Adverse Selection in Competitive Search Equilibrium

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study adverse selection in environments with search frictions

competitive search: principals compete to attract agents

here: uninformed principals compete to attract heterogeneous agents

principals form rational beliefs about matching probability and *composition of agents* associated to any contract
Objective

- study adverse selection in environments with search frictions
- competitive search: principals compete to attract agents
- here: uninformed principals compete to attract heterogeneous agents
- principals form rational beliefs about matching probability and *composition of agents* associated to any contract

**ISSUE:** interaction of adverse selection and competitive search

1. adverse selection affects the search equilibrium
2. search affects the set of contracts offered in equilibrium
Results

- existence and uniqueness of equilibrium
- equilibrium may be constrained inefficient
- private information may distort terms of trade or market tightness
- three examples:
  - layoff insurance
  - asset market
  - rat race


Roadmap

- general model
  - environment
  - equilibrium definition
- example:
  - layoff insurance
- general results:
  - existence and uniqueness
- other examples:
  - asset market
  - rat race
Model
large measure of ex-ante homogeneous principals
continuum of measure 1 of heterogeneous agents
\[ \pi_i > 0 \text{ agents of type } i \in \{1, 2, \ldots, I\} \equiv I \]
agent’s type is his own private information
principals and agents have single opportunity to match
Timing

- each principal can post a contract $C$ at a cost $k > 0$
- agents observe the set of posted contracts $\bar{C}$
- agents direct search to their preferred one
- principals and agents match in pairs
- matched principals and agents implement the contract
- agents who fail to match get their outside option $= 0$
Contracts

- $y \in \mathcal{Y}$ is an action
- $\mathcal{Y}$ is a compact metric space (allows for lotteries)
- if a principal and a type-$i$ agent match and undertake $y$:
  - agent gets $u_i(y)$, continuous
  - principal gets $v_i(y)$, continuous
- WLOG, contracts are revelation mechanisms
- a contract is a vector of actions $C \equiv \{y_1, \ldots, y_I\}$, where $y_i$ is prescribed if matched agent reports type $i$
a contract $C = \{y_1, \ldots, y_I\}$ is incentive-compatible iff

$$u_i(y_i) \geq u_i(y_j) \text{ for all } i, j$$

let $\mathcal{C}$ be the set of incentive-compatible contracts ($\bar{\mathcal{C}} \subset \mathcal{C}$)
Matching

- constant returns to scale matching function
- $\Theta(C)$: principal/agent ratio associated to contract $C$
- $\gamma_i(C)$: share of agents of type $i$ seeking $C$
- $\Gamma(C) = (\gamma_1(C), \ldots, \gamma_I(C))$
- $\mu(\Theta(C))$: probability an agent seeking $C$ matches
- $\eta(\Theta(C))\gamma_i(C)$: probability a principal posting $C$ matches with a type $i$ agent
- $\mu$ and $\eta$ are continuous functions, $\mu(\theta) = \eta(\theta)\theta$
Expected Utilities

- expected utility of principal offering $C = \{y_1, \ldots, y_I\}$:

\[ \eta(\Theta(C')) \sum_{i=1}^{I} \gamma_i(C') v_i(y_i) - k \]

- expected utility of type-$i$ agent seeking $C = \{y_1, \ldots, y_I\}$:

\[ \mu(\Theta(C')) u_i(y_i) \]
A CSE is a vector $\bar{U} = \{\bar{U}_1, \ldots, \bar{U}_I\}$, a measure $\lambda$ on $\mathbb{C}$, and two functions $\Theta(C)$ and $\Gamma(C)$ on $\mathbb{C}$ s.t.

(i) **profit maximization and free entry**: $\forall C \in \mathbb{C}$,

$$\eta(\Theta(C)) \sum_{i=1}^{I} \gamma_i(C)v_i(y_i) - k \leq 0, \text{ with equality if } C \in \bar{C};$$

(ii) **optimal search**: $\forall C \in \mathbb{C}$ and $i$,

$$\mu(\Theta(C))u_i(y_i) \leq \bar{U}_i \equiv \max \left\{ 0, \max_{C' = \{y'_1, \ldots, y'_I\} \in \bar{C}} \mu(\Theta(C'))u_i(y'_i) \right\},$$

with equality if $\Theta(C) < \infty$ and $\gamma_i(C) > 0$;

(iii) **market clearing**
Layoff Insurance
(Rothschild and Stiglitz 1976)
Model

principals = homogeneous risk-neutral firms
- cost $k$ to search for a worker
- can hire at most one worker
- productive match produces 1 unit of output
- unproductive match leads to a layoff

agents = heterogeneous risk-averse workers
- $p_i = \text{probability of a productive match for type } i$
- 2 types with $k < p_1 < p_2$, share $\pi_i$
- $p_i$ is private info, firms only verify ex-post realization
- utility of workers never employed is normalized to 0
Contracts and Payoffs

- action: \( y = (c^e, c^u) \) with
  - \( c^e \) = consumption if productive
  - \( c^u \) = consumption if unproductive

- if firm and type-\( i \) worker match and undertake \((c^e, c^u)\):
  - worker gets \( u_i(c^e, c^u) = p_i U(c^e) + (1 - p_i) U(c^u) \)
  - firm gets \( v_i(c^e, c^u) = p_i (1 - c^e) - (1 - p_i) c^u \)

- contract: \( C = \{ (c^e_1, c^u_1), (c^e_2, c^u_2) \} \)

- matching function: \( \mu(\theta) = \min\{\theta, 1\} \)
there exists a unique separating equilibrium

all workers find a job with probability 1

private info distorts contracts (relative to full info)

\[ c_1^e = c_1^u = p_1 - k \rightarrow \text{full insurance} \]

\[ c_2^e > c_1^e \quad \text{and} \quad c_2^u < c_1^u \rightarrow \text{partial insurance} \]
in RS if there is an equilibrium, least cost separating

BUT if there are few bad agents, a pooling contract is profitable deviation \( \Rightarrow \text{non-existence result} \)

if offering pooling contract, optimal offer full insurance

consider a contract \( C^P = \{ (c, c), (c, c) \} \) such that

\[
U(c) \geq p_2 U(c_2^e) + (1 - p_2) U(c_2^u) > p_1 U(c_1^e) + (1 - p_1) U(c_1^u)
\]

if \( \pi_1 < 1 - c \) then the deviation is profitable given that

\[
(1 - \pi_1) - c > 0
\]
Existence

why in our model this deviation is not profitable?

consider the same pooling contract $C^P = \{(c, c), (c, c)\}$

to attract both types need

$\mu (\Theta(C^P)) U(c) \geq \bar{U}_1$

$\mu (\Theta(C^P)) U(c) \geq \bar{U}_2$

as long as one is a strict inequality, $\Theta(C^P)$ would adjust up to

$\mu (\Theta(C^P)) U(c) = \bar{U}_1$

$\mu (\Theta(C^P)) U(c) < \bar{U}_2$

BUT then bad types don’t go $\rightarrow$ not profitable
equilibrium may be constrained inefficient

few low types $\rightarrow$ cross-subsidization Pareto dominant

consider a planner that restrict firms to post pooling contracts

in this case, the associated market tightness will be equal to 1

an equilibrium is constrained inefficient when it does not exists in RS!
Characterization
define $\tilde{Y}_i \equiv \{ y \in Y \mid \bar{\eta}v_i(y) \geq k \text{ and } u_i(y) > 0 \}$

**A1. Monotonicity:** for all $y \in \bigcup_i \tilde{Y}_i$

$$v_1(y) \leq v_2(y) \leq \ldots \leq v_I(y)$$

**A2. Local non-satiation:** for all $i$, $y \in \tilde{Y}_i$, and $\varepsilon > 0$

$$\exists y' \in B_\varepsilon(y) \text{ s.t. } v_i(y') > v_i(y)$$

**A3. Sorting:** for all $i$, $y \in \tilde{Y}_i$, and $\varepsilon > 0$, $\exists y' \in B_\varepsilon(y) \text{ s.t.}$

$$u_j(y') > u_j(y) \text{ for all } j \geq i$$
$$u_j(y') < u_j(y) \text{ for all } j < i$$
Optimization Problem

☐ consider the constrained maximization problem

\[
\bar{U}_i = \max_{\theta \in [0, \infty], y \in Y} \mu(\theta) u_i(y)
\]

\[\text{s.t. } \eta(\theta) v_i(y) \geq k, \quad \mu(\theta) u_j(y) \leq \bar{U}_j \text{ for all } j < i.\]

☐ call a solution to the collection of (P-\(i\)) a solution to (P)

☐ if for some \(i\) the constraint set is empty or the problem has a negative maximum set \(\bar{U}_i = 0\)

☐ Lemma: (P) has a solution and \(\bar{U}\) is unique

⚠️ recursive structure of (P) \(\Rightarrow\) solve (P-1) first...
Proposition 1: assume A1-A3, let \( \{\bar{U}_i\}, \{\theta_i\}, \) and \( \{y_i\} \) be a solution to (P) there exists a CSE \( \{\bar{U}, \lambda, \bar{C}, \Theta, \Gamma\} \) with

1. \( \bar{U} = \{\bar{U}_i\} \)
2. \( \bar{C} = \{C_i\} \), where \( C_i = (y_i, \ldots, y_i) \)
3. \( \Theta(C_i) = \theta_i \)
4. \( \gamma_i(C_i) = 1 \)
Proposition 2: assume A1-A3, let \( \{\bar{U}, \lambda, \bar{C}, \Theta, \Gamma\} \) be a CSE

Let \( \{\bar{U}_i\} = \bar{U} \)

Take any \( \{\theta_i, y_i\} \) s.t. \( \exists C_i = \{y_1, \ldots, y_i, \ldots, y_I\} \in \bar{C} \) with
\[
\theta_i = \Theta(C_i) < \infty, \gamma_i(C_i) > 0
\]

\( \{\bar{U}_i\}, \{\theta_i\}, \) and \( \{y_i\} \) solve (P)
Proposition 3: assume A1-A3,
\[ \{ y \in \mathbb{Y} | \eta(0) v_i(y) > k \text{ and } u_i(y) > 0 \} \neq \emptyset \text{ for all } i \]

in equilibrium, \( \bar{U}_i > 0 \) for all \( i \)

NOTE: positive gains of trade for some \( i \) do not guarantee \( \bar{U}_i > 0 \) (next example)
Asset Market
(Akerlof 1970)
Model

- sellers own heterogeneous apples
- buyers value apples more than sellers
- an action is $y = (\alpha, t)$:
  - $\alpha = \text{probability seller gives up apple}$
  - $t = \text{transfer from buyer to seller}$
- if buyer and type-$i$ seller match and undertake $(\alpha, t)$:
  - seller’s payoff: $u_i(\alpha, t) = t - \alpha a_i^S$
  - buyer’s payoff: $v_i(\alpha, t) = \alpha a_i^B - t$
Assumptions

- Assume there are only two types $I = 2$
- Type 2 sellers have a better apple: $a_2^S > a_1^S \geq 0$
- Preferences of buyers and sellers aligned: $a_2^B > a_1^B \geq 0$
- For now assume gains from trade: $a_i^B > a_i^S + k$
- Matching $\mu(\theta) = \min\{\theta, 1\}$
there exists a CSE with:

1. $\alpha_i = 1$ and $t_i = a^B_i - k$ for all $i$

2. $\theta_1 = 1$ and $\theta_2 = \frac{a^B_1 - a^S_1 - k}{a^B_2 - a^S_1 - k} < 1$

NOTE: private information affects market tightness

rationing through $\alpha$ would be more costly due to $k$

Pareto improvement if $\pi_1 < \frac{a^B_2 - a^S_2 - k}{a^B_2 - a^S_1 - k}$
now suppose $a_1^B \leq a_1^S + k$

then, $\bar{U}_1 = \bar{U}_2 = 0$, no contracts are posted

bad asset shuts down the market for a good one
Rat Race
(Akerlof 1976)
workers heterogeneous in preferences and productivity

homogeneous firms need to hire a worker to produce

an action is \( y = (c, h) \):

- \( c \) = wage
- \( h \) = hours worked

if a firm and a type-\( i \) worker match and undertake \( (c, h) \)

- worker’s payoff: \( u_i(c, h) = u_i(c, h) \)
- firm’s payoff: \( v_i(c, h) = f_i(h) - c \)
**Assumptions**

- assume there are only two types \( I = 2 \)
- wlog type 2 is more productive: \( f_2(h) > f_1(h) \) for all \( h \)
- single crossing assumption:
  \[
  - \frac{\partial u_2/\partial h}{\partial u_2/\partial c} < -\frac{\partial u_1/\partial h}{\partial u_1/\partial c}
  \]
- matching \( \mu(\theta) = \min\{\theta, 1\} \)

"Adverse Selection and Search"
full info equilibrium determined by three equations:

- optimality for hours

\[-\frac{\partial u_i(c_i, h_i)/\partial h}{\partial u_i(c_i, h_i)/\partial c} = f_i'(h_i)\]

- optimality for vacancy creation:

\[\mu'(\theta_i) \left( f_i(h_i) - c_i + \frac{u_i(c_i, h_i)}{\partial u_i(c_i, h_i)/\partial c} \right) = k\]

- free-entry condition

\[\frac{\mu(\theta_i)}{\theta_i} (f_i(h_i) - c_i) = k\]
there is a unique separating equilibrium

private info distorts contracts:

- low type not distorted
- high type distorted (overemployment):

\[-\frac{\partial u_2(c_2, h_2)/\partial h}{\partial u_2(c_2, h_2)/\partial c} > f_2'(h_2)\]

market tightness may be distorted in either direction
there is a unique separating equilibrium

private info distorts contracts:

- low type not distorted
- high type distorted (overemployment):

\[ \frac{-\partial u_2(c_2, h_2)/\partial h}{\partial u_2(c_2, h_2)/\partial c} > f_2'(h_2) \]

market tightness may be distorted in either direction

**NOTE**: equilibrium may be constrained inefficient

few low types or high cost of screening, cross-subsidization may Pareto dominate
Conclusions
Conclusions

- general framework combining search frictions and adverse selection
- existence and uniqueness
- general algorithm to characterize equilibrium
- private information can affect contracts and/or matching
- equilibrium may be Pareto dominated
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Incentive Feasibility
An allocation is

- a vector $\vec{U}$ of expected utilities for the agents
- a measure $\lambda$ over $\mathcal{C}$ with support $\bar{C}$
- a function $\tilde{\Theta} : \bar{C} \mapsto [0, \infty]$
- a function $\tilde{\Gamma} : \bar{C} \mapsto \Delta^I$
Incentive-Feasibility

An allocation is incentive feasible if

1. \[
\int \left( \eta(\tilde{\Theta}(C')) \sum_{i=1}^{I} \tilde{\gamma}_i(C') v_i(y_i) - k \right) d\lambda(C') = 0;
\]

2. for any \( C \in \bar{C} \) and \( i \) s.t. \( \tilde{\gamma}_i(C) > 0 \) and \( \tilde{\Theta}(C) < \infty \),
   \[
   \mu(\tilde{\Theta}(C)) u_i(y_i) = \bar{U}_i = \max_{C' \in \bar{C}} \mu(\tilde{\Theta}(C')) u_i(y'_i)
   \]

3. \[
\int \frac{\tilde{\gamma}_i(C')}{\tilde{\Theta}(C')} d\lambda(C') \leq \pi_i, \text{ with equality if } \bar{U}_i > 0
\]
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