MIT Libraries | DSpace@MIT

MIT Open Access Articles

Collaborative multi-vehicle localization and mapping in high clutter environments

The MIT Faculty has made this article openly available. ***Please share*** how this access benefits you. Your story matters.

Citation: Moratuwage, M. D. P., W. S. Wijesoma, B. Kalyan, Nicholas M. Patrikalakis, and Peyman Moghadam. Collaborative Multi-vehicle Localization and Mapping in High Clutter Environments. In 2010 11th International Conference on Control Automation Robotics & Vision, Singapore, 7-10th December 2010, pp.1422-1427. © Copyright 2010 IEEE.

As Published: http://dx.doi.org/10.1109/ICARCV.2010.5707778

Publisher: Institute of Electrical and Electronics Engineers

Persistent URL: http://hdl.handle.net/1721.1/79055

Version: Final published version: final published article, as it appeared in a journal, conference proceedings, or other formally published context

Terms of Use: Article is made available in accordance with the publisher's policy and may be subject to US copyright law. Please refer to the publisher's site for terms of use.

Massachusetts Institute of Technology

2010 11th Int. Conf. Control, Automation, Robotics and Vision
Singapore, 7-10th December 2010

Collaborative Multi-Vehicle Localization and Mapping in High Clutter Environments

Moratuwage M.D.P.*, Wijesoma W.S.*, Kalyan B.*, Nicholas M. Patrikalakis†, Peyman Moghadam*
*School of Electrical and Electronic Engineering
Nanyang Technological University
50, Nanyang Avenue, Singapore 639798
†Department of Mechanical Engineering
Center for Ocean Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139

***Abstract*—Among today's robotics applications, exploration missions in dynamic, high clutter and uncertain environmental conditions is quite common. Autonomous multi-vehicle systems come in handy for such exploration missions since a team of autonomous vehicels can explore an environment more efficiently and reliably than a single autonomous vehicle (AV). In order to improve the navigation accuracy, especially in the absence of a priori feature maps, various simultaneous localization and mapping (SLAM) algorithms are widely used in such applications. As for multi-vehicle scenarios, collaborative multi-vehicle simultaneous localization and mapping algorithm (CSLAM) is an effective strategy. However use of multiple AVs poses additional scaling problems such as inter-vehicle map fusion, and data association which needs to be addressed. Although existing CSLAM algorithms are shown to perform quite adequately in simulations, their performance is much less to be desired in high clutter scenarios that is inevitable in actual environments. In this paper, we present an approach to improve the performance of a CSLAM algorithm in the presence of high clutter, by combining an effective clutter filter framework based on Random Finite Sets (RFS). The performance of the improved CSLAM algorithm is evaluated using simulations under varying clutter conditions.**

***Index Terms*—SLAM, Multi-vehicle, Localization, PHD Filter, Random Finite Sets, RFS, FIIST**

I. Introduction

Autonomous vehicles (AV) are making a significant impact in a diverse set of applications, including exploration of unstructured environments, various surveillance missions, search and rescue opertions to name a few. Multiple AV deployments have become common in such missions, due to the sheer complexity of assignment, extent of the environment and performance gains that can be achieved in termns of speed and accuracy.

In order to effectively utilize AVs in such missions, it's essential to localize themselves in possibly unknown and unstructured environment, in which they are deployed. For example if it's a hydrographic survey in shallow waters, multiple autonomous surface crafts (ASC) might be deployed to collaboratively scan a marine environment using multi-beam sonar scanners for extracting features and objects on the sea bed. The features that are extracted should be combined with accurate positional information in order to build accurate maps to be used for further examination. Even though GPS data (in surface and ground vehicles) can be used for localization, there is still a possibility that due to changing atmospheric and field conditions, GPS measurements might be unavailable or inaccurate. To overcome the limitations, uncertainties and inaccuracies caused by sensors, Simultaneous Localization and Mapping (SLAM) algorithm was introduced [1]. SLAM [2] [3] [4] algorithm exploits artificial and/or naturally occuring features in the operating environment for mapping and localization of the vehicles. Using the well known SLAM framework as a basis, few collaborative localization and mapping algorithms (CSLAM) were proposed for multi-vehicle autonomous vehicles.

Although such algorithms perform adequately well in simulated environments, in field conditions they tend to perform poorly, because of high clutter returns from the environment and also inaccurate sensor models used and sensor noises. Out of these, clutter produced by sensors can significantly degrade navigation performance, because such data, if not processed correctly cause SLAM algorithms to produce inaccurate and inconsistent results. As an example multi-beam sonar sensor used in shallow sea-bed mapping applications produces measurement data with high clutter near the overlapping areas of beam scans. This requires that the sensor data be filtered from clutter before being used for feature extraction. In this paper, we present an approach to improve the performance of a CSLAM algorithm in the presence of high clutter, by combining an effective clutter filter framework based on Random Finite Sets (RFS).

This paper is organized as follows. In Section II we describe the related work that lead to multi-vehicle SLAM and clutter filtering approach using an RFS framework. In Section III we briefly describe the probabilistic building blocks of the CSLAM algorithm and how to extend those building blocks to solve multi-vehicle SLAM problem. In Section IV we briefly introduce the concepts in Random Finite Sets (RFS) based filtering, clutter reduction and the improvements that are made to the CSLAM algorithm. In Section V simulation results are presented and discussed and Section VI concludes the paper.

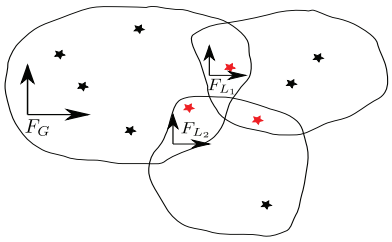


Fig. 1. Two robots start mapping independently, with respect to their local frames of reference. F_G refers to the global reference frame while F_{L_1} and F_{L_2} refers to the local reference frame of the two robots. Black stars in the local frames of reference correspond to the features mapped by each vehicle and the red ones correspond to the overlapping features.

II. Multi-Vehicle SLAM

Decorrelated nature of local sub-maps and delayed data association made CLSF [5] an ideal methodology for Multi-Vehicle SLAM [6]. The collaborating vehicles perform SLAM independent of each other and produce local sub-maps by using the features available in their vicinity. These sub-maps are fused into the existing global map and a consistent single global map is recovered at a later stage. Fig. 1 is a pictorial representation of this scenario.

In order to demonstrate this approach, let's consider the case of two robots collaboratively performing SLAM in an unknown environment (Fig. 1). Both of the robots, start from two arbitrary locations and continue to map with respect to their frame of reference. The origins of both robots with respect to the global reference frame are stored for later use. Once the decision is made to fuse the local maps into the global map, the local sub-maps are first transformed into the global frame of reference. Data association is then performed between features in the global map and local sub-maps to identify common (over lapping) features (red colored in Fig. 1). These common or duplicate features are used as constraints to obtain improved map and vehicle position estimates. After removing the duplicate features from these updated local sub-maps, they are fused into the global map, after which the robots can continue mapping again. The formulation of CLSF based Multi-Vehicle SLAM [6] approach is briefly discussed below.

Suppose the robots are at locatioin estimates, ${}^G\hat{x}^+_{v1}(k-1)$ and ${}^G\hat{x}^+_{v2}(k-1)$ and the map of the feature estimates is denoted by ${}^G\hat{x}^+_m(k-1)$ with respect to the global coordinate frame F_G. The superscript G indicates that the estimates are with respect to the global reference frame. The composite state vector is given by,

$$\hat{x}^+ = \begin{bmatrix} {}^G\hat{x}^+_{v1}(k-1) \\ {}^G\hat{x}^+_{v2}(k-1) \\ {}^G\hat{x}^+_m(k-1) \end{bmatrix} \tag{1}$$

The covariance matrix of the state with respect to the frame F_G is given by,

$$P^+ = \begin{bmatrix} {}^GP^+_{v1v1}(k-1) & {}^GP^+_{v1v2}(k-1) & {}^GP^+_{v1m}(k-1) \\ {}^GP^{+T}_{v1v2}(k-1) & {}^GP^+_{v2v2}(k-1) & {}^GP^+_{v2m}(k-1) \\ {}^GP^{+T}_{v1m}(k-1) & {}^GP^{+T}_{v2m}(k-1) & {}^GP^+_{mm}(k-1) \end{bmatrix} \tag{2}$$

At this point, the decision is made by both robots to build independent sub-maps and perform SLAM within their sensors' field of view. First, two new coordinates frames, F_{L_1} and F_{L_2}, are defined centered at the current vehicle estimates and then, each vehicle initializes a sub-map at the new origin, with no position uncertainity, and continue to perform SLAM. At some later time, both vehicles decide to combine the local sub-maps into the global map. Now the combined state vector is given by,

$$\hat{x}^+_{ism}(k) = \begin{bmatrix} {}^G\hat{x}^+(k) \\ {}^{L_1}\hat{x}^+(k) \\ {}^{L_2}\hat{x}^+(k) \end{bmatrix} \tag{3}$$

with,

$${}^G\hat{x}^+(k) = \begin{bmatrix} {}^G\hat{x}^+_{L_1}(k) \\ {}^G\hat{x}^+_{L_2}(k) \\ {}^G\hat{x}^+_m(k) \end{bmatrix} \tag{4}$$

$${}^{L_1}\hat{x}^+(k) = \begin{bmatrix} {}^{L_1}\hat{x}^+_{v_1}(k) \\ {}^{L_1}\hat{x}^+_m(k) \end{bmatrix} \tag{5}$$

$${}^{L_2}\hat{x}^+(k) = \begin{bmatrix} {}^{L_2}\hat{x}^+_{v_2}(k) \\ {}^{L_2}\hat{x}^+_m(k) \end{bmatrix} \tag{6}$$

where the subscript 'ism' stands for independent sub-maps, which is an aggregated state vector that combines both composite state vector (which contains global vehicle position estimates and global feature estimates) and decorrelated local state vectors (vehicle position estimation and feature estimations) produced by individual vehicles performing SLAM. Vehicle state estimation and feature estimations are given by ${}^{L_i}\hat{x}^+_{v_i}(k)$ and ${}^{L_i}\hat{x}^+_m(k)$ with respect to the frame of reference F_{L_i}. The covariance matrix of the combined state vector is given by,

$$P^+_{ism}(k) = \begin{bmatrix} {}^GP^+(k) & 0 & 0 \\ 0 & {}^{L_1}P^+(k) & 0 \\ 0 & 0 & {}^{L_2}P^+(k) \end{bmatrix} \tag{7}$$

with,

$${}^GP^+(k) = \begin{bmatrix} {}^GP^+_{L_1L_1}(k) & {}^GP^+_{L_1L_2}(k) & {}^GP^+_{L_1m}(k) \\ {}^GP^{+T}_{L_1L_2}(k) & {}^GP^+_{L_2L_2}(k) & {}^GP^+_{L_2m}(k) \\ {}^GP^{+T}_{L_1m}(k) & {}^GP^{+T}_{L_2m}(k) & {}^GP^+_{mm}(k) \end{bmatrix} \tag{8}$$

$${}^{L_1}P^+(k) = \begin{bmatrix} {}^{L_1}P^+_{v_1v_1}(k) & {}^{L_1}P^+_{v_1m}(k) \\ {}^{L_1}P^{+T}_{v_1m}(k) & {}^{L_1}P^{+T}_{mm}(k) \end{bmatrix} \tag{9}$$

$${}^{L_2}P^+(k) = \begin{bmatrix} {}^{L_2}P^+_{v_2v_2}(k) & {}^{L_2}P^+_{v_2m}(k) \\ {}^{L_2}P^{+T}_{v_2m}(k) & {}^{L_2}P^{+T}_{mm}(k) \end{bmatrix} \tag{10}$$

As can be observed, the covariance matrix of the combined state vector is block diagonal, due to the decorrelated sub-mapping approach. Now the combined state vector is transformed into the global reference frame, by a suitable transformation matrix.

$${}^G\hat{x}^+_{ism}(k) = T_G(k).\hat{x}^+_{ism}(k) \tag{11}$$

The corresponding transformed covariance matrix is given by,

$$^{G}P^{+}_{ism}(k) = \nabla T_G(k).P^{+}_{ism}(k).\nabla T^{T}_{G}(k) \tag{12}$$

Now data association is performed in order to identify duplicate (overlapping) features present in local sub-maps and global map. These duplicate features are treated as constraints and a constraints minimization approach is used to recover a more robust estimates for combined state vector and covariance matrix. The constraints can be written in the form,

$$C.^{G}\hat{x}^{+}_{ism}(k) = b. \tag{13}$$

This is solved using linear constraint minimization approach [7] to obtain a better estimation of the combined state vector and covariance matrix. Now a suitable transformation is applied to remove duplicate features. Standard composite state vector and covariance matrix for Multi-Vehicle SLAM is recovered by applying another suitable transformation.

III. CSLAM IN HIGH CLUTTER ENVIRONMENTS

A clutter filter, developed by employing the concepts in multi-target filtering using random finite sets (RFS) introduced by Mahler [8], was combined with conventional extended kalman filter based SLAM (EKF-SLAM) algorithm as shown in Fig. 5. The developed filer is capable of removing clutter from the measurements prior to the update of EKF-SLAM filter. The modified SLAM algorithm, is used in each vehicle performing sub-mapping in high clutter. Subsection A presents an overview of multi-target filtering approach. Subsection B presents a viable approximation to the full multi-target filtering process, and Subsection C presents an implemtation of this approximation that we adopted in our simulations.

A. Random Finite Set (RFS) multi-target Filtering

In RFS multi-target filtering, multi-target state and multi-target measurement at time k are represented as random sets X_K and Z_k. Assume that at time k we have T_k targets and N_k measurements. Then, state and measurement sets can be written as,

$$X_k = (x_{k,1}, x_{k,2}, ..., x_{k,T_k}) \subseteq E_s \tag{14}$$

$$Z_k = (z_{k,1}, z_{k,2}, ..., z_{k,T_k}) \subseteq E_o \tag{15}$$

Analogous to single-target tracking, where uncertainity is characterized by modeling state and observations as random vectors, uncertainity in multi-target tracking systems is characterized by modeling the multi-target state and multi-target measurement as random finite sets (RFS) Ξ_k and Σ_k of respective state and observation spaces E_s and E_o.

Assuming a realization X_{k-1} of Ξ_{k-1} at time $k-1$, the multi-target state at time k can be modeled by the RFS,

$$\Xi_k = S_k(X_{k-1}) \cup \Gamma_k \tag{16}$$

where $S_k(X_{k-1})$ denotes the RFS of targets that have survived from time $k-1$ and Γ_k denotes the spontaneous targets appeared at time k. Similarly measurements at time k can be modelled by the RFS

$$\Sigma_k = \Theta_k(X_k) \cup C_k(X_k) \tag{17}$$

where $\Theta_k(X_k)$ denotes the RFS of measurements generated by the targets having the state X_k, and $C_k(X_k)$ denotes the RFS of clutter.

Using these RFS models, recursion of the optimal multi-target Bayes filter is given by,

$$\begin{aligned} &p_{k|k-1}(X_k|Z_{1:k-1}) \\ &= \int f_{k|k-1}(X_k|X_{k-1}) p_{k-1}(X_{k-1}|Z_{1:k-1}) dX_{k-1} \end{aligned} \tag{18}$$

$$p_k(X_k|Z_{1:k}) = \frac{g_k(Z_k|X_k) p_{k|k-1}(X_k|Z_{1:k-1})}{\int g_k(Z_k|X_k) P_{k_k|k-1}(X_k|Z_{1:k-1}) dX_k} \tag{19}$$

where analogous to the Markov transition density in single target tracking, the statistical behaviour of the RFS Ξ_k is characterised by the conditional probability density $f_{k|k-1}(X_k|X_{k-1})$, and similar to the likelihood function in single target tracking, statistical behaviour of the RFS Σ_k is described by the conditional probability density $g_k(Z_k|X_k)$.

B. The Probability Hypothesis Density (PHD) Filter

The full multi-target posterior recursion equations contains multiple integrals making it computationally intractable. As a remedy, Mahler proposed propagating the first moment instead of the full posterior. Analogous to the expectation of a random vector, the first order moment of the RFS Ξ_k is called the Probability Hypothesis Density (PHD), which is denoted by D_Ξ. The PHD is a function, whose integral over some region S yeilds the expected number of targets (elements of Ξ) present in S, which is given by $\hat{N}$,

$$\hat{N} = \int_S D_\Xi(x) dx \tag{20}$$

Assume, $D_{k|k}$ and $D_{k|k-1}$ denote the densities correspond to full multi-target posterior p_k and multi-target predicted prior $p_{k|k-1}$ respectively. Then under the assumption of Poison distribued clutter, PHD prediction and update equations are given by,

$$\begin{aligned} &D_{k|K-1}(x) = \gamma_k(x) \\ &+ \int P_{S,k}(x_{k-1}) f_{k|k-1}(x_k|x_{k-1}) D_{k-1}(x_{k-1}) dx_{k-1} \end{aligned} \tag{21}$$

$$\begin{aligned} D_k(x) &= [1 - P_{D,k}(k)] D_{k|k-1}(x) \\ &+ \sum_{z \in Z_k} \frac{P_{D,k}(x) f_k(z|x) D_{k|k-1}(x)}{\kappa_k(z) + \int P_{D,k(\zeta)} f_k(z|\zeta) D_{k|k-1}(\zeta)} \end{aligned} \tag{22}$$

where, PHD of RFS Γ_k, which corresponds to spontaneous targets appeared at time k, is given by γ_k. $P_{S,k}$ denotes the probability of survival of a target, while $P_{D,k}$ is the probability of detection. κ_k is given by, $\kappa_k = \lambda_k c_k$ where c_k denotes the clutter probability density and λ_k denotes the average number of Poison clutter points per time step.

Although PHD filter is computationally cheaper than full multi-target posterior (18,19), due to multiple integrals, it can't be readily used in implementations. Vo et. al [9] presented an eligent approximation of the PHD filter using Sequential Monte Carlo (SMC) methods, which can easily be implemented in various tracking applications.

C. Sequential Monte Carlo implementation of PHD Clutter Filter

The PHD clutter filter was implemented using Sequencial Monte Carlo (SMC) approach proposed by Vo et al [9]. The algorithm works as follows,

Step 1: Initialization

For time $k \geq 0$ the initial set of particles is given by, $\alpha_k = \{w_k^i, x_k^i\}_{i=1}^{L_k}$, which represents the PHD $D_{k|k}$ representing the estimates of targets, while w_k representing the associated weights of the particles. PHD is recursively propogated for $k \geq 1$.

Step 2: Prediction

Sample $\tilde{x}_k^i$ and predict the weight of each particle according to the motion model. In here we have adoptod the typical target tracking problem to suit SLAM community by assuming targets are moving according the vehicle's motion model, while vehicle (sensor) is stationary. Where $i = 1, ..., L_{k-1}$, and additionally Jk amount of particles are added to represent the PHD γ_k of spontaneously appeared targets.

Step 3: Correction

For each observation $z \in Z_k$, likelihoods $g_k(z|\tilde{x}_k^{(i)})$ are computed for each particle and weights are updated according to,

$$\tilde{w}_k^{(i)} = \left[(1 - P_{D,k}(\tilde{x}_k^{(i)})) + \sum_{z \in Z_k} \frac{P_{D,k}(\tilde{x}_k^{(i)}) g_k(z|\tilde{x}_k^{(i)})}{\kappa_k(z) + C_k(z)}\right] \tilde{w}_{k|k-1}^{(i)} \tag{23}$$

where,

$$C_k(z) = \sum_{j=1}^{L_{k-1}+J_k} P_{D,k}(\tilde{x}_k^{(j)}) f_k(z|\tilde{x}_k^{(j)}) \tag{24}$$

The estimated number of targets are given by the sum of the weights;

$$\tilde{N}_{k|k} = \sum_{j=1}^{L_{k-1}+J_k} \tilde{w}_k^{(j)} \tag{25}$$

Step 4: Resampling

Particles are resmapled to obtain $\{w_k^{(i)}/\tilde{N}_{k|k}, x_k^{(i)}\}_{i=1}^{L_k}$. Then the weights are rescaled by $\tilde{N}_{k|k}$ to get $\{w_k^{(i)}, x_k^{(i)}\}$

Step 5: Estimation of target locations

Maximum weighted particles are chosen and using k-means clustering algorithm, exact feature locations are estimated.

IV. Results

Simulations were conducted to verify the utility, feasibility and performance gains of the new multi-vehicle SLAM algorithm with the PHD filter, in high clutter environments. Two identical vehicles (ASCs) were deployed in the in-house build simulator environment with artificially placed landmarks (Fig. 2). Both vehicles were equipped with 2D Laser rangers with identical noise parameters, providing range and bearing measurements. The vehicles were driven on two overlapping trajectories in a rectangular area with randomly placed landmarks (Fig. 3), while performing online multi-vehicle SLAM. Clutter was assumed to be Poison distribued and generated uniformly over the field of view (FOV) of ranging sensor, with an average rate of λ_c.

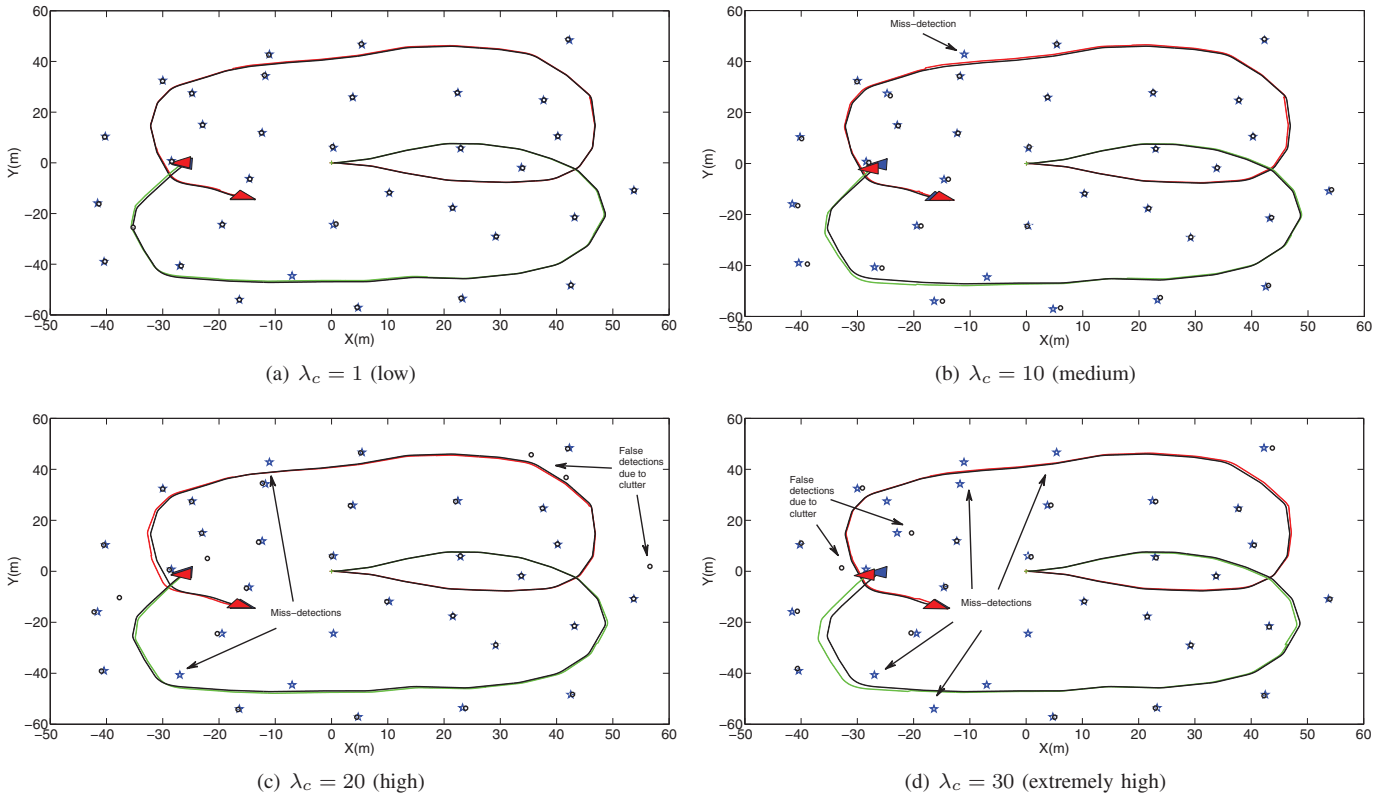
Experiments were conducted for low, medium, high and extremly high clutter scenarios, and the resulting estimated vehicle trajectories are shown in Fig. 4. The errors (in X direction of a vehicle) and runtimes against each clutter density is shown in Fig. 6 and Fig. 7 respectively. It's clear from the results that, except for the high and extremely-high clutter cases, the proposed algorithm can eliminate the effects of clutter, and perform adequately well. In high and extremely high clutter conditions, an increase of miss-detections can be observed in addition to the errors in trajectory estimations and false detections. Moreover, as the clutter density is increased, total running time of same simulation increases as shown in Fig. 7. This is due to the fact that, each clutter point is initially considered as a valid measurement, and being tracked until discarded as a false alarm.

Fig. 2. Two ASCs performing CSLAM in a simulator developed in-house for testing and validation of algorithms before actual field trials.

V. Conclusion

In this paper, we have presented a multi-robot SLAM [6] algorithm applicable in high clutter environments and evaluated it's performance. First we presented the formulation of CLSF based multi-robot SLAM algorithm, using the fact that, CSLAM problem can be solved as several mono-SLAM problems. The ability of delayed fusion of local sub-maps into the global map made CLSF an ideal and natural tool for solving the CSLAM problem. Then we discussed the improvements that can be obtained by combining Random Finite Set (RFS) filtering approaches with observation and feature extraction processes.

The performance of the proposed algorithm was evaluated using various clutter conditions. It is evident from the results that, multi-robot SLAM algorithms can greatly benefit from



(a) $\lambda_c = 1$ (low)

(b) $\lambda_c = 10$ (medium)

(c) $\lambda_c = 20$ (high)

(d) $\lambda_c = 30$ (extremely high)

Fig. 4. Estimated vehicle trajectories (in red and green), superimposed on ground truth (in black) with estimated features (black circles) and actual features (blue stars), under various clutter conditions.

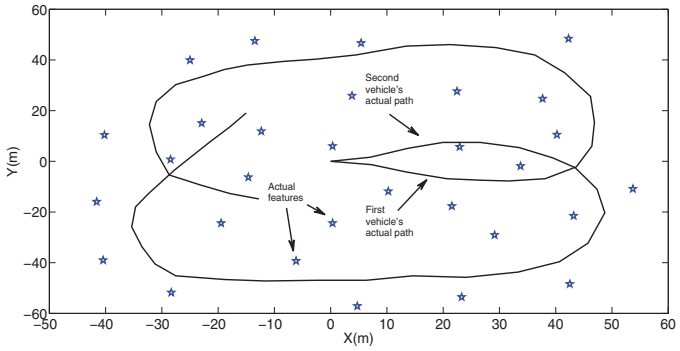


Fig. 3. The ground truth of vehicle paths (in black) and features present in the simulation environment (blue stars).

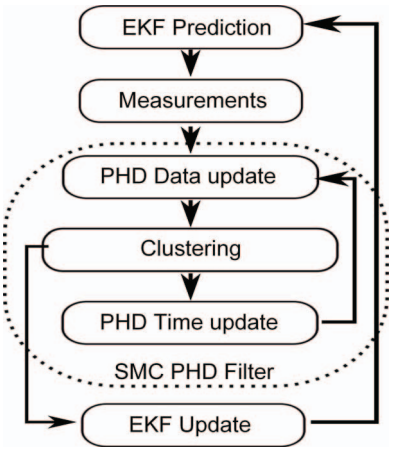


Fig. 5. EKF-SLAM algorithm with PHD clutter filter

RFS based clutter reduction approach for improved mapping and localization. The current drawback of the algorithm is the heavy computational requirement, which can be overcome by employing high end processors with good software engineering approaches. The proposed method is believed to be beneficial for practical implementations of autonomous multi-vehicle exploration, serveillance or search and rescue missions under various feild conditions.

ACKNOWLEDGMENT

This work was supported by National Research Foundation (NRF), Singapore and Center for Environmental Sensing and Modeling (CENSAM) under the auspices of the Singapore-MIT Alliance for Research and Technology (SMART).

REFERENCES

[1] R. Smith, M. Self, P. Cheeseman, "A stochastic map for uncertain spatial relationships," in *Proc. IEEE International Conference on Robotics and Automation*, 1987.

[2] Durrant-Whyte, H. and Bailey, T., "Simultaneous Locaization and Mapping: part I," *Robotics and Automation Magazine, IEEE*, vol. 13, pp. 99–110, Jan 2006.

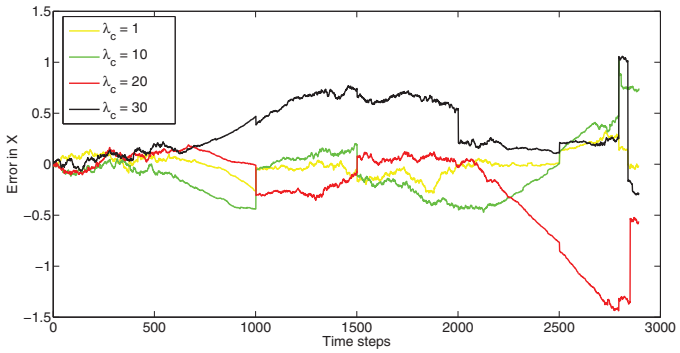


Fig. 6. Track errors of one vehicle, in X direction.

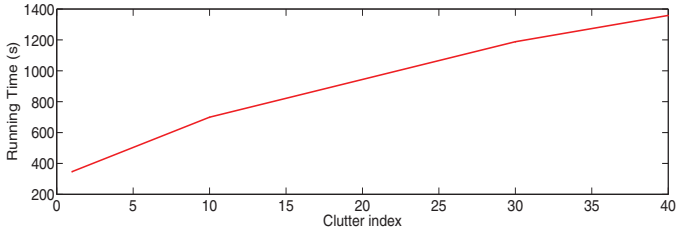


Fig. 7. Total running time of the same simulation under various clutter densities.

[3] Bailey, T. and Durrant-Whyte, H., "Simultaneous Locaization and Mapping: part II," *Robotics and Automation Magazine, IEEE*, vol. 13, pp. 99–110, 2006.

[4] "SLAM summer school," 2002. [Online]. Available: http://www.cas.kth.se/SLAM

[5] Williams, S.B., Dissanayake, G. and Durrant-Whyte, H., "An efficient approach to the simultaneous localization and mapping problem," in *Proc. International Conference on Robotics and Automation (ICRA'02)*, 2002.

[6] Williams, S.B., Dissanayake, G. and Durrant-Whyte, H, "Towards multi-vehicle simultaneous localization and mapping," in *Proc. International Conference on Robotics and Automation (ICRA'02)*, 2002.

[7] P.Newman, "On The Structure and Solution of the Simultaneous Localisation and Map Building Problem," Ph.D. dissertation, University of Sydney, Australian Center for Field Robotics, 1999.

[8] R. Mahler, "An introduction to multisource-multitarget statistics and its applicatioins." Lockheed Martin, Tech. Rep., 2000.

[9] Ba-Ngu Vo, Singh S. and Doucet A., "Sequential monte carlo methods for multi-target filtering with random finite sets." *Trans. on Aerospace and Electronic Systems, IEEE*, vol. 41, pp. 1224 – 1245, 2005.