CONTROL FOR LARGE SCALE AND UNCERTAIN SYSTEMS

(INTERIM REPORT)

by

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SUMMARY

This report presents an overview of the research carried out by faculty and students of the Decision and Control Sciences Group of the M.I.T. Electronic Systems Laboratory with partial support provided by the United States Air Force Office of Scientific Research under grant AF-AFOSR-72-2273. The grant monitor was Walter J. Rabe, Lt. Col. USAF.

The time period covered in this report is from January 1, 1975 to January 1, 1976.

The following faculty members received partial salary support under the above grant: Professor Michael Athans, Professor Sanjoy K. Mitter; Professor Alan S. Willsky, Professor Nils R. Sandell, Jr. and Professor Pravin Varaiya. Professor Timothy L. Johnson and Professor Adrian Segall also participated in the research, but did not receive direct salary support from the grant.

Several students participated in the research effort. The research assistant was Mr. D. Teneketzis. In addition, the following students participated in the research effort with no direct salary support from the grant: Mr. S. Marcus, Mr. D. Castagnon and Mr. R. Kwong.
0. Introduction

The research activities carried out during the current grant period are described in the sequel under the following headings. We remark that the descriptions of the research projects are brief, since many of the technical details can be found in the cited references.

1. Stochastic Systems
   1.1. Modelling and Identification
   1.2. Dynamic Hypothesis Testing for Estimation and Control
   1.3. Filtering for Delay Systems
   1.4. Analysis and Estimation for Non-linear Stochastic Systems
   1.5. Bounds for Non-linear Filtering

2. Stochastic Control of Linear Systems by the Multiple Time Scales Method

3. Adaptive Control
   3.1. Introduction
   3.2. Robustness of Linear Quadratic Designs
   3.3. Probing and Learning in Adaptive Control
   3.4. Adaptive and Communications Aspects of Non-classical Control Theory

4. Topics in Infinite Dimensional Systems
1. **Stochastic Systems**

1.1 **Modelling and Identification**

One of the fundamental problems in control of uncertain systems is the modelling of the input and sensor noise processes. To apply techniques of control theory which have been developed in the last few years it is necessary to model these stochastic processes as stochastic dynamical systems. Professor Mitter has initiated work on stochastic realization theory with a view to understanding the fundamental problems involved here. The goal of this work is to construct a stochastic realization theory (analogous to deterministic realization theory for linear systems) and computational algorithms for realizations.

Professor Mitter has also continued his work on the modelling of noise and filtering in the presence of such noise. The motivation for the original work (reported in the doctoral thesis of Larry Horowitz entitled "Optimal Filtering of Gyroscopic Noise") was to understand the gyro noise. Since then we have discovered that noise processes with these characteristics arise in a variety of other situations (e.g., flicker noise frequency fluctuations in accurate clocks). Furthermore, a better understanding of the filter is needed from the implementation point of view. Spectral factorization methods may also be appropriate in the solution of these problems. A joint paper with Larry Horowitz is now being prepared.

The problem of identification is undoubtedly one of the central problems of estimation and control theory. Its practical importance stems from the fact that application of the many tools of modern control and estimation theory is conditioned on the availability of adequate mathematical models.
The detection problem can be considered a specialization of the identification problem to the case where the sought mathematical model is known to belong to a finite set of known models. In spite of the considerable amount of work devoted to the identification problem in recent years, some essential questions are still unanswered. The question of the necessary or sufficient conditions on the statistical structure of the observation process for identifiability of parameters characterizing the process has only partly been answered. Statisticians have been mostly concerned with the case of independent observations, which is of very little interest to system engineers. The treatment of the problem in the system theoretic literature is in general inadequate, as proofs are often incorrect or excessive in the assumptions and the conditions imposed on the problem.

Mr. Y. Baram and Prof. N. R. Sandell, Jr., have initiated an investigation of the problem of identifiability and detectibility of uncertain systems and random signals in a methodical and systematic manner. Since identifiability of parameters is defined as consistency of the parameter estimates, it seems natural to enquire into the consistency of parameter estimation techniques. The consistency of maximum likelihood and least squares estimates for classes of dependent observations is currently being investigated. Determination of necessary and/or sufficient conditions for consistency of parameter estimates will provide clues to questions of system identifiability and signal detectibility.

1.2 Dynamic Hypothesis Testing for Stochastic Estimation and Control

The long range research of Professor Athans and Dr. K.-P. Dunn on dynamic hypothesis algorithms (or multiple model models as we call them)
has paid off in three distinct application areas. The first application area deals with the design of adaptive stability augmentation systems for high performance aircraft and was illustrated in a case study involving the F8-C Digital Fly By Wire Aircraft. The second application area involved a collaborative effort with Dr. R. H. Whiting and Dr. M. Gruber of the M.I.T. Lincoln Laboratory, in which dynamic hypothesis testing ideas were used to design an improved tracking algorithm for dealing with the wake phenomena inherent in the tracking and discrimination of reentry vehicles. The reason that this was done in collaboration with the M.I.T. Lincoln Laboratory was that they had actual radar measurements and could provide the necessary algorithmic support. The third application area dealt with the design of a tracking and discriminating algorithm that could be used in maneuvering reentry vehicle applications; once more this was a collaborative effort with Drs. C. B. Chang and R. H. Whiting of the M.I.T. Lincoln Laboratory.

The pragmatic experience obtained from these three application areas has been extremely valuable, because they demonstrated the tremendous power and shortcomings of the available theoretical results. They have provided valuable insight into future relevant theoretical and algorithmic research in the area of dynamic hypothesis testing and its use in adaptive estimation and control.
1.3 Filtering for Delay Differential Systems

Work on filtering for linear and nonlinear time delay systems has continued over this past year. This work has been done by Dr. R. W. Kwong, Professor A. S. Willsky, and Professor S. K. Mitter. Briefly, the problem under consideration is described by the functional differential equation

$$\frac{d\tilde{x}(t)}{dt} = f(\tilde{x}_t) + dw(t)$$

where \( \tilde{x}_t \) is the state of the system defined by

$$\tilde{x}_t(\theta) = x(t + \theta) \quad -\tau \leq \theta \leq 0$$

Thus, (1) includes some time delay effects. Similarly, the measurement equation is

$$dz(t) = h(\tilde{x}_t)dt + dv(t)$$

and in this model one can consider "multipath" or "echo" effects -- e.g.

$$h(\tilde{x}_t) = ax(t) + bx(t - \tau)$$

The major results obtained are as follows. A general integral representation formula has been obtained for the conditional estimate

$$E[\phi(\tilde{x}_t) | z(s), 0 \leq s \leq t]$$

of any function \( \phi(\tilde{x}_t) \) of the trajectory piece \( \tilde{x}_t \). Recursive filtering equations are shown to exist in the linear case and in the nonlinear case when \( h \) and \( f \) satisfy certain smoothness conditions (e.g., pure point delays cannot be accommodated and one needs some type of "blurring"

$$h(\tilde{x}_t) = \int_{-\tau}^{0} p(\theta)x(t + \theta)d\theta$$
In the linear-point delay case, however, a full set of filter equations are rigorously derived, and, in addition, the stability of the steady-state filter is shown when the only delays are in the dynamics (1) and when certain observability-type conditions are satisfied.

Presently, Prof. Willsky and Mr. P. Y. Kam are extending some of these results to the extremely important problem when the time delay \( \tau \) is not known. Examples of this arise in various signal processing problems in which one is trying to estimate the distance from transmitter to receiver (or reflector) given that multipath effects may corrupt the received signal.

Using methods to study the infinite-time quadratic cost problem for infinite dimensional systems Professor Mitter (in collaboration with Dr. R. B. Venter, formerly at M.I.T. and now at Imperial College) has been able to prove the stability of the filter (in the linear case) using hypotheses of stabilizability and detectability. The stabilizability and detectability conditions can be verified using matrix tests. The filter obtained is however infinite dimensional and work is now in progress to understand how the filter can be implemented using adders, multiplier and delay elements.

1.4 Analysis and Estimation of Nonlinear Stochastic Systems

During this past year, work has continued in the area of stochastic analysis for nonlinear systems possessing certain types of structure. This work has been carried out under the supervision of Prof. Willsky, with the aid, in particular, of Dr. Steven I. Marcus who completed his Ph.D. thesis in this area. The major results obtained in this area arose out of the consideration of following bilinear estimation problem
\[ \dot{x}(t) = \left[ A_0 + \sum_{i=1}^{N} A_i \xi_i(t) \right] x(t) \]  
\[ \dot{\xi}(t) = F\xi(t) + \dot{v}(t) \]

where we observe

\[ z_1(t) = H\xi(t) + v_1(t) \]
\[ z_2(t) = Hx(t) + v_2(t) \]

The case in which we observe the process \( z_1 \) is motivated by strapdown navigation problems, in which \( \xi \) can be interpreted as an angular velocity vector. This class of problems has yielded one of the first large class of nonlinear filtering problems for which there exist explicit, finite-dimensional optimal solutions. The basic method behind the work reported in [10-12] is the expansion of the solution of (1) in a Volterra series

\[ x(t) = g_0(t) + \sum_{i=1}^{N} \int_{0}^{t} \mu_{1i}(s) \xi_1(s) ds + \sum_{i,j=1}^{N} \int_{0}^{t} \int_{0}^{T} \mu_{2ij}(\tau, s) \xi_1(s) \xi_j(t) ds d\tau + \ldots \]  

One of the major results obtained is a set of finite-dimensional nonlinear stochastic equations for the optimal estimate of each term in this series. Thus if the series is finite -- which is equivalent to an algebraic conditions on the \( A_i \) in (1) -- one obtains a finite-dimensional optimal estimate for \( x \). Generalizations of this to some cases in which (5) need not be finite are given in [10-12]. In addition, in these references it is shown that many nonlinear systems have representations as Volterra series as in (5), and optimal estimation schemes for these are derived.
In the cases in which the series (5) does not lead to finite-dimensional optimal estimators, one can use an approximation technique to truncate the series. The utility of this -- especially for the strapdown problem -- is being investigated.

If one wants to consider estimation given the measurement (4), the problem is much more difficult. These models arise in synchronous communication, satellite tracking, and incorporation of measurements such as from a star tracker into an estimate of attitude. The problem (1), (2), (4) leads immediately to infinite-dimensional filter equations. For the phase tracking problem, Willsky had earlier used a Fourier series method to obtain suboptimal estimators. Work is presently continuing in this area -- performed by Mr. J. Eterno and Mr. V. Klebanoff -- in order to obtain a priori performance bounds for the phase tracking systems and to develop alternative designs. In addition, Marcus [10] has extended these concepts to the design of harmonic-analysis-based nonlinear filters for satellite tracking and attitude estimation. The simulation performance of these filters is presently under investigation.

1.5 Bounds for Non-Linear Filtering

Professor Mitter has continued his work on obtaining upper and lower performance bounds for non-linear filtering. As reported last year, various results were obtained in the doctoral dissertation of Dr. Jorge Galdos but there were many questions unresolved. Work this year has been done in trying to understand whether martingale methods and martingale inequalities can be used to evaluate approximately (or bound) various
integrals that arise in the expressions for the lower bounds. A paper with Dr. Galdos is now in preparation.
2. **Stochastic Control of Linear Systems by the Multiple Time Scales Method**

Natural phenomena in engineering systems often evolve in widely separated time scales. Specifically, one can point to the separation of the phugoid and short period modes in the longitudinal dynamics of an aircraft; the separation of the rigid body and flexure modes of aircraft, spacecraft, and launch vehicles; and the separation of the Schuler and earth-rate frequencies in inertial navigation systems. Effects of this type are often exploited in other engineering sciences, but have only recently been introduced into control science by Kokotovic and a few other workers.

Mr. D. Teneketzis and Prof. N. R. Sandell, Jr. have investigated the application of multiple time scale methods to the stochastic linear regulator problem. Using the methodology of singular perturbation theory, a hierarchically structured controller has been obtained that is optimal in the limit as the separation of the time scales becomes infinite. The higher levels of the controller deal with the slower system dynamics and do not require any information from the lower levels. The lower levels of the controller deal with the faster system dynamics and require information from the higher levels. Thus the higher level portion of the control algorithm can be iterated at a much slower rate than would be possible by a single time scale design with an attendant saving in on-line computation. Significantly, this savings is not achieved at the expense of additional off-line computation. Indeed, the off-line computations of the multiple time-scale design are reduced in size and are better conditioned than those of the single time scale design.

The results will be documented in the M.S. thesis of Mr. Teneketzis, which should be ready shortly.
3. Adaptive Control

3.1 Introduction

The need for adaptive control is necessitated by the need to take into account explicit variations in the parameters of the open loop system. Several adaptive techniques are available in the literature each requiring different amounts of real time computation.

What is not clear from an engineering viewpoint is when adaptive control should be used. If one follows the modern trend in evaluating the performance of an adaptive stochastic control algorithm not only by its identification accuracy, but rather by the performance improvement (as measured by a well defined performance index) of the overall system, then it is desirable to have an idea of

(a) the robustness of a non-adaptive, i.e. fixed gain, stochastic control algorithm

(b) a "worst-case" type of performance that might be expected if parameter identification cannot be carried out.

To properly answer these questions is the subject of a long range research program. During the past year we have initiated two basic research efforts. We feel that partial results we have obtained provide a basis for understanding not only the issue of when one should use adaptive control, but hopefully shed some light upon the development of adaptive control algorithms which are computationally more efficient than present dual methods.

Of fundamental importance is the interaction of system uncertainty, as exhibited by large parameter variations in the open loop dynamics, and the trade-off parameters that appear in the performance index to be optimized.
3.2 Robustness of Linear Quadratic Designs

In a study carried out by Professor Athans and Mr. P.K. Wong we investigated the robustness of a deterministic linear-quadratic (LQ) (cf. Ref. 4, List of Publications) optimal design, with respect to guaranteed stability and with respect to changes in the gains of the control system (which may be caused by changes in the weighting matrices in the quadratic performance index and/or changes in the nominal values of the matrices that define the open loop dynamics). Two main results have been obtained.

(a) A Multivariable Infinite Gain Margin Theorem, which states that in a LQ-optimal design all control gains can be arbitrarily increased and yet the closed loop system remains stable

(b) A Multivariable Gain Reduction and Robustness Theorem, which provides sufficient conditions for a limited reduction in the loop gain, while still guaranteeing the stability of the closed loop system.

At the present time we are examining closely the implication of these "robustness" results in the context of adaptive systems.

3.3 Probing and Learning in Adaptive Control

Related work has been done by Professor M. Athans and Mr. R. Ku, which deals with a discrete-time version of a linear quadratic controller for a system with uncertain parameters. In this study, we are considering a worst-case type of analysis in the sense that we want to "tell the mathematics" in a rigorous manner that the stochastic control algorithm cannot identify the uncertain parameters of the dynamic system, so that the dual effects of the control are eliminated. One way of accomplishing this is to model the unknown parameters as being white in time with known means and covariances. We are examining the implication of this stochastic control problem by considering all possible correla-
tions of the parameters with themselves and the instantaneous values of the state variables. Technically this requires the extension of some previous results due to Aoki. However, when one considers all the possible cross-correlations one obtains additional insight about the issues of

(a) caution and probing due to the parameter uncertainties
(b) instantaneous vs long-range learning.

No formal documentation of these results are available as yet.

Our long range goal is to attempt to combine the robustness results with the "white noise" parameter results so as to obtain at the very least some bounds on the improvement in performance by adaptive control, which would serve as a basis for dual control suboptimal adaptive control algorithms with less stringent computational requirements.

3.4 Adaptive and Communications Aspects of Non-classical Control Theory

Mr. David Castagnon with Professors Athans and Sandell have been investigating adaptive and communication aspects of non-classical stochastic control theory. These problems are characterized by information patterns available to the controller which are not necessarily of the type occurring in stochastic control.

The problem considered was one where perfect recall is not present. This is a stochastic control problem whereby, through memory limitations or other factors, the controller may lose information from time to time. A mathematical framework similar to Witsenhausen's was developed, using this framework, general Necessary Conditions were developed for the existence of a solution. Similarly, several results based on duality properties were established which produced some feasible techniques to obtain solutions to the problem. Prior to these
developments, Nils Sandell and M. Athans [Ref. 4] had taken similar approaches in solving some Non-classical stochastic control problems characterized by Finite-state, finite-memory problems.

4. **Topics in Infinite Dimensional Systems**

   Over the past five years, Professor Mitter has been working on various aspects of infinite dimensional systems. The motivation for this has been to develop a theory of modelling, estimation and control for systems with time delay and distributed parameter systems. The theory for delay systems is quite complete. An account of this work is presented in [Refs. 16-18] which has been transmitted to Colonel Rabe. It is hoped that a monograph will be published based on these reports and we have had preliminary discussions with Professor Bellman and Academic Press about it.
LIST OF PUBLICATIONS

The following papers, reports and preprints have appeared during this time period supported in part or in full by the above grant. Copies have been transmitted to the grant monitor, Lt. Colonel W.J. Rabe.


17. A. Bensoussan, M.C. Delfour, and S.K. Mitter, "Optimal Control of Linear Systems with a Quadratic Cost Criterion", ESL-P-603, June 1975, Part II (this report forms a chapter of a forthcoming monograph entitled: "Representation and Control of Infinite Dimensional Linear Systems").
