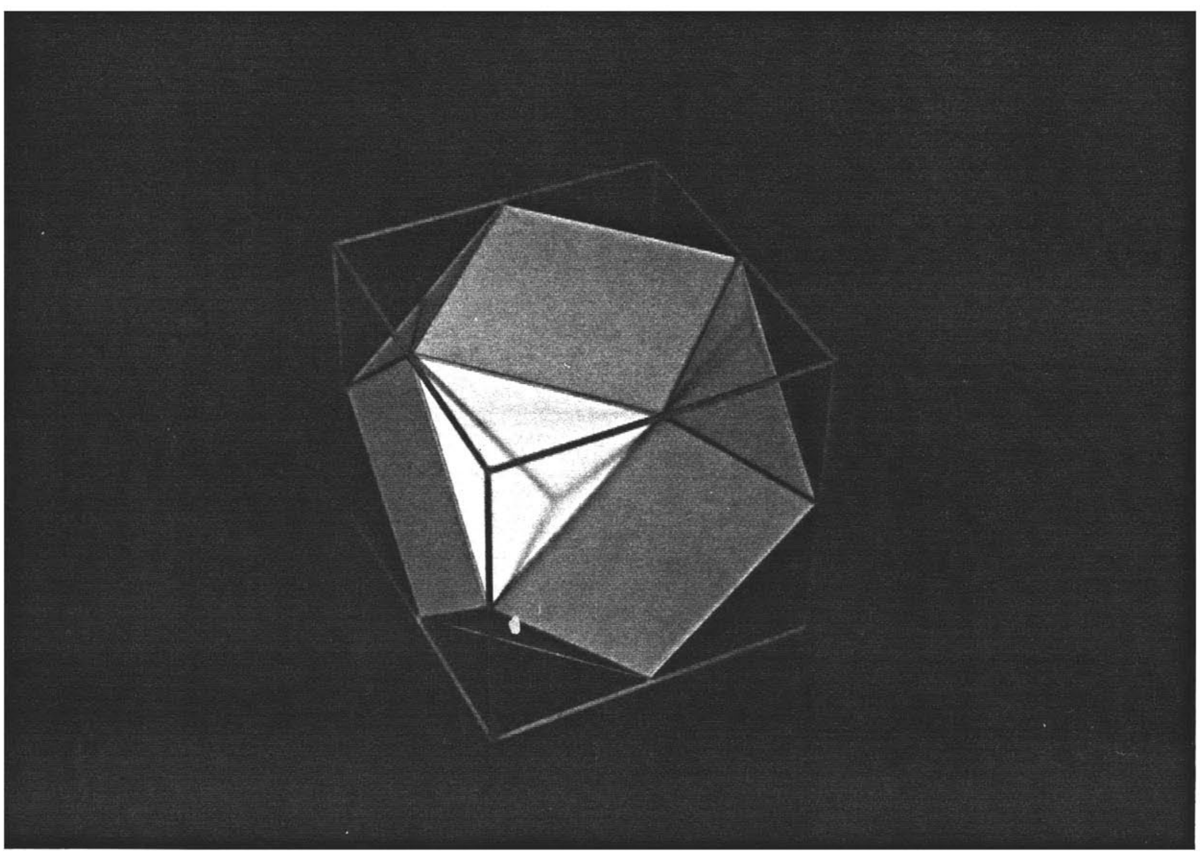
Recursive subdivision of the triangular faces of the tetrahedron generates the octahedron which in turn generates the cuboctahedron. The four corner tetrahedra may also be recursively subdivided in the same sequence. This is the general mechanism enabling the intertransformability and growth of all spatial configurations.



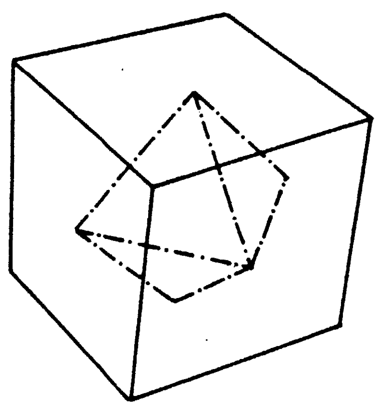


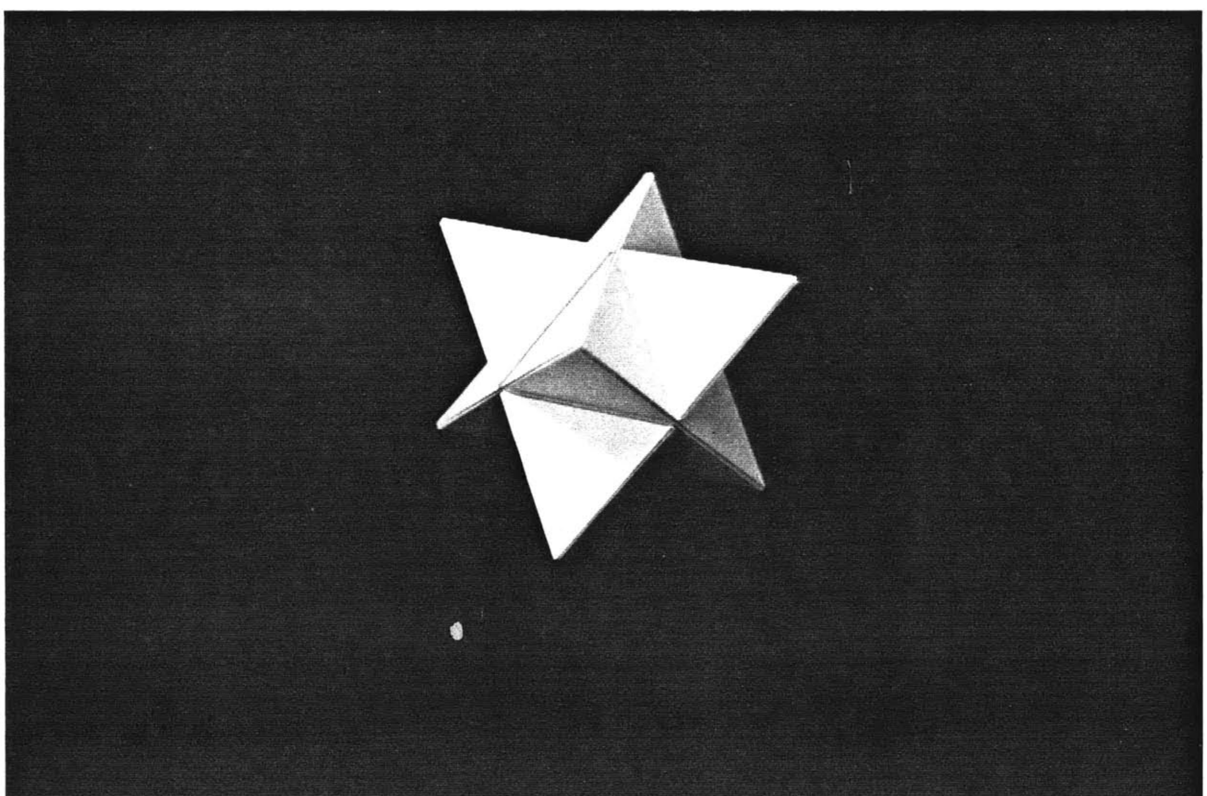
Recursive subdivision of the square faces of the cube gives rise to the cuboctahedron.



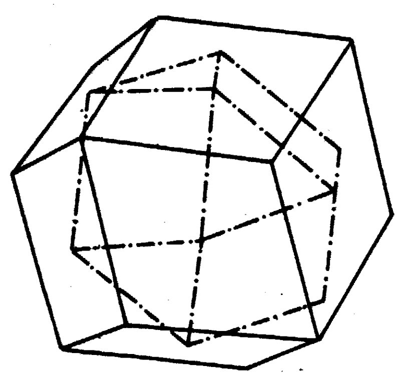


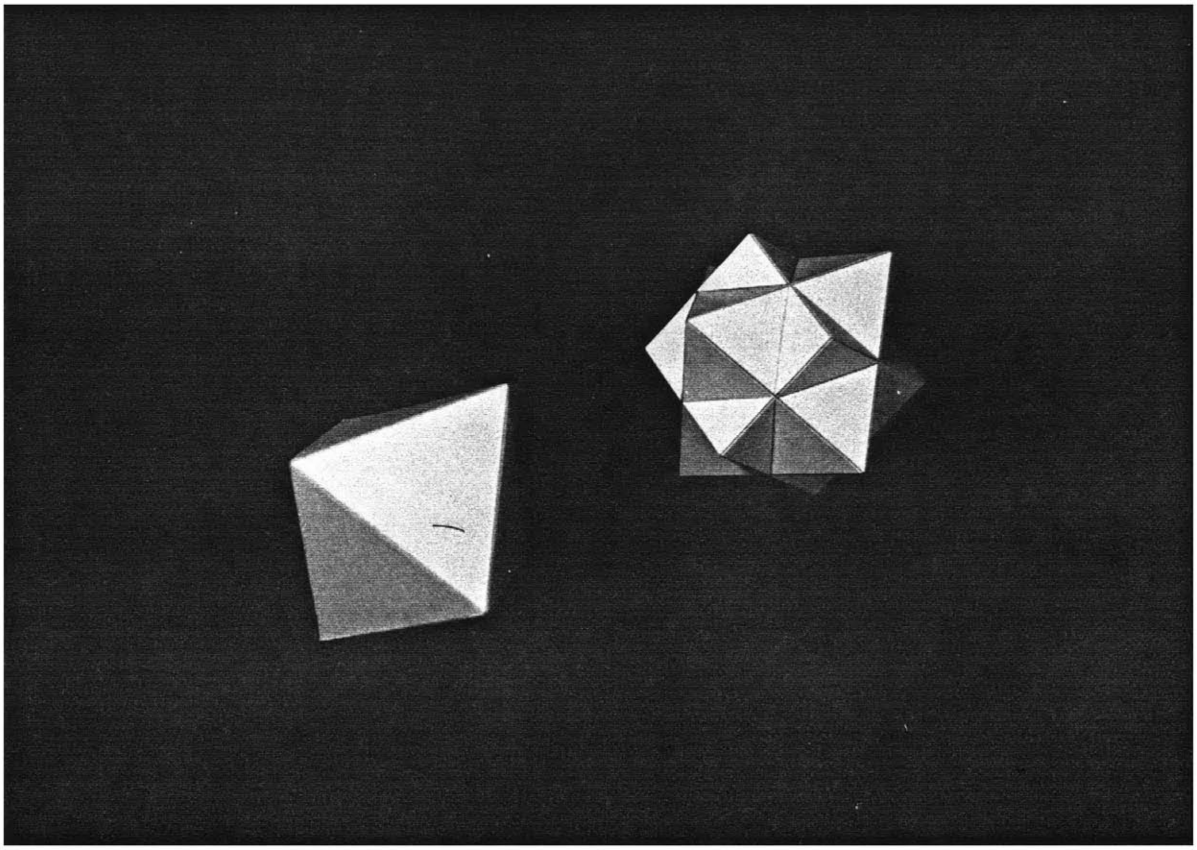
Tetrahedron is it's own dual. The compound
figure generates the octahedron at the
intersection and the cube as a vertex figure.

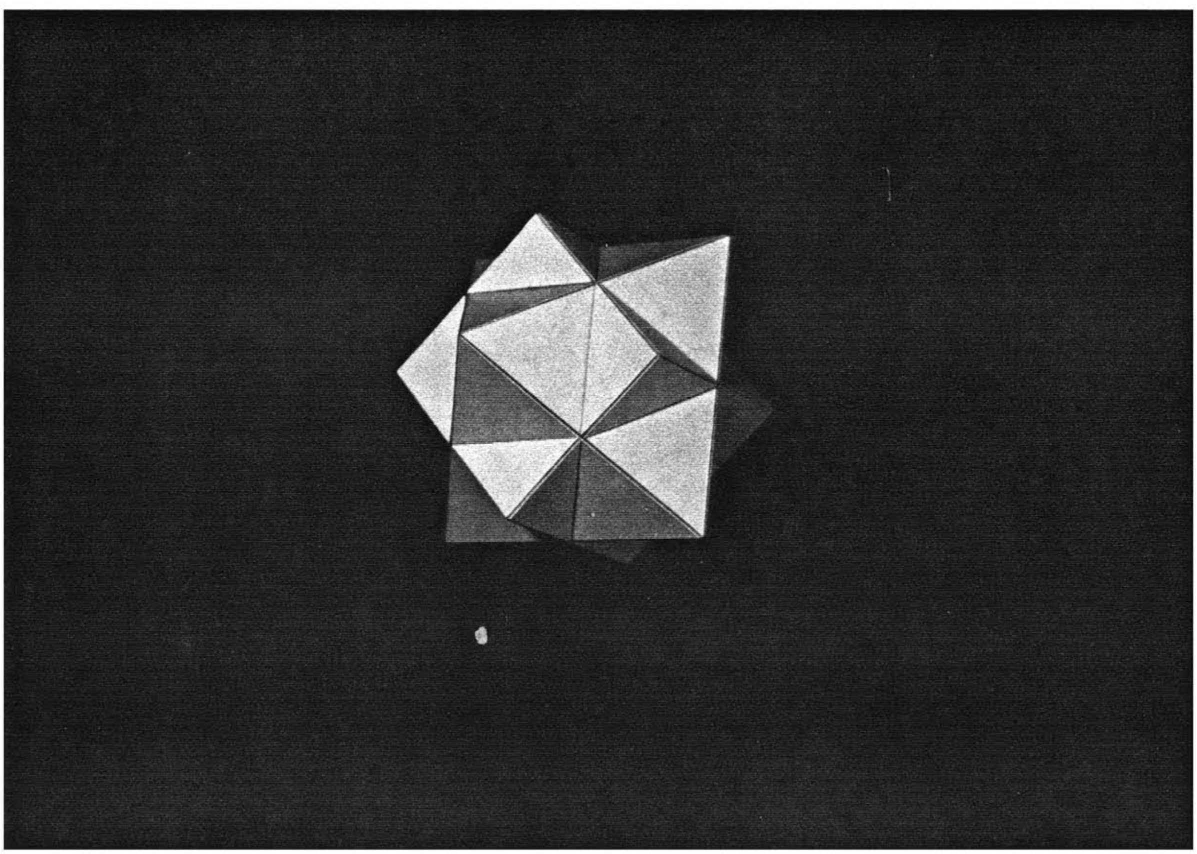




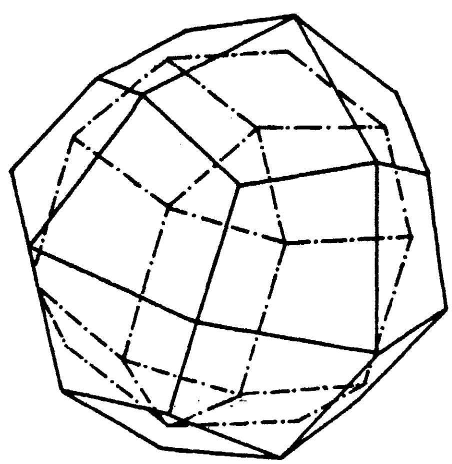
Octahedron is the dual of the cube. Note the characteristic automorphism of each of the 'faces' of the octahedron involved in generating the dual. The intersection figure here is the cuboctahedron and the vertex figure is the rhombic dodecahedra. These two figures are also dual.

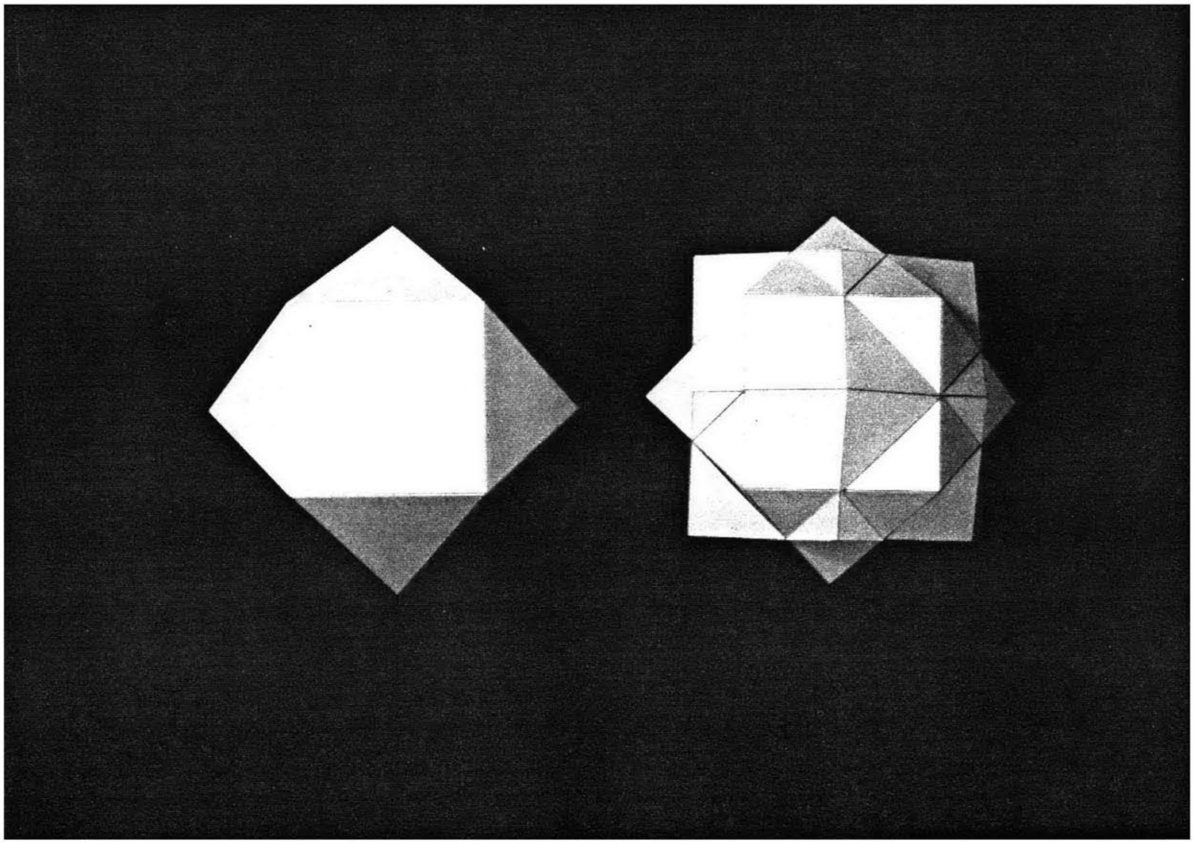


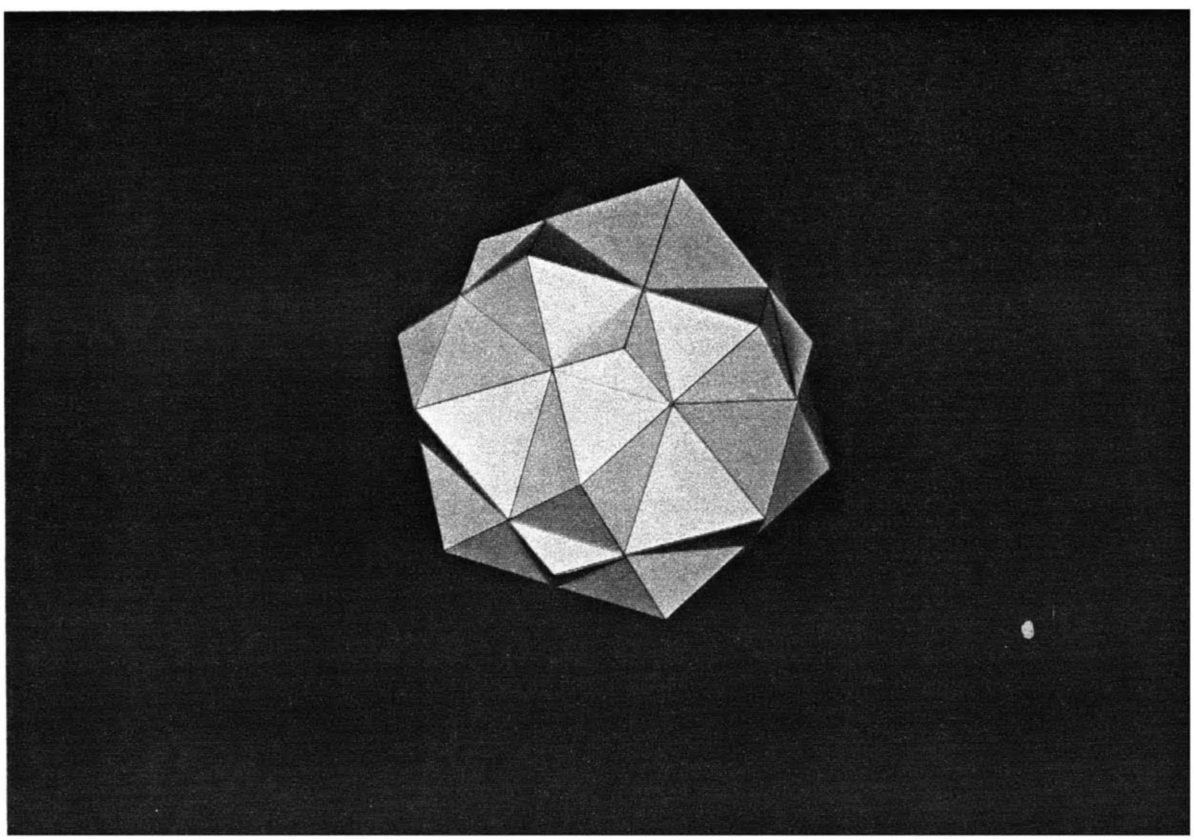


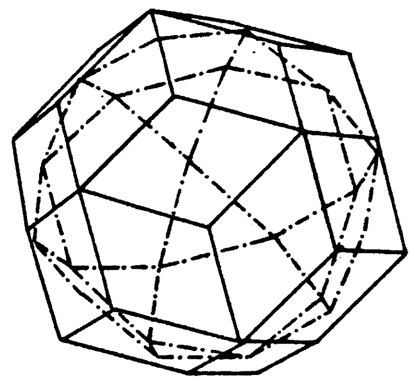


The compound of the cuboctahedron and the rhombic dodecahedron produces a rhombicuboctahedron at the intersection and a trapezoidal icosatetrahedron at the apices.

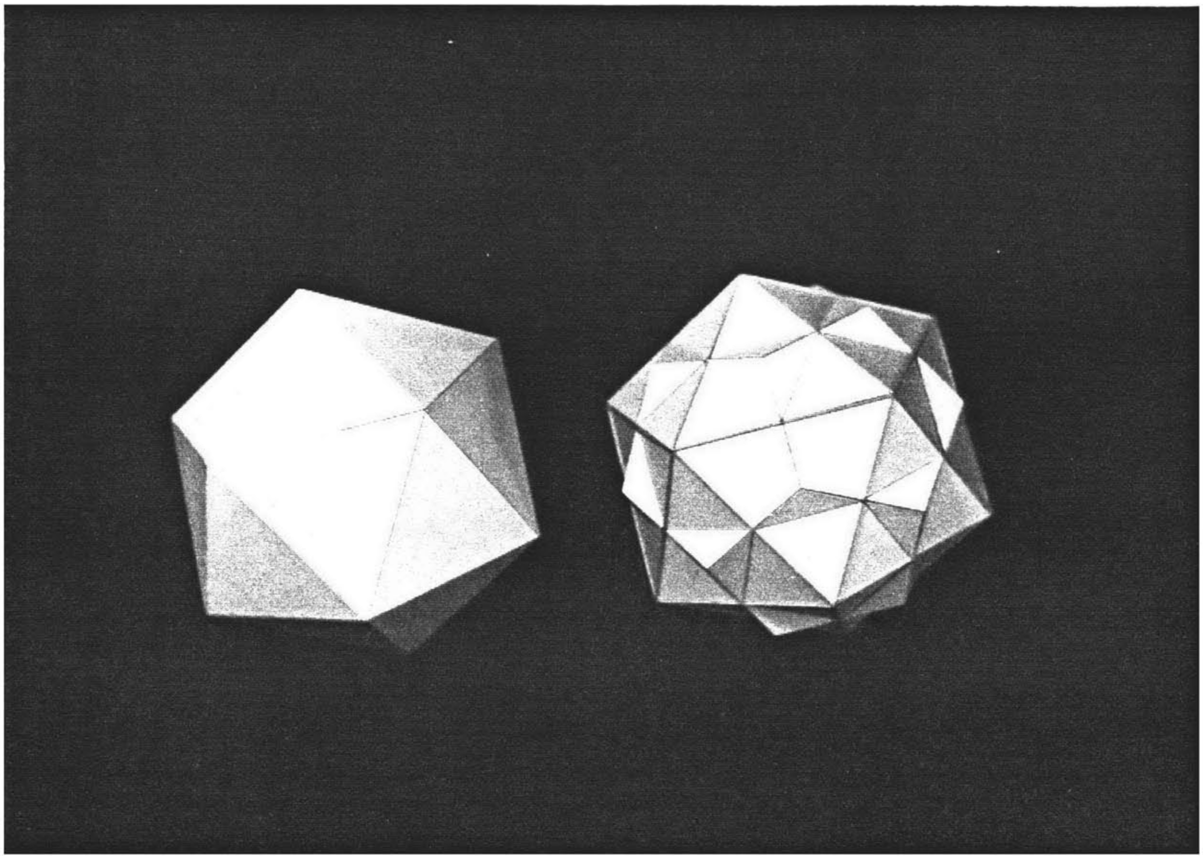




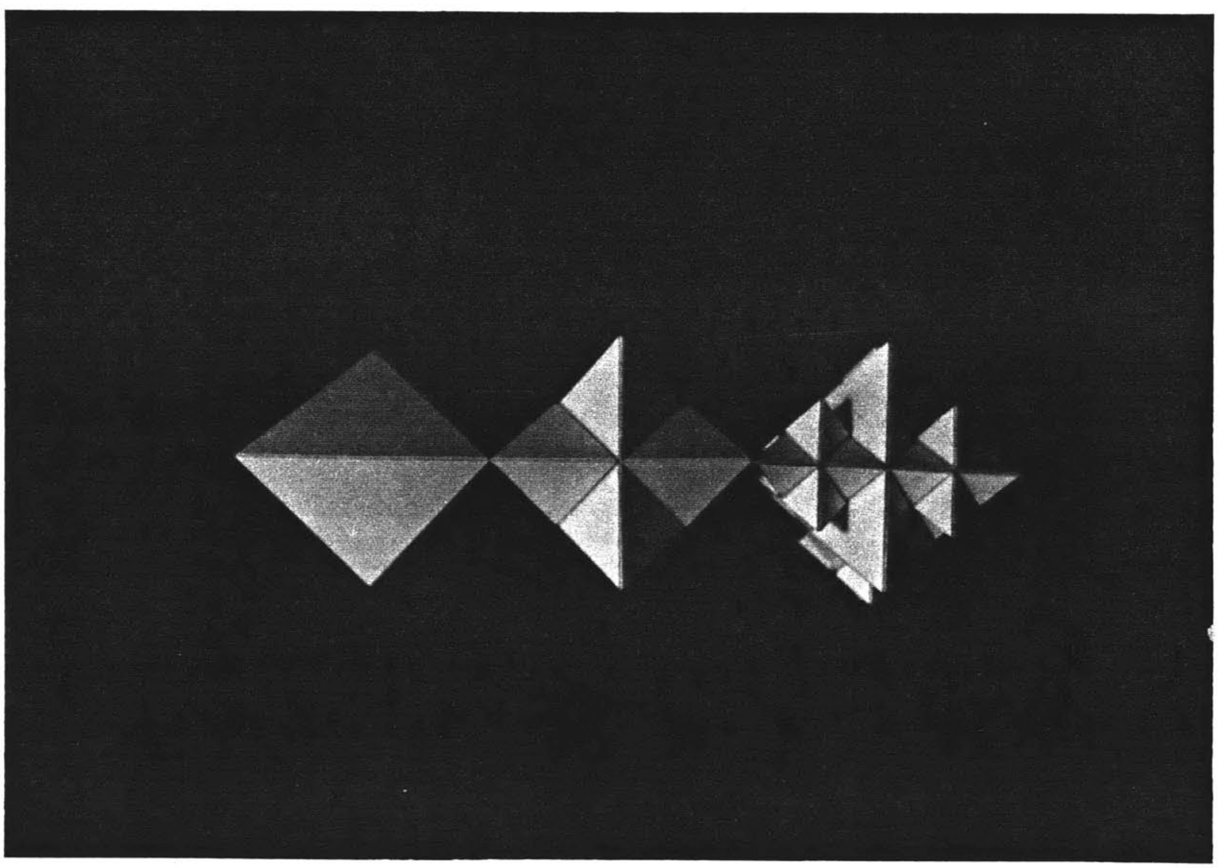


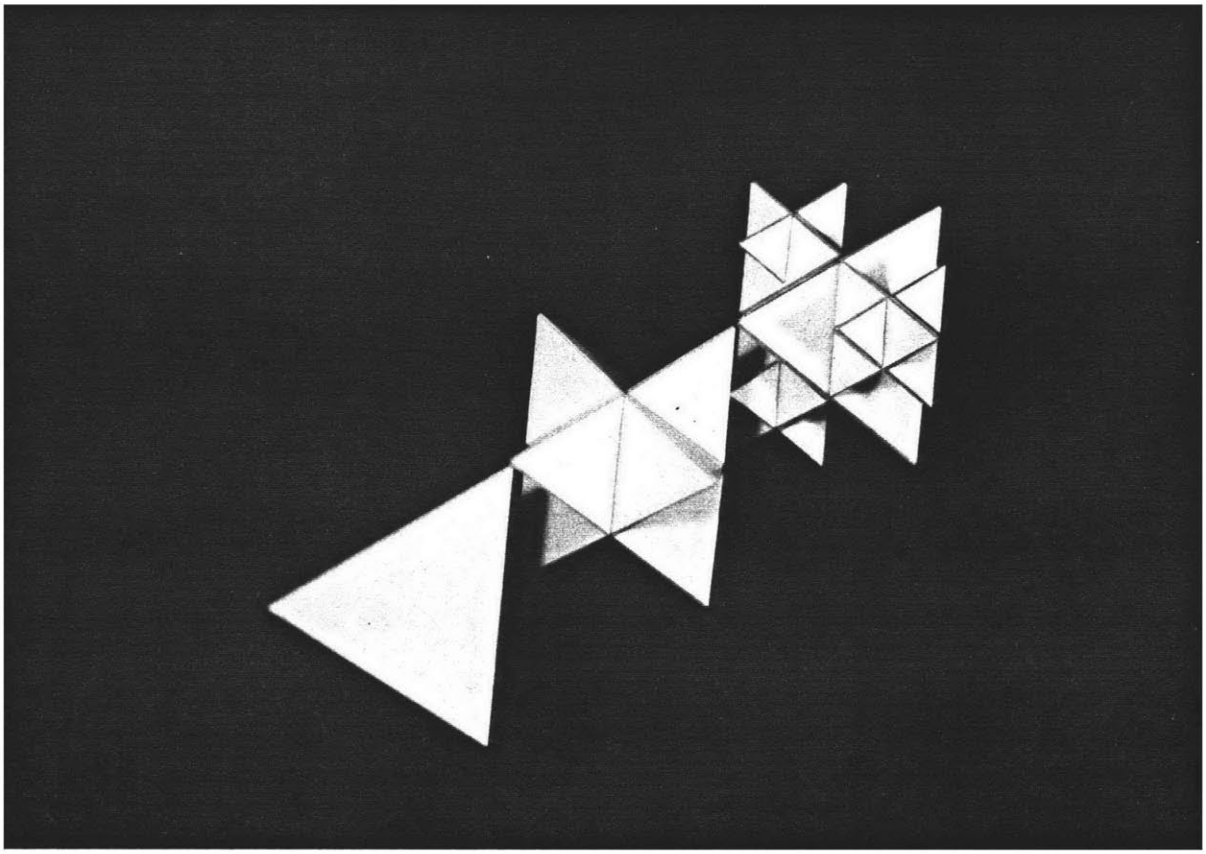


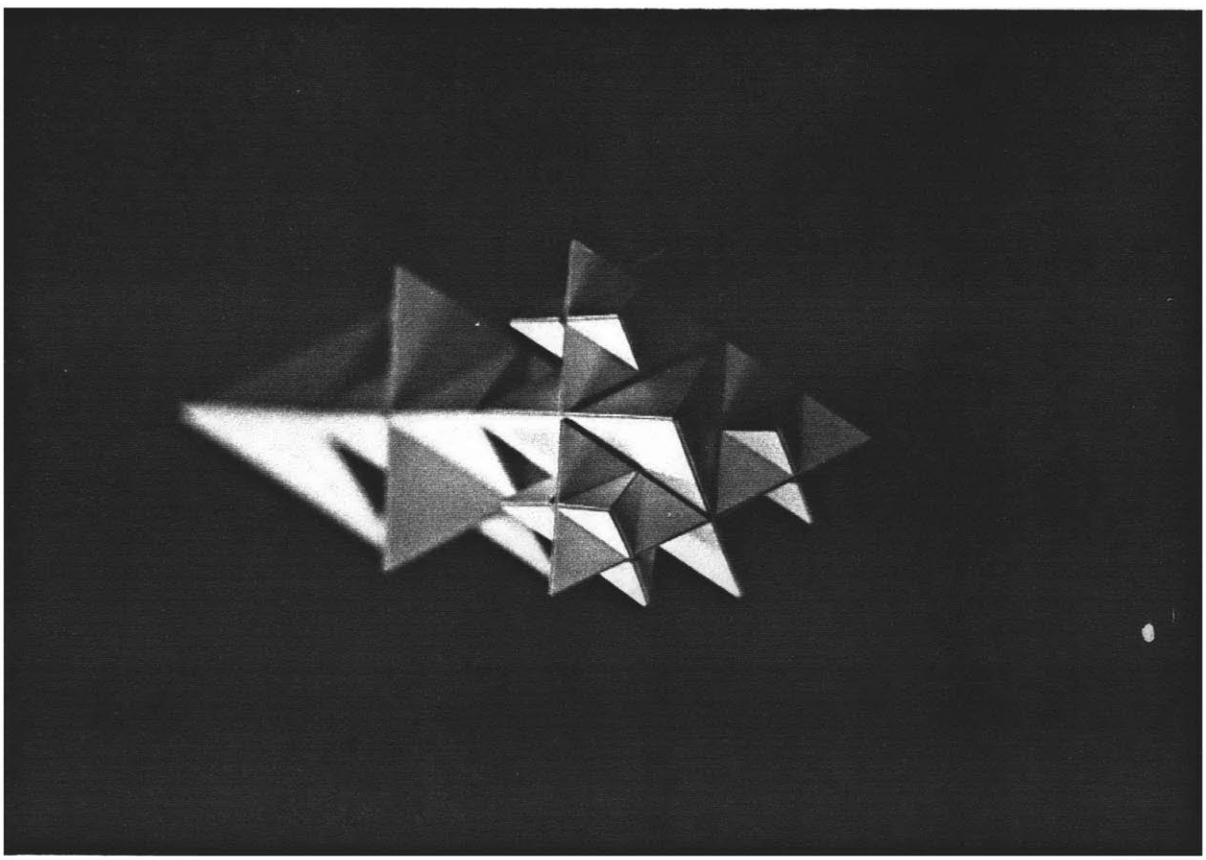
The compound of the ilosahedron and the
pentagonal dodecahedron produces a
icosadodelahedron at the intersection and a
rhombic triacontahedron as a vertex figure.



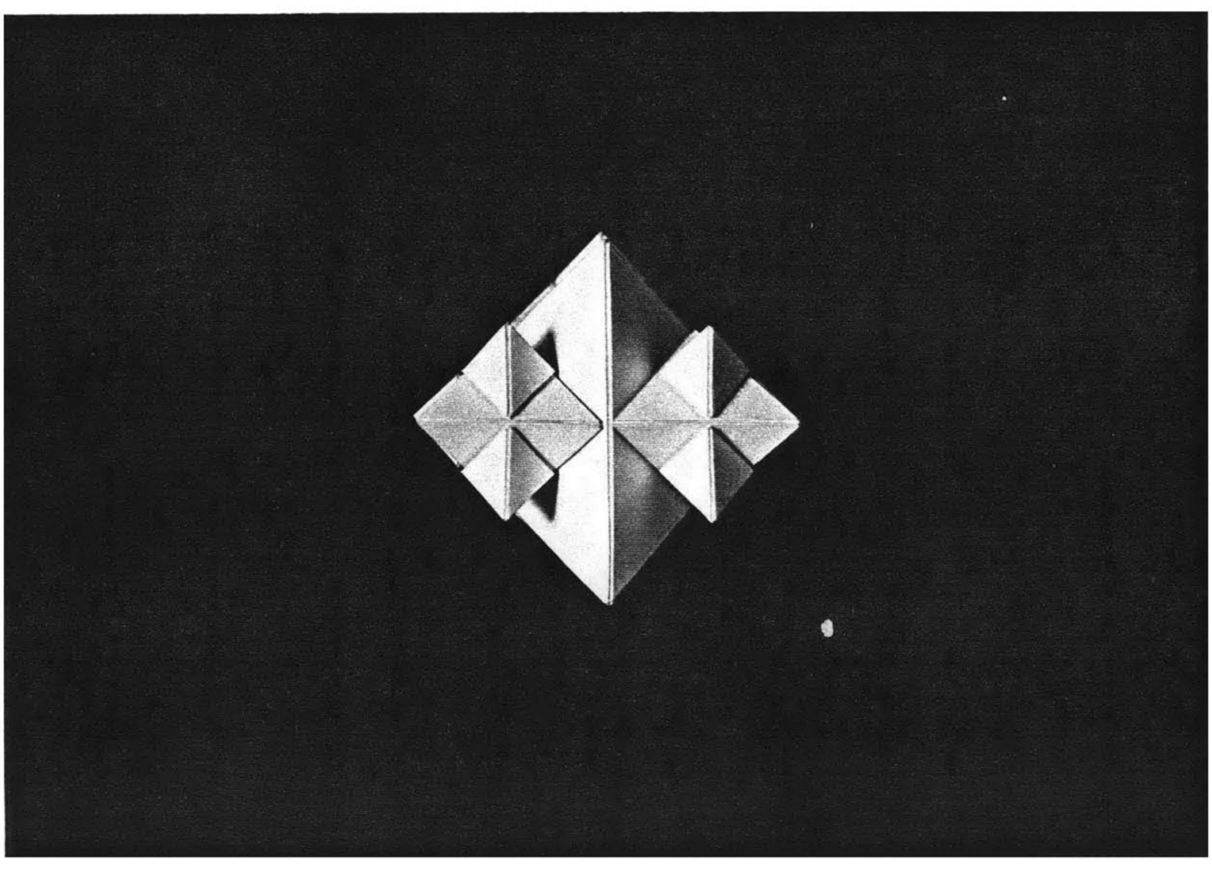
Tetrahedral growth sequence moves to a cubic limit figure. Edge lengths of successive generations occur as a $^1/_2$ progression.

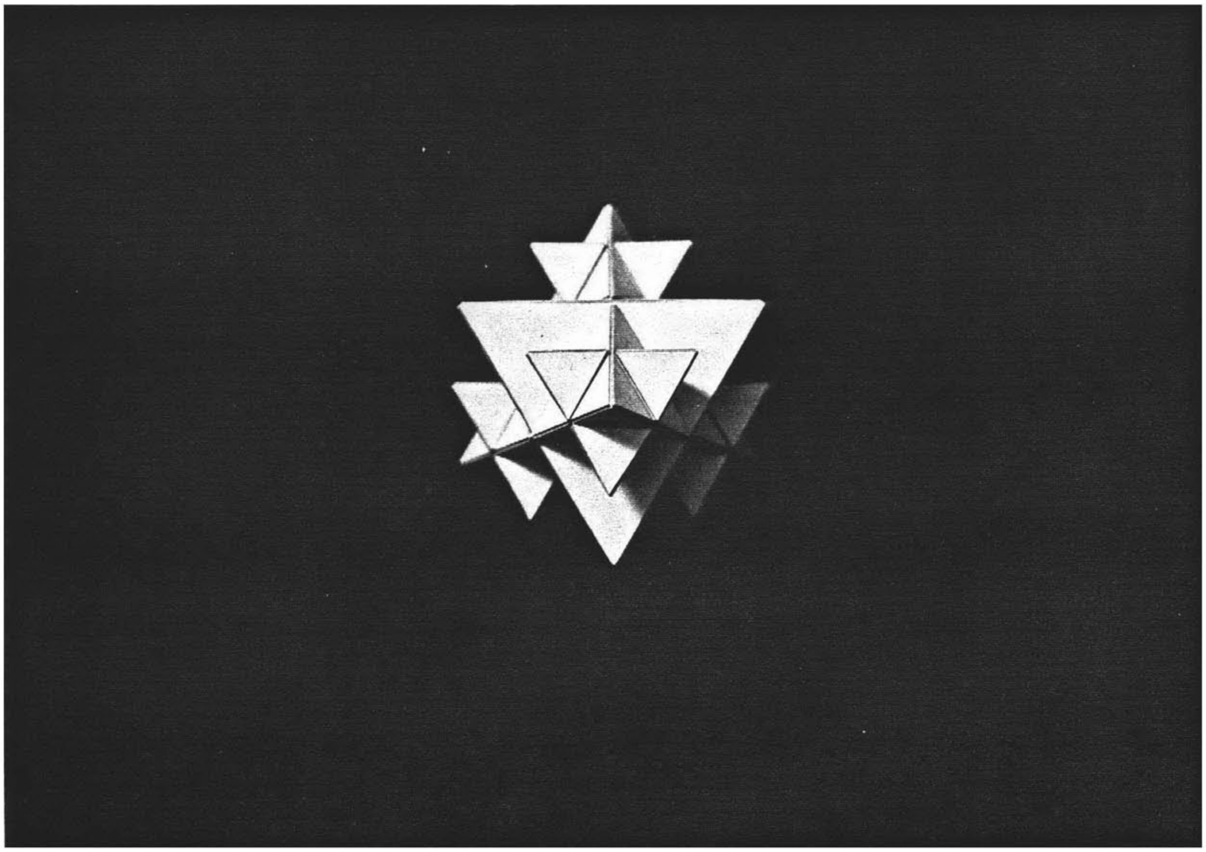


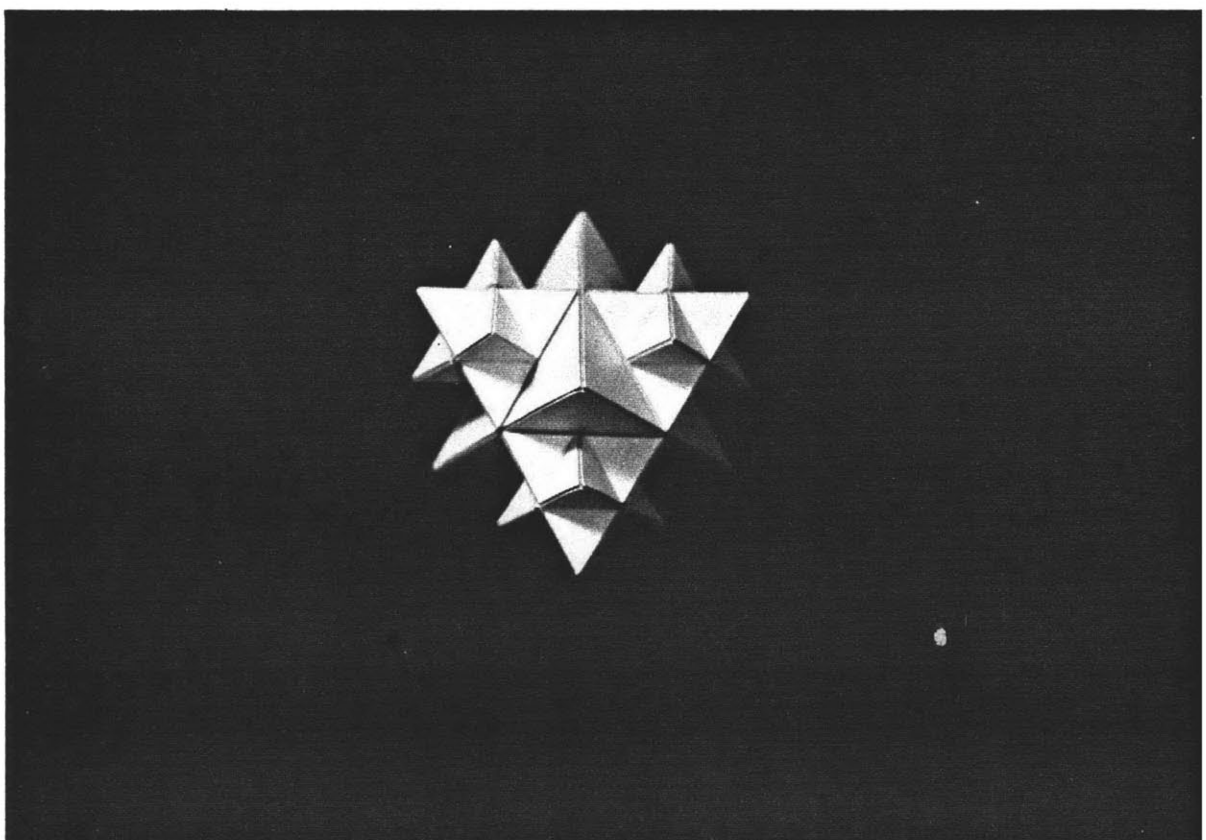




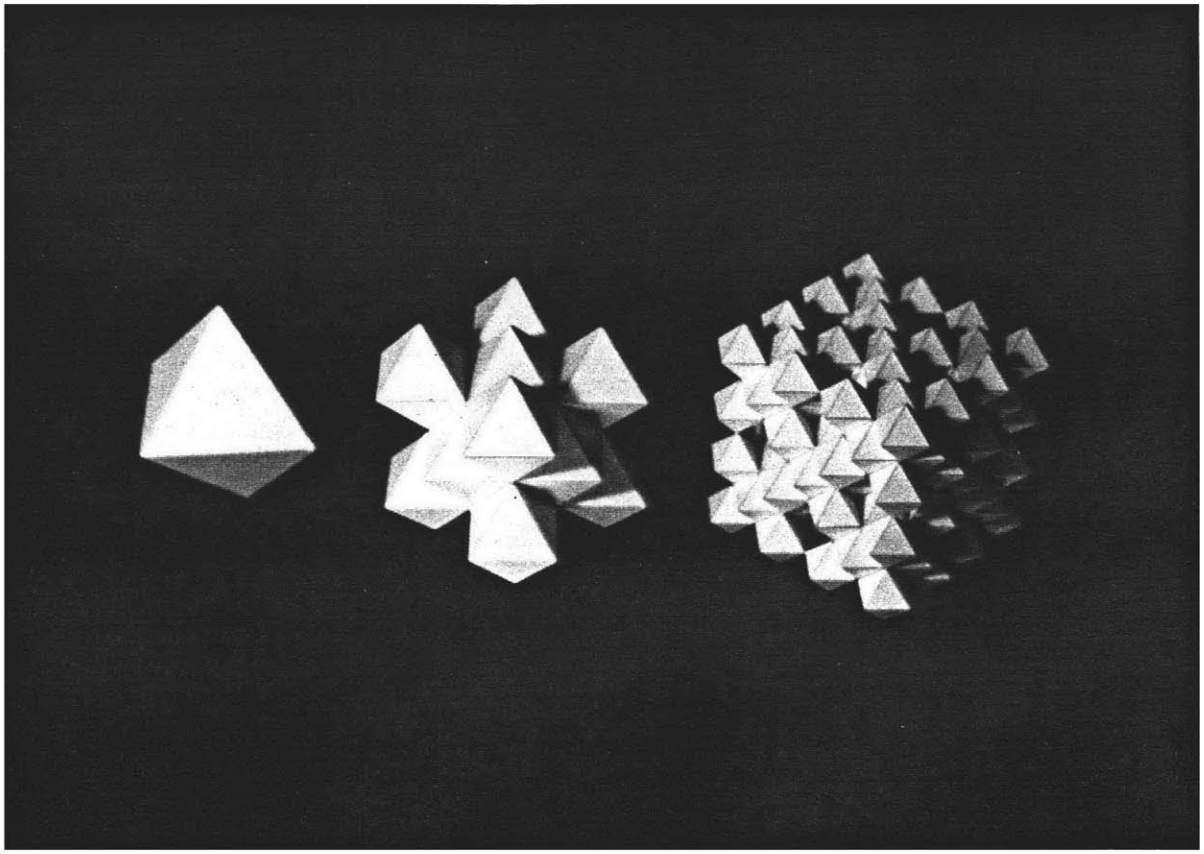
3 axial views (vertex-axis, edge-axis, face-axis)
of 2nd level tetrahedral growth structure.

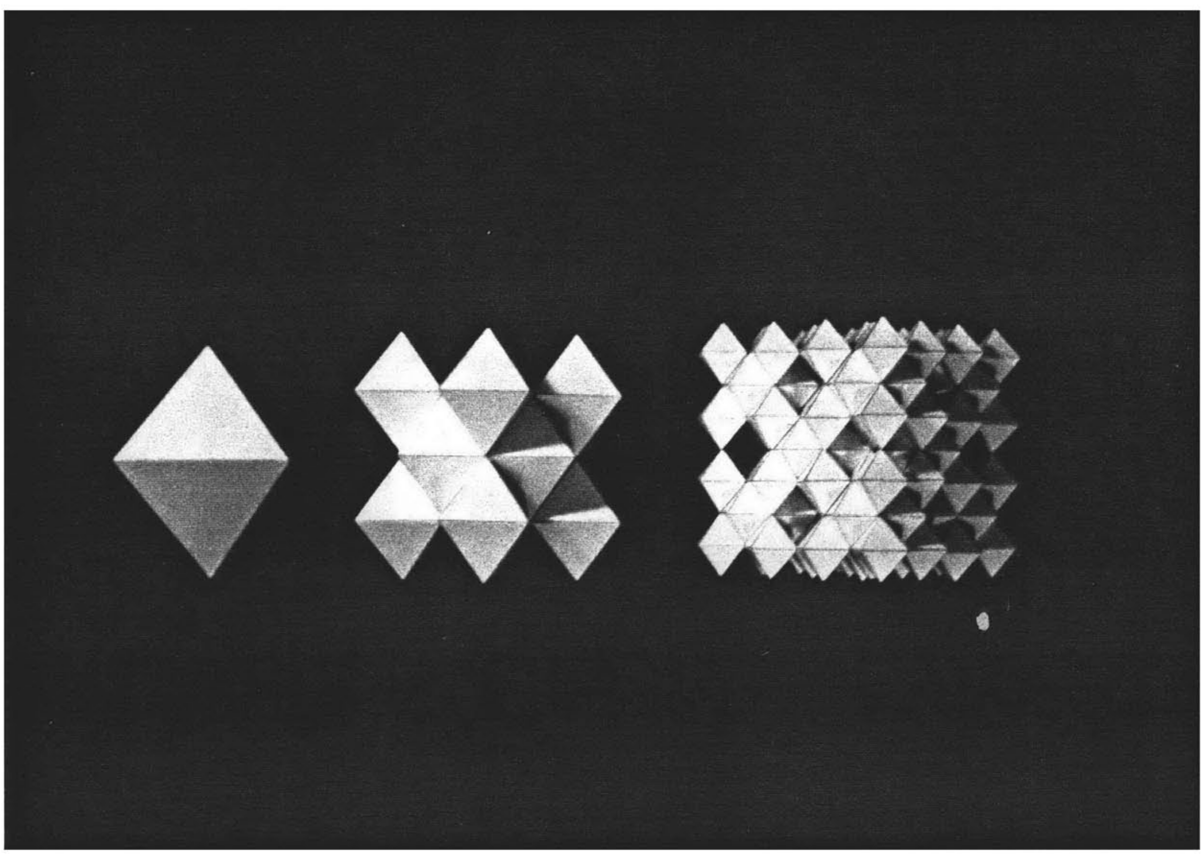




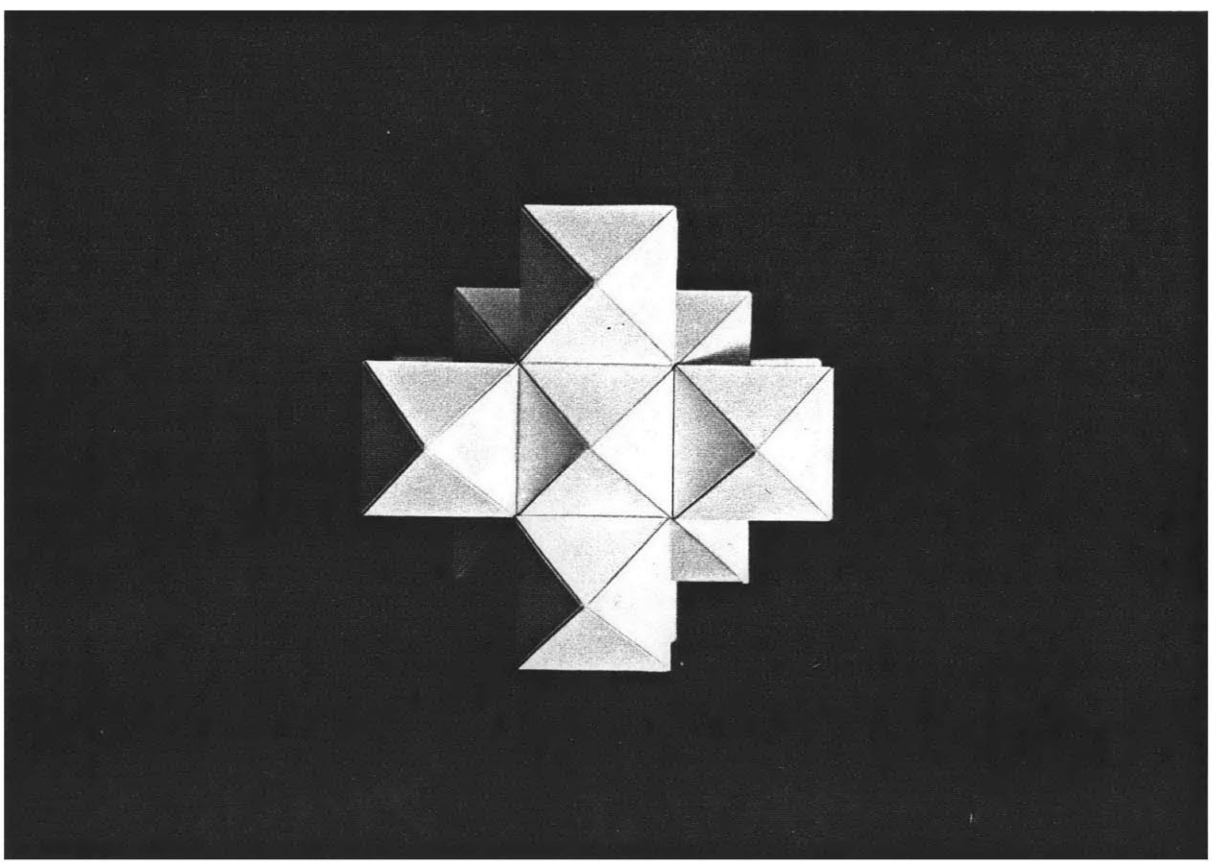


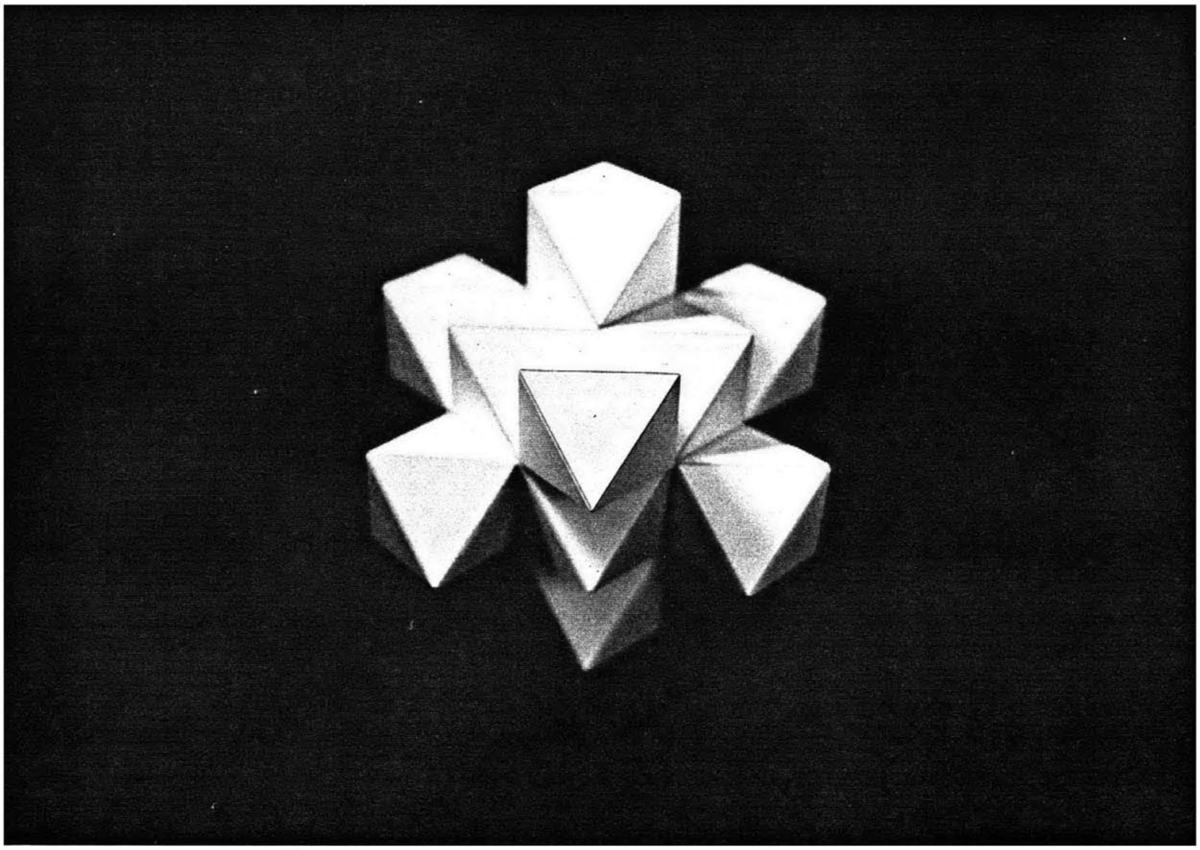
2 axial views of automorphic octahedral growth
sequence progressing to a cubic limit figure.
Edge lengths occur in a ½ progression.

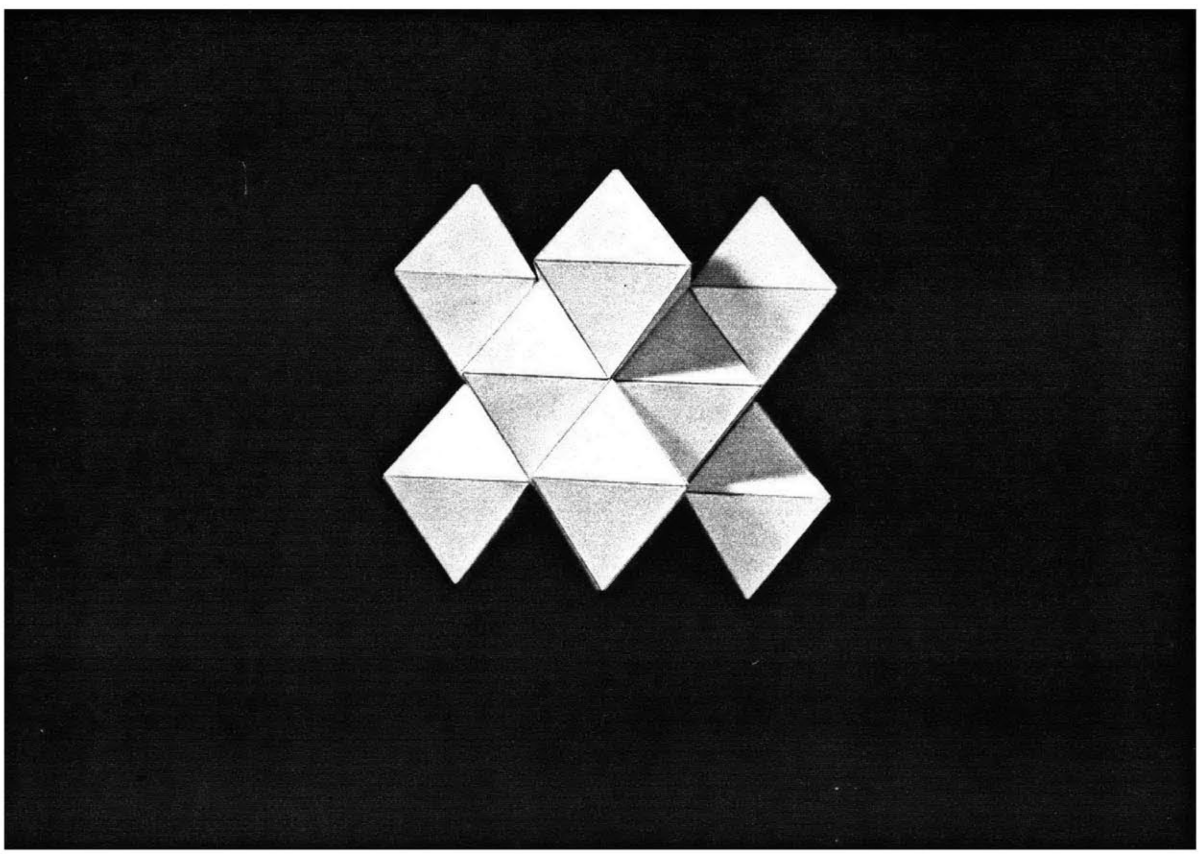




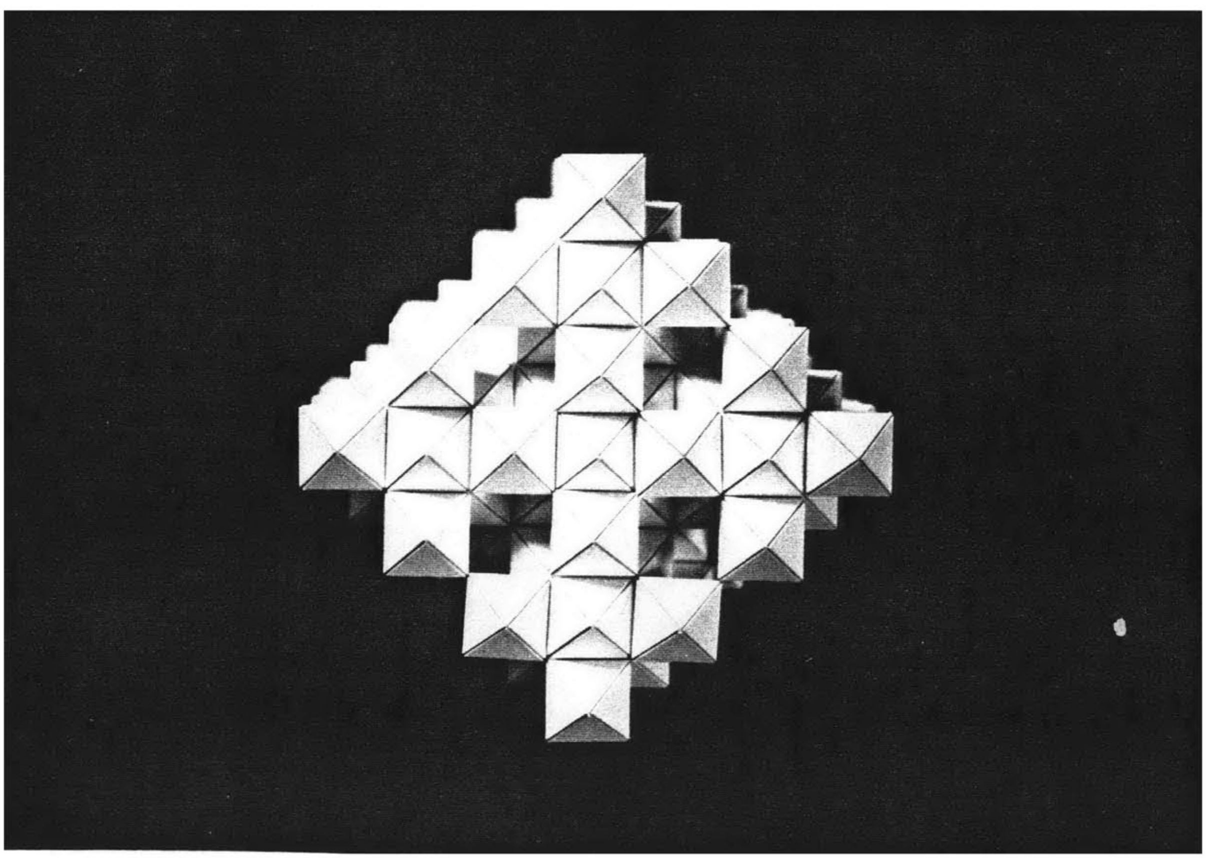
3 views of 1st level octahedral growth.

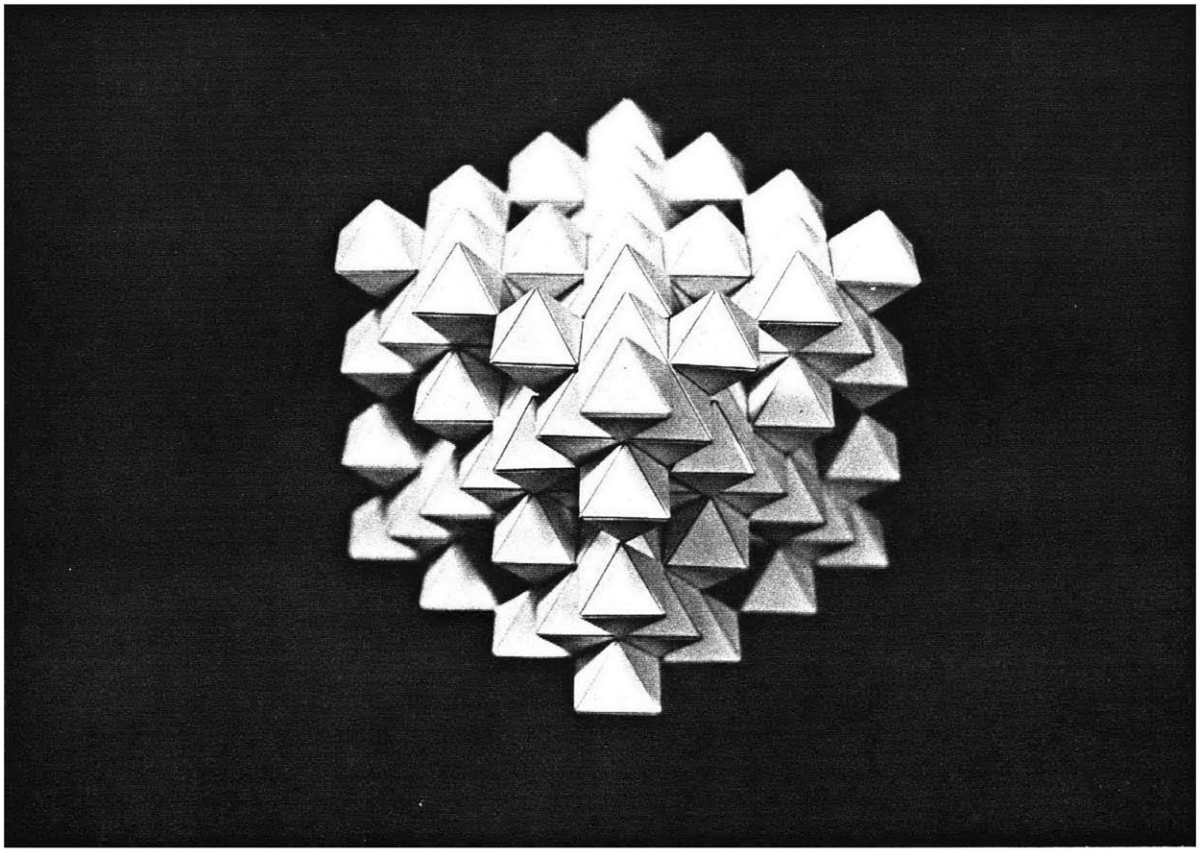


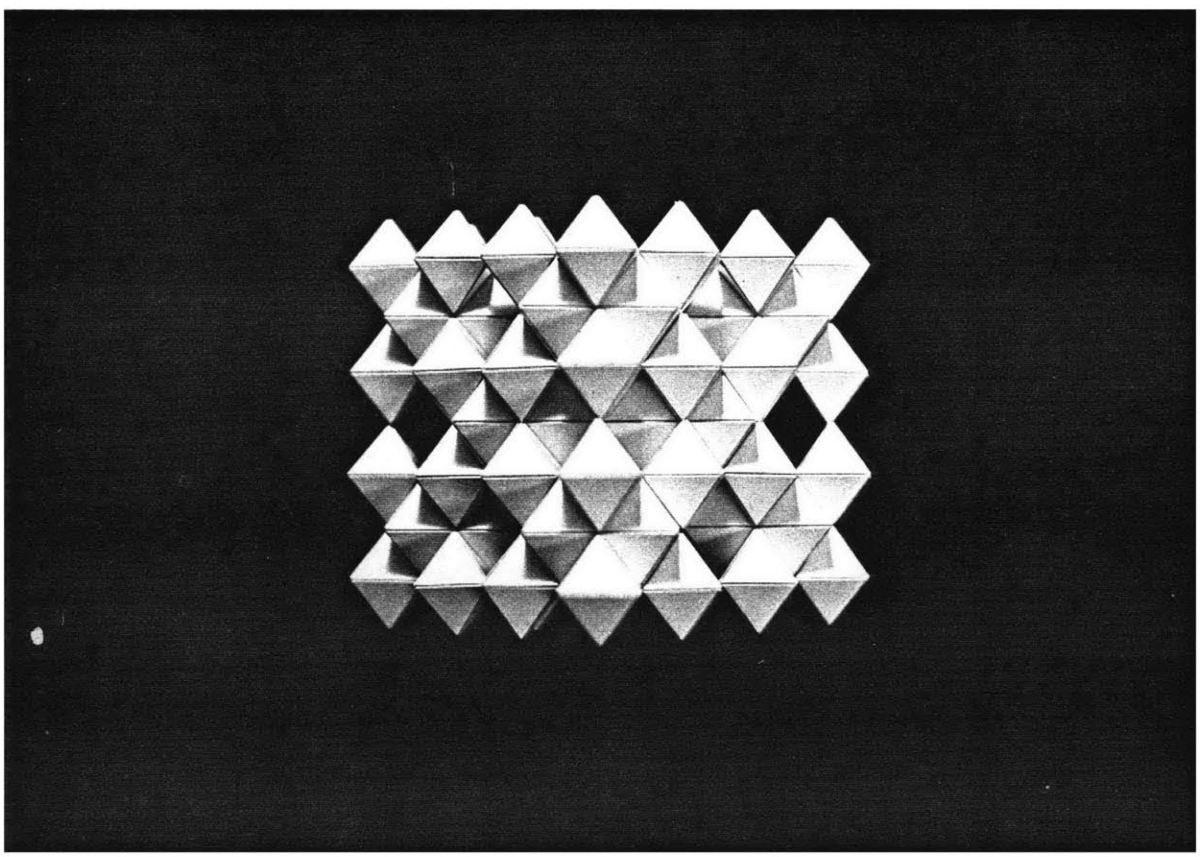




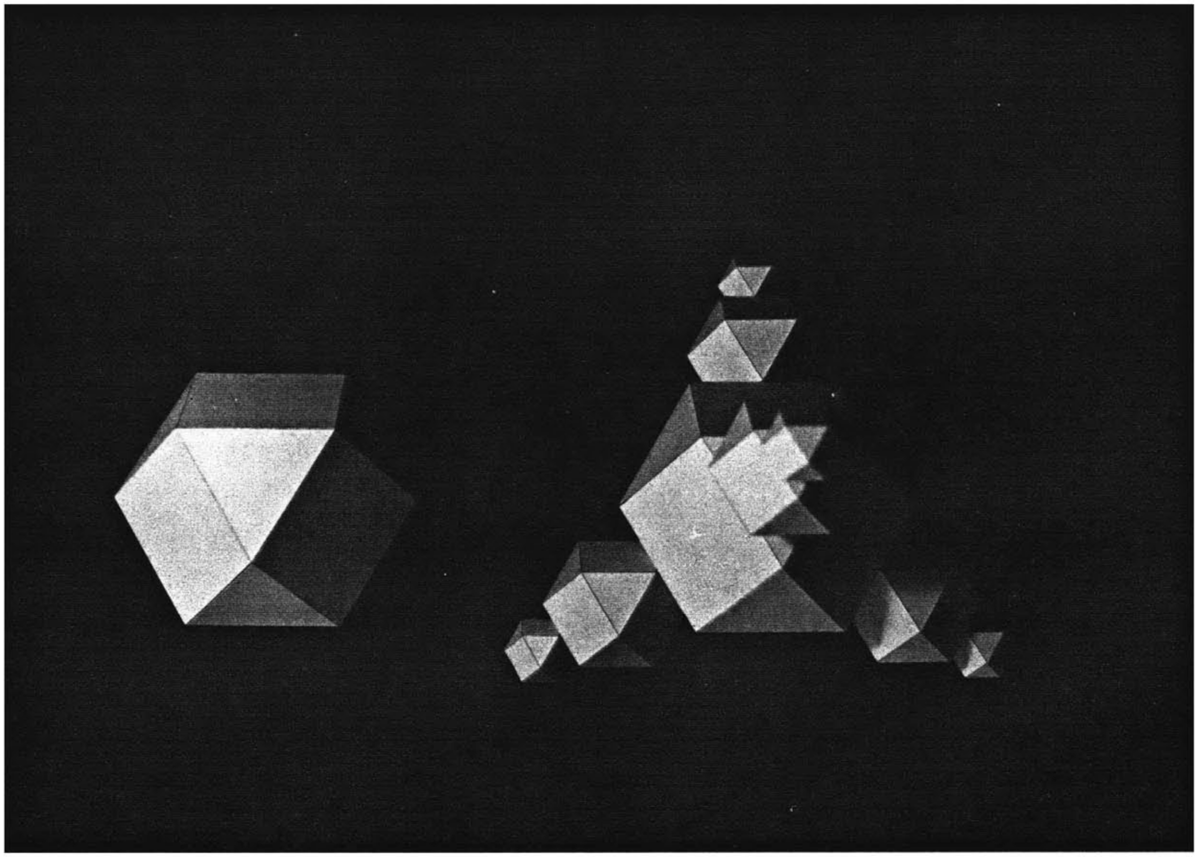
3 axial views of 2nd level octahedral growth structure.

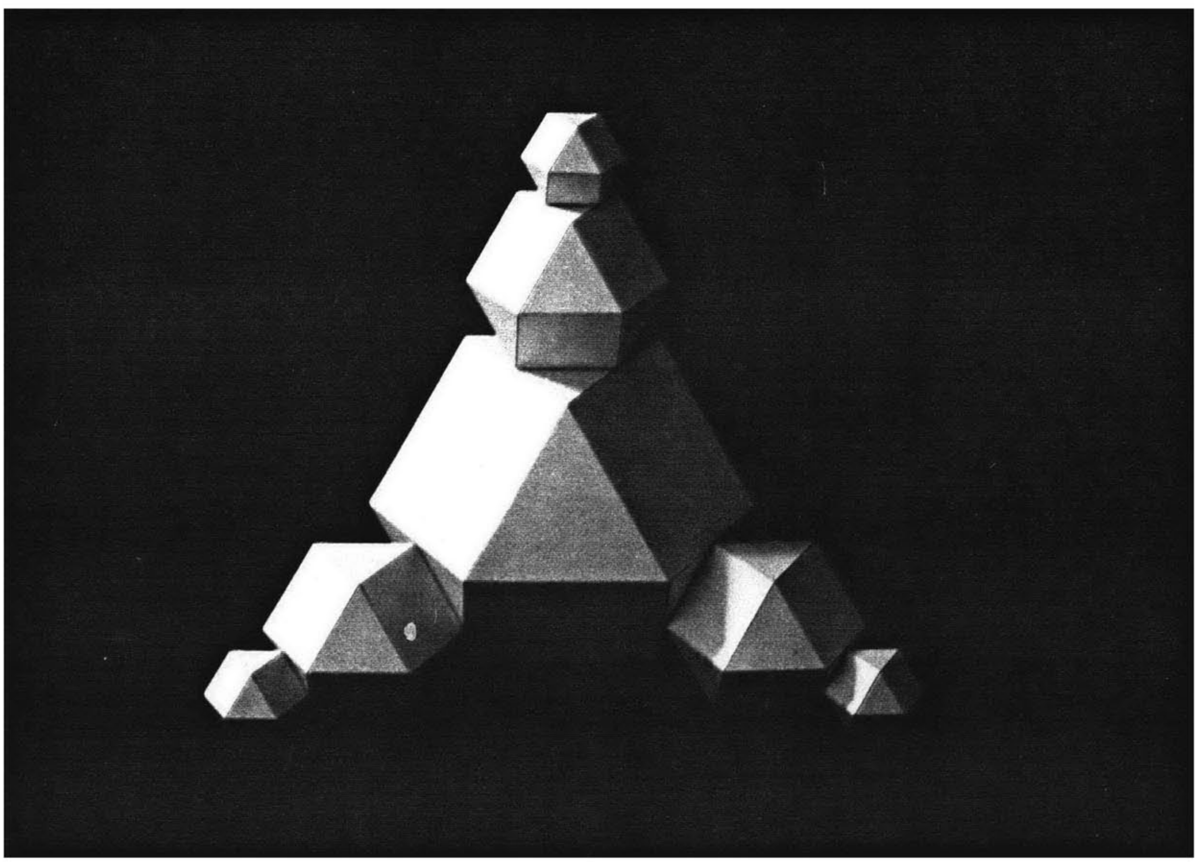


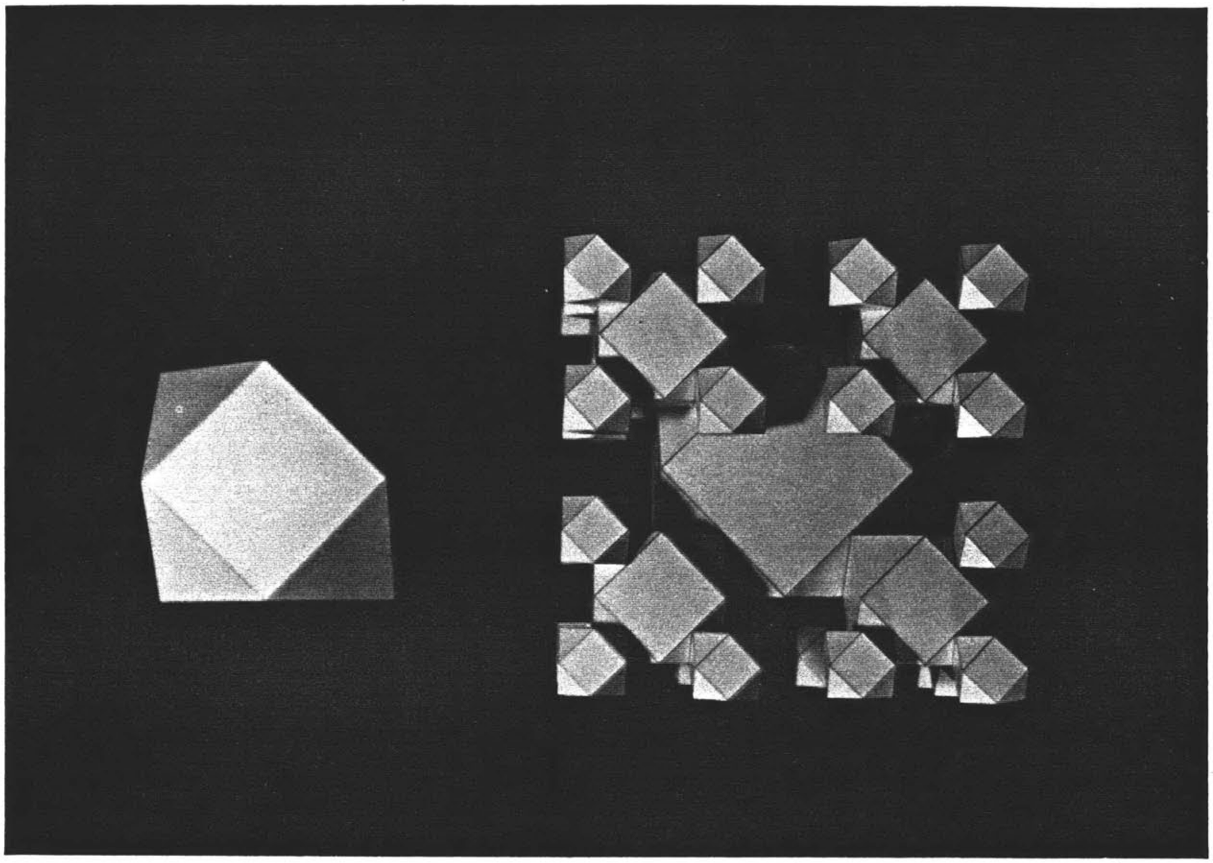




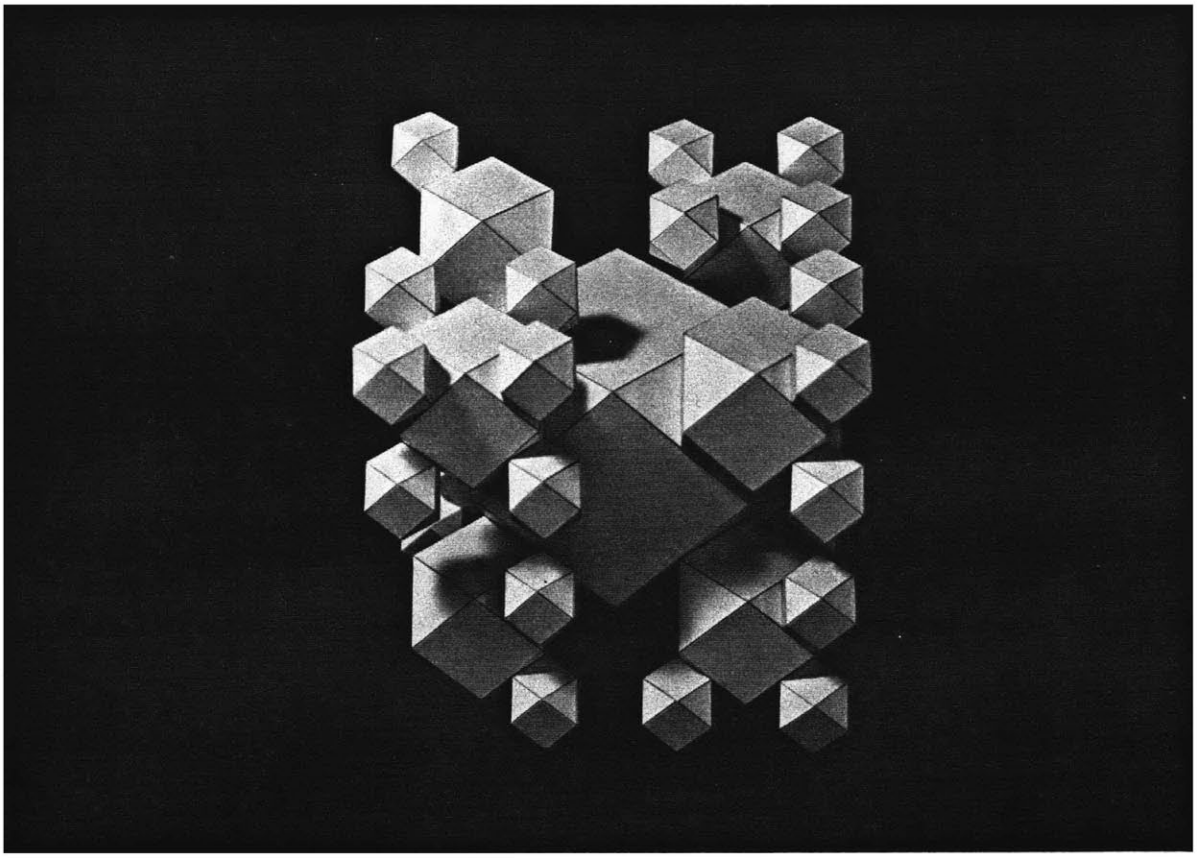
Automorphic growth along 4 axis of the cuboctahedron proceeds to a tetrahedral limit figure. Edge lengths of successive generations are in a 1/ 2 progression.

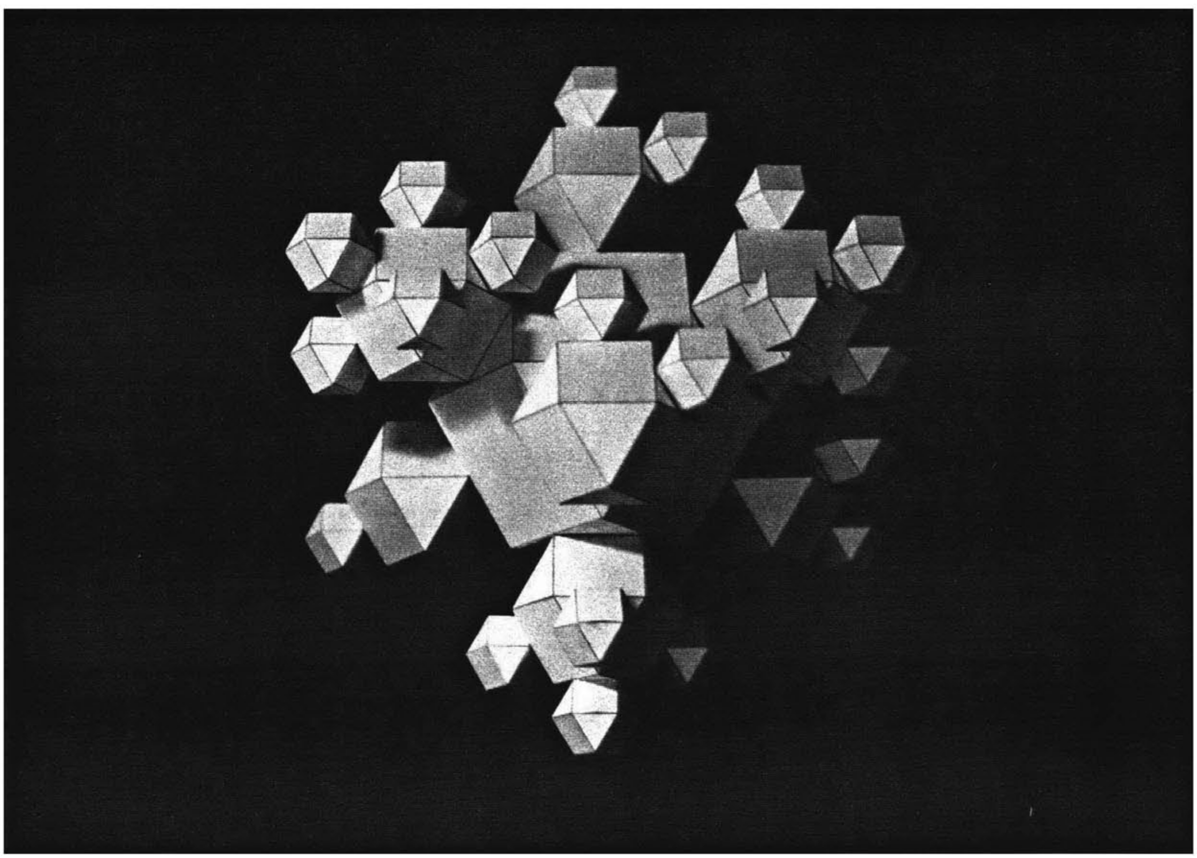


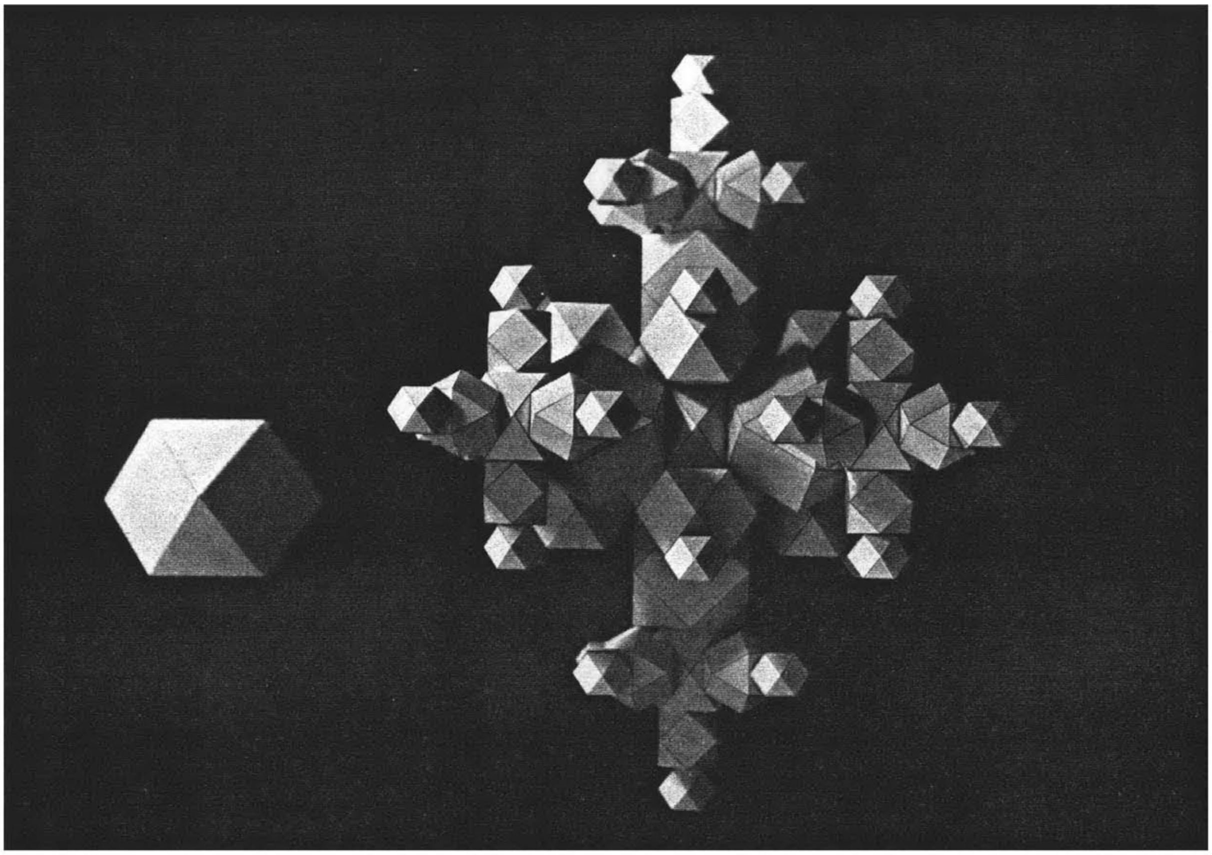




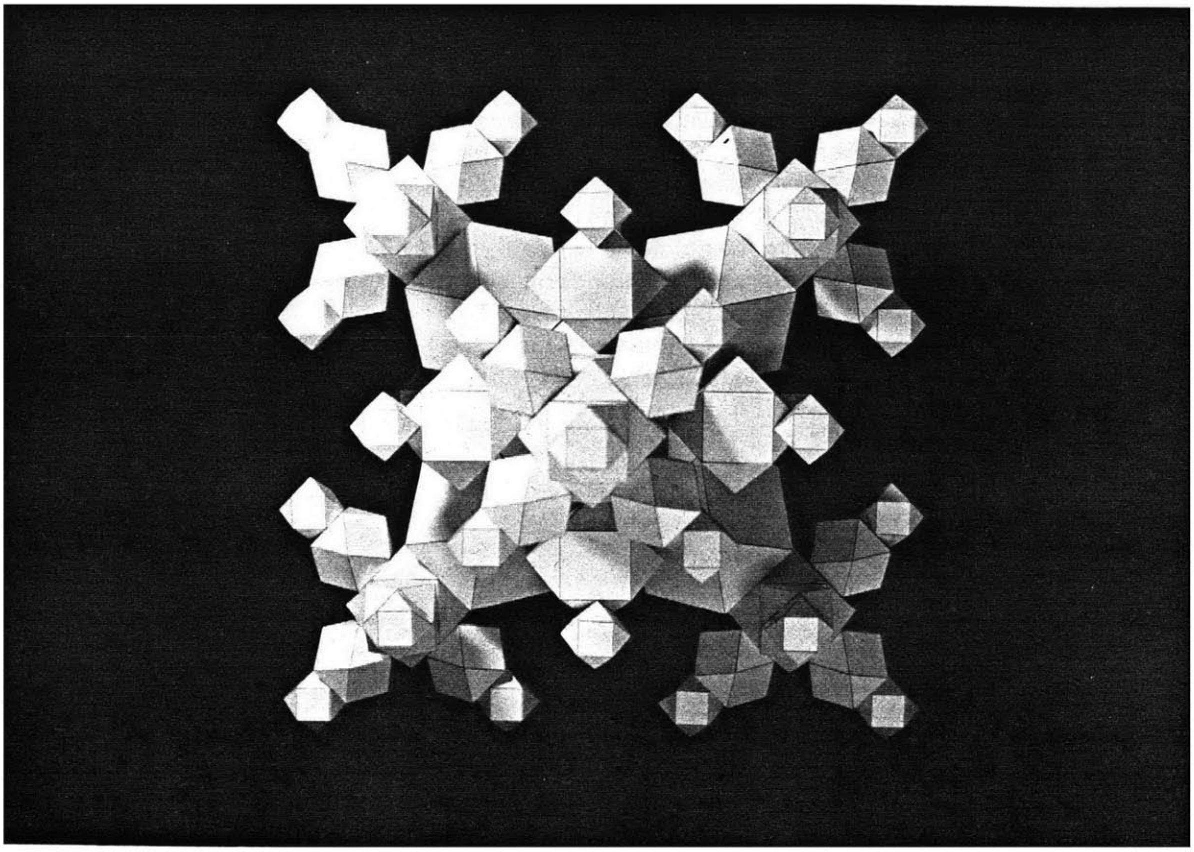
Automorphic growth along 8 axis of the
cuboctahedron proceeds to a cubic limit figure.

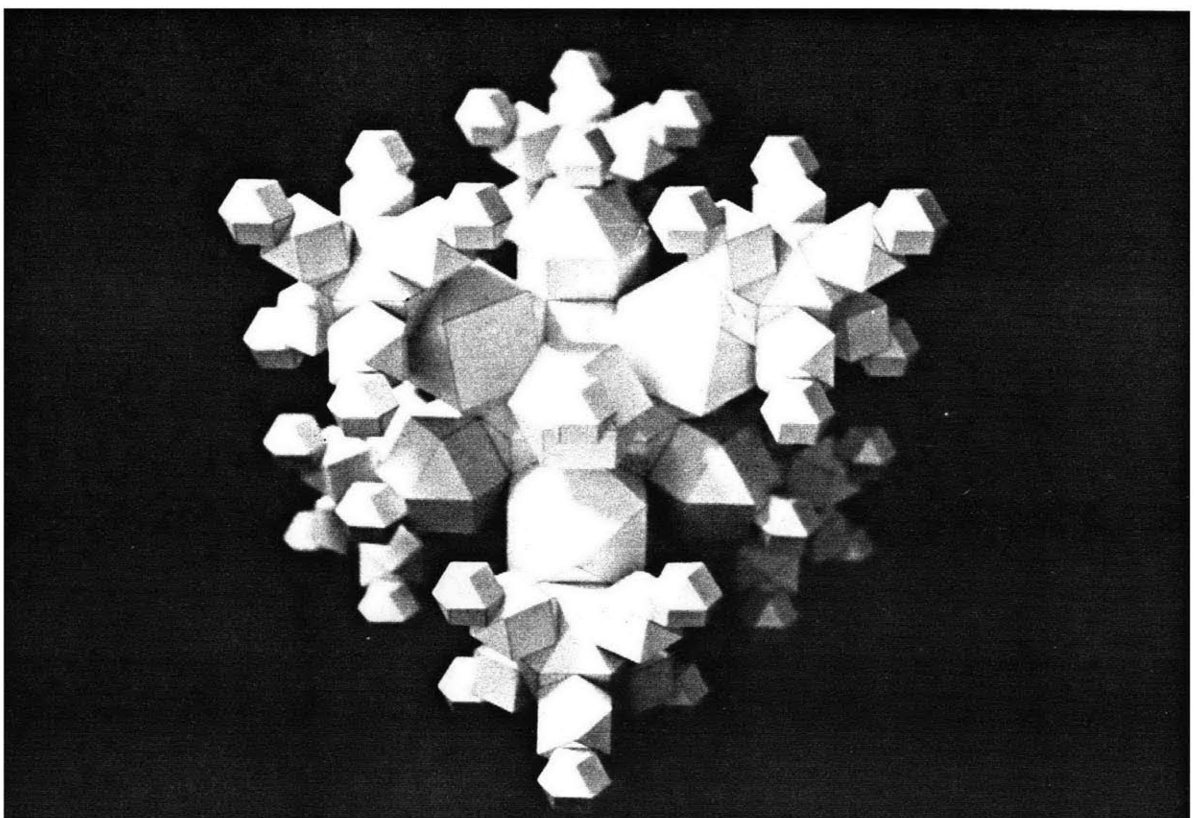




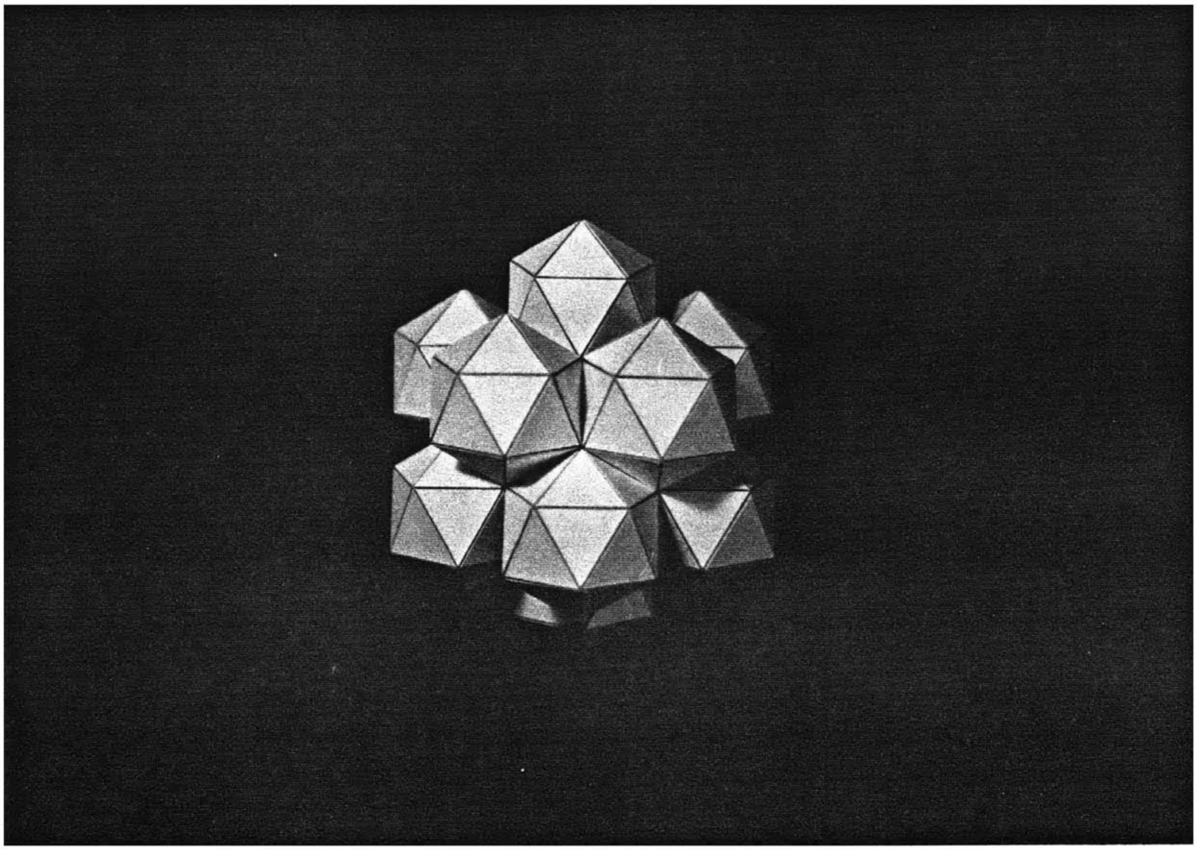


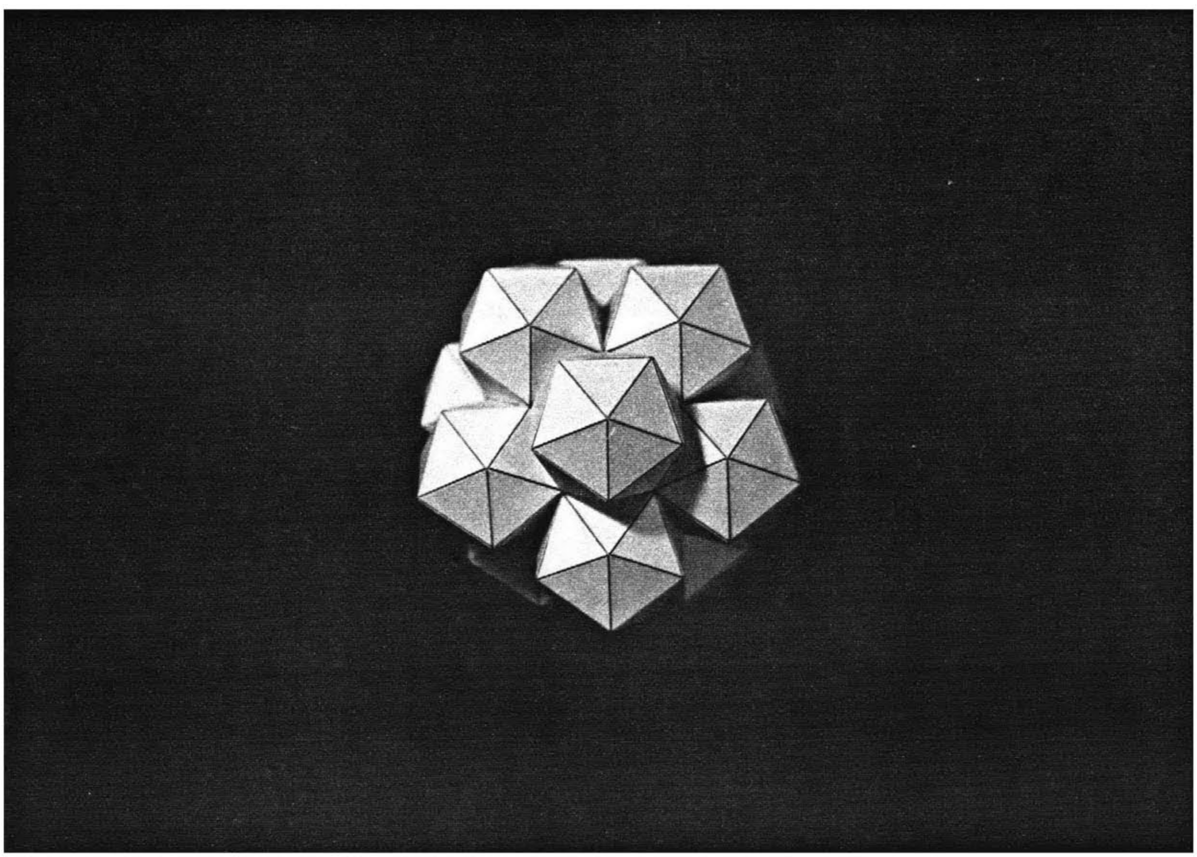
Growth along the axial directions of the cuboctahedron reaches a compound limit figure of an octahedron and cube. Growth along the six axis defined by the triangular faces proceeds in a $1/2$ progression. Growth along the eight remaining axis is in a $1/\sqrt{2}$ progression.



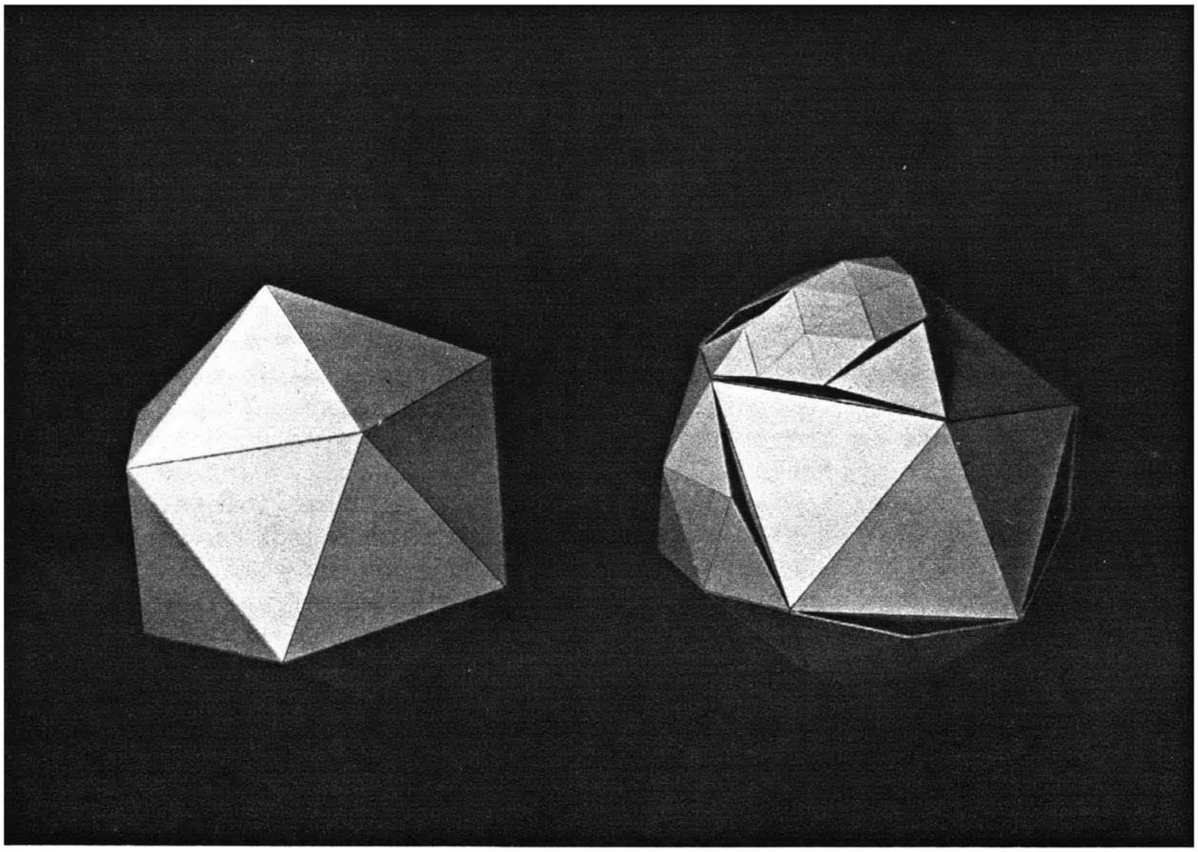


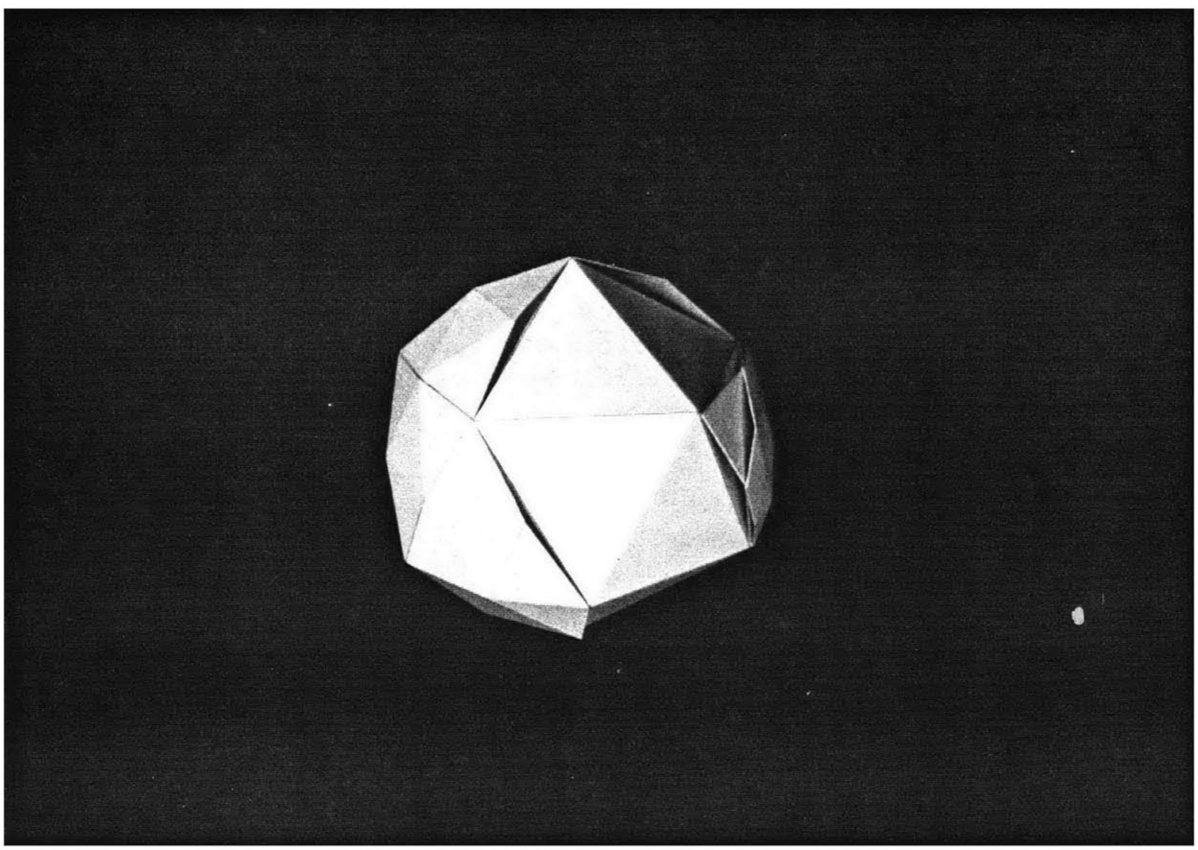
Self replicating growth is most directly evidenced in the icosahedron. Smaller icosahedrons aggregate by sharing edges to form a larger icosahedron which may in turn aggregate to form even larger icosahedrons. One may think of growth in this manner as dimensional substitution as opposed to increases in the extent properties (length, mass, etc.). For each edge of an icosahedron is substituted an icosahedron of the same size. Analogous procedures with the tetrahedron and octahedron are possible but to replicate a larger version of themselves the vertexes must transform into figures.





Recursive subdivison of the 'faces' of the icosahedron produces the family of high frequency geodesics. The accompanying plates show a (pseudo) random frequency modulation of an initial icosahedron. As a development of R. B. Fuller's dymaxion air-ocean world map one would not depict the continental masses as inscriptions within the triangular facial units but in fact model them by increasing the frequency of articulation. This particular model shows three different levels of articulation each one hierarchically dependent on each other -- reminiscent of the core, mantle, and crust configuration of our so-called 'globe.'





POSTSCRIPT

This thesis attempts to "delineate" automorphism as a principle attribute of form or structure. Although I don't think I have fully qualified it (or can hope for full qualification) my position is that all persistant structure (thus all "form") is so, due to self-replication at some level(s) (scales). I have tried to show some fragments of contemporary physical theory which are not only most visually to the "point" but which lend tthemselves most easily to this generalization. Well, one might say, there is an example of form which does not display any sort of automorphism--that is the opint. But this is so only relative to a bottoming out of our perceptual mechanism. Conceptually (and/or through instrument extension) it is composed of many, many other points. In another sense the point hast the potential if "set" in motion for the description of all form ("set" conceptually because the point is already in motion, already "being" some form or another--possibly a diagram of a "point" absolute stationarity is only a conceptual possibility--a belief in "the place where nothing ever hapens"). Like the seed or embryo it contains within it virtual presences not (yet) superficially manifest. It contains within it dimension(s) which "given time" will emerge.

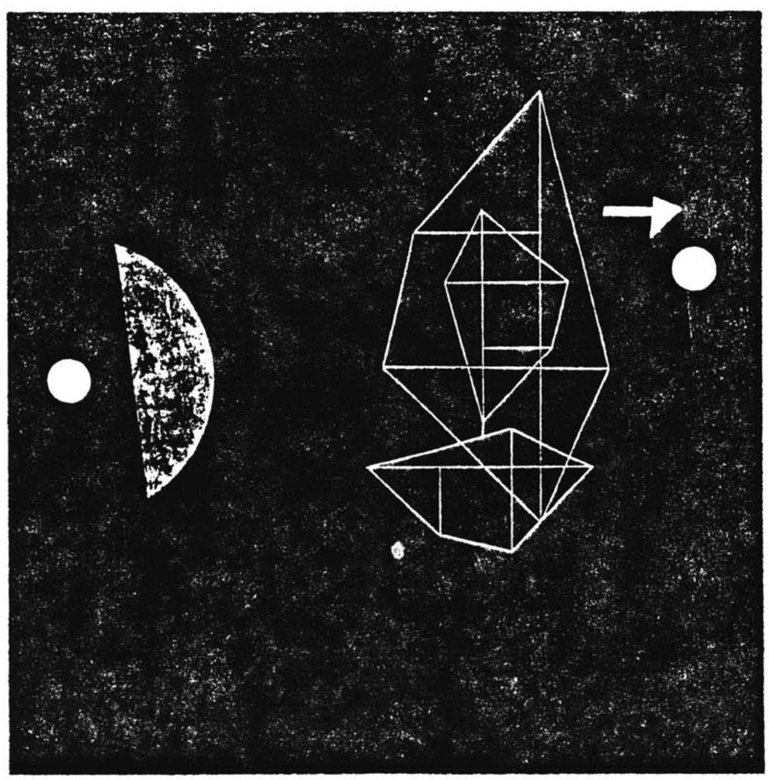
The fact of (that there is) form is one of temporal "stability". We observe all those things which persist "long" enough for our perceptual mechanisms to register. Those things with "not enough time" apparently do not exist. There is an equation (an equivalence of terms) suggested by all of this--that is that,

Movement is time is dimension is stability is self replication

so there are no "things" which do not display automorphic behavior (as yet, or up to this "point").

If I were to suggest a model, (a conceptual repository for those notions) that is to begin with/to order, a first approximation would be of a general case intertransformable uniform cellular space. Cellular-based on the assumption of modularity, uniform--assuming space is homogenous because the simplest "physics" arises from this condition. (Of course, no one would maintain that our world is strictly homogenous, but it works well as first approximation), intertransformable--to the extent that only those transformations which occur "automorphically" are admitted, the "simplest" of which are outlined in the previous section. Being the "simplest" means to imply they would also be the fastest (occupy the minimal time, dimension) the more symmetrical, simple, nuclear, the transformations (or given state of the cellular space) the more frequently reoccurring the configuration is. The maximum speed (minimum time) by which these transformations) take palce is the "clock" giving coherence (of a modular order) to "turbulence" or the "highly complex" fluctuations apparent in natural phenomenon. The suggestion is that if our perceptual mechanisms were able to operate at much greater speeds, more coherence would be apparent.

This suggests a means by which the preliminary model could be improved, that is by operating at greater perceptual speed, by increasing resolution (minimizing the "size" of the cells), and by the "building up to" inhomogeneity. This is in fact the means by which the "physical sciences" are presently advancing themselves, and it is possible that this is so because this type of model is so condusive to the use of electronic computers--the most important characteristic of which is the amazing capacity for repitition or repeated application of given operations--their capacity for automorphism. And as the speed of computation is increased, the models accuracy approximates or "world" much more closely. I suppose the present limit is the speed of light (although this may be only because we haven't noticed anything faster), and as we see the introduction of machines operating on light impulses instead of electronic we approach even this limit yet probably only to see a not yet noticed other "point" "off in the distance".



PROJECTION

The following projections I hope will give some indication of the possible directions of the previously mentioned form generalizations application to built form. These possibly indicate a specific personal direction but should by any means act as limits. I cannot even claim them to be complete or limited within themselves. They are beginning approximations. I suppose if one were to suggest a type of limit to the generation of form it would be recognition of our own human conceptual limits -- a limit whose first approximation is the awareness of self.

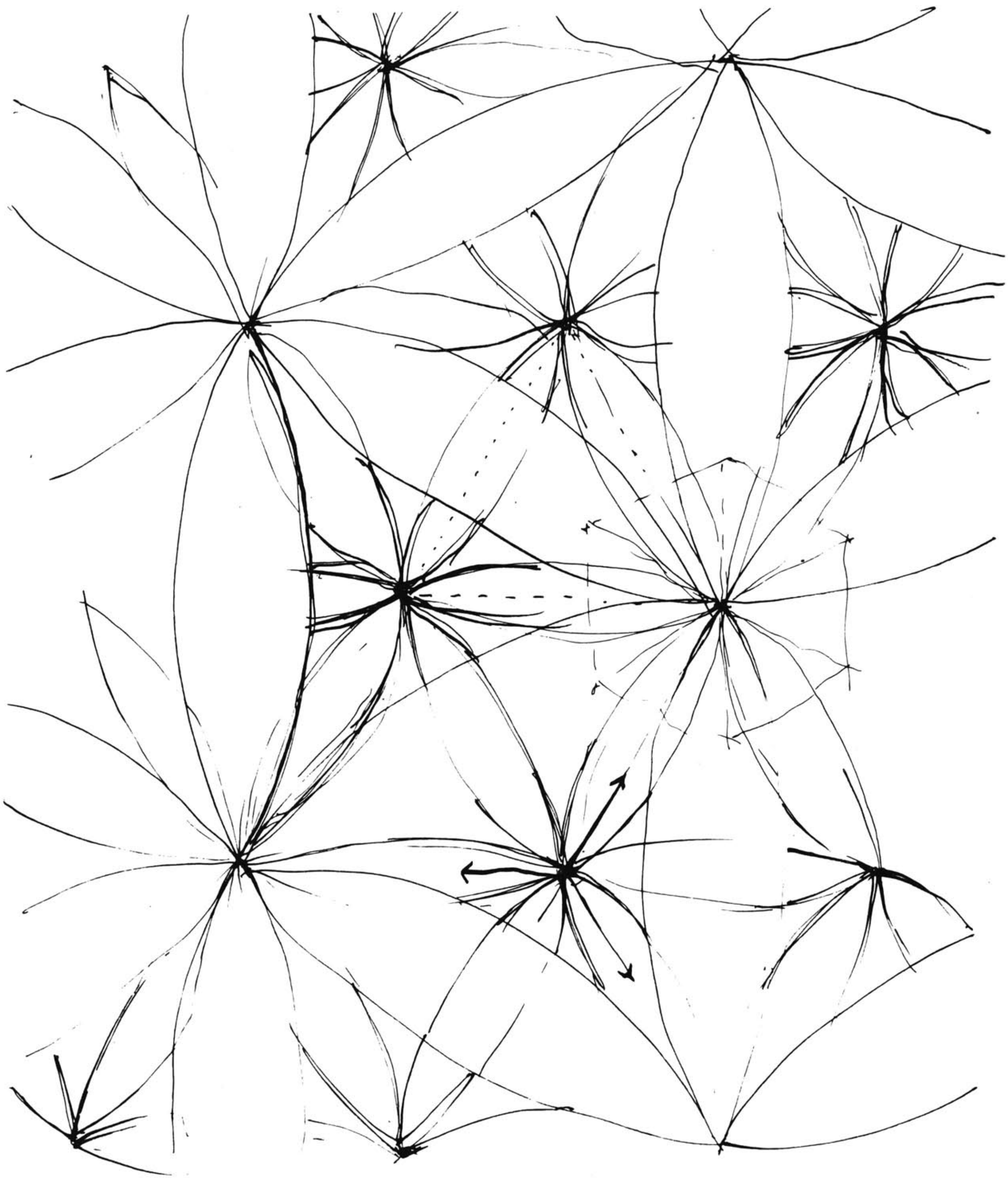
a place to become

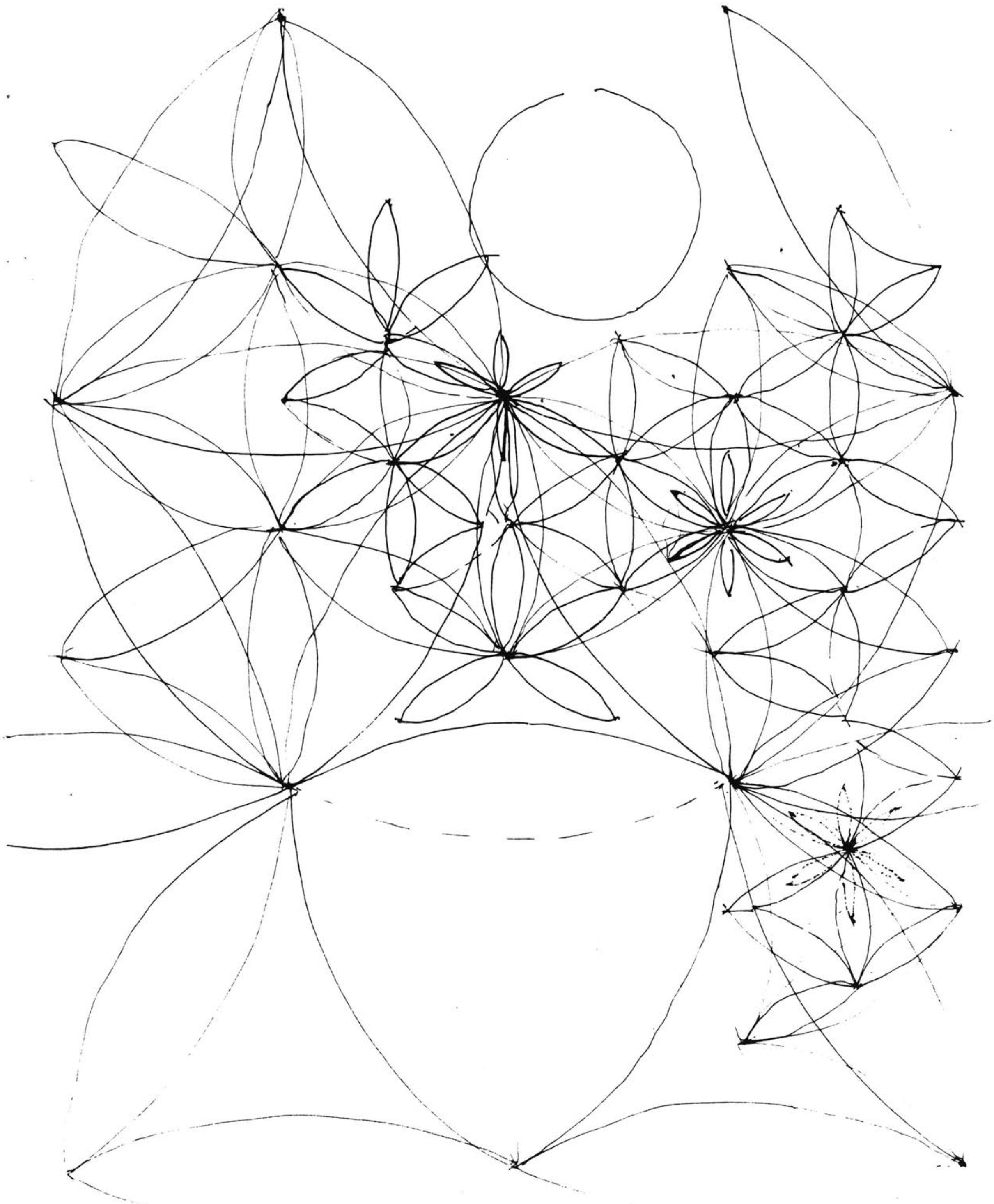
This first projection was done before this discourse was written and to some extent remains the most pleasing. This is possibly because, while demonstrating self similarity it does not do so in a strict sense. Scaling on the other hand does occur in a strict ratio.

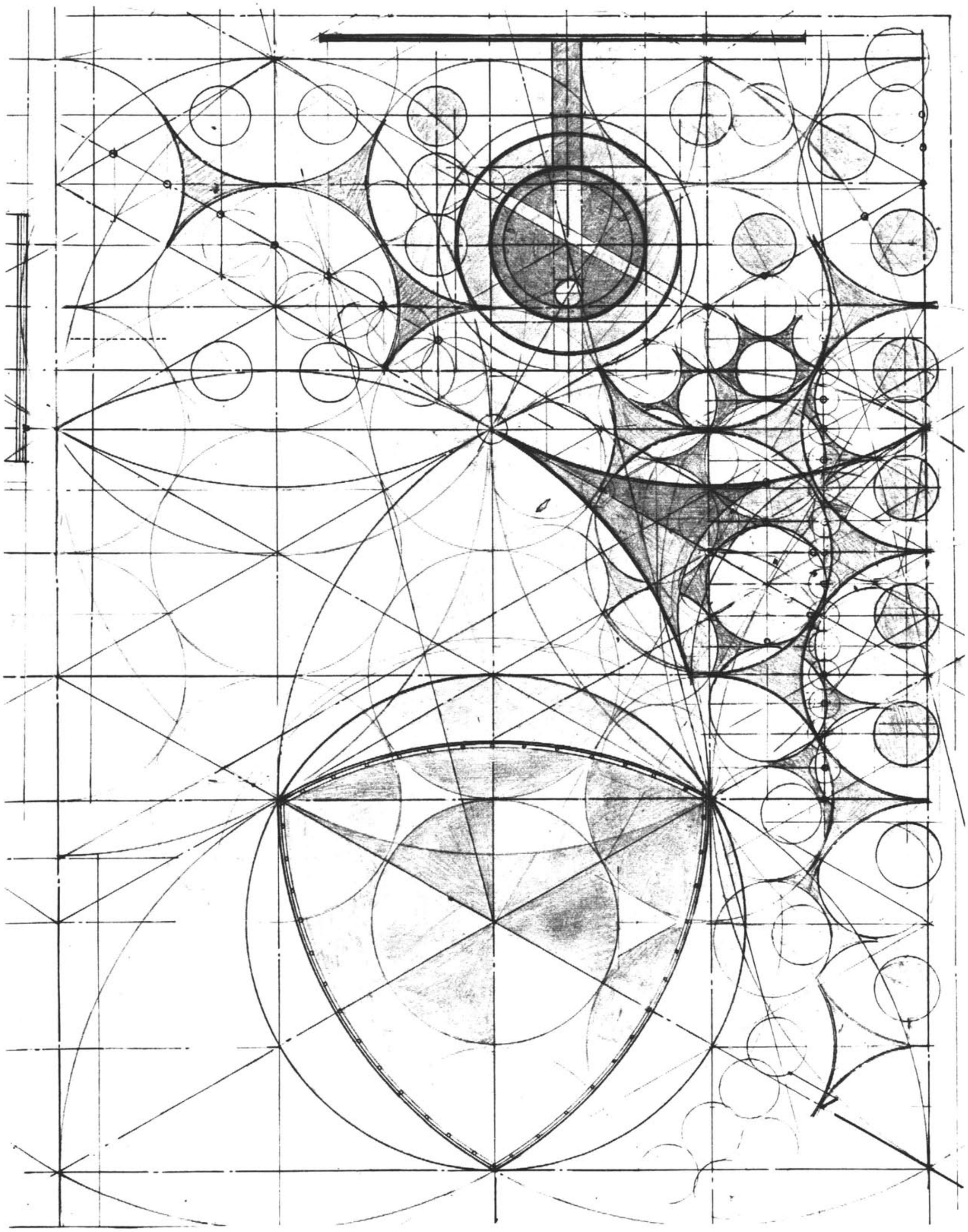
The context here (which is the Kresge oval on the M.I.T. campus) generates a scaling circular 'net.' 'Net' as opposed to tesselation because circles (or spheres depending on which dimension one happens to be 'seeing' in) do not fill space. The built artifact, a tensile structure, occupies the slack in the substrate.

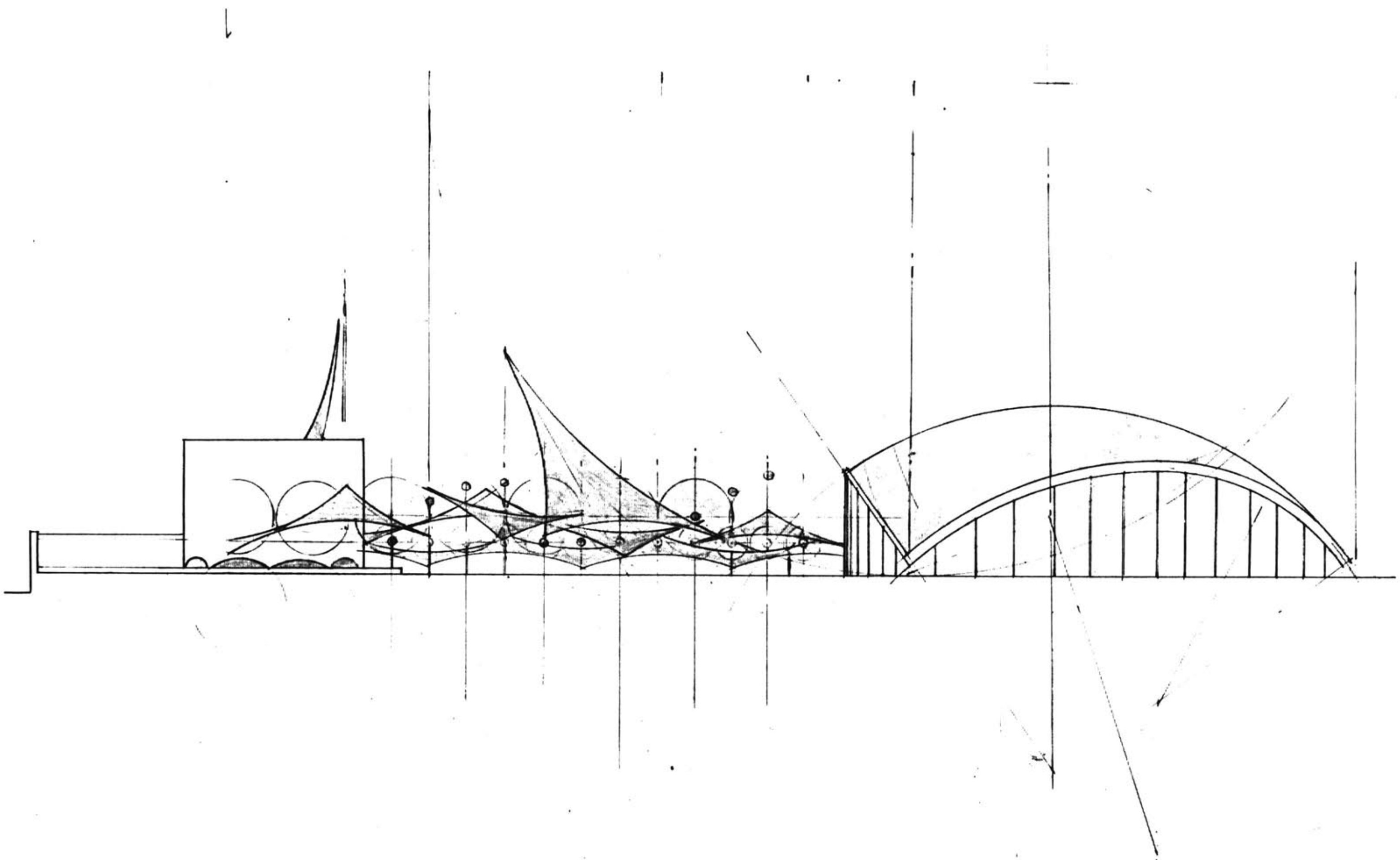
The collective dimension of Kresge Auditorium is scaled down 4 times towards a lower limit of the personal sized tensile structure -- that is the umbrella. A large number of steps in this progression are not built, (it would take approximately 30 steps to complete) but the first four are. In such a public context I believe it is much more important to start the progression and have it complete itself conceptually at the individual dimension than to complete it in an explicitly physical manner. In fact the project attempts to minimize its physical presence by providing only ceiling definition. It establishes itself as a place to move through -- physically and conceptually.

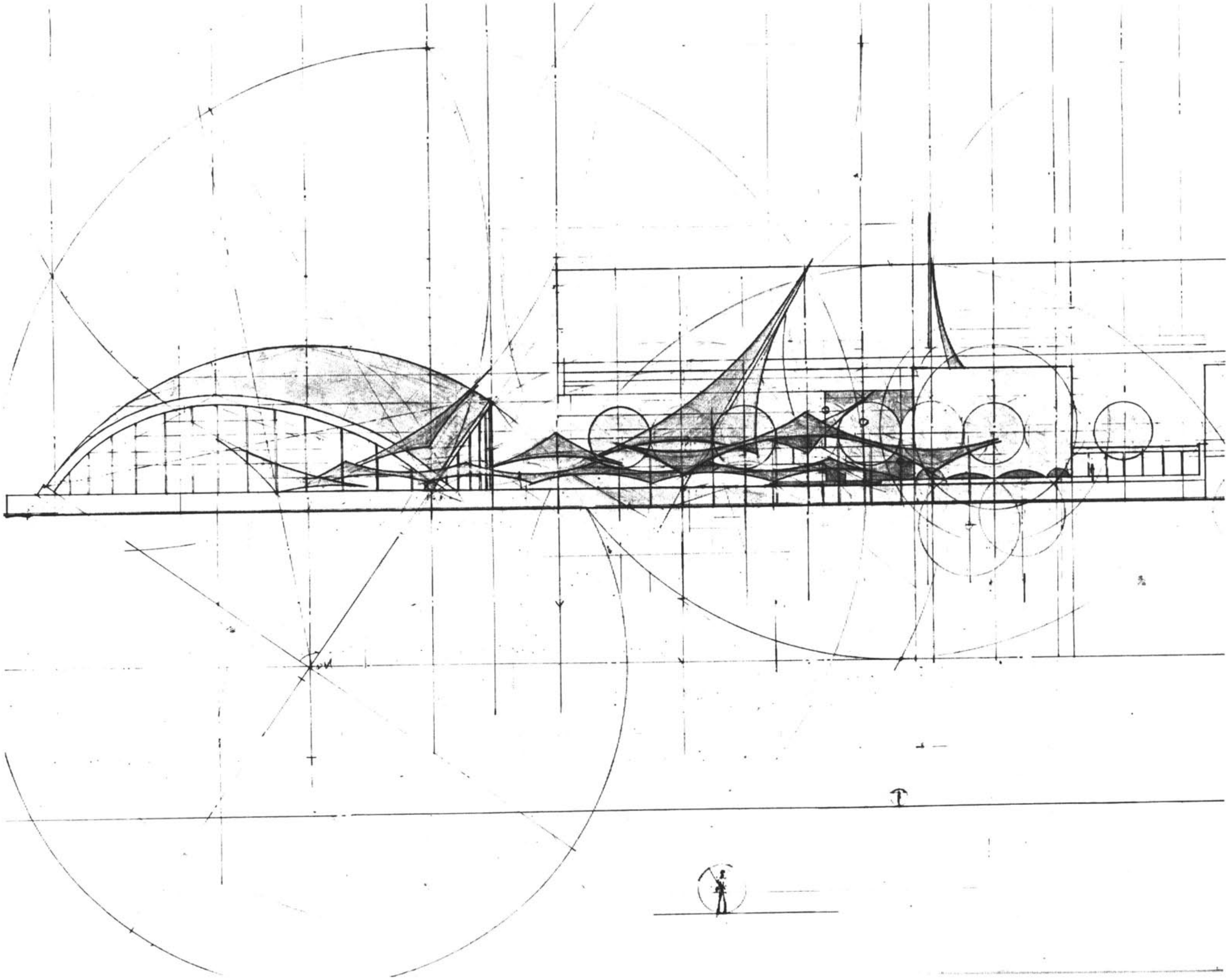
> "Intimate immensity is within ourselves. It is attached to a sort of expansion of being that life curbs and caution arrests, but which starts again when we are alone. Immensity is the movement of a motionless man."
>
> Gaston Bachelard

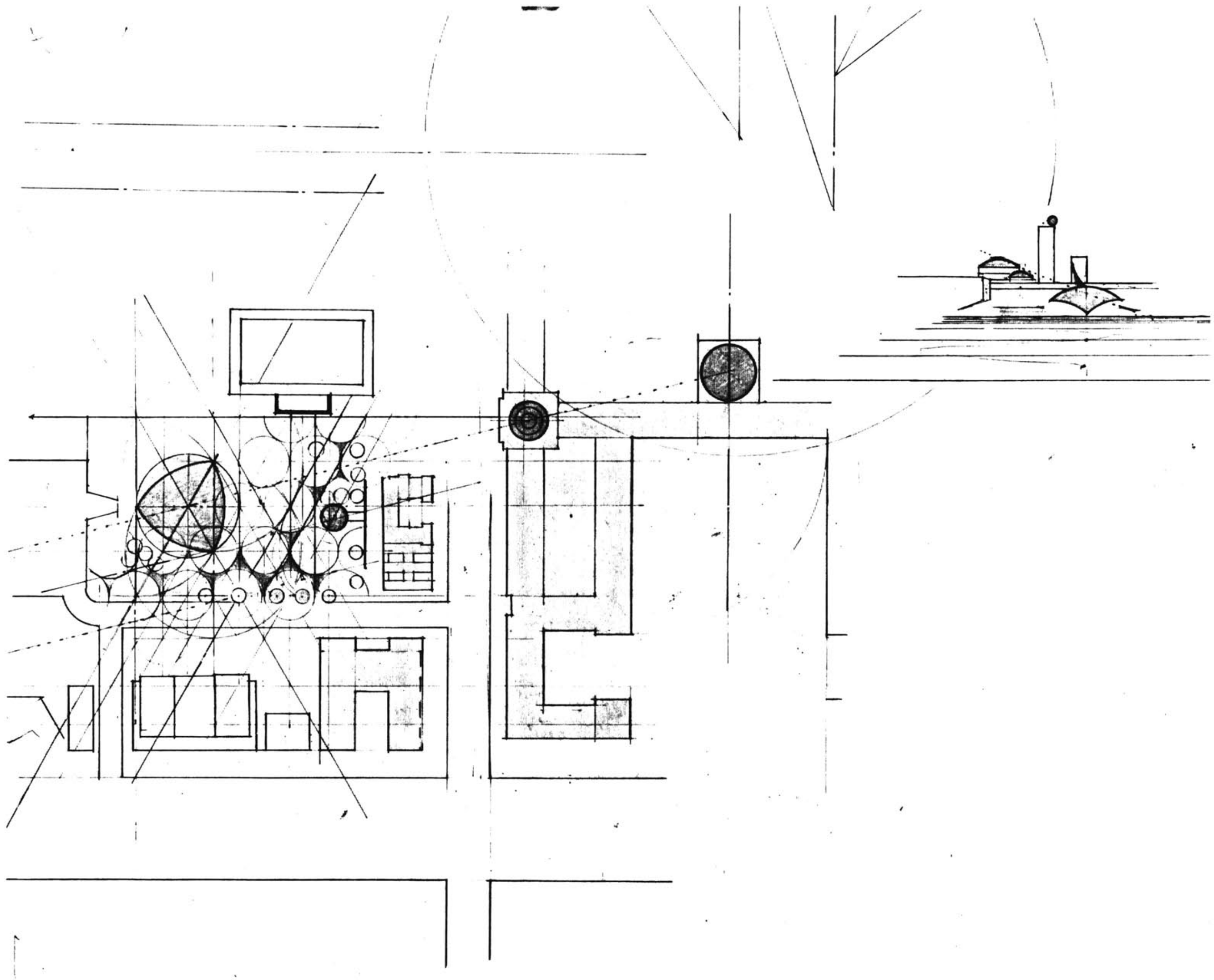












island

This projection for a hypothetical island structure is based on a type of Koch curve termed a 'dragon sweep.' 'Sweep' because it is the trace of a moving point. Which after a certain number of recursive generations closes on itself (self-contacts), therefore delineating a number of identical planar units. Thus, its dimension increases with time. This fact reitterates the notion of dimension (and certainly extent) as being a time-related X, a function of frequency modulation.

The dragon is self-similar but in addition is segmented by 'wasp waists' into portions which are among themselves self-similar but not to the entire dragon itself. There are several other properties of the dragon which make it fascinating. While apparently complex we see that the mechanism of its generation is simple, and while displaying a very articulated edge and a complex symmetry it will tile (fully pack) the plane. I have shown 10 different adjancency diagrams for two dragons which are completely congruous (i.e. will not disrupt the tesselation). The most interesting of these is the configuration which aggregates to form a larger dragon or a dragon spiral.

The dragon acts as a portion of a uniform cellular space. I have added dimension to it based on an extrusion -- the extent of each cells extrusion corresponding to the number of neighbors (adjacent cells) it has in the plane. The inverse is also possible, i.e. if the cells adjacency number is 4 it would be extruded 5 units. The general formula is extrusion = 9 - adjacency.

The unit habitation cells take the initial form of the cuboctahedron.

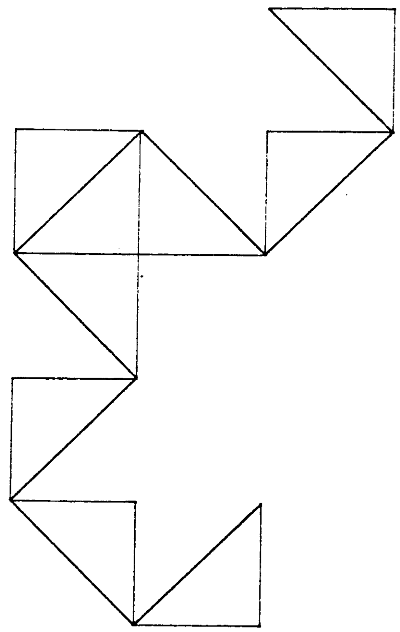
Each one of these unit cells (21 ft. to a side) may transform according to the set of spatial transformations outlined earlier, and if its 'faces' are free (no adjacencies) these may initiate automorphic growth. If its exterior 'faces' are not free, internal growth may occur.

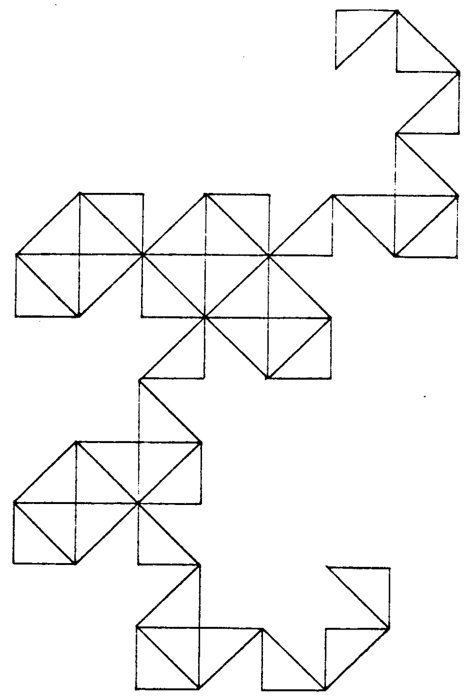
In both the external and internal case, growth occurs in a 1/ 2 progression along the axis defined by the square faces an a 1/2 progression along those subtended by the triangular faces. The lower limit to these progressions is the structural node which also happens to be a cuboctahedral configuration (or its dual-rhombic iodecahedral).

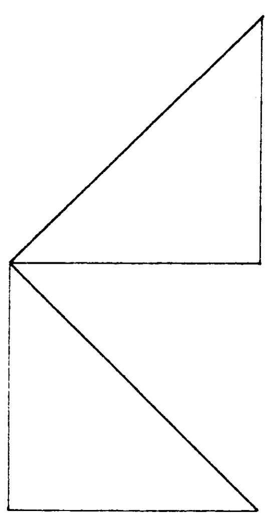
The unit cells may aggregate and transform to higher frequency geodesics accommodating larger volume communal settings. If 9 unit cells aggregate they will transform to a 3 frequency geodesic, thus keeping the bar lengths approximately the same. Given the energetic and adjacency considerations these volumes should occur internally. These internal cells have the highest degree of stability socially and energetically.

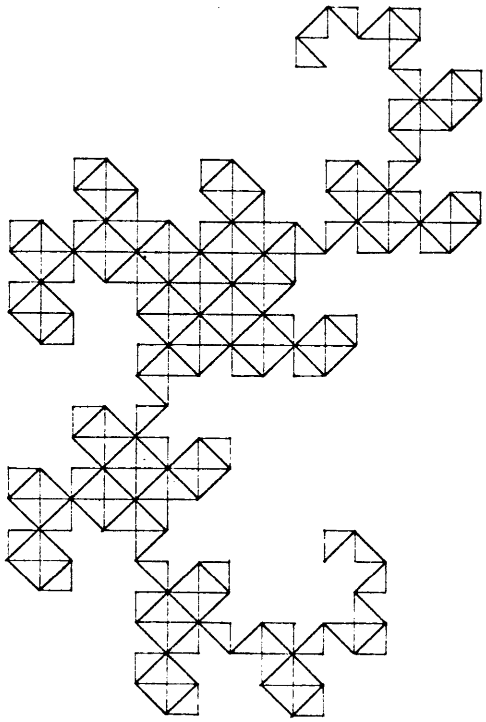
The cells along the edge of the dragon by contrast exhibit the least degree of stability (i.e. the possibility of transformation is very high) and in the extreme case transform themselves into interface vehicles.

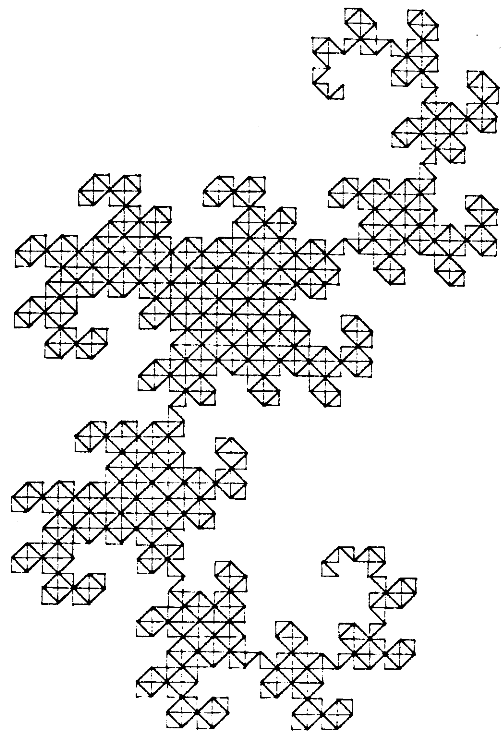
Circulation through the dragon is defined by a lower frequency version of the dragon itself and is built by octahedral and tetrahedral cells.

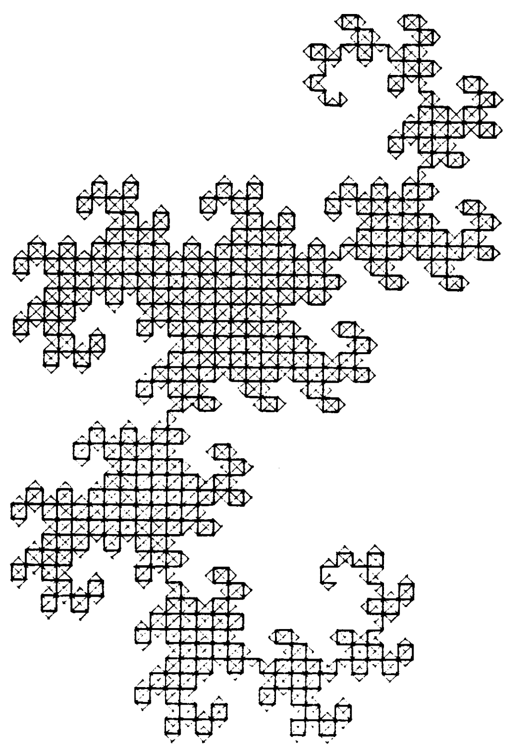


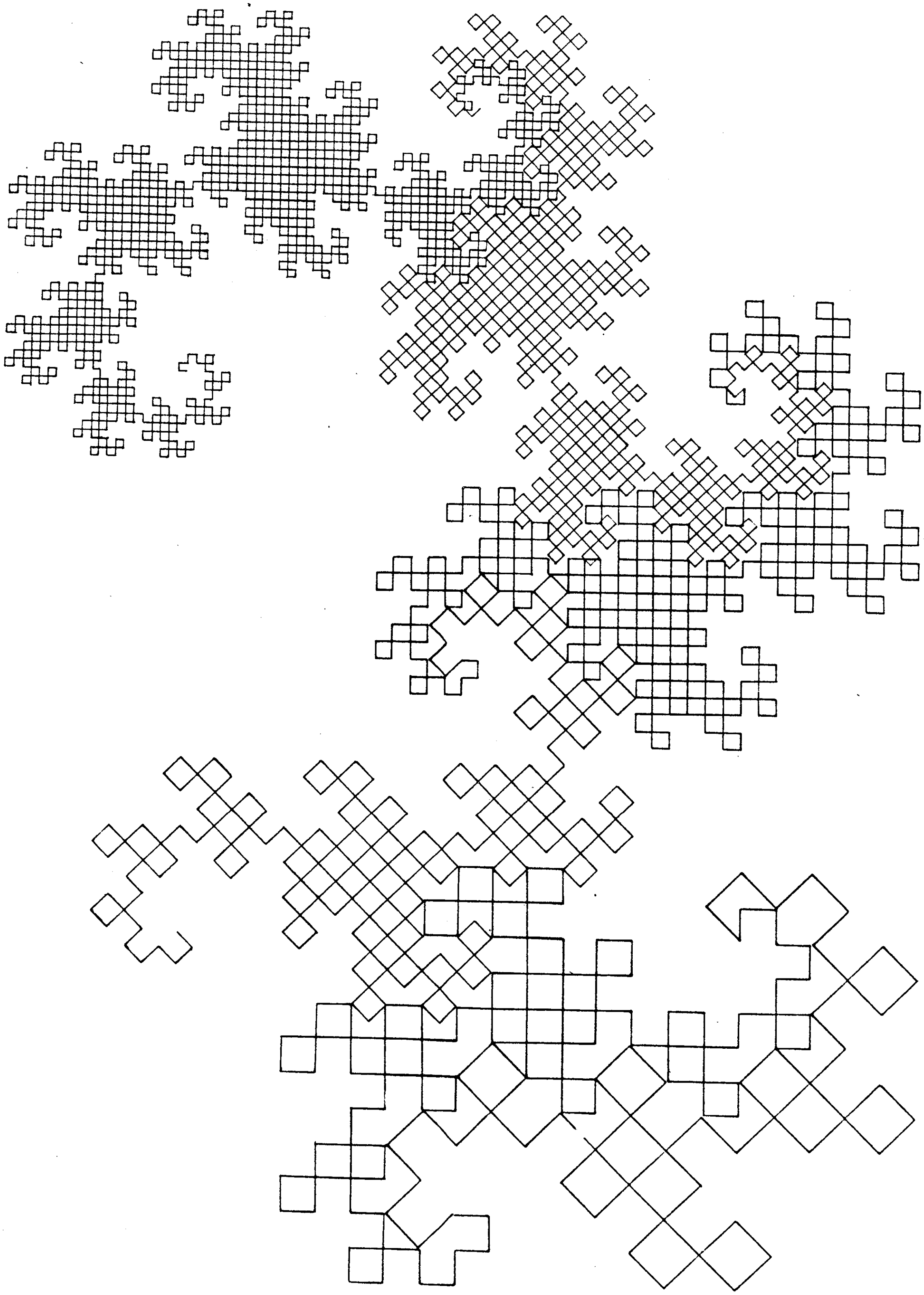


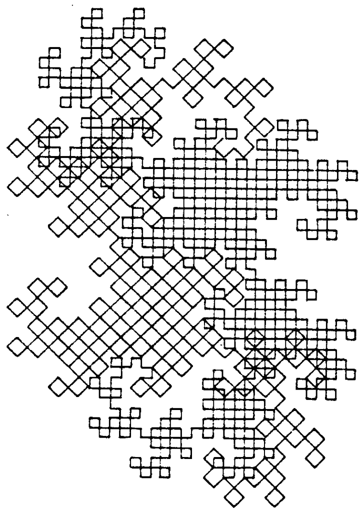


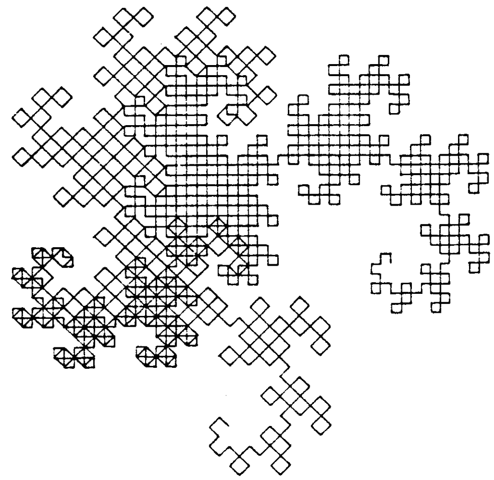


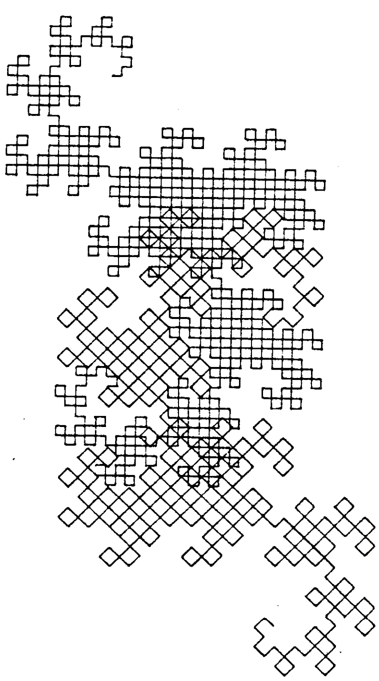


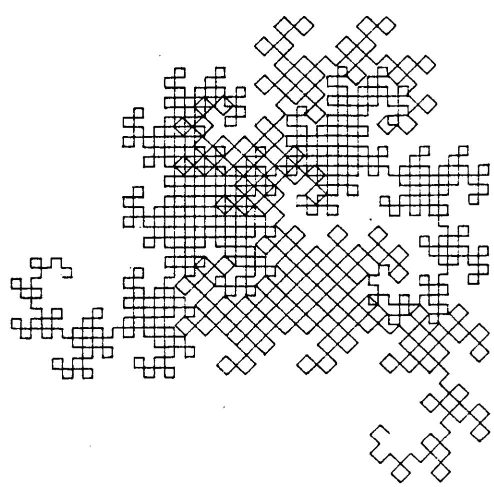


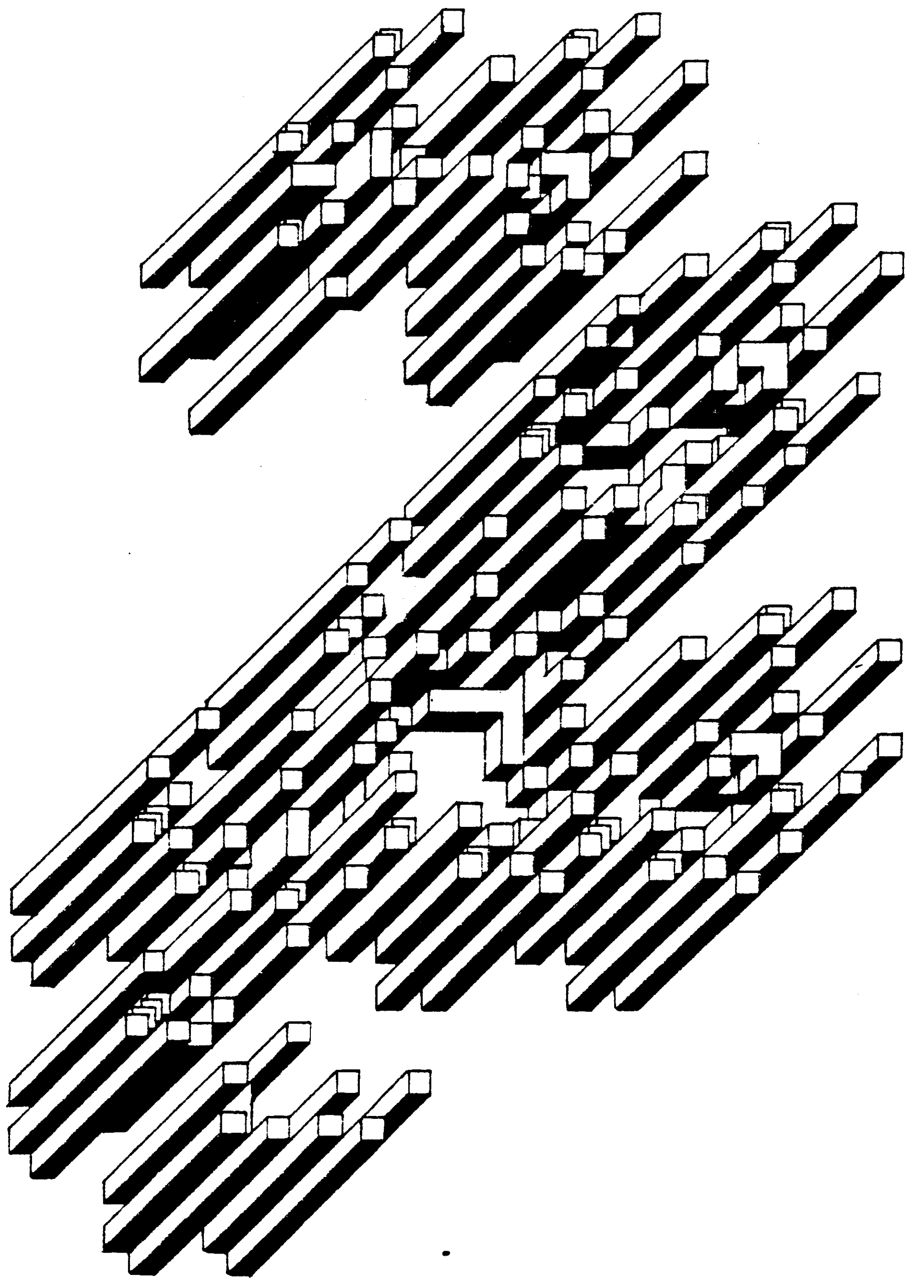


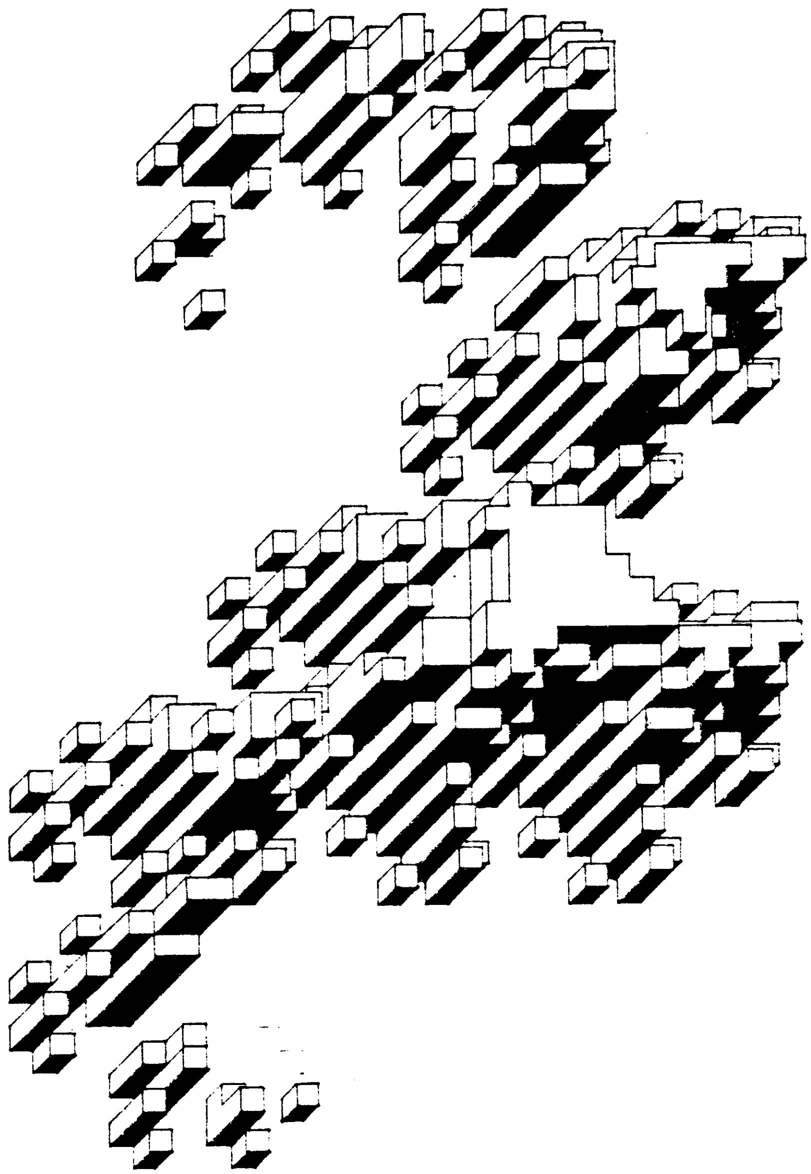


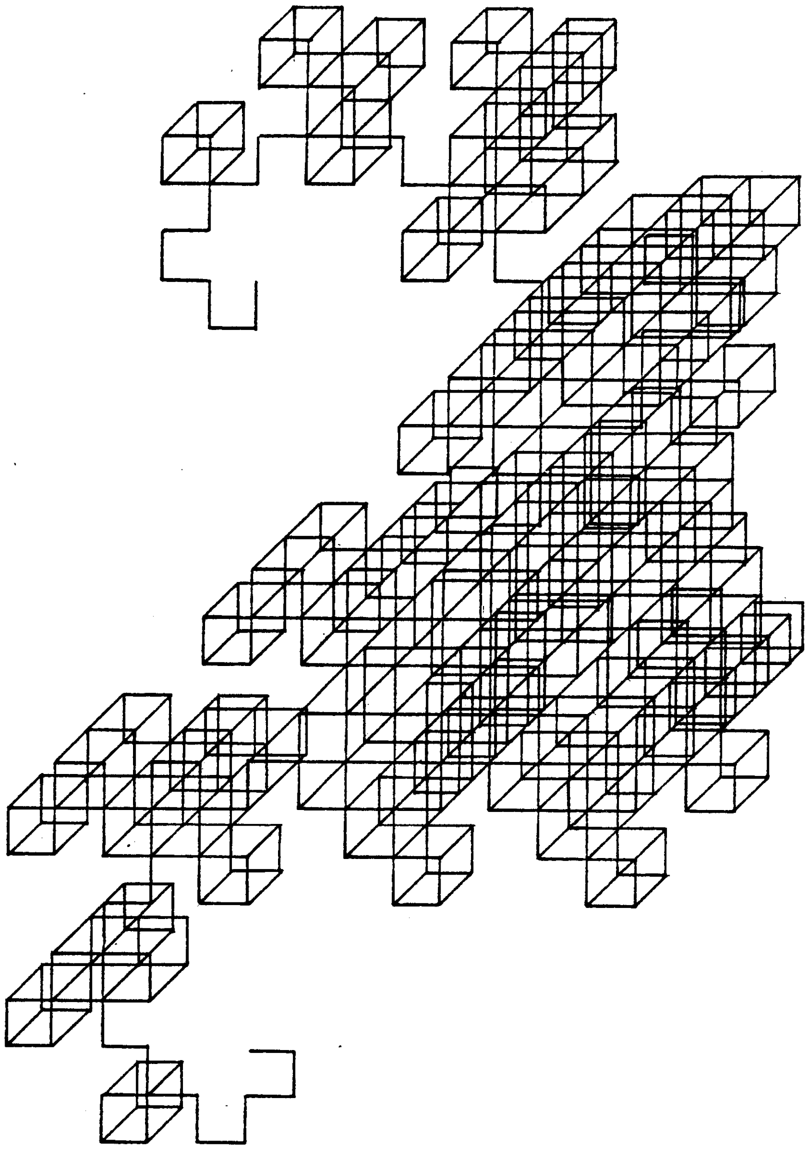


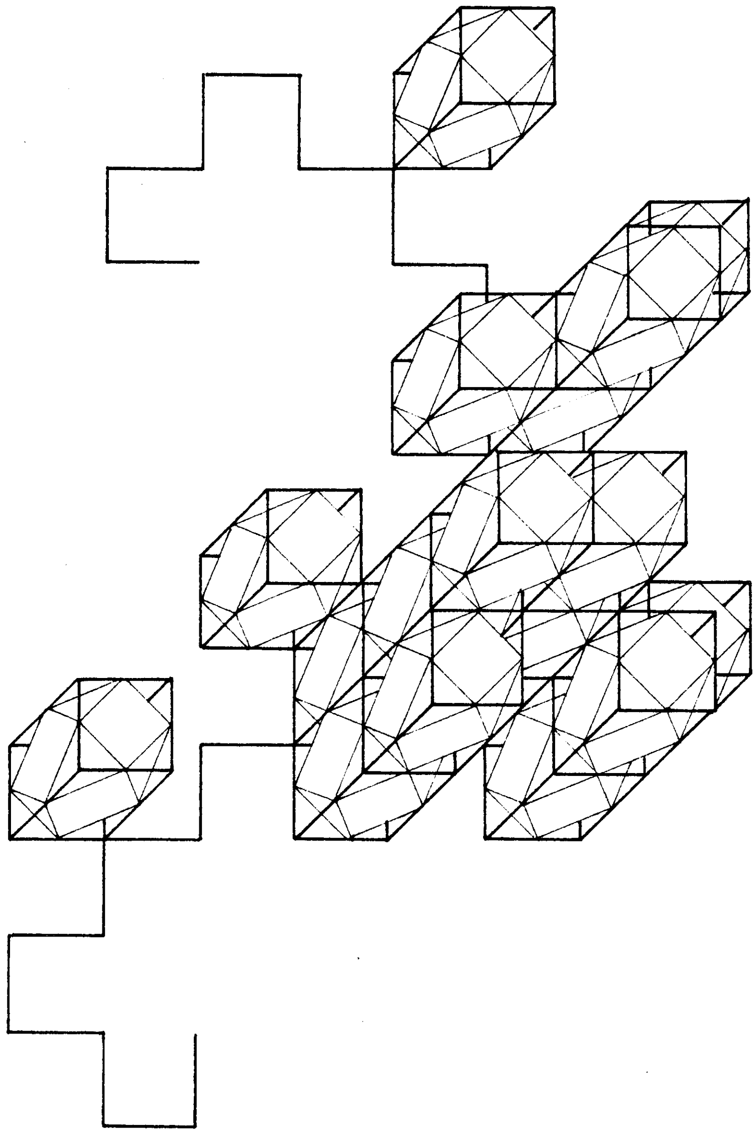


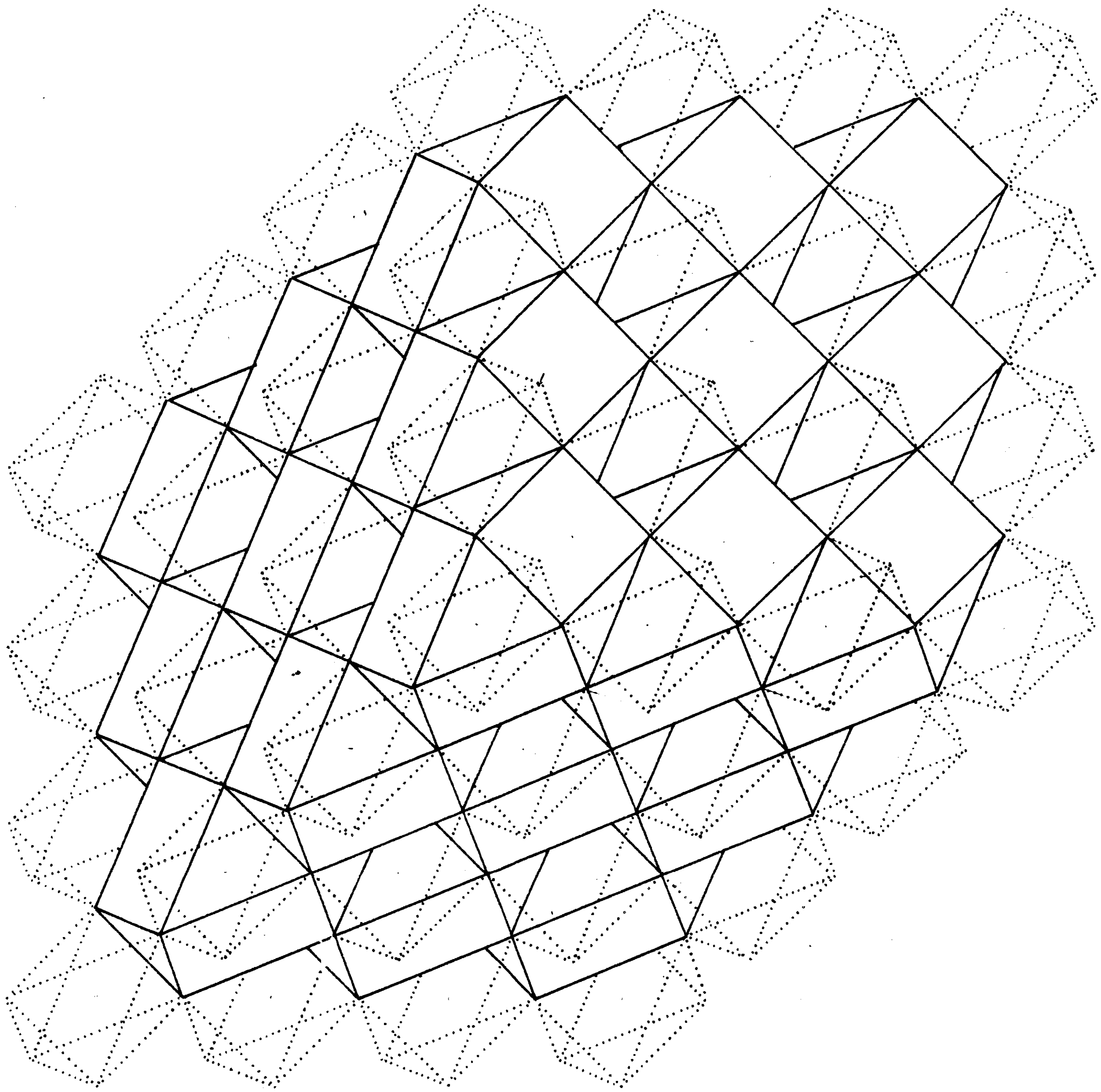


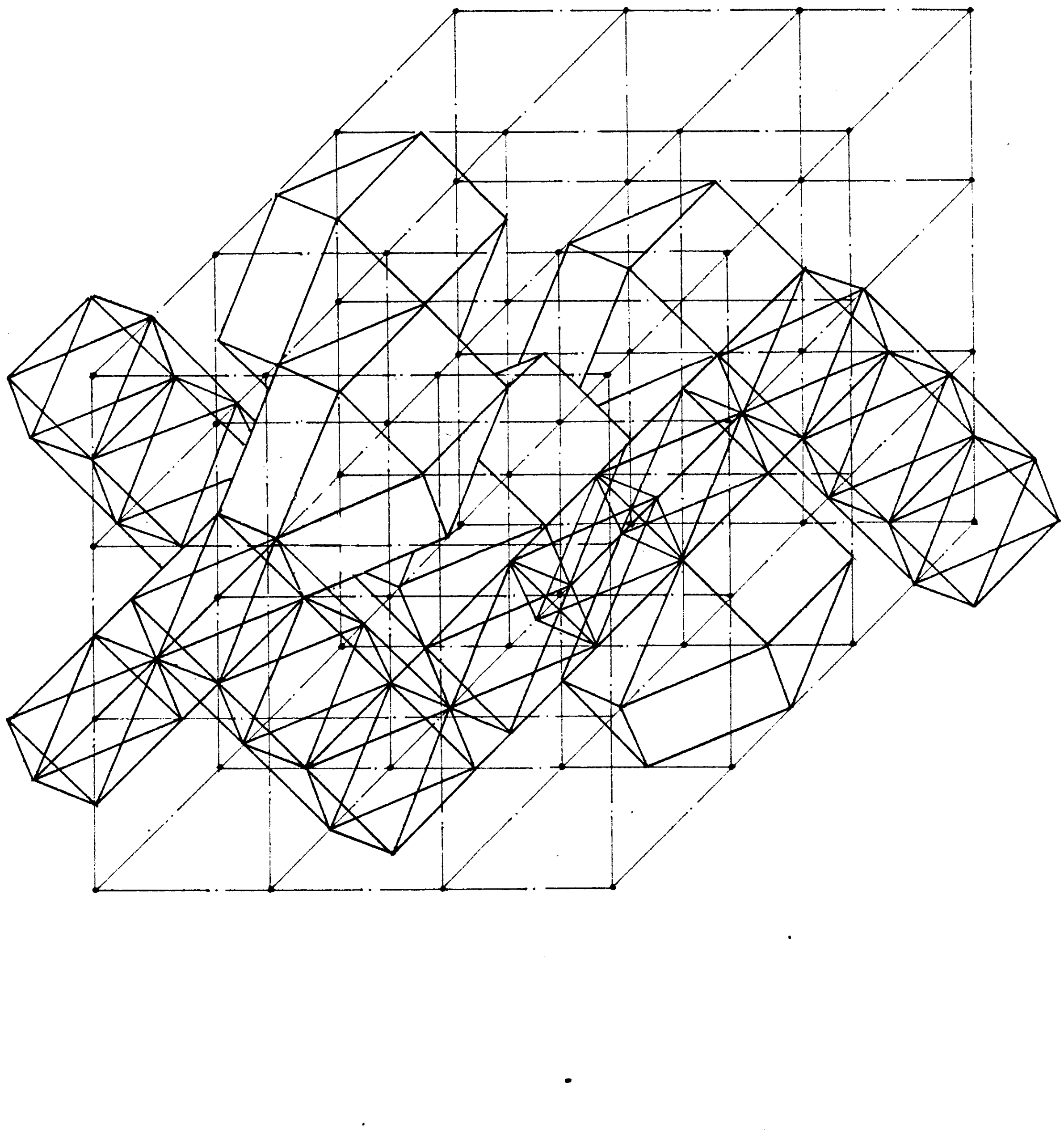


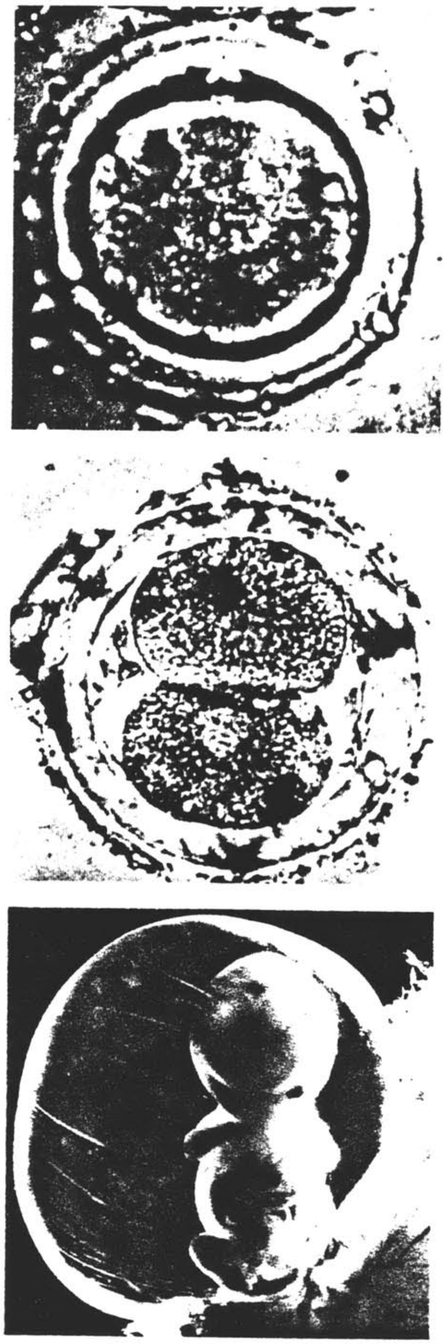


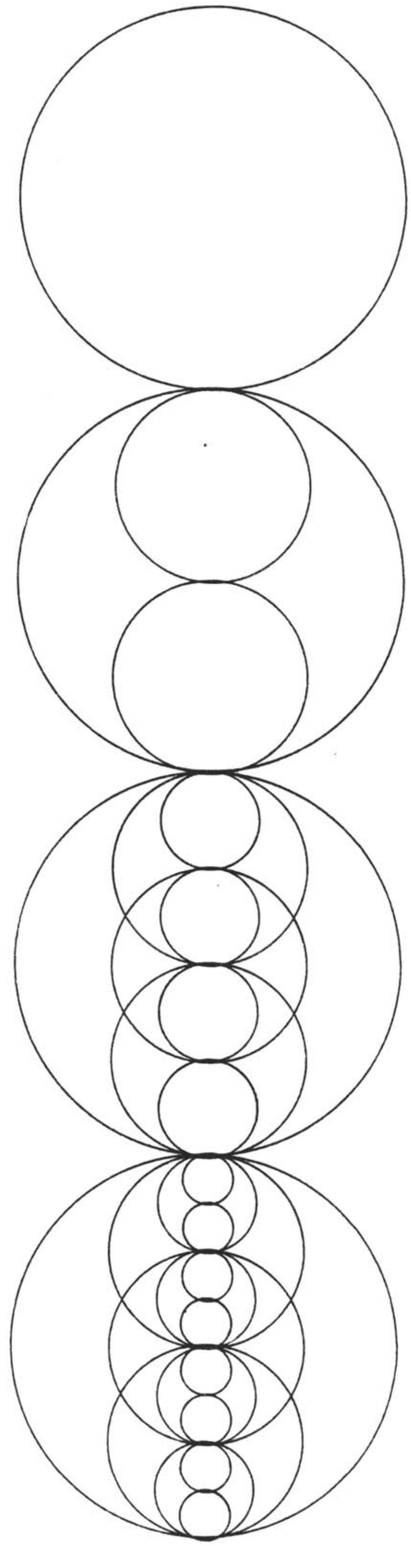


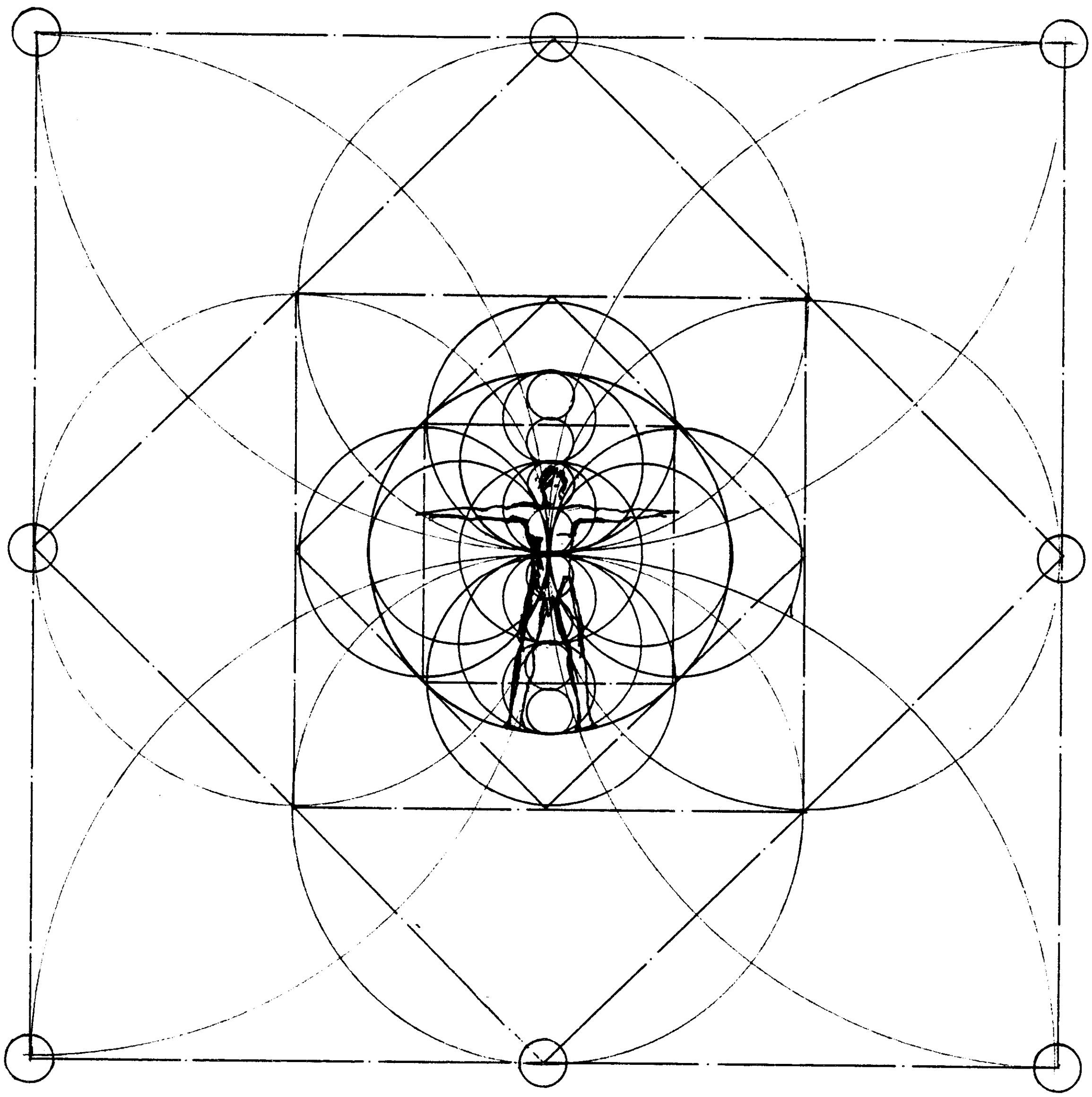


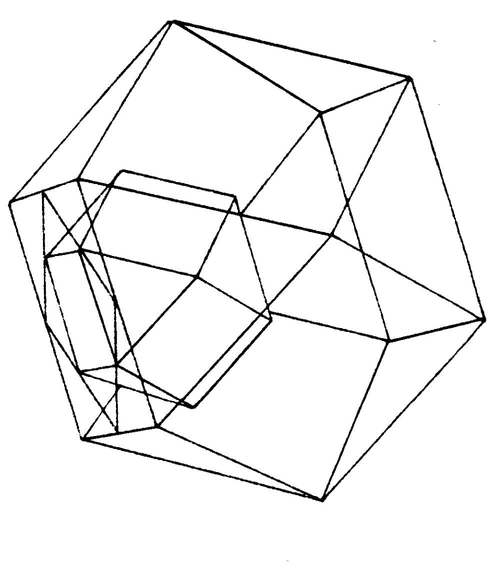


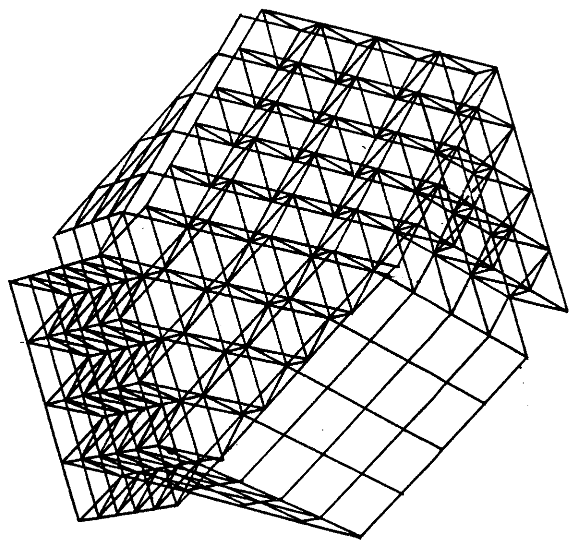


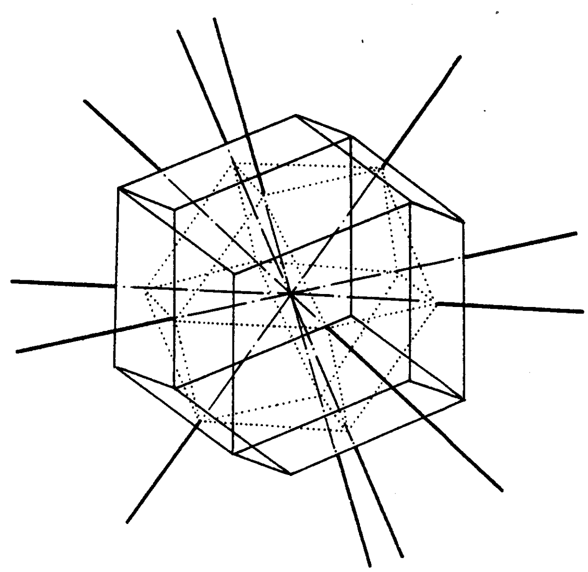


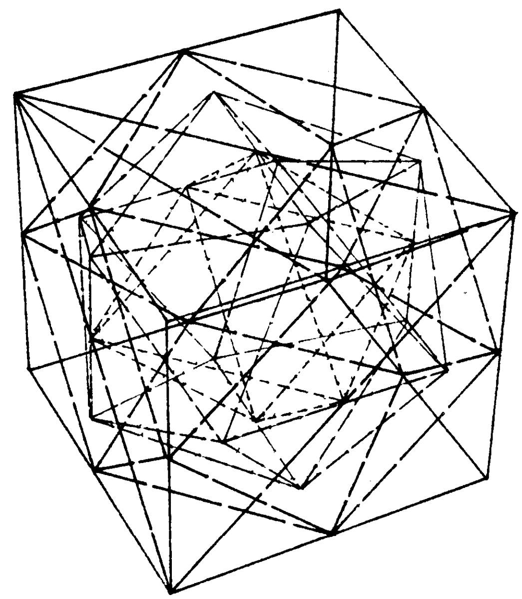


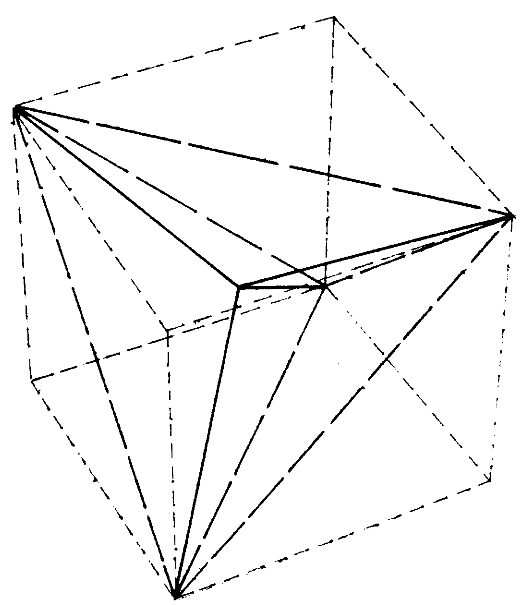


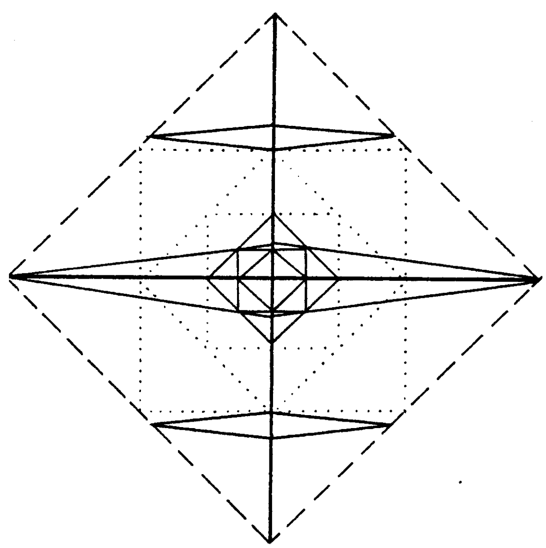


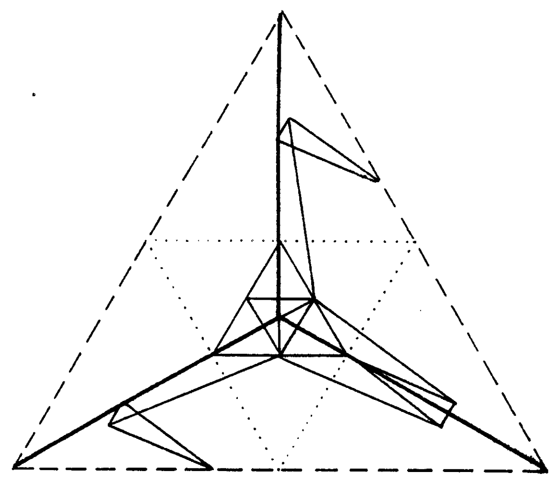


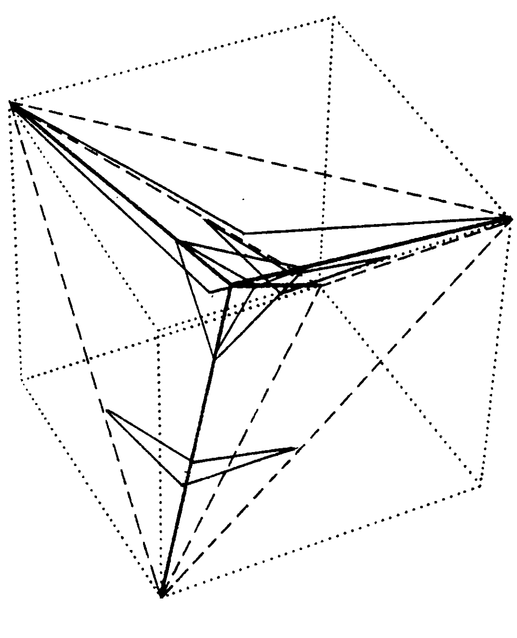


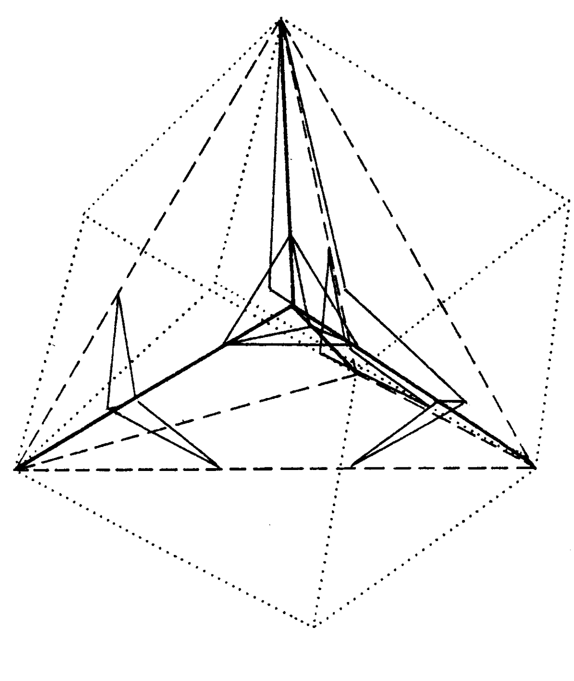


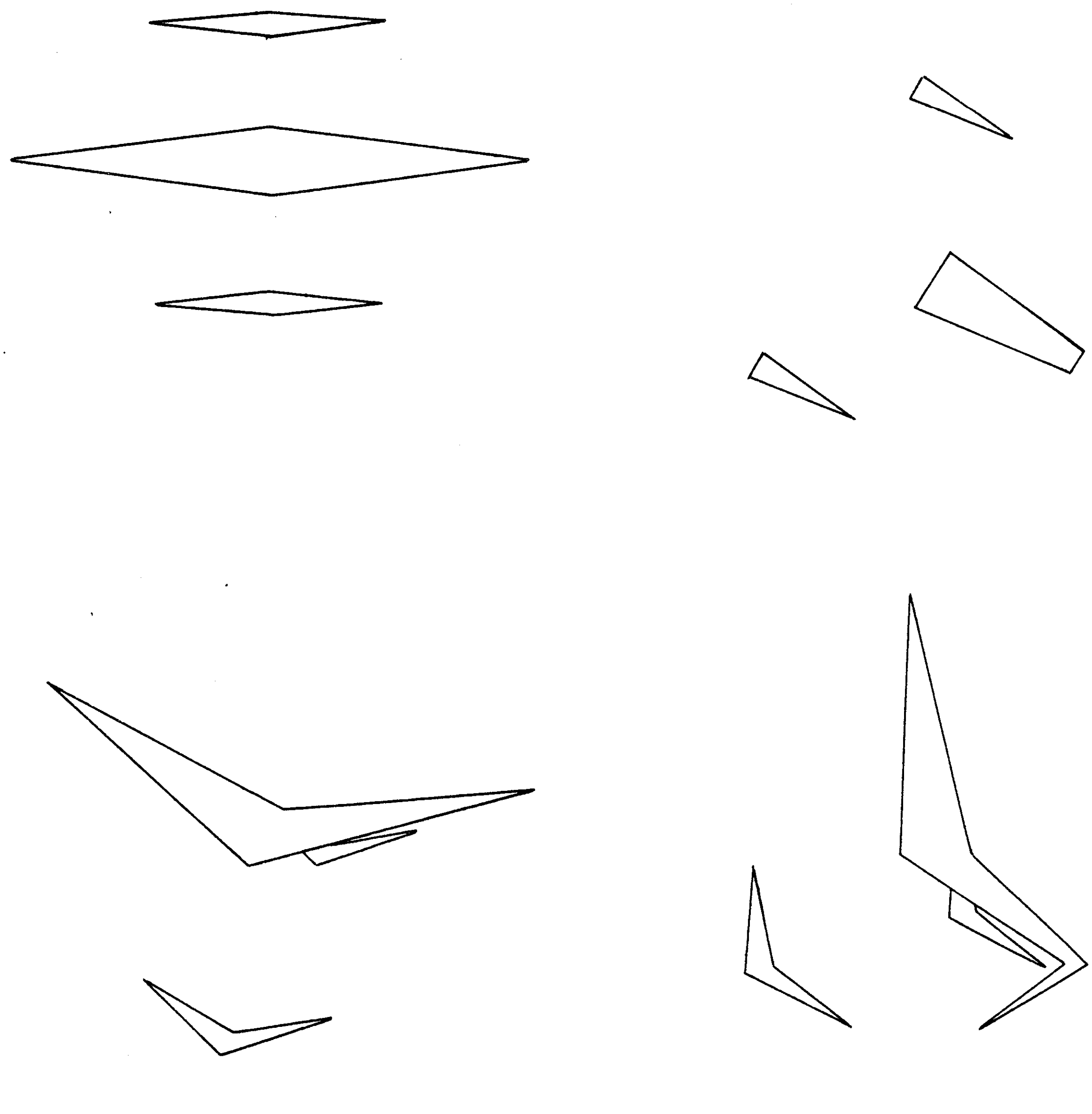












BIBLIOGRAPHY

Abelson, H. and A. Disessa. Turtle Geometry, The Computer as a Medium for Exploring Mathematics. (M.I.T. Press, Cambridge, 1980).

Bronstein, Leo. Fragment of Life, Metaphysics, and Art. (Bond Wheelwright Company, 1953).

Critchlow, Kieth. Islamic Patterns, an Analytical and Cosmological Approach. (Schocken Books, New York, 1976).

Critchlow, Kieth. Order in Space. (Viking Press, New York, 1971).

Dawkins, Richard. The Selfish Gene. (Oxford University Press, New York, 1976).

Eames, Ray and Charles Morrison, Philip and Phylis. Powers of Ten. (W.H. Freeman and Company, New York, 1982).

Eigen, M. and R. Winkler. Laws of the Game; How the Principles of Nature Govern Chances. (Harper Colophon, New York, 1983).

Foucault, Michael. The Archeology of Knowldge, and the Discourse on Language. (Pantheon, New York, 1982).

Fuller, R.B. Synergetics I, Explorations in the Geometry of Thinking. (MacMillan Publishing Co., New York, 1978).

Fuller, R.B. Synergetics II, Further Explorations in the Geometry of Thinking, (MacMillan Publishing Co., New York, 1979).

Gardner, Martin. Wheels, Life, and Other Mathematical Amusements. (W.H. Freeman and Company, New York, 1983).

Hofstadter, Douglas R. Godel, Escher, Bach: An Eternal Golden Braid. (Basic Books, New York, 1979).

Hofstadter, Douglas R. and Daniel L. Dennet. The Mind's I: Fantasies and Reflections on Self and Soul. (Basic Books, New York, 1981).

Kepes, Gyorgy, ed. The New Landscape. (George Brazilier, New York, 1956).

Kepes, Gyorgy, Ed. Structure in Art and Science (George Brazilier, New York, 1965).

Keapes, Gyorgy ed. Module, Proportion, Symmetry, Rythm. (George Brazilier, New York, 1966).

Knowles, Ralph L. Energy and Form. (M.I.T. Press, Cambridge, 1979).

Lalvani, Haresh. Transpoloyhedra. (Haresh Lalvani, New York, 1977).

Leavitt, Ruth. Artist and Computer. (Creative Computing, Morristown, 1976).

LeRicolais, Robert. Towards a Technique of Forms and Forms and Structures in Zodiac 22: Light Structures (Milan, 1973).

Loeb, Arthur L. Space Structures, Their Harmony and Counterpoint. (Addison-Wesley Publishing Company, Reading, Mass.)

Mandelbrot, Benoit. Fractals: Form, Chance, Dimension. (W.H. Freeman and Company, New York, 1977).

Mandelbrot, Benoit. The Fractal Geometry of Nature. (W.H. Freeman and Company, New York, 1980).

Otto, Frei. Naturliche Konstruktionen. (Deutsche Verlags-Anstalt Gmbh, Stuttgart, 1982).

Pearce, Peter. Strucutre in Nature is a Strategy for Design. (M.I.T. Press, Cambridge, 1978).

Plato. Timaeus and Critius. (Penguin Books, New York, 1965).

Rebbi, Claudio. Lattice Theory of Quaric Confinement in Scientific American, Feb. '83. (Scientific American, New York, 1983).

Silk, J., A.S. Szalay and Y.B. Zel'Dovich. Large Scale Structure of the Universe in Scientific American, Oct. '83. (Scientific American,New York, 1983).

Thompson, Sir D'Arcy. On Growth and Form. (Cambridge Univeristy Press, Cambridge, 1961).

Tyng, Ann Griswold. Resonance Between Eye and Archetype in VIA 6 Architecture and Visual Perception. (M.I.T. Press, Cambridge, 1983).

Weschler, Judith. On Aesthetics in Science. (M.I.T. Press, Cambridge, 1978).

Wiener, Norbert. Cybernetics (M.I.T. Press, Cambridge, 1965).

Wiener, Norbert. God and Golem. (M.I.T. Press, Cambridge, 1969).

Wiener, Norbert. The Human Use of Human Beings. (Avon Books, New York, 1976).

GLOSSARY

Automorphism-- The term s not new but like most of the other crucial terms in this thesis, it's conventional meaning is somewhat vague. Crystalographers have used it to imply strict isometries while people like Le Ricollet have defined it in topological terms, that is, an equivalence of incidence. Like the term "symmetry" its meaning depends on who is using it and for what purposes. I have tried to maintain the generality implied by its roots "autos" meaning self and "morphe" meaning form although most of the examples in this thesis are those which are identity, isometry, or similarity transformations.

Homogeneity-- Refers to invariance under displacement and change of scale. The assumption of spatial homogeneity of homogenous distributions of some quantity is the assumption of modular coordination.

Symmetry-- Refers to richer properties of invariance than those included under the term "homogenous" (translation) specifically invariance under rotation and reflection. Symmetry operations are those which preserve length, the identity and isometry transformations. There are various other mappings or transformations which can be tabulated showing progressive removal of the restraints of invariance.

Identity: invariance of position, length, angles, ratios of angles and sides, parallelism, point-order or connectivity.

Isometry: iso-equal, metry-lengths, transformations of reflection, rotation and translation.

Similarity: neither length nor position are preserved, but angles, the ratios of angles and sizes, parallelism, and point-order are invariant.

Affinity: only parallelism and point-order remain invariant. Shadows cast by the sun onto a flat surface are affinities because the sun's rays are for most intents and purposes parallel, and consequently parallels are mapped onto parallels.

Topology: only point-order or connectivity is invariant. An example is a square mapped onto the cortex via the retina as a curvilinear quadrangle.

Periodic (Spatial Systems)-- quantization of space through repetive spatial units. More specifically it refers to regular and remiregular polyhedrons (platonic and archimedian).

Stability-- refers to a "persistence" of structure in time (space) due to self replicaiton. This stable structures are those which possess H high degree of symmetry. Refers to the minimization of potential energy