14.12 Game Theory
Prof. Muhamet Yildiz
Fall 2005

Homework 5

Due on 11/23/2004 (in class)

Note: This is a very short homework. I expect that you will submit your solutions on time. But since it is holiday, if you cannot turn in on time, you can submit your solutions when you come back—without any penalty.

1. Consider a two player game with payoff matrix

	L	R
X	3,θ	0,0
Y	2,2θ	2,θ
Z	0,0	3,$-\theta$

where $\theta \in \{-1, 1\}$ is a parameter known by player 2. Player 1 believes that $\theta = -1$ with probability 1/2 and $\theta = 1$ with probability 1/2. Everything above is common knowledge.

(a) Write this game formally as a Bayesian game.

(b) Compute the Bayesian Nash equilibrium of this game.

(c) What would be the Nash equilibria in pure strategies (i) if it were common knowledge that $\theta = -1$, or (ii) if it were common knowledge that $\theta = 1$?

2. (Problem 3 of Homework 1—revisited) In a college there are n students. They are simultaneously sending data over the college's data network. Let $x_i \geq 0$ be the size data sent by student i. Each student i chooses x_i himself or herself. The speed of network is inversely proportional to the total size of the data, so that it takes $x_i \tau (x_1, \ldots, x_n)$ minutes to send the message where

$$\tau (x_1, \ldots, x_n) = x_1 + \cdots + x_n.$$

The payoff of student i is

$$\theta_i x_i - x_i \tau (x_1, \ldots, x_n),$$

where $\theta_i \in \{1, 2\}$ is a payoff parameter of player i, privately known by himself or herself. For each $j \neq i$, independent of θ_j, player j assigns probability 1/2 to $\theta_i = 1$ and probability 1/2 to $\theta_i = 2$. Everything described so far is common knowledge.

(a) Write this game formally as a Bayesian game.

(b) Compute the symmetric Bayesian Nash equilibrium of this game.
Hint: symmetric means that $x_i (\theta_i) = x_j (\theta_j)$ when $\theta_i = \theta_j$. In the symmetric equilibrium one of the types will choose zero, i.e., for some $\theta \in \{1, 2\}$, $x_i (\theta_i) = 0$ whenever $\theta_i = \theta$. The expected value $E [x_1 + \cdots + x_n]$ of $x_1 + \cdots + x_n$ is $E [x_1] + \cdots + E [x_n]$.