Study of H-mode Access Conditions on the Alcator C-Mod Tokamak

by

Yunxing Ma

B.S. Physics (2006)
University of Science and Technology of China

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Abstract

Usually when sufficient heating power is injected, tokamak plasma will make an abrupt transition into a state with improved confinement, known as the high-confinement mode, or H-mode. Given the greatly enhanced fusion yield, H-mode is foreseen as the baseline scenario for the future plasma operation of the International Thermonuclear Experimental Reactor (ITER). Many research efforts have been given to understand the criteria for H-mode access. To further contribute to this research, a primary focus of this thesis is characterizing the H-mode access conditions in the Alcator C-Mod tokamak, across a broad range of plasma density, magnetic field, and plasma current. In addition, dedicated experiments were designed and executed on C-Mod, to explore the effects of divertor geometry, ICRF resonance location, and main ion species on H-mode access conditions. Results from these experiments will be included in this thesis.

The underlying physics of H-mode access is very complex, and the critical mechanisms remain largely unresolved. To promote our understanding, some models proposed for the H-mode transition are tested, using well documented local plasma conditions, obtained in C-Mod experiments. In particular, this thesis pioneers the test of a recently developed model for H-mode threshold power predictions.

Thesis supervisor: Earl S. Marmar
Title: Senior Research Scientist, Department of Physics

Thesis supervisor: Jerry W. Hughes
Title: Research Scientist, Plasma Science and Fusion Center

Thesis reader: Jan Egedal-Pedersen
Title: Associate Professor of Physics, Department of Physics

Thesis reader: Nuh Gedik
Title: Assistant Professor of Physics, Department of Physics
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Chapter 1

Introduction

Thesis goal and outline – Due to the continually increasing demand and rapidly exhausting fossil fuel reserves, the global energy problem has become an urgent issue to be faced by human beings in this century. Because of the vast fuel supply and absence of dangerous by-product, nuclear fusion is anticipated as a potential ultimate solution for the world’s energy crisis [1]. Among many scientific endeavors, a device called a tokamak [2] is considered a very promising experiment to realize controlled nuclear fusion. To accomplish this objective and further yield sufficient energy for the fusion reactions to be self-sustained, a tokamak is preferred to be operated with high performance fusion plasma. However, the most common high performance mode, called H-mode [3], can only be achieved when certain conditions are met. The goal of this thesis is to study the conditions required for H-mode access and better understand the physics behind it. This thesis study was carried out on the Alcator C-Mod tokamak [4] at MIT Plasma Science and Fusion Center.

The main parts of this thesis consist of seven chapters, arranged as follows:

Chapter 1 introduces the basic concept of nuclear fusion, plasma, magnetically controlled fusion, and covers some fundamental tokamak physics. A brief description of the Alcator C-Mod tokamak and required plasma diagnostics for the thesis study is given at the end.
Chapter 2 overviews some important results in previous L-H transition experiments from the main tokamaks in the world, followed by a brief introduction to models developed to explain H-mode transition triggering and predict H-mode threshold power.

Chapter 3 presents the principle results from a comprehensive survey on global threshold power and local plasma edge conditions for H-mode access conducted in a large L-H transition database assembled from dedicated C-Mod experiments.

Chapter 4 describes the detailed experimental setup of a dedicated C-Mod experiment exploring the effect of divertor geometry on H-mode threshold conditions and presents some remarkable discoveries from this experiment.

Chapter 5 shows the main results from dedicated C-Mod experiments designed to study the dependence of H-mode threshold conditions on ICRF resonance location and main ion species of plasma.

Chapter 6 shows comparison of C-Mod experiments with models for the L-H transition and H-mode threshold power that are introduced in Chapter 2.

Chapter 7 summarizes the main conclusions of previous chapters. Prospective future explorations in this research area are proposed at the end of this chapter.
1.1 Nuclear fusion and plasma†

1.1.1 Definition of nuclear fusion and plasma

*Nuclear fusion* refers to the reactions of two nuclei fusing together to form a new, heavier nucleus and releasing net energy. It is the kind of nuclear reaction that powers the active stars, like our sun, and is envisioned as a vast reservoir of clean energy to be potentially harnessed in the future.

Yet, making fusion happen is extremely difficult on the earth. The reason is that the nuclear force which holds the nuclei together is short-ranged in nature, effective only when the two nuclei are very close, so that the distance between the centers of the two nuclei centers, \( r \), is of sub-Angstrom order. However, at such a small distance, the Coulomb repulsive force (proportional to \( 1/r^2 \)) between the two positively charged nuclei will rise sharply, preventing the two nuclei from further approaching each other. So in order to make fusion happen, the fusing nuclei must carry sufficient energy to overcome this unfavorable Coulomb barrier. One approach to achieve such high energy is by heating the fusion fuel to a very high temperature, so that the high-energy tail of the velocity distribution of the fuel ions contains enough particles to induce significant fusion reactions. When fusion reactions are realized in this way, it is called the *thermonuclear fusion*.

The reaction rate \( R_f \) (the number of reactions per unit volume per unit time) of a thermonuclear fusion reaction is given by:

\[
R_f = n_i n_j \langle \sigma v \rangle
\]  

(0.1)

where \( n_i \) and \( n_j \) are the density for the two species involved in the reaction, \( i \) and \( j \), respectively. \( \langle \sigma v \rangle \) is the fusion cross section, averaged over the velocity distribution of each ion species, which is approximately the Maxwellian distribution \( f_M(\vec{v}) \)

† Unless otherwise specified, the main references for section 1.1 and 1.2 are [2], [5]-[9].
\[ f_m(\bar{v}) = n \left(\frac{m}{2\pi T}\right)^{3/2} \exp(-mv^2/2T) \]  

(0.2)

with \( m \) the mass, \( n \) the density, and \( T \) the temperature of the nuclei ions. Typically, \( \langle \sigma v \rangle \) increases rapidly with \( T \) before it peaks at an optimum temperature and then decreases. For example, the maximum \( \langle \sigma v \rangle \) for the D-T reaction is \( \sim 10^{-21} \text{m}^{-3}/\text{s} \), at \( T \sim 70 \text{KeV} \).

Fusion reactions involving light nuclei are good candidates for exploration, including

\[
\begin{align*}
\text{D+D} & \rightarrow ^3\text{He}+\text{n}+3.27\text{MeV} \\
\text{D+T} & \rightarrow ^4\text{He}+\text{n}+17.6\text{MeV} \\
\text{D}+^3\text{He} & \rightarrow ^4\text{He}+\text{p}+18.3\text{MeV}
\end{align*}
\]

Here D, T, n, and p represent deuterium, tritium, neutron, and proton, respectively. Among these reactions, the D-T reaction is considered most promising because of its high energy yield and larger fusion cross section (usually by an order of magnitude compared to other reactions for same temperature). But even for the D-T reaction, the temperature still needs to reach several (~10) KeV, in order to generate significant fusion productions. At such high temperature, the gas fuel is fully ionized into unbounded electrons and ions, a new substance called plasma. Unlike the other three states of matter (gas, liquid, solid), particles in plasma interact with each other mainly through the long-range electromagnetic interaction; this gives plasma a number of distinct characters.

Rigorously speaking, a group of charged particles can be called plasma, only when their density is large enough to satisfy the plasma approximation,

\[ n_e \lambda_D^3 >> 1 \]  

(0.3)

where \( n_e \) is the electron density. \( \lambda_D \) is called the Debye length, defined as

\[ \lambda_D = V_{te} / \omega_{pe} = 7400(T/n)^{1/2} \]  

(0.4)

where \( V_{te} \) is the electron thermal velocity,

\[ V_{te} = (T_e / m_e)^{1/2} \]  

(0.5)

\( \omega_{pe} \) is known as the plasma frequency.
\[ \omega_{pe} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}} \]  \tag{0.6}

which is the characteristic frequency for electrons to recover from small spatial perturbations. If the spatial and temporal scales of interest, \( L \) and \( \tau \), are such that

\[
L \gg \lambda_p \quad \quad \tau \gg \omega_{pe} \tag{0.7}
\]

then plasma can be considered as *quasi-neutral*, which means the total positive and negative charges are roughly equal, *i.e.*

\[
n_e \approx \sum_j Z_j n_j \tag{0.8}
\]

for a plasma with multiple ion species. \( Z_j, n_j \) are charge and density for the ion species \( j \).

### 1.2.2 Collisions in plasma

Collision in fully ionized plasma is not a true ‘head-on’ collision. Instead, it refers to the binary small-angle Coulomb interaction that scatters the particles off their single-particle trajectories, a process also known as the Coulomb scattering. Parameters applied to characterize this process, for electron \( (e) \) and ion \( (i) \), are the thermal averaged *collision frequency* (number of collisions per unit time)

\[
\nu_e = 4.36 \times 10^{-11} n_e T_e^{-3/2} \tag{0.9}
\]

\[
\nu_i = 6.75 \times 10^{-13} Z_i^4 n_i (m_i / m_p)^{-1/2} T_i^{-3/2} \approx (m_e / m_i)^{1/2} Z_i^4 \nu_e \tag{0.10}
\]

and the *mean free path* (m.f.p., distance for a particle to travel before a binary collision event occurs)

\[
\lambda_e = V_e / \nu_e \approx 10^{16} T_e^2 / n_e \tag{0.11}
\]

\[
\lambda_i = V_i / \nu_i \approx 10^{16} T_i^2 / n_i Z_i^4 \tag{0.12}
\]

with temperature \( (T) \) in the unit of eV, and density \( (n) \) in the unit of \( m^{-3} \). Due to the electron-ion collisions, plasma is conductive to electric current, *i.e.*
\[ E = \eta_{\text{plasma}} J \]  

where \( \eta_{\text{plasma}} \) is the Spitzer resistivity, given by  

\[ \eta_{\text{plasma}} \approx 2.5 \times 10^{-8} / T_e^{-3/2} \]  

Note that \( \eta_{\text{plasma}} \) is independent of plasma density and inversely proportional to electron temperature \( \propto T_e^{3/2} \), which means plasma becomes more conductive (less resistive) as temperature increases. This is in contrast to our experience that resistivity should increase with temperature, as in the case of metal, e.g., copper. Due to resistivity, plasma can be heated ohmically, at a power  

\[ P_{\text{out}} = \int_{V} \eta J^2 dV \]  

Plasma collisions can also induce particle and heat diffusion via the random walk process. With no externally applied magnetic field \((B=0)\), and assume \( Z=1, n_e \approx n_i, T_e \approx T_i \), the particle and heat diffusion coefficients, \( D \) and \( \chi \), due to the electron and ion Coulomb collisions, are given by  

\[ D_e \approx D_i = \lambda_e^2 \nu_e \propto T_e^{5/2} / n_e \]  

\[ \chi_e = \lambda_e^2 \nu_e = D_e \]  

\[ \chi_i = \lambda_i^2 \nu_i \approx (m_e / m_i)^{1/2} \chi_e \]  

For a typical fusion plasma with \( T_e \sim 1 \text{KeV} \) and \( n_e \sim 10^{20} \text{m}^{-3} \), a rough estimation for \( \lambda_e \) and \( \nu_e \) gives \( \lambda_e \sim 10^2 \text{m}, \nu_e \sim 10^5 \text{s}^{-1} \), and a remarkably high \( D_e \), of the order of \( 10^9 \text{m}^2/\text{s} \)! At such a high diffusion rate, fusion plasma in a reactor-scale machine \((L \sim 1 \text{m}, L \) is the minor radius\) will almost immediately escape to the chamber wall, within just a few ns (roughly estimated using \( t \sim L^2 / D \)). The focus of fusion energy research is to address the challenging problem of how to effectively hold the high temperature plasma, such as using a strong magnetic field, as will be discussed in section 1.2.
1.2 Magnetic confinement and tokamak

The extremely high temperature basically excludes the possibility of using any known solid materials as direct container for fusion plasma. Among the many alternatives, using a strong magnetic field to confine fusion plasmas is a promising solution; this is called the magnetical confinement fusion (MCF), or simply, magnetic fusion. The basic idea behind MCF is illustrated in Fig.1-1: with a strong magnetic field, the Larmor radii of charged plasma particles are reduced, and their orbits are squeezed and more closely tied to the magnetic field lines. Yet, although the perpendicular motion can be effectively constrained, the motion parallel to the magnetic field remains free, and is largely independent of the strength of magnetic field. Therefore, plasma can still leak out through the open ends of magnetic field lines, as in the case of a linear device.

![Diagram of magnetic confinement fusion](image1)

Fig.1-1: This figure illustrates the basic concept of magnetically confined fusion: (a) without external magnetic field, plasma particles move freely and collide with each other, resulting in significant diffusion towards the chamber wall, (b) with an external magnetic field, motions of plasma particles in the direction perpendicular to the magnetic field lines become more constrained.
A solution to avoid the end losses of plasma particles is making the magnetic field lines closed, by fabricating a machine with a torus shape, called a toroidal device. A tokamak\(^\dagger\) is one kind of toroidal magnetic fusion device. Research on tokamak experiments first started in the 1950s, in the Kurchatov Institute of the former Soviet Union. After more than a half century’s exploration, the tokamak is considered to be promising for realizing controlled thermonuclear fusion. As the largest and the most advanced tokamak, the International Thermonuclear Experimental Reactor (also known as the *ITER*) [10] was proposed, and is now under construction. The objective of ITER is to demonstrate the possibility of achieving \(Q=10\), *i.e.* to produce fusion power that is ten times of the total input power.

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\(^\dagger\) acronym for Russian words “toroidal'naya kamera s magnitnymi katushkami”, which stand for toroidal chamber with magnetic coils

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Fig.1-2: (left) schematic drawing of a tokamak: plasma volume, central solenoid, toroidal magnetic field (TF) coils, and shaping coils. ([http://www.efda.org/2011/09/tokamak-principle-2/](http://www.efda.org/2011/09/tokamak-principle-2/)). (right) tokamak geometry can be nearly represented by a cylindrical-like \(R\theta\phi\) coordinate. \(S_p\) and \(S_T\) indicate the integration area for poloidal and toroidal magnetic flux, \(\psi_p\) and \(\psi_T\), respectively.
1.2.1 Tokamak geometry

The basic geometry of a tokamak is an axisymmetric torus, as shown in Fig.1-2, which can be nearly represented by a cylindrical-like coordinate spanned in radial (R), poloidal (θ), and toroidal (φ) directions. The major and minor radius of the torus, \( R_0 \) and \( a_0 \), are illustrated in Fig.1-2. The aspect ratio of a tokamak is defined as \( A \equiv 1/\varepsilon = R_0/a_0 \), which typically ranges between 1.3 and 4. Geometrically, \( R \) can be related to \( r \), the radius relative to the center of the tube (or minor radius), as \( R \approx R_0 + r \cos \theta \). The horizontal plane coming through the center of the torus (the \( Z=0 \) plane) is called the midplane. The outboard and inboard midplane refer to the place where \( \theta = 0 \) and \( \theta = \pi \), respectively.

Three key parameters for tokamak plasma operation are toroidal magnetic field (\( B_T \)), plasma current (\( I_p \)), and plasma density (\( \bar{n}_e \), averaged electron density over a line through plasma core). \( B_T \) is the principle magnetic field, created using a series of toroidal field (TF) coils surrounding the torus, and evenly distributed in \( \phi \) (see Fig.1-2). Generated in this way, the strength of \( B_T \) is intrinsically inhomogeneous, which decays with \( R \), as \( B_T \sim 1/R \), inside the torus. The low-field-side (LFS) and high-field-side (HFS) refer to the outer and inner part of torus (or plasma), i.e. \( R_0 < R < R_0 + a_0 \) and \( R_0 - a_0 < R < R_0 \), respectively.

Plasma current is primarily in the toroidal direction, driven inductively by a central solenoid (\( I_p \sim V_\phi/\bar{\eta} \)). Here, \( V_\phi \) is the toroidal loop voltage, and \( \bar{\eta} \) is the volume averaged plasma resistivity. The plasma current, in turn, produces a magnetic field in the poloidal direction, \( B_p \). This magnetic field (\( B_p \)) is typically much smaller than \( B_T \) (typically, for a high-aspect ratio tokamak, \( B_p/B_T \sim \varepsilon^2 \), i.e. \( \sim 1/10 \), for \( A=3 \)), but is indispensible to tokamak plasma operation (e.g. in establishing global equilibrium, as will be seen later in section 1.2.3). With \( B_p \), magnetic field lines become helical and form infinite layers of nested (like onion skins) closed surfaces, called magnetic flux surfaces, or simply flux surfaces. These flux surfaces are generically shifted concentric circles, when projected to a poloidal cut of the torus (poloidal cross section).
The flux surfaces can be described using flux labels. Two widely adopted choices are the \textit{poloidal flux}

\[ \psi_p = \int_{S_p} \mathbf{B} \cdot \mathbf{n} dS \quad (0.19) \]

and the \textit{toroidal flux}

\[ \psi_T = \int_{S_T} \mathbf{B} \cdot \mathbf{n} dS \quad (0.20) \]

The integration areas, \( S_p \) and \( S_T \), are illustrated in Fig.1-2. According to this definition, \( \psi_p \) generally increases with minor radius \( r \), and peaks in the edge of plasma volume. The \textit{normalized poloidal flux}, defined by

\[ \psi = (\psi_p - \psi_a) / (\psi_p - \psi_b) \quad (0.21) \]

is often employed to label the flux surfaces. Here, \( \psi_a \) and \( \psi_b \) are the poloidal flux of the magnetic axis (magnetic center of plasma) and the plasma boundary, respectively. By this definition, \( \psi=0 \) on the magnetic axis, and \( \psi=1.0 \) at the plasma boundary.

From \( \psi_p \) and \( \psi_T \), one can define the \textit{safety factor} (named because this parameter is also related to tokamak plasma stability)

\[ q = d\psi_T / d\psi_p \quad (0.22) \]

as a measure of the inverse pitch of the magnetic field. This parameter also quantifies the number of toroidal turns for a magnetic field line to make before it closes upon itself. For a tokamak with large aspect ratio and nearly circular or elliptical plasma cross section, \( q \) can be approximated by

\[ q(r) = \frac{r B_r}{RB_p} \quad (0.23) \]

which takes the cylindrical limit

\[ q_{\text{cyl}} = \frac{2\pi ab B_r}{\mu_0 RI_p} \quad (0.24) \]
near the plasma boundary. Here, \( a, b \) are the half-width of plasma volume in radial and vertical direction. It can be proven that if \( q \) is not a rational number (i.e. \( q \neq m/n \), with \( m \) and \( n \) arbitrary positive integers), then a magnetic field line can never close to itself, and will eventually map out the entire flux surface on which it lies. The \( q \) value in tokamak generally increases monotonically with minor radius (or the \( \psi \)) and peaks near the plasma boundary. The \( q \) values near the magnetic axis (\( q_0 \)) are typically close to unity.

1.2.2 Single particle motion in tokamak plasma

Plasma dynamics is a many-body problem with complex interactions between particles, which can be treated statistically, and studied using the Boltzmann plasma kinetic equation, or its moment equations, the Braginskii plasma fluid equations. Details about these equations can be found in Appendix A. However, many tokamak plasma behaviors can often be described from the motion of a single charged particle. This topic is considered in this section.

Consider in general the motion of a charged particle (species \( j \)), in the presence of spatially inhomogeneous electric and magnetic field. This problem can be described as

\[
m_j \frac{dv_j}{dt} = Z_j e(\mathbf{E}(\mathbf{x}_j) + \mathbf{v}_j \times \mathbf{B}(\mathbf{x}_j)), \quad \mathbf{v}_j = \frac{dx_j}{dt}
\]

This differential equation rarely has an analytical solution for a complex geometry, as in the case of a tokamak. However, when the magnetic field is strong, the single particle motion can be nearly decomposed into a fast cyclotron motion about the \textit{guiding center} (the center of the cyclotron motion), and a slow cross-field drift motion of the guiding center, because the temporal and spatial scale of the cyclotron motion, \textit{i.e.} the \textit{cyclotron frequency}, \( \omega_c \), and the \textit{Larmor radius} (or \textit{gyro radius}), \( \rho \), are much smaller compared to the scales of other dynamics. Here, \( \omega_{cj} \) (\( \omega_c \) for the species \( j \)) is defined as

\[
\omega_{cj} = \frac{Z_j eB}{m_j}
\]
where $Z_j$ is the charge (negative for electron, by convention), and $B$ is the strength of magnetic field. $\rho_j$ is defined as

$$\rho_j = \frac{V_{j,\perp}}{\omega_{cj}}$$  \hspace{1cm} (0.27)$$

where $V_{j,\perp}$ is the particle velocity in the directions perpendicular to the magnetic field. Statistical average of $\rho_j$ over a Maxwellian velocity distribution gives

$$\rho_j = \frac{V_j}{\omega_{cj}} = \sqrt{\frac{m_j T_j}{Z_j e B}}$$  \hspace{1cm} (0.28)$$

Guiding center drift is relevant to many problems, e.g. plasma turbulence and transport [11]. These are important to this thesis. The drift motion along the magnetic field, to the lowest order, is essentially free streaming, with

$$V_{gc,i} = V_{j,i} b$$  \hspace{1cm} (0.29)$$

where $b$ is the unit vector along $B$. The cross-field drifts (drifts perpendicular to the magnetic field) mainly include two types. The first type is associated with the electric field, including the $E \times B$ ($E$-cross-$B$) drift

$$V_{E-B} = \frac{E \times B}{B^2}$$  \hspace{1cm} (0.30)$$

and the polarization drift (for time varying electric field only)

$$V_p = \frac{1}{\omega_{cj} B} \frac{dE}{dt}$$  \hspace{1cm} (0.31)$$

The second kind is caused by the magnetic geometry, including the $B \times \nabla B$ (grad-$B$) drift

$$V_{VB} = \frac{V_{j,\perp}^2 \times B}{2 \omega_{cj} B^2}$$  \hspace{1cm} (0.32)$$

and the magnetic curvature drift

$$V_c = \frac{V_{j,\perp}^2 \times (b \cdot \nabla b)}{\omega_{cj} B} = \frac{V_{j,\perp}^2}{\omega_{cj} B} B \times \kappa$$  \hspace{1cm} (0.33)$$

where $\kappa = b \cdot \nabla b$ is the curvature of magnetic field lines (magnetic curvature).
Fig. 1-3: The particle trajectories for ions (red) and electrons (blue), in the presence of the $E \times B$ and the $\text{grad-}B$ drift. The direction of the magnetic field is assumed to be pointing out of the paper.

In case of the so-called ‘vacuum field’ (i.e. when the volume current $J \sim \nabla \times B$ is zero or negligible), the magnetic curvature drift can be written in a form that is close to the grad-$B$ drift, as

$$V_x = \frac{V^{2}_{j,l}}{\omega_{ij}} \frac{B \times \nabla B}{B^2}$$  \hspace{1cm} (0.34)

Thus the two drifts can be combined and give rise to the \textit{magnetic drift},
\[ V_B = \frac{V_{j,ii}^2 + V_{j,\perp}^2/2}{\alpha_{ij}} \mathbf{B} \times \nabla B \]  

(0.35)

Averaged over a Maxwellian velocity distribution, eqn.(0.35) becomes

\[ \langle V_B \rangle = \frac{T_{j,ii} + T_{j,\perp}}{Z_j e} \mathbf{B} \times \nabla B \]  

(0.36)

where \( T_{ji} \) and \( T_{\perp} \) are the temperature parallel and perpendicular to the magnetic field. For isotropic plasma, \( T_{\parallel} = T_{\perp} = T \).

Note from (0.30) that the \( \mathbf{E} \times \mathbf{B} \) drift does not depend on charge or mass of the particle; therefore, it does not give rise to charge separation. Unlike the \( \mathbf{E} \times \mathbf{B} \) drift, both polarization drift and magnetic (\( \text{grad-B} \) and curvature) drifts care about the charge sign, so their directions are opposite for electrons and ions. The polarization drift for electrons is much smaller (by a factor of \( m_e/m_i, \text{i.e. } 1/3600 \) for deuterium) compared to that of ions; therefore, it is often neglected. A drawing of the particle trajectories in the presence of \( \mathbf{E} \times \mathbf{B} \) and \( \text{grad-B} \) drift for electrons and ions is shown in Fig.1-3.

### 1.2.3 Tokamak equilibrium and stability

Global equilibrium and steady state plasma operation are fundamental requirement for a tokamak experiment. However, the huge difference (typically 4-5 order of magnitudes) in plasma pressure \( (P=nT) \) between the hot core and the cooler edge produces a force that tends to expand the fusion plasma. This force is balanced by the Lorentz force in a tokamak (see Fig.1-4) to establish a global equilibrium, described by

\[ \rho \frac{d \mathbf{V}}{dt} = \nabla P - \mathbf{J} \times \mathbf{B} \]  

(0.37)

with \( \rho = m_n + m_i n \) the mass density, \( \mathbf{V} = (m_n \mathbf{V}_n + m_i n \mathbf{V}_i)/\rho \approx \mathbf{V}_i \) the center-of-mass velocity (approximately the ion flow velocity due to small electron mass), \( P = P_c + P_i \) the plasma pressure, \( \mathbf{B} = \mathbf{B}_r + \mathbf{B}_p \) the total magnetic field, and \( \mathbf{J} \) the plasma current density.
In a quasi-steady state \( (d/dt \approx 0) \), (0.37) becomes
\[

\nabla P \approx J \times B
\]  

(0.38)

Taking \( \mathbf{b} \cdot \) on both sides of (0.38) gives
\[

\mathbf{b} \cdot \nabla P \approx 0
\]  

(0.39)

Eqn.(0.39) means that plasma pressure is nearly constant along a magnetic field line, and therefore, is a function of poloidal flux, i.e. \( P = P(\psi) \). The fast streaming and frequent collisions along magnetic field make the thermal relaxation process very fast (especially for electrons); therefore, it is reasonable to assume that temperature and density individually are also flux function for \( \psi < 1 \), i.e. \( T = T(\psi) \) and \( n = n(\psi) \). Substituting \( J \) in (0.38) by the Ampère’s law, \( \mu_0 J = \nabla \times \mathbf{B} \), gives
\[

\nabla \left( P + \frac{B^2}{2\mu_0} \right) = \frac{B^2}{\mu_0} \kappa
\]  

(0.40)
where $\kappa$ is the magnetic curvature, as defined in (0.33). Without a poloidal magnetic field ($\mathbf{B}=B_\parallel \mathbf{e}_\parallel \sim (1/R)\mathbf{e}_\parallel$, $\nabla (B^2/2) = -(B_r^2/R)\mathbf{e}_r$, $\kappa = -\mathbf{e}_r$, then $\nabla P \approx 0$, and a pressure gradient in minor radius can not be sustained in this case. From (0.40), one may also define *plasma beta*, the ratio of kinetic energy to magnetic energy, as

$$\beta = \frac{P}{B^2 / 2\mu_0}$$  \hspace{1cm} (0.41)

The $\beta$ is a key dimensionless parameter in plasma physics.

However, the global equilibrium in tokamak plasma is not necessarily stable. The radial pressure gradient and plasma current, although force balanced, are also sources of free energy for plasma instabilities [12]. In fact, tokamak plasma can never be completely free from instabilities, but not all of them are devastating, some may be even benevolent [13,14]. Of major concerns to tokamak operation are those which can cause macroscopic collapse of plasma, the *plasma disruption*. These concerns set a number of constrains in tokamak plasma operation, so that not any combinations of $B_T$, $I_p$, and plasma density are permitted. It is widely accepted that the primary cause of major disruptions is largely due to the global kink instabilities. To avoid this, the safety factor near tokamak plasma boundary ($q_a$) must be larger than about 2.0 (normally, tokamaks are operated with $q_a > 3.0$, taking into account this constraint).

The enhanced outward radial particle flux at high density driven by plasma edge microinstabilities may also lead to plasma disruptions. This kind of disruption is associated with an upper limit in plasma density for tokamak operation, known as the *Greenwald density limit* ($n_G$) [15], given by

$$\bar{n}_e < n_G = \frac{I_p}{\pi a_0^2}$$  \hspace{1cm} (0.42)

where $I_p$ is in the unit of MA, $a_0$ in the unit of $m$, and $n_G$ in the unit of $n_{20}=10^{20}m^{-3}$

The plasma shaping is also known to have a strong influence on steady state operation. Due to its importance, nowadays, many tokamaks installed a number of built-in shaping
coils (see Fig.1-2) for magnetic geometry control. Instead of the shifted-circular geometry, most devices, including Alcator C-Mod, have adapted the vertically elongated D-shape magnetic configuration (Fig.1-5), optimized for tokamak operation. This configuration can be nearly described by five parameters: $a$, $b$, elongation ($\kappa$), upper and lower triangularitis ($\delta_{\text{upper}}$, $\delta_{\text{lower}}$)

\[
\kappa = \frac{b}{a} \\
\delta_{\text{upper}} = \frac{c}{a} \\
\delta_{\text{lower}} = \frac{d}{a}
\]

(0.43)

1.2.4 Plasma boundary of tokamak

The plasma boundary of tokamak [16] refers to the narrow region where plasma begins to make contact with the vessel wall, or other solid components inside the vacuum chamber. This region includes many important physical phenomena, e.g. plasma-surface interaction, material sputtering and erosion, particle fueling and recycling, impurity and neutral transport, etc. Early tokamak research could not easily diagnose this region. Later, with improved plasma diagnostic capability and more research effort, it was realized that the boundary layer can strongly affect, sometimes even determine, the performance of bulk plasma. Now it is widely believed that an optimized plasma boundary is critical to the success of a burning plasma experiment, such as a tokamak.

Direct contacts of tokamak plasma with a large area of metallic wall will cause a number of unwanted consequences. For example, the incident plasma could damage or erode on materials; sputtered high-Z impurities may result in severe contamination when they enter the confined plasma, greatly degrade the plasma performance, or even induce major disruptions. To avoid direct plasma-wall contact, a component called a limiter was proposed to be implemented. A poloidal limiter is a non-axisymmetric surface, protruding from the vessel wall (Fig.1-5(a)). Tokamak plasma is restricted in front of the limiter surface. The limiter position defines the last closed flux surface (LCFS) for limiter
plasma. Inside the LCFS is the main (or confined) plasma, contained on closed magnetic flux surfaces. The thin layer beyond the LCFS is called the scrape-off layer (SOL), where the magnetic field lines are open, and strike on the limiter surface.

Fig.1-5: D-shaped elongated tokamak magnetic geometry for (a) limiter, and (b) divertor plasma. Definitions of elongation, triangularities, confined plasma, scrape-off layer (SOL), separatrix, X-point, and upstream location are shown.

Another component called a divertor was proposed to better handle the impurity and heat leakage from the main plasma. Typically, a divertor is a toroidally closed ribbon, placed distant from main plasma, and inside (usually near the bottom and/or top) the vacuum chamber, as shown in Fig.1-5(b). Divertor operation is often realized by creating one or more points with $B_p=0$, called X-points, inside the vessel, while diverting the magnetic field lines in plasma boundary to connect with the divertor surface. The flux surface
crossing the X-point is called the *separatrix*, which is essentially the LCFS for divertor plasma. The narrow layer beyond the separatrix with open magnetic field lines is defined as the divertor SOL, in analogy to the limiter plasma case, which connects the divertor target plate and the plasma *upstream* (the region far from the divertor plate, see Fig.1-5).

The *SOL connection length* \((L_{\parallel})\) can be defined as *half* the distance between the two points where a SOL magnetic field line makes contact with limiter or divertor surface, called *strike points*, if the poloidal geometry of the SOL is nearly symmetric [16]. An estimation of \(L_{\parallel}\) for limiter plasma is

\[
L_{\parallel}^{\text{lim}} \approx \frac{\pi R_0}{N_{\text{lim}}}
\]  

(0.44)

where \(N_{\text{lim}}\) is the number of wall limiters, and for divertor plasma

\[
L_{\parallel}^{\text{div}} \approx \frac{\pi R_0 q_{\text{cyl}}}{N_{\text{xpt}}}
\]  

(0.45)

with \(N_{\text{xpt}}\) the number of X-points. One may note that \(L_{\parallel}\) in divertor plasma is longer by a factor of \(q_a (>3)\) than that of limiter plasma. This significant increase in \(L_{\parallel}\) makes divertor and limiter plasma different, especially with respect to the SOL transport and edge plasma profiles. In case the poloidal geometry of the SOL is asymmetric, it can be convenient to define \(L_{\parallel}\) as the parallel length of the SOL on the LFS, since plasma transport mainly takes place in the LFS region, due to ballooning-like turbulence (described later in chapter 2.2.1).

When plasma makes contact with a divertor, a very thin layer (typically with thickness of just a few Debye lengths) called *plasma sheath* (or ‘sheath’ for simplicity), is formed in front of the solid surface of the divertor target plate. Plasma inside a sheath is non-neutral and constitutes net positive charges, *i.e.* with \(Z_i n_i > n_e\); the electric potential is negative and drops sharply towards the surface; therefore ions get accelerated, and speed of plasma flow in a sheath is of the order of ion sound speed \(C_s\), which is

\[
C_s = \sqrt{\frac{(T_e+T_i)}{m_i}}
\]  

(0.46)
Because \( p + m_i V^2 = \text{const} \) (see Appendix C, (C.4)), the pressure at divertor target plate (‘\( t \)’) is only about half the pre-sheath (‘pre-sh’, refers to the entrance of a sheath) or upstream (‘\( u \)’) values, \textit{i.e.}

\[
P_{\text{pre-sh}} \approx P_u \approx 2P_t
\]  

(0.47)

when \( V_u \ll V_t \). The sheath region sets up the boundary conditions for the SOL, under certain conditions it might even directly determine the SOL character.

\subsection*{1.2.5 Plasma transport in tokamak}

After stable plasma operation, another major problem for tokamak experiments to address is how to confine plasma for a sufficiently long period to achieve fusion \textit{ignition}, \textit{i.e.} the point at which the fusion reactions becomes self-sustained, with output fusion energy greater than the total input energy. The condition for ignition is given the Lawson criteria

\[
n_e \tau_E T_i > 10^{21} \text{KeV} \cdot \text{s} / \text{m}^3
\]  

(0.48)

The variable \( \tau_E \) is the global energy confinement time, defined as

\[
\tau_E = \frac{W_p}{P_{\text{loss}}} = \frac{W_p}{P_{\text{in}} - \frac{dW_p}{dt}}
\]  

(0.49)

where \( W_p \) is the stored plasma energy; \( P_{\text{loss}} \) is the power loss across the separatrix for divertor plasma; \( P_{\text{in}} \) is the total input power, including the ohmic heating \( (P_{\text{oh}} \sim I_p V_{\text{loop}}) \), and any auxiliary heating \( (P_{\text{aux}}) \). In steady state, with \( dW_p/dt \sim 0 \)

\[
\tau_E = \frac{W_p}{P_{\text{in}}}
\]  

(0.50)

Global confinement is largely determined by the local plasma transport in the boundary layer, including the edge of confined plasma (hereafter called \textit{plasma edge}) and the SOL. However, transport in these two regions can be rather different, due to their distinct magnetic configuration.
Because the magnetic field lines are closed, cross-field plasma transport is presumably dominant in the plasma edge. There are two main origins of cross-field transport. The first one is the neoclassical (NC) transport [17,18], which address the plasma diffusion due to Coulomb collisions in real tokamak magnetic geometry. Plasma diffusion becomes highly anisotropic in the presence of a magnetic field. Since the particle motions along magnetic field lines do not quite ‘sense’ the existence of magnetic field, the diffusion process parallel to $B$ can still be characterized by eqn.(0.16)-(0.18), as if $B=0$. The diffusion perpendicular to $B$, however, is significantly modified: the mean free paths in eqn.(0.16)-(0.18) should be replaced by the Larmor radii, yielding (for $T_e \sim T_i$)

$$D_{e\perp} \approx D_{i\perp} = \rho_e^2 \gamma_e \propto n_e / B^2 T_e^{1/2}$$

(0.51)

$$\chi_{e\perp} \approx \rho_e^2 \gamma_e = D_{e\perp}$$

(0.52)

$$\chi_{i\perp} \approx \rho_i^2 \gamma_i = \left(m_i / m_e\right)^{1/2} \chi_{e\perp}$$

(0.53)

Note from eqn.(0.51)-(0.53) that the cross-field thermal diffusion is mainly carried by ions, as as opposed to the parallel heat diffusion, which is mainly carried by electrons. For typical C-Mod plasma edge conditions in deuterium majority plasmas: $T_e \sim 100$eV, $n_e \sim 10^{19}$ m$^{-3}$, $B=5$T, one will have $D_e \sim \chi_e \sim 10^{-5}$ m$^2$/s, and $\chi_{i\perp} \sim 10^{-3}$ m$^2$/s. The NC diffusion coefficients, taking into account the tokamak magnetic geometry, are generally larger, by about one order of magnitude compared to these estimations; but still too low compared to experimental values, which is typically $D \sim \chi \sim 1$ m$^2$/s. Neoclassical diffusion sets the baseline for cross-field plasma transport in a tokamak, and can not be further reduced.

The gap between NC diffusion and experiment is mainly contributed by the ‘anomalous’ transport due to plasma turbulence [11,19,20,21]. Effective diffusion coefficients for turbulence transport, estimated using the mixing length theory, are

$$D_{\text{turb}} \sim D_B \sim \frac{T}{16eB}$$

(0.54)

where $D_B$ is known as the Bohm diffusion coefficient [22, 9]. There is experimental evidence that the tokamak plasma core transport could be Bohm or gyro-Bohm like [23-
26]. It is widely accepted that plasma turbulence is responsible for most measured cross-field transport in tokamak plasma edge. However, due to its complexity, accurately knowing turbulent transport is very difficult, which is still an area under active research.

On the other hand, plasma transport in the SOL region may be dominated by parallel conduction along the open magnetic field lines. This process carries (mainly by electrons) the particle and heat leakage from the confined plasma to the divertor region, where particles are recycled and heat is exhausted. Compared to cross-field transport, the physics of parallel conduction is relatively simple and well-defined; it can be approximately modeled using a two-point SOL transport model [16] in many cases (details about the two-point model can be found in Appendix C).

The two-point model indicates that the key parameter for the SOL-divertor plasma transport is the SOL collisionality

$$\nu_{SOL} \equiv \frac{L_{\perp}}{\lambda_e} \approx 10^{-16} n_e L_{\parallel} / T_e^2$$  \hspace{1cm} (0.55)$$

The character of the SOL is distinctive for different ranges of $\nu_{SOL}$. The SOL is said to be in the sheath-limited regime when $\nu_{SOL} < 10$ (this is usually the case for low density, high temperature). In this scenario, the parallel heat conduction is fast, so that the temperature gradient along a magnetic field line is small, i.e. typically $T_u / T_d < 1.5$ (‘$u$’ for upstream, ‘$t$’ for divertor target). Therefore, the behavior of $T_e$ at upstream is largely determined by that at the divertor target plate. In case of higher collisionality ($10-15 < \nu_{SOL} < 70-85$), the SOL is called in the conduction-limited regime. The parallel conduction becomes less effective in this case, establishing a substantial temperature difference between upstream and divertor, with $T_u / T_d > 1.5$. For very high collisionality ($\nu_{SOL} \approx O(10^5)$), the divertor becomes detached, with $T_u$ reduced to only a few eV. Typically, the condition for divertor detachment is $T_u < 2$eV.
1.2.6 Confinement regimes in tokamak and the L-H transition

So far four major plasma confinement regimes have been discovered in tokamak experiments. They are Ohmic, L-mode (L for ‘low’), H-mode (H for ‘high’) and I-mode (I for ‘improved’).

The *Ohmic* regime is associated with plasmas that are heated purely by ohmic power (or called ohmic plasma). The plasma density dependence of the global energy confinement time is nearly linear at low density in this regime, given by the neo-Alcator or linear ohmic confinement (LOC) scaling [27,28], as

\[ \tau_{E}^{\text{ohm}} \approx 0.07(\bar{n}_e / n_{20})a_0R_0^2q \]  

where \( n_{20} = 10^{20} \text{ m}^{-3} \). The LOC scaling becomes saturated when plasma density is above a threshold value \( n_{\text{sat}} \), given by [2]

\[ n_{\text{sat}} \approx 0.06n_{20}J_pR_0^4A\kappa^{-1}a_0^{-2.5} \]  

where \( A \) is the atomic mass of ions. \( \tau_E \) depends weakly on density when \( \bar{n}_e > n_{\text{sat}} \).

The L-mode confinement regime was discovered in the early days of tokamak research, with the addition of auxiliary heating in ohmic plasma. A distinct feature of the L-mode regime is that the quality of plasma confinement degrades, when more auxiliary heating power is injected [29,30]. A widely used scaling for the L-mode energy confinement time is the *ITER89-P* scaling [31], given by

\[ \tau_{E}^{\text{ITER89-P}} = 0.048J_p^{0.85}R_0^{1.2}a_0^{0.3}(\bar{n}_e / n_{20})^{0.1}B^{0.2}A^{0.5}P^{-0.5} \]  

where \( P \) is the total input heating power in the unit of MW. Eqn. (0.58) suggests that

\[ W_p = \tau_{E}P \propto P^{0.5} \]  

It means in L-mode, the important measure of fusion reactions, \( nT \), is basically unchanged with increasing input power, since \( nT = P \sim W_p \), and \( n\tau_{E}T \sim W_p \tau_E \propto P^0 \)
The turning point is the discovery of H-mode on the ASDEX tokamak in 1982 [3]. This new regime was accessed when input heating power in L-mode plasma exceeded a certain threshold, the value of which is reduced by the implementation of divertor. The \textit{ITER-98(γ,2)} scaling [32] is widely used for the H-mode energy confinement time, given by

\[
\tau_{E,\text{ITER98}(\gamma,2)} = 0.145 I_p^{0.93} R_0^{1.39} a_0^{-0.58} \kappa^{-0.78} (\bar{n}_e / n_{20})^{-0.41} B^{0.15} A^{0.19} P^{-0.69}
\]  

(0.60)

According to eqn.(0.60), the confinement time in H-mode plasma is typically about twice of that in L-mode \((\tau_{E,H}/\tau_{E,L} \sim 2)\), for similar engineering parameters. Due to the improved confinement, H-mode plasma stores more energy than in L-mode for same heating power. As a result, \(n \tau_E T\) can be enhanced by almost one order of magnitude relative to L-mode, making H-mode a promising candidate for high performance tokamak plasma operation.

Another distinct feature for H-mode is the appearance of both temperature and density pedestals (‘pedestal’ refers to a region with sharp radial gradient) in the tokamak plasma edge; by contrast, edge pedestals are not found in L-mode plasma. Plasma transport is significantly reduced inside the edge pedestal, making it also a transport barrier. Examples of typical electron temperature and density profiles in L-mode and H-mode are shown in Fig.1-6.

Nowadays, H-mode can be routinely achieved and operated by most major tokamaks [33-41] in the world. Unlike ohmic or L-mode, there are many kinds of H-mode, varying with machines and plasma conditions. Some well-known examples include ELM\(^\S\)-free H-mode [34], ELMy H-mode [34-36], EDA (Enhanced D-Alpha) H-mode [13,34], Quiescent H-mode [37], etc. With numerous advantages, especially improved plasma confinement and enhanced fusion yield, H-mode is currently considered to be the baseline scenario for ITER plasma operation.

---

\(^\S\) ELM for \textit{Edge Localized Mode} [33], a type of plasma edge instability.
Fig. 1-6: Examples of $T_e$ and $n_e$ profiles in plasma edge for L-mode (black), I-mode (red) and H-mode (blue), as a function of normalized poloidal flux $\psi$. Edge pedestals are seen in H-mode for both $T_e$ and $n_e$, in I-mode for $T_e$ but not $n_e$, and not found in L-mode (courtesy of J.W. Hughes).

The I-mode regime has very recently been developed in the Alcator C-Mod tokamak [42]. This regime was obtained with ion grad-B drift in the direction that is unfavorable for H-mode access (i.e. the case with higher H-mode threshold power), and input heating power below the H-mode threshold power. I-mode is featured by improved, H-mode like energy confinement, while maintaining L-mode like particle confinement, which allows appropriate particle transport to avoid impurity accumulation in plasma core. Thus I-mode plasma profiles show a ‘mixture’ of both L- and H-mode characteristics, featured by the existence of temperature pedestal, but no density pedestal in plasma edge, as shown in Fig.1-6. Because of these merits, I-mode was proposed as a potential alternative plasma scenario for ITER.
Fig. 1-7: Shown in this plot are time histories of (a) plasma density, (b) $D_\alpha$, (c) ICRF power, and (d) volume integrated plasma $\beta$, for a plasma discharge with a clear L-H transition (marked by the dashed line).

Due to the significance of H-mode, knowing its access conditions has become an important issue in tokamak research, which includes the study of the L- to H-mode (L-H) and the I- to H-mode (I-H) transition. This thesis only aims to study the L-H transition. Research on the I-H transition has just begun on C-Mod, and some preliminary results were reported in [43]. Behavior associated with the L-H transition across different tokamaks usually include a sudden drop of $D_\alpha$ signals (indicating a reduction of edge particle transport), and a break-in-slope increase in plasma density (Fig.1-7). In addition, edge pedestals in $T_e$ and $n_e$ are found to form shortly following the L-H transition.
1.3 The Alcator C-Mod tokamak

Alcator C-Mod [4] is a compact divertor tokamak operated at MIT Plasma Science and Fusion Center since 1993. Among the existing tokamaks, C-Mod operates at the highest magnetic field (regular operation at 5.4T), and the highest plasma energy density (plasma pressure). Ranges of some main parameters for C-mod plasmas are listed in table 1.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_0$ [m]</td>
<td>0.68</td>
</tr>
<tr>
<td>$a_0$ [m]</td>
<td>0.22</td>
</tr>
<tr>
<td>$B_T$ [Tesla]</td>
<td>2.7-8</td>
</tr>
<tr>
<td>$I_p$ [MA]</td>
<td>0.3-1.4</td>
</tr>
<tr>
<td>$\bar{n}_e$ [$10^{21} m^{-3}$]</td>
<td>0.3-3.0</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.3-1.6</td>
</tr>
<tr>
<td>$\delta_{lower}$</td>
<td>0.4-0.8</td>
</tr>
<tr>
<td>$\delta_{upper}$</td>
<td>0.2-0.4</td>
</tr>
</tbody>
</table>

Table 1.1: Typical ranges of some main parameters on C-Mod.

C-Mod pioneered the implementation of the vertical-plate divertor [44]. This design enjoyed advantages in better power handling, and tight neutral baffling. C-Mod is normally operated with divertor plasma (limiter plasma can also be made); X-point(s) can be created near the lower and/or the upper part of the vessel, allowing three possible X-point (null point) configurations, namely, the lower single null (LSN), the upper single null (USN), and the double-null (DN) configurations. Plasma shaping control and X-point positioning are realized by a series of built-in plasma shaping coils. Pick-up coils were buried behind the vessel wall to measure the local magnetic field. With these measurements, C-Mod equilibrium magnetic geometry can be reconstructed by solving the Grad-Shafranov equation (i.e. (0.38) in tokamak geometry) [12] using the numerical
code EFIT [45]. Shown in Fig.1-8 is the standard C-Mod LSN divertor plasma magnetic geometry.

Fig.1-8: A poloidal cross section showing the standard C-Mod LSN divertor plasma geometry, overlaid with the measurement locations of the key diagnostics used for the thesis. The flux contours come from EFIT equilibrium magnetic reconstruction.

High performance plasma operation, including H-mode relevant physics, is a highlighted research area on C-Mod. H-mode plasma can be routinely achieved on C-Mod with ion
cyclotron range of frequency (ICRF) wave heating. The C-Mod ICRF system [46,47] was designed with 8MW maximum total source power, which is sufficient to ensure H-mode access under an extensive range of plasma conditions. The ICRF antenna frequency is tunable in the range of 50-80MHz. For regular C-Mod plasma operation, it is tuned at the value of

$$2\pi f \approx \omega_{eH} = eB(R_0)/m_{eH}$$

(2.61)

to perform on-axis fundamental hydrogen (H) minority heating [48] (in this case ICRF power is directly deposited on the H minority species, then transferred to electrons and the main deuterium ions through collisional damping). The main dependency of ICRF heating efficiency in the minority heating scenario is the minority ion concentration relative to that of the main ion, i.e. $n_{H}/n_{D}$. With optimum minority concentration of $n_{H}/n_{D}=5\sim10\%$, ICRF power absorption rate in plasma can reach a maximum of \sim 90\% [49,50]. Under certain conditions, H-mode can also be induced in C-Mod with ohmic heating alone, by ramping the $B_T$ and/or $I_p$. H-mode of this type is called ohmic H-mode [51,52], which stands out as a different topic, and will not be covered in this thesis.

C-Mod is also equipped with a large set of plasma diagnostics, taking measurements of key parameters across the entire plasma volume [53]. Of particular relevance to L-H transition study are those probing the plasma edge and the SOL-divertor region. Among the edge diagnostics, the most crucial ones to this thesis study include the high spatial resolution edge Thomson scattering (ETS) [54], divertor Langmuir probes (LP) [55], and Charge-Exchange Recombination Spectroscopy (CXRS) [56]. The locations where these diagnostics take measurements are shown in Fig.1-8, over a C-Mod poloidal cross section. The measurement locations are mapped to outboard midplane using EFIT.

By alternatively launching two pulsed Nd:YAG lasers, each fired repeatedly at 30Hz, into plasma, ETS measures upstream $T_e$ and $n_e$ profiles in the edge pedestal and near-SOL region (typically for $a_0-2cm<r<a_0+5mm$ at outboard midplane), with nominal radial resolution of 1.3mm, and time resolution of 16.67ms. Embedded behind divertor tiles, divertor Langmuir probes provide measurements of SOL $T_e$ and $n_e$ profiles on divertor target plates, with ~mm spatial resolution (when mapped to midplane), and ~10ms time
resolution. CXRS measures $B^{5+}$ impurity ion temperature, poloidal and toroidal plasma rotation in the edge pedestal region ($a_0-2cm<r<a_0$), with $\sim 10ms$ time resolution. Due to the very short thermal equilibrium time (i.e. the impurity-ion collision time, $\tau_i = 1/\nu_i$) between impurity ions and main ions in plasma edge, it is reasonable to assume $T_i \approx T_e$.

Other key plasma diagnostics for this thesis study involve:

1. A chord of multi-channel two-color interferometer (TCI) system, applied to record the line average electron density $\bar{n}_e$ (called plasma density in this thesis);
2. ECE polychromators, providing sub-ms $T_e$ measurements on outboard midplane, with reduced spatial resolution ($\sim 1cm$ in plasma edge);
3. A photodiode array, measuring $D_\alpha$ emission along a line of sight that is close to the midplane;
4. Phase contrast imaging [57], measuring the line integrated electron density fluctuation;
5. B-dot magnetic pick-up coils, recording the fluctuations in poloidal magnetic field.

Measurement locations of (1)-(5) are illustrated in Fig.1-8.
Chapter 2

Overview of L-H transition experiments and theories

Ever since the discovery of H-mode, many research efforts have been given to study the L-H transition physics, due to its importance to high-performance tokamak plasma operation. These studies have greatly expanded our knowledge in this area. Using well documented measurements from a large number of tokamaks, empirical scaling laws for H-mode threshold power ($P_{th}$) were produced to project the design of future devices.

On the theory side, a large variety of models have been proposed, to explain the trigger mechanism for the L-H transition. Some of them were extensively compared against experiments, and yielded promising agreement. Despite these advances, there is still no widely accepted theory to explain the L-H transition, due to its complex nature.

This chapter will briefly review some important results from previous research on the L-H transition. Main experimental observations on the behavior of $P_{th}$, and local plasma edge conditions at the L-H transition, are first presented in section 2.1. Following this in section 2.2, basic properties of tokamak edge turbulence are discussed; two promising models for the L-H transition, based on the suppression of tokamak plasma edge turbulence, are introduced. Finally, section 2.3 introduces a recently developed model for $P_{th}$ predictions, based on edge and SOL transport physics.
2.1 Overview of L-H transition experiments

2.1.1 H-mode threshold power

Dedicated experimental studies of the L-H transition have been carried out in many tokamaks over the past decades. These studies were initially motivated by a real engineering problem, faced in the design of a burning plasma device (e.g., a tokamak), that is, how much heating power will be required for a tokamak to access H-mode at its target operational plasma conditions. This power is called the H-mode threshold power, \( P_{\text{th}} \). To address this issue, many early studies on L-H transition were focused on developing a tool to project \( P_{\text{th}} \) for future devices. These efforts gave birth to a multi-machine L-H transition database [58-65], which assembled numerous data of \( P_{\text{th}} \), from a wide range of tokamaks in the world, including Alcator C-Mod. The \( P_{\text{th}} \) data in this database are values of \( P_{\text{loss}}=P_{\text{in}}-dW/dt \), evaluated at the L-H transition times. This large database covers a broad range of machine size (major and minor radius), plasma density, magnetic field, plasma current, auxiliary heating methods, divertor geometry, etc.

Regression analyses were carried out as the database expanded. These analyses generated a variety of power-law based empirical scaling laws. Among them, the one given by Martin et al [65]

\[
P_{\text{th}}[\text{MW}] = 0.049 \bar{n}_e^{0.72} B_T^{0.8} S^{0.94}
\]

(2.1)

is the most recent version, and is widely used. Here, \( \bar{n}_e \) is in \( 10^{20} \text{m}^{-3} \), \( B_T \) is in Tesla, and \( S \) is the surface area of plasma volume in \( \text{m}^2 \). The three main dependencies suggested by eqn.(2.1) are also consistent with many earlier works [58-64]. It should be stressed that only data from deuterium plasma, with the ion \( B \times \nabla B \) drift pointing towards the active X-point (this is called in the favorable direction for H-mode access), and not too low density (threshold values were set for each devices), were selected in deriving the Martin scaling.

The accuracy of the scaling law predictions was examined in [65]. The data were very scattered, with large departure from experimental values, as much as a factor of 3-5. Yet
without a better model, the empirical scaling remains the primary predictive tool for ITER $P_{th}$, at present.

![Graph showing U-shaped plasma density scaling of $P_{th}$ in DIII-D deuterium (D) and helium (He) plasmas](image)

**Fig.2-1:** U-shaped plasma density scaling of $P_{th}$ in DIII-D deuterium (D) and helium (He) plasmas [from (70)].

Attention was also given to examine the detailed behaviors of $P_{th}$ in a *single* device, trying to identify the origins of the large scatter in the scaling law predictions. Dedicated experimental studies were carried out on many tokamaks, with primary focus given to explore the plasma density dependence of $P_{th}$ (since for most tokamaks, $B_T$ and $I_p$ are usually varied only in a narrow range). Reported by many tokamak experiments: JT60 [66], COMPASS-D [67], ASDEX Upgrade (AUG) [68], JET [69], DIII-D [70], and Alcator C-Mod [71,72], the plasma density dependence of $P_{th}$ typically exhibits a ‘U’ shape, rather than the approximately linear correlation with $\bar{n}_e$, as eqn.(2.1) suggests ($P_{th} \propto \bar{n}_e^{0.72}$). An example from the DIII-D tokamak is shown Fig.2-1. In this case, there exists a local minimum in $P_{th}$, and correspondingly an ‘optimum’ density ($\bar{n}_{\text{min}}$) for H-
mode access. Reasonable agreement of scaling law predictions with experimental results was mainly found at moderated densities, around $n_{\text{nim}}$, while experimental values were seen ‘abnormally’ high (relative to the scaling) at both low and high densities.

The appearance of the low and high density regimes for H-mode access became a concern for ITER, since ITER power system design was based solely on the scaling law projection; therefore, if ITER target operation densities happen to be in either of the two regimes, it is likely for the scaling law to significantly underestimate the actual power requirement for ITER H-mode access (the Martin scaling predictions for ITER $P_{\text{th}}$ in deuterium plasma, with $B_T=5.3\, \text{T}$ and $S=679m^2$, are $P_{\text{th}} \approx 52\, \text{MW}$ for $\bar{n}_e=0.5 \times 10^{20} \, m^{-3}$, and $P_{\text{th}} \approx 86\, \text{MW}$ for $\bar{n}_e=1.0 \times 10^{20} \, m^{-3}$). So a main task of the L-H transition research is to address this issue, telling: (1) whether the low-density branch will exist on ITER, and (2) if yes, under what conditions it will appear. However, empirical scaling can not provide compelling answers to these questions.

The $B_T$ dependence of $P_{\text{th}}$ in a single device has been less extensively studied, because for most tokamaks, $B_T$ can only be varied in a relatively small range. Therefore, the dedicated C-Mod experiments [71-73] made a unique contribution to this study. The C-Mod experiments showed that the actual $B_T$ dependence is more complex than the empirical scalings suggest. In addition, C-Mod experiments also confirmed a speculation from an earlier analysis of the multi-machine database, that $n_{\text{nim}} \propto B_T$ [65]. Results from the C-Mod experiments will be shown in Chapter 3.

Aside from density and magnetic field, it was found that other factors that are not explicitly included in the empirical scalings, could also strongly influence $P_{\text{th}}$. Two well known examples are quoted here. The first one refers to the ion grad-B drift direction relative to the active X-point(s). This effect has been systematically studied on C-Mod [74]. In single-null divertor plasma, $P_{\text{th}}$ were found significantly enhanced, typically by a factor of 2-4, when the ion grad-B drift was switched from being in the favorable (i.e. pointing towards the X-point), to the unfavorable direction for H-mode access (this can be experimentally realized by flipping the null points, from LSN to USN, or by reversing
the direction of $B_T$), while keeping all other parameters nearly constant. The grad-B drift dependence has also been investigated on other tokamaks [58,75], and similar results were reported. Since $P_{th}$ is very sensitive to this parameter, for all cases included in this thesis, the ion grad-B drift is in the favorable direction. The second example refers to the effects of X-point/divertor configuration, including the X-point position relative to the divertor target plate, and the divertor geometry. This will be discussed in Chapter 4.

Given its relative simplicity, it is clear that the empirical scaling law may not make reliable prediction of $P_{th}$ for ITER. This appeals to understanding the physics of the L-H transition, and physics-based models for $P_{th}$.

### 2.1.2 Local plasma edge conditions for L-H transition

There is plenty of experimental evidence (e.g., [76-79]) indicating that the L-H transition begins in the plasma boundary near the separatrix, and then propagates into the plasma core region, accompanied by the formation of edge pedestal. Compared to $P_{th}$, local plasma parameters were considered to be more relevant to the physics of the L-H transition. Studies on the L-H threshold local plasma conditions were conducted in many tokamaks, with favorable ion grad-B drift. The parameters of interest varied a lot, but one that was routinely characterized is the edge electron temperature, in analogy to the concept of ‘critical temperature’ in phase transition physics. There was no unified standard, applied to all tokamaks, that explicitly indicates where this parameter should be measured. Typically, the L-H threshold $T_e$ was evaluated at the radial location near the top of H-mode $T_e$ pedestal, $T_{e,ped}$. It was found in a wide range of tokamak experiments that $T_{e,ped}$ is generally a good measure for different confinement regimes [80-85]: it is consistently lower in L-mode than in H-mode, and the two regimes are clearly separated by a ‘belt’, containing the values taken at the L-H threshold. Generally, $T_{e,ped}$ near the L-H threshold varies weakly with density, and increases with $B_T$. An exception is found in the low-density branch of $P_{th}$, where $T_{e,ped}$ increases substantially at reduced density, akin to the behavior of $P_{th}$ in this regime. Some studies [81-83] also suggested empirical
scalings for $T_{e,\text{ped}}$, based on data from their devices, as shown below in Table.2.1. A multi-machine scaling of $T_{e,\text{ped}}$, given by \[59\], is: $T_{e,\text{ped}}[eV] = 0.39(\overline{n}_e/10^{20})^{-0.69} B_T^{0.69} q_{95}^{-0.68} A^{-0.14}$.

<table>
<thead>
<tr>
<th></th>
<th>$B_T$ [T]</th>
<th>$I_p$ [MA]</th>
<th>$\overline{n}_e[10^{20} \text{m}^{-3}]$</th>
<th>L-H threshold $T_{e,\text{ped}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUG</td>
<td>1.5-3</td>
<td>0.6-1.2</td>
<td>0.23-1.3</td>
<td>$T_{e,\text{ped}}(eV) = 145\overline{n}_e^{-0.3}(\pm0.2,\pm0.1) B_T^{0.6}(\pm0.2,\pm0.1) I_p^{0.5\pm0.2}$</td>
</tr>
<tr>
<td>C-Mod</td>
<td>5.3-8</td>
<td>0.8-1.3</td>
<td>1.0-2.5</td>
<td>$T_{e,\text{ped}} \approx 0.15\overline{n}_e^{-0.57\pm0.1} B_T^{0.16\pm0.15} I_p^{0.17\pm0.2}$</td>
</tr>
<tr>
<td>JET</td>
<td>1.8-2.7</td>
<td>2.2-2.5</td>
<td>0.1-0.2</td>
<td>$T_{e,\text{ped}}(eV) = 390\overline{n}<em>e^{-0.64\pm0.15} B_T^{0.69\pm0.18} A^{-0.14\pm0.19} q</em>{95}^{-0.38\pm0.57}$</td>
</tr>
</tbody>
</table>

Table.2.1: Scaling of the L-H threshold $T_{e,\text{ped}}$ on AUG [83], C-Mod [81] and JET [82] tokamaks. The ranges of $B_T$, $I_p$, and plasma density in which these scaling were derived are also given.

As a companion of $T_{e,\text{ped}}$, the behavior of ion pedestal temperature ($T_{i,\text{ped}}$), however, was less extensively studied, largely because measurements of this quantity was not routinely available on many tokamaks. According to previous studies in DIII-D [80], JET [84] and JT-60 [85], $T_{e,\text{ped}}$ and $T_{i,\text{ped}}$ are close (perhaps $T_e$ is slightly lower, but not significant) in L-mode and at the L-H threshold (see Fig.2-2), and hardly can be decoupled, even at the low densities of these experiments. This is somehow anticipated, because the high collisionality in the pedestal region tends to bring different plasma species into thermal equilibrium very quickly.

Surprisingly, a recent AUG experiment [86] shows that this ‘robust’ equilibrium can be broken in low density plasma ($\overline{n}_e \approx 2 \times 10^{20} \text{m}^{-3}$), with electron cyclotron resonance heating (ECRH), which directly heat the electrons. Observed in this experiment is a remarkable, continually amplified separation between $T_{e,\text{ped}}$ and $T_{i,\text{ped}}$ with increasing ECRH power, from $T_{e,\text{ped}} \sim T_{i,\text{ped}}$ in ohmic plasma, to a maximum of $T_{e,\text{ped}} \sim 3T_{i,\text{ped}}$ just before the L-H transition. Unlike $T_{e,\text{ped}}$, which peaks towards low density, $T_{i,\text{ped}}$ near the L-H threshold only weakly vary with density, and is close to $\sim 300eV$. Based on this evidence, the authors suggested that the ion channel could play a crucial role in the L-H transition.
Fig. 2-2: $T_e$ and $T_i$ near the pedestal top at the L-H threshold in the JT-60U tokamak [from (85)].

The parameter widely considered to be most relevant to the L-H transition is the radial electric field $E_r(r)$ in the edge pedestal region [87]. According to some theories (e.g. [88, 89]), the sheared poloidal $\mathbf{E} \times \mathbf{B}$ flow ($V_{pol} = E_r/B_r$, sheared means $dV_{pol}/dr \neq 0$) can effectively suppress the edge turbulence transport, and possibly lead to L-H transition, when such suppression becomes significant. Experimentally, $E_r$ can be measured directly using Langmuir probes or heavy ion beam, or can be inferred from charge exchange spectroscopy (CXS) measurements, via the radial force balance of impurity ions [90].
\[ E_r = \frac{1}{n_i Z_i e} \frac{dP_i}{dr} - V_{\rho i} B_r + V_{\varphi i} B_\varphi \]  

(2.2)

where \( n, P, V, Z \) are the density, pressure, velocity, and charge; subscript \( T, p \) stand for the toroidal and poloidal direction, respectively. Behavior of \( E_r \) in plasma edge and its relation with the L-H transition were explored in many tokamaks experiments [91-100]. Commonly seen in these experiments is that \( E_r \) formed a ‘well’ in the edge pedestal region, which is moderate in L-mode, and becomes more negative in H-mode, as shown in Fig.2-3.

![Fig.2-3](image_url)

Fig.2-3: This figure shows the radial profiles of (a) \( E_r \), (b) the \( E \times B \) shearing rate, \( \omega_{E,B} = (r/\rho) d (q E_r/r B)/dr \), measured by the charge exchange recombination spectroscopy (CXRS) diagnostics in C-Mod L-mode and H-mode plasma [from (96)].

However, since \( E_r \) is covariant with pressure gradient (eqn. (2.2)), it is difficult to tell whether the deepening of \( E_r \) well is a cause, or a consequence, of the L-H transition.
Experimental test of this causality requires accurate measurements $E_r$ in plasma edge with excellent temporal and spatial resolution; therefore, this has only been attempted on few tokamaks. Some experiments showed evidence in support of the $\mathbf{E} \times \mathbf{B}$ shear causing the L-H transition. For example, the biased H-mode experiment on the TEXTOR tokamak revealed an oscillation in $\nabla E_r$ leading that in $\nabla n_e$ by 4-5ms, at the L-H threshold [99]. The CXS measurements in a DIII-D experiment showed that at $R-R_{sep} \sim 8$mm (inside the separatrix), $E_r$ started to fall off at $\sim 5$ms before the L-H transition (the time scale is similar to that of the TEXTOR experiment), while ion temperature gradient and density gradient varied slowly in time before the L-H transition [100]. At the same time, the DIII-D experiment also found that $E_r$ measured at $R-R_{sep} \sim 4$mm (outside the separatrix, in the SOL) did not show such behavior, and did not change very much as approaching the L-H transition, implying that the L-H transition is mainly associated with the plasma inside and near the separatrix [100]. However, because the existing evidence is quite limited (some observations are also very difficult to reproduce), the causal relation between $E_r$ and the L-H transition is still under considerable discussion.

2.2 Overview of models for L-H transition

2.2.1 The $\mathbf{E} \times \mathbf{B}$ shear suppression paradigm

A number of models have been proposed to explain the L-H transition triggering, many of which can be found in a comprehensive review article by Connor and Wilson [101]. Most of these models considered the suppression of plasma edge turbulence transport by the $\mathbf{E} \times \mathbf{B}$ shear flow as a leading mechanism for the L-H transition, and suggest that this could occur when the $\mathbf{E} \times \mathbf{B}$ shearing rate

$$
\alpha_{E_x B} = \frac{r}{q} \frac{d}{dr} \left( \frac{q E_r}{r B} \right) \approx \frac{1}{B} \frac{dE_r}{dr} + \frac{E_r}{B} \frac{d \ln q}{dr}
$$

(2.3)
exceeds the linear growth rate of the most unstable fluctuation mode, \(i.e\).

\[
\omega_{E \times B} > \gamma_{\text{max}}
\]  \hspace{1cm} (2.4)

Shown in Fig.2-4 is a simple picture for this paradigm. The sheared poloidal flows tilt, stretch, and break the large, long-range correlated turbulence eddies into small ones, reducing transport compared to the case without shearing.

Fig.2-4: A simple picture for the \(E \times B\) shear suppression of plasma turbulence.

Similar to eqn.(2.2), \(E_r\) can be written as

\[
E_r = \frac{1}{Z_en_i} \frac{dP}{dr} + V_{pi}B_T + V_{Ti}B_p
\]  \hspace{1cm} (2.5)

for the main ions \((i)\). According to theories, \(E_r\) is mainly generated by neoclassical effect and plasma turbulence. The neoclassical term is given by [17, 18, 86]

\[
E_{r,\text{neo}} \approx \frac{1}{n_iZ_i e} \frac{dP}{dr} + \alpha_r(V_\ast) \frac{dT_i}{dr}
\]  \hspace{1cm} (2.6)
where $\alpha_r$ is a numerical factor, which is sensitive to the neoclassical collisionality

$$\nu_e \sim qR/\lambda_e$$

(2.7)

Note that $E_{r,\text{neo}}$ is closely related to the diamagnetic term ($\nabla P$) in eqn.(2.5), i.e.

$$E_{r,\text{dia}} = (1/n_eZ_e)e dP_i/dr$$

(2.8)

According to (2.6), $E_{r,\text{neo}}$ varies in the transport time scale, i.e. the time scale for plasma equilibrium (e.g. pressure) to evolve, typically of the plasma (energy and/or particle) confinement time, e.g. 20-40ms for C-Mod L-mode plasma. Ion orbit loss near plasma boundary, especially in the X-point region [102], could amplify the magnitude of $E_{r,\text{neo}}$ in the plasma edge, relative to the value given by eqn.(2.6). However, the main parametric dependence will not be qualitatively changed by this effect.

The significance of $E_{r,\text{neo}}$ is that it provides an irreducible baseline for $E_t$. Some studies also suggested that the neoclassical (including the ion orbit loss) effect may trigger the L-H transition [103,104]. The recent AUG experiment [86] found that the minimum in $E_{r,\text{neo}}(r)$ for L-H transition is almost constant, at $\sim 15eV/mm$, and nearly independent of plasma density, which hints that the neoclassical $E_t$, or the ion channel may play a crucial role in the L-H transition.

The contribution from plasma turbulence ($E_{r,\text{turb}}$) is more complicated. Unlike $E_{r,\text{neo}}$, plasma turbulence affects $E_t$ mainly through changing the ion poloidal velocity ($V_{pi}$), via the turbulence Reynold stress. This can be described by the poloidal ion momentum balance equation

$$m_i n_i \frac{dV_{pi}}{dt} \approx j_r B_r + \frac{m_i n_i}{r} \left\langle \hat{V}_i \hat{V}_{pi} \right\rangle - m_i n_i (\mu_p + \nu_n) V_{pi}$$

(2.9)

where $j_r$ is the radial electric current, $\mu_p$ is the poloidal viscosity, $\nu_n$ is the ion-neutrals collision frequency, $\hat{V}$ the ion velocity fluctuation; $\left\langle \ldots \right\rangle$ means averaging over time. The Reynold stress term on the RHS of eqn.(2.9) could either drive or damp $V_{pi}$, depending on its sign.
A distinct feature of $V_{pi}$ is that it could grow very fast, in a time scale much shorter compared to the transport time scale, when the nonlinear coupling of turbulence (given by the Reynold stress term in eqn.(2.9)) is strong; therefore, it could possibly explain why the onset of the L-H transitions is very sudden. There was experimental evidence [91,100] showing that $V_{pi}$ starts to change prior to the L-H transition, while the $T_i$ profile remained less varied, suggesting that $E_{r,neo}$ alone may not be sufficient for the L-H triggering, and $V_{pi}$ could play a crucial role in giving a final ‘hit’.

Recent experiments on various tokamaks (e.g. AUG [105], DIII-D [106], EAST [107]) observed limit-cycle oscillation of poloidal flow prior to L-H transition in plasma edge, which is evidence that the plasma turbulence driven oscillatory flow, and its interaction with the mean equilibrium flow and plasma turbulence may play a key role in triggering the L-H transition. It is also noteworthy that the nonlinear effect might suppress plasma turbulence via generating substantial plasma flow that lead to a change in $E_r$. Due to its importance to the L-H transition, some basic properties of tokamak edge turbulence will be discussed later in 2.2.3.

2.2.2 The ion-orbit-loss (Itoh-Itoh) model for the L-H transition

The ion orbit loss near the tokamak plasma boundary was considered as a leading mechanism for the L-H transition in the early days of H-mode research. A number of models were proposed based on this. A similarity of these models is that the key plasma parameters are the ion-ion collision frequency, $v_{ii}$ (eqn. (0.10)), and the ion poloidal gyro radius, $\rho_{pi}$, defined as

$$\rho_{pi} = V_{ni}/(Z_i e B_p / m_i)$$  \hspace{1cm} (2.10)

The model proposed by Itoh and Itoh [104] is introduced in this section.
The Itoh-Itoh model focuses on the edge $E_r$, generated by the non-ambipolar** (NA) component of the electron and ion radial particle flux, in the narrow region of $|r - a_0| < \rho_{pi}$ (i.e. where the model is applied), and predicted a bifurcation in transport and $E_r$, when a critical condition is satisfied. The ion transport is assumed neoclassical. Therefore, the NA ion particle flux ($\Gamma_{i,NA}$) is mainly induced by the ion particle loss in the narrow layer, $|r - a_0| < \rho_{pi}$, due to the ion-ion Coulomb collisions that scatter the particles into this region, given by

$$\Gamma_{i,NA} = \frac{F}{\sqrt{\epsilon}} n_{i,i} \rho_{pi} \exp \left\{ - \left( \frac{\rho_{pi} e E_r}{T_i} \right)^2 \right\} \tag{2.11}$$

where $0 < F < 1$ is a numerical factor of the order of unity, $\epsilon = a_0 / R_0$ is the inverse aspect ratio of the tokamak.

The NA electron loss is dominated by the anomalous transport by plasma turbulence, in the region of $|r - a_0| < \delta$, where $\delta = \rho_{pi} (qk/\epsilon) < \rho_{pi}$. The NA electron flux depends on $E_r$, and is given by

$$\Gamma_{e,NA} = -D_e n_e \left\{ \frac{1}{n_e} \frac{d n_e}{dr} + \alpha \frac{1}{T_e} \frac{d T_e}{dr} + \frac{e}{T_e} E_r \right\} \tag{2.12}$$

where $\alpha \sim 1$ is a constant, $D_e$ is the anomalous electron diffusivity.

The balance between $\Gamma_{e,NA}$ and $\Gamma_{i,NA}$ (i.e. $\Gamma_{e,NA} = \Gamma_{i,NA}$) yields the following equation for $E_r$:

$$\exp\{-X^2\} = d(\lambda - X) \tag{2.13}$$

where,

$$X = \rho_{pi} e E_r / T_i \tag{2.14}$$

$$d = \sqrt{\epsilon} D_e / n_{i,i} F \rho_{pi}^2 \tag{2.15}$$

$$\lambda = -\frac{T_e}{T_i} \rho_{pi} \left\{ \frac{1}{n_e} \frac{d n_e}{dr} + \alpha \frac{1}{T_e} \frac{d T_e}{dr} \right\} \tag{2.16}$$

** ambipolar here means the diffusion of electrons and ions are at the same rate
This equation is parameterized by $\lambda$. There exist multiple solutions of $\Gamma$, *i.e.* a bifurcation in transport, when $\lambda=\lambda_c$, where $\lambda_c$ is approximately

\[ \lambda_c d \approx 1 \]

(2.17)

It means, for similar equilibrium profiles, different levels of transport can coexist, akin to the case of the L-H transition. For $\lambda < \lambda_c$ (*i.e.* with less steep profiles), the transport level is high, corresponding to the L-mode. For $\lambda > \lambda_c$, the transport level is significantly reduced, corresponding to the H-mode.

At the same time, a bifurcation in $E_r$ is found, when $\lambda=\lambda_c$. The model predicts that $E_r$ is negative and increases with $\lambda$ in L-mode ($\lambda < \lambda_c$), and ‘jumped’ to a *positive* values

\[ E_{r,H} \approx \frac{\lambda T_i}{e \rho_{pi}} \]

(2.18)

in the H-mode branch ($\lambda \geq \lambda_c$). However, it has been observed in many tokamaks that edge $E_r$ becomes more *negative* in H-mode, inconsistent with the model prediction. This discrepancy is a main difficulty for this model.

In a subsequent work [108], the authors refined the model by including the effect of neutral particles, impurities, plasma rotation, radial derivative of $E_r$, *etc.* The resulting L-H threshold condition is not qualitatively changed by adding these effects.

### 2.2.3 Linear tokamak plasma edge instabilities

Another main group of L-H transition models are those involve the tokamak plasma edge turbulence. Plasma turbulence is considered to arise from the nonlinear evolution of a spectrum of plasma instabilities, and is associated with the fluctuations in plasma parameters (*e.g.* density, temperature, velocity, *etc.*) measured in experiments. The characteristics of plasma turbulence and turbulence induced transport can be studied using the Braginskii-Maxwell equations (see Appendix A for details). These equations
admit a large variety of plasma instabilities, ranging from the ion temperature gradient driven modes (ITG modes, [109,110]), which are dominant in the hot, collisionless tokamak core, to the resistive-type modes [111,112], which are dominant in the cold, collisional tokamak edge (The ITG modes are either stabilized or insignificant compared to other modes in tokamak edge). Tokamak plasma edge turbulence is believed relevant to the L-H transition, and thus is of particular interest to this thesis. To better interpret the results shown later in section 2.2.4 and 2.2.5, it is important to first demonstrate some important characteristics of linear plasma instabilities (details about how to perform linear stability analysis can be found in Appendix B).

The discussion given in this section follows closely the works by Scott [111] and Zeiler [112]. As a simple case, temperature (for both electron and ions) and parallel ion velocity perturbations in the Braginskii-Maxwell equations are neglected, to reduce mathematical complexity and highlight the key physics; while other terms, especially plasma collisions, complete parallel electron dynamics, and magnetic geometry (which gives grad-B and curvature drift), are retained. The reduced equations give the linear dispersion relation

$$\omega^2 = (k_{\parallel} V_A)^2 \hat{\beta} \frac{\omega(1+k_{\perp}^2 \rho_i^2) - \omega_s}{(\omega - \omega_s) \hat{\beta} + (\omega + iv_c)k_{\perp}^2 \rho_i^2 \hat{\mu}} - \frac{\omega_c \omega_d}{k_{\perp}^2 \rho_i^2}$$

(2.19)

In eqn.(2.19), $k_{\parallel}$, $k_{\perp}^2$, $\omega$ are parallel and perpendicular wave number, and angular frequency (the imaginary part of $\omega$ represent the linear growth rate), for a normal mode ($\omega$, $k$); $v_c$ here is the electron collision frequency; $\rho_\omega$, which is defined as

$$\rho_\omega = C_{se} / \omega_d = \sqrt{m_i T_e / eB}$$

(2.20)

is the ion gyro radius at electron temperature. $C_{se}$, in eqn.(2.20), is the ion sound speed at electron temperature

$$C_{se} = \sqrt{T_e / m_i}$$

(2.21)

$\omega_c$ and $\omega_d$ are the frequency of the electron drift-wave and the shear-Alfv\é n wave,

$$\omega_c = k_{\perp} V_d = (k_{\perp} T_e / eB) / L_\perp = k_{\perp} \rho_i C_{se} / L_\perp$$

(2.22)

$$\omega_d = k_{\parallel} V_A = k_{\parallel} B / \sqrt{\mu_i m_i n}$$

(2.23)
where \( V_d, V_A \) are electron diamagnetic velocity and Alfvén velocity, respectively; \( L_\perp \) is the radial gradient scale length of electron density profile,

\[
L_\perp = \left| \frac{d \ln n_e}{dr} \right|^{-1}
\]

(2.24)

\( \omega_d \) is a characteristic frequency associated with the ion grad-B drift, given by

\[
\omega_d = k_\perp V_{cb}
\]

(2.25)

where \( V_{cb} \) is the ion grad-B drift velocity, as defined in eqn.(0.32).

\( \hat{\beta} \) and \( \hat{\mu} \) are normalized beta and electron inertia (mass), defined as

\[
\hat{\beta} = \frac{\mu_e n T_e}{B^2 \left( \frac{q R}{L_\perp} \right)^2} = \left( \frac{C_{se}/L_\perp}{V_A/q R} \right)^2 \sim \left( \frac{\omega_e}{\omega_A} \right)^2
\]

(2.26)

\[
\hat{\mu} = \frac{m_e}{m_i} \left( \frac{q R}{L_\perp} \right)^2 = \left( \frac{C_{se}/L_\perp}{V_e/q R} \right)^2 \sim \left( \frac{\omega_e}{\omega_i} \right)^2
\]

(2.27)

the estimation is taken for \( k_\parallel \rho_s \approx 1 \) and \( k_\parallel \approx 1/q \) (\( q \) is the safety factor here); \( \omega_i = V_e/q R \), in eqn.(2.27), is the parallel electron transit frequency.

The two terms on the right hand side of eqn.(2.19) give rise to different types of waves and/or instabilities. Consider the two asymptotic limits:

1). When the first term dominates, so that the second one can be nearly dropped, then eqn.(2.19) is reduced to

\[
\omega^2 \left[ (\omega - \omega_e) \hat{\beta} + (\omega + i \gamma_c) k_\parallel \rho_s^2 \hat{\mu} \right] = k_\parallel V_A \hat{\beta} \left[ \omega (1 + k_\parallel \rho_s^2) - \omega_e \right]
\]

(2.28)

This recovers the drift-Alfvén wave instabilities, the electromagnetic version of the electron drift waves.

In case both \( \hat{\beta} \) and \( \hat{\mu} \) are very small, \( i.e. \) when the electromagnetic and electron inertia effects are weak, the solution of (2.28) is nearly

\[
\omega = \omega_e + i \gamma_c \approx \omega_e + i \hat{\mu} k_\parallel \rho_s^2 V_e
\]

(2.29)
\[ \omega_b = \omega_e / (1 + k^2 \rho_s^2) \]  
\hspace{2cm} (2.30)

Eqn.(2.29) recovers the dispersion relation for the electrostatic *resistive electron drift wave*, which is featured by \( \omega_e \sim \omega_\alpha \), and destabilized by electron collisions.

Electromagnetic (EM) effect becomes no longer negligible when \( \hat{\beta} \sim 1 \). In the strong EM limit, the solution of eqn.(2.28) turns into the *kinetic shear Alfvén wave*:

\[ \omega \approx k_e V_A (1 + k_{\parallel} \rho_s) - i \mu k_{\parallel} \rho_s^2 v_e \]  
\hspace{2cm} (2.31)

which has a characteristic frequency of \( \sim \omega_A \), and is damped by collision.

2). On the other hand, if the second term on the RHS of eqn.(2.19) is much larger compared to the first one, then neglecting the first term, eqn.(2.19) is reduced to

\[ \omega^2 = -\frac{\omega_e \omega_d}{k^2 \rho_s^2} \approx \frac{2C_{sc}^2 \text{sgn}(\nabla P, \nabla B)}{L_R} \]  
\hspace{2cm} (2.32)

One may note that \( \omega_e \omega_d \) changes sign from being positive on the low-field-side (LFS) in a tokamak, where pressure gradient and magnetic field gradient are in the same direction \( (\text{sgn}(\nabla P, \nabla B) = 1) \), to being negative on the high-field side (HFS), where their directions are opposite. So, the instability described by eqn.(2.32) is highly inhomogeneous, changing from being unstable and purely growing on the LFS, to being stable on the HFS. This type of instability is called the *ballooning mode instability*, which tend to localize near the outboard midplane, where the instability drive is maximum.

The ballooning mode instability can be stabilized by parallel electron streaming, or shear Alfvén wave, which can be shown by reinstating the first term on the RHS of eqn.(2.19). Now, consider two simple cases:

A). in the quasi-electrostatic limit (i.e. \( \hat{\beta} \sim 0 \)), the term with \( \hat{\beta} \) in the denominator is then dropped, which yields
\[
\omega^2 \approx -\gamma_b^2 + \frac{k_p^2 V_e^2 \omega}{k_m^2 \rho_i^2 V_t} = -\gamma_b^2 + (k_p^2 \omega_e \omega_i / k_m^2 V_t) \omega \tag{2.33}
\]

where \(\gamma_b\) is the ideal ballooning mode growth rate, given by

\[
\gamma_b = \sqrt{2C_{ni}^2 / L_m R} \tag{2.34}
\]

Assuming the mode is strongly ballooning, \(i.e.\) for \(\gamma_b > \omega_i\), then eqn.(2.33) describes the linear resistive-ballooning mode instability, with the linear growth rate given by

\[
\gamma \approx i\gamma_b (1 - k_p^2 \omega_e \omega_i / k_m^2 V_t \gamma_b) \tag{2.35}
\]

Note that \(\gamma < \gamma_b\), and therefore, this mode is less unstable than the ideal ballooning mode, given by eqn.(2.32). Define \(L_{RB}\), the characteristic resistive ballooning length, as

\[
L_{RB} = (V_e \gamma_b / k_p^2 \omega_e \omega_i)^{1/2} \tag{2.36}
\]

Then eqn.(2.35) can be written as

\[
\gamma \approx i\gamma_b (1 - \lambda_{\perp}^2 / L_{RB}^2) \tag{2.37}
\]

where \(\lambda_{\perp} = 1/k_{\perp}\) is perpendicular wavelength. Eqn.(2.37) indicates that the resistive-ballooning mode is more stable for long-wavelength fluctuations (with larger \(\lambda_{\perp}\)). For \(k_{\perp} \rho_i \sim 1\) and \(k_m \sim 1/qR\)

\[
\gamma \approx i\gamma_b (1 - V_e^2 / q^2 R^2 V_t \gamma_b) \tag{2.38}
\]

which means that the resistive-ballooning mode can be stabilized by parallel electron streaming, and destabilized by electron collision.

B). in case the electromagnetic effect is strong (when \(\hat{\beta}\) is large), the dispersion relation can be simplified, by neglecting the electron inertia terms (those with \(\hat{\mu}\)) in eqn.(2.19), and becomes

\[
\omega^2 \approx k_p^2 V_a^2 - \gamma_b^2 \tag{2.39}
\]
Eqn.(2.39) recovers the linear dispersion relation for the **MHD ballooning mode**. This mode is stabilized by the shear Alfvén wave. Accordingly, one can define an MHD ballooning parameter

$$\alpha \equiv \frac{\gamma_b}{k_b^2 V_A^2} \sim q^2 R \beta / L_\perp$$

(2.40)

Then, the condition for the MHD ballooning mode to be linearly unstable is translated into that $\alpha > 1$. For $\alpha < 1$, the mode is linearly unstable.

An estimation of the key parameters in above equations, using typical C-Mod L-mode plasma edge conditions: $T_e = 100 eV, n_e = 5 \times 10^{19} m^{-3}, B \sim 5T, L_\perp \sim 1.0 cm, qR \sim 2 - 3m$, is given below, as

$$C_s \sim 7 \times 10^4 m/s, \ V_A \sim 1.5 \times 10^7 m/s, \ V_e \sim 4 \times 10^6 m/s,$$

$$\omega_A \sim V_A / qR \sim 5 \times 10^6 s^{-1}, \ \omega_e \sim C_s / L_\perp \sim 7 \times 10^6 s^{-1}, \ \gamma_b \sim 10^6 s^{-1},$$

$$\beta \sim 10^{-4}, \ \alpha \sim 0.1 - 0.2, \ \hat{\beta} \sim 1, \ \hat{\mu} \sim 10$$

From this estimation, we note that both electromagnetic and electron inertia effects are important, and therefore, the solution of eqn.(2.19) should represent a strong interaction between drift-wave and shear Alfvén-wave. The ideal ballooning growth rate is about the same order of magnitude as $\omega_e$ and $\omega_A$; therefore, the edge turbulence could also exhibit strong ballooning character.

In summary, the analysis reveals that tokamak edge turbulence is fundamentally drift-wave (including drift-resistive and drift-Alfvén) type of instabilities, with ballooning character (**i.e**. the mode tends to be more unstable in the LFS than in the HFS), due to the tokamak magnetic geometry. In case the drift-wave character prevails, the weak ballooning effect will only produce a downshift (on the LFS) or an upshift (on the HFS) in $\omega_r$ (mode frequency). On the other hand, if the ballooning-mode instability dominates, the parallel electron streaming and the shear Alfvén-wave essentially play a role of stabilizing the ballooning-mode instability. Conditions for strong ballooning mode instability are $\lambda_\perp < L_{RB}$ in the electrostatic case, and $\alpha > 1$ in the electromagnetic case.
Although linear stability analysis may illuminate some fundamental characteristics of plasma turbulence, a caveat must be put here, that understanding the behavior of plasma turbulence based on their linear stability properties is usually very insufficient, especially when strong nonlinear effects exist (e.g. mode-mode coupling, self-generation of plasma flow by turbulence, etc.). Some numerical studies discovered that strong nonlinear effects can cause significant turbulence suppression, and lead to the L-H transition. A number of L-H transition models were proposed based on these findings. Two of them – the DAW (drift-Alfvén wave) model and the RDZ model – will be introduced in 2.2.4 and 2.2.5.

2.2.4 The drift-Alfvén wave (DAW) model for the L-H transition

The DAW (drift-Alfvén wave) model discussed here is an analytical model, developed by Kerner, Cordey, et al [113], based on their studies of turbulence transport by the drift-Alfvén wave type of plasma instabilities. The analytical nature of the DAW model merits its use in studying the qualitative behavior of edge turbulence.

The DAW model neglects, in the derivation, the parallel ion velocity and ion temperature perturbations terms in Braginskii-Maxwell equations, while keeping complete electron dynamics and ion density perturbation (treated kinetically). The model adapted a simple slab model, which represents the tokamak geometry using a Cartesian coordinate xyz (x,y,z is equivalent to the radial, poloidal, and toroidal direction, respectively). In this case, magnetic geometry effect (grad-B and curvature drifts) and poloidal mode structure (ballooning character) are not taken into account. X-point and SOL are also not included; so the DAW model can only be applied to confined plasma, on closed flux surfaces.

The two control parameters of the DAW model are normalized beta ($\beta_n$) and normalized electron collision frequency ($\nu_n$)

$$\beta_n = \frac{m_i}{m_e} \frac{\beta_e}{\mu} = \frac{2\mu_0 P_e}{B^2} \left( \frac{m_i}{m_e} \right)^{1/2} \frac{qR}{L_{pe}}$$

(2.41)
\[ V_n = \frac{V_{ei} L_{pe}}{C_{se} \lambda_e} = \left( \frac{m_i}{m_e} \right)^{1/4} \left( \frac{q R L_{pe}}{\lambda_e} \right)^{1/2} \quad (2.42) \]

where \( \mu \) is the normalized electron inertia, given by

\[ \mu = \frac{V_{ei}}{C_{se} L_{pe}} = \sqrt{\frac{m_i}{m_e} L_{pe}} / qR \quad (2.43) \]

which is related to the \( \hat{\mu} \) (eqn. (2.27)), by \( \hat{\mu} = \mu^2 \); \( L_{pe} = \| d \ln P_r / dr \|^{-1} \) is the gradient scale length of the electron pressure profile; \( \lambda_e \) is the mean-free-path for electron Coulomb collisions (eqn. (0.9)); \( \beta_n \) is related to \( \hat{\beta} \) (eqn. (2.26)) by

\[ \beta_n = \frac{qR}{2L_{pe}} \left( \frac{m_i}{m_e} \right)^{1/2} \hat{\beta} \quad (2.44) \]

The DAW model derived a linear dispersion relation for the drift-Alfvén wave instability, which is a fourth order algebraic equation in \( \omega \). Taking results from the linear stability analysis, plasma transport induced by this instability is estimated using a quasi-linear theory (see Appendix C). The resulting particle diffusion coefficient is given by

\[ D_\perp = \frac{D_{GB}}{\sqrt{\mu}} \times (\beta_n, V_n) \quad (2.45) \]

\( D_{GB} \) in (2.45) is the Gyro-Bohm diffusivity, given by

\[ D_{GB} = C_{se} \rho_s^2 / L_{pe} = \frac{V_{ei}^2 \rho_e}{qR \omega_{ri}} \beta_n \]

\[ \propto T_e^{3/2} m_i^{1/2} / B^2 L_{pe} \quad (2.46) \]

which is essentially proportional to \( \beta_n \). \( \bar{\chi} \) is the normalized diffusivity, given by

\[ \bar{\chi} = \frac{\nu_c^{1/3} [1 + (\nu_n / \nu_c)^2]^{1/2}}{[1/ \nu_c^2 + (\nu_n / \nu_c)^4]^{1/2}} \quad (2.47) \]

where \( \nu_c = 1/(1 + \beta_n^2)^{3/2} \). Combining (2.45) and (2.46) gives
\[D_\perp = \frac{D_c b}{\sqrt{\mu}} \bar{\chi}(\beta_n, \nu_n) = \frac{V_A^2 \rho_e}{q R \omega_i \sqrt{\mu}} \beta_n \bar{\chi}(\beta_n, \nu_n) \propto \beta_n \bar{\chi}(\beta_n, \nu_n)\]  

(2.48)

The behavior of \( \bar{\chi} \) was explored in various asymptotic limits, and illustrated on a \( \beta_n - \nu_n \) phase diagram, as shown in Fig.2-5. This diagram was divided into four regions by the four lines: \( \beta_n = 1, \nu_n = 1, \beta_n \sim 1/\nu_n^{1/3}, \beta_n \sim \nu_n^{2/3} \). Dependence of \( \bar{\chi} \) on \( \beta_n \) and \( \nu_n \) is different in each regions.

Fig.2-5: The \( \beta_n - \nu_n \) phase diagram of the DAW model. The model suggests that the L-H transition should occur in zone (III) or (IV) of the diagram [adapted from (113)].
1). Low-beta cases (Zone I, II)

Zone I: $\beta_n < 1$ and $\nu_n < 1$, i.e. for low density and low temperature. $\overline{X}$ is independent of $\beta_n$ or $\nu_n$, and reads

$$\overline{X} \sim 1$$

$$D_\perp \propto \beta_n$$ (2.49)

Zone II: $\beta_n < \nu_n^{2/3}$ and $\nu_n > 1$, i.e. for high density and low temperature. $\overline{X}$ is independent of $\beta_n$ and weakly depends on $\nu_n$

$$\overline{X} \sim \nu_n^{1/3}$$

$$D_\perp \propto \beta_n \nu_n^{1/3}$$ (2.50)

So, in both cases (for low beta), transport ($D_\perp$) increases with pressure gradient ($\beta_n \propto P_e / L_{ne}$), and also with collisionality at high density.

2). High-beta cases (Zone III, IV)

Zone III: a narrow region bounded by $\beta_n = 1$, $\nu_n = 1/\nu_n^{1/3}$, and $\nu_n < 1$, i.e., for low density and moderate temperature. $\overline{X}$ shows a strong inverse dependence on $\beta_n$, but is independent of $\nu_n$,

$$\overline{X} \sim 1/\beta_n^4$$

$$D_\perp \propto 1/\beta_n^3$$ (2.51)

Zone IV: $\beta_n > 1/\nu_n^{1/3}$ ($\nu_n < 1$) or $\beta_n > \nu_n^{2/3}$ ($\nu_n > 1$), i.e. for high density, high temperature. $\overline{X}$ depends on both $\beta_n$ and $\nu_n$ in this regime, as

$$\overline{X} \propto \nu_n / \beta_n$$

$$D_\perp \propto \nu_n$$ (2.52)
A big difference between low-beta and high-beta is: in the former case, $\mathcal{R}$ does not depend on $\beta_n$; for the later, $\mathcal{R}$ shows an inverse dependence on $\beta_n$, i.e. $\mathcal{R}$ decreases with increasing pressure gradient for a given $B_T/I_p$. Based on these properties, the DAW model suggested that L-H transition occurs, when

$$\beta_n \geq 1 + \nu_n^{2/3} \quad (2.53)$$

This condition corresponds to Zone III, IV on Fig.2-5. The two limits of eqn.(2.53) are: $\beta_n > 1$ for $\nu_n < 1$ (low collisionality), and $\beta_n > \nu_n^{2/3}$ for $\nu_n > 1$ (high collisionality). The model also hints that $\beta_n = 1 + \nu_n^{2/3}$ is satisfied inside and near the separatrix, at the L-H transition.

### 2.2.5. The RDZ model for the L-H transition

The RDZ model [114] is named after the initials of its principle authors: Rogers, Drake, and Zeiler. It is a close ‘relative’ to the previously discussed DAW model, both of which were based on essentially the same transport equations. The RDZ model was developed from a series of numerical investigation on the nonlinear tokamak edge turbulence, with a focus on the nonlinear drift-wave and the resistive-ballooning mode instabilities. The model takes the Braginskii-Maxwell equations (Appendix A), and applies the following normalization scales

$$t \rightarrow t\gamma_b$$
$$\nabla_\perp \rightarrow L_{RB}\nabla_\perp$$
$$\nabla_\parallel \rightarrow 2\pi qR\nabla_\parallel \quad (2.54)$$

for space and time, with $\gamma_b$ and $L_{RB}$ defined in eqn.(2.34) and eqn.(2.36), respectively.

Two key control parameters of the RDZ model are a diamagnetic parameter

$$\alpha_d = \frac{T_e}{eBL_nL_{RB}\gamma_b} = \frac{\omega_e}{\gamma_b L_{RB}}$$

$$= \frac{1}{8\pi q}(2m_i/m_e)^{1/4}(\lambda_c/\sqrt{RL_n})^{1/2} \quad (2.55)$$

and an MHD ballooning parameter.
\[ \alpha_{\text{MHD}} = q^2 R \beta / L_{pe} \]  

(2.56)

\( \alpha_{\text{MHD}} \) is essentially the \( \alpha \) in (2.40), with \( L_\perp \) replaced by \( L_{pe} \). Here, \( L_\perp = |d \ln n_e / dr|^{-1} \) and \( L_{pe} = |d \ln P_e / dr|^{-1} \) in eqn.(2.55) and eqn.(2.56), are the gradient scale lengths of electron density and electron pressure profile; \( \beta \) is the plasma \( \beta \) \( \alpha \), as defined in (0.41).

\( \alpha_{\text{MHD}} \) is a measure of the MHD ballooning mode instability, the onset condition of which is \( \alpha_{\text{MHD}} > -1 \). Similarly, \( \alpha_d \) is a measure of the resistive-ballooning mode instability, which becomes significant when \( \alpha_d \ll 1 \). \( \alpha_{\text{MHD}} \) and \( \alpha_d \) essentially scale with \( \beta \) and inverse plasma collisionality, respectively. In addition, \( \alpha_{\text{MHD}} \) and \( \alpha_d \) are functionally equivalent to \( \beta_n \) and \( \nu_n \), as in the DAW model (eqn.(2.41), (2.42)), related by

\[ \alpha_{\text{MHD}} = 2 q \left( \frac{m_e}{m_i} \right)^{1/2} \beta_n \quad \nu_n \alpha_d^2 = \frac{1}{630 q^{3/2}} \left( \frac{m_i}{m_e} \right)^{3/4} \left( \frac{2 L_{pe}}{L_{\text{ie}}} \right)^{1/2} \]  

(2.57)

The authors (RDZ) performed a number of 3D nonlinear numerical simulations, in a simplified \( \text{shift-circular} \) magnetic geometry, with \( \text{closed} \) flux surfaces (i.e. shaping, X-point and SOL were not included in these simulations). A \( \text{phase diagram} \) in terms of \( \alpha_{\text{MHD}} \) and \( \alpha_d \) for tokamak edge turbulence is generated from these studies, as shown below in Fig.2-6.

Seen in Fig.2-6, the diagram is divided into several sections, each one corresponding to a different tokamak operation regime:

1). On the left of the phase diagram where \( \alpha_d \ll 0.5 \) to \( 0.6 \). Turbulence transport was found to be dominated by the \( \text{nonlinear resistive-ballooning mode} \) in this regime. Linear instability properties of this mode were discussed in section 2.2.2, the focus here is characterizing the induced plasma transport. For \( \alpha_d \ll 0.3 \), a significant \( \text{enhancement} \) in particle flux (\( \Gamma \)) with \( \text{increasing} \) \( \alpha_{\text{MHD}} \) was found in the simulations, even when \( \alpha_{\text{MHD}} \) is far below the ideal ballooning limit: \( \alpha_{\text{MHD}} \sim 1.0 \). As \( \alpha_{\text{MHD}} \) further increases, the significant outward radial transport limits the rise of \( \alpha_{\text{MHD}} \) (or plasma pressure), and makes plasma...
collapse. This was thought to be potentially relevant to the density limit in tokamak plasma operation. Consequently, the upper left corner of the $\alpha_{\text{MHD}}$-$\alpha_d$ diagram is inaccessible for steady plasma operation.

Fig. 2-6: The $\alpha_{\text{MHD}}$-$\alpha_d$ ‘phase diagram’ for tokamak plasma edge, produced by the RDZ model. The model-predicted L-mode, H-mode domains are shown on the diagram. ‘$\Gamma$’ in this plot represent the turbulence driven particle flux [adapted from (114)].

2). When $\alpha_d$ is raised to $\alpha_d\sim 0.5$, the influence of $\alpha_{\text{MHD}}$ on transport becomes less remarkable. It is also likely that the transport level could even slightly decrease, when $\alpha_{\text{MHD}} \sim 0.2$. Compared to the case of $\alpha_d=0.25$, the particle flux for $\alpha_d = 0.5$ was found consistently lower, over a wide range of $\alpha_{\text{MHD}}$, meaning that the turbulence transport was
reduced with increasing \( \alpha_d \). This trend qualitatively agrees with that the linear growth rate of resistive-balloonning mode is reduced with \( T_e \) (eqn. (2.38)), or with \( \alpha_d \), since \( \alpha_d \sim T_e \).

3). When \( \alpha_d \) is further increased, to \( \alpha_d \sim 1.0 \), the resistive-balloonning mode is suppressed, and the dominant edge turbulence is transformed into the nonlinear drift-wave instabilities. Strikingly, the effect of \( \alpha_{\text{MHD}} \) on turbulence transport is reversed, compared to that in the resistive-balloonning mode (low \( \alpha_d \)) case, \( i.e. \) \( \Gamma \) decreases with \( \alpha_{\text{MHD}} \), as seen from computation. The reduction of plasma transport will in turn further enhance the pressure gradient (\( i.e. \) \( \alpha_{\text{MHD}} \) when \( B_T, I_p \) are fixed), leading to more turbulence suppression. Numerical simulations suggested that this positive feedback loop could cause a spontaneous formation of edge transport barrier (pedestal), when \( \alpha_{\text{MHD}} \) exceeds a certain threshold value, akin to what is seen following the L-H transition in experiment.

Based on these simulations, the authors proposed an approximate L-H threshold condition: \( \alpha_{\text{MHD}} \sim 0.3-0.4 \), \( \alpha_d \sim 0.5-0.6 \), as shown by the ‘L’ shaped boundary on the phase diagram in Fig.2-6. This condition, when expressed in terms of \( \beta_n \) and \( \nu_n \), becomes

\[
\beta_n = 2-3, \quad \nu_n < 0.3-0.6
\]

(2.58)

for typical values of \( q \sim 3-4 \), \( L_p \sim 2-3L_n \). Eqn.(2.58) nearly corresponds to the transition between zone (III) and zone (IV) on the \( \beta_n-\nu_n \) diagram (Fig.2-6). Guzdar et al studied the zonal flow generation by the nonlinear drift-wave turbulence with finite-beta effect, using analytical theories, and obtained a criteria for the L-H transition as: \( \alpha_{\text{MHD}}\alpha_d^2 \sim \text{const} \) \([115]\). This mathematical relation seems to nearly fit the L-H threshold boundary suggested by the RDZ model, if the constant is chosen as \( \sim 0.35 \), \( i.e. \) \( \alpha_{\text{MHD}}\alpha_d^2 \sim 0.35 \).

The RDZ model uncovers some main characters of nonlinear tokamak edge turbulence and predicted conditions for the onset of many related phenomena, \( e.g. \) the density limit and the L-H transition, which are important to tokamak research. However, it was built for the simple shifted-circular geometry and does not include some essential edge physics (\( e.g. \), X-point, SOL, divertor, vessel wall, etc), therefore, the RDZ model could be used to tell some trend on edge transport, but may not be quantitatively accurate.
2.3 A model for L-H threshold power

Reliable predictions of the H-mode threshold power for given plasma conditions are important to ITER and future fusion reactor design. As already seen in section 2.1, the power-law based empirical scaling was found insufficient for this purpose, therefore a physics-based model for \( P_{th} \) prediction is desired. Recently, a model was developed by Fundamenski et al (named the FM\(^3\) model, [116]), which makes some advances in this area.

The FM\(^3\) model predicts the scaling of \( P_{th} \) with magnetic and plasma variables based on essential tokamak plasma edge and SOL-divertor physics. It assumes the L-H transition begins in the region just inside the separatrix, and therefore \( P_{th} \) can be expressed as a function of upstream near-separatrix plasma conditions. The model is based on a core Ansatz that the L-H transition takes place when the Wagner number (\( W_a \)) exceeds a certain critical value, of the order of unity, near the separatrix, i.e.

\[
W_a > W_{a,L-H} \sim 1 \tag{2.59}
\]

Here, \( W_a \) was introduced by the authors and defined as

\[
W_a = \frac{\tau_{\parallel,A}}{\tau_{\perp,Q}} \tag{2.60}
\]

where \( \tau_{\parallel,A} \) is the parallel Alfvén transit time, and \( \tau_{\perp,Q} \) is the perpendicular cross-field electron heat convection time, given respectively by

\[
\tau_{\parallel,A} = \frac{L_{\parallel}}{V_A} \approx \pi q_a R/V_A \tag{2.61}
\]

\[
\tau_{\perp,Q} = \frac{L_{\perp,corr}}{V_{\perp,Q}} \tag{2.62}
\]

where \( L_{\perp,corr} \) is the turbulence radial correlation length and \( V_{\perp,Q} \) is the radial velocity for electron energy convection. The authors linked the Ansatz to the \( \text{E} \times \text{B} \) shear flow paradigm (section 2.2.1), suggesting that when eqn. (2.59) is satisfied, the nonlinear interaction between electron drift-waves and shear Alfvén-waves becomes significant, generating a strong \( \text{E} \times \text{B} \) shear flow which quenches the edge turbulence and triggers the L-H transition.
$V_{\perp Q}$ in (2.62) is related to $P_{\perp}$, the power across separatrix, by tokamak edge power balance, as

$$V_{\perp Q} = P_{\perp} / \left( \frac{3}{4} S_{\perp} P_{e} \right)$$  \hspace{1cm} (2.63)$$

where $S_{\perp}$ is the surface area of plasma volume, $P_{e}$ is the electron pressure. Combining (2.63) with (2.60), (2.61), (2.62) yields an expression for L-H threshold power as

$$P_{th} \sim \frac{3}{4} P_{e} S_{\perp} V_{A} L_{\perp, corr} / L_{\eta}$$  \hspace{1cm} (2.64)$$

with all local parameters evaluated just inside the separatrix.

In addition to the core Ansatz, the following assumptions on local plasma conditions are further made by the $FM^3$ model:

1). $L_{\perp, corr} \approx \alpha_{\perp} \sqrt{L_{\rho} \rho_{s}}$ \hspace{1cm} (2.65)$$

where $\alpha_{\perp} \sim O(1)$ is a free dimensionless parameter varied with machines and plasma conditions.

2). $L_{p} \propto a_{0} q_{sep} A^{-1/2} \propto a_{0} q_{cyl}^{1/2} A^{-1/2}$ \hspace{1cm} (2.66)$$

where, $q_{sep} \sim q_{cyl}^{1/2}$, assuming the presence of a stochastic layer near the separatrix. $A=m_{i}/m_{p}$ is the mass number for the main ions. It was seen in experiments that $L_{p}$ could also vary with density.

3). The Mach number for perpendicular electron convection, $M_{\perp} = V_{\perp, Q} / C_{s}$, is nearly constant, thus the $e$-folding length of SOL heat flux, $\lambda_{Q}$, is estimated by

$$\lambda_{Q} \propto R q_{cyl} (Z / A)^{1/2} v_{SOL}$$  \hspace{1cm} (2.67)$$

where $v_{SOL} \propto L_{\eta} / \lambda_{e}$ is the plasma collisionality near the separatrix in the upstream plasma.
With the above assumptions, $P_{th}$ is related to upstream $T_e$ and $n_e$ by

$$P_{th} = \alpha_n (n_{e,sep} B \kappa / q_{sep})^{1/2} a_0^{3/2} Z^{1/4} T_{e,sep}^{5/4}$$  \hspace{1cm} (2.68)$$

Here $\alpha_n$ is a free parameter depending on specific device and plasma conditions; $\kappa$ is the plasma elongation. $T_{e,sep}$, $n_{e,sep}$ are $T_e$ and $n_e$ evaluated in the upstream plasma just inside the separatrix. Finally, using a two-point SOL transport model (see Appendix C) to relate $T_{e,sep}$ with $P_{th}$, one obtains the following equation for $P_{th}$

$$P_{th} \propto (n_{e,sep} B \kappa / q_{sep})^{1/2} a_0^{3/2} Z^{1/4} \left[ \left( \frac{P_{th}}{8 \zeta S_{\parallel} n_{el}} \right)^{7/3} + \frac{7}{4} \frac{P_{th} L_{n\parallel}}{\kappa_{ce} S_{\parallel}} \right]^{5/4}$$ \hspace{1cm} (2.69)$$

where $\zeta$ is the sheath constant, $\kappa_{ce}$ is the Spitzer-Harm conductivity, and $S_{\parallel} = 4 \pi a_0 \kappa^{1/2} \lambda_{ce} / q_{cyl}$ is the SOL cross section for parallel heat conduction.

The two terms inside the bracket on the RHS of (2.69) correspond to different SOL regimes:

1). the first one prevails in the sheath-limited regime, i.e. $\nu_{SOL} \ll 10$. In this case, (2.69) becomes

$$P_{th}^{sh} = c_1 n_{e,sep} B^3 S^{-0.5} (a / R)^{9/2} \kappa^{0.75} A^2 Z^{-9/4} a q_{sep}^{-1.5} \nu_{SOL}^{-5}$$ \hspace{1cm} (2.70)$$

2). For the conduction-limited regime, i.e. $15 < \nu_{SOL} < 70$, the second term dominates and (2.69) becomes

$$P_{th}^{cd} = c_2 (n_{e,sep} B / \sqrt{A q_{sep}})^{7/9} a^{16/9} \kappa^{0.5} q_{cyl}^{5/9} Z^{2/3} \nu_{SOL}^{-5/9} A^{1/9}$$ \hspace{1cm} (2.71)$$

where $c_1$ and $c_2$ in (2.70) and (2.71) are undetermined numerical factors resulting from the assumptions made by the model, and are functions of $\alpha_\perp$, $\alpha_0$.

Given that the variation in $\nu_{SOL}$ is not very significant (typically $\nu_{SOL} \approx 5-40$ if the divertor is not detached), these equations suggest an inverse scaling of $P_{th}$ with density in the sheath-limited regime and a positive one in the conduction-limited regime, corresponding to the low- and high-density branch of the U-shaped density dependence of $P_{th}$. 

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respectively. In addition, a strong $B_T$ dependence ($P_{th} \propto B^3$) is manifested in (2.70) for the sheath-limited regime, while the $B_T$ dependence is relatively weak in the conduction-limited regime.

A key objective of L-H transition studies is to characterize the scaling of $n_{min}$ (the density of minimum $P_{th}$) and to find the optimum density for H-mode access. From the discussions above, a significant implication of the FM$^3$ model is that a minimum of $P_{th}$ in density occurs as the SOL transport transitions from the sheath-limited to the conduction-limited regime, i.e. when $v_{solv}=10^{-15}$, which yields a scaling for $n_{min}$ as

$$n_{min} \propto B^{4/5} R^{-1/5} \varepsilon^{4/5} A^{9/10} Z^{-3/5} (Z + 1)^{-9/10} q_{cyl}^{3/5} V_{SOL}^{8/5} \ (2.72)$$

$\varepsilon = a/R$ is the inverse aspect ratio. Or in terms of $f_{G, min} \equiv n_{min}/n_G$, the Greenwald fraction

$$f_{G, min} \propto B^{4/5} \varepsilon^{4/5} A^{9/10} Z^{3/1} q_{cyl}^{2/5} V_{SOL}^{8/5} \ (2.73)$$

This scaling implies that $n_{min}$ decreases with $B_T$ or machine size, $R$, while the Greenwald fraction is independent of $R$ and only weakly decreases with $B_T$.

Yet another important aspect – the dependence of $P_{th}$ on ion grad-B drift direction – was not explicitly given in the FM$^3$ mode. However, the authors suggested that this effect was embedded in the core Ansatz of the model and argued that the critical Wagner number for L-H transition tends to be higher, or that the values of $\alpha_\perp$ are different with unfavorable ion grad-B drift, thus resulting in larger $P_{th}$ in this scenario.

Finally, the relation between the FM$^3$ model and models for L-H transition (e.g. the RDZ, DAW model) is briefly discussed below. Assuming the cross-field electron heat transport is dominated by drift-wave type instabilities, so that $\tau_{\perp,Q}$ should essentially scale inversely with linear growth rate, i.e.

$$\tau_{\perp,Q} \sim 1/\gamma_{DW} \propto L_{pe}/C_s \ (2.74)$$

Then the core Ansatz of the FM$^3$ model, $\tau_{\perp,Q} \sim \tau_{\perp,A}$, is translated into

$$qR/\sqrt{B^2/m_n n} \sim L_{pe}/C_s \ (2.75)$$
which can be written as

\[ nT_e (qR / L_{pe})^2 / B^2 \propto \beta (qR / L_{pe})^2 = \hat{\beta} \sim c \] (2.76)

The variable \( \hat{\beta} \) here is the normalized \textit{beta} as defined in eqn.(2.26), and is similar to \( \alpha_{\text{MHD}} \) or \( \beta_n \). Thus the \textit{Ansatz} of the FM\(^3\) model can be translated into the condition that the onset of the L-H transition is associated with a critical \( \hat{\beta} \) value. This is consistent with what was suggested by Guzdar \textit{et al} in [115], and shows a connection with the RDZ and DAW models.
Chapter 3

Scaling of H-mode threshold conditions
– dependence on density, plasma current, and magnetic field

As shown in Chapter 2, the multi-machine scaling law is insufficient to make accurate projections of \( P_{\text{th}} \) to future devices, e.g. ITER. To better understand the behaviors of \( P_{\text{th}} \) and resolve their disagreement with scaling law predictions, dedicated L-H transition/H-mode pedestal experiments were carried out in Alcator C-Mod lower single null plasmas, with favorable ion grad-B drift, over a wide range of plasma density, magnetic field, and plasma current. The C-Mod experiments confirm that the plasma density dependence of \( P_{\text{th}} \) is U-shaped, while this trend shows no clear dependence on plasma current. The local minimum of \( P_{\text{th}} \) in density (\( i.e. n_{\text{min}} \)) decreases as magnetic field is reduced. This investigation also revealed that the actual behavior of \( P_{\text{th}} \) is more complex than what is indicated by the empirical scaling, and can not be simply expressed by any power law based relation of the type \( P_{\text{th}} \propto \bar{n}_e^x B_i^y I_p^z \).

Local conditions of key plasma parameters preceding L-H transition were documented in the plasma edge and on the divertor target. Scaling of these parameters was systematically studied. Compared to previous C-Mod research, this survey covers a much broader range of tokamak operation conditions, especially with an extension to the low-
density regime of H-mode access, and to lower magnetic field. Also, more plasma parameters were characterized; in particular the local spatial gradient of plasma profiles was considered, at more than one radial location in the plasma edge region. These results will be employed to test various models for L-H transitions and for $P_{th}$, as discussed in Chapter 6.
3.1 L-H transition database on Alcator C-Mod

3.1.1 Data selection rules

A large number of carefully selected plasma discharges from dedicated L-H transition/ H-mode pedestal experiments in the 2005-2011 Alcator C-Mod experimental campaigns were assembled into an L-H transition database for this study. According to the data selection rules, each discharge in this database must contain at least one unambiguous L-H transition during the $I_p$ and $B_T$ flattop (i.e. the phase in which both $I_p$ and $B_T$ are nearly constant) and no L-H transition during the initial $I_p$, $B_T$ ramp up (e.g. for $t<0.5\text{sec}$ as in Fig.3-1). An unambiguous L-H transition is defined when the following phenomena are simultaneously observed in experiment:

1). Sudden drop of $D_\alpha$ emission, which signals the change in edge particle transport and the onset of L-H transition;

2). Beginning of a steep, break-in-slope rise in plasma density, confined plasma energy, confinement time, and neutron production rate;

3). Fast rise in edge $T_e$ measured with electron cyclotron emission (ECE);

4). Sudden reduction of edge fluctuation amplitude (e.g. density, magnetic fluctuations)

In addition, edge $T_e$ and $n_e$ pedestals start to form right after the L-H transition, typically indicated by Thomson scattering measurements. $D_\alpha$ signals are used to precisely determine the L-H transition time. In order to keep the analysis self-consistent, only the initial (first) L-H transition event is included if a discharge contains multiple ones. Time histories of key plasma parameters for a typical C-Mod plasma discharge with a clear L-H transition, followed by multiple transient L-H/H-L cycles, are shown in Fig.3-1.
Fig.3-1: Time histories for some key plasma parameters of a typical C-Mod plasma discharge with a defining L-H transition (marked by dashed line), followed by multiple L-H/H-L cycles (one of these cycles is highlighted by the yellow shadow): (a) plasma current (b) line averaged density (c) Dα emission (d) ECE measured $T_e$ near pedestal top (e) total heating power and RF power (f) stored plasma energy derived from EFIT (g) global energy confinement time (h) PCI measured line integrated density fluctuation amplitude in the frequency range of 10-60KHz.
The ranges of plasma density, \( B_T \), and \( I_p \) covered by this database are

\[
0.6 \times 10^{20} \text{ m}^{-3} < \bar{n}_e < 2.5 \times 10^{20} \text{ m}^{-3}
\]

\[
B_T = 3.5 \text{T}, \ 5.4 \text{T}
\]

\[
0.6 \text{MA} < I_p < 1.4 \text{MA}
\]

All discharges included in the database are lower single null (LSN) deuterium (D) plasma with ion grad-B drift in the favorable direction for H-mode access, i.e. pointing downwards towards the active X-point. ICRF is the sole auxiliary heating source. For each discharge, the ICRF antenna was tuned at the corresponding frequency to perform on-axis fundamental hydrogen minority heating. The hydrogen minority concentration for the analyzed discharges is typically in the range of 5-10%, which is about optimum for high efficiency plasma heating with power absorption fraction of ~90%. Magnetic geometry was nearly identical and fixed in the standard C-Mod LSN divertor plasma magnetic configuration (i.e., \( \kappa \sim 1.6 \), \( \delta_{\text{lower}} \sim 0.55 \), \( \delta_{\text{upper}} \sim 0.35 \), strike point on the vertical divertor plate, see Fig.1-8).

Effects of divertor geometry on H-mode threshold conditions were studied in a dedicated C-Mod experiment, and will be shown in Chapter 4. Dependences of H-mode threshold on ICRF-plasma resonance location and on main ion species were also studied in dedicated C-Mod experiments, and will be discussed in Chapter 5.

### 3.1.2 Plasma measurements and profile analysis

Reliable local plasma edge measurements of key parameters are crucial to this study. Among the involved C-Mod diagnostics for this thesis study (Fig.1-8), TCI [53], ECE [53], Edge Thomson scattering (ETS) [54], and divertor Langmuir probes [55] have been routinely operated on C-Mod and available for most (>95%) included discharges. The CXRS [56] measurements are only available for a limited number of discharges, while the C-Mod diagnostic neutral beam (DNB) was in operation. The EFIT code was
executed in each discharge to reconstruct equilibrium magnetic geometry and employed to map the plasma measurements to the outboard midplane for local profile analysis.

**Radial profile fitting**

Data of profile measurements first need to be fitted by smooth radial profiles, before detailed local analysis can be deployed. A mathematical model widely adapted in the edge profile fitting is a modified hyperbolic tangent ($\text{mtanh}$) function, defined [117] as

$$y(x) = A \frac{(1+\alpha z)e^z - e^{-z}}{e^z + e^{-z}} + B,$$

where $z = (x_0 - x) / d$ 

![Graph showing edge pedestal fitting using a modified hyperbolic tangent function](image)

**Fig.3-2:** This figure shows an example of edge pedestal fitting using a modified hyperbolic tangent function (solid curve). Shown in open squares are TS measured $n_e$. Error bars for the data are also displayed.
The $mtanh$ function was initially developed to fit the H-mode edge pedestal, and is generalized to be implemented for profile analysis at L-H threshold in this thesis (a similar approach was also adapted in prior L-H studies, e.g. [80]). Shown in Fig.3-2 is an example of pedestal fitting using this function. Basic characteristics of the fitting can be described with these variables: the center of the pedestal is given by $x_0$, the width by $2d$, the height of the pedestal ‘top’ by $A+B$, the height of the pedestal ‘foot’ by $A-B$. The main difference between (3.1) and the standard tanh function is that the exponential term in the numerator is multiplied by $(1+\alpha z)$, which allows a linear rise beyond the pedestal top into plasma core, with $\alpha$ related to the slope. For the case shown in Fig.3-2, $A=1.4m^{-3}$, $B=1.1m^{-3}$, $d=4mm$, $x_0=0.22m$, $\alpha=0.268$.

**Time averaging in profile analysis**

Limited by the time resolution of involved diagnostics, most local plasma parameters are not measured at the exact L-H transition times. Therefore, the term ‘local L-H threshold conditions’ will hereafter refer to those proceeding but very close to L-H transition. To reduce the measurement (both statistical and systematic) errors, data from the edge profile diagnostics (*i.e.* ETS, LP, CXRS) within a finite time window, rather than just at the last time slice before L-H transition, are utilized in analysis.

Specifically, edge TS measurements from the last two laser pulses prior to the L-H transition time are included in profile analysis, fitted by the $mtanh$ function given by (3.1). The two laser pulses are $16.67ms$ apart and the earlier one is typically $20-30ms$ before L-H transition. CXRS measurements of $T_i$ within $30ms$ before L-H are taken, also fitted using the $mtanh$ function. Langmuir probe data in the SOL region of $0<\rho<15mm$ ($\rho=R-R_{sep}$, referenced to midplane values) within $30ms$ before L-H are first converted into logarithm scale, then these data are fitted using a linear function of $y(r)=a_0+a_1r$, by assuming the SOL $T_e$ and $n_e$ profiles decay exponentially in minor radius. These time ranges ($\sim30ms$) are approximately equal to typical C-Mod L-mode energy/particle confinement times, which are roughly $25-40ms$. We assume the plasma equilibrium is not
significantly changed over these time scales, and thus the obtained profiles should be close to those just at the L-H transition times.

Fig.3.3. Constructed radial profiles from fitting midplane (left) and divertor target (right) measurements (open circles) within 30ms before L-H transition for two C-Mod 5.4T/0.8-0.9MA discharges with low (0.8×10^{20} m^{-3}, in blue) and moderate (1.8×10^{20} m^{-3}, in black) density: (a)-(b) TS measured T_e and n_e, (c) CXRS measured T_i, (d)-(f) divertor LP measured T_e, n_e, P_e = n_e T_e. Dashed lines and shaded bars represent the approximate radial location of the separatrix and the estimated range of uncertainties.
Shown in Fig.3-3 are measurements from the above stated diagnostics alongside the constructed radial profiles in a low density discharge ($\bar{n_e}=0.8\times10^{20} m^{-3}$), which is taken to be representative of the reliability of diagnostic performance and profile fitting (low density is chosen because it is more challenging for many diagnostics to produce accurate measurements in these conditions). Profiles from a higher density discharge ($\bar{n_e}=1.8\times10^{20} m^{-3}$) are shown for comparison on the same plots.

Shown in Fig.3-3(a) and (b), although the uncertainty on individual TS measurements are generally larger compared to the higher density case, the use of multiple time points and high spatial resolution ensure the profile fitting is still reasonable at low density. Local uncertainties in fitted TS profile (not the plotted error for individual data point bars in this figure) are estimated using a Monte Carlo (MC) profile error analysis technique (see Appendix D for details). Profile fitting in CXRS measurements of $T_i$ (Fig.3-3(c)) is good for both cases. Divertor LP measurements, as shown in Fig.3-3(d)-(f), are generally well fitted by the chosen model, although the fitting in the low density case might be not as reliable as in the higher case, due to the absence of data for $0<\rho<3$ mm.

**Midplane separatrix registration**

Combined uncertainties in EFIT separatrix finding, profile mappings, and the position calibration of ETS could give rise to uncertainties in midplane separatrix registration on TS radial profiles of a few mm. Compared to the small spatial scale of the C-Mod plasma edge (which is of the order of 1~2 cm in L-mode), this level of uncertainty is not negligible and must be taken into account in the profile analysis. Thus accurate determination of the midplane separatrix locations within the TS radial profiles is crucial to this thesis study. Yet, this is also a very challenging. The following two alternate approaches were adapted in previous studies:

1) Define the separatrix as the radial location where upstream $T_e$ is consistent with what is expected from the two-point SOL transport model ((C.10)) for given plasma conditions;
2). or, if the divertor is not detached, then separatrix can be defined as the flux surface where the upstream $P_e$ matches the value projected from the strike point on the divertor target, i.e. $P_{eu} = 2n_{et}T_{et}$ (see eqn. (1.47) or Appendix C, eqn. (C.7)).

Fig.3-4: Reliability check on the midplane separatrix registration technique introduced in this thesis. Comparison of the upstream conditions of (a) $T_e$ and (b) $P_e$ evaluated at the separatrix on the TS radial profiles, shifted according to the technique introduced by this thesis, with (a) the expected values of upstream $T_e$ from a two-point SOL-divertor transport model (b) divertor target $P_e$ ($P_{eu}=2n_{et}T_{et}$) evaluated on LP profiles near separatrix. Agreement is generally good, suggesting the separatrix registration on TS profile after applying these shifts is accurate to within 1.5mm.

An approach different from the two methods stated above is introduced and applied in this thesis. This method defines the separatrix as the location where the second radial derivative of upstream $P_e$ (i.e. $d^2P_e/dr^2$) is a maximum (or approximately at the pedestal ‘foot’ of the $P_e$ profile using $mtanh$ fitting). The assumption behind this method is that the pressure gradient ($dP_e/dr$) changes sharply across the separatrix, since it divides the confined plasma ($J\times B$-$dP_e/dr$) and the unconfined SOL, making $d^2P_e/dr^2$ a maximum. Note that the diamagnetic velocity shear $V_d \propto d(\nabla P_e/n_e)/dr$ is nearly proportional to
\(d^2P_e/dr^2\), so this assumption also implies that \(V'_d\) is a maximum near the separatrix. Required radial shifts in TS profiles are generally minor; for most cases the values are less than 3mm. Implementation of these shifts can greatly reduce data scatter, making data trends more easily identified, while not qualitatively changing the key results.

Reliability of this technique is examined in Fig.3-4 by a cross-check with the other two methods stated before. Upstream conditions \((P_{eu}, T_{eu})\) are evaluated on shifted TS profiles at the separatrix, \(i.e. P_{eu}=n_{e,sep}T_{e,sep}, T_{eu}=T_{c,sep}\). Predictions of upstream \(T_e\) using a two-point SOL transport model are assessed using the experimentally measured \(P_{loss}\) (\(i.e. the power loss across the separatrix, defined in eqn.(0.49)\)) and divertor target density at \(\rho=0.5\text{mm}\) for \(n_{et}\). Divertor target pressures \(P_{et}=2n_{et}T_{et}\) are evaluated at \(\rho=0.5\text{mm}\). The plots in Fig.3-4 indicate good agreement among the three methods, which suggests the midplane separatrix registration on TS radial profiles is accurate to within 1.5mm using the introduced technique, which is satisfactory for this study.

**Local profile analysis and spatial averaging**

Local conditions of the key plasma parameters (section 3.3 and 3.4) are assessed at two radial locations on these radial profiles (TS profiles after radial shift):

1. near pedestal top at \(\psi=0.95\) \((r~a_0-8\text{mm})\);
2. inside and near the separatrix at \(\psi~1.0\).

Plasma parameters to be characterized include:

1. upstream (midplane) \(T_e, n_e, T_i, a_0/L_T=|(a_0/T_e)(dT_e/dr)|, a_0/L_n=|(a_0/n_e)(dn_e/dr)|\);
2. near-separatrix \(T_e, n_e\) at divertor target.

Here, \(a_0/L_T\) and \(a_0/L_n\) are the normalized inverse gradient scale lengths of \(T_e\) and \(n_e\) profiles, which quantitatively describe the local profile shape, namely, larger \(a_0/L_T\) (\(a_0/L_n\)) means the \(T_e\) \((n_e)\) profile is steeper.
To reduce data scatter in $a_0/L_T$ and $a_0/L_n$, mean values in the shadowed bars shown on Fig.3-5 are taken as the values at $\psi=0.95$ and $\psi=1.0$, respectively. Upon examination, this technique can effectively reduce data scatters in these quantities, without qualitatively changing the general trend and scaling properties. Magnitudes of midplane $T_e$, $T_i$, and $n_e$ are evaluated at the corresponding radial locations as marked by dashed lines on Fig.3.5.

![Radial profiles of (a) $a_0/L_T=|a_0/T_e(dT_e/dr)|$, and (b) $a_0/L_n=|a_0/n_e(dT_e/dr)|$ derived from TS radial profiles for the same two cases as in Fig.3.3. The profiles are seen sharply peaked in the boundary layer, thus mean values in the two shadowed regions are taken as the local conditions for these parameters at $\psi=0.95$ and $\psi=1.0$, respectively. Local $T_e$, $T_i$, and $T_i$ are evaluated at the dashed lines.](image-url)
3.2 Scaling of H-mode threshold power

Scaling of H-mode threshold power ($P_{th}$) is examined in this section. Consistent with the standard definition, $P_{th}$ in this study is taken as the value of $P_{loss}$

$$P_{loss} = P_m - dW/ dt = P_{OH} + P_{aux} - dW/ dt$$

at the L-H transition time. Both $P_{OH}$ (ohmic power) and $dW/dt$ (time derivative of plasma stored energy) here are derived from EFIT outputs; $P_{aux} = P_{RF}$ as ICRH is the sole auxiliary heating source, with $P_{RF}$ the net input RF power measured by directional waveguide couplers, assuming 100% absorbed by plasma. Although in reality ~90% absorption rate seems a more reasonable estimate suggested by prior studies, the 10% difference only results in minor variation in $P_{th}$, of about 0.1-0.3MW, which will not fundamentally change the scaling of $P_{th}$. Some literature also suggests that a more appropriate parameter to quantify the power loss should be $P_{net} = P_{loss} - P_{rad,main}$, with $P_{rad,main}$ the plasma radiation from main plasma (inside the separatrix). For the included cases, $P_{rad,main}$ is typically 0.2-0.5MW in L-mode and before L-H transition, which is small compared to $P_{loss}$ for most cases. The density dependence of $P_{th}$ and $P_{net}$ mostly differ by an offset, of about 0.3MW.

Shown in Fig.3-6 are $P_{th}$ for 5.4T and 3.5T cases versus $\bar{n}_e$, evaluated at L-H transition times. For these data, $P_{OH}$ are close to 0.8~1.2MW and $dW/dt$ are typically 0.3-0.5MW; therefore the strong variation in $P_{th}$ as $\bar{n}_e$ is scanned is dominated by differences in $P_{RF}$.

First, the plasma density dependence of $P_{th}$ is examined for data with $B_T=5.4T$. A U-shaped scaling is clearly noted in Fig.3-6, indicating that the low-density branch of $P_{th}$ also exists in C-Mod, and appears when $\bar{n}_e \leq 1.1 \times 10^{20} m^{-3}$ at this toroidal magnetic field. In this regime, $P_{th}$ increases sharply with reduced density, approximately $P_{th} \propto 1/\bar{n}_e^{2.0}$. The high-density branch occurs when $\bar{n}_e \geq 1.8 \times 10^{20} m^{-3}$, where $P_{th} \propto \bar{n}_e^{3.0-3.5}$. Between the two limits, $P_{th}$ is nearly independent of density, and exhibits a local minimum of $P_{th} \approx 1.5$MW. Despite a factor of two variation in plasma current, these data do not show a clear $I_p$ dependence, which means $P_{th}$ is not strongly correlated with Greenwald fraction, $\bar{n}_e / n_G$.
Recall from Chapter 2.1 that a similar U-shaped density dependence of $P_{th}$ was identified on other devices, including AUG, COMPASS-D, DIII-D, JET, and JT-60U. However, the specific density of minimum $P_{th}$ (i.e. $n_{min}$) varies significantly, from about $1.2 \times 10^9 m^{-3}$ on JET and JT-60U, $3.4 \times 10^9 m^{-3}$ on AUG and DIII-D, to $1.1-1.8 \times 10^{20} m^{-3}$ on C-Mod. It seems $n_{min}$ generally decreases with machine size (in terms of major radius, $R$), the ordering is $R_{JET}=2.96 m \sim R_{JT-60U}=3.0 m \sim R_{AUG}=1.65 m \sim R_{DIII-D}=1.66 m \sim R_{C-Mod}=0.67 m$.

The broad coverage of $B_T$ in C-Mod experiments made it possible to investigate the $B_T$ scaling of $P_{th}$ in a single device. Recall that the ICRF antenna frequency was tuned
accordingly, as $B_T$ is varied to maintain the same heating scheme. First, looking at the 3.5T data in Fig.3.6 shows that the U-shaped density scaling is preserved at reduced $B_T$. However, the trend is shifted towards lower density compared to $B_T=5.4T$. There is clear evidence that $P_{th}$ scales strongly with $B_T$, roughly as $P_{th} \propto B_T^{2.0-3.0}$, for density range of $\bar{n}_e \leq 1.1 \times 10^{20} m^{-3}$. However, the $B_T$ dependence at higher density ($\bar{n}_e > 1.1 \times 10^{20} m^{-3}$) appears to be considerably weaker. The lower bound of $n_{min}$ is reduced from $\bar{n}_e \sim 1.1 \times 10^{20} m^{-3}$ at 5.4T, to $\bar{n}_e \sim 0.8 \times 10^{20} m^{-3}$ at 3.5T, which confirms an earlier postulation, based on an analysis of the multi-machine database, that the value of $n_{min}$ could increase with magnetic field.

Scaling law predictions are also shown on the same plot for comparison. Quantitative agreement with experimental data is only seen at $B_T=5.4T$ with moderate densities of $\bar{n}_e \sim 1.5 \times 10^{20} m^{-3}$, while remarkable departures are detected at many places, especially for low densities or low magnetic field (the 3.5T cases). Qualitatively, the dependences given by the scaling law are insufficient to capture the actual complex behavior of $P_{th}$, which is clearly not well described by any power-law relation of the type $P_{th} \propto \bar{n}_e B_T^x I_p^y$.

### 3.3 Scaling of local plasma conditions for L-H transition

A group of TS radial profiles measured prior to L-H transition for discharges with low density ($\bar{n}_e < 1.1 \times 10^{20} m^{-3}$) and various $B_T$, $I_p$ are shown in Fig.3-7 to give a sense of how these profiles look in different plasma conditions. Some features are clearly manifested, e.g. Fig.3-7(a) and (d) show that the $T_e$ and $a_0/L_T$ profiles near the separatrix are nearly identical for cases with similar density (e.g. $\bar{n}_e = 1.0 \times 10^{20} m^{-3}$), regardless of differences in $B_T$ or $I_p$. Progressing radially inward to $\psi=0.95$, $T_e$ profiles with different $I_p$ but similar density and $B_T$ clearly diverge, such that those with higher $I_p$ become much steeper, manifested by larger $a_0/L_T$. The generality of these properties will be verified later.
Fig. 3.7. Composite edge $T_e$ and $n_e$ profiles prior to L-H transition from some low density ($\bar{n}_e < 1.0 \times 10^{20} \text{ m}^{-3}$) plasmas, obtained by fitting the TS measurements from the last two laser pulses prior to L-H, and radially shifted according to section 3.1.2. Profiles of $a_0/L_T$ and $a_0/L_m$ are derived using the $T_e$ and $n_e$ profiles, respectively. Dashed lines represent the separatrix ($R=R_{sep}$) and the approximate midplane radial location of $\psi=0.95$. Shaded areas are the used regions for spatial averaging in evaluating $a_0/L_T$ and $a_0/L_m$. 
3.3.1. Scaling of L-H threshold conditions near pedestal top

Shown in Fig.3-8(a)-(d) are data of $T_e$, $T_i$, $n_e$, and $P_e$ at $\psi=0.95$ prior to L-H transition. Error bars on some data in this figure (also for the subsequent figures in this chapter) represent the estimated uncertainties in profile fitting and in determining radial locations of the corresponding flux surfaces. It is remarkable that $T_{e,95}$ ($T_e$ at $\psi=0.95$) for the 5.4T cases increase substantially with reduced density when $\bar{n}_e \leq 1.1 \times 10^{20} m^{-3}$, following a $T_{e,95} \propto 1/\bar{n}_e$ correlation, as indicated in Fig.3-8(a). Note that this is also the density range for the low-density branch of $P_{th}$ at $B_T=5.4T$. On the other hand, most data points for higher densities, with $\bar{n}_e > 1.1 \times 10^{20} m^{-3}$, are scattered in the range of $150eV \pm 30eV$, without showing notable dependence on density, which is consistent with previous studies.

Shown in Fig.3-8(b), $T_{i,95}$ are close to $T_{e,95}$ even in the low-density branch. This result is consistent with JET, JT-60U, but is in clear contrast with AUG [86], where pedestal $T_e$ and $T_i$ were found to be strongly decoupled ($T_{e,ped} \sim 3T_{i,ped}$) at low density ($\bar{n}_e \approx 2 \times 10^{19} m^{-3}$) before L-H transition. Shown in Fig.3-8(c), $n_{e,95}$ is basically linearly correlated with $\bar{n}_e$ over the entire density range. As a result, $P_{e,95}$ is nearly flat for the low density branch and peaks up for $\bar{n}_e > 1.5 \times 10^{20} m^{-3}$, as shown by Fig.3-8(d).

Similar to $P_{th}$, no clear $I_p$ dependence was identified in these parameters. A $B_T$ dependence is mainly seen in $T_{e,95}$ at low density ($\bar{n}_e \leq 1.0 \times 10^{20} m^{-3}$), where $T_{e,95}$ drops as $B_T$ is reduced to 3.5T. However, the $B_T$ dependence becomes weak or perhaps even vanishes at higher density ($\bar{n}_e > 1.0 \times 10^{20} m^{-3}$), where $T_{e,95}$ at 3.5T and 5.4T, for similar densities, are seen to be almost the same. Interestingly, previous C-Mod studies showed that $T_{e,95}$ for $B_T=8T$ are considerably higher than for 5.4T, at moderate densities ($\bar{n}_e=1.0-2.0 \times 10^{20} m^{-3}$) [73]. Based on this evidence and Fig.3-8(a), it is likely that the density scaling of $T_{e,95}$ also shifts to a lower value of density at reduced $B_T$, as in the case of $P_{th}$. However, the heating scheme in the 8T experiment (dominantly mode conversion heating of electrons) is different than for 5.4T and 3.5T; therefore the higher $T_e$ at 8T could also be a result of this difference, leaving the $B_T$ dependence of $T_{e,95}$ not completely resolved.
Fig. 3-9 shows data for \( a_0/L_T \), \( a_0/L_n \), and \( a_0/L_p = a_0/L_T + a_0/L_n \), prior to L-H, against plasma density. A remarkable difference with Fig. 3-8 is that a clear \( I_p \) dependence is seen in \( a_0/L_T \) (also \( a_0/L_p \) since \( a_0/L_p \approx a_0/L_T \)), nearly as \( a_0/L_T \sim I_p \), while the density scaling (with \( I_p \) fixed) becomes weak. A similar \( I_p \) dependence is also identified in \( a_0/L_n \), but mainly at low density (\( \bar{n}_e \leq 1.0 \times 10^{20} \text{ m}^{-3} \)). At higher density, data points for \( a_0/L_n \) with different \( I_p \) are tightly grouped and decrease towards higher \( \bar{n}_e \). Due to the presence of \( I_p \) dependence, the \( B_T \) scaling is examined using data with similar \( I_p \) (0.6MA) but different \( B_T \) (3.5T and 5.4T).
5.4T), as shown in Fig.3-9(d)-(f). These plots suggest an inverse linear $B_T$ dependence at low density for both $a_0/L_T$ and $a_0/L_n$. The $B_T$ scaling at high density is not clear due to the relative lack of data.

Fig. 3-9: L-H threshold conditions at $\psi=0.95$: (a) $a_0/L_T$, (b) $a_0/L_n$, (c) $a_0/L_p$ in 5.4T, and: (d) $a_0/L_T$, (e) $a_0/L_n$, (f) $a_0/L_p$ at 3.5T/0.6MA and 5.4T/0.6MA. Dashed lines are intended to approximately guide the eyes.
Taking the above evidence together, it appears that $a_0/L_T$, $a_0/L_n$ are correlated with the local safety factor, $q_{95}$ ($q_{95} \sim rB_T/R^l_p$), at low density. To elucidate this correlation, data of $a_0/L_T$ and $a_0/L_n$ in the low-density branch of $P_{th}$ (i.e. $\bar{n}_e < 1.1 \times 10^{20} m^{-3}$ for 5.4T, $\bar{n}_e < 0.8 \times 10^{20} m^{-3}$ for 3.5T) are displayed against $q_{95}$ in Fig.3-10. These data are seen to generally track the $\sim 1/q_{95}$ scaling.

**Fig.3-10:** Data of L-H threshold conditions at $\psi = 0.95$ for (a) $a_0/L_T$, and (b) $a_0/L_n$ in the low-density branch of $P_{th}$ ($\bar{n}_e < 1.1 \times 10^{20} m^{-3}$ for 5.4T, $\bar{n}_e < 0.8 \times 10^{20} m^{-3}$ for 3.5T), displayed against $q_{95}$. These data generally track the $\sim 1/q_{95}$ scaling as shown by the dashed lines.
Fig.3-11 shows data for $a_0/L_T, a_0/L_p$ against $\bar{n}_i/n_0$. Note that data of different $I_p$ or $B_T$ are clustered, with a trend that $a_0/L_p$ generally decreases with $\bar{n}_i/n_0$. Similar behavior was also discovered in the SOL of C-Mod ohmic plasma [118], evidence that SOL transport is regulated by electromagnetic turbulence. This resemblance suggests that the entire edge-SOL region should be treated as an integrated system, governed by similar physics.

Fig.3-11: Data of L-H threshold conditions at $\psi=0.95$ of (a) $a_0/L_T$ and (b) $a_0/L_p$ displayed against $\bar{n}_i/n_0$. Using $\bar{n}_i/n_0$ for the independent variable significantly decreases the scatter in data when compared to Fig.3-9, where the data are plotted versus $\bar{n}_e$. 

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3.3.2 Scaling of L-H threshold conditions near the separatrix: upstream and at the divertor target

*Upstream* $T_e$, $T_i$, $n_e$, and $P_e$, inside and near the separatrix, preceding L-H are shown in Fig.3-12. These conditions are important to the test of the FM$^3$ model, as described later in section 6.4. Examining these plots reveals little or no dependence on $B_T$ or $I_p$; therefore emphasis is given to examining their density dependences.

![Upstream L-H threshold conditions near separatrix (ψ=1.0): (a) $T_e$, (b) $T_i$, (c) $n_e$, (d) $P_e$ against plasma density. Dashed lines in these plots (except in (b)) are approximate curve fitting to 5.4T data with $\tilde{n}_i < 1.5 \times 10^{20} m^{-3}$. No clear magnetic field or plasma current dependence is seen in these plots.](image-url)

Fig.3-12: Upstream L-H threshold conditions near separatrix ($\psi=1.0$): (a) $T_e$, (b) $T_i$, (c) $n_e$, (d) $P_e$ against plasma density. Dashed lines in these plots (except in (b)) are approximate curve fitting to 5.4T data with $\tilde{n}_i < 1.5 \times 10^{20} m^{-3}$. No clear magnetic field or plasma current dependence is seen in these plots.
Shown in Fig.3-12(a), $T_{e,\text{sep}}$ (i.e. $T_e$ near the separatrix) is nearly independent of $\tilde{n}_e$, without showing a remarkable rise at low density as already identified in $T_{e,95}$. Values of $T_{e,\text{sep}}$ for most data points are in the range of 50-100eV, consistent with what is expected from the two-point SOL transport model (a comparison between the two was shown in Fig.3-4).

Compared to $T_{e,\text{sep}}$, $T_{i,\text{sep}}$ shown in Fig.3-12(b) is consistently higher. Their values are mostly close to 100-150eV (a few low density cases could be even higher with $T_{i,\text{sep}} \sim 200eV$, but the uncertainties for these data could be larger). This range is also generally in agreement with recent measurements of upstream $T_i$ on C-Mod using ion sensitive probes [119]. A possible reason for $T_{i,\text{sep}} > T_{e,\text{sep}}$ is that the parallel conduction in the SOL, which removes the heat from confined plasma to divertor, is dominantly carried by electrons (i.e., power loss is mainly through the electron channel); thus presumably, $T_e$ should be lower near the separatrix. Scaling of $n_{e,\text{sep}}$ with density is weak for $\tilde{n}_e < 1.5 \times 10^{30} m^{-3}$, while it becomes stronger than linear ($n_{e,\text{sep}} \sim \tilde{n}_e^{3.0}$) for $\tilde{n}_e > 1.5 \times 10^{20} m^{-3}$.

Data of near-separatrix $a_0/L_T$ and $a_0/L_n$ are shown in Fig.3-13. Contrary to what was seen near the pedestal top, data of $a_0/L_T$ near the separatrix exhibit an inverse correlation with density, going nearly as $a_0/L_T \propto 1/\tilde{n}_e$, but does not seem to clearly depend on $B_T$ or $I_p$. Similar behavior is also observed in Fig.3-13(b) for $a_0/L_p$. Since $T_{e,\text{sep}}$ is only weakly variant with density (Fig.3-12(a)), it is the significant increase in near-separatrix $a_0/L_T$ (i.e. the steepening of $T_e$ profile in pedestal) that gives the dramatic enhancement in $T_{e,95}$ at low density.

Shown in Fig.3-13(c), the density scaling of $a_0/L_n$ for 5.4T peaks around $\tilde{n}_e = 1.3 \times 10^{20} m^{-3}$, and the falls off at higher density. Similar to $a_0/L_T$, no clear $I_p$ dependence is observed in $a_0/L_n$. However, as shown in Fig.3-13(d), the peak of $a_0/L_n$ moves to a lower value of density ($\tilde{n}_e \sim 0.9 \times 10^{20} m^{-3}$) as $B_T$ is reduced to 3.5T. Interestingly, the density at the peak of $a_0/L_n$ also seems nearly aligned with $n_{\text{min}}$, and its movement with $B_T$ is also consistent with what has been seen in $P_{\text{th}}$. 

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Fig. 3-13: Upstream L-H for threshold conditions near the separatrix: (a) $a_0/L_T$, (b) $a_0/L_p$, (c) $a_0/L_n$ at 5.4T, (d) $a_0/L_n$ for 5.4T/0.6MA and 3.5T/0/6MA. No clear $B_T$ or $I_p$ dependences are seen in $a_0/L_T$ or $a_0/L_p$. $a_0/L_n$ shows a peak with density; the peak moves to a lower density at reduced $B_T$.

Fig. 3-14 shows the near-separatrix (at $\rho=0.5\text{mm}$) conditions of $T_e$, $n_e$, and $P_e$ on the divertor target plate. Comparing Fig. 3-14(a) to Fig. 3-14(b), it is found that $T_{et}$ (i.e. divertor target $T_e$ near separatrix) correlates much better with $f_G = \bar{n}_e/n_G$ than $\bar{n}_e$. Values of $T_{et}$ are mostly in the range of 30-50eV and weakly variant with $f_G$, for $f_G < 0.25$, then followed by a sharp drop at $f_G \sim 0.25-0.3$, which implies the SOL transport transitions from the sheath-limited to the conduction-limited regime near this normalized density. This behavior is also very similar to that for ohmic plasma in an earlier C-Mod study [44].
Fig. 3-14: Divertor target L-H threshold conditions near separatrix (ψ=1.0): (a) $T_e$ against plasma density, (b) $T_e$ against $f_G=\bar{n}/n_e$, (c) $n_e$, (d) $P_e$ against plasma density. Dashed lines in these plots (except in (b)) are approximate curve fits to the 5.4T data with $\bar{n}_e<1.5\times10^{20}m^{-3}$. $T_e$ is seen much better correlated with $f_G$ than plasma density. No clear magnetic field or plasma current dependencies are seen in local $n_e$. Scaling of divertor $P_e$ is similar to that of the upstream near-separatrix $P_e$.

In contrast to $T_{e,t}$, $n_{e,t}$ (Fig. 3-14(c)) strongly correlates with density (exponentially). As a result, the density scaling of $2T_{e,t}n_{e,t}$ (Fig. 3-14(d)) is seen to be very similar to that of upstream pressure $P_{eu}=T_{e,sep}n_{e,sep}$ (Fig. 3-8(d)). Quantitative comparison of $2T_{e,t}n_{e,t}$ and $P_{eu}$ was shown in Fig. 3-4(b), where good agreement was seen, confirming the upstream (midplane) seperatrix determination in this study is satisfactory.
3.3.3 Scaling of local conditions in ohmic/low-power L-mode

A plasma discharge just before L-H transition is an L-mode with maximum allowed input power. Then it is natural to ask: what are the differences of this special state compared to L-mode with low heating power, e.g. the ohmic baseline plasma. To address this problem, local plasma edge conditions in low power L-mode phases (hereafter called ‘L-mode’ in this chapter for simplicity) are also characterized for the included discharges. Here, ‘L-mode’ refers to the first 50ms of $I_p$ flattop, i.e. typically 0.55-0.6sec for a C-Mod plasma discharge. Most included cases are ohmic plasma in this phase, with only a few exceptions, which have low ICRF power injection ($P_{RF}<0.5$MW). The local conditions in L-mode are obtained by averaging plasma measurements within this time interval, following the same profile analysis scheme as described in 3.1.2.

![Graph](image)

Fig.3-15: L-mode (blue) and L-H threshold (red) TS radial profiles for three 5.4T/0.8-0.9MA discharges with line-averaged density of (a) 0.7, (b) 1.0, (c) $1.5 \times 10^{20}$ m$^{-3}$

Some TS midplane $T_e$ and $n_e$ profiles from three L-mode phases, with similar $B_T/I_p$ and different plasma density, are shown in Fig.3-15. The L-H counterparts for these cases are
displayed together to contrast the differences. For the two lower density cases ($\bar{n}_e$=0.7, $1.0 \times 10^{19} m^{-3}$, referred to the values at L-H transition), the L-mode edge $T_e$ profiles tend to be less steep than those at the L-H threshold, while the $n_e$ profiles look similar. For the higher density case ($\bar{n}_e$=1.5$ \times 10^{19} m^{-3}$), the differences in $T_e$ profiles become much smaller. The difference in edge $n_e$ could be mainly due to the $\sim$15% higher $\bar{n}_e$ at L-H threshold than in L-mode.

Fig.3-16: L-mode conditions near the pedestal top ($\psi$=0.95): (a) $T_e$, (b) $T_i$, (c) $n_e$, (d) $P_e$ against plasma density. Dashed lines in these plots (except the one in (b), which is an approximate curve fitting to the L-H data of $T_i,95$) are taken from the L-H counterparts of these plots. No clear $B_T$ or $I_p$ dependence is seen. The arrow nearly represents the typical direction for plasma discharge evolution in going from L-mode to L-H. Values of $T_e$ in L-mode are generally lower compared to those of L-H.
L-mode conditions near the pedestal top are shown in Fig.3-16 and Fig.3-17. Note that the density range for the L-mode data is not the same as that of L-H (typically the value of $\bar{n}_e$ at L-H is 10-15% higher than in L-mode for a given discharge), primarily due to a small increase in density associated with wall outgassing during ICRF operation. The arrow on Fig.3-16(a) represents the typical direction for a plasma discharge to evolve from L-mode to L-H. Ideally, if plasma density is maintained constant, then it should point vertically upward. The dashed lines are taken from the L-H counterpart of each plot.

Shown in Fig.3-16(c), the density scaling of $n_{e,95}$ in L-mode and for L-H transition are almost identical. Distinguishable differences are mainly seen in $T_{e,95}$ (Fig.3-16(a)), with L-mode values consistently lower compared to the L-H threshold values, especially at low density. In addition, $T_{i,95}$ and $T_{e,95}$ remain well coupled in L-mode, as well as for the L-H threshold (Fig.3-8).

![Fig.3-17: L-mode conditions near pedestal top ($\psi=0.95$) of (a) $a_0/L_T$; (b) $a_0/L_P$. Dashed lines in these plots are taken from the L-H counterparts of these plots. The arrow represents the typical direction for plasma discharges to evolve as they go from L-mode to L-H. No remarkable difference between L-mode and L-H is seen.](image-url)
Shown in Fig. 3-17 are L-mode $a_0/L_T$ and $a_0/L_p$ near the pedestal top. These parameters are strongly correlated with $\bar{\tau}_i/n_e$ as in Fig. 3-11 for L-H transition. The magnitudes are found to be close to the L-H threshold values for similar $\bar{\tau}_i/n_e$. It means $L_T$ (also $L_p$) is not remarkably different at this radial location between L-mode and L-H.

![L-mode $\psi=1.0$](image)

Fig. 3-18: Upstream L-mode conditions near the separatrix ($\psi=1.0$): (a) $T_e$, (b) $n_e$, (c) $T_i$, (d) $a_0/L_T$, (e) $a_0/L_n$, (f) $a_0/L_p$ against plasma density. Dashed lines in these plots are taken from the L-H counterparts of these plots. L-mode values are notably lower than the L-H values for $a_0/L_T$ and $a_0/L_p$. 
Near-separatrix upstream L-mode conditions are shown in Fig.3-18. As in Fig.3-14 for L-H transition, Fig.3-18(a) shows that $T_{e,\text{sep}}$ in L-mode remains weakly variant with density and independent of $B_T$ and $I_p$. In contrast to $T_{e,\text{95}}$, L-mode values of $T_{e,\text{sep}}$ are only slightly (generally by $\sim 10-20eV$) lower than L-H threshold values. Shown in Fig.3-18(c), L-mode $T_{i,\text{sep}}$ are basically flat with density and generally $\sim 50eV$ lower compared to L-H threshold values, except for the few very low density cases, where the difference could be as large as 100eV or more. Scaling of $n_{e,\text{sep}}$ (Fig.3-18(b)) in L-mode is very similar to that for the L-H transition.

By contrast, Fig.3-18(d) and (f) show that L-mode $a_\ell/L_T$ and $a_\ell/L_p$ are dramatically lower than L-H threshold values, over the entire density range. This steepening of near-separatrix profiles leads to an enhancement in $T_{e,\text{95}}$ and pedestal pressure gradient ($dP_e/dr$) as the L-H transition is approached. Similar features are also found in $a_\ell/L_n$ (Fig.3-18(e)), especially for densities near $n_{\text{min}}$ (i.e., near the peak of the density trends for $a_\ell/L_n$), where L-mode $a_\ell/L_n$ appears notably lower compared to the L-H threshold cases. These local behaviors can also be verified by examining the radial profiles in Fig.3-15.

Near-separatrix L-mode conditions on the divertor target are shown in Fig.3-19. Seen in Fig.3.19-(a), $T_{e}$ in L-mode remains inversely correlated with $\bar{n}_e/n_e$ and exhibits a sharp fall-off at $\bar{n}_e/n_e \sim 0.2$, a little lower compared to that of L-H, which is $\bar{n}_e/n_e \sim 0.25$ (Fig.3-14(b)). Values of $T_{e}$ in L-mode are only slightly lower compared to those of L-H threshold, typically by just a few eV. Fig.3-19(b) suggests that there is no distinguishable difference in $n_{e}$ between L-mode and L-H.

Taken together, it can be concluded that the electron pressure gradient ($P'_e = dP_e/dr = P_e/L_p$) for L-H transition is generally larger compared to the L-mode values at a given density, both at the pedestal top and at the separatrix, but by different means: near the pedestal top, the enhancement in $P'_e$ is mainly due to the rise in $T_e$, while the profile shape ($L_T$, $L_n$) is little changed; near the separatrix, the enhancement is primarily due to the increase in $1/L_p$, i.e. the steepening of the pressure profile.
3.4 Plasma behaviors at low density before L-H transition

A striking discovery in a few very low density ($\bar{n} \leq 0.8 \times 10^{20} m^{-3}$) 5.4T discharges is that a clear edge $T_e$ pedestal can develop in the L-mode plasma, well before L-H transition (see e.g. Fig.3-15(a)), whereas $n_e$ and $T_i$ remain in L-mode shape without a signature of edge pedestal formation. Time histories of key plasma parameters for one such discharge are shown in Fig.3-20. The phase, as just stated, lasts from $\sim 0.8 sec$ to $1.24 sec$, when the L-H transition takes place. In this period, $B_T$, $I_p$ and density are nearly fixed, and $P_{RF}$ is slowly increased.

Shown in Fig.3-20(d), $T_e$ near the pedestal top, as measured by ECE, rises from $\sim 200 eV$ in the ohmic phase to $\sim 300 eV$ at the L-H transition, accompanied by an increase in stored energy ($W_{MHD}$, Fig.3-20(g)). Despite the increasing heating power, global energy confinement time (Fig.3-20(i)) is essentially unchanged, which seems to violate the normal L-mode scaling ($\tau_e \propto P_n^{0.5}$).
Fig.3-20: Time histories of some key plasma parameters for a low density ($\bar{n}_e \sim 0.6 \times 10^{20} \text{m}^{-3}$) 5.4T/0.9MA discharge. The L-mode phase with $T_e$ pedestal lasts from 0.8 sec to 1.24 sec. H-mode was accessed at 1.24 sec.
The normalized confinement factor $H_n = \tau_e/\tau_{\text{H\textsc{ern}}}$ (Fig.3-20(h)) rises with time and reaches ~1.0 prior to the L-H transition. In addition, a highly coherent electromagnetic fluctuation with frequency of ~250KHz was detected by PCI and magnetic pickup coils, shown in Fig.3-20(j) and Fig.3-20 (k).

Shown in Fig.3-21(a)-(b) are 100ms averaged $T_e$ and $n_e$ profiles in low-power L-mode, pre L-H, and H-mode phase for the case quoted in Fig.3-20. Seen in Fig.3-21(a), the pre L-H $T_e$ profile is very similar to that in the subsequent H-mode; both exhibit a clear edge pedestal; the one just before L-H is perhaps slightly wider, but the two $T_e$ pedestals generally have very similar radial structure. By clear contrast, edge $T_e$ pedestal is absent in the low-power L-mode case. Shown in Fig.3-21(b), the shape of the pre L-H $n_e$ profile is similar to that of the low-power L-mode, and is less steep than that of the H-mode, in the plasma edge region.

L-mode and pre L-H $T_e$ and $T_i$ profiles from this low-density case and another discharge with identical $B_T$ and $I_p$, but higher density are shown together in Fig.3.21(c)-(d) for comparison. A remarkable difference is that there is no clear edge $T_e$ pedestal seen before L-H transition in the higher density case, as apposed to the low density case. Shown in Fig.3-21(d), the behavior of $T_i$ is quite different than $T_e$. It seems for both cases $T_i$ profiles before L-H are generally globally elevated from those of L-mode, *i.e.* the separatrix value of $T_i$ is enhanced by similar magnitude as that at the pedestal top; by contrast the near-separatrix values of $T_e$ are closely pinned to ~50eV in various conditions. Formation of $T_e$ pedestal before L-H transition was also observed in AUG low density plasma. However, in the AUG experiment, $T_e$ and $T_i$ near the pedestal top were seen strongly decoupled with $T_{e,\text{ped}} \sim 3 T_{i,\text{ped}}$, whereas $T_{e,\text{ped}}$ and $T_{i,\text{ped}}$ are well coupled in C-Mod (Fig.3-21(c),(d)). Plasma behaviors in low density plasma before L-H transition also resemble those of the I-mode, particularly in that both see edge $T_e$ pedestal but no density pedestal. More detailed discussions can be found in [72].
Fig. 3.21. 100ms averaged (a) $T_e$ and (b) $n_e$ profiles in L-mode (black), pre L-H (blue) and H-mode (red) phase of C-Mod shot 1070807008. TS measurements associated with these profiles are shown in open circles. Shown in (c) and (d) are 100ms averaged $T_e$ and $T_i$ profiles for this shot and a higher density case in L-mode and pre L-H.
Chapter 4

Effects of divertor geometry on H-mode threshold conditions

The importance of the divertor to the H-mode transition was noticed since the first discovery of H-mode in 1982. In early ASDEX experiments, H-mode was found to be only accessible in divertor plasma, while in limiter plasma, no H-mode was successfully achieved with the available heating power. Although it was proved later that H-mode can be accessed with a wall limiter, the threshold power was found to be much higher than in a divertor plasma with similar engineering parameters and favorable ion grad-B drift (towards the active X-point). So it can be said that the accidental discovery of the H-mode confinement regime benefited from the implementation of the poloidal divertor.

Later, many tokamak experiments found even for divertor plasma with nearly constant conditions (e.g. density, $B_T$, $I_p$, ion grad-B drift, wall condition, etc), a slight change in the divertor/X-point configuration could give rise to unusually low or high $P_{th}$ compared to normal values under the same conditions [70, 85, 123]. The resulting variation in $P_{th}$ is a possible source of the large scatter in the multi-machine H-mode threshold database. It was seen on many tokamaks that with an optimized divertor/X-point configuration, megawatts of heating power sometimes can be saved in getting H-mode, which is comparable or even more than can be achieved by varying main engineering parameters (e.g. plasma density, magnetic field). The appearance of a divertor effect offers a
potential for further reduction in H-mode threshold power when other operational parameters have been optimized and fixed.

This chapter first reviews the main results from previous studies of divertor/X-point effects on the H-mode threshold in various tokamaks. Then a dedicated C-Mod experiment, conducted to explore the divertor geometry dependence of H-mode threshold, is described, and the main results are presented. The most striking observation is a significant (>50%) reduction in \( P_{th} \) when the C-Mod divertor is operated in a ‘slot’ configuration, while edge pedestal electron temperature is very little changed.
4.1 Review of divertor effects on H-mode threshold

Divertor effects on H-mode threshold mainly include two types – one is associated with divertor geometry, the other X-point location relative to divertor plate.

Fig.4-1: Divertor geometries for various tokamaks: (a)-(b) ASEDX Upgrade LYRA (closed) and open divertor, (c)-(d) JET Septum (W-shaped, closed) and SRP (open) divertor, (e)-(f) JT60-U W-shaped (closed) and open divertor, (g) Alcator C-Mod vertical-plate divertor, (h) DIII-D flat-plate divertor [from (120)].
Divertor geometry effects on H-mode threshold were first investigated on PDX in 1983 \[121\]. It was indicated in this study that H-mode access is only possible in PDX with a ‘closed’ divertor, \textit{i.e.} the configuration in which the backflow of neutral particles (\textit{e.g.} hydrogen molecules) into main plasma is reduced\footnote{It should be pointed out here that this definition is qualitative, and there is no parameter that quantitatively characterizes the degree to which a divertor is closed.}. Later, it was explored on many tokamaks including AUG \[122\], JET \[69\], JT-60U \[85\], in open versus closed divertor configurations. AUG observed a 20\% increase in $P_{\text{th}}$ with the closed divertor compared to the open divertor (Fig.4-2(a)), accompanied by a ~20-50\% increase in seperatrix density for a given line averaged density (Fig.4-2(c)). This difference disappeared when $P_{\text{th}}$ of both open and closed divertor are shown against $n_{e,\text{sep}}$ instead of $n_e$ (Fig.4-2(b)), which hints that local parameters could be more relevant than global parameters to H-mode threshold physics.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4-2.png}
\caption{AUG experiments show a 20\% increase in $P_{\text{th}}$ with closed divertor (DIV-II, panel (a)), while the seperatrix density is higher by 20-50\% (panel (c)). The discrepancies disappeared if data for both divertor geometries are plotted against seperatrix density (panel (b)) \[\text{from (122)}\]}
\end{figure}
In JET, $P_{th}$ is seen to be $\sim25\%$ lower with the more closed Septum divertor (Fig.4-1(c)) compared to the open divertor (Fig.4-1(d)) over a wide density range, as shown in Fig.4-3(a). Limited by the ICRF antenna performance, the minimum density for H-mode access in JET experiments with the open divertor is about $\bar{n}_e \sim1.1 \times 10^9 m^{-3}$. Above this density, $P_{th}$ is found to increase linearly with plasma density. With the closed divertor, however, the minimum accessible density for H-mode is extended to a lower value of $\bar{n}_e \sim0.8 \times 10^9 m^{-3}$. U-shaped density scaling is manifested in this case. Despite the differences in $P_{th}$, $T_e$ at pedestal top in Fig.4-3(b) is very similar for the two divertor geometries.

Fig.4-3: JET results show (a) $P_{th}$ is lower with the more closed Septum divertor (b) no clear difference in pedestal $T_e$ between the two divertor geometries [from (69)]

The JT-60U results are similar to those of JET, where $P_{th}$ was found to be consistently lower, by about 30% with the more closed W-shaped divertor, as shown in Fig.4-4.(a) and (b). Unlike JET, a U-shaped density scaling of $P_{th}$ was also observed in the JT-60U open-divertor plasma. The density of minimum $P_{th}$ is increased from $\bar{n}_e \sim1.2-1.5 \times 10^9 m^{-3}$ for the open divertor, to $\bar{n}_e \sim2.1-2.5 \times 10^9 m^{-3}$ for W-shaped divertor. Shown in Fig.4-4(c) and (d), the behavior of $T_i$ at the pedestal top ($T_{i,95}$) for the two divertor configurations
seems quite different. For the open divertor (Fig.4-4(c)), the density scaling of $T_{i,95}$ is ‘V’-shaped, whereas the one for the closed divertor (Fig.4-4(d)) is in an ‘L’ shape.

![Graphs showing plasma parameters for open and closed divertor configurations.](image)

Fig.4-4: Experiments in JT-60U show (a)-(b) both $P_{th}$ and $P_{net}$ are higher in the open divertor, especially at high densities, (c)-(d) the density scaling of pedestal $T_i$ looks quite different between the open and the W-shaped divertor [from (85)]

Another divertor-related effect is the location of X-point relative to the divertor plate. Of particular interest is the so-called X-point height ($H_{xpt}$), which is defined as the vertical distance between the lower X-point and the horizontal (flat) divertor plate. The $H_{xpt}$
dependence was studied in a number of tokamaks with flat-plate divertor, *e.g.* JET [123], DIII-D [70], and recently EAST [124]. It was consistently observed in these experiments that $P_{\text{th}}$ can be substantially lowered by reducing $H_{\text{xpt}}$, *i.e.* by bringing the X-point closer to divertor plate, while other plasma parameters were kept nearly constant. Shown in Fig.4-5 for example are results from JET and DIII-D. It is worth pointing out that in the JET experiment, the separatrix strike points were on the horizontal divertor plate for $H_{\text{xpt}}$ (relative to the septum top) between -6 and 6cm, while for $H_{\text{xpt}}$>6cm, the strike points were on the vertical divertor plate. The JET experiments also uncovered that pedestal $T_e$ is correlated with $H_{\text{xpt}}$, and decreases as $H_{\text{xpt}}$ is reduced (Fig.4-5).

![Fig.4-5: (left) JET, (right) DIII-D and EAST found $P_{\text{th}}$ decrease with the X-point height. The JET experiments also show a trend of increasing pedestal $T_e$ with X-point height. [from (123), (70), (124)]](image-url)
The mechanism(s) responsible for the divertor effects on H-mode threshold is still not very clear. It was often speculated that this effect could be related to neutrals in the divertor and plasma edge, which, as some theoretical studies suggested, could strongly affect the H-mode transition, either by damping the poloidal plasma flow via ion-neutral collisions [125], or by reducing ion-orbit loss near the X-point through ion-neutral charge exchange [126].

For example, a proposed mechanism [127] to explain why H-mode was only accessible with divertor but not limiter in the ASEDX experiments is that the injection of neutral particles into the divertor plasma could shape the boundary layer plasma profiles in a favorable way for H-mode transition, whereas neutral injection in limiter plasma always seems to deteriorate plasma confinement and impede H-mode transition. DEGAS [128] simulations for the JT-60U divertor geometry experiments indicate that the poloidal distribution of neutral density (n_0) for the W-shaped divertor is more asymmetric than for the open divertor [85]. The midplane n_0 are similar for both geometries, but the divertor n_0 is about one order of magnitude higher with the W-shaped divertor. It was hypothesized in [85] that n_0,mid/n_0,div, or in general the poloidal inhomogeneity of neutrals, could be relevant to the observed variation in P_{th}. JET [123] and DIII-D [70] observed a variation in divertor D_α emission (presumably proportional to divertor n_0) with H_{xpt}, but the trends seem opposite: in JET, D_α decreases with H_{xpt} and shows an inverse correlation with P_{th}, while this relation is reversed in DIII-D. These results suggest that both P_{th} and neutrals are sensitive to the X-point/divertor configuration, but can not serve as direct evidence that the variation in P_{th} is actually caused by neutrals.

The effect of neutrals was also explored experimentally on Alcator C-Mod [60][129]. Neutral pressure (P_0) was externally varied by closing divertor bypass gaps, which reduced the gas leakage and thus increased P_0 in divertor, or by midplane gas puffing which directly raised P_0 in midplane. Interestingly, changes in P_0 resulted in very little variation in P_{th}. Admittedly, C-Mod is distinctly different than other machines in density, B_T, divertor structure, and wall material, all of which could possibly reduce the importance of neutrals in C-Mod. Nevertheless, due to the inconsistent experimental
observations, and lack of a compelling theoretical model, the effects of divertor geometry and the role of neutrals are still unclear, and are usually considered as ‘hidden variables’ in H-mode threshold physics [66].

4.2 Divertor geometry effects on H-mode threshold in C-Mod

One may note from Fig.4-1(g) that the divertor geometry of C-Mod differs from that of the other tokamaks. It is the first so-called vertical-plate divertor [44], in which divertor plates are arranged in a compact way to form a more closed, slot-like structure. This configuration is more tightly baffled and more effective for divertor pumping. The vertical divertor plate can also withstand more heat flux, due to the small incident angle between SOL magnetic field lines and the divertor plate, which makes the incident heat more broadly distributed. Due to these merits, a similar geometry is adapted in the ITER divertor design.

Another main advantage of this design is that it makes it possible to compare different divertor plate geometries in a single device. C-Mod can operate in three different divertor geometries named flat-plate, vertical-plate, and slot by placing the outer strike point (OSP, i.e. where the separatrix surface intersects with the outer divertor plate) on different divertor plates, labeled as ‘F’, ‘V’, ‘S’ in Fig.4-6, respectively. This unique capability motivated dedicated C-Mod experiments [130] to look into the divertor geometry dependence of H-mode threshold conditions.

4.2.1 Experimental arrangement

The experiment was performed within a single day of C-Mod plasma operation (C-Mod run #1101221). The toroidal magnetic field and plasma current were fixed at $B_T=5.4-5.5T$ and $I_p=0.9MA$, with $q_{95} \sim 4.0$. All discharges were deuterium (D) majority plasmas,
created in lower single null magnetic configuration with ion grad-B drift in the favorable direction for H-mode access (ion grad-B drift towards the active X-point). Line averaged plasma density at the L-H transition was controlled between $1.3 \times 10^{20} m^{-3}$ and $1.6 \times 10^{20} m^{-3}$ for the included discharges. Correspondingly, $\bar{n}_e/n_{ec} = 0.25-0.28$, and the SOL-divertor transport was in the high-recycling regime [44]. An important reason for choosing this density range is to minimize the influence of plasma density on $P_{th}$, since it was already known (Chapter 3.2) that $P_{th}$ is minimized and weakly variant with density while $1.2 \times 10^{20} m^{-3} < \bar{n}_e < 1.7 \times 10^{20} m^{-3}$ and $B_T=5.4T$.

All L-H transitions in this experiment were achieved with the addition of ICRF as the sole auxiliary heating power. The ICRF antenna frequency was tuned at ~80MHz, to facilitate on-axis fundamental hydrogen minority heating. The Hydrogen minority concentration ($n_H/n_D$) was approximately constant, and close to 5-10% throughout the experiment, allowing high-efficiency ICRF power absorption [50]. ICRF power was injected during the $I_p$ flattop of each discharge; the power was increased either in small steps or in a slow continuous ramp-up pattern as shown in Fig.4-8.

The lower X-point was set at a different location for each plasma discharge, while plasma shaping was attempted to be maintained nearly fixed in this experiment. Of course, the shaping and X-point control could not be made perfectly independent. As a result, plasma shaping parameters at the L-H transition are subject to some variations, yielding $\kappa \sim 1.5-1.7$, $\delta_{lower} \sim 0.45-0.65$, $\delta_{upper} \sim 0.27-0.45$, where $\kappa$, $\delta_{lower}$, $\delta_{upper}$ are elongation, lower and upper triangularity, respectively, defined in eqn.(0.43). Gaps between plasma and inner or outer wall limiter were ~1cm on midplane. Moving the X-point also caused a shift of the OSP on the divertor plate. Compared to shaping parameters, the variation in OSP location is more dramatic: it was scanned from the top of the vertical plate down to the horizontal floor of the slot, while divertor geometry was changed from vertical-plate to slot.

Accompanied with this change is a significant variation in outer divertor leg length (OLL), defined as the poloidal distance between the lower X-point and the OSP. The OLL was substantially incrementated, from ~10 cm when the OSP is near the top of the
vertical plate, to ~25 cm when it is on the floor of the slot. Locations of the X-points, inner and outer strike points just before the L-H transition for the analyzed discharges, are shown in Fig.4-6. Separatrix contours for a typical slot (red) and short-leg vertical-plate (blue) case are also shown on this figure. Standard C-Mod magnetic geometry (black) is overlaid to for comparison.

Fig.4-6: C-Mod poloidal cross-section showing locations of X-point (+), outer (X) and inner (open circles) separatrix strike point prior to L-H for discharges in run 1101221. Separatrix contours for short-leg vertical-plate (blue), slot (red), and standard C-Mod LSN (black) configurations are also shown. C-Mod can operate in three different divertor geometries: flat-plate, vertical, and slot, by placing the outer strike point on the divertor plate(s) labeled “F”, “V”, and “S”, respectively. The experiment was conducted in the vertical-plate and the slot geometries. Measurement locations for Thomson scattering (solid black circle), ECE (orange), the viewing cord for midplane $D_\alpha$ emission (dashed line), and the integration cord for an interferometer channel (solid line) are also displayed.
Most key C-Mod diagnostics described in Chapter 1.4 were used for this experiment. Thomson scattering and divertor Langmuir probes (LP) measurements of $T_e$ and $n_e$ profiles, in upstream and at the divertor target respectively, are available for most discharges. ECE polychromators provide sub-ms $T_e$ measurements on outboard midplane to resolve fast dynamics and document the values of $T_e$ at L-H transition times. As usual, plasma line averaged density was measured by a TCI chord, and $D_\alpha$ emissions were detected by filtered photodiodes with a line of sight close to midplane. Unfortunately, edge $T_i$ and plasma rotation measurements with CXRS were not available for this experiment. In addition, divertor LP measurements are not usable in the slot geometry since: (1) no probes are installed on the floor of the slot (*i.e.* the plate ‘S’ in Fig.4-6) and the probe closest to the separatrix (probe 1, Fig.1-8) is at $R-R_{sep}$~5mm when mapped to midplane, and (2) the incident SOL magnetic field lines are nearly tangential to the vertical divertor plate, which is unfavorable for reliable LP measurements.

4.2.2 Reduction of H-mode threshold power with the slot divertor

The most remarkable result from this experiment is that $P_{th}$ is strongly reduced with slot divertor operation. An example is shown in Fig.4-7. The two discharges quoted here are of similar density (in L-mode), $B_T$, and $I_p$, while the divertor geometry is different: one with the vertical-plate divertor, the other with the slot divertor. L-H transition times are determined using $D_\alpha$ signals. The time base for the two discharges has been shifted so that the origin is aligned with the L-H transition times. One may note in the slot divertor case (shown in red in Fig.4-7) a ‘dithering’ in $D_\alpha$ signals, lasting for ~10-20ms, at the H-mode transition. This feature was also observed in other slot divertor discharges in this experiment, but not seen in plasmas with the vertical-plate divertor. The appearance of $D_\alpha$ dithering makes it difficult to tell exactly when L-H transition occurs. Here, we choose to define the L-H transition as the time of the first notable $D_\alpha$ drop in the dithering phase.

Fig.4-7 indicates that the reduction in $P_{th}$ is mainly associated with a reduction in auxiliary heating power ($P_{RF}$) required for H-mode access. It is seen in Fig.4-7(e) and (g)
that $P_{OH}$ and $dW_{MHD}/dt$ at L-H transition are very close for the two cases. However, the difference in $P_{RF}$ (Fig.4-7(f)) is remarkable: with the slot divertor, H-mode could be induced with a tiny amount of $P_{RF}$ (~0.2MW), whereas $P_{RF}$ ~1.0MW was typically required for the vertical-plate cases. It is also noticed in Fig.4-7(b), as well as other cases in this experiment, that $D_{a}$ signals tend to be consistently lower with the slot divertor. The relation between $D_{a}$, neutral pressure (or neutral density), and $P_{th}$ will be discussed later, but the reason for the $D_{a}$ difference remains unclear, which needs further investigation in future experiments.

Fig.4-7: Comparison of two discharges with the vertical-plate (blue) and slot (red) divertor geometry: (a) line averaged density (b) midplane $D_{a}$ emission (c) $T_{e}$ ($\psi$=0.95) from ECE (d) radiated power from bulk plasma $P_{rad,main}$ (e) ohmic power $P_{OH}$ (f) ICRF power $P_{RF}$ (g) $dW_{MHD}/dt$ (h) $P_{loss}=P_{OH}+P_{RF}-dW_{MHD}/dt$. The time base has been shifted so that the origin is aligned with the L-H transition time.
Shown in Fig.4-8 are data of $P_{th}$ and $P_{th}/P_{\text{scaling}}$ as a function of OLL. It is clearly noticed that $P_{th}$ and $P_{th}/P_{\text{scaling}}$ in slot divertor (red) are consistently lower than in the vertical-plate cases (blue). The minimum $P_{th}$ obtained in this experiment is around 0.7MW, which is approximately 1/3 of the normal value ($P_{th}$=1.5~1.8MW in standard C-Mod magnetic geometry for these plasma conditions), and only about 40% of the multi-machine scaling law prediction. In addition, there is barely any overlap in the range of OLL between the two configurations, with OLL for slot cases seen consistently larger in Fig.4-8. The significant $P_{th}$ reduction in slot divertor is promising for H-mode access at reduced power.

Fig.4-8: (a) $P_{th}$ and (b) $P_{th}/P_{\text{scaling}}$, as a function of the OLL. $P_{th}$ is systematically lower for slot divertor (red) than in vertical-plate (blue) cases.
4.2.3 Correlation of H-mode threshold power with various parameters

To investigate the dependence of $P_{th}$ on plasma geometry, data of normalized L-H threshold powers, $P_{th}/P_{scaling}$, are displayed in Fig.4-9 against a number of potentially relevant shaping parameters other than the OLL. These parameters are covariant to various degrees, for example, $\delta_{lower}$ exhibits a generally inverse correlation with $RXL$ (the major radius of the lower X-point), and $L_{o,div}$ (the SOL connection length between the outboard midplane and the outer divertor plate) is strongly correlated with $OLL$.

As shown in Fig.4-9(i), data with different divertor geometry are separated in $L_{o,div}$, yielding an overall correlation between $P_{th}$ and $L_{o,div}$ (a similar trend is also seen in Fig.4-8 between $P_{th}$ and OLL), such that $P_{th}$ generally decreases with $L_{o,div}$. However, it seems the trend ‘jumps’ at the intersection of the two groups of data (divided in terms of divertor geometry), i.e. for $L_{o,div}\sim7$m (Fig.4-9(i)) or OLL~20cm (Fig.4-8). Moreover, the data taken in a single divertor geometry (e.g. for the slot divertor) are scattered, and do not manifest a clear scaling with $L_{o,div}$ or OLL. Interestingly, similar data separation and correlation, as in Fig.4-9(i), is not observed in Fig.4-9(h) between $P_{th}$ and $L_c$ – the SOL connection length between inner and outer separatrix strike point. The main reason is that the remaining part of $L_c$ (i.e. the SOL connection length from outboard midplane to the inner divertor plate, which is largely covariant with the inner leg length) is on average ~2m longer for the vertical-plate cases. So the differences in $L_{o,div}$ between the two divertor geometries are basically cancelled when the two components are added, giving a similar range in $L_c$ of 22-26m for both cases. Correlation of $P_{th}$ with other geometric parameters is weak.

Surprisingly, if results from the X-point height experiments on JET, DIII-D (see Fig.4-5) are also interpreted in the context of OLL or $L_{o,div}$, then $P_{th}$ is found increase with OLL in these experiments, which is opposite to the C-Mod result. This could be associated with differences in divertor configuration, wall materials, density, or $B_T$ between C-Mod and other tokamaks, but the exact reason is unclear so far.
Fig. 4-9: Data of $P_{th}/P_{\text{scaling}}$ are displayed as a function of various plasma geometric parameters: (a) inner divertor leg length, (b) major radius of X-point, (c) vertical position of X-point relative to the midplane, (d) elongation, (e) lower triangularity, (f) upper triangularity, (g) vertical position of OSP, (h) SOL connection length between inner and outer strike point, (i) SOL connection length between outboard midplane and OSP. Among these parameters, $P_{th}/P_{\text{scaling}}$ shows the clearest correlation with $L_{\alpha,\text{div}}$. 
Shown in Fig.4-10 are $P_{\text{th}}$ displayed as a function of normalized $D_{\alpha}$ (*i.e.*, $D_{\alpha}/\bar{n}_e$), neutral pressure in the divertor ($P_{0,\text{div}}$) and at the midplane ($P_{0,\text{mid}}$), and their ratio $f_{\text{div-mid}}=P_{0,\text{div}}/P_{0,\text{mid}}$. A strong positive correlation of $P_{\text{th}}$ with $D_{\alpha}/\bar{n}_e$ is identified in Fig.4-10(a). This correlation seems even clearer than with OLL or $L_{\alpha,\text{div}}$, and unlike in Fig.4-8 or Fig.4-9(h), the trend manifested in Fig.4-10(a) looks rather continuous without a jump at the place where the two groups of data overlap. This plot also shows that values of $D_{\alpha}$ with slot divertor are distinguishably lower compared to vertical-plate cases, which means $D_{\alpha}$ is sensitive to divertor geometry. This is similar to the observation in JET and DIII-D experiments that $D_{\alpha}$ varies with X-point height.

By contrast, no strong correlation of $P_{\text{th}}$ with $P_{0,\text{mid}}$, $P_{0,\text{div}}$, or $f_{\text{div-mid}}$ is clearly identified in Fig.4-10. Data from both divertor geometries are largely scattered (although the vertical-plate data in Fig.4-10(c) may show a slight positive correlation between $P_{\text{th}}$ and $P_{0,\text{mid}}$). Ranges of $P_{0,\text{div}}$, $P_{0,\text{mid}}$, and $f_{\text{div-mid}}$ covered by the vertical-plate data are generally in agreement with results in previous C-Mod studies [131]. Unlike $D_{\alpha}$, the range of $P_{0,\text{div}}$ (also $P_{0,\text{mid}}$ and $f_{\text{div-mid}}$) for both divertor geometries largely overlap and are not clearly affected by the change of divertor geometry.

The systematic differences in $D_{\alpha}$ between slot and vertical divertor geometries imply that $D_{\alpha}$ emission could be dramatically changed at some place(s) in plasma. Unfortunately, the C-Mod vessel wall is highly reflective to $D_{\alpha}$ radiation, making it difficult to trace exactly where the measured $D_{\alpha}$ emission comes from and further relate this to local quantities. If the $D_{\alpha}$ emission dominantly comes from $e^-$ impact excitation, such that $D_{\alpha}\sim n_e n_p <\sigma_n v>(T_e)$ [90], where $\sigma_n$ is the cross section for $e$-$D^0$ collision; then since $P_{0,\text{mid}}$ and $P_{0,\text{div}}$ are less sensitive to divertor geometry (Fig.4-10), and edge $n_e$ and $T_e$ are not affected by divertor geometry (at least for the outboard midplane, as will be seen in Fig.4-11), the differences in $D_{\alpha}$ can not be attributed solely to differences in the poloidal distribution of neutral density near the LFS. In this case, it is possible that the $D_{\alpha}$ emissions from the HFS near the inner wall may vary with divertor geometry, probably caused by a change in: (1) edge $n_e$, $T_e$ on the HFS, and/or (2) HFS neutral density due to the variation of main-chamber recycling and/or X-point fueling, with divertor geometry.
Experimental determination of the HFS neutral density is the subject of future work. However, this is not a complete picture for \( \mathrm{D}_\alpha \) emission, since recombination can also be significant. The physics of recombination is more complicated, and beyond the scope of this thesis.

![Graphs showing data of \( P_{th}/P_{\text{scaling}} \) against various parameters.]

Fig.4-10. Data of \( P_{th}/P_{\text{scaling}} \) are displayed against (a) \( \mathrm{D}_\alpha/\bar{n}_e \), (b) neutral pressure in the divertor \( P_{0,\text{div}} \), (c) neutral pressure at the midplane \( P_{0,\text{mid}} \), (d) \( f_{\text{div-mid}} = P_{0,\text{div}}/P_{0,\text{mid}} \). A positive correlation with \( \mathrm{D}_\alpha/\bar{n}_e \) is identified.

### 4.2.4 Local edge conditions

Shown in Fig.4-11 are radial profiles of \( T_e, n_e, a_0/L_T, a_0/L_m \), preceding and close to L-H transitions, in the edge and near scrape-off layer (SOL) region for the cases in Fig.4-8. Again, these profiles are obtained from the last two TS pulses prior to L-H transition (within 20-30ms), and analyzed according to the same scheme as stated in section 3.1.
The required radial shifts for these profiles to better align with EFIT separatrix are typically 1.5~3 mm. Upon examination, the resulting $T_{e,\text{sep}}$, after the profile shifting are consistent with expectations from the two-point SOL transport model. Cross-check against divertor LP measurements were done for the vertical-plate discharges with reliable LP data, where reasonable agreement is also seen.

Despite the dramatic change in $P_{th}$, no distinguishable difference in midplane $T_e$ or $n_e$ profiles is found between slot and vertical-plate configurations, as shown in Fig.4-11(a) and (b). Their shapes (gradient scale lengths) for the two divertor geometries are also very similar, as seen in Fig.4-11(c) and (d).

**Fig.4-11:** Midplane radial profiles of (a) $T_e$, (b) $n_e$, (c) $a_0/L_T$, (d) $a_0/L_n$ in the plasma edge before L-H transitions. These profiles are obtained by fitting the EFIT-mapped Thomson scattering measurements of $T_e$ and $n_e$ from the last two laser pulses prior to L-H transition, and shifted as described in section 3.1, to better register separatrix locations. Shaded bars represent the approximate radial locations for $\psi=0.95$ and the separatrix ($\psi=1.0$).
Fig.4-12: Data of $T_{e,95}$ at L-H threshold, measured by an ECE channel viewing the top of the pedestal region, are shown against $P_{th}$. Although $P_{th}$ is lower with slot divertor, no clear difference in the range of $T_{e,95}$ is seen between the two divertor geometries.

Data for $T_{e,95}$ at the L-H threshold measured by one ECE channel viewing the pedestal top, are plotted in Fig.4-12 versus $P_{th}$. Uncertainties in pinning down the radial locations of $\psi=0.95$, and the transient rise of edge $T_{e}$ due to the sawtooth heat pulses from the plasma core (see Fig.4-8(c)), could give rise to estimated errors in the ECE measurements of 30-50eV. Strikingly, although $P_{th}$ with slot divertor is consistently much lower, the range of $T_{e,95}$ for both divertor geometries is very similar, namely, 100-200eV for most data. This range is also consistent with the TS measurements (Fig.4-11(a)), meaning that $T_{e}$ is not abruptly changing just before the L-H transition.

Uncertainties in defining the L-H times could result in additional variations for the ‘dithering’ transition cases (which are seen only with the slot divertor). One may note from one such case, shown in Fig.4-8(c), that $T_{e,95}$ rises continuously in the dithering phase. If the L-H times are defined as the end of the dithering (e.g. at the last notable $D_\alpha$ drop in the dithering phase), then values of $T_{e,95}$ for some slot data in Fig.4-12 could be slightly higher. In no event is $T_{e,95}$ lower in the slot geometry lower than in the vertical-plate geometry.
Since $T_e$ and $n_e$ are not clearly affected by divertor geometry, then one may naturally ask: what quantities are significantly changed that lead to the dramatic variation in $P_{th}$? Potential candidates include SOL collisionality ($\nu_{SOL}=L_{ni}/\lambda_e$) and edge poloidal plasma flow ($V_p$) in relation to neutral density, $n_0$. Values of $\nu_{SOL}$ are consistently larger for the slot geometry due to the longer upstream-divertor (or LFS) connection length associated with this configuration. In the FM$^3$ model (section 2.4), $P_{th}$ and $\nu_{SOL}$ are related by $P_{th} \propto \nu_{SOL}^{-5/9}$ in the conduction-limited SOL regime (the case for this experiment); thus the model implies that $P_{th}$ should be lower with the slot divertor due to the larger $\nu_{SOL}$, consistent with the experimental results. More about the FM$^3$ model predictions for divertor geometry effects will be discussed in Chapter 5.

The reduced $D_\alpha$ in the slot geometry might imply a smaller $n_0$ in plasma, most likely on the HFS near the inner wall, as already discussed. The damping of $V_p$ by ion-neutral collisions (see e.g. eqn. (2.9)) would be weaker with lower $n_0$, possibly resulting in larger poloidal flow (and also flow shear), which in turn could facilitate H-mode access. Unfortunately, measurements of edge plasma flow are not available in this experiment; therefore, its behavior remains largely unknown, and should be explored in the future.

### 4.3 Plasma density scaling of H-mode threshold conditions in the slot divertor geometry

Plasma density scaling of H-mode threshold in the slot divertor geometry was studied in a subsequent C-Mod experiment ([132], C-Mod run #1120223). This experiment was set up with $B_T=5.4T$ and $I_p=1.0MA$. Plasma density was scanned over a broad range ($\bar{n}_e=0.7-2.2 \times 10^{20} \text{ m}^{-3}$). Equilibrium plasma geometry was nearly fixed, as shown in red in Fig.4-6, with the OSP maintained on the floor of the divertor slot. As usual, all H-mode transitions in this experiment were induced with the addition of ICRF as the sole auxiliary heating, which was set up in the fundamental hydrogen minority configuration,
with \( n_{\text{H}}/n_{\text{D}}=5-10\% \), optimum for high single-pass absorption. Thomson scattering data are analyzed in the same scheme as before, and used to determine the local L-H threshold conditions.

Fig.4-13: Data of (a) \( P_{\text{th}} \) and (b) \( T_{\text{e,95}} \) from C-Mod experiment 1120223 (\( B_T=5.4\text{T}, I_p=1.0\text{MA}, \) slot divertor) are plotted against line averaged density, together with C-Mod database points (blue) for \( 5.4\text{T}, 0.8-1.0\text{MA} \).

Shown in Fig.4-13(a) are \( P_{\text{th}} \) for data from this experiment, alongside those from the C-Mod L-H transition database with \( B_T=5.4\text{T}, I_p=0.8-1.0\text{MA} \), and the vertical-plate divertor geometry. Note that the density scaling of \( P_{\text{th}} \) remains ‘U’ shape in the slot divertor...
geometry, with $n_{\text{min}}$ similar to that for the vertical-plate divertor. Reduction of $P_{\text{th}}$ with the slot divertor operation is mainly found for moderate and high densities ($\bar{n}_e > 1.1 \times 10^{20} \text{ m}^{-3}$), consistent with the observation in section 4.2. Interestingly, the divertor geometry effect is diminished or perhaps even disappears at low density ($\bar{n}_e < 1.1 \times 10^{20} \text{ m}^{-3}$), i.e. essentially the low-density branch of $P_{\text{th}}$. Shown in Fig.4-13(b) are TS measured pedestal temperature, $T_{e,95}$, prior to the L-H transition. This figure suggests that the plasma density scaling of $T_{e,95}$ is very similar between the two divertor geometries.

For a consistency check, $P_{\text{th}}$ data from this experiment with density in the range of $\bar{n}_e = 1.3-1.6 \times 10^{20} \text{ m}^{-3}$, which is nearly the same coverage as that of the previous experiment (C-mod run 1120221), are shown in Fig.4-14. These data overlay the trend exhibited in Fig.4-8, which confirms that results from the two experiments are consistent.

![Figure 4-14](image-url)

**Fig.4-14:** Data from C-Mod experiment 1120223, with $\bar{n}_e = 1.3-1.6 \times 10^{20} \text{ m}^{-3}$, overlay the trend in Fig.4-8.
Chapter 5

Dependence of H-mode threshold conditions on ICRF resonance location and main ion species

The influence of power deposition profile on the H-mode access is an issue relevant to ITER [133]. Since ITER will employ multiple auxiliary heating schemes, with varied power deposition profiles in different plasma conditions, it is important to know how far off-axis the power can be deposited in plasma, while still achieving robust H-mode access. This topic was experimentally studied on C-Mod to specifically address the dependence of H-mode threshold conditions on the ICRF resonance location in plasma. A considerable enhancement in $P_{th}$ is observed only when the resonance layer is placed in plasma edge near the inner wall, possibly associated with a degradation of single-pass ICRF power absorption in this case. Local plasma edge conditions, however, are not sensitive to the ICRF power deposition, suggesting that they could be more relevant to the L-H transition physics.

Knowing the H-mode access conditions in plasmas of different main ion species is another important issue for ITER and future fusion reactors [58]. This was investigated on C-Mod in helium versus deuterium plasmas [134]. It was found that $P_{th}$ is substantially higher in helium plasma, associated with a strong core plasma radiation. Pedestal (and core) $T_e$ in helium plasmas are higher than that in deuterium, by ~30%, but the total pressure ($P_{e}+P_{i}$) may be similar in both cases. This evidence could imply that both ions and electrons play important roles in the L-H transition, and neither of them should be overlooked.
5.1 Dependence of H-mode threshold conditions on ICRF resonance location

A dedicated experiment was designed and executed on Alcator C-Mod to investigate the effect of ICRF resonance location on H-mode threshold conditions [133]. The experiment was carried out in a single day (C-Mod run #1110114) of C-Mod plasma operation with deuterium majority plasmas. The ICRF system was configured to operate at two different frequencies: the antennas at D- and E-port were tuned at $f=80\text{MHz}$ and $f=80.5\text{MHz}$; the ones at J-port were tuned at $f=70\text{MHz}$. The fundamental hydrogen minority heating remained the main heating scenario of ICRF waves. The hydrogen minority concentration was maintained nearly constant, at approximately 5-10\% throughout the experiment, which is optimal for efficient single-pass ICRF power absorption. The $\sim10\text{MHz}$ difference in antenna frequencies could give rise to a $\sim\pm8\text{cm}$ shift in the major radius of the ICRF resonance layer ($R_{\text{res}}$). For example, for $B_T=5.4\text{T}$, the $80\text{MHz}$ ICRF waves resonate with the hydrogen minorities at $R_{\text{res}}\sim68\text{cm}$, which allows on-axis heating, while for $70\text{MHz}$, the resonance layer is shifted towards the LFS, with $R_{\text{res}}\sim75\text{cm}$, which is about midway in minor radius.

Three different combinations of $B_T/I_p$ were scanned in this experiment, which are 5.3T/1.0MA, 4.6T/0.9MA, and 4.0T/0.8MA. The values of $q_{95}$ were kept nearly constant in these cases at $\sim4.0$. The locations of $R_{\text{res}}$ for the $80\text{MHz}$ and $70\text{MHz}$ wave at each $B_T$ are illustrated in Fig.5-1. ICRF waves with single frequency ($80$ or $70\text{MHz}$) were injected in the $I_p$ flattop phase of each plasma discharge to induce H-mode transitions. Plasma densities at L-H transitions are in the range of $\bar{n}_e=0.7-1.7\times10^{20}\text{ m}^{-3}$, basically covering the low density branch and the local minima of $P_{\text{th}}$. The equilibrium magnetic configuration was fixed in the standard C-Mod LSN divertor plasma geometry (with the outer separatrix point on the vertical divertor plate). The ion grad-B drift was pointed in the favorable direction for H-mode access.
Fig. 5-1: ICRF resonance locations at the hydrogen ion cyclotron frequency ($2\pi f_{\text{RF}} = eB(R)/m_i$) for different $B_T$ and ICRF wave frequencies.

Fig. 5-2(a)-(c) shows data of H-mode threshold power with 80MHz and 70MHz ICRF heating at different $B_T$. One may note from Fig. 5-2(a) and (b) that there is no clear difference in $P_{\text{th}}$ between the cases of on-axis heating (i.e. $R_{\text{res}}$~67cm), and those of off-axis heating with the resonance layer near the middle of the minor radius (i.e. $R_{\text{res}}$~57cm or $R_{\text{res}}$~75cm). However, Fig. 5-3(c) shows a substantial enhancement in $P_{\text{th}}$ for $R_{\text{res}}$~50cm, near the inner wall. These are the cases when the ICRF resonance layer is far off-axis, placed near the plasma edge on the HFS (see Fig. 5-1). Values of $P_{\text{th}}$ for these cases are about 2MW higher compared to the cases with $R_{\text{res}}$~58cm. However, despite the large difference in $P_{\text{th}}$, local plasma edge conditions (e.g. $T_{e,95}$ as shown in Fig. 5-2(d)-(f)) still look similar, and are not clearly affected by the variation of the resonance layer location.
Fig. 5-2 also confirms the previously identified $B_T$ dependence of $P_{\text{th}}$ (Fig.3-6), while adding some data with intermediate $B_T$, which bridges the gap between the 5.4T and 3.5T data group in the database, and helps uncover a trend of continuous reduction of $P_{\text{th}}$ with $B_T$ at low density.

Fig. 5-2: This figure shows (a)-(c) $P_{\text{th}}$, (d)-(f) $T_{e,95}$ with 80MHz and 70MHz ICRF wave heating at different $B_T$ (5.3, 4.6, 4.0T). $P_{\text{th}}$ is not affected by ICRF resonance location, unless it is placed on the HFS near plasma edge ($R_{\text{res}}=50\text{cm}$), in which case $P_{\text{th}}$ is substantially increased by $\sim 2\text{MW}$. $T_{e,95}$ is not affected by ICRF resonance location. Shown in dashed lines are approximate curve fittings to 5.4T (black) and 3.5T (green) data in the C-Mod database, taken from Fig.3-6 ($P_{\text{th}}$) and Fig.3-16 ($T_{e,95}$).
A detailed power balance analysis for two 4.0T/0.8MA cases in Fig.5-2(c), with $\bar{n}_e \sim 1.5 \times 10^{20} m^{-3}$ (referred to the values at L-H) but different $R_{\text{res}}$ are shown in Fig.5-3. Seen in Fig.5-3(e), the value of $P_{\text{rad,main}}$ in the $R_{\text{res}}=50\text{cm}$ case is about 1MW higher, but even after this quantity is subtracted, the net power loss, $P_{\text{net}}=P_{\text{th}}-P_{\text{rad,main}}$, for this case, is still $\sim$1MW higher compared to the $R_{\text{res}}=58\text{cm}$ case. Moreover, one may note that although $\sim$3MW ICRF power with $R_{\text{res}}=50\text{cm}$ was injected, the stored plasma energy is little changed and looks similar to that in the ohmic phase (perhaps being even slightly lower), which implies that ICRF was not well absorbed by the plasma when the resonance layer is placed near the plasma edge and close to the inner wall. This degradation of ICRF-plasma coupling may possibly be associated with a drop in the single pass ICRF power absorption rate ($\alpha$), as a result of the lower $T_e$ at the resonance location. Detailed RF modeling is required to demonstrate this conclusively, but some physics insight can be gained from an analytical expression of $\alpha$, which reads \[48\]

\[\alpha = 1 - e^{-2\eta} \]

\[2\eta = \frac{\pi}{2} \frac{\omega_{pM} R}{c} \frac{n_m Z_m}{n_M Z_M} \frac{(1 - \omega / \omega_{eM})^2}{1 + \sigma_1^2} \]

\[\sigma_1^2 = \frac{\pi}{2} \left( \frac{n_m Z_m^2 M}{n_M Z_M^2 m} \right)^2 \left( 1 - \frac{\omega_{eM}^2}{\omega^2} \right) \left( \frac{\omega}{k_B V_{\text{tm}}} \right)^2 \]

where subscripts $M$ and $m$ stand for the main and minority ion species, respectively. $\omega=2\pi f_{\text{ICRF}}$ and $k_B$ in (5.1) are the angular frequency and the parallel (to $B$) wave number of the launched ICRF wave; $V_{\text{tm}}$ is the thermal velocity of the minority ions. The temperature dependence of $\alpha$ comes from $\sigma_1^2$ via $V_{\text{tm}}$. According to (5.1), $\alpha$ is generally larger with higher temperature (\textit{i.e.} for smaller $\sigma_1$), which is qualitatively consistent with the above speculation.
Fig. 5-3: Time histories of some key parameters for two 4.0T/0.8MA discharges quoted in Fig. 5-2(c) with similar density but different ICRF wave frequency (also $R_{res}$). L-H transition times are marked by dashed lines. The shaded area represents the ohmic phase during the $I_p$ plat top of these plasmas.
5.2 Dependence of H-mode threshold conditions on main ion species

The dependence of H-mode threshold conditions on main ion species was studied in a dedicated Alcator C-Mod experiment ([134], C-Mod run #1090821). This experiment was conducted at $B_T/I_p=5.4T/1.0\text{MA}$. The equilibrium magnetic configuration was fixed at the standard C-Mod LSN divertor plasma geometry, with ion grad-B drift in the favorable direction for H-mode access. The first half of the experiment was carried out in helium ($\text{He}, Z=2, A=4$) majority ion plasma, and then switched to deuterium ($\text{D}, Z=1, A=2$) majority ion plasma in the second half. Line averaged plasma densities at the L-H transitions are in the range of $\bar{n}_e=1.3-2.1\times10^{20}\text{m}^{-3}$.

ICRF is the sole auxiliary heating source employed in this experiment. The L-H transitions were induced with the addition of ICRF power during the $I_p$ flattop of each plasma discharge. ICRF antennas were tuned at 78-80MHz to allow on-axis fundamental hydrogen minority heating at 5.4T. The hydrogen minority concentration was optimal for high efficiency single-pass ICRF power absorption in D plasma. According to eqn.(5.1), the single-pass absorption rate depends on the charge-to-mass ratio, $Z/A$. So theoretically, the single-pass absorption rate in He plasmas should be similar to that in D for similar hydrogen minority concentration, since $Z/A$ is identical for these two species.

Shown in Fig.5-4(a) are H-mode threshold power in He and D plasmas. It is clearly seen that $P_{\text{th}}$ in He ($\sim3.5\text{MW}$) are substantially higher than in D for similar densities. The D data are also found close to the assembled L-H transition database values at 5.4T (Fig.3.6, as shown by dashed line here), and mostly on the high density branch of the $P_{\text{th}}$ curve. On the other hand, $P_{\text{th}}$ for the He data does not vary strongly with density. The U-shaped density dependence of $P_{\text{th}}$ for D is not seen in He over the obtained density range. The net power loss $P_{\text{net}}=P_{\text{th}}-P_{\text{rad,main}}$ is shown in Fig.5-5(b). Comparing Fig.5-4(a) and (b), one may note that $P_{\text{rad,main}}$ for He is consistently larger than for D. After subtracting $P_{\text{rad,main}}$ from $P_{\text{th}}$, the two groups are brought closer, in particular for the lower density cases ($\bar{n}_e\leq1.5\times10^{20}\text{m}^{-3}$). However, $P_{\text{net}}$ for He still seems to be consistently higher compared to D, by $\sim1\text{MW}$. Enhancement of $P_{\text{th}}$ in He plasma was also seen in DIII-D [70], and AUG
[68] before 2008. Interestingly, this feature seemed to disappear on AUG after 2008, when AUG changed from a carbon (C) to a tungsten (W) wall [68]. The reason remains unclear so far, but it may suggest that wall conditions could play a role.

Fig. 5-4: Data of (a) $P_{th}$ (b) $P_{net}$=$P_{th}$-$P_{rad,main}$ in 5.4T/1.0MA D (red circles) and He (green stars) plasma are shown as a function of plasma density. The values of $P_{th}$ for the He data are significantly higher than in D. The dashed line represents the values from the C-Mod database for $B_T$=5.4T. The multi-machine scaling predictions, derived solely from D majority plasmas, are shown by the solid line.
Fig.5-5: Time histories of some key parameters for He (green) and D (red) discharge with similar densities at L-H transition (marked by dashed lines) and in the initial ohmic phase (in the shaded bar).
A detailed power balance analysis for He ($P_{th} \sim 3.8$MW) and D ($P_{th} \sim 1.8$MW) discharges with similar densities in the ohmic phase ($\bar{n}_e \approx 1.5 \times 10^{20} \text{m}^{-3}$) and at L-H transitions ($\bar{n}_e \approx 1.7 \times 10^{20} \text{m}^{-3}$) is shown in Fig.5-5. ICRF power injection in both cases started at 0.65 sec. The D$_\alpha$ signals are employed to determine the L-H transition times.

Seen in Fig.5-5(d), the ohmic power ($P_{OH}$) at L-H is approximately 0.8MW for both cases; therefore the significant difference in $P_{th}$ is mainly manifested by the large discrepancy in ICRF power, which is 1.2MW for D and 3.2MW for the He case, as shown in Fig.5-5(c). One may note in Fig.5-5(f) that the stored energy ($W_p$) in the ohmic phase (0.55-0.65 sec) is similar for He and D, both with $W_p \sim 50$KJ. As ICRF power was launched (see e.g. the first 100ms), $W_p$ increased by ~50KJ for He and ~20KJ for D. The increment in net input power ($\Delta P_n \sim \Delta (P_{id}+P_{OH})-\Delta P_{rad,main}$) is about 2.5MW for He and 1MW for D. It suggests that the ICRF heating efficiency is similar in He and D plasma and the observed difference in $P_{th}$ is not due to bad ICRF-plasma coupling.

Note in Fig.5-5(f) and (g) that the central $T_e$ in He ($T_{e,0} \sim 2.0$KeV) is about 0.5KeV or 25% higher than in D in the ohmic phase, although $W_p$ for these two cases look similar. The explanation is that $W_p$ essentially scales with plasma pressure $P=P_e+P_i$. Given that $Z$(He)=2, the quasi-neutrality condition says $n_e \sim 2n_i$, thereby $P \sim 1.5P_e$ for He, assuming $T_i \sim T_e$ (this assumption is generally valid for the density range of this experiment); while $P \sim 2P_e$ for D plasmas since $Z$(D)=1. So similar $W_p$ implies that $T_e$ in D is ~75% of that in He, consistent with what is shown in Fig.5-5(g). Note that in this analysis, we assumed that impurities do not contribute significantly to the plasma ion charge density.

Radial profiles of $T_e$ and $n_e$ in the plasma edge prior to L-H transition for the two discharges in Fig.5-5, are shown in Fig.5-6(a) and (b). It is seen in Fig.5-6(a) that the $T_e$ profile in He is more peaked inside the separatrix. Values of $T_e$ at the separatrix are close to 70eV for both cases, while $T_e$ at $\psi \sim 0.95$ in He is slightly higher compared to D, by ~50eV. Shown in Fig.5-6(b), the $n_e$ profiles for the two cases are very similar. The minor difference between the two curves is mainly due to the fact that $\bar{n}_e$ for the D case is ~10% higher. The $T_e$ and $n_e$ profiles in the ohmic phase (0.55-0.65sec) of the same discharges
are shown in Fig.5-6(c)-(d). Compared to those just prior to L-H (see Fig.5-6(a)), $T_e$ in the ohmic phase seems to be globally lower. Moreover, edge $T_e$ in He is higher by about 30\% than in D, consistent with the results for central $T_e$, as shown in Fig.5-5(g). The ohmic $n_e$ profiles near the pedestal top look similar between D and He, while near the separatrix, $n_e$ in the D case appears to be steeper than in He.

Fig.5-6: Radial TS profiles in the plasma edge of: (a) $T_e$, (b) $n_e$ from the last two laser pulses prior to L-H; (c) $T_e$ (d) $n_e$ in ohmic phase (0.55-0.65sec) for the two cases in Fig.5-5. TS measurements are shown in circles, and the profile fits to these data are shown by solid lines. The shaded bars represent the approximate locations for $\psi=0.95$ and the separatrix ($\psi=1.0$).
Fig. 5-7: Local plasma edge conditions prior to the L-H transition for He (green stars) and D (red circles) of: (a) $T_e$ at $\psi$=0.95, (b) $n_e$ at $\psi$=0.95, (c) $T_e$ at $\psi$=1.0, (d) $n_e$ at $\psi$=1.0. Shown in black triangles are data from the assembled L-H transition database with $B_T$=5.4T, $I_p$=0.8-1.0MA.

Local edge conditions at $\psi$=0.95 and $\psi$=1.0, prior to the L-H transition, are shown in Fig. 5-7 as a function of plasma density. These results are obtained from the TS radial profiles, analyzed with the same scheme as already described in Chapter 3. Data from the C-Mod L-H transition database with $B_T$=5.4T, $I_p$=0.8-1.0MA are also presented. It is seen
in Fig.5-7(a) that values of $T_{e,95}$ in He are nearly constant and close to 220eV (+/-20eV) for the covered density range, which are a little higher than in D by about 50eV. The scaling of $T_{e,\text{sep}}$ and local $n_e$ (both $n_{e,95}$ and $n_{e,\text{sep}}$) in He are found to be similar to those in D, as shown in Fig.5-7(b)-(d).

Assuming $T_i\sim T_e$ and quasi-neutrality (so that $n_e=2n_i$ for He), then one may find that the total pressure ($P_e+P_i$) could be similar (although $P_e$ or $P_i$ individually are different) for He and D plasmas. This implies that both electrons and ions may play an important role in the L-H transition, and neither of them should be overlooked.
Chapter 6

Comparison with models for L-H transition and H-mode threshold power

In this chapter, the experimental results obtained in Chapters 3-5 will be used to test various models relevant to H-mode transition as previously introduced in Chapter 2.

First, the RDZ and DAW models are compared to C-Mod experiments. The primary goal of this comparison is to examine how well the L-H threshold criteria predicted by each model agree with experimental data. Reasonable agreement between model predictions and experiments is seen, which suggests that the finite-β effect, or namely the nonlinear interaction of electron drift waves and shear Alfvén waves, may influence the L-H transition.

In the second part of this chapter, the FM³ model for H-mode threshold power predictions is tested against C-Mod experiments. The model successfully reproduced the complex $B_T$ dependence and the divertor geometry effects, as observed in experiment, which is encouraging. Another significant implication of this model is that the SOL transport should have a big impact on $P_{th}$, such that the density for minimum $P_{th}$, for a given $B_T$, occurs as the SOL transitions from the sheath-limited to the conduction-limited regimes. This also agrees with experiment.
6.1 Comparison with the RDZ model

The RDZ model (section 2.2.5) has been tested against some tokamak experiments [81-83,95,135]. Previous comparisons with C-Mod experiments [81] were conducted in intermediate and high magnetic field ($B_T/I_p=5.3T/0.8MA$, $8T/1.0-1.25MA$) with moderate densities ($n_e=1.0-2.0 \times 10^{20} m^{-3}$) in deuterium plasmas, focused on the behavior near the radial location of the $\psi=0.95$ surface. This study showed encouraging agreement between model predictions and C-Mod data. The comparisons done in this thesis are an extension of [81] to a broader parameter space of tokamak plasma operation, in particular to the low-density regime of H-mode access (i.e. for $n_e \leq 1.0 \times 10^{20} m^{-3}$) and to lower magnetic fields ($B_T=3.5, 4.6T$), which are visited for the first time. Data from helium plasmas (section 5.2) are also included to test the model predictions for a different main ion species. In addition, with improved edge profile diagnostics, the spatial region inside and near the separatrix is also explored.

The two key parameters of the RDZ model are $\alpha_d ((2.55))$ and $\alpha_{MHD} ((2.56))$

$$\alpha_d = (1/8 \pi q_e) (2 m_i / m_e)^{1/4} (\lambda_e / \sqrt{R L_n})^{1/2}$$

$$\alpha_{MHD} = q_a^2 \beta / L_{pe}$$

Here, $\alpha_{MHD}$ is essentially the plasma pressure gradient for a given $B_T/I_p$, and is closely related to $\beta$, which characterizes the ‘strength’ of the nonlinear interaction between drift-waves and shear-Alfvén waves; $\alpha_{MHD}$ essentially scales inversely with electron collision. The model proposed an ‘L’-shaped boundary on the $\alpha_{MHD}$-$\alpha_d$ diagram (see Fig.2-6) as the L-H threshold condition, approximately $\alpha_{MHD} \sim 0.3-0.4$, $\alpha_d \sim 0.6$.

To test this model, the variables in $\alpha_{MHD}$ and $\alpha_d$, which are related to $T_e$ and $n_e$, including $\beta=\beta_e+\beta_i$, $L_p$, $L_n$, $\lambda_e$, are computed using the constructed and radially shifted TS radial profiles, as described in previous chapters. Similar data analysis scheme as before is adapted here: $\alpha_{MHD}$ and $\alpha_d$ are spatially averaged in the shaded areas illustrated in Fig.3-5 (~5mm wide), and the mean values are taken as the values at $\psi \sim 0.95$ and $\psi \sim 1.0$. Measurements of $T_i$ are not available for all included cases, therefore some assumptions
need to be made. \( T_i = T_e \) is assumed at \( \psi \sim 0.95 \), which is generally valid as already seen (Fig. 3-8, 16). According to some experimental evidence (Fig. 3-12, 18), \( T_i = 2T_e \) is assumed at \( \psi \sim 1.0 \). The quasi-neutrality assumption is applied, so that \( n_e = n_i \) for deuterium, and \( n_e = 2n_i \) for helium plasmas, neglecting the contributions from impurities. Other input variables include: \( R = 0.68 \text{ m}, a = 0.22 \text{ m}, q_a = 2\pi a^2 \kappa B_T / \mu_0 R I_p \) is the cylindrical approximation of local safety factor.

Fig. 6-1: Shown in this figure are data just before L-H and in ohmic phase from C-Mod experiments, plotted on the \( \alpha_{\text{MHD}} - \alpha_d \) plane: (a) \( \psi \sim 0.95 \), prior to L-H (b) \( \psi \sim 0.95 \), ohmic (c) \( \psi \sim 1.0 \), prior to L-H (d) \( \psi \sim 0.95 \), ohmic. The dashed lines on these plots represent the L-H threshold boundary predicted by the RDZ model. These data are evaluated using two-pulse time averaged TS radial profiles and are radially averaged over \( \sim 5 \text{ mm} \) (Fig. 3-5) at the corresponding locations.
Shown in Fig.6-1(a) and (c) are $\alpha_{\text{MHD}}$-$\alpha_d$ prior to L-H at $\psi \sim 0.95$ and $\psi \sim 1.0$. First, it is noticed that data for different density, $B_T$, $I_p$, and main ion species are tightly clustered on these plots, suggesting $\alpha_{\text{MHD}}$ and $\alpha_d$ are generally a good set of control parameters for the tokamak plasma edge. Data taken at $\psi \sim 0.95$ and $\psi \sim 1.0$ are clustered near the two ‘arms’ of the L-shaped L-H boundary, as predicted by the RDZ model, showing reasonable agreement. However, there are some differences between these two plots. As seen in Fig.6-1(a), data taken at $\psi \sim 0.95$ exhibit a large span in $\alpha_d$ ($\alpha_d = 0.4 - 2.5$ for most data), while the range of $\alpha_{\text{MHD}}$ is relatively narrow, with most data at $\alpha_{\text{MHD}} \sim 0.2 - 0.4$. The broad coverage in $\alpha_d$ is largely due to the variation in density, seen later in Fig.6-3. By contrast, shown in Fig.6-1(c), data from $\psi \sim 1.0$ are dominantly vertically distributed, i.e. spread in $\alpha_{\text{MHD}}$. Data at $\psi \sim 1.0$ also generally tend to have lower $\alpha_d$ compared to data at $\psi \sim 0.95$, which is due to the higher collisionality in the near-separatrix region ($\alpha_d \sim 1/\sqrt{T_e^{1/2} - v_c^{1/2}}$).

Shown in Fig.6-1(b) and (d) are $\alpha_{\text{MHD}}$-$\alpha_d$ data for ohmic/low-power L-modes at $\psi \sim 0.95$ and $\psi \sim 1.0$. These plots provide a contrast to above ones for L-H transition. Comparing Fig.6-1(b) with Fig.6-1(a), one may note that the ohmic data of $\psi \sim 0.95$ are consistently below the model-predicted L-H boundary, and generally have lower $\alpha_{\text{MHD}}$ compared to the L-H data. This enhancement in $\alpha_{\text{MHD}}$ (i.e. $dP_e/dr$ for a given $q_a = B_T/I_p$) just before the L-H transition is primarily associated with the rise in $T_e$ at $\psi \sim 0.95$, seen in Fig.3-8(a). The range in $\alpha_d$ for the ohmic data is not notably different than that of L-H. As seen in Fig.6-1(d), the ohmic data at $\psi \sim 1.0$ are mainly found in the region below and to the left of the L-shaped L-H boundary. The values of ohmic data at $\psi \sim 1.0$ are generally lower in $\alpha_{\text{MHD}}$ compared to those prior to L-H (Fig.6-1(c)). Unlike at $\psi \sim 0.95$, the enhancement in $\alpha_{\text{MHD}}$ at $\psi \sim 1.0$ is largely caused by the steepening of near-separatrix $P_e$ profile (i.e. the increase in $1/L_p$) prior to the L-H transition.

The $\alpha_{\text{MHD}}$-$\alpha_d$ data in the L- and H-mode phases of the included discharges are shown in Fig.6-2, in order to examine the model predicted domains for the two different confinement regimes. Each data point on these plots is evaluated using $T_e$ and $n_e$ profiles constructed from the TS measurements at a single time slice, then spatially averaged near
$\psi \sim 0.95$ and $\psi \sim 1.0$, as have been described previously (section 3.1.2). Here, the L-mode data have excluded those from the last two TS pulses prior to L-H transition.

Fig. 6-2: Data of $\alpha_{\mathrm{MHD}} - \alpha_{\ell}$ from C-Mod experiments in L-mode (blue) and H-mode (red) phase at (a) $\psi \sim 0.95$ (b) $\psi \sim 1.0$. Each data point in this figure is assessed using the measured TS profiles in a single laser pulse. The RDZ model predicted boundaries for L-H threshold and density limit are shown by dashed lines on these plots. The L- and H-mode domains indicated by the model are labeled on (a). The L-H data from Fig. 6-1 are also shown (yellow).
As seen in these plots, experimental data of L- and H-mode basically appear in the corresponding domains predicted by the RDZ model. The two groups of data are separated by the L-shaped L-H boundary on the \(a_{\text{MHD}}-a_d\) diagram, mainly in \(a_{\text{MHD}}\) (i.e. H-modes tend to have much higher pressure gradient) while the range in \(a_d\) is relatively narrow. Data prior to L-H, from Fig.6-1, are also shown on these plots, and generally lie in between the L- and H-mode data clusters, with values of \(a_{\text{MHD}}\) larger than most L-modes seen at both radial locations.

As shown in Fig.6-2, \(a_d>0.3\) for most data, therefore according to [114], the resistive ballooning mode is largely stabilized by diamagnetic effect and is not very likely to be the dominant instability. It is seen in Fig.6-2(a) that \(a_d>0.6\) for almost all L-modes, and for the L-H data taken at \(\psi\sim0.95\), except for the few with very high density. This is also the case for most low and moderate density data taken at \(\psi\sim1.0\). Plasma turbulence in this range of \(a_d\) is considered to be dominated by the nonlinear electromagnetic drift-ballooning mode instability. The transport induced by this type of instability, as shown in section 2.2.5, might be reduced by finite-\(\beta\) (or electromagnetic) effect, i.e. with increasing \(a_{\text{MHD}}\). Some higher density cases for \(\psi\sim1.0\) with \(a_d=0.3\)-0.6, could exist near the transition from the resistive-ballooning to the nonlinear drift-wave dominant regime. In this case, turbulent transport may also be reduced by the diamagnetic effect, i.e. with increasing \(a_d\). Also seen in Fig.6-2, almost all data are outside the forbidden ‘density limit’ zone.

To demonstrate how plasma density affects the evolution of plasma edge conditions of a discharge in L-mode before the first L-H transition, \(a_{\text{MHD}}-a_d\) data from four deuterium plasmas with \(B\tau/I_p=5.4T/0.8\text{-}1.0\text{MA}\) and different densities (\(\bar{n}_e=0.7, 1.0, 1.5, 2.1\times10^{20}\text{ m}^{-3}\), referenced to the values at L-H, with \(\bar{n}_e\) being \(\sim10\sim15\%\) lower in ohmic phase) are shown in Fig.6-3. It is apparently seen from Fig.6-3(a) that as the discharge density increases, data from \(\psi\sim0.95\) basically moves horizontally towards lower \(a_d\). The reason is that \(a_d\sim T_e/n_e^{0.5}\) for given \(B\tau/I_p\), and since \(T_e\) at \(\psi\sim0.95\) is nearly independent of density (e.g. in ohmic phase, Fig.3-8), \(a_d\) is reduced as \(n_e\) increases. This confirms that the horizontal data span in Fig.6-1 is largely due to the variation in plasma density. For \(\psi\sim1.0\), as shown in Fig5-3(b), the data not only move to lower \(a_d\), but also to higher \(a_{\text{MHD}}\).
\((\alpha_{\text{MHD}} \sim q_{a}^{2} n_{e} T_{e} B^{2} L_{p})\) as density increases. This trend is mainly due to the stronger than linear density dependence of \(n_{\text{e,sep}}\) (i.e., \(n_{\text{e,sep}} \sim \bar{n}_{e}^{3.0}\), as seen in Fig.3-12(c)), especially at higher density. Interestingly, one may also note in Fig.6-3(b) that qualitatively, the case with data from \(\psi \sim 1.0\) being closer to the model-predicted L-H boundary also seems to correspond to lower H-mode threshold power. This might hint at a potential relation between \(P_{\text{th}}\) and the near-separatrix plasma conditions.

Effects of magnetic field and plasma current are examined in Fig.6-4, using five low density deuterium discharges \((\bar{n}_{e} \leq 1.0 \times 10^{20} \text{ m}^{-3})\) with different \(B_{T}\) or \(I_{p}\). The \(q_{a}\) values for the two low-\(B_{T}\) cases (i.e. \(B_{T}/I_{p}=3.5\text{T}/0.6\text{MA}\)) are close to those of the two 5.4T/0.9MA cases. Note that for similar \(q_{a}\) and density (take for example the two \(2.0 \times 10^{20} \text{ m}^{-3}\) cases), reducing \(B_{T}\) leads to lower \(\alpha_{d}\). This is seen clearly at \(\psi \sim 0.95\), mainly due to the lower \(T_{e}\) at reduced \(B_{T}\) (Fig.3-8,16), and becomes less distinguishable at \(\psi \sim 1.0\), since the differences in \(T_{e,\text{sep}}\) for different values of \(B_{T}\) are smaller (Fig.3-12,18). Reducing \(B_{T}\) also results in higher \(\alpha_{\text{MHD}}\) for \(\psi \sim 1.0\), as shown in Fig.6-4(b). The reason is that \(L_{p}, n_{e}, T_{e}\) taken at \(\psi \sim 1.0\) mainly depend on density, and are less sensitive to \(B_{T}\) and \(I_{p}\) (Fig.3-12,13,18), and therefore \(\alpha_{\text{MHD}} \sim B^{2}\), which is lower for higher \(B_{T}\). However, this is not clearly seen in Fig.6-4(a) for \(\psi \sim 0.95\). This is because \(L_{p} \sim q_{a5}\) for \(\psi \sim 0.95\) at low density, and thus \(\alpha_{\text{MHD}} \sim \beta \sim P_{e}/B^{2}\). Since \(P_{e}\) at \(\psi \sim 0.95\) tends to be lower at reduced \(B_{T}\), ranges of \(\alpha_{\text{MHD}}\) turn out to be similar for different \(B_{T}\). Comparing the two \(2.0 \times 10^{20} \text{ m}^{-3}/5.4\text{T}\) cases with those at different plasma current, one will see that reducing \(I_{p}\) gives rise to lower \(\alpha_{d}\) and higher \(\alpha_{\text{MHD}}\), primarily due to the enhancement in \(q_{a}\) as a result of the reduced \(I_{p}\).

Although the densities are nearly identical, \(P_{\text{th}}\) for the 3.5T/0.7\( \times 10^{20} \text{ m}^{-3}\) case is ~1MW lower than that of 5.4T/0.7\( \times 10^{20} \text{ m}^{-3}\), and close to that of 5.4T/1.0\( \times 10^{20} \text{ m}^{-3}\) \((P_{\text{th}}=1.8\text{MW})\). Interestingly, Fig.6-4(b) shows that data at \(\psi \sim 1.0\) for the lower \(P_{\text{th}}\) cases \((P_{\text{th}} \sim 1.8\text{MW})\) largely overlap, and are closer to the L-H boundary than the higher \(P_{\text{th}}\) case. It seems the relation between the ‘distance’ of near-separatrix conditions to the L-H boundary and \(P_{\text{th}}\), observed in Fig.4-3(b), still basically holds here.
Fig. 6-3. This figure shows the evolution of edge conditions in $\alpha_{\text{MHD}}-\alpha_d$ plane from the ohmic phase to the first L-H transition in four deuterium plasmas with $B_T/I_p=5.4T/0.8-1.0\text{MA}$ and $\bar{n}_e=0.7, 1.0, 1.5, 2.1 \times 10^{20} \text{ m}^{-3}$ (densities refer to the values at the L-H transition) for (a) $\psi \sim 0.95$, (b) $\psi \sim 1.0$. The RDZ model-predicted L-H threshold boundary is shown by the dashed lines.
This figure shows the evolution of plasma edge conditions in $\alpha_{\text{MHD}} - \alpha_d$ plane from the ohmic phase to the first L-H transition in five low density deuterium plasmas ($n_e \leq 1.0 \times 10^{20} \text{m}^{-3}$) with various $B_T/I_0$ for (a) $\psi \sim 0.95$, (b) $\psi \sim 1.0$. The RDZ model-predicted L-H threshold are shown as in Fig.6-3.
Uncertainties in the above comparisons mainly result from (1) the value of the safety factor $q$, and (2) the ratio $T_i/T_e$ at $\psi \sim 1.0$. The cylindrical $q$ was used in the comparison, as the model was derived for shifted-circular plasma geometry, and was not intended to handle the D-shape. If local $q$ values from EFIT (i.e. taking into account the real tokamak plasma geometry, typically $\sim$20-30% higher than the cylindrical approximations) were used, then values of $\alpha_{\text{MHD}}$ could be raised by $\sim$50% and $\alpha_d$ slightly reduced by $\sim$20% (e.g. $\alpha_{\text{MHD}}$ at $\psi \sim 0.95$ prior to L-H, as shown in Fig.6-1(a), would be $\sim$0.5). However, the relative positions of the L-, L-H, and H-mode data on the $\alpha_{\text{MHD}}$-$\alpha_d$ diagram will not be strongly affected, and the general conclusions still hold. It was assumed that $T_{i,\text{sep}}/T_{e,\text{sep}}=2$ in the comparison. If other values are used, for example, $T_{i,\text{sep}}/T_{e,\text{sep}}=3$ or $T_{i,\text{sep}}/T_{e,\text{sep}}=1$, then correspondingly, $\alpha_{\text{MHD}}$ will be raised or reduced by $\sim$30%, while the values of $\alpha_d$ will not change. Since data taken at $\psi \sim 1.0$ are mainly distributed in the vertical direction on the phase diagram (i.e. in $\alpha_{\text{MHD}}$), such differences do not significantly change the general agreement with the model predicted L-H boundary. Finally, because the RDZ model does not include the complete edge physics (e.g. shaping, X-point, SOL and divertor, etc.), it is necessary to point out that while the comparison with this model could only provide some physical insight in the L-H transition, it may not be quantitatively accurate, even though some agreement was found.

6.2 Comparison with the drift-Alfvén wave model

The drift-Alfvén wave (DAW) model was introduced in section 2.2.4. The two key dimensionless parameters of this model are $\beta_n$ ((2.41)) and $\nu_n$ ((2.42))

$$\beta_n = \sqrt{m_i / m_e} \beta_c q_a R / L_p$$

$$\nu_n = (m_i / m_e)^{1/4} (q_a R L_{pe})^{1/2} / \lambda_c$$

which are essentially pressure gradient and collisionality for given $B_T / I_p$. These variables are also basically equivalent to the $\alpha_{\text{MHD}}$ and $\alpha_d$ of the RDZ model. The correspondence between the two set of parameters is given in eqn.(2.57).
The DAW model suggests that L-H transition should occur when the condition
\[ \beta_n \geq 1 + \nu_n^{2/3} \]
is satisfied near the tokamak plasma edge, \textit{i.e.} essentially when the gradient of the electron \( \beta \) exceeds a threshold value. This predicted L-H threshold condition has been tested in some tokamaks (\emph{e.g.} [82,95,135]), and some agreement with experiments was found. The DAW model has now been tested using C-Mod experiments (the same set of cases as in section 5.1), and the results are described in this section. The two parameters, \( \beta_n \) and \( \nu_n \), are computed in the same manner as for \( \alpha_{\text{MHD}} \) and \( \alpha_d \) as described in section 5.1. Again, \( q_a \) here is the cylindrical safety factor.

Shown in Fig.6-5 on the \( \beta_n-\nu_n \) diagram are data preceding the L-H transition, and in ohmic/low-power L-mode phases, at \( \psi \approx 0.95 \) and \( \psi \approx 1.0 \). This diagram can be thought as essentially an \( \alpha_{\text{MHD}}-\alpha_d \) diagram, reflected in the horizontal direction (since \( \nu_n \propto 1/\alpha_d^2 \)).

First, one may note from these plots that data of different density, \( B_T, I_p, \) and main ion species are tightly clustered, similar to the results in Fig.6-1, suggesting that \( \beta_n-\nu_n \) is also a good parameterization of plasma edge physics. As shown in Fig.6-5(c), at \( \psi \approx 1.0 \), the data prior to L-H are found to be closely tied to the \( \beta_n = 1 + \nu_n^{2/3} \) curve (shown by the dashed line), which is approximately the marginal condition for L-H transition predicted by the DAW model. Fig.6-5(a) shows that prior to L-H, most data for \( \psi \approx 0.95 \) are generally above the \( \beta_n = 1 + \nu_n^{2/3} \) line, also basically consistent with the model predicted L-H transition criterion (\emph{i.e.} \( \beta_n \geq 1 + \nu_n^{2/3} \)). Fig.6-5(a) also shows a nearly flat trend in \( \nu_n \), similar to what is shown in Fig.6-1(a). Values of \( \nu_n \) at \( \psi \approx 0.95 \) are, on average, lower compared to those taken at \( \psi \approx 1.0 \).

The ohmic \( \beta_n-\nu_n \) data for \( \psi \approx 0.95 \) and \( \psi \approx 1.0 \) are displayed in Fig.6-5(b) and (d). It is seen in Fig.6-5(d) that the ohmic data at \( \psi \approx 1.0 \) are mostly below the DAW model-predicted marginal L-H threshold condition: \( \beta_n = 1 + \nu_n^{2/3} \). Fig.6-5(b) shows that \( \beta_n \) of the ohmic data at \( \psi \approx 0.95 \) is in the range of approximately 1.0–2.0, and varies weakly with \( \nu_n \). Compared to Fig.6-5(a) and (c), \( \beta_n \) of the ohmic data is generally lower than the L-H counterparts, while the range in \( \nu_n \) looks similar.
Fig. 6-5. Shown in this figure are $\beta_n$-$\nu_n$ data from C-Mod experiments: (a) $\psi$~0.95, prior to L-H; (b) $\psi$~0.95, ohmic; (c) $\psi$~1.0, prior to L-H; (d) $\psi$~0.95, ohmic. The dashed lines on these plots represent $\beta_n = 1 + \nu_n^{2/3}$. The DAW model predicts that L-H transition should occur when $\beta_n \geq 1 + \nu_n^{2/3}$. As in Fig.6-1, the data displayed here are also evaluated using the two-pulse time averaged TS radial profiles, with radial averaging over ~5mm (ref. Fig.3.5) at the corresponding locations.
Shown in Fig.6-6 are the $\beta_n$-$\nu_n$ data in the L- and H-mode phases for the included cases. As in Fig.6-2, each data point shown on these plots is evaluated using the $T_e$ and $n_e$ profiles constructed from measurements using single TS pulses. Again, L- and H-mode data are clearly separated in $\beta_n$, both at $\psi \sim 0.95$ and $\psi \sim 1.0$, while the ranges of $\nu_n$ between L- and H-mode data are not very different. In addition, as shown in Fig.6-6(b), most L-mode data for $\psi \sim 1.0$ are found to be below the $\beta_n = 1 + \nu_n^{2/3}$ boundary, consistent with the model predictions. Data prior to L-H, taken from Fig.6-5, are shown together on these plots. Values of $\beta_n$ for these (pre L-H) data are generally larger compared to most L-mode data, at both radial locations.

The four cases in Fig.6-3, and the 3.5T/0.7 x 10^20 m^-3 case in Fig.6-3, are also shown on a $\beta_n$-$\nu_n$ diagram, in Fig.6-6. It is seen that raising discharge density or reducing magnetic field leads to an increase in $\nu_n$ (i.e. a reduction in $\alpha_d$), consistent with the trend identified in Fig.6-3, 4. Similarly, cases with lower $P_{th}$ seem to be closer to the model-predicted L-H boundary at $\psi \sim 1.0$. Interestingly, one may note in Fig.6-6(a) that most L-mode data for $\psi \sim 0.95$, in the two low density 5.4T cases (especially the one with $\bar{n}_e = 0.7 \times 10^{20}$ m^-3), appear in the region that is bounded between $\beta_n = 1.0$ and $\beta_n = 1/\nu_n^{1/3}$ (i.e. zone III on Fig.2-6), where the DAW model predicts the transport should be strongly reduced with increasing $\beta_n$ ( $\sim 1/\beta_n^i$, eqn.(2.51)). This could be related to the appearance of a $T_e$ pedestal before L-H transition at low density, as shown in Fig.3-21.

In summary, the comparisons with the RDZ and the DAW models suggest that: (1) the dominant turbulence in the L-mode plasma edge could be nonlinear drift-ballooning/drift-Alfvén wave type of instabilities, where the key relevant parameters are essentially $\beta$ gradient ($d\beta/dr$) and plasma collisionality ($\nu$); and (2) the L-H transition could be related to the nonlinear suppression of edge turbulence associated with an increase in edge $d\beta/dr$ (or $dP/dr$ for a given $B_T/I_p$). In addition, the control parameters of these models, $\alpha_{MHD}$ and $\beta_n$, are also slight variations of another important parameter, $\hat{\beta}$ (see eqn.(2.26)), which by definition is the ratio of the electron drift frequency to the shear Alfvén frequency. So these results also imply that the enhancement in the nonlinear interaction between
electron drift waves and shear Alfvén waves may play a crucial role in triggering the L-H transition.

Fig. 6-6. $\beta_n$-$\nu_n$ data from C-Mod experiments in L-mode (blue) and H-mode (red) at: (a) $\psi \sim 0.95$ (b) $\psi \sim 1.0$. Each data point in this figure is assessed using the TS profiles measured in a single laser pulse. The dashed lines on these plots represent $\beta_n = 1 + \nu_n^{2/3}$, i.e. the marginal L-H threshold condition suggested by the DAW model. The L-H data from Fig.6-5 are also shown (in yellow).
Fig.6-7. This figure shows the evolutions of a plasma discharge on the $\beta_n$-$n_n$ diagram, from the ohmic phase to the first L-H transition, for the four cases in Fig.6-3 and the 3.5T/0.7×10^{20} m^{-3} case in Fig.6-4 at (a) $\psi$~0.95 (b) $\psi$~1.0. The dashed lines on these plots represent $\beta_n = 1 + n_n^{2/3}$, i.e. the DAW model suggested marginal L-H threshold condition.
6.3 Comparison with the Itoh-Itoh model

The Itoh-Itoh model (section 2.2.3) is based on an ion orbit loss theory and predicts a threshold condition for the L-H transition as

$$\lambda = \lambda_c \approx 1/d$$

where \( d \) and \( \lambda \) and are dimensionless variables introduced by the model, as defined in (2.15) and (2.16), respectively. Taking \( \alpha = 1, F = 1 \) (\( \alpha, F \) are numerical factors in \( \lambda \) and \( d \), of the order of unity), the threshold condition becomes

$$\xi_i \equiv v_i \rho_{pi} \approx \sqrt{\varepsilon D_e T_e} \frac{L_{pe}}{T_i}$$

where \( L_{pe} \) is the gradient scale length of the \( P_e \) profile.

The input parameters required to test this model include local \( n_e, T_e, T_i, \) and \( L_{pe} \), evaluated at the time just prior to L-H transition, as already described in Chapter 3. The CXRS measured \( T_i \) profiles were available for a limited number of discharges at \( B_T = 5.4T \), and therefore the comparison is done only for these cases. The anomalous electron diffusivity, \( D_e \), is difficult to determine. In the following comparison, \( D_e \) is taken as the transport coefficient from the DAW model (given by (2.45), section 2.2.4), which is essentially proportional to the Gyro-Bohm diffusion coefficient, i.e.

$$D_{Gh} = (T_e/eB)(\rho_i/L_{pe})$$

The comparisons are carried out at \( \psi = 0.95 \) and \( \psi = 1.0 \). Results are shown in Fig.6-8. The model predicted L-H threshold conditions for \( 0.5 < F < 1 \) are represented by the shaded area. It is seen in these plots that \( \lambda / \lambda_c \) generally increases with \( \xi_i \), inconsistent with the model predicted trend that \( \lambda / \lambda_c \sim const \). Quantitative agreement is only seen in a narrow range of \( \xi_i \): for example, \( \xi_i = 30 - 60 \) at \( \psi = 1.0 \) (Fig.6-8(b)), corresponding to the moderate density cases. The inconsistency between experimental data and model predictions could be partially due to the specific transport model chosen in the comparison. In order to reproduce the model prediction, \( D_e \) needs to scale as

$$D_e \propto v_i \rho_{pi} L_{pe} (T_i/T_e) \propto n_e L_{pe} / L_i T_e$$

which is quite different than the Gyro-Bohm scaling. If this is real, it may suggest that a
rather distinct mechanism (other than EM drift-wave turbulence) should dominate the edge transport, which according to existing evidence, seems to be unlikely.

Fig. 6-8: Comparisons between the C-Mod experiments and predictions of the Itoh-Itoh model. The shaded bars (in yellow) indicate the model predicted conditions for the L-H transition, assuming $\alpha=1$, $0.5<F<1$ ($\alpha$, $F$ are free parameters in this model, of the order of unity), and $D_e$ the diffusion coefficient from the DAW model (section 2.2.4).
6.4 Comparison with the model for H-mode threshold power

The recently developed FM$^3$ model (section 2.3) is a model for H-mode threshold power ($P_{th}$) prediction. It relates $P_{th}$ to near-separatrix upstream plasma conditions, then further with divertor target conditions using a two-point SOL transport model, and finally generates an algebraic equation for $P_{th}$ (eqn.(2.69)). Asymptotic solutions of this equation in sheath-limited (eqn.(2.70)) and conduction-limited (eqn.(2.71)) regimes are

$$P_{th}^{sh} = c_1 n_{e,sep}^{-2} B^3 (aR)^{-0.5} (a/R)^{9/2} \kappa^{-0.5} A^{-2} Z^{-9/4} q_{sep}^{-1.5} v_{SOL}^{-5}$$
$$P_{th}^{cd} = c_2 (n_{e,sep}B / \sqrt{A q_{sep}})^{7/9} a^{16/9} \kappa^{-0.5} q_{cyln}^{5/9} Z^{2/3} v_{SOL}^{-5/9} / A^{1/9}$$

Here, $c_1$, $c_2$ are undetermined numerical factors, resulting from a series of assumptions in the FM$^3$ model (see eqn.(2.65)-(2.67)), and are related to various free parameters (e.g., $\alpha_\perp$ as in the radial correlation length assumption, eqn.(2.65), which depends on the specific behavior of edge turbulence, and may vary for different devices and plasma conditions).

This thesis pioneers the comparison of this model with experiments. To reduce mathematical complication and highlight the important physics, the model-predicted $P_{th}$ is approximated by $P_{th} = P_{th}^{sh} + P_{th}^{cd}$ in the comparisons. As seen later in Fig.6-11, the $P_{th}^{sh}$ term decays to almost zero at high density, where $P_{th} \approx P_{th}^{cd}$, while at low density, $P_{th}^{sh}$ becomes significant and begins to dominate; this confirms that $P_{th}$ is reduced to the corresponding asymptotic limits for different SOL (density) regimes. In addition, differences between these approximations and the exact solutions are typically less than 15% upon examination; so quantitatively, the accuracy is also satisfactory. The fundamental input parameters include: $B=B_T$, $\kappa$ (elongation), $q_{cyln}=2\pi a^2 k B_T / \mu_0 R I_p$ (the cylindrical approximation of local safety factor), and $q_{sep}=q_{cyln}^{0.5}$ as the model suggests.

Upstream near-separatrix conditions of $T_e$ and $n_e$ prior to L-H transition obtained in C-Mod experiments (described in Chapters 3-5) are taken as input for $n_{e,sep}$, and used to evaluate $v_{SOL} = L_n / \lambda_e \sim \pi q_{cyln} R / \lambda_e$. In addition, $R=0.68 m$, $a=0.22 m$, $Z=1$ (2), $A=2$ (4) for deuterium (helium) plasmas.
Fig.6-9. Shown in this figure are (a) $P_{th}$ predicted by the FM$^3$ model, (b) $P_{th}$ measured in experiments, (c) upstream $v_{SOL}$ evaluated from near-separatrix plasma conditions prior to L-H for the discharges included in the C-Mod L-H transition database (Chapter 3). The dashed lines in (a) and (b) represent approximate curve fitting to experimental data for $B_T=5.4T$ (black) and $B_T=3.5T$ (green). The shaded bar in (c) shows $v_{SOL}=10-15$, i.e. roughly the range in $v_{SOL}$ for SOL transition from the sheath- to the conduction-limited regime.
Determining the values of \( c_1, c_2 \) from first principle theories or direct experimental measurements is very difficult and has not been successfully attempted so far. Therefore, a different strategy is applied here: the model is first ‘calibrated’ using the 5.4T data of the C-Mod L-H transition database, \( i.e. \) by adjusting the values of \( c_1, c_2 \) to best fit the model-predicted \( P_{th} \) with experimental results\(^\dagger\). The optimum combination is found as \( c_1=120, c_2=10 \). The agreement, after the implementation of these numbers, is excellent. Shown in Fig.6-9(a), the model predicted \( P_{th} \) for 5.4T cases match almost exactly with the experimental data, both in trend \( i.e. \) the U-shaped density scaling) and magnitudes. Later, \( c_1, c_2 \) will be fixed at these values in the comparison. The model predictions for the dependence of \( P_{th} \) on \( B_T, I_p, \) and divertor geometry will be tested.

Data of model predicted \( P_{th} \) in 3.5T are shown in Fig.6-9(a) together with the 5.4T data. It was shown in Chapter 3 that the \( B_T \) scaling of \( P_{th} \) is more complex than that suggested by the simple scaling law \( i.e. \) \( P_{th} \propto B_T^{0.8} \), and in reality the scaling of \( P_{th} \) generally moves towards lower density at reduced \( B_T \) (see Fig.6-9(b) or Fig.3-6). Comparing Fig.6-9(a) and (b), one may note that the model successfully reproduces the complex \( B_T \) scaling; in particular, the \( B_T \) dependence observed in experiments at low densities is also clearly captured by the model predictions, as shown in Fig.6-9(a). The FM\(^3\) model suggests that the \( I_p \) dependence of \( P_{th} \) is not strong: scaling of each component with \( q \) \( i.e. \) with \( 1/I_p \) when \( B_T \) is fixed) is \( P_{th}^{th} \propto q^{0.75} \) and \( P_{th}^{cd} \propto q^{1/6} \). It is seen in Fig.6-9(a) and (b) that neither model predictions, nor the experimental results exhibit a strong \( I_p \) dependence. Another significant implication of the FM\(^3\) model is that the minimum of \( P_{th} \) with density appears as the SOL transport transitions from the sheath-limited to the conduction-limited regime, or roughly when \( v_{SOL} =10\text{--}15 \). This prediction is also in general agreement with experimental results, as seen in Fig.6-9(c). Shown in Fig.6-10(a), the quantitative agreement between experimental data and the FM\(^3\) model predictions is excellent, better than the multi-machine scaling law (Fig.6-10(b)), especially for low and high densities, and lower magnetic field \( (B_T=3.5T)\).

\(^\dagger\dagger\) One may also calibrate the model using data of \( P_{net} \), and then compare to predictions for these data. This results in slightly different values of \( c_1, c_2 \), but the general conclusions do not changed.
Fig. 6-10: $P_{th}$ measured in experiment are plotted against the $P_{th}$ predicted by (a) the FM$^3$ mode, (b) the multi-machine scaling law, for the discharges included in the database. The agreement between experimental data and the FM$^3$ model predictions is seen to be excellent.

Fig. 6-11. Shown in this figure are each component of FM$^3$ model-predicted $P_{th}$ for the discharges included in the C-Mod L-H database: (a) $P_{th}^{ch}$, (b) $P_{th}^{cd}$. The dashed lines represent approximate curve fitting to the experimental data of $B_T=5.4T$ (black) and $B_T=3.5T$ (green), as seen in Fig. 6-9.
Shown in Fig. 6-11 are data of each component of the model-predicted $P_{th}$. This figure shows that the low-density branch of $P_{th}$, and the strong $B_T$ dependence of $P_{th}$ at low density, are caused by the $P_{th}^{sh}$ term: note in Fig. 6-11(a) that the magnitude of $P_{th}^{sh}$ drops very rapidly with density and almost approaches zero when $\bar{n}_e > 1.0 \times 10^{20} m^{-3}$, which is due to the inverse correlation of $P_{th}^{sh}$ with $n_{e,sep}$ and $\nu_{SOL}$, both of which increase with density, especially when $\bar{n}_e > 1.5 \times 10^{20} m^{-3}$; values of $P_{th}^{sh}$ at reduced $B_T$ (3.5T) are distinctly lower when compared to cases with higher $B_T$ (5.4T) and similar densities.

As seen in Fig.6-11(b), $P_{th}^{cd}$ increases substantially with density when $\bar{n}_e > 1.0 \times 10^{20} m^{-3}$, mainly because $P_{th}^{cd} \propto n_{e,sep}$, and $n_{e,sep}$ scales strongly with density, as $n_{e,sep} \sim \bar{n}_e^{1/2}$ in this range. The strong increase of $P_{th}^{cd}$ gives the high density branch of $P_{th}$. Interestingly, the magnitude of $P_{th}^{cd}$ remains appreciable ($\sim$1MW) at low density and seems weakly variant with $\bar{n}_e$ and $B_T$, which may set a baseline (or minimum) for $P_{th}$.

Clear divertor geometry effects on H-mode threshold were demonstrated in Chapter 4, with a significant reduction in $P_{th}$ seen in the C-Mod ‘slot’ divertor configuration. The model-predicted $P_{th}$ for the cases from the divertor geometry experiment presented in section 4.2 are shown in Fig.6-12. Model predictions not only reproduced the trend of $P_{th}$ with outer diverter leg length (OLL), but also seem to quantitatively agree with the experimental data. Shown in Fig.6-13(a) and (b) are the upstream near-separatrix $T_e$ and $n_e$ from this experiment, which exhibit little variation with divertor geometry. However, because the slot configuration is associated with longer OLL or LFS-SOL connection length, as shown in Fig.6-13(c), values of $\nu_{SOL}=L_\eta/\lambda_e$ for those slot cases tend to be larger compared to the vertical-plate cases, as seen in Fig.6-13(d) (here, $L_\eta$ is taken as the SOL connection length in the LFS; for vertical-plate cases, these values are found to be close to those estimated from $L_\eta \sim \pi q_{cyh} R$). Given that the leading term in the model-predicted $P_{th}$ is $P_{th}^{cd}$ for the density range of this experiment ($i.e.$, $\bar{n}_e = 1.3-1.6 \times 10^{20} m^{-3}$), and that $P_{th}^{cd} \propto \nu_{SOL}^{5/9}$, $P_{th}$ should be consistently lower in the slot geometry, according to the model. It is noteworthy that the plasma edge physics included in this model seems to be
sufficient to account for the experimentally observed divertor geometry effect, and no ‘hidden variables’, such as neutrals, need to be invoked to explain the experimental observations.

Fig. 6-12. This figure shows (a) $P_{th}$ predicted by the FM$^3$ model, (b) measured $P_{th}$ for discharges from the divertor geometry experiment (section 4.2), plotted versus outer divertor leg length. Results for the two different divertor configurations are color coded: slot in red and vertical-plate in blue.
The FM$^3$ model predictions of $P_{th}$ for the He plasmas (Chapter 5.2) are shown in Fig.6-14(a), alongside the predictions for the D cases from the same experiment. The main ion species dependence comes into $P_{th}^{sd}$ (which is the dominant term in $P_{th}$ for the density range covered by this experiment) as $P_{th}^{sd} \propto A^{-1/2} Z^{2/3} v_{SOL}^{-5/9}$, where $v_{SOL} \propto Z$, and thus $P_{th}^{sd} \propto A^{-7/18} (Z/A)^{1/9}$. Note that the model predicted $P_{th}$ for the D cases are close to the experimental values. However, the model predictions for the He cases are in the range of
Fig. 6-14: Shown in this figure are (a) $P_{th}$ predicted by the FM$^3$ model, (b) $P_{th}$ (c) $P_{net}$ measured in experiment, for the He and D plasmas in the experiment described in section 5.2. The dashed lines represent the approximate curve fitting to the 5.4T data of the assembled L-H transition database.
1~1.3MW, which are much lower than the experiment values (for both $P_{th}$ and $P_{net}$). This large discrepancy is likely due to the fact that the values of $c_1$ and $c_2$, obtained by fitting the model prediction to the 5.4T deuterium cases, may not apply to He plasmas, probably because some assumptions made by this model vary with main ion species.

Finally, some of the underlying assumptions made by the FM3 model are briefly examined. It was demonstrated in Chapter 2.3 that the core Ansatz of the model, i.e. that the L-H transition occurs when the Wagner number reaches a critical value, could be translated into a critical $\hat{\beta}$, assuming that the edge transport is dominated by the drift-wave type of plasma turbulence. This hypothesis is found to be generally consistent with what was seen in the comparison between C-Mod experiments and models for the L-H transition (e.g., the RDZ and DAW models, see section 6.1 and 6.2). Another assumption of the FM$^3$ model is that $L_p \propto q_{cyl}^{1/2}$, in the region that is inside and near the separatrix. As already seen in C-Mod experiments, $L_p \propto q_{cyl}$ at $\psi=0.95$, while no $q$ dependence of $L_p$ was clearly seen at $\psi=1.0$. Therefore, the $L_p \propto q_{cyl}^{1/2}$ dependence could make sense, if the entire boundary layer is treated as an integrated system, and $L_p$ is taken as the average over this region.
Chapter 7

Conclusions and future directions

Summary of main conclusions

This thesis presents a comprehensive study of H-mode access on the Alcator C-Mod tokamak in LSN divertor plasma with favorable ion grad-B drift. Some main conclusions from Chapters 3-6 are summarized below.

Chapter 3 consists of a survey of the scaling of global H-mode threshold power ($P_{th}$), and local plasma edge conditions for H-mode access, conducted in a large L-H transition dataset, which was assembled from dedicated C-Mod L-H threshold/H-mode pedestal experiments. The scaling of $P_{th}$ with plasma density was found to be U-shaped and basically independent of plasma current. A strong $B_T$ dependence of $P_{th}$ is mainly seen at low density. The local minimum of $P_{th}$ moves towards lower density at reduced $B_T$, confirming the speculation from a previous analysis of the multi-machine L-H database. Local plasma edge conditions just before the L-H transition were examined at $\psi=0.95$ and $\psi=1.0$. At $\psi=0.95$, the L-H threshold $T_e$ manifested a remarkable inverse correlation with $\bar{n}_e$ and a strong $B_T$ dependence at low density, while no clear $I_p$ dependence was identified; $T_e$ and $T_i$ are well coupled ($T_e \approx T_i$) over the entire density range. The gradient scale lengths, $L_T$ and $L_p$, show notable $I_p$ and $B_T$ dependences and a strong correlation with $\bar{n}_e/n_G$. At $\psi=1.0$, the L-H threshold $T_e$ is insensitive to both $B_T$ and $I_p$, and only
weakly change with density; $L_T$ and $L_p$ also exhibit a strong inverse correlation with density, whereas the dependences on $B_T$, $I_p$ were seen to be weak. In all, neither $P_{th}$, nor local edge conditions for L-H transition, can be well described by a power-law relation of the form $\sim \pi e^T B_i^\gamma I_p^\gamma$. Compared to the ohmic baselines, values of pressure gradient at L-H threshold tend to be consistently larger, mainly due to the increase in $T_e$ at $\psi=0.95$ and the steepening of $P_e$ profiles (i.e. the increase in $1/L_p$) at $\psi=1.0$. Finally, an edge $T_e$ pedestal was seen to form in low density L-mode plasma before the L-H transition.

Chapter 4 described the experimental arrangement, and covered the main results of a dedicated C-Mod experiment exploring the effects of divertor geometry on H-mode threshold. A significant reduction in $P_{th}$ was clearly identified when the outer separatrix strike point was lowered from being on the vertical-plate divertor to being in the slot divertor, accompanied by a substantial increment in outer divertor leg and the SOL connection length in the LFS. No clear correlation of the $P_{th}$ reduction with other plasma geometric parameters was identified. The divertor geometry effect was clear at moderate-high densities, and became weak, or even possibly vanished, at low density. Despite the large variation in $P_{th}$, edge $T_e$ and $n_e$ were little affected by the change of divertor geometry.

Chapter 5 showed the main results from dedicated C-Mod experiments studying the dependence of H-mode access conditions on ICRF resonance location and main ion species. It was found that $P_{th}$ is not affected by the ICRF resonance location, unless it is placed in the plasma edge near the inner wall; in this case, a considerable ($\sim 2$MW) increase in $P_{th}$ was seen, possibly due to a degradation in single-pass ICRF power absorption by the plasma under these conditions. Local plasma edge conditions at the L-H transition are not sensitive to ICRF resonance location. Compared to deuterium, $P_{th}$ in helium plasmas was seen to be significantly higher, associated with strong radiation from the bulk (core) of the helium plasma. The L-H threshold pedestal $T_e$ in helium plasmas is $\sim 30\%$ higher that in deuterium, but the total pressure ($P_e+P_i$) may be similar in both cases.
Chapter 6 showed a comparison of C-Mod experimental results with various models for the L-H transition and H-mode threshold power. Reasonable agreement with the RDZ and the DAW model was found. Experimental data at the L-H threshold were seen to be close to the L-H threshold conditions predicted by these models. The L-mode, L-H, and H-mode data were clearly separated in $d\beta/dr \sim \beta/I_p$ (for a given $B_T/I_p$), implying that finite-$\beta$ (electromagnetic) effect, or the nonlinear interaction between drift-waves and shear Alfvén-waves, might play a crucial role in the L-H triggering. The L-H threshold condition predicted by the Itoh-Itoh model, based on the ion orbit loss theory, seemed inconsistent with the experimental results. Comparison with the FM$^3$ model for H-mode threshold power yielded promising agreement. The FM$^3$ model reproduced the plasma density, $B_T$, and divertor geometry dependences of $P_{th}$. The model also indicated that the minimum of $P_{th}$ as a function of density, occurs as the SOL transport transitions from the sheath-limited to the conduction-limited regime, in general agreement with experiment.

**Future directions of H-mode threshold study**

H-mode threshold physics is still far from being completely understood. A main direction of this research is moving towards diagnosing and understanding the plasma dynamics close to (preceding and following) the L-H transition. Of particular interest is the dynamic evolution of plasma flow, which is speculated to play a key role in triggering the L-H transition. Measurements from some earlier studies (e.g. [91,100]) seemed to support this. Recently, there was more experimental evidence [105-107, 136] from multiple tokamaks indicating that zonal flows [137] or geodetic acoustic modes [138] driven by plasma turbulence could exist in plasma edge before L-H transition, in particular at low density, which were speculated to have a potential relation with the L-H triggering and the formation of edge $T_e$ pedestal in low density L-mode plasma.

Another important issue brought up recently is the relation between the local and the global picture of the L-H transition, or namely, between the models for L-H transition
associated with local plasma conditions and models for global H-mode threshold power. The FM$^3$ model made some significant progress in this respect. However, more research efforts are needed to systematically test this model on more tokamaks, including careful experimental tests of the numerous assumptions made by this model.

As a final remark, the ultimate goal of H-mode threshold research is to help optimize the H-mode access and benefit the H-mode plasma operation. Therefore, attention should always be paid to the significance of physical results in real application, e.g. how to reduce $P_{th}$, how could L-H transition affects the subsequent H-mode plasma performance, what could be the L-H threshold conditions in a future tokamak or fusion reactor, etc. As an example, some implications of the C-Mod results to the H-mode access on ITER are discussed in the following.

A big question for ITER is: will its target L-mode density (potentially at $\bar{n}_e=0.6\times10^{20} m^{-3}$) be in the low density branch of $P_{th}$? The answer, based on C-Mod experiments and the FM$^3$ model, is: not very likely. The reason is that the low density branch of $P_{th}$ may largely coincide with the sheath-limited SOL regime (i.e. roughly for $\bar{n}_e/n_G \leq 0.2$), according to the analysis in previous chapters. However, $\bar{n}_e/n_G$ at ITER L-mode target density is $\sim 0.5$ ($n_G \sim 1.2\times10^{20} m^{-3}$ for 5.3T/15MA ITER plasma operation), much higher than that for the sheath-limited regime, so it is considered that ITER will basically not operate in the sheath-limited SOL regime, thereby will not see the low density branch of $P_{th}$. At the same time, the potential ITER L-mode target density is probably not at the optimum value for minimum $P_{th}$ either (expected to be at $\bar{n}_e/n_G \sim 0.2-0.3$, i.e. where the SOL transitions from the sheath- to the conduction-limited regime); therefore, the high density branch of $P_{th}$, or even the detached divertor regime, might be a potential concern for ITER. The C-Mod divertor geometry experiment (chapter 4) suggested that plasma operation with a longer LFS SOL connection length may lead to a reduction in $P_{th}$. This observation may have a potential application on ITER, in case its power system design is marginal or insufficient for H-mode access.
Appendices

In addition to the main contents, the thesis also includes four appendices (Appendix A-D), arranged as follows:

Appendices A and B show the equations describing tokamak plasma dynamics, and linear and quasi-linear stability analysis of tokamak plasma, respectively. These two appendices mainly support the discussion of fundamental tokamak plasma physics (section 1.2), and the linear instability of tokamak edge turbulence (section 2.2.3).

Appendix C introduces a two-point model for the SOL transport. This model was used when discussing the plasma boundary of tokamak (section 1.2.4), and the model for H-mode threshold power (the FM3 model, section 2.3).

Appendix D explains the algorithm for global radial profile fitting, and the Monte Carlo profile error analysis technique. These are important to the data analysis of this thesis.
Appendix A

Equations describing tokamak plasma dynamics

The basic equation describing the behavior of high temperature plasma in a tokamak is the kinetic (Boltzmann) equation

\[ \frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + Z_s e (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_s f_s = C(f_s) \]  \hspace{1cm} (A.1)

for species \( s \). Here, \( f=f(x,v,t) \) is the velocity space distribution function. \( \mathbf{E} \) and \( \mathbf{B} \) are electric and magnetic field in space. The RHS of eqn.(A.1) is the Fokker-Planck (F-P) collision operator, developed to characterize the small-angle cumulative Coulomb collisions in plasma. The exact form of the F-P collision operator is complicated, and qualitatively scales as

\[ C(f_s) \sim v_s f_s \]  \hspace{1cm} (A.2)

where \( v_s \) is the collision frequency.

The kinetic equation, (A.1), is a seven-dimensional partial differential equation, which is extremely difficult to solve, and does not convey much information to help understand the characteristics of the system. Compared to eqn.(A.1), a set of fluid-like equations are more often used. These equations are the moment equations of eqn.(A.1). Here, the \( k \)-th moment of an equation \( \varepsilon(x,v,t) = 0 \) refers to velocity space integration.
\[
\int [\mathcal{E}(\mathbf{x}, \mathbf{v}, t) = 0] \mathbf{v} \cdot \ldots \mathbf{v} d\mathbf{v}
\]  \hspace{1cm} (A.3)

The zeroth, first, and second order moment of eqn. (A.1) give the conservation of mass (particle), momentum, and energy, given by

mass:
\[
\frac{dn_s}{dt} + n_s \nabla \cdot \mathbf{u}_s = S_{s,\text{ext}} \hspace{1cm} (A.4)
\]

mom.:
\[
m_s n_s \frac{d\mathbf{u}_s}{dt} = Z_s e n_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) - \nabla p_s - \nabla \cdot \mathbf{\Pi}_s + \mathbf{R}_s + \mathbf{R}_{s,\text{ext}} \hspace{1cm} (A.5)
\]

energy:
\[
\frac{3}{2} n_s \frac{dT_s}{dt} = -\nabla \cdot \mathbf{q}_s - p_s \nabla \cdot \mathbf{u}_s - \mathbf{\Pi}_s \cdot \nabla \mathbf{u}_s + Q_s + Q_{s,\text{ext}} \hspace{1cm} (A.6)
\]

with \( \frac{d}{dt} = \partial / \partial t + \mathbf{u}_s \cdot \nabla \) the total time derivative operator, \( n_s = \int f_s(\mathbf{x}, \mathbf{v}, t) d\mathbf{v} \) the particle density, \( \mathbf{u}_s = \int \mathbf{v} f_s(\mathbf{x}, \mathbf{v}, t) d\mathbf{v} \) the flow velocity, \( T_s = m_s \int d\mathbf{v} f_s (\mathbf{v} - \mathbf{u}_s)^2 / 3 \) the temperature, \( p_s = n_s T_s \) the pressure, \( \mathbf{q}_s = n_s m_s \int d\mathbf{v} f_s (\mathbf{v} - \mathbf{u}_s)^2 / 2 \) the heat flux, \( \mathbf{\Pi}_s = m_s n_s \int d\mathbf{v} f_s (\mathbf{v} - \mathbf{u}_s)(\mathbf{v} - \mathbf{u}_s) \cdot \mathbf{u}_s \) the viscosity tensor.

\( S_{\text{ext}}, R_{\text{ext}}, \) and \( Q_{\text{ext}} \) represent the external particle, momentum, and energy source, e.g. for a tokamak, \( S_{\text{ext}} \) may consist of external gas puff, particle pumping, or charge exchange process (mainly for ions); \( R_{\text{ext}} \) could be associated with neutral beam injection or current/flow drive by RF waves; \( Q_{\text{ext}} \) includes ohmic (for electrons) and auxiliary heating.

One may note that eqn.(A.4)-(A.6) are not closed, i.e. the lower order equation contains higher order moment of \( f_s \). For example, to solve \( n_s \) from eqn.(A.4), one needs to first know \( u_s \), which is determined by the next order moment equation, eqn.(A.5), so on and so forth. Therefore, finding a proper closure to truncate this infinite hierarchy is a key issue for the fluid description. A set of closures, valid for strong magnetic field, when the condition \( \omega_s \tau_s >> 1 \) is satisfied, was suggested by Braginskii [139], as
\[
\mathbf{R}_{ei} = -\mathbf{R}_{ne} = \nu_{ei} ne(\mathbf{u}_e - \mathbf{u}_i) - 0.71 \nabla T_e + (3n_e / 2 \omega_{ce} \tau_e) \mathbf{b} \times \nabla T_e
\]  
(A.7)

\[
q_e = 0.71 \nabla T_e u_{e,i} - (3n_e T_e / 2 \omega_{ce} \tau_e) \mathbf{b} \times \nabla u_e - \kappa_{\parallel} \nabla T_e - \kappa_{\perp} \mathbf{b} \times \nabla T_e - \kappa_{\perp} \nabla T_e
\]  
(A.8)

\[
q_i = -\kappa_{\parallel} \nabla T_e - \kappa_{\perp} \mathbf{b} \times \nabla T_e - \kappa_{\perp} \nabla T_e
\]  
(A.9)

\[
Q_i = -Q_e - \mathbf{R}_{ei} \cdot \mathbf{u}_e = 3n_e m_e / m_i \tau_e (T_e - T_i)
\]  
(A.10)

where \(\nu_{ei}, \tau_e\) are electron collision frequency and collision time, \(\kappa\)'s are collisional heat conductivity, given by

\[
\kappa_{\parallel} = 3.16 n_e T_e \tau_e / m_e \quad \kappa_{\perp} = -5n_e T_e / 2m_e \omega_{ce} \quad \kappa_{\perp} = 4.66 n_e T_e / 2m_e \omega_{ce}^2 \tau_e
\]  
(A.11)

\[
\kappa_{\parallel} = 3.9 n_i T_i \tau_i / m_i \quad \kappa_{\perp} = 5n_i T_i / 2m_i \omega_{ci} \quad \kappa_{\perp} = 2n_i T_i / m_i \omega_{ci}^2 \tau_i
\]  
(A.12)

Closure of the viscosity tensor (i.e., \(\tilde{\mathbf{\Pi}}\)) is more complicated, which can be found in [139]. All terms in the viscosity tensor are small (by a factor of \(r_L / L_{\perp}\), with \(r_L\) the Lamor radius, and \(L_{\perp}\) the perpendicular scale length) compared to other terms in these equations, so to the lowest order (especially when magnetic field is strong and \(r_L\) is small), viscosity is neglected. Eqn.(A.4)-(A.10) are called the Braginskii equations, which are fundamental to the study of tokamak plasma physics. They are coupled with the four Maxwell equations, eqn.(A.13)-(A.16), to form a self-consistent system

\[
\nabla \cdot \mathbf{E} = e(n_i - n_e) / \varepsilon_0
\]  
(A.13)

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}
\]  
(A.14)

\[
\nabla \cdot \mathbf{B} = 0
\]  
(A.15)

\[
\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 e(n_i - n_e) + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}
\]  
(A.16)
Depending on the temporal and spatial scale of interest, the Braginskii-Maxwell equations can be used to study various plasma dynamics, including the magnetohydrodynamics (MHD) equilibrium, transport, MHD- and micro-instabilities, and plasma turbulence. For example, the macroscopic equilibrium can be described by a reduced, one-fluid MHD model, as follows

\[
\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{V} = 0 \tag{A.17}
\]
\[
\rho \frac{d\mathbf{V}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla P \tag{A.18}
\]
\[
d(P / \rho^\gamma) = 0 \tag{A.19}
\]
\[
\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} \tag{A.20}
\]
\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \tag{A.21}
\]

where \( \rho = m_i n_i + m_e n_e \approx m_i n_i \), \( \mathbf{V} = (m_i n_i \mathbf{V}_i + m_e n_e \mathbf{V}_e) / \rho \approx \mathbf{V}_i + (m_i / m_e) \mathbf{V}_e \), \( P = P_e + P_i \), \( \eta \) the plasma resistivity, \( \gamma \) the adiabatic constant, which can often be taken as \( \gamma = 5/3 \).
Appendix B

Linear and quasi-linear analysis of plasma instabilities

Linear stability analysis addresses the problem of how a system responds to small-amplitude perturbations. To perform this analysis, a quantity in the Braginskii-Maxwell (B-M) equations (Appendix A), $\xi$, is first separated, according to the time scale, into a slowly varying equilibrium part, $\xi_0$, and a fast varying fluctuation part, $\tilde{\xi}$, such that $\xi = \xi_0 + \tilde{\xi}$. Assuming the fluctuating term, $\tilde{\xi}$, is a small perturbation, as

$$\tilde{\xi}(x,t) = \tilde{\xi} \exp(-i\omega t + ik \cdot x) \quad \text{(B.1)}$$

where $\omega$ in eqn.(B.1) can be a complex number, $\omega = \omega_r + i\gamma$. In this case, $\tilde{\xi}$ becomes

$$\tilde{\xi}(x,t) = \xi e^{\gamma t} e^{-i(\omega_r t - k \cdot x)} \quad \text{(B.2)}$$

If $\gamma$ is positive, then the small perturbation will grow exponentially at a linear growth rate, $\gamma$, and becomes linearly unstable. If $\gamma$ is negative, the perturbation decays exponentially in time, and is linearly stable. If $\gamma = 0$, the small perturbation is a pure oscillation, at frequency $\omega_r$. The $k$ is the wave number. Likewise, if the imaginary part of $k$ is positive, it means the perturbation is evanescent, i.e. it decays in space.
In this representation, the B-M equations can be *linearized*, by making the following substitutions

\[
\frac{\partial l}{\partial t} \to -i\omega
\]
\[
\nabla \to ik
\]

(B.3)

(B.4)

Then the partial differential equations are transformed into linear algebraic equations

\[
D(\omega, k, n, T, \nabla n, \nabla T, B, I_p, ...) = 0
\]

(B.5)

with \( \omega \) and \( k \) the unknowns to be solved. From eqn.(B.5), one can derive a set of linear dispersion relations, each corresponding to a plasma instability *mode*. Solving these dispersion equations, then \( \omega \) is expressed as a function of \( k \) and other equilibrium plasma parameters (e.g. \( n, T, \nabla n, \nabla T, B, I_p, \) etc.), as

\[
\omega_r = \omega_r(k, n, T, \nabla n, \nabla T, B, I_p, ...)
\]
\[
\gamma = \gamma(k, n, T, \nabla n, \nabla T, B, I_p, ...)
\]

(B.6)

Therefore, giving an arbitrary \( k \), the value of \( \omega \) can be uniquely determined. This is also known as the *spectral method* for stability analysis. However, because the system (e.g. a tokamak) is in fact with finite size, not any \( k \), but only those that are the eigenvalues of the equations are allowed.

The fluctuation induced cross-field (radial) convection is usually dominated by the fluctuating \( E \times B \) drift motion, \( \tilde{V}_{E_r} \). In this case, the radial particle flux is given by

\[
\Gamma = \left\langle \tilde{n} \tilde{V}_{E_r} \right\rangle + c.c.
\]

(B.7)

where \( \left\langle ... \right\rangle \) means averaging over time, c.c. for complex conjugate. Accurately knowing \( \Gamma \) requires the information of both amplitude and phase of \( \tilde{n} \) and \( \tilde{V}_{E_r} \), which is difficult to obtain, and needs large scale numerical simulations to determine. However, if one is only interested in the qualitative behavior of \( \Gamma \), then the quasi-linear method can be used. The quasi-linear method suggests using the linear response, \( \tilde{n}(\omega,k) \) and \( \tilde{V}_{E_r}(\omega,k) \), as in eqn.(B.7), to estimate \( \Gamma \). To demonstrate how it works, assuming for simplicity that the mode is electrostatic (\( A = 0, \, \tilde{E} = -\nabla \tilde{\phi} \)), then

\[
\tilde{V}_{E_r} = -ik_y \tilde{\phi}_x / B
\]

(B.8)
Further assume the mode is drift-wave type, with $\omega >> k, c$, then

$$\tilde{n}_i \sim n_0 \frac{\omega_e}{\omega} \frac{e\tilde{\phi}}{T_e}$$  \hspace{1cm} \text{(B.9)}$$

where $\omega_e = k, T/eBL_n = k, c, \rho_s / L_n$ is the electron diamagnetic frequency. Substituting eqn.(B.8) and (B.9) to eqn. (B.7) gives

$$\Gamma = n_0 \frac{T_e}{eB} \sum_k i k_y \frac{\omega_e}{\omega} \left| \frac{e\tilde{\phi}_k}{T_e} \right|^2 + \text{c.c.}$$ \hspace{1cm} \text{(B.10)}$$

with $\omega_k = \omega + i \gamma$, eqn.(B.10) can be written as

$$\Gamma = 2n_0 \frac{T_e}{eB} \sum_k \left| \frac{e\tilde{\phi}_k}{T_e} \right|^2 \frac{k_y \gamma_k}{\omega_r^2 + \gamma_k^2} \frac{\omega_e}{\omega}$$ \hspace{1cm} \text{(B.11)}$$

From eqn.(B.11), an effective diffusion coefficient is derived, as

$$D = \Gamma / (n_0 / L_n) = 2 \frac{T_e}{eB} \sum_k \left| \frac{e\tilde{\phi}_k}{T_e} \right|^2 \frac{k_y \gamma_k c_s}{\omega_r^2 + \gamma_k^2} \frac{k, \rho_s}{k, c_s}$$ \hspace{1cm} \text{(B.12)}$$

Knowledge of $\tilde{\phi}$ is needed, in order to further the analysis. For an estimation, suppose the nonlinear $E \times B$ convection, $\tilde{V}_E (\partial \tilde{n} / \partial t)$, saturates the linear growth of density perturbations, when the two competing effects are balanced, i.e. when $\gamma \sim k, \tilde{V}_E$, or

$$\frac{e\tilde{\phi}}{T_e} \sim \frac{1}{k, \rho_s} \frac{\gamma}{k, c_s}$$ \hspace{1cm} \text{(B.13)}$$

Then substituting eqn.(B.13) to eqn.(B.12) gives

$$D = 2 \frac{T_e}{eB} \sum_k \frac{\gamma_k^2}{\omega_r^2 + \gamma_k^2}$$ \hspace{1cm} \text{(B.14)}$$

In the limit that a single mode with $\gamma \sim \omega_r$ dominates, then eqn.(B.14) becomes

$$D \sim \frac{T_e}{eB}$$ \hspace{1cm} \text{(B.15)}$$

Eqn.(B.15) recovers the Bohm diffusivity.
It is noteworthy that the quasi-linear estimation given by eqn.(B.11) and eqn.(B.12) may also be applied, when the perturbation is electromagnetic only in the direction parallel to $B$ [113], i.e.

$$
\begin{align*}
\tilde{E}_\| &= -\hat{\nabla}_\| \tilde{\phi} - \frac{\partial \tilde{A}_\|}{\partial t} \\
\tilde{E}_\perp &= -\nabla_\perp \tilde{\phi} \\
\tilde{B}_\| &= 0 \\
\tilde{B}_\perp &= \nabla_\perp \times (\tilde{A}_\| b)
\end{align*}
$$

so that the magnetic field lines can be wiggled, but can not be compressed. This includes the case of the shear Alfvén-wave, but excludes the compressional Alfvén-wave.
Appendix C

A two-point model for scrape-off layer (SOL) transport

The two-point model introduced in this appendix describes the parallel transport in the SOL. The reason why this model is called ‘two-point’ is that it only involves the plasma conditions at two locations – the upstream (with subscript ‘u’), and the divertor target (with subscript ‘t’) – and relates them through mathematical equations. Therefore, the two-point model does not model the entire SOL, such as giving a distribution of $T_e$ along magnetic field lines.

The building blocks of this model are a set of parallel transport equations derived from the Braginskii fluid equations, given below by (C.1)-(C.3)

$$\partial_{\mu} (nV) = S_p \quad (C.1)$$

$$\partial_{\mu} (m_i V^2 + P) = -m_i V n n_e \langle \sigma_{in} \rangle \quad (C.2)$$

$$\partial_{\mu} \left( \frac{1}{2} m_i V^2 + 5 p_e \right) n V - \kappa_{ie} T_e^{5/2} \partial_{\mu} T_e = Q \quad (C.3)$$

where $n=n_e \approx Z n_i$, $p=p_e + p_i$, $V \approx V_i$ is parallel plasma flow, $\kappa_{ie} = (4 \pi e_0^2/m_i^{1/2} e^4 Z \ln \Lambda$ is the Spitzer-Harm conductivity. On the RHS of these equations are the source terms: $S_p$ is the
particle source, \( m_s \langle \sigma v_n \rangle \) is the momentum source due to charge-exchange with cold neutrals, and \( Q \) is the heat source in the SOL.

Consider a simple case: ignore the momentum source term, then (C.2) reads

\[
\frac{\partial}{\partial r} \left( m_s n V^2 + p \right) = 0
\]

or,

\[
m_s n V^2 + p = \text{const}
\]  

(C.4)

which suggests a conservation of total pressure along a magnetic field line. Evaluating (C.4) at upstream and divertor target gives

\[
p_u + m_s n u V^2_u = p_t + m_s n V^2_t
\]  

(C.5)

In ideal (Bohm) sheath, plasma becomes sonic with \( V_t \) reaching the ion sound speed ( \( V_t = c_s = \sqrt{k_B (T_e + T_i) / m_i} \) ), due to the negative potential that accelerates ions. \( V_u \) is considered small compared to \( V_t \), and therefore (C.5) becomes simply

\[
p_u = 2 p_t
\]  

(C.6)

If one assumes \( T_e \approx T_i \), or the ratio \( T_e / T_i \) is maintained nearly constant along \( B \), then

\[
n_u T_{eu} \approx 2 n_i T_{ei}
\]  

(C.7)

More rigorously, (C.7) should be written as

\[
n_u T_{eu} = 2 f_{mom} n_i T_{ei}
\]  

(C.8)

where \( f_{mom} \) (\( 0 < f_{mom} < 1 \)) is a correction factor for the momentum losses. However, it was found in many cases that with \( T_{ei} > 5 \text{eV} \), \( f_{mom} \) is actually very close to unity (\( 0.8 < f_{mom} < 1 \)); thus the pressure conservation is generally valid.

Ignoring the convection terms in the energy equation, (C.3), yields

\[
-k_{oe} T_e^{5/2} \frac{\partial}{\partial r} T = q_r
\]  

(C.9)

where \( q_r = \int Q(r) dr \) is the parallel heat flux. If one assumes \( q_r \) in the SOL comes entirely from upstream, due to the cross-field heat loss from the main plasma, and is removed at divertor, then integrating (C.9) from upstream to divertor gives
\[ T_{eu}^{7/2} = T_{et}^{7/2} + \frac{7q_\parallel L_y}{2\kappa_0 e} \]  
\[ (C.10) \]

where \( L_y \) is the length between upstream and divertor target in the SOL along a magnetic field line (the SOL connection length). The upstream parallel heat flux, \( q_\parallel \), is related to the power loss from the main plasma, \( P_L \), by

\[ q_\parallel = L_y P_L / S_\perp \lambda_p \]
\[ \approx q_a P_a / 4\pi a \kappa^{5/2} N \lambda_p \]  
\[ (C.11) \]

where \( S_\perp = 2\pi R^2 \kappa a \kappa^{5/2} \) is the surface area of the LCFS; \( L_y \sim \pi q_a R / N \), where \( N \) is the number of X-points (a X-point is a point with \( B_y = 0 \)); \( \lambda_p \) is the e-folding decay length for the SOL pressure profile, defined as \( \lambda_p = p_e / (dp_e / dr) \). Note in (C.10) that \( T_{eu} \) is rather insensitive to input power, magnetic geometry, machine size, and pressure profile. For most tokamaks, \( T_{eu} \) is in the range of 30-200eV.

According to the Bohm sheath criterion, the heat flux reaching divertor target, \( q_t \), is

\[ q_t = \gamma n_e k_w T \gamma c_s \]  
\[ (C.12) \]

where \( \gamma \) is the sheath heat transmission coefficient, usually taken as \( \gamma \approx 7 - 8 \). \( q_t \) is related to \( q_\parallel \) by

\[ q_t = (1 - f_m) q_\parallel \]  
\[ (C.13) \]

where \( f_m \) is introduced to account for the volumetric power losses due to radiation and charge exchange. Combining (C.12) and (C.13) gives

\[ q_\parallel = \gamma n_e k_w T \gamma c_s / (1 - f_m) \]  
\[ (C.14) \]

In summary, the key equations for the two-point model are (C.8), (C.10), (C.14), i.e.,

\[ n_a T_{eu} = f_{mon} 2n_t T_{et} \]
\[ T_{eu}^{7/2} = T_{et}^{7/2} + \frac{7q_{ij}L_{ij}}{2\kappa_{0e}} \]

\[ q_{ij} = \gamma n_i k_B T_i c_i / (1 - f_m) \]

Solving these equations gives the temperature gradient factor, \( f_T = f_{eu}/f_{et} \), as a function of upstream conditions, which reads

\[
\frac{T_{eu}}{T_{et}} = 10^8 \left( \frac{7e\gamma}{8k_{0e}(m_i / 2e)^{0.5}} \right) \left( \frac{v_{SOL}}{1 - f_m} \right)^2 \approx \frac{1}{400} \left( \frac{v_{SOL}}{1 - f_m} \right)^2
\]

(C.15)

where \( v_{SOL} \equiv L/n_\lambda \) is the upstream SOL collisionalitiy. Depending on the value of \( f_T \), the SOL transport can be divided into three different regimes, called the sheath-limited (\( f_T < 1.5 \)), the conduction-limited (\( f_T > 1.5 \) and \( T_{et} > 1.5 \)eV), and the detachment regime (\( f_T > 1.5 \) and \( T_{et} < 1.5 \)eV), respectively. For \( f_m \sim 0.5 \) and \( T_{eu} \sim 100 \)eV, the range of \( v_{SOL} \) for the sheath-conduction transition is \( v_{SOL_{sh-cd}} \approx 10-15 \), for the conduction-detachment transition is \( v_{SOL_{cd-detach}} \approx 70-85 \).
Appendix D

Global radial profile fitting and error analysis

Global radial profiles of $T_e$ and $n_e$ are crucial to tokamak transport and turbulence modeling. As part of this thesis study, an efficient and robust algorithm was developed to construct such profiles by fitting the Thomson scattering measurements. This algorithm has been numerically realized using the IDL code ‘quickfit’, which has been incorporated in the C-Mod data visualization system, and contributes to C-Mod physics research. Some key issues of this algorithm are covered in this appendix.

Since the shape of $T_e$ (also $n_e$) profile changes dramatically from the core to the edge of tokamak plasma (e.g. in H-mode there is a sharp edge pedestal), it is challenging to find a mathematical model, which can well characterize these profiles over the entire radial range. So, what this algorithm does, step by step, are: (1) divide the plasma minor radius into several regions (2) choose an appropriate mathematical model for each region (3) fit the TS data in each region using the selected model (4) connect the curve fittings of different regions using smooth functions.

These procedures are explained in more details below. First, the plasma minor radius is divided into three regions: (1) center ($0<r<1.5r_{inv}$). $r_{inv}=a_0/q_{95}$ is the sawtooth inversion radius, typically, $r_{inv} \approx 3-5cm$ for C-Mod), (2) core ($1.5r_{inv}<r<a_0-2cm$, where $a_0$ is the minor radius for the separatrix at outboard midplane), (3) edge ($a_0-2cm < r< a_0-5mm$).
TS measurements of $n_e$ and $T_e$ in the plasma center are fitted with a parabolic function, peaked at the magnetic axis, $r_{\text{mag}}$ (typically $r_{\text{mag}} \sim 0 - 1 \text{cm}$ for C-Mod)

$$y_1 = a(r - r_{\text{mag}})^2 + b$$  \hspace{1cm} (C.16)

An $n$-th order polynomial function (one can designate the degree) is applied to fit the data in the core region

$$y_2 = f_{\text{pol}}(r, n) = c_0 + c_1 r + c_2 r^2 + \cdots + c_n r^n$$  \hspace{1cm} (C.17)

In the plasma edge, the edge TS data are fitted with a modified hyperbolic tangent ($\text{mtanh}$) model, which can handle the steep edge pedestals in H-mode plasmas

$$y_3 = A \frac{(1 + \alpha z)e^z - e^{-z}}{e^z + e^{-z}} + B, \quad z = (r_0 - r)/d$$  \hspace{1cm} (C.18)

ECE data, as supplemental $T_e$ measurements, can also be included in $T_e$ fitting.

The profile fits in two adjacent regions are smoothly connected, using hyperbolic tangent functions, centered at their joint location. These function are essentially continuous weighting factors, which effectively damp the model profiles outside the region where they are relevant. The center-core is chosen to join at $r_{12} = 1.2r_{\text{inv}}$, using

$$W_{12,21} = 0.5 \left[ 1 \mp \tanh \left( \frac{r - r_{12}}{\Delta_{12}} \right) \right] \hspace{1cm} (C.19)$$

$$r_{12} = 1.2r_{\text{inv}}$$

for $y_1$ ($W_{12}$) and $y_2$ ($W_{21}$).

The core-edge joint is set at the radial location, $5\text{mm}$ inside the $\text{mtanh}$ fitting returned pedestal top location, $r_{\text{ped}}$, i.e. $r_{23} = r_{\text{ped}} - 5\text{mm} \approx r_0 - d - 5\text{mm}$, using

$$W_{23,32} = 0.5 \left[ 1 \mp \tanh \left( \frac{r - r_{23}}{\Delta_{12}} \right) \right] \hspace{1cm} (C.20)$$

$$\Delta_{23} = 5\text{mm}$$

for $y_2$ ($W_{23}$) and $y_3$ ($W_{32}$).
The global profile is then given by

\[ y = W_{12}y_1 + W_{21}W_{23}y_2 + W_{32}y_3 \]  

(C.21)

An example of the global profile fitting using this algorithm is shown in Fig.D-1, for an H-mode, with edge pedestals.

Fig.D-1: Shown in solid lines are global radial profiles of (a) \( n_e \), (b) \( T_e \), obtained by fitting the midplane mapped TS and ECE measurements, using the algorithm described in this Appendix, for an H-mode. The red and black dashed lines represent the radial locations of the magnetic axis and the separatrix, respectively.
Uncertainties of the obtained global profile fitting, at a specific radial location, can be estimated using Monte Carlo (MC) error analysis technique. This technique assumes that the distribution of a quantity $\xi$ about the measured value $\bar{\xi}=\bar{X}$, is Gaussian, i.e. the possibility for $\xi=X$ is given by

$$p(\xi = X) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(X - \bar{X})^2}{2\sigma^2}\right)$$  \hspace{1cm} (C.22)

where $\sigma$ is the estimated experimental uncertainty for $\xi$. From a set of experimental measurements, $\bar{\xi}=\bar{X}$ ( $\bar{X}$ can be the TS measured $n_e$ or $T_e$ data at a certain time slice), one can randomly generate a group of hypothetical measurements, $\xi=\bar{X}_i$ ($i$ is the index of MC events), assuming Gaussian distribution (eqn.(C.22)). Then, $\xi=\bar{X}_i$ are fitted into a global radial profiles $y = \bar{y}_i(r)$, using the above described algorithm. Repeat these procedures for $N$ (typically $N>30$, to be statistically meaningful) times, then a bundle of hypothetical profiles $\bar{y} = \{\bar{y}_1, \bar{y}_2, \ldots \bar{y}_N\}$ are produced. The mean ($\mu$) and uncertainty ($\sigma$) of the profile fitting, at location $r=r_0$, for a quantity $\theta = f(\xi)$, can be estimated as

$$\mu_{MC}(\theta) = \frac{1}{N} \sum_{i=1}^{N} f(\bar{y}_i(r_0))$$  \hspace{1cm} (C.23)

$$\sigma_{MC}(\theta) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (f(\bar{y}_i(r_0)) - \mu_{MC})^2}$$  \hspace{1cm} (C.24)

Fig.D-2 shows an example of MC error analysis for the case in Fig.D-1. Note that the profile fits, using the described algorithm, are very close to those from averaging the MC profiles. Shown in Fig.D-3 are $a_0/L_n$ and $a_0/L_T$ at $r=0.215m$, calculated using the profiles generated in each MC events. The uncertainties of $a_0/L_n$ and $a_0/L_T$ are estimated using these data.
Fig.D-2: An example of randomly generated MC profiles (with $N=100$ MC events), for the case in Fig.D-1: (a) $n_c$, (b) $T_e$. Each profile shown in blue is a fitting to the randomly generated hypothetical measurements in an MC event. The red curves represent the average of these MC (blue) profiles. The black curves are global profile fittings to the experimental measurements, taken from Fig.D-1.
Fig.D-3: This figure shows (a) $a_0/L_n$, (b) $a_0/L_T$, at the radial location $r=0.215m$, versus the index of MC events. These data are evaluated using the MC (blue) profiles, as shown in Fig.D-2. On each plot, the red solid line and the shaded area represent the mean value and estimated uncertainty for the plotted data, respectively. Dashed lines in (a) and (b) are values of $a_0/L_n$ and $a_0/L_T$, evaluated using the profiles in Fig.D-1, i.e. the profiles from fitting the experimental data, using the described algorithm.


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