14.12 Game Theory
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Homework 4

Due on 11/7/2005
(You may return your homework until 11/9 in class,
but your homework will be graded before the midterm only if you submit it by 11/7.)

1. Consider the game in which the following stage game is repeated five times:

	L	R
A	2,1	0,0
B	0,0	1,2

   Assume that the payoffs are the sum of the stage payoffs. For each strategy profile in the stage game, find a pure-strategy subgame-perfect equilibrium of the repeated game in which the players play the strategy profile in the first day.

2. Goliath Software is a large software company that sells an internet browser called X. Each year a new startup comes along. The startup can either produce another browser that is comparable to X or a search engine. Seeing what the startup produced, Goliath Software either updates X or not. At the end of that year, independent of the outcome, the start up disappears, leaving its place to the next start up. The annual profits for each contingency are as in the following table, where the first entry is the profit of Goliath Software:

	Browser	Search Engine
Update	1,0	3,1
No Update	2,2	4,1

   (The above is just a table, not a game.) If Goliath Software gets annual profit of π_0 in year 0, π_1 in year 1, ..., π_t in year t, ..., then its overall payoff is

$$\pi_0 + .9\pi_1 + (.9)^2 \pi_2 + \cdots + (.9)^t \pi_t + \cdots .$$

   The payoff of each startup is its own annual profit. The entire history is observable, and the game never ends. For each strategy profile below, check whether it is a subgame-perfect equilibrium, and identify the observed outcome of the strategy profile.

   (a)
   - If the startup produces a browser and Goliath Software has updated X every time a startup produced a browser in the past,[1] then Goliath Software updates its product;
   - otherwise, it does not update.
   - A startup produces a search engine if Goliath Software has updated X every time a startup produced a browser in the past;
   - otherwise, the startup produces a browser.

[1] At $t = 0$, it is vacuously true that Goliath Software has updated X every time a startup produced a browser in the past.

(b) At each t, Goliath Software updates its product if and only if the startup produces a browser (at t). Startups always produce a search engine.

(c) The years that are multiples of 100 are *holy years*. In holy years, Goliath Software does not update its browser (independent of what startup does) and the startup produces a browser. For the years that are not holy, we have the following:

- If the startup produces a browser and Goliath Software has updated X every time a startup produced a browser in the past except for holy years, then Goliath Software updates its product;
- otherwise, it does not update.
- A startup produces a search engine if Goliath Software has updated X every time a startup produced a browser in the past except for holy years;
- otherwise, the startup produces a browser.

(d) In (c) replace the holy years with non-holy years, i.e., now years 100, 200, ... are not holy, but the remaining years are holy.

3. Consider a Firm and a Worker. The firm first decides whether to pay a wage $w > 0$ to the worker (hire him), and then the worker is to decide whether to work, which costs him $c > 0$ and produces π to the firm where $\pi > w > c$. The payoffs are as follows:

	Firm	Worker
pay, work	$\pi - w$	$w - c$
pay, shirk	$-w$	w
don't pay, work	π	$-c$
don't pay, shirk	0	0

(a) Find all Nash equilibria.

(b) Now consider the game where this stage game is repeated infinitely many times and the players have a discount factor of δ. The following are strategy profiles for this repeated game. For each of them, check if it is a subgame-perfect Nash equilibrium for large values of δ, and if so, find the lowest discount rate that makes the strategy profile a subgame-perfect equilibrium.

1. No matter what happens, the firm always pays and the worker works.
2. At any time t, the worker works if and only if he is paid at t, and the firm always pays.
3. At $t = 0$, the firm pays and the worker works no matter what. At any time $t > 0$, the firm pays if and only if the worker worked at all previous dates, and the worker works if and only if he has worked at all previous dates.
4. At $t = 0$, the firm pays and the worker works if and only if he is paid. At any time $t > 0$, the firm pays if and only if the worker worked at all previous dates at which the firm paid, and the worker works if and only if he is paid at t and he has worked at all previous dates at which he was paid.
5. There are two modes: Employment, and Unemployment. The game starts at Employment. In this mode, the firm pays, and the worker works if and only if he has been paid at this date. If the worker shirks we go to Unemployment

mode; otherwise we stay in Employment. In Unemployment the firm does not pay and the worker shirks no matter what the firm does in that period. After $T > 0$ days of Unemployment we always go back to Employment. (Your answer should cover each $T > 0$.)